Diversification possibilities in the Swedish financial markets
- A correlation analysis of the returns of stock, bonds and real estate

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Acknowledgments

*To learn is to live*

*To live is to read*

*To read is to learn*

- “The authors”

We would like to provide our sincerest thanks first and foremost to the seminar opposition, whom, with brilliant and crucial insight, steered us away from rocky cliffs and into the safe haven of true scientific inquiry. Our supervisor, Anders Boman, also deserves deep gratitude for the efforts of pushing us towards an independently written thesis.

We hope with this thesis not only contribute with another pair of shoulders to stand on, but also spark further interest for practical scientific studies regarding the Swedish financial markets.

Stay diversified, especially around the Swedish holidays.

Pleasant read,

Navid Haddad & Jacob Wergeland

Gothenburg, 2017-06-06
Abstract

The purpose of this study is to analyse the risk-adjusted returns of Swedish stocks, Swedish fixed income gov.- bonds and real estate return over the last 8 years in combination with the time-varying aspects of the correlation between the asset classes. The method used is a time-series analysis of OMXS30, SWEGOVTV115, and SX8600PI. For each of the asset types, an index is calculated, as well as the risk-adjusted returns for Sharpe Ratio and Treynor Ratio. Correlation coefficients for different time periods are also computed. The results show that the correlation remained stable between the OMXS30 and SX8600PI, at approx. 0.7, while the correlation between SWEGOVTV115 and OMXS30, and SX8600PI fluctuated wildly with a yearly average of approx. -0.25 and -0.2 respectively. Increased exposure towards the bond market could have provided increased risk-adjusted returns for the stock-portfolio, and the real estate market provided the higher overall returns.
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1. Introduction
Every investor seeks to create wealth by utilizing high returns. The conundrum presented to investors lies therein how to achieve these high returns: the fundamental problem of what asset to invest in. There exists a myriad of asset types (assets with similar characteristics), but there are some main classes of assets: stocks, bonds, money market instruments, real estate, and commodities (Investopedia, 2018a). Naturally, it follows that a rational investor, in the simplest sense, chooses what asset to invest into based on the returns, while also taking risk into account (Engle, 1982). Historically, stock returns have, in the sense of strictly highest returns, performed best, bonds second, and direct real estate investments worst (Szumilo et al., 2017).

The concept of risk is of utmost importance for investors. To reduce the risk, or the perceived risk, investors turn to the concept of diversification; the classic saying of “do not put all your eggs in the same basket”1 is the embodiment of this philosophy. For a successful portfolio investor, the most crucial aspect to understand is that the success of the portfolio depends on the correlations of the returns of the portfolio assets (Case, Yang, & Yildirim, 2012). Similarly, Yang, Zhou, & Leung (2012) argues that risk management and asset allocation is significantly impacted on how these return correlations varies across time. And, of course, the asset classes have a dynamic relationship in that the correlations and linkages between depend on macroeconomic conditions which have evolved over time (Yang et al., 2012).

The financial crisis of 2008 had a profound effect for the real estate investments in Germany, the direct investments had elevated from both domestic and foreign investors (Szumilo et al., 2017). Furthermore, as Jung & Maderitch (2014) stated, the volatility in the bond market increased after the financial crisis of 2007/2008. Enhanced by the fact that obligations (as in financial contracts) between financial institutions highly links the them to each other (Zhou & Li, 2017), the increased volatility together with the indications that some structural changes of pricing have occurred in the bond market could be a signal that, temporarily or permanently, the allocation between these three asset classes has changed. Furthermore, the public belief has

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1 This note is in principle light years beyond this study's relevance and scope, but in good Christian spirit, or rather perhaps the Swedishification of this specific holiday, all economists should unite in the belief that Easter is the only acceptable time to put all your eggs in the same basket.
been that elements that affect the yield of real estate and other investment form has been the same and hence investing in real estate offered little to non-diversification benefits (Szumilo et al., 2017).

Szumilo et al. (2017) showed that in Germany, during the period from 2009 until 2015, that various factors affected the direct property investment index (MSCI index) and the stock returns; this realization follows from that the returns had a small and negative correlation. Even though this MSCI index cannot be bought, the negative correlation implies that stock investments could be diversified, on average, by investing into real estate (Szumilo et al., 2017). Of both economic and financial, and scientific interest is thus how the returns of the three main asset classes have evolved over time and how this potentially dynamic relationship between them have changed.

Some studies has been done on this subject: according to the study conducted in Germany by Szumilo et al. (2017), yield returns on real estate has always been lower than that of the stock market, and compared to bonds, real estate returns has been higher while at the same time offering the same risk-to-return ratio; Clayton and Mckinnon (2003) showed that from 1978 to 1998 the volatility of NAREIT was largely dominated by large cap stocks; Yang et al. (2012) analysed the correlations between the stocks, bonds, and securitized real estate (REIT index) in the US market from 1999 to 2008 and found an asymmetrical (conditional) correlation between the real estate and stock market.

As shortly discussed in this introductory section, we now understand that the return and the risk of different assets changes over time, but more importantly that the correlation of returns are ever-changing, especially volatile following market-wide disruptions. This leads us to an interesting question: how have the correlations between stocks, bonds, and real estate changed in the recent years following the financial crisis of 2007/2008? Szumilo et al. (2017) provided us with an answer regarding the German market, Clayton & Mckinnon (2003) and Yang et al. (2012) for the US market, but no such study has been conducted for the Swedish markets. Consequently, this study aims to fill this research gap.
1.1 Purpose of the study
The purpose of this study is to analyse the risk-adjusted returns of Swedish stocks, Swedish fixed income gov.-bonds and real estate return over the last 8 years in combination with the time-varying aspects of the correlation between the asset classes.

1.2 Limitations
With the prevalence of similar studies done on foreign markets, this study focuses on the Swedish asset markets. This limitation is done for simplicity’s sake, as a more thorough and comprehensive study would require a scope that is not for this study possible. Regarding the time-span, monthly data from 2010 to 2018 are analysed; the reason for this temporal limitation is the availability of relevant time series data as well as the limited time frame this study is conducted in. This limitation is rather unfortunate but presents an opportunity for a more thorough study to be done with more frequent data and a larger time-span for those researchers with other, more abundant resources.

1.3 Previous literature
Case et al. (2012) did a study on the American market that explored the dynamic correlation between the US stock market and the publicly traded US REITs companies (REITs are companies that own and manage real estate assets). This study used a multivariate DCC-GARCH model for estimating the correlations between the years of 1972-2008 but divided the period into two structurally different time spans: the pre-modern era of 1972 until August 1991, and the modern era of August 1991 until 2008. Case et al. (2012) found that the implied dynamic correlation between REITs and stocks fell from, on average, 0,687 during the pre-modern era down to an average of around 0,5 during the first half of the modern era, and subsequently rose again to around 0,6 September 2008. Stocks against bonds had a rockier journey with larger fluctuations and an average implied correlation of around 0,3 during the pre-modern era with a slow decrease, ending with a correlation around 0.
Contrary, REITs against bonds had a stable correlation of around 0,275 until 2001 with a following correlation of around 0,18/0,19.

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2 Due to this being a bachelor thesis, both time and large time-span data were scarce. For a masters or even higher research conditions, larger data series can be collected to strengthen the study.
On a similar note but different note, Clayton & Mackinnon’s (2003) study of how the importance of the stock and bond market in explaining the REITs return, and thus examines the degree of the stock and bond market returns on the REIT returns. They conclude that the factors that drive the large cap stocks also mainly explain the volatility in REITs up until 1990 and afterwards the real estate factors emerge as the key factor. Thus, the REIT value, from 1990 and onwards, more closely mimics the value of the underlying real estate asset.

A more recent study was conducted by Szumilo et al. (2017) examined the relationship between the German real estate returns and stock and bond returns during the period 1998-2015. Similar to Case et al. (2012), Szumilo et al. (2017) also used a multivariate DCC-GARCH model on a de-smoothed MSCI total property return index (as direct property returns), RX Real Estate Index returns (as indirect property returns), DAX (a german stock market index), and REX Gesamt Kursindex (an index containing 30 German government bonds). Interestingly, they found three distinctly different time periods, corresponding to stages of the business cycle: before the crisis of 2008, during the crisis (2008-2012), and after the crisis (2012-). Bonds and direct real estate investments were closely related and exhibited signs of the same driving factors especially during and after the crisis, with the argument that the increased volatility of the stock market drove investors to safer alternatives but were discouraged by the low interest rates of the bond market and thus invested in real estate. The underlying asset did not seem to provide shelter from a volatile stock market as the indirect real estate investments followed closely the stock market index.

Both Szumilo et al. (2017) and Case et al. (2012) discuss asset allocation with regard to correlation and driving factors of the returns of the asset classes. Case et al. (2012) concluded that from 2001-2008, as the correlation between REITs and stocks increased, optimal portfolio allocation of both REITs and stocks decreased (from 44%-40% for REITs and 39%-36% for stocks) while the importance of bonds increased (from 17%-24%). Lastly, Szumilo et al. (2017) found that real estate provided a statistically significant, although barely positive, diversification possibility for stocks; moreover, the diversification benefits were improbable.
2. Theoretical review

In this section we present the theoretical foundation required for and is of interest for this study. First, we present how an index is calculated and the theoretical price for our asset classes; we also include a brief discussion of why looking at indices is relevant and what the purpose of the index is. Second, we look at different risk measurements (standard deviation and Beta values) and conjoined with this we also discuss two types of risk-adjusted return measurements: Sharpe ratio and Treynor ratio measurement. Lastly, we present the theory of Pearson’s correlation coefficient.

2.1 Calculating return indices and modelling future returns

The most basic way of calculating returns on any investment, be it stocks, bonds, or real estate, is by dividing the price difference of two distinct separate time instants by the price of the first time instant; or put differently, the difference between the gain and the cost divided by the cost (Investopedia, 2018b). More formally, this could be shown in the following way:

\[
\text{Return on investment} = \frac{(Price_t + 1 - Price_t)}{Price_t} \]

This method calculates the growth (in percentage terms) in price of investment object in relation to its original price bought at. This is a very easy and straightforward way to construct an index. By setting \( Price_t \) at a specific start time (could be year, month, day, or any other time instant) as a base-price, and then calculating the returns by using the above model for different time instants and prices, a index will be created where all of the time periods after the start date will have investment growth number. To be noted is that it is possible to calculate returns of not realized profits; that is, if there exist a price at any point in time after the original price, a return can be calculated. Moreover, this method does not account for the time periods frame (that is, how long has passed between \( Price_t \) and \( Price_{t+1} \), and thus does not account for the time value of the investment\(^3\)) or any other macroeconomic factors such as inflation or capital mobility. Calculating an index of this sort works the same way for all three of our asset types and there is no need to discern any specific index characteristics for each respective asset type. The only

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\(^3\) What is meant here is that the formula does not weigh into account the subjective factor of time (people like to have the money today rather than later). The formula just strictly calculates the return of the investment.
thing to add in the index construction is to multiply the return on investment by a factor of 100; thus, the formula can be shown as:

\[ \text{Index value}_t = \left( \frac{\text{Price}_t - \text{Price}_{t-1}}{\text{Price}_{t-1}} \right) + 1 \times \text{Index value}_{t-1} \]

or, alternatively:

\[ \text{Index value}_{t+k} = \left( \frac{\text{Price}_{t+k} - \text{Price}_{t=0}}{\text{Price}_{t=0}} \right) + 1 \times 100 \]

This way, the first year of the index structure becomes 100 (the base year), and the following time \((t+k)\) as a percentage change represented by a deviation from 100 (that is the percentage change from the base year). As we now have seen, calculating an index type structure is less than problematic.

Should there be any difference in the returns of stocks, bonds, and real estate? In retrospect, there clearly are differences in returns and risk (SE); but how are these differences explained by theory, or, put differently, how can future returns be modelled by theory and how does these modelling techniques differ between the asset classes?

First, let’s explore the theoretical evaluations, and how they differ from each other, of our asset classes of importance (stocks, bonds, and real estate). This approach enables us to provide a foundation on which to examine the differences in returns between the asset classes.

### 2.2.1 How do we value bonds (issued by governments)?

Brealey, Myers, & Allen (2014) provides a simple model for calculating bonds:

\[ PV = \sum_{i=1}^{t} \frac{\text{payments \ [coupon+principal]}}{(1+r)^t} \]

The model discounts future cash flow, the bond coupons and the principal payment, for each year at the opportunity cost of capital. This model can also, to better show the relationship between the coupon rate and the interest rate, be written in the following way:

\[ PV = \frac{1}{r} - \frac{1}{r(1+r)^t} + \frac{N}{(1+r)^t} \]
where \( C \) is the coupon rate, \( r \) is the opportunity cost of capital, and \( N \) is the principal amount.

This opportunity cost of capital could/should be interpreted as the cheapest feasible way to finance the bond, which should correspond to the interest rate of the theoretical “risk-free” government bonds. One important aspect of this evaluation model is that the bond price (PV) has an inverse relation with interest rate since the coupon rate is constant through the bonds maturity: a rise in interest rates drives down the bond prices and a decrease in interest rates drive up the bond prices. Furthermore, in a practical sense, the interest rate, \( r \), is often called YTM (yield to maturity) instead and is often the unknown variable calculated for the market price. YTM in that sense is the payoff for the bond at the current market price.

Note that this model calculates present value of future cash flows; that is, the price of the bond at the buying of the bond \((t=0)\). To retroactively calculate the return of the investment, Brealey et al. (2014) provides us with the following simple model:

\[ \text{Rate of return} = \frac{\text{coupon income} + \text{price change of the bond}}{\text{investment price of the bond}} \]

The observant reader (also with some mathematical abilities) should notice that this is a similar formula as formula 1 described above in section 2.1, but with added coupon income.

2.2.2 How do we value stocks?

Stocks are a bit trickier to evaluate than bonds since stocks lack both the fixed time periods and a specific maturity date that bonds have, but also that stocks lack the fixed-income construction of bonds (both the coupon rate and principal amount is fixed). But likewise, bonds, the income types are fixed, which for stocks are dividends and capital gains; but as the investment horizon (or investment terminal date) is pushed forward towards infinity, the capital gains part of the price becomes negligible and we are left with the following model:

\[ P_0 = \sum_{t=1}^{\infty} \frac{DIV_t}{(1+r)^t} \]
This is a well know model most often called dividend discount model (other names are DCF [discounted cash flow] model or Gordons formula\textsuperscript{4}).

2.2.3 How do we value real estate?
The evaluation of real estate is a bit trickier than for stocks and bonds. Since real estate is often purchased with the purpose of establishing living arrangements (in contrary to that of stocks and bonds, whereas the purpose is solely of capital gains), real estate investments exhibits other intangible values than just returns. On a similar note, a distinction could be made of two types of real estate investors: the private investors who buy real estate for housing purposes, and the institutional investors who buys real estate for the purpose of capital gains. The institutional investors produce their capital gains by buying the real estate and then either invest to enhance the property to later sell it off for capital gains or invest to enhance the property to manage and administer for more continual capital gains. This highlights one of the more fundamental differences for real estate as a financial asset compared to stock and bonds: real estate depreciates over time, and thus real estate needs repairs to keep the condition of the property intact\textsuperscript{5}. This does not however mean that real estate needs to be maintained for a positive return to occur, as the real estate prices tends to increase over time.

The capitalistic rule of that the market sets the price is more applicable regarding real estate than for stocks and bonds. The difference is that real estate traditionally lacks the periodic capital gains that stocks (dividends) and bonds (coupon payment) provides. Thus, the inherent value of real estate is often appraised by a real estate broker, who often provide a theoretical market price. Obviously, this market price includes the condition of the property (such as design and quality) but also the location of the property (where many various factors are included), but in the end, the market sets the price.

\textsuperscript{4} Gordons formula, or Gordon growth model is in fact a similar model in which the future dividends are expected to grow at a constant rate of \( g \), and thus the Gordon growth model is modelled as \( P_0 = \frac{D_1}{r-g} \).

\textsuperscript{5} This is not entirely true as land property also counts as real estate. This kind of land real estate does not depreciate in the same sense as real estate that contains buildings.
The formula for calculating the return of a real estate investment is formulated in a similar fashion as for stocks and bonds, but for real estate we also add maintenance cost:

\[
Return = \frac{[Price_t - Price_{t-1}] - Maintenance costs}{Price_{t-1}}
\]

2.2.4 Why do we look at indices?
Instead of using the actual price of the asset, it is beneficial in many cases to use an index. When using an index, it is the relative change that is of interest rather than the actual index-level. For example, an index with a starting value of 1000 is now worth 3300; this means that the index value has now more than tripled. As indices are a composite of several underlying assets, often weighted, the various assets do not have a uniform price structure; in other words, different assets have different prices. Monitoring actual price change of a basket with assets with different prices does not really provide any useful information as the increase could be solely due to one of the assets. To prevent this, an index is created to monitor the relative change in the price of all the assets.

2.3 What is risk?
The most common way to calculate risk is to look the standard deviation of the past prices of that investment, often called historical volatility (Flexscore, 2018; Brooks, 2008). Brooks (2008) also describe several other type of volatility measurements, such as implied volatility models, exponentially weighted moving average models and autoregressive volatility models.
In this study two risk measurements are calculated; standard deviation and the beta value. The beta value describes the volatility of one specific investment in relation to index (Investopedia, 2018c). In this case the investments volatility is compared to the Standard and Poor’s index, S&P 500, volatility.

2.3.1 Standard deviation
In its most fundamental definition of standard deviation, often expressed as \( \sigma \) (the Greek symbol, sigma), explains how much the historical values differs or deviates from the historical mean value (Bland & Altman, 1996). A high \( \sigma \) implies that the historical values are far away from the mean and low \( \sigma \) implies that the historical values are close to the mean. For a
stochastic variable, which a market transaction could be approximated as, the standard deviation is calculated as:

\[ \sigma = \sqrt{E(X - \mu)^2} \]

where \( E(X) = \mu \) is the mean value of the price of asset X.

### 2.3.2 Beta

Another way to determine risk for a financial instrument is to calculate the beta value. The beta value measures the difference in volatility of an asset compared to a benchmark index (Investopedia, 2018c). The mathematical definition of beta of a financial instrument is:

\[ \beta = \frac{\text{cov}(r,R)}{\text{var}(R)} \]

Where \( r \) is the return of the investment and \( R \) is the return of the benchmark index.

The beta value measures the co-variation of an asset compared to the benchmark asset divided by the variance of the benchmark asset. What this effectively measures is how much of the risk the asset shares with the whole market; that is, how much of the assets risk or volatility is not diversifiable and cannot be reduced. Depending on the sign and size of \( \beta \) information is received about the risk of the instrument, see table 1.
Table 1. Interpretation of different values of beta (β).

<table>
<thead>
<tr>
<th>Value of Beta</th>
<th>Interpretation</th>
</tr>
</thead>
<tbody>
<tr>
<td>β&lt;0</td>
<td>The movement between the assets are negative</td>
</tr>
<tr>
<td>β=0</td>
<td>The assets are not cointegrated</td>
</tr>
<tr>
<td>0&lt;β&lt;1</td>
<td>The assets and index comove in a positive direction but the volatility of the asset is less than index</td>
</tr>
<tr>
<td>β=1</td>
<td>The asset and index comove in a positive direction and the volatility is the same</td>
</tr>
<tr>
<td>β&gt;1</td>
<td>The asset and index comove in a positive direction but the volatility of the asset is larger than the index</td>
</tr>
</tbody>
</table>

Note: Explanation of the different value beta. Source: Wikiinvest

2.4 Risk-adjusted return
A big part of risk, and why it is important and should be included in any financial/economic evaluation is to measure returns in comparison and relative to risk. One way to do this is to estimate risk-adjusted returns. In this part two different measures of risk-adjusted return will be presented: the Sharpe ratio and the Treynor ratio. The Sharpe ratio is the most widely used risk-adjustment return measurement and was chosen due to its popularity. Contrary to the criticism of the Sharpe ratio, the Treynor ratio was chosen to address this and to include the comparison to a benchmark as the systematic risk.

2.4.1 Sharpe ratio
The Sharpe ratio is a method used to calculate risk-adjusted return and was developed by the American economist, William Sharpe. Sharpe ratio is one of the most common methods to
calculated risk-adjusted return (Corporate Finance Institute, 2018). The formula is quite straightforward:

\[ SR = \frac{r - r_f}{\sigma} \]  

Where \( r \) is the return of the instrument, \( r_f \) is the risk-free rate and \( \sigma \) is the standard deviation of the instrument. The numerator in the formula shows the risk premium, the extra compensation the investor receives for holding a risky asset; and thus, the Sharpe ratio measures how much compensation the investor receives per extra volatility the asset provides. A Sharpe ratio of a positive number suggest that the asset gives higher return than the risk taken. A negative Sharpe ratio on the other hand indicated that the asset performs worse than the risk-free rate given the extra risk taken.

The downside to this method is that the Sharpe ratio assumes that the return of the investments is normally distributed (the sigma \( \sigma \) in the denominator) while the empirical result shows that financial instruments deviate from normal distributions (Investopedia, 2018d).

2.4.2 Treynor ratio

The Treynor ratio and the Sharpe ratio are quite similar methods. The only difference is that in the Treynor ratio there is a beta instead of a sigma in the denominator (The Economic Times, 2018). The beta value, as explained prior, is a measurement of the co-movement between an asset and a benchmark index.

\[ TR = \frac{r - r_f}{\beta} \]  

Where \( r \) is the return of the instrument, \( r_f \) is the risk-free rate and \( \beta \) is the beta value of the instrument. The Treynor ratio gives an understanding of how well an asset perform relative to the risk-free rate given how cointegrated the asset is with the market as a hole.

A limitation to the Treynor ratio is that the ratio is a ranking criterion between different assets and the number does not give performance measure of an asset. Furthermore, if a comparing
asset with fluctuating total risk but similar systematic risk the value of the TR will be the same (Wallstreetmojo, 2018).

2.5 Pearson's correlation coefficient
To measure whether assets co-moves, the correlation between the assets are calculated. In this section a method for calculating correlations are presented: Pearson's correlation coefficient.

The Pearson's correlation coefficient is defined as:

\[
\rho_{xy} = \frac{\text{cov}(x_t, y_t)}{\sigma_x \sigma_y}
\]

According to this formula, the correlation between to variables can be between -1.0 and 1.0, where 1 is perfectly positive correlation and -1.0 perfectly negative correlation. If two assets have a correlation of 1, the assets perfectly co-move in the same direction: if one of the assets price increases by 10 (units) the other assets price also increase by 10; and if the correlation is -1, the relationship is inverse: if one assets price rise by 10, the other asset price fall by 10\(^6\).

To successfully compute an accurate Pearson correlation coefficient, the data needs to meet the following requirements (Laerd Statistics, 2018):

- Continuous variables
- Linear relationship
- The variables are independent
- The distribution of the data approximately normal
- Random sampling
- No outliers

If the data do not satisfy the above criterion and is not normally distributed, the significance level using a t-test will not be able to be calculated correctly. The most difficult requirement to satisfy is that the data is normally distributed, and thus, ensuring the data is normally distributed is crucial.

\(^6\) To be noted here is that the notion of causality is not implied by the correlation. From a correlation coefficient, nothing can be said of how the two assets affect each other, only how they co-move.
The t-test is defined as:

\[ t - value = \frac{(\bar{x} - \mu)}{\sigma / \sqrt{n}} \]

where \( \bar{x} \) is the sample mean (or coefficient in terms of correlation), \( \mu \) is the null hypothesis, \( \sigma \) is the standard deviation, and \( n \) is the number of observations.

The t-value is used to calculate the significance level of the coefficient, where a high t-value (bigger than/or equal to |1.64|) indicates a significance level at 10% or lower. A significant coefficient confirms that the coefficient significantly differs from the null hypothesis.
3. Methodology & Data

To model our research problem, a quantitative and deductive study needs to be undertaken. This includes a deterministic view where a causality approach is the main focus (Söderbom & Ulvenblad, 2016).

For financial times series, like the data presented in this study, it is implausible to presume that the homoscedasticity assumptions are satisfied as it is not realistic that the volatility of the error term does not exhibit signs of non-constant volatility (Brooks, 2008). Moreover, as financial asset portfolios are allocated with regard to the assets return means and variances, and that any change in asset demands, and therefore also portfolio allocation, is associated with a change of the return or variance of an asset. The variance of the return should be constant over time if the data series are assumed to adhere to the traditional OLS assumptions (Engle, 1982); if not, the homoscedasticity assumption is violated as the changes in variances of the returns depend on exogenous variable (Brooks, 2008; Engle, 1982).

The heteroscedasticity and the fact that the distribution of the return of the assets in a market are not normally distributed, makes calculation of the Pearson's correlation coefficient problematic since the assumptions for the model are violated. Though, for any dataset, if the number of observations are large, regardless of distribution, the data can be approximated with a normal distribution according to the central limit theorem (Rice, 2007). The central limit theorem implies that if the number of observations increases, our assets data could be approximated with a normal distribution, which in turn makes the use of the Pearson's correlation coefficient plausible. When the data exhibits signs of heteroscedasticity, the only other viable alternative is to, except to use complex models such as DCC-GARCH (Brooks, 2008), split the data into prudent intervals to minimize the effect of the varying variance. This procedure limits the effect of the volatility clustering and centralizes the variance into more stable periods.

The time series data for the analysis of the yield and risk of stocks, bonds and real estate are all three gathered from Nasdaq: the stock data used is the OMXS30 (the 30 biggest actors on the Swedish stock exchange), the bond data from Swedish Government Bond 1-15 (SWEGOVT115). SWEGOVT115 is a compilation of the seven different outstanding Swedish
government bonds RGKB -1047, -1054, -1056, -1057, -1058, -1059, and -1060\(^7\). For the real estate data we use the index OMX Stockholm Real Estate PI (SX8600PI) as a proxy for the real estate market. This real estate index is formed of the biggest real estate companies on the Swedish stock market, and thus we posit that the share price of the individual companies mirrors the true value of the real estate holdings. The use of a company share index as a proxy for the real estate market warrants some argumentation/explanation. First, several studies have been done with a similar approach regarding the US real estate market where the REIT (Real estate investment trust fund) is used as the proxy. Clayton & Mckinnon (2001) showed that during the 1990's the REIT market matured and became more integrated with the underlying real estate market and Hoesli & Oikarinen (2012) argued that REIT and real estate markets are in the long run tightly linked; Clayton & Mckinnon (2003), Case et al. (2010), Yang et al. (2012) used REIT in their studies, and similarly this study uses SX8600PI as a proxy for the Swedish real estate market. To calculate the beta value of each asset, the Standard & Poor 500 index (S&P500) is used as the benchmark market. The reason for the use of S&P500 as the benchmark is that shocks occurring in the United states affects the Swedish market as well to some extent. Lastly, the Swedish 10Y government bond is used as the risk-free return, which is also gathered from Nasdaq.

### 3.1 Data preparation
Since the data from Nasdaq are the closing prices for each asset, the first step is to turn the data into daily, monthly and yearly returns; \( \Delta p = \frac{p_t - p_{t-1}}{p_{t-1}} \), where \( p_t \) is the price of the stock time \( t \). The reason for this is to remove the autoregressive part as well as the time component (the linear time trend). With the daily returns of OMXS30, SX8600PI and SWEGOV115 each asset are indexed, with a start date of February 14th 2010 (this date is chosen due to limitations in data). Summary statistics are calculated with appropriate commands in Python\(^8\).

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\(^7\) These seven swedish government bonds have different characteristics, such as different starting date, maturity date, coupon rate, and principal value. Since their starting dates doesn't coalign, we found it rather uninteresting to publish their de jure length as it could only confuse.

\(^8\) Python is an open source programing languages mainly used for data analysis and machine learning.
To identify whether the data is normally distributed, the Shapiro–Wilk test is conducted (see Appendix B) and to identify if the data is linear a scatter plot is done with each of the assets (see Appendix A).
4. Empirical results and analysis/discussion
In this section we present relevant tables and graphs of interest. First, we present historical return and summary statistics for each of the assets, OMX30, SWEGOVTL115 and SX8600PI. Then look at the risk-adjusted return and lastly the correlation between assets are presented.

4.1 Index
In graph 1 we present the indexes for our asset classes jointly. With a base year of 2010 (February 10th) and an end date of 2018 (May 4th)\(^9\), we clearly see that the real estate index (SX8600PI) has had the highest overall returns, seconded by the stock index (OMXS30), and lowest the bond index (SWEGOVTL115). Generally, for the sample period, the real estate and the stock market have followed the same economic patterns with the same booms and recessions, while the bond market increased substantially more well behaved and smoothly. Interestingly the stock market, and nearly the real estate market, fell below the steady-going bond market during the second half of 2011, caused by the fear that the European sovereign debt crisis would spread to Italy and Spain as well as the downgrading of the credit rating of the US and the potential downgrading of France’s and Britain’s (Reuters, 2011). The stock market dipped below the bond market once more during 2012, caused by the still ongoing European sovereign debt crisis. From 2013 and onwards both the real estate- and stock market outperformed the bond market, but with some heavy dips from mid-2015 to mid-2016 for the stock market, caused by several world-wide economic factors, but the main one was a Chinese stock market crash in Shanghai (Harwell, Denyer & Rauhala, 2015, august 25th), as well as the British EU-referendum produced the continued volatility during the early parts of 2016.

From 2014 and onward, the Swedish Central Bank made the historical and unprecedented decision to lower the interest rate to the negative value of -0,1. This negative interest rate together with the housing industry’s structural problems and lacklustre regulations is part of the cause to the boom in the real estate market (Focus Economics, 2016, 22 august), which can be clearly seen in graph 1.

\(^9\) May the fourth be with you! (A pleasing note for the sci-fi fans).
Graph 1. Indices of the asset classes.

Note: The indices are calculated with the method presented in the theoretical review. The real estate market, stock market, and government bonds corresponds to SX8600PI, OMXS30, and SWEGOVT115 respectively.

We present the daily-frequency change in percentage in graph 2 and the descriptive statistics in table 3. To be noted are primarily two things; first, the approximately zero-mean daily returns of all the asset classes, which can be seen with “eye-ball economics” in graph 2, and also supported by table 2; and second, the volatility clustering (low/high volatility is often followed by low/high volatility: volatility happens in bursts) and the seemingly strong exogenous factors impact on the volatility. It would seem from graph 2 that: we have heteroscedastic variables.

---

10 Both the volatility clustering and the strong exogenous impact is more or less a defining characteristics of an autoregressive time series. Due to the strong correlation between t and t-1, an exogenous shift/disruption have a long-lasting impact and effect.
**Graph 2.** Daily frequency change of the asset classes measured in percentage.

![Graph showing daily frequency change of asset classes](image)

*Note: To clarify, the daily frequency change in percentage could also be called the first-difference of the index; that is, the graph shows the increase/decrease in percentage terms for day $t$ compared to day $t+1$ of the index returns.*

**Table 2.** Descriptive statistics for the asset classes.

<table>
<thead>
<tr>
<th>Descriptive statistics</th>
<th>SX8600PI</th>
<th>SWEGOV115</th>
<th>OMXS30</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>0.000628</td>
<td>0.000124</td>
<td>0.000325</td>
</tr>
<tr>
<td>Median</td>
<td>0.00081</td>
<td>0.00168</td>
<td>0.000599</td>
</tr>
<tr>
<td>Std</td>
<td>0.010541</td>
<td>0.001952</td>
<td>0.011814</td>
</tr>
<tr>
<td>Min</td>
<td>-0.005151</td>
<td>-0.015004</td>
<td>-0.08424</td>
</tr>
<tr>
<td>Max</td>
<td>0.051109</td>
<td>0.011671</td>
<td>0.064336</td>
</tr>
<tr>
<td>N. obs</td>
<td>2070</td>
<td>2070</td>
<td>2070</td>
</tr>
</tbody>
</table>

*Note: What is here measured is the returns of the asset classes at daily frequency.*
4.1 Risk-adjusted return

In table 3 we present the descriptive statistics regarding the different risk measurements, presented in the theoretical review, for the whole sample period (2010-2018) calculated at daily frequency.

For the real estate index (SX8600PI), the standard deviation was 0.010541 (≈ 1.05%), the beta value 1.937733, the Sharpe ratio 1.533047, and the Treynor ratio 0.01949. For the Swedish government bonds (SWEGOV115), the standard deviation was 0.001952 (≈ 0.19%), the beta value 0.078755, the Sharpe ratio 0.969688, and the Treynor ratio -0.028442. For the Swedish stock market index (OMXS30), the standard deviation was 0.011814 (≈ 1.18%), the beta value 1.178865, the Sharpe ratio 0.919865, and Treynor ratio 0.004636. As expected, the standard deviation was the lowest for the bonds, but fairly equal for the real estate and stock market. Similarly, the beta value was lowest for the bonds, followed by stock market, and highest for the real estate market. The higher beta value for the real estate than for the stocks is due to the higher covariance between the real estate market and the benchmark (S&P500) than for the stock market. The negative Treynor ratio for the bonds is due to the higher average returns of the risk-free rate compared to the average return of the bonds during the sample period.

Table 3. Descriptive statistics for the risk-measurements at daily frequency.

<table>
<thead>
<tr>
<th>Descriptive statistics</th>
<th>SX8600PI</th>
<th>SWEGOV115</th>
<th>OMXS30</th>
</tr>
</thead>
<tbody>
<tr>
<td>STD (σ)</td>
<td>0.010541</td>
<td>0.001952</td>
<td>0.011814</td>
</tr>
<tr>
<td>Beta (β)</td>
<td>1.937733</td>
<td>0.078755</td>
<td>1.178865</td>
</tr>
<tr>
<td>Sharpe ratio</td>
<td>1.533047</td>
<td>0.969688</td>
<td>0.819865</td>
</tr>
<tr>
<td>Treynor ratio</td>
<td>0.01949</td>
<td>-0.028442</td>
<td>0.004636</td>
</tr>
</tbody>
</table>

Note: For clarification regarding the formulas for how the values were calculated, see section 2.3 Risk in our theoretical review. STD is short for standard deviation. The values are calculated using non-indexed data at daily frequency, which means that the different measurements also show the daily values.
4.1.1 Sharpe ratio

In graph 3 we show the Sharpe ratio for the asset classes calculated with a monthly frequency. The real estate market (SX8600PI) had relatively small fluctuations compared to the other two asset classes, where the ratio moved mostly around the zero mark but with some large positive movements; that is, the real estate markets upward spikes were larger and had a bigger impact than the small and somewhat inconsequential negative movements, which contributed significantly for the average Sharpe ratio of 1.5533047 for the whole period. The bonds (SWEGOV115) seem to have the largest volatility with alternating periods, often in groups of approximately six months, of positive and negative ratios. The positive spike periods fluctuated approximately between +7 and 0, while the negative periods fluctuated between -7 and 0. The stock market (OMXS30) had periods of stable volatility between -3 and +3 up until the second half of 2013, followed by a period of large positive fluctuations for one and a half year. From the second quarter of 2015, the fluctuations slowed down with a calmer period until the second half of 2017 with heavy fluctuations for the rest of the sample period.

Some positive spikes are shared by all three asset classes and seems to have been caused by some exogenous inter-market variable. Interestingly, the real estate market does not seem to suffer as severe as the bond and stock market on the ratios downturns. This “characteristic” of only participating on the booms most definitely contributes to the real estate markets higher overall Sharpe ratio.
Graph 3. The Sharpe ratio displayed for the asset classes with a monthly frequency.

Note: For clarification regarding how the values were calculated, see section 2.3.3.1 Sharpe ratio. The graph is displayed at monthly frequency; the return variable was calculated as the absolute return for the month, the risk-free return as the average risk-free return for each month, and the standard deviation as the mean deviation from the monthly return mean using the daily data.

4.1.2 Treynor ratio
In graph 4 we present the Treynor ratio for the asset classes calculated at a monthly frequency. The bond ratio is mostly negative; the volatility fluctuates around the zero mark, but with some large negative spikes. While never reaching above 0.15, the bonds Treynor ratio moves as low as -0.3. To the contrary, the real estate market never moves below -0.1 but reaches the heights of around +0.25. The graphs seem to imply a mirrored relationship between the bonds ratio and real estate market ratio\textsuperscript{11}. Moreover, the stock market and bonds seem to share the same booms and recession, although the stock market booms and bond market recession are greater than their respective counterpart.

\textsuperscript{11} This is not entirely true since the bond ratio suffers from more and larger negative ratios than the real estates positive ratios.
Graph 4. The Treynor ratio displayed for the asset classes.

Note: For clarification regarding how the values were calculated, see section 2.3.3.2 Treynor ratio. The graph is displayed at monthly frequency; the return variable was calculated as the absolute return for the month, the risk-free return as the average risk-free return for each month, and the beta as the covariance between the absolute monthly return and the absolute monthly benchmark return divided by variance of the daily benchmark return calculated for each month.

4.2 Correlations

In graph 5 we present how correlation between the asset classes changes over time. We also present the correlation coefficients for the whole sample period, table 4.

The correlation between the real estate (SX8600PI) and the stock market (OMXS30) [correlation 1] have remained relatively high during the whole period with often small, incremental fluctuations disrupted by larger short anomalies. The correlation [1] fluctuated around 0.6-0.7, which can be further corroborated by looking at table 4 for the correlation for the whole period but went as low as around 0.25 twice in 2010 and late 2016/early 2017, and even negative for a brief period in mid-2014. The reason for this high correlation could be explained by the fact that we use real estate stocks as proxies for the real estate and that part of the real estate stocks are a subset of OMXS30. The correlation between the real estate (SX8600PI) and the government bonds (SWEGOVVT115) [correlation 2], and the stock market
(OMXS30) and the government bonds (SWEGOV115) [correlation 3] are strikingly similar with no clear trend and large periods of high volatility. During the most stable period 2010-2013, the correlations [2,3] moved rapidly from values close to 0 down to -0.75, and from 2013 and onward fluctuated with increased vigour, still seemingly equally interlinked.

**Graph 5. Pearson’s correlation coefficient over time**

Note: The graph displays the Pearson’s correlation coefficients computed using the correlation equation [1] at a monthly frequency; that is, the covariance and standard deviation are both estimated using daily data into a monthly value.

**Table 4. Yearly Pearson’s correlation coefficient for the whole period**

<table>
<thead>
<tr>
<th></th>
<th>SX8600PI</th>
<th>SWEGOV115</th>
<th>OMXS30</th>
</tr>
</thead>
<tbody>
<tr>
<td>SX8600PI</td>
<td>1</td>
<td>-0.21389</td>
<td>0.69825</td>
</tr>
<tr>
<td>SWEGOV115</td>
<td>-0.21389</td>
<td>1</td>
<td>-0.25913</td>
</tr>
<tr>
<td>OMXS30</td>
<td>0.69825</td>
<td>-0.25913</td>
<td>1</td>
</tr>
</tbody>
</table>

Note: The table displays the Pearson’s correlation coefficients for the average year for the whole sample period. The coefficients are computed by using the correlation equation [1]. All the specific yearly correlations used in the computation of the average yearly correlation are significant at 10% level, except for 2013, 2015,2016 between SX8600PI and SWEGOV115.
The coefficients displayed in table 4 are all significant except for the correlation between SWEGOVТ115 and SX8600PI for the years 2013, 2015, 2016. For these years the coefficients have a high p-value that corresponds to a significance level at above 10% and are thus not significant in any practical scientific sense. The reason for the high p-value (low significance level) for the correlation between the bonds and the stocks, and between the bonds and the real estate is due to the calculation of the t-value: since the major upwards spike seen in mid-2013 for both of the bonds correlation, is a major volatility swing, the t-value becomes small and the p-value becomes large, and thus no significant coefficients are calculated.

Since some of the yearly coefficients are not significant, we also compute the correlation coefficient for the whole period to understand the sign and size of the significant correlation coefficients at 1%. By comparing the coefficients between table 4 and table 5, we see that they are somewhat similar except for the correlation between OMXS30 and SWEGOVТ115. Moreover, the signs of the coefficients are the same, but we can deduce that the yearly average coefficients are closer to the correct value. The reason for this is, as we argued in section 3, that the procedure of limiting the window or period affects the volatility clustering and centralizes the variance into more stable periods.

**Table 5.** Pearson’s correlation coefficient for the whole period

<table>
<thead>
<tr>
<th></th>
<th>SX8600PI</th>
<th>SWEGOVТ115</th>
<th>OMXS30</th>
</tr>
</thead>
<tbody>
<tr>
<td>SX8600PI</td>
<td>1</td>
<td>-0.27187</td>
<td>0.74138</td>
</tr>
<tr>
<td>SWEGOVТ115</td>
<td>-0.27187</td>
<td>1</td>
<td>-0.40317</td>
</tr>
<tr>
<td>OMXS30</td>
<td>0.74138</td>
<td>-0.40317</td>
<td>1</td>
</tr>
</tbody>
</table>

*Note: The table displays the Pearson’s correlation coefficients for the whole sample period. The coefficients are computed by using the correlation equation [1]. All correlations are significant at 1% level.*
5. Conclusions
The correlation between the stock and the real estate market have remained relatively stable around 0.7, and the government bonds had negative correlations with both stocks and real estate with an increasingly volatile tendency and no clear trend. As the real estate market is highly illiquid, the short-term volatility in the correlations cannot be thought of as a trend change, instead a rather stable one. Although the bond market is also illiquid, the correlation with both stock and real estate market seems to be trending upwards, towards positive coefficients. Thus, this indicates a trend towards a higher degree of integration between the bonds and the other asset markets; this study has not performed any estimations regarding this seemingly increasing trend, but his could provide grounds for further research.

The standard deviation, as seen in table 3, for the bonds was about one tenth of both the stock and real estate market; a result which conforms to existing literature. Furthermore, this lower standard deviation contributed to the, interestingly, higher monthly average Sharpe ratio for the bonds than for the stocks. This result indicates that bonds could be a good diversification tool for a stock portfolio, with a correlation of approx. -0.26, and actually could provide the portfolio with increased risk-adjusted returns. Moreover, the abnormal and seemingly exponential development of real estate returns, which can be seen in graph 1 and by observing the higher Sharpe and Treynor ratios, could have provided the more traditional stock portfolio investors with increased return if they expanded their exposure to the real estate markets in late 2014.

To conclude this study, the real estate market has had the highest overall risk-adjusted return, with a Sharpe ratio of 1.533, followed by the high 0.969 for the bond market, and lastly 0.819 for the stock market. The correlation between the stock and real estate market remained relatively stable with an overall correlation of around 0.7 and no signs of a trend-change; on the other hand, the correlations between the bond and the other two asset classes had a close,
almost interlinked relationship, with overall coefficients of -0.1583 and -0.2827 for the real estate and stock market respectively.\textsuperscript{12}

\textsuperscript{12} To carry on the spirit of the glorious Roman empire, the authors would like to end this paper with the following quote from Cato the Elder: "Ceterum/Praeterea censeo Carthaginem esse delendam" (Furthermore, Carthago must be destroyed).
References


**Database sources:**


- Riksbanken. Risk-free rate (10Y GOV BOND). [Retrieved 2018-05-03] https://www.riksbank.se/sv/statistik/sok-rantor--valutakurser/?g7-SEGVB10YC=on&g130-SEKNOKPMI=on&from=2010-02-08&to=2018-05-03&f=Day&c=cAverage&s=Comma
Appendix A: Scatter plot

Scatter plot of SX8600P and SWEGOV'T115. Indicates a linear relationship

Scatter plot of SX8600PI and OMXS30. Indicates a linear relationship
Scatter plot of SWEGOV115 and OMXS30. Indicates a linear relationship
Appendix B: Shapiro–Wilk test

<table>
<thead>
<tr>
<th>Asset</th>
<th>Test statistik</th>
<th>P-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>SX8600PI</td>
<td>0,97526</td>
<td>0</td>
</tr>
<tr>
<td>SWEGOV1T115</td>
<td>0,97015</td>
<td>0</td>
</tr>
<tr>
<td>OMXS30</td>
<td>0,9599</td>
<td>0</td>
</tr>
</tbody>
</table>

The Shapiro-Wilk test to identify if the distribution is normal. The p-value column confirms that we can reject the null-hypothesis that the distribution is not normal.