A Petri Nets Semantics for Privacy-Aware Data Flow Diagrams

Master's thesis in Computer Science

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Abstract
Privacy of personal data in information systems is gaining importance rapidly. Although data flow diagrams (DFDs) are commonly used for designing information systems, they do not have appropriate elements to address privacy of personal data. Privacy-aware data flow diagrams (PA-DFDs) were introduced recently to tackle this issue. However, they lack the concrete semantics to be formally verifiable. On the other hand, Petri net is a well-known mathematical modeling language that has all the necessary supporting elements for formal verification. In this work, we present appropriate transformations for PA-DFDs to Petri nets and therefore, provide a Petri nets semantics for them. Firstly, we clearly identify different elements of PA-DFDs. Then, we take a modular approach where for each element of PA-DFDs we present an algorithm which transforms that element to a Petri nets representation. Secondly, we demonstrate the effectiveness of the transformations on a case study, where we transform a PA-DFD to a corresponding Petri nets model. The case study is quite elaborate and covers most of the important aspects of PA-DFDs. Finally, we perform verification tasks on the obtained Petri nets model from the case study where we check privacy properties such as purpose limitation and accountability of the data controller. The Petri nets semantics along with the rest of the supporting work constitute a step forward when it comes to privacy of personal data in an information system.

Keywords: privacy policy, verification, privacy by design, data flow diagrams, privacy-aware data flow diagrams, Petri nets
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1 Introduction

Human beings consider privacy as an important aspect of their day-to-day life. As new systems gather more and more information from its users, the importance of privacy of the users’ personal data is also increasing rapidly. Research awareness of privacy engineering has also improved significantly after 2012 implying its value in modern information systems [34]. Although these research introduced many sophisticated privacy solutions, their integration with everyday engineering practice has been slow thus far. As a result, recent history shows lots of concerning reports related to privacy violations of various kinds. For instance, Facebook app allowing the sharing of users’ friend networks with advertisers, Snapchat’s violation of user expectation by not deleting users’ messages, and NoScript’s (Firefox extension) defaults leading to de-anonymization attacks on Tor users are some of the notable examples worth mentioning [34].

In general practice, privacy was (and still is) more of an afterthought when it came to designing a system. However, the concept of Privacy by Design (PbD) is gaining importance and needs to be addressed with utmost gravity. It is an approach to systems engineering which takes privacy into account from the earliest of stages of designing a system and throughout the whole engineering process. This will be required by the next coming European General Data Protection Regulation (GDPR1). However, taking privacy into account while designing information systems is not a straightforward task for software architects. They are far from being an expert in this area. It is a common practice while designing software architectures to use modeling languages which are based on graphical representations of the system such as graphs or diagrams (e.g., UML) ([26], [13], [9]).

One of the most common approaches to design information systems is using Data Flow Diagrams (or DFDs). It is an approach to model the flow of data in an information system. Due to the way DFDs are defined, there is no room to take privacy into account when designing an information system. A privacy-enhancing transformation ([3], [4]) of DFDs has been proposed to tackle this issue by introducing some new and useful annotations, which ensures a certain amount of privacy for personal data.

Although Privacy-Aware DFDs (PA-DFDs) are a promising step towards preserving the privacy of users’ personal data, they lack concrete semantics. This makes it difficult to verify the intended usefulness of them. Formal verification is an appropriate approach to gain confidence in a model’s intended behavior. If formal verification is possible for PA-DFDs, it can be used to formally guarantee and check that no privacy violation occurs.

Unfortunately, the proposed PA-DFDs cannot be used for formal verification or reasoning. A more precise model is required to be formulated in order to perform verification regarding privacy-related properties. One such model is Petri nets (PN) ([5], [6]). A number of tools are also available to perform verification in a PN model. Transforming PA-DFD models into meaningful and correct PN models in order to perform verification is one step closer to the right direction when it comes to putting privacy in the forefront of designing information systems.

1.1 Thesis Overview

Apart from Chap. 1 (introduction), 2 (literature review), 6 (discussion), and 7 (conclusion), the organization of the thesis is done in three parts.

The main focus of the thesis is to give a suitable transformation from PA-DFD models to PN models so that verification can be carried out. In order to do that, we needed to decide upon a variant of Petri nets first. The Colored Petri Nets (CPN) [6] is chosen for this purpose. In Chap. 3, we present the transformation in a detailed manner. For each component of PA-DFDs, a corresponding CPN transformation is given. CPN are formally introduced in Chap. 2.

Furthermore, it is essential to decide upon a tool where a CPN model can be implemented. CPN Tools [32] is chosen to carry out this task. In Chap. 4, a case study is conducted. An example DFD model is presented and according to the transformation proposed in [4], it is transformed into a PA-DFD model. Afterward, according to the transformation presented in Chap. 3, the PA-DFD model is transformed into a CPN model, which is implemented manually in CPN Tools. The transformation for the case study is presented in detail in Chap. 4.

Finally, in Chap. 5, verification is done for the CPN model obtained from the case study by checking some privacy related properties. The essential know-hows for verification in CPN Tools such as state space calculation, state space queries and functions, etc. ([6], [32], [33]) are discussed beforehand in the literature review (Chap. 2).
1.2 Scope and Limitations

The primary aim of the thesis is to give a suitable transformation for PA-DFD models to CPN models. Although, PA-DFDs deal with concepts such as retention time of personal data, due to certain shortcomings of the implementation environment (CPN tools), this aspect of PA-DFDs is kept out of the scope of the thesis. Therefore, the CPN models to be obtained after the transformation from PA-DFD models are non-timed. The implementation of the CPN model in the case study is done manually. The automatic implementation will need a lot more time to develop and thus, kept out of the scope of the thesis. Due to the limited time frame of the thesis work, verification carried out in the CPN model is far from being exhaustive. Nevertheless, the transformation lays the foundation to further explore this area in future works. The reasons behind the limitations are further discussed in more detail in Chap. 6 (discussion).
2 Literature Review

In order to carry out the thesis work, knowledge in DFDs, PA-DFDs and Petri nets are essentials. Therefore, we consider these in section 2.1, 2.2 and 2.3. Furthermore, in section 2.3, we go deeper into the topic of Petri nets, introducing different variants of it, as well as familiarizing with certain tools and their use in verification.

2.1 Data Flow Diagrams (DFDs)

System development life cycle (SDLC) is a process used for development of software system starting from planning to the implementation phase. SDLC mainly consists of four phases, which are analysis, design, implementation, and testing [2]. Data flow diagrams (DFDs) are usually used during the analysis phase to produce the process model [1]. The process model plays an important role defining the requirements in a graphical view, which makes its reliability a key factor for improving the performance of the following phases in SDLC [2].

DFDs are graphical notations used to design how data flows in an information system. DFDs are simple to understand. There are four basic components in a DFD: external entity (user or an outside system that sends or receives data), process (it can be any action or computation that modifies the data), data store (which is a database like entity for storing data) and data flow (a route that carries data from and to the other components). Complex process is another component representing a complex functionality or computation that is detailed in an additional DFD [4]. It can be refined into a sub-process [3]. In [4], an extension to the standard DFDs was proposed. The extension is another type of flow, which is called data deletion. This acts as an

![Diagram of DFD components with data deletion](image)

**Figure 2.1:** Standard symbols of DFD components with the extension ‘data deletion’ [4].
incoming flow to data stores. This incoming flow carries the reference of the data that is stored in the data store so that the data can be deleted from it using the reference to it. The standard graphical symbols of DFD components, as well as the extension, are presented in Fig. 2.1.

DFDs are created with the composition of the aforementioned symbols and should obey a set of rules in order to be well-formed and consistent [35]:

- Each process should have no less than one incoming data flow and one outgoing data flow.
- All processes should have unique names.
- Each data store should have at least one input data flow and one output data flow.
- Two different data stores cannot be directly connected to each other with a data flow.
- Two different external entities also cannot be directly connected to each other with a data flow.
- Data cannot directly move from a data store (or external entity) to an external entity (or data store). There should be a process between the data store (external entity) and the external entity (data store). The data should flow from the data store (external entity) through the process to the external entity (data store).

In Fig. 2.2, a simple DFD of an ATM (Automated Teller Machine) system is presented. It has one external entity named “User”, three processes named “Log in to system”, “See Account Information”, and “Withdraw Money”. The system involves the user providing credentials, receiving confirmation, accessing information, making a request, and withdrawing money from the account database.

**Figure 2.2:** A simple DFD for an ATM system.
Information”, and “Withdraw Money”, and a data store named “Account Database”. A “User” can log in to the system by giving his credentials and get confirmation for the login. They can make a request to the process “See Account Information” and the process will provide the “User” with the information after getting it from the data store “Account Database”. They can also withdraw money by means of providing the process “Withdraw Money” with the amount he wants to withdraw and the process will communicate with the data store to provide the “User” with the money. Here, credentials, confirmation, request, information, amount, and money are the flow of data inside the system.

In summary, the user can log in to the ATM system, check account information and withdraw money from their account.

### 2.2 Privacy-Aware Data Flow Diagrams (PA-DFDs)

Most of the data flows in DFDs may carry personal data. This is when the privacy of personal data comes into consideration. However, DFDs do not have necessary elements to tackle issues regarding privacy of personal data. This is one of the primary reasons for proposing an extension of the standard DFDs with privacy-aware annotations so that software designers can take privacy principles into account. A DFD of such kind with privacy-aware annotations is named Privacy-Aware DFD (PA-DFD), the outline of which was introduced in [3]. In a later work [4], the approach of designing PA-DFDs was stated in detail.

In order to enrich a standard DFD with privacy-aware annotations, first, the designer of the DFD needs to provide a classification of the data flows where it mentions whether a data flow is personal or non-personal. The designer of the model provides the following additional information for each personal data flow [4]: (i) name of external entity the data belongs to, (ii) the purpose for the data to flow (which will be checked against the consents of the user), and (iii) the retention time (or expiration time) for the personal data. This slightest level of annotation on top of the standard DFD is needed to detect parts in the model (also called hotspots [4]) which are impacted by privacy principles (in this case the European GDPR). The next task is to transform these identified hotspots with privacy-aware annotations to obtain a PA-DFD.

There are mainly six operations corresponding to a step of the personal data lifecycle: data collection, disclosure, usage, recording, retrieval, and erasure according to the GDPR [4]. From Fig. 2.3, we can see which parts of a DFD (on the left-hand side of both figures) are considered
hotspots for potential privacy violations and how they correlate with the six operations stated above. Appropriate modifications in the DFD model need to be made where these hotspots are detected so that certain privacy properties are entailed by construction. These properties related to each hotspot are later mentioned in Chap. 5 and listed in Table 5.1 [4].

![Figure 2.3: Privacy-sensitive parts (i.e., hotspots) of a DFD and their corresponding privacy-aware transformations [4].](image)

In the model-to-model transformations shown in Fig. 2.3, we see the hotspots for a DFD on the left-hand side and the corresponding privacy-aware transformations for each hotspot on the right-hand side. Application of these transformations on the required hotspots in a DFD results in a PA-DFD. Two main differences can be spotted when comparing the PA-DFD symbols with the DFD ones described earlier in Section 2.1. One of which is, five subtypes for the Process element: Limit, Reason, Request, Log, and Clean. Moreover, a notion of “priority” was introduced, where a process labeled with ‘p’ needs to be executed before any other process. In this figure, ‘d’ is personal data.

As can be seen from Fig. 2.3, all the transformations share some common elements. They are the processes Limit, Request, Log, and a store also named Log. Each of them plays their own specific roles when personal data d goes through them. The process Limit is always the first
step for $d$ to go through. The task of Limit is to restrict the processing of $d$ in accordance with the consent given for it in a policy $pol$. This $pol$ needs to be provided earlier by another process Request in order for the process Limit to perform the restriction on the processing of $d$ later. In addition to these two processes, Log is another common element of all the transformations. Its only task is to perform a log operation and store a trace of the data processing on $d$ in accordance with its $pol$ to a Log store. Then, it just forwards $d$ on a data flow to the rest of the model [4].

Transformations stated in Fig. 2.3 are different from each other and perform different tasks on the personal data according to the consent of the external entity. In a collection we see personal data $d$ and the policy (which includes the consents) $pol$ corresponding to it being received from an External entity. Then it goes through the common elements and is forwarded to the rest of the model. A disclosure can be said to be the opposite of a collection, where it takes both personal data $d$ along with its corresponding policy $pol$ and forwards it to an External entity. A usage similarly takes personal data $d$ along with its corresponding policy $pol$ and applies the process named Process on $d$ to get a computed data $d'$, which is then forwarded to the rest of the model. On the other hand, the process Reason also forwards a changed policy $pol'$ which corresponds to $d'$ to the rest of the model. Recording takes personal data $d$ and its policy $pol$ and stores them respectively in a Data store and a Policy store. Moreover, process Clean, which ensures the erasure of $d$ (which has a reference $ref$) from the Data store if and when the policy $pol$ corresponding to it conforms to it. Typically $d$ is erased when its retention time expires or a consent in its policy $pol$ that indicates to its erasure. A retrieval takes personal data $d$ and its policy $pol$ respectively from a Data store and a Policy store and forwards them to the rest of the model after using the common elements on them. Lastly, an erasure takes a reference $ref$ and the policy $pol$ which is related to the referenced data. It then performs the erasure of said data from the Data store.

2.3 Petri Nets

Petri nets [5] are considered a powerful modeling formalisms not only in computer science but also in system engineering and many other disciplines [6]. The theoretical part of Petri nets makes it possible to do precise modeling and analysis of system behavior. On the other hand, the graphical representation of Petri nets helps visualize the state changes of a modeled system [6]. Petri nets have been used to model really complex and dynamic event-driven systems.
Important examples of such include, manufacturing plants ([7],[8],[6]), command and control systems [10], computers networks [11], workflows [12], logistic networks [14], real-time computing systems ([15], [16]), and communication systems [17].

There exists a number of different variants of Petri nets. There are mainly two kinds of Petri nets: low-level (basic) and high-level. The Petri nets relevant to this thesis are explained with their formal definitions in the following sections of this chapter, starting with the low-level or basic one.

2.3.1 Basic Petri Nets

A Petri net is a bipartite graph and consists of just three types of basic elements. These are places, transitions, and arcs. This graph has two types of nodes. A place is represented as a circle; a transition is represented as a bar or box. Arcs can directly connect places to transitions as well as transitions to places, but not transitions to transitions or places to places. A token is another primitive concept of a Petri net. Tokens are represented as black dots residing inside places of a Petri net graph. Tokens can be present or absent in certain places, which, for instance, can indicate whether conditions associated with those places are true or false [6]. Elements for Petri nets graphical representation can be seen in Fig. 2.4.

![Elements of Petri nets](image)

**Figure 2.4: Elements of Petri nets.**

**Definition 1.** A Petri net is formally defined [6] as a five-tuple $N = (P, T, I, O, M_0)$, where

1. $P = \{p_1, p_2, \ldots, p_m\}$ is a finite set of places;
2. $T = \{t_1, t_2, \ldots, t_n\}$ is a finite set of transitions, $P \cup T \neq \emptyset$, and $P \cap T = \emptyset$;
3. $I: P \times T \to N$ is an input function which defines directed arcs connecting places to transitions. Here, $N$ is a set of nonnegative integers;
4. $O: T \times P \to N$ is an output function defining directed arcs from transitions to places;
5. $M_0: P \to N$ is the initial marking.

---

1 We will be using ‘=’ for assignment, ‘≡’ for equality, and ‘≠’ for inequality throughout the document.
As stated earlier, *arcs* directly connect places to transitions or transitions to places. Assume, there is a place $p_j$ and a transition $t_i$. If there is an arc directed from $p_j$ to $t_i$, according to Def. 1, $p_j$ is an input place of $t_i$, and is denoted by $I(p_j, t_i) = 1$. On the other hand, if there is an arc directed from $t_i$ to $p_j$, according to Def. 1, $p_j$ is an output place of $t_i$, and is denoted by $O(t_i, p_j) = 1$. If $I(p_j, t_i) = k$ (or $O(t_i, p_j) = k$), it means there exist $k$ arcs connecting $p_j$ to $t_i$ (or $t_i$ to $p_j$) in parallel. However, in the graphical representation, parallel arcs connecting a place (transition) to a transition (place) are usually represented by a single directed arc with the multiplicity or weight of $k$.

A *marking* of a Petri net is represented by the distribution of tokens over places. A Petri net has an initial marking which assigns a nonnegative integer to each place. Marking changes depending on the execution of Petri nets and movement of tokens from one place to another. It is also referred to as change of state.

![Figure 2.5: A simple Petri net.](image)

Fig. 2.5 shows an example of a simple Petri net, from where we have the following information:

- $P = \{p_1, p_2, p_3, p_4\}$
- $T = \{t_1, t_2, t_3\}$

- $I(p_1, t_1) = 1, I(p_1, t_2) = 1, I(p_1, t_i) = 0$ for $i = 3$;
- $I(p_2, t_3) = 1, I(p_2, t_i) = 0$ for $i = 1, 2$;
- $I(p_3, t_3) = 1, I(p_3, t_i) = 0$ for $i = 1, 2$;
- $O(t_1, p_2) = 2, O(t_1, p_j) = 0$ for $j = 1, 3, 4$;
$O(t_2, p_3) = 2, O(t_2, p_j) = 0 \text{ for } j = 1, 2, 4;  \\
O(t_3, p_4) = 2, O(t_3, p_j) = 0 \text{ for } j = 1, 2, 3;  \\
M_0 = (2, 0, 0, 0)$

An execution of a Petri net happens by the firing of a transition. It changes the marking of the Petri net. However, it is not always possible for a transition to fire. Before a transition can fire, it needs to be enabled first. There are some basic rules that are followed for enabling and firing of transitions [6].

i. Enabling Rule: If each input place $p$ of a transition $t$ contains at least the number of tokens which is equal to the weight of the arc directly connecting $p$ to $t$, i.e.,  
$\forall p \in P: M(p) \geq I(t, p)$, then $t$ is enabled.

ii. Firing Rule: A transition can fire only when it is enabled. When an enabled transition $t$ is fired, from each input place $p$ it consumes the number of tokens equal to the weight of the arc connecting $p$ to $t$. On the other hand, if $t$ has one or more output places, it then also deposits in each output places $p'$ the number of tokens which is equal to the weight of the arc directly connecting $t$ to $p'$. If at the same time more than one transition is enabled, the firing of transition is nondeterministic. Firing a transition results in a new marking. If a transition $t$ fires with the marking $M$, we will get a changed marking $M'$. Formally, it can be written as follows:

$\forall p \in P: M'(p) = M(p) - I(p, t) + O(t, p)$

If there is a transition that does not have any input place, then that transition is called a source transition. A source transition is always enabled. On the other hand, a transition without any output place called a sink transition. A sink transition consumes tokens but does not produce any. If a transition $t$ has the same input and output place $p$, then the pair $p$ and $t$ is called a self-loop. A Petri net without any self-loop is called pure [6].

If we take a look at the Petri net shown in Fig. 2.5, we can see that both the transition $t_1$ and $t_2$ are enabled and ready to be fired. The initial marking of the Petri net is:

$M_0 = (2, 0, 0, 0).$

If $t_1$ is fired, the marking changes and we get a new marking according to the firing rule (see Fig. 2.6):
$M_1 = (1, 2, 0, 0)$.

**Figure 2.6:** Petri net from Fig. 2.5 after firing transition $t_1$.

At this point, still $t_1$ and $t_2$ both are enabled according to the enabling rule. As the firing rule states, any of them can be fired. For the sake of this example, we say $t_2$ is fired. Accordingly, we get a new marking (see Fig. 2.7):

$M_2 = (0, 2, 1, 0)$.

**Figure 2.7:** Petri net from Fig. 2.5 after firing transition $t_2$.

Finally, the transition $t_3$ is the only transition that is enabled. Following the firing of $t_3$, we get (see Fig. Figure 2.8):

$M_3 = (0, 1, 0, 1)$. 


2.3.2 Colored Petri Nets

In a basic (low-level) Petri net, it is not possible to distinguish tokens from each another. Most of the times this leads to a significantly large and unstructured model. In that case, it becomes troublesome understanding the model as well as checking properties in the model. To tackle such issues, high-level Petri nets were developed. We will focus on a type of high-level Petri net where different pieces of information can be identified. These are called Coloured Petri nets.

Colored Petri Nets (CPN) were introduced by Kurt Jensen [18]. Unlike a basic Petri net, the tokens in a CPN are distinguishable from each other. For simplicity of understanding, it can be said that all the tokens of a basic Petri net are uncolored (or black) and all the tokens of a CPN have different colors. In addition to tokens, a place of a CPN is attached with a set of colors. Upon firing of a transition in CPN it removes tokens from its input places and adds them to its output places just like a basic Petri net. However, the enabling and firing of a transition here is associated with colors. A transition may remove tokens of different colors from its input places and add entirely new colors of tokens to its output places. In the later sections, we will see that the concept of colors actually represents complex data-values. It is efficient to model systems in a more compact and well-mannered fashion using CPN. An example CPN model is presented in the following section.

A CPN model of a system is not only state oriented but also action-oriented. From a CPN model of a system, we can get the information about different states of the system depending on different actions (transitions) taken. CPN models are executable and it is possible to perform
simulations of the model of a system to learn about different states and behaviors of the system [21].

CPN is considered a discrete-event modeling language. It has been under development since 1979 by the CPN group at Aarhus University, Denmark. Along with the characteristics of basic Petri nets, it possesses the power of a high-level programming language. The CPN ML programming language is based on the functional programming language Standard ML ([19], [20]). Like functional programming, CPN ML provides primitives for defining data types and data manipulation, which in turn help models to be compact [21]. There are quite a lot of modeling languages [22] developed for concurrent and distributed systems and CPN is one of them. Other notable examples like such include the Calculus of Communicating Systems [23] as supported by, for example, the Edinburgh Concurrency Workbench [24], Statecharts [25] as supported by, for example, the VisualState tool [20], Promela [27], as supported by the SPIN tool [28], and Timed Automata [29] as supported by, for example, the UPPAAL tool [30]. The CPN group at Aarhus University, Denmark has developed industrial-strength tools like Design/CPN [31] and CPN Tools [32], which support CPN. In this thesis, we chose to work with CPN Tools to create and simulate CPN models as well as checking certain properties in them.

2.3.2.1 CPN ML Programming

Before presenting the formal specification of CPNs in section 2.3.2.2, in this section, we take an introductory look at how to use CPN ML programming language (to define color sets and functions, declare variables, and write inscriptions) when creating a CPN model. This will help understand the formal specification of CPNs, as we will be illustrating that with the help of an example model where this programming language is used. We will not cover every aspect of CPN ML in this section as it is an enormous topic to explain. We are only focusing on parts of CPN ML essential to support the understanding of the thesis work done. The reader is referred to [21] where the CPN ML programming language is discussed in a more detailed manner.

There are some predefined set of basic types in CPN ML, which are inherited from Standard ML (SML). The basic types are as follows:

- Basic types:
  - int (the set of integers)
  - string (the set of all text strings)
o bool (has two values, true and false)
o real (the set of all real numbers)
o unit (has only one value, written (). It is used to represent uncolored tokens.)

A color set is a type, which is defined using a color set declaration \texttt{colset ...=...}. Basic types are used to define simple color sets. Simple color sets can then be used further to create structured color sets using a set of color set constructors such as, \texttt{with}, \texttt{product}, \texttt{record}, \texttt{list} [21]. When a color set is declared, it can later be used to type places in a CPN model. Furthermore, the color set constructor \texttt{with} can be used to define sub color sets or entirely new color sets. Some examples:

- Defining simple color sets:
  o \texttt{colset I = int;}
  o \texttt{colset S = string;}
  o \texttt{colset B = bool;}

- Defining sub color sets using \texttt{with}:
  o \texttt{colset Price = int with 0..1500; (this means Price can only have integer values from 0 to 1500)}
  o \texttt{colset Letter = string with "a".."z";}
  o \texttt{colset BinaryBool = bool with (zero,one);}

- Defining new color sets using \texttt{with}:
  o \texttt{colset Sports = with Football | Cricket | Handball;}
  o \texttt{colset SmartPhones = with Android | IPhone | Windows;}

- Defining new color sets using \texttt{product}, \texttt{record} and \texttt{list}:
  o \texttt{colset PhonePrice = product SmartPhones * Price;}
    Possible values: (Android, 1000), (IPhone, 1300), ...
  o \texttt{colset listOfSports = list Sports;}
    Possible values: [Cricket], [Handfall, Football], ...
  o \texttt{colset ItemOffer =}
    record item:S * regularCost:Price * discount:Price
Possible values: {item="Oil", regularCost=15, discount=3}, ...

There are some basic operations on lists, records, and products. [] denotes an empty list. Concatenation of lists can be made using the operator ^^, e.g., \( [5,4,3]^{\mathbin{\mathbf{\kern-1pt^\kern-1pt\kern-1pt\mathbin{-}}}} [7,1,8] \) evaluates to \([5,4,2,7,1,8]\). To add an element in front of a list, the operator :: can be used, e.g., \( "n" :: ["o","i","c","e"] \) evaluates to \(["n","o","i","c","e"]\). We can use the operator \# to extract a field of a record, e.g., \#n{m=13,n=2} evaluates to 2. The same operator can be used to extract an element from a product value, e.g., \#3("a","b","c","d") evaluates to "c".

Furthermore, there are some other basic operators, such as \(\sim\) (unary minus), \(+\) (addition for reals and integers), \(-\) (subtraction for reals and integers), \(*\) (multiplication for reals and integers), \(/\) (division for reals and integers), \(\text{div}\) (division for integers), \(\text{mod}\) (modulo for integers), \(=\) (equal to), \(<\) (less than), \(>\) (greater than), \(\langle\) (less than or equal to), \(\rangle\) (greater than or equal to), \(<>\) (not equal to) and \(^\) (string concatenation). There are also some logical operators available; these include \(\neg\) (negation), \(\text{andalso}\) (logical AND), \(\text{orelse}\) (logical OR) and \(\text{if then else}\) (if takes a boolean argument; upon its evaluation to \text{true}, it returns the value after \text{then}; upon its evaluation to \text{false}, it returns the value after \text{else}; values after \text{then} and \text{else} are of the same type). The use of some of these operators are presented with the following examples:

- \(\neg(1>2)\) \(\text{andalso} (1=1 \text{ orelse} 2<1)\)
  \(\text{value: true.}\)
- \(\text{If } 2<>2 \text{ then } \text{"hi" else } \text{"hi"}^{\mathbin{\mathbf{\kern-1pt^\kern-1pt\kern-1pt\mathbin{-}}}} \text{"there"}\)
  \(\text{Value: "hi there"}\)

We also declare variable and constants. To declare a variable, the keyword \text{var} is used. On the other hand, the keyword \text{val} is used to declare a constant. Examples of variable and constant declarations are as follows:

- \text{var a:Price;}
- \text{val b = 1132;}

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Now, we turn our attention to some of the important aspects of CPN ML programming language which deal with the graphical representation of the CPN model. The first thing that comes to mind when talking about any kind of Petri net model, is the number of tokens and their availability in different places in the model. A place can either be empty or non-empty with one or more tokens. In order to initialize places with multiple tokens, we need multiset [21]. Multisets in CPN are denoted using this notation: \( a_1 \cdot v_1 \cdot + + a_2 \cdot v_2 \cdot + + \ldots + + a_n \cdot v_n \) where \( v_1 \) represents one of the elements and \( a_1 \) represents the number of times it occurs in the multiset. Another important thing to keep in mind while initializing tokens in a place is that the type or color set of that place needs to be the same as the tokens’.

We can look at the examples given in Fig. 2.9 to get a clear idea of how tokens are initialized in a place of a CPN model. The place \( p_1 \) is empty. On the other hand, the place \( p_2 \) has only one token with an integer value of 1 and \( p_3 \) has 9 tokens. In CPN, multisets are implemented as lists, e.g., \( 3 \cdot 7 + + 2 \cdot 5 + + 4 \cdot 1 \) is equivalent to [7, 7, 7, 5, 5, 1, 1, 1, 1]. Therefore, it is useful when using list functions on multisets.

One of the most important elements of any kind of Petri net is the arc or directed arc that connects a place to a transition and vice versa. An arc inscription plays a major role while connecting places and transitions to each other. It essentially represents the value of a token that goes through an arc. An inscription of an arc possesses the ability to change the value of a token that goes through that arc before being forwarded to a place or a transition. An arc inscription can be a variable, a constant, a multiset, a function (which we will be talking about later in this section) or some other expressions. We can see different types of arc inscriptions in the CPN model given in Fig. 2.10. Later in this section, there is an explanation for the model which covers the functionalities of the different arcs used in the model.

Another concept that is necessary to get familiarized with to understand enabling and firing of transitions in CPN, is binding. If there is a transition \( t \) and in its input and output arcs have

![Figure 2.9: Initialization of tokens in places of a CPN model.](image-url)
variables $a_1, a_2, \ldots, a_n$, then a binding of $t$ assigns a concrete value $v_1, v_2, \ldots, v_n$ to each of these variables. The assigned values should be of the same type as the variables they are assigned to. A binding is enabled when there are tokens whose values match the values of the variables on the input and output arcs. After necessary bindings are enabled, it can occur, i.e., the transition can fire, consuming and producing tokens respectively from the input and to the output places. If an arc has a variable to which the transition cannot assign any value, then that variable is considered unbound. The pair $(t, \langle \text{bindings} \rangle)$ is called a binding element. Here, for the transition $t$, the binding element is $(t, \langle a_1=v_1, a_2=v_2, \ldots, a_n=v_n \rangle)$.

Enabling and firing of a transition also depend on the evaluation of a guard used attached to that transition. A guard is a Boolean expression, which needs to be evaluated to true in order for the transition to enable and fire. Only those binding elements are enabled which evaluate the guard to true. A guard is denoted by square brackets. In the model shown in Fig. 2.10, we see the transition withdraw has a guard $[y<x]$, which means if the value of $y$ is less than the value of $x$, then the transition is enabled and ready to fire.

In addition to binding and guards, a transition can enable and fire depending upon the priority attached to it. In CPN Tools there are three standard priorities given:

```plaintext
val P_HIGH = 100;
val P_NORMAL = 1000;
val P_LOW = 10000;
```

As can be seen, they are simply three constants with integer values, where lower value suggests a higher priority. By default, all transitions in a CPN model are attached with the P_NORMAL priority. If two transitions $t_1$ and $t_2$, both have all of their bindings enabled and guards evaluated to true, but $t_1$ is attached with the priority P_NORMAL and $t_2$ is attached with P_HIGH, then only $t_2$ will be enabled and ready to be fired. Only after $t_2$ is fired, if there are no necessary bindings enabled for it or no guards evaluated to true, $t_1$ will be enabled and ready to be fired. We can declare our own priorities in CPN Tools.

Like other programming languages, CPN ML programming language also supports the definition and use of functions. Most of the times when modeling a complex system, the use
of complex expressions or calculations are required. It is troublesome to include these long and complex expressions as arc inscriptions or guards in the graphical representation of the CPN model because they take a lot of spaces as well as make the model unnecessarily hard to understand [21]. Sometimes several complex expressions with a similar functionality are needed in different parts of the model. For these reasons and much more definition of functions and their use is more than necessary. The modeler needs to define the function once with a meaningful name and refer to it whenever it is needed in the model. It makes the model look easy to read and update if needed. Functions in CPN ML are defined using the keyword fun followed by the name of the function and the list of parameters separated by commas [21]. Functions can be used for arc inscriptions, guards and even initializing tokens in a place.

Example function declarations in CPN ML are as follows:

```plaintext
colset INT = int;

fun IsItEven(x:INT) = if (x mod 2) = 0 then true else false;

fun fact(y:INT) = if y>1 then y*fact(y-1) else 1;
```

The functions IsItEven given an integer determines if it is even or not. On the other hand, fact, a recursive function, calculates the factorial of a given integer.

Figure 2.10: A simple CPN model for ATM Machine designed in CPN Tools.
We will end this section by giving a simple yet proper example of a CPN model to demonstrate most of the aspects in CPN ML. In Fig. 2.10, we show a simple CPN model for an ATM machine, where one can withdraw and deposit money with the identification of their account number. For this model we need to define the following color sets and variables:

\[
\begin{align*}
\text{colset AccountNo} &= \text{int with } 1..10000; \\
\text{colset Amount} &= \text{int with } 1..50000; \\
\text{colset Balance} &= \text{int}; \\
\text{colset AvailableBalance} &= \text{product AccountNo} \times \text{Balance}; \\
\text{colset AcAm} &= \text{product AccountNo} \times \text{Amount}; \\
\text{var an:AccountNo; }
\text{var am:Amount; }
\text{var ba:Balance; }
\end{align*}
\]

From the model in Fig. Figure 2.10, we can see that an arc inscription can be a tuple and we can perform addition or subtraction in those too. The places AmountToDeposit, AmountToWithdraw and Balance respectively have 8, 3 and 4 tokens. Also, notice how each place is associated with a color set (specified bottom right of each place). Both of the
transitions are enabled (CPN Tools marks the transition border with green color when it is enabled). There is no guard attached to the transition Deposit. However, the transition Withdraw has a guard attached to it, which is \([\text{ba>}\text{am}]\). It means, if the available balance of an account is less than what is asked to be withdrawn from that account, then for that account money cannot be withdrawn. The final marking of the model (where, no transition is enabled) is shown in Fig. Figure 2.11.

The reason for the transition Deposit to be disabled is pretty straightforward: its input place AmountToDeposit does not have any token left in it. On the other hand, the reason for the transition Withdraw to be disabled is quite different. Although its input place AmountToWithdraw has more than the necessary amount of tokens for it to be enabled, the guard \([\text{ba>}\text{am}]\) does not allow Withdraw to be enabled. The available balance for both the accounts 1234 and 3456 are less (can be seen in the place Balance) than the amount requested to be withdrawn (can be seen in the place AmountToWithdraw). Therefore, the guard \([\text{ba>}\text{am}]\) evaluates to false and disables the transition Withdraw.

2.3.2.2 Formal Definition of Colored Petri Nets

In section 2.3.1, we have looked at the formal definition of basic Petri nets. In this section, we will take a look at the formal definition of Colored Petri Nets (non-hierarchical), which we will be using for this thesis work. When presenting a definition we will use the example in Fig. Figure 2.10 for illustration. The color sets definition, as well as the variable for the example, are already given in the previous section.

First, we give the definition [6] of multisets, which is used in the definition for Colored Petri Nets. Example of multisets can be the following three multisets \(m_{AD}, m_{AW}, \text{ and } m_B\) over the color sets AcAm and AvailableBalance corresponding to the markings of AmountToDeposit, AmountToWithdraw, and Balance are in Fig. Figure 2.10:

\[
\begin{align*}
  m_{AD} &= 4'(1234,500) + 1'(3456,700) + 3'(9101,2000) \\
  m_{AW} &= 1'(1234,5000) + 1'(3456,2700) + 2'(9101,200) \\
  m_B &= 1'(1234,500) + 1'(3456,1700) + 1'(9101,2000)
\end{align*}
\]

**Definition 2.** Assume there is a non-empty set \(S = \{s_1, s_2, s_3, \ldots\}\). A multiset over \(S\) is a function \(m : S \rightarrow N\) that maps each element \(s \in S\) into a non-negative integer \(m(s) \in N\) called
the number of appearances (coefficient) of $s$ in $m$ [6]. A multiset $m$ can also be written as a sum:

$$\sum_{s \in S} m(s)'s = m(s_1)'s_1 + m(s_2)'s_2 + m(s_3)'s_3 + \cdots$$

Next, we will start defining various elements of a Colored Petri Net. The net structure consists of a finite set of places, $P$, a finite set of transitions, $T$, and a finite set of directed arcs, $A$ [6]. For the example given in Fig. Figure 2.10, $P$ and $T$ are defined as follows:

$$P = \{ \text{AmountToDeposit}, \text{AmountToWithdraw}, \text{Balance} \}$$

$$T = \{ \text{Deposit}, \text{Withdraw} \}$$

$P$ and $T$ are disjoint, i.e., $P \cap T = \emptyset$. The set of directed arcs, $A$ connecting places and transitions is defined as a set of pairs. In this pair, the first component is the source of the arc and the second component is the destination of the arc. Here, $A$ needs to be a subset of $(P \times T) \cap (T \times P)$ to make sure that an arc connects a place to a transition or a transition to a place [6]. In the example from Fig. Figure 2.10, we have the following arcs:

$$A = \{(\text{AmountToDeposit}, \text{Deposit}), (\text{Deposit}, \text{Balance}), (\text{Balance}, \text{Deposit}), (\text{Balance}, \text{Withdraw}), (\text{Withdraw}, \text{Balance}), (\text{AmountToWithdraw}, \text{Withdraw})\}$$

After net structure, we will turn our attention to defining net inscriptions, i.e., color sets, arc expressions, guards and initial markings. We denote the set of expressions provided by the inscription language (which is, CPN ML in the case of CPN Tools) as $EXPR$. Also, by using $Type[e]$ we denote the type of the expression $e \in EXPR$. The free variables in an arc expression $e$ is denoted by $Var[e]$, where the type of a variable $v$ can be denoted as $Type[v]$ [6]. For the arc expressions from the CPN model in Fig. Figure 2.10, we have the following free variables:

$$Var[e] = \begin{cases} \{\text{an, am}\} & \text{if } e = (\text{an, am}) \\ \{\text{an, ba, am}\} & \text{if } e \in \{ (\text{an, ba + am}), (\text{an, ba - am})\} \\ \{\text{an, ba}\} & \text{if } e = (\text{an, ba}) \end{cases}$$

We use $\Sigma$ to define the finite set of non-empty color sets for a CPN model. In the case of our example from Fig. Figure 2.10, it is as follows:

$$\Sigma = \{\text{AccountNo, Amount, Balance, AvailableBalance, AcAm}\}$$
Set of variables can be denoted by \( V \). Each variable should have a type that is in \( \mathbb{\Sigma} \). For the CPN model in Fig. 2.10, we have the following variables:

\[
V = \{ \text{an: AccountNo, am: Amount, ba: Balance} \}
\]

The color set function \( C : P \rightarrow \mathbb{\Sigma} \) assigns to each place \( p \) a color set \( C(p) \), which belongs to the set \( \mathbb{\Sigma} \). For the example model in Fig. 2.10, it is defined as

\[
C(p) = \begin{cases} 
\text{AcAm} & \text{if } p \in \{ \text{AmountToDeposit, AmountToWithdraw} \} \\
\text{AvailableBalance} & \text{if } p = \text{Balance}
\end{cases}
\]

There is also a guard function \( G : T \rightarrow \text{EXPR}_V \), which assigns to each transition \( t \in T \) a guard \( G(t) \), which needs to be a boolean expression, i.e., \( \text{Type}[G(t)] = \text{Bool} \). Here, \( \text{EXPR}_V \) means that there exists \( e \in \text{EXPR} \) such that \( \text{Var}[e] \subseteq V \). That means, the set of free variables appearing in the guard expression \( e \) is required to form a subset of \( V \). For that reason, \( G(t) \in \text{EXPR}_V \). For the model in Fig. 2.10, the guard expressions are as follows:

\[
G(t) = \begin{cases} 
\text{ba} > \text{am} & \text{if } t = \text{Withdraw} \\
\text{true} & \text{for all } t \in T \text{ where } t \neq \text{Withdraw}
\end{cases}
\]

In a CPN model, when a transition does not specify a guard explicitly, that means there is an implicit constant guard \text{true}.

There are two more functions, arc expression function and initialization function. First, we take a look at the former one. It is defined as \( E : A \rightarrow \text{EXPR}_V \), which assigns to each arc \( a \in A \) an expression \( E(a) \). An arc expression is essentially a multiset. For example, in Fig. 2.10, the arc expression \((\text{an, am})\) of the arc connecting the place AmountToDeposit to the transition Deposit can be alternatively written as \( 1\cdot(\text{an, am}) \), which is the same thing. Here, \( 1\cdot \) is implicit. However, if it was written as \( 2\cdot(\text{an, am}) \), that would have meant that the arc was carrying two identical copies of the same token. Therefore, for an arc \((p, t) \in A \), connecting a place \( p \in P \) to a transition \( t \in T \), it should be the case that the type of the arc expression is the multiset type over the color set \( C(p) \) of the place \( p \), i.e., \( \text{Type}[E(p, t)] = C(p)_{\mathbb{MS}} \). Similarly, for an arc connecting a transition \( t \in T \) to a place \( p \in P \), \( \text{Type}[E(t, p)] = C(p)_{\mathbb{MS}} \). The arc expression function for the example model in Fig. 2.10 is given as follows:
Finally, we have the initialization function $I: P \rightarrow EXPR_\emptyset$, which assigns to each place $p \in P$ an expression $I(p)$. Here, $EXPR_\emptyset$ means that there exists $e \in EXPR$ such that $Var[e] \subseteq \emptyset$. That means, there should not be any free variables in the expression $e$, i.e., it needs to be a closed expression. $I(p)$ must belong to $EXPR_\emptyset$. Also, $Type[I(p)] = C(p)_{MS}$, meaning the type of $I(p)$ is the multiset type over the color set of the place $p$. Initialization function for the example from Fig. Figure 2.10 is as follows:

$$I(p) = \begin{cases} 
4'(1234,5000) + + 1' (3456,7000) & \text{if } p = \text{AmountToDeposit} \\
+ + 3'(9101,2000) & \\
1'(1234,5000) + + 1' (3456,27000) & \\
+ + 2'(9101,2000) & \text{if } p = \text{AmountToWithdraw} \\
1'(1234,5000) + + 1' (3456,1700) & \\
+ + 1'(9101,2000) & \text{if } p = \text{Balance} 
\end{cases}$$

**Definition 3.** A Colored Petri Net (non-hierarchical) can be represented as a nine-tuple $CPN = (P, T, A, \Sigma, V, C, G, E, I)$ [6], where:

- The finite set of places is denoted by $P$.
- The finite set of transitions is denoted by $T$.
- The finite set of directed arcs is denoted by $A \subseteq (T \times P) \cup (P \times T)$.
- The finite set of color sets is denoted by $\Sigma$.
- The finite set of typed variables is denoted by $V$, where $\forall v \in V. Type[v] \in \Sigma$.
- A guard set function which assigns a color set to each place is denoted by $C: P \rightarrow \Sigma$.
- A guard function which assigns a guard to each transition $t$ is denoted by $G: T \rightarrow EXPR_\emptyset$ such that $Type[G(t)] = Bool$.
- An arc expression function is denoted by $E: A \rightarrow EXPR_\emptyset$. For each arc $a \in A$, this function assigns an expression such that $Type[E(a)] = C(p)_{MS}$. Here, $p \in P$ is connected to the arc $a$.
- An initialization function is denoted by $I: P \rightarrow EXPR_\emptyset$. The task of this function is to assign initialization expression to each $p \in P$, such that $Type[I(p)] = C(p)_{MS}$. 

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2.3.2.3 Verification of CPN Models Using CPN Tools

As mentioned earlier, CPN Tools [32] was chosen to verify the CPN model for this thesis work. CPN Tools provides options to calculate the State space of a CPN model. After the calculation, the model then becomes ready to be verified. CPN Tools needs to be downloaded\(^1\) and installed in the machine in order to open a CPN model and perform further tasks on it.

Calculation of state space for a CPN model refers to the calculation of all the reachable states (markings) and state changes (occurring binding elements) of that model. The state space is represented as a directed graph, where the nodes correspond to set of reachable markings and the arcs correspond to occurring binding elements [6]. One of the prerequisites for calculating the state space of a model in CPN Tools is that all the transitions and places in that model need to be uniquely named. Otherwise, the tool cannot calculate the state space. It is therefore important to keep in mind to generate unique names for all the transitions and places in the CPN model obtained after transforming a PA-DFD model.

Let us consider the simple CPN model\(^2\) stated in Fig. 2.12 with the stated initial markings. Upon firing of the transition \(T\) it consumes one token from each of its input place (\(A\) and \(B\)) and outputs the summation of those tokens to its output place \(C\). We can reach different markings from this initial marking.

\(\text{Figure 2.12: Simple CPN example for integer sum.}\)

\(^1\)http://cpntools.org/download includes the necessary instructions to install CPN Tools in supported platforms.
\(^2\)https://www.dropbox.com/s/z2t0enfvjfp8ce4/IntegerSum.cpn is the CPN model for downloading.
Let us calculate and inspect the state space of this model. After the calculation\(^1\) of the state space, we can draw the state space graph using the state space tool. In Fig. 2.13, we can see the state space graph for this model. The graph has seven nodes, i.e., seven reachable markings including the initial marking. The nodes in the figures are numbered, where “1” is the number of the node and it denotes the initial marking. For the node 1, we see that it also has another numbering “0:4”. Here, “0” means, this node has zero, i.e., no predecessor node and “4” means, it has 4 successor nodes (which is easily understandable from the figure).

We present the state space graph again in a different figure (Fig. 2.14), this time with markings for each node. This will help visualize the idea of reachability (from one marking to another).

\(^1\) http://cpnptools.org/documentation/gui/palettes/state_space_tools/start contains instructions on how to use the state space tools.
Here, \textit{SumOfIntegers} is the name of the page the CPN model is drawn on. Therefore, \textit{SumOfIntegers'} \textit{A}, \textit{SumOfIntegers'} \textit{B}, and \textit{SumOfIntegers'} \textit{C} are the three places of the model. We can see the number of tokens for each place beside its name in different markings.

After the calculation of the state space, it is possible to start checking properties for the model. It is also possible to automatically generate and save a state space report after the state space calculations are finished. The report contains information, such as the total number of nodes and arcs in the state space for that model, home properties, liveness properties, boundedness properties, fairness properties, etc. The reader is referred to [32] to find more about the use of state space tool in CPN Tools, where relevant documentations\(^1\) on its usage are provided.

\begin{figure}[h]
\hspace*{1cm}
\includegraphics[width=\textwidth]{figure.png}
\caption{Screenshot of CPN tools and some of its options.}
\end{figure}

There are some standard query functions available to check properties in a CPN model. We can use them to make state space queries\(^2\). One such function is \textit{Reachable}, which takes as argument a pair of integers \((m, n)\) and returns \textit{true} if there exists a path from node \(m\) to node \(n\) in the state space graph and \textit{false} otherwise. For the state space graph in Fig. 2.13, the first of the following queries returns \textit{true} and the other one returns \textit{false}:

\begin{itemize}
\item \textit{Reachable}(3, 4)
\item \textit{Reachable}(4, 3)
\end{itemize}

\(^1\) http://cpntools.org/documentation/tasks/verification/calculate_a_state_space\_includes\_relevant\_documentations\_regarding\_calculation\_of\_state\_space\_in\_CPN\_Tools.

\(^2\) http://cpntools.org/documentation/tasks/verification/make_state_space_queries\_contains\_information\_on\_how\_to\_make\_state\_space\_queries\_in\_CPN\_Tools.
Reachable (1,7);

Reachable (2,7);

We can also inspect the markings of places in the model for certain nodes of the state space using the query which has the following ML structure:

fun Mark.<PageName>’<PlaceName> Inst -> (Node -> CS ms)

Here, <PageName> refers to the name of the page the model is drawn on, <PlaceName> refers to the name of the place we are investigating, Inst is the instance of the page (which is 1 in our case), Node is the number of the node in the state space graph for that model, and CS ms is the multiset type of the color set of <PlaceName>. The multiset type of a color set is simply the list of that color set. For the model in Fig. 2.12, we can write:

Mark.SumOfIntegers’C 1 7;

to get the multiset of the tokens on the place C on the first instance of the page SumOfIntegers in the marking of the node 7 in the state space graph. It returns [17, 39]: INT ms, which we can confirm from Fig. 2.14, is the correct value.

Another useful function is PredAllNodes: (Node -> Bool) -> Node List. This function takes as an argument, a predicate (a function that takes as an argument a node of the state space graph and returns bool) and returns the list of nodes of the state space graph for which the predicate is true. For example, we may want the list of nodes in the state space graph (Fig. 2.13), where multiset of tokens of place C includes 28. To achieve that, we first write the predicate as follows:

fun Predicate1 (CPN'n:Node) = contains (Mark.SumOfIntegers'C 1 CPN'n) [28];

The function Predicate1 takes as an argument a state space node n. It is not enough to only write n as the argument to the function. We have to explicitly mention it as CPN’n:Node. The body of the function checks whether the multiset of tokens in the place C for the node n contains 28. The function contains is a predefined list function\(^1\) in CPN ML. Now we give

---

\(^1\) We will be using other predefined list functions such as hd and tl in Chap. 5. They are provided here: http://cpntools.org/documentation/concepts/colors/declarations/colorsets/list_colours

\(^2\) http://cpntools.org/documentation/concepts/colors/declarations/colorsets/implementation_of_list_fu

28
Predicate1 as an argument to PredAllNodes, which searches the entire state space and returns a list of nodes for which Predicate1 returns true. In other words, we get a list of nodes (reachable markings), for which the place C contains a token with the value 28. It is written as follows:

PredAllNodes Predicate1;

This returns [6,5,2] : Node List, which we can confirm from Fig. 2.14, is correct.

The query can also be written in a different form as follows:

PredAllNodes (fn (CPN'n:Node)

=> contains (Mark.SumOfIntegers'C 1 CPN'n) [28])

In this form, we directly write the body of the function Predicate1 in the query using the notation (fn arguments => body). This way, we do not have to define separate functions each time we need to change something in the body of the function.

It is also possible to check properties using non-standard queries, which can be created by writing CPN ML functions. There is a good amount of predefined state space functions available in [32] and mostly in [33], which can be used to check properties in CPN models.

Queries are written using the text (marked as “3” in Fig. 2.15) option provided by the Auxiliary tool palette under the tool box in CPN Tools. After writing the query, it can be evaluated (to boolean values: true or false) using the Evaluate ML (marked as “4” in Fig. 2.15) option in the Simulation tool palette under the tool box. However, before making any queries for a CPN model, the calculation of state space of that model is a prerequisite (using the options marked as “1” and “2” in Fig. 2.15). A screenshot of the graphical user interface of CPN Tools is provided along with the aforementioned options and tool palettes in Fig. 2.15.

---

**Figure 2.16:** Instructions on how to change the options for calculating state space of a CPN model.
It is possible to change options for calculating the state space. In Fig. 2.16 we show how to access those. Sometimes, it is the case that the state space for a model is really big. For such cases, it is convenient to change some options for calculating the state space. In Fig. 2.16, the first four options (nodesstop, arcsstop, secsstop, and predicatestop) are called stop options and the last three options are called branching options. The stop options help decide when a state space should end. In the figure, we see the default values for these options, where nodesstop : 0 indicates that calculation will not stop until all the nodes are calculated. Similarly, for arcsstop 0 indicates that calculation will not stop until all arcs are calculated. Any non-zero positive value given to either nodesstop or arcsstop will result in the stoppage of calculation of the state space when that number is reached. For secsstop, the value is by default set to 300, which means the state space calculation will stop after 300 seconds, even if it is partially calculated (not fully calculated). Therefore, this option is helpful when calculating a really large state space. If the option is set to zero for secsstop, the calculation will not stop until the state space is fully calculated [33]. In Chap. 5, when we perform verification on the CPN model (which has a really large state space), we use this option in order to ensure full calculation of the state space for the model.
3 Transformation from PA-DFD Models to CPN Models

In this chapter, we describe an algorithm to transform a PA-DFD to a CPN model. The formal definition of Colored Petri Nets, discussed in section 2.3.2.2, is used for the algorithm. After the transformation, we will have a CPN represented by the nine-tuple \((P, T, A, \Sigma, V, C, G, E, \mathcal{I})\). The algorithm is presented using pseudo code which uses a syntax similar to that of current programming languages and thus facilitates its future implementation.

Let us define a set,

\[
\text{Component} = \{\text{ExternalEntity}, \text{Process, DataStore, Limit, Request, Log, LogStore, Reason, PolicyStore, Clean}\}
\]

of all the components of PA-DFDs. For the reader’s convenience, all the components are separately presented in Fig. 3.1.

![Figure 3.1: Different components of PA-DFDs.](image)

It is also apparent that flows in PA-DFDs carry a lot more different information than DFDs. For the sake of identifying each flow separately on the basis of what information it carries, let

\[
\text{FlowsType} = \{RFlow_d, RFlow_p, RFlow_{dp}, RFlow_r, RFlow_{rp}, DFlow_r\}
\]

be the set of all the different kinds of flows. Here,

- \(RFlow_d\) represents regular directed flow carrying data (see (a) from Fig. 3.2);
- $\text{RFlow}_p$ represents regular directed flow carrying a policy (see (b) from Fig. 3.2);
- $\text{RFlow}_{dp}$ represents regular directed flow carrying a tuple of data and its corresponding policy (see (c) from Fig. 3.2);
- $\text{RFlow}_r$ represents regular directed flow carrying reference to certain data (see (d) from Fig. 3.2);
- $\text{RFlow}_{rp}$ represents regular directed flow carrying a tuple of reference to certain data and its corresponding policy (see (e) from Fig. 3.2);
- $\text{DFlow}_r$ represents data deletion flow carrying reference to certain data (see (f) from Fig. 3.2);

The type of flows can be further classified into two separate sets, $\text{RegularFlows}$, which contains all the regular directed flows, and $\text{DeleteFlows}$, which contains all the deletion flows:

$$\text{RegularFlows} = \{\text{RFlow}_d, \text{RFlow}_p, \text{RFlow}_{dp}, \text{RFlow}_r, \text{RFlow}_{rp}\}$$

$$\text{DeleteFlows} = \{\text{DFlow}_r\}$$

![Figure 3.2: Different kinds of flows in PA-DFDs.](image)

Before going into details about the transformation from PA-DFD models to CPN models, we need to clearly state some of the aspects of DFDs to PA-DFDs transformation because they will still be needed later for the transformation to CPN. As mentioned in section 2.2, when transforming a DFD model to a PA-DFD model, the designer of the DFD model provides a data flow classification for each personal data flow. Let this classification be represented as a table named $\text{DFClass}$ with four columns: $\text{FlowLabel}$, $\text{DataSub}$, $\text{PurpOfFlow}$, and $\text{RetentionTime}$. $\text{FlowLabel}$ stores the label (inscription) of the flow (which is here the label of the data). $\text{DataSub}$ stores the name of the data subject (external entity) that the data belongs to. $\text{PurpOfFlow}$ stores the purposes of the flow. $\text{RetentionTime}$ stores the retention time for the data. The first two column $\text{FlowLabel}$ and $\text{DataSub}$ stores their information as a string. The third column $\text{PurpOfFlow}$ stores information as a list of strings where each element represents
a single purpose. The stored information in RetentionTime is represented as a non-negative integer.

Along with the aforementioned four columns, we add a fifth column PolList for the convenience of PA-DFD to CPN transformation. This column stores the list of labels of all the flows of type \( RFlow_p \) that correspond to the personal data in the row. This will help us to identify which policy flow corresponds to which personal data flow at the time of transformation. An example row of the table is as follows:

\[
("d","user","research","advertise"),10, ["p1","p2"]
\]

From this example, we can say the personal data flow labeled “d” has two corresponding policy flows \( (RFlow_p) \) labeled “p1” and “p2”.

### 3.1 Parsing the PA-DFD Model and Storing Information

In what follows, we define the sequence of steps required to transform a PA-DFD model into a CPN model. The PA-DFD model is parsed and each component is marked or identified with a unique identifier, which is a non-negative integer. We denote this identifier as ID. Let there be a table named ComponentTable, where we record each component against the ID it is identified with. The table ComponentTable has four columns, namely, IdColumn containing the ID of the components, CompColumn containing the Component for that ID, CompName containing the name of the component as a string, and SubCompColumn which will contain a string value. Later in this section, we will discuss the SubCompColumn and what is stored in it, but for now, the column remains empty. If each row of the table is defined as a four-tuple, then it can be stated as follows:

\[
\text{ComponentTable} = (\text{IdColumn}, \text{CompColumn}, \text{CompName}, \text{SubCompColumn})
\]

In Fig. 3.3, a small subset of a PA-DFD model is given as an example, where each component is identified with a unique ID; in this case, the ExternalEntity (Patient) is identified with 1 and the Limit, with 2. According to this example, we can have two rows for the ComponentTable. They are as follows:

\[
(1, \text{ExternalEntity}, "Patient",""
\]
\[
(2, \text{Limit},"",""
\]
Like *ComponentTable*, another table is necessary for all the flows in the model too. As all flows start from a *Component* and end to another, where each of them is uniquely identified, it is convenient to define a table *FlowTable*, where for each flow, its source component and destination component’s *ID* are stored. In that case, there will be four columns for this table. The first column namely *SourceID*, stores the *ID* of the source component of a flow; the second column, namely *DestID*, stores the *ID* of the destination component of a flow; the third column, namely *TypeOfFlow*, stores the type of the flow (as *FlowsType*); the fourth column named *FlowLabel*, stores the label of the flow (as string). If each row of the table is defined as a four-tuple, then it can be stated as follows:

\[
\text{FlowTable} = (\text{SourceID}, \text{DestID}, \text{TypeOfFlow}, \text{FlowLabel})
\]

An example row of *FlowTable* stated as a four-tuple according to Fig. 3.3 is:

\[
(1, 2, \text{RFlow}_d, "d")
\]

We define a set,

\[
\text{Connection} = \{(x_1, x_2) | x_1, x_2 \in ID \text{ and } x_1 \neq x_2\}
\]

Here, *Connection* is a set of two-tuples, where the first element of the tuple is the *ID* of the source component of the flow and the second element is the *ID* of the destination component of the flow. In *FlowTable*, each row represents a unique flow. Therefore, we get a unique *Connection* for each flow. In Fig. 3.3, the *ID* of the source component of the flow is 1 and the *ID* of the destination component of the flow is 2. Therefore, the *Connection* regarding the flow is (1,2).

![Diagram](image)

**Figure 3.3:** Identifying PA-DFD components and flows.

As can be seen from Fig. Figure 2.3, most of the PA-DFD components have more than one combination of incoming and outgoing flows connected to them. For example, at the time of **recording** and **erasure** of personal data, a *Request* is only connected with one outgoing *RFlow*_\(_p\) and one incoming *RFlow*_\(_p\). On the other hand, when at the time of **collection**, **disclosure**,
retrieval, and usage of personal data, Request is connected with two outgoing RFlow_p and one incoming RFlow_p. Similarly, for some other components, there can be more than one combination of incoming and outgoing flows connected to them. Keeping this in mind, it is, therefore, convenient to define one or more sub-components for each component for clarity. There can be a lot more sub-components for all the components. However, we will define sub-components that are convenient for the transformation only.

Thus far, the column SubCompColumn from the table ComponentTable remains empty. It needs to be filled up. For each row in the table, starting from the first, we will check what PA-DFD component it is and the ID of it. Let’s assume the component is c_1. Then, with the help of the table FlowTable, it is possible to compute the combination of flows c_1 connected with. Further, we define Algorithm 1 later in this section (which uses this computation) where a sub-component is obtained and the column SubCompColumn is updated with it, in the row of the table ComponentTable, against the ID in question. All the sub-components under the column SubCompColumn are represented as a string.

Before stating the algorithm for defining different sub-components, it is necessary to declare few functions that will be used in the rules. The classical ‘dot’ notation is used to access table values needed for some of the functions. For a PA-DFD component associated with i \in ID, the following functions are listed:

- The function \textit{ird}(i) and \textit{ord}(i) returns respectively the number of incoming and outgoing \textit{RFlow}_d carrying personal data connected to the component. The information of whether data carried by the flow is personal or not can be known with the help of the tables ComponentTable, FlowsTable and DFClass.
- The function \textit{irp}(i) and \textit{orp}(i) returns respectively the number of incoming and outgoing \textit{RFlow}_p connected to the component.
- The function \textit{irdp}(i) and \textit{ordp}(i) returns respectively the number of incoming and outgoing \textit{RFlow}_{dp} connected to the component.
- The function \textit{conToLog}(x, DFlow_r), where x \in ID, returns true if the component associated with x is connected to the PA-DFD component Log with a DFlow_r.
- The function \textit{selComp}(i) returns ComponentTable.CompColumn where ComponentTable.IdColumn \equiv i.
• The function `upSubComp(i,s)` updates `ComponentTable.SubCompColumn` with a string `s`, where `ComponentTable.IdColumn ≡ i`.

Let us now go through each row of the table `ComponentTable` and update the `SubCompColumn` column by adding a sub-component obtained using Algorithm 1. Let the value of the column `IdColumn` be `i` for the row.

Algorithm 1 Obtaining sub-components for each PA-DFD components

1: if `selComp(i) ≡ ExternalEntity` then
2:     `upSubComp(i,"EE")`
3: else if `selComp(i) ≡ Limit` then
4:     if `(ird(i) ≥ 1) && (irp(i) ≡ 1)) && (ordp(i) ≡ 1))` then
5:         `upSubComp(i,"LimG")`
6:     else
7:         `upSubComp(i,"LimE")`
8:     end if
9: else if `selComp(i) ≡ Request` then
10:    if `(irp(i) ≡ 1) && (orp(i) ≡ 2))` then
11:       `upSubComp(i,"RG")`
12:    else
13:       `upSubComp(i,"RRE")`
14:    end if
15: else if `selComp(i) ≡ Log` then
16:    if `(irdp(i) ≡ 1) && (ord(i) ≡ 1)) && (ord(i) ≡ 1))` then
17:       `upSubComp(i,"LogG")`
18:    else
19:       `upSubComp(i,"LogE")`
20:    end if
21: else if `selComp(i) ≡ LogStore` then
22:    if `(irdp(i) ≡ 1)` then
23:       `upSubComp(i,"LSG")`
24:    else
In section 2.3.2.3, the importance of naming all the transitions and places in a CPN model uniquely was discussed. Keeping that in mind, the names of the sub-components are kept short relating to the initials of the original components as well as the privacy hotspots they correspond to. A brief explanation behind the naming of the sub-components are as follows:

- The component Limit is assigned two sub-components: “LimE” (when it corresponds to erasure) and “LimG” (when it corresponds to anything but erasure).

- The component Request is assigned two sub-components: “RRE” (when it corresponds to recording or erasure) and “RG” (when it corresponds to anything but recording or erasure).

- The component Log is assigned two sub-components: “LogE” (when it corresponds to erasure) and “LogG” (when it corresponds to anything but erasure).

- The component LogStore is assigned two sub-components: “LSE” (when it corresponds to erasure) and “LSG” (when it corresponds to anything but erasure).
- The component *DataStore* is assigned two sub-componens: “DSE” (when it corresponds to *erasure*) and “DSG” (when it corresponds to anything but *erasure*).

- For each of the components *ExternalEntity*, *Process*, *Reason*, *PolicyStore* and *Clean*, there is no more than a single sub-component. They are respectively named as “EE”, “Pr”, “Rs”, “PS”, and “Cl”. We assign sub-components to these also in order to be consistent. As we will be going through the *ComponentTable* and reading the column for *SubCompColumn* for performing the transformation, we want all components to have a sub-component. We also use the name of the sub-components (because of their short naming) while naming the places and transition in the CPN model that we obtain after the transformation.

After applying the Algorithm 1 for each row in *ComponentTable*, we are ready to start transforming the given PA-DFD model to a CPN model. However, we need to set up the environment for the CPN model by defining appropriate color sets, functions and variable declarations. This is discussed in the next section.

### 3.2 Definition of Color Sets, Functions and Variable Declaration

When constructing a CPN model, the prerequisites are to have the necessary color sets, functions and variables declared. These are the building blocks to express a model effectively. To get a meaningful CPN model after transforming a PA-DFD model, necessary color sets, functions and variables need to be defined beforehand too. The color sets for the model are as follows:

```plaintext
colset REF = INT;
colset EE_NAME = STRING;
colset D_LABEL = STRING;
colset PR_HISTORY = list STRING;
colset DATA = product REF * EE_NAME * D_LABEL * PR_HISTORY;
colset CONS = list STRING;
colset POL = product REF * CONS;
```
The very first thing that needs to be decided for the CPN model is how to represent data as well as the policy as a token. Each token of a CPN model has a color set attached to it. Therefore, the color set DATA is defined to represent data. It is a four-tuple that includes a unique identifier REF (reference) for each data, the name of the data subject (external entity) EE_NAME, which the data belongs to, the label of the data D_LABEL, and PR_HISTORY, which is a list of strings representing the list of processes the data has been through. On the other hand, the color set POL is defined to represent a policy. It is a two-tuple consisting of REF, a unique identifier, for each policy and CONS, a list of user consents, which is represented as a list of strings.

A policy belongs to personal data. When the REF of a policy \( p \) and the REF of data \( d \) are equal, we say, \( p \) is a policy of \( d \).

Let us also define how to distinguish personal data from non-personal data in the CPN model obtained after the transformation. We say, when the REF of data is a non-negative integer, it is considered personal data. Therefore, the policies should also have a REF which is a non-negative integer. Alternatively, when a REF of data is \( \sim 1 \) (as mentioned in section 2.3.2, \( \sim \) is the unary minus operator in CPN ML), we say it is a non-personal data.

In addition to the color sets DATA and POL, some other color sets are also defined. The color set DATAxPOL is defined in order to support the places in the CPN model where data and policy are together. Similarly, REFxPOL is defined for places where the reference of data and policy are together. On the other hand, the color set LOG is defined to store relevant information from certain data and its corresponding policy in a place of the CPN model which corresponds to the LogStore of the PA-DFD model.

As mentioned in section 2.3.2, a Colored Petri Net is represented as a nine-tuple \( CPN = (P,T,A,\Sigma,C,G,E,I) \) (Def. 3). The set of color sets for the model is as follows:

\[
\Sigma = \{\text{REF, EE_NAME, D_LABEL, PR_HISTORY, DATA, CONS, POL},
\text{DATAxPOL, REFxPOL, LOG}\}
\]
Variables in a CPN model are as important as the color sets themselves. They carry the value of the color sets from one place to another. The following variables are defined for the CPN model obtained from the transformation:

\[
\text{var id:REF;}
\]
\[
\text{var d1, d2:DATA;}
\]
\[
\text{var p1, p2:POL;}
\]

The set of typed variables in the model is as follows:

\[
V = \{ \text{id:REF, d1:DATA, d2:DATA, p1:POL, p2:POL} \}
\]

In addition to color sets and variables, some functions are defined to carry out necessary tasks in the model. They are defined as follows:

\[
\text{fun logInfo(d:DATA, p:POL) = (#1d, #2d, #3d, #4d, #2p);}
\]
\[
\text{fun dataLab(d:DATA, label:STRING) = (#1d, #2d, label, #4d);}
\]
\[
\text{fun processData(d:DATA, PrName:STRING) = (#1d, #2d, #3d, PrName::(#4d));}
\]

Each of the aforementioned functions has different uses on data and policy in the model. The function \text{logInfo} is used when the relevant information regarding a DATA token and its corresponding POL token are stored in a place which corresponds to \text{LogStore}. The place corresponding \text{LogStore} in the CPN model has the color set \text{LOG} assigned to it. Therefore the function \text{logInfo} returns a value of color set \text{LOG}. On the other hand, the \text{dataLab} function takes a DATA and a STRING value as arguments and replaces the value of the D\_LABEL of the data with the STRING value. Finally, the function \text{processData} adds to the existing PR\_HISTORY of a DATA token, the name of the process that it goes through. It is done so to have the track of the usage of the data in question. Both \text{dataLab} and \text{processData} are used when data go through a transition in the CPN model which corresponds to a process in the PA-DFD model. Both functions return a value of color set \text{DATA}. In CPN ML, \#nd is used to extract the n\text{th} element of the tuple d.

As mentioned in section 2.3.2, priorities are used to control the enabling and firing of transitions. The priorities requiring for the transformed CPN model are as follows:
In addition to the standard priorities available in CPN Tools, we declare a new priority \texttt{P EXTRA \_HIGH}, which is higher than the already defined ones.

### 3.3 Transformations for Sub-components

This section explains the details of the transformation for each sub-components, which in turn ensures the transformation for all the PA-DFD components. We will make use of the various tables defined earlier that store relevant information of the PA-DFD model. Details regarding that were already covered in previous sections of this chapter.

The general idea behind the transformation for each component has one thing in common: each component of the PA-DFD model, when transformed into corresponding parts of a CPN model, should contain one or more transitions. As we will later see in section 3.4 when a flow of the PA-DFD model is transformed into the corresponding parts of a CPN model it contains a single place with an incoming directed arc and an outgoing directed arc. By doing so, it ensures that sub-components, after being transformed into corresponding parts of a CPN model, will be connected to each other in ways they are connected in the PA-DFD model.

In order to make the reader visualize the concept of the transformation, a small and general illustration of such is shown in Fig. 3.4 with an example. On the left-hand side, some PA-DFD parts are shown and on the right-hand side, their corresponding CPN transformation. Here, we see two sub-components \texttt{a} and \texttt{b} of a PA-DFD model. We assume the CPN transformation of \texttt{a} consists of a place \texttt{ap} and a transition \texttt{at} which are connected by a directed arc. On the other hand, we assume, the CPN transformation of \texttt{b} is slightly different and consists of two transitions, \texttt{bt1} and \texttt{bt2}, which are connected to a place \texttt{bp} by two directed arcs. Then, we see the CPN transformation for the flow labeled \texttt{d}, which consists of a place \texttt{dp} and is connected to one incoming and one outgoing arc. Finally, at the bottom left of the figure, when sub-components \texttt{a} and \texttt{b} are connected by the flow labeled \texttt{d}, on the right-hand side it is shown
how their corresponding CPN parts are connected to each other. Although this is just an example, it precisely illustrates the main idea of the transformation for each component.

![Diagram showing PA-DFD and CPN models](image)

**Figure 3.4:** Example of the general concept behind the transformation.

It is important to remember that the name of each transition and place in the transformed CPN model needs to be unique. Furthermore, we need to connect each transition correctly with the places where it is required. For these reasons, we define a table *TransTable* having three columns: *Transition*, which stores the transition; *IdSubTrans*, which stores the ID of the sub-component (component), whose CPN transformation the transition belongs to; *ConnectionList*, which stores the list of *Connection*. The column *ConnectionList* is required to identify with which flows’ CPN transformation the transition is connected to. If each row of the table is defined as a three-tuple, then it can be stated as follows:

\[
\text{TransTable} = (\text{IdSubTrans}, \text{Transition}, \text{ConnectionList})
\]

The algorithm starts by going through each row of the table *ComponentTable* (defined in section 3.1) and for the sub-component stored in that row, we give a suitable CPN transformation (with the help of other information of the same row as well as other earlier defined tables). Before applying the transformation, we state that the CPN model has an empty set of places, \( P \), an empty set of transitions, \( T \), and an empty set of arcs, \( A \). When the transformation takes place for each sub-component of each row of the table *ComponentTable*,
we add suitable elements to these sets. In the transformations, we will also make use of the color set function $C: P \rightarrow \Sigma$ to assign a color set to a place, the guard function $G: T \rightarrow EXP_{\nu}$ to assign a guard to a transition, and the arc expression function $E: A \rightarrow EXP_{\nu}$ to assign an expression to an arc.

In the transformation, a few new functions are used. For a sub-component associated with $i \in ID$, the following functions are listed:

- The functions $irf(i)$ and $orf(i)$ respectively returns the list of $Connection$ for input and output RegularFlows connected to the sub-component.
- The functions $idf(i)$ and $odf(i)$ respectively returns the list of $Connection$ for input and output DeletionFlows connected to the sub-component.
- The functions $ipf(i)$ and $opf(i)$ respectively returns the list of $Connection$ for input and output RegularFlows that carry personal data and are connected to the sub-component.
- The functions $inflow(i, ft)$ and $outflow(i, ft)$ respectively return the list of $Connection$ for input flows and list of $Connection$ for output flows that have the type $ft$.
- The function $pford(d, lp)$, given a $Connection$ $d$ of a personal data flow and a list of $Connection$ $lp$ of flows of type $RFlow_{\nu}$ (flows that carry policies), returns a single element from $lp$ whose policy corresponds to the personal data flow of $d$.
- We use the function $length(l)$, which given a list $l$, returns the length of the list.
- The function $fl((x, y))$, given a $Connection$ as an argument, returns $FlowTable.FlowLabel$ where $FlowTable.SourceID \equiv x$ and $FlowTable.DestID \equiv y$.
- The function $pl(x)$, given a label of a flow (which is a string), returns $DFClass.PurpOfFlow$ where $DFClass.FlowName \equiv x$.
- The function $selSubComp(i)$ returns $ComponentTable.SubCompColumn$ where $ComponentTable.IdColumn \equiv i$.
- The function $np(a, b, c)$ is used for generating a $place$ for the CPN model. Here, $a$ is a string and $b$ & $c$ both are integers. After applying this function on these arguments, the output is a $place$, which gets added to the set of places $P$ (if it is already not added before). The name of the place is the concatenation of all the arguments to the function.
For example, \( np("AP", 10,1) \) should return \( AP101 \) where \( AP101 \in P \). By doing so, it is ensured that the generated place’s name is unique.

- The function \( nt(c, y) \) is used for generating a transition for the CPN model. Here, \( x \) is a string and \( y \) is a list of \( Connection \). After applying this function on these arguments, the output is a transition, which gets added to the set of transitions \( T \) (if it is already not added before). The name of the transition is the concatenation of all the arguments to the function. For example, \( nt("AT", [(10,1), (20,2)]) \) should return \( AT101202 \) where \( AT101202 \in T \). By doing so, it is ensured that the generated transition’s name is unique.

- If \( l \) is a non-empty list, the function \( hd(l) \) returns the first element of the list. On the other hand, \( tl(l) \) returns the rest of the list after the first element. For example, \( hd([1,2,3]) \) returns 1, and \( tl([1,2,3]) \) returns [2,3].

- The function \( rm(l_1, l_2) \) removes all the elements of \( l_2 \) which are also members of the list \( l_1 \) and returns the list \( l_2 \) with the remaining elements. For example, \( rm([1,2], [1,3,4,2,1,4]) \) returns [3,4,4].

- The function \( insT(x, y, z) \) is used insert a row in the table \( TransTable \), where \( x \) represents the value to be added in column \( IdSubTrans \), \( y \) represents the value to be added in column \( Transition \) and \( z \) represents the value to be added in column \( ConnectionList \).

- The function \( conTo(i, c, cmp) \), given \( i \in ID \), \( c \in Connection \) and \( cmp \in Component \), returns the name of \( cmp \). Basically, it returns the name of \( cmp \) if it is connected by \( c \) to the component which has the \( ID i \). This can be computed with the help of \( FlowTable \) and \( ComponentTable \).

- The function \( conToTo(i, c, cmp_1, cmp_2) \), given \( i \in ID \), \( c \in Connection \) and \( cmp_1, cmp_2 \in Component \), returns the name of \( cmp_2 \). Basically, it returns the name of \( cmp_2 \) if it is connected to \( cmp_1 \) which is connected by \( c \) to another component which has the \( ID i \). This can be computed with the help of \( FlowTable \) and \( ComponentTable \).

- The function \( cpd(l) \) given \( l \in list of Connection \), returns the first occurrence of the \( Connection \) which carries personal data. This can be computed with the help of the tables \( FlowTable \) and \( DFClass \).

- The function \( eqdl(c, lc) \), given \( c \in Connection \) and \( lc \in list of Connection \), returns a list of \( Connection \) from \( lc \) whose flows have the same data label as the data label of the flow of \( c \).
• The function `conWith(lc, cmp)`, given `lc ∈ list of Connection` and `cmp ∈ Component` returns a list of `Connection` from `lc` which are connected with `cmp`.

• The function `selTrans((x, y), z)`, given `(x, y) ∈ Connection` and `z ∈ ID`, returns `TransTable.Transition` where `TransTable.IdSubTrans ≡ z` and `(x, y)` is an element of the list `TransTable.ConnectionList`.

• There is no function available in the formal definition of a Colored Petri Net to assign a `priority` to a transition. However, we need to assign certain priorities to different transitions in the CPN model obtained after the transformation to implement it correctly in CPN Tools. Therefore, we declare a function `Prio: T → Priority` that assigns a priority to a transition, where `Priority` is the set of all the defined priorities. The set with its elements are as follows:

```
Priority = {P_EXTRA_HIGH, P_HIGH, P_NORMAL, P_LOW}
```

We present the algorithms applied for transforming each sub-components of PA-DFDs to Colored Petri Nets from the sub-section 3.3.1 to the sub-section 3.3.9. Some of the algorithms may seem too complex. In such cases, we provide example figures. As we will be going through each row of `ComponentTable` and apply the appropriate transformation algorithm, let the value of the column `IdColumn` be `i` for that row.

### 3.3.1 Transformations for Sub-components of `ExternalEntity`

The PA-DFD component `ExternalEntity` has a single sub-component, “EE”. For its transformation, we have to capture several cases where it is connected with only personal data or only non-personal data or both at the same time. Furthermore, it can be the case where it is connected with only `collection` of data or `disclosure` of data or both at the same time. Keeping that in mind, the Algorithm 2 is presented capturing all the aforementioned cases.

```
Algorithm 2 Transformation for sub-component “EE”

1: if `selSubComp(i) ≡ "EE"` then
2:    l_1 = ipf(i)
3:    l_2 = opf(i)
4:    l_3 = inflow(i, RFlow_p)
5:    l_4 = outflow(i, RFlow_p)
6:    l_5 = irf(i)
```
7: \( l_6 = orf(i) \)
8: \( l_7 = l_1^{\oplus} l_3 \) \hspace{1em} /* "\oplus" is used to denote concatenation of lists */
9: \( l_8 = l_2^{\oplus} l_4 \)
10: \( l_9 = rm(l_7, l_5) \) /* list of Connection for incoming non-personal data flow */
11: \( l_{10} = rm(l_9, l_6) \) /* list of Connection for outgoing non-personal data flow */
12: \( p_1 = np("EEP", i, 1) \) /* The ‘P’ is added to denote place */
13: \( C(p_1) = DATAxPOL \)
14: \textbf{while} (length(l_1) > 0) \textbf{do}
15: \( h_1 = hd(l_1) \)
16: \( hp_1 = pford(h_1, l_3) \)
17: \( hhp_1 = [h_1]^{\oplus}[hp_1] \) /* "[\]" is used to denote list */
18: \( t_1 = nt("EET", hhp_1) \) /* The 'T' is added to denote transition */
19: \textit{inst}(i, t_1, hh_1p_1)
20: \{\( (t_1, p_1) \}) \cup A
21: \( E((t_1, p_1)) = (dl, p1) \)
22: \( G(t_2) = ((\#1d1)=(\#1p1)) \)
23: \( l_1 = tl(l_1) \)
24: \textbf{end while}
25: \textbf{while} (length(l_2) > 0) \textbf{do}
26: \( h_2 = hd(l_2) \)
27: \( hp_2 = pford(h_2, l_4) \)
28: \( hhp_2 = [h_2]^{\oplus}[hp_2] \)
29: \( t_2 = nt("EET", hhp_2) \)
30: \textit{inst}(i, t_2, hh_2p_2)
31: \{\( (p_1, t_2) \}) \cup A
32: \( E((p_1, t_2)) = (dl, p1) \)
33: \( G(t_2) = ((\#3d1)=fl(h_2)) \)
34: \( l_2 = tl(l_2) \)
35: \textbf{end while}
36: \textbf{while} (length(l_3) > 0) \textbf{do}
The algorithm starts with checking whether the sub-component for that row is “EE”. If it is, then it proceeds with the rest of the algorithm and transforms it into a CPN representation. Otherwise, it is some other sub-component and a different algorithm will be applied depending on which sub-component it is. This is done for all the other algorithms too.

In lines 2-11, necessary lists of Connection for all the different flows that are connected with the sub-component are stored in different variables separately. An ExternalEntity needs a place to hold the data and the policies together. In order to facilitate that, a place \( p_1 \) is defined in lines 12-13.

In lines 14-24, we work with all the incoming flows carrying personal data to the sub-component (which corresponds to a disclosure). The variable \( l_1 \) stores a list of all the
Connection of such flows. A Connection $h_1$ from that list is selected in each iteration of the while loop until the list is empty. We also identify the Connection $hp_1$ of the corresponding policy flow for the personal data flow that has the Connection $h_1$. A list $hhp_1$ is created containing $h_1$ and $hp_1$ as its only elements (line 17). A transition $t_1$ is created and is recorded along with the list of Connection $hhp_1$ and the ID of the sub-component in the table TransTable (lines 18-19). This transition will later be identified in order to connect with the CPN representations of the two flows having the Connection $h_1$ and $h_2$. We create an arc connecting $t_1$ to $p_1$ which is added to the set of arcs $A$ (line 19) and assign an arc expression to that arc (line 20). In line 21, we assign a guard to the transition $t_1$ which specifies in order for a DATA token and a POL token to go through that transition, they need to have the same reference.

Similarly, In lines 25-35, we work with all the outgoing flows carrying personal data from the sub-component (which corresponds to a collection). In line 33, the guard on the transition checks the label of the data $d_1$ in order to make sure that it is indeed the data that is allowed to go through the transition.

Thus far, in the algorithm, we have been dealing with flows that carry personal data. Flow carrying non-personal data can also be connected with the sub-component. Let us first take into consideration the incoming flows carrying non-personal data to the sub-component. The variable $l_9$ stores a list of all the Connection of such flows. Then, like before we create a transition $t_9$ and an arc connecting that to the place $p_1$. The arc expression (line 41) is quite different than the aforementioned two cases regarding personal data flows (lines 21 and 32). As we are using a place of color set DATAxPOL, we have to make sure the tokens that reside there also have the same color set. However, non-personal data do not have corresponding policy that they can make tuple with and form a token of color set DATAxPOL. We have to take care of that in the arc expression ($t_9, p_1$) by adding an empty policy with the data. This ensures that everything type-checks.

Similarly, in lines 44-52, we perform the transformation for any outgoing flows carrying non-personal data from the sub-component. It is quite similar to the transformation for outgoing flows that carry personal data from the sub-component, except it does not have to deal with corresponding policy flows because it does not have one.
In Fig. 3.5, we show an example of this transformation. On the left-hand side of the figure, we have an ExternalEntity (sub-component “EE”) named “User”, which is identified with 1 as an ID (written in blue font inside the component). This is connected with two incoming flows carrying personal data with the labels pd1 and pd2 which respectively have corresponding incoming policy flows with labels pp1 and pp2. Similarly, it is also connected with two outgoing flows carrying personal data with the labels pd3 and pd4 which respectively have corresponding outgoing policy flows with labels pp3 and pp4. It is also connected with one incoming and one outgoing flows carrying non-personal data respectively with the labels nd1 and nd2. The respective Connection for each flow is written alongside with its label (in blue font). We assume each Connection for the sake of a clear and complete example.

On the right-hand side of the figure, we can see how the ExternalEntity (sub-component “EE”) looks like when transformed into a CPN representation. Keep in mind, the place EEP11 is assigned the color set DATAxPOL. Due to the lack of appropriate space in the figure, the color set is not mentioned along with the place. We can also see the guards assigned to the transitions EET151, EET1415 and EET1213. We are also storing each transition in the TransTable with suitable Connection for each flow it is supposed to be connected with (after we transform all the flows to a CPN representation).

Notice, the flows are not transformed here. The flows (on the left-hand side of the figure) are shown here because the transformation of the component itself depends on how it is connected to different flows. Same is applicable for the transformation of all the other sub-components.
3.3.2 Transformations for Sub-components of Limit

The PA-DFD component Limit is assigned two sub-components: “LimG” and “LimE”. “LimE” is present in the model whenever erasure operation takes place. On the other hand, “LimG” is always present in the model whenever other operations take place.

Algorithm 3 Transformation for sub-component “LimG”

1: if selSubComp(i) ≡ "LimG" then
2:   l₁ = irf(i)
3:   l₂ = orf(i)
4:   h₂ = hd(l₂)
5:   l₃ = l₁^^l₂ /* "^^" is used to denote concatenation of lists */
6:   prn = conTo(i,h₂,Process)
7:   een = conToTo(i,h₂,Log,ExternalEntity)
8:   t₁ = nt("LimGT",l₃)
9:   inst(i,t₁,l₃)
10:  if prn ≠ null
11:     G(t₁) = (((#1d1)=(#1p1))
               andalso ((intersect (#2p1) [prn])<>[]))
12: else if een ≠ null
13:     G(t₁) = (((#1d1)=(#1p1))
               andalso ((intersect (#2p1) [een])<>[]))
14: else
15:     G(t₁) = (((#1d1)=(#1p1))
               andalso ((intersect (#2p1) pl(fl(cpd(l₁))))<>[]))
16: end if
17: else
18:     Apply Algorithm 4
19: end if

We present the Algorithm 3 for the transformation of the sub-component “LimG”. Lines 2-5 take care of storing different lists of Connection for different incoming and outgoing flows
connected to *Limit* ("LimG"). In line 6, we store the name of the *Process* that is connected to it in a variable. On the other hand, in line 7, we store the name of the *ExternalEntity* where the personal data is being disclosed. The name of each *Process* and *ExternalEntity* (where personal data are disclosed) serves as a single purpose (which are saved against certain personal data flows in the table *DFClass*). A policy token (color set POL) includes a list of user consents (CONS) where each consent should be a name of the *Process* or a name of the *ExternalEntity* or simply as “erase” (which is checked when erasure operation takes place).

In line 8, we create a transition $t_1$. This is the only component that is used to represent “LimG” in CPN. However, it is the guard assigned to it that differs from one case to another.

If the *Limit* is connected to a *Process*, which means it is a part of the usage operation, the lines 11-12 of the algorithm will execute. Here, we assign a guard to the transition $t_1$. It first checks whether the policy and the data corresponds to each other by evaluating their reference. Then, it checks whether the consents given by the user in the policy for the corresponding data include the name of the *Process* (i.e., whether it is allowed for the data to be processed by that particular *Process* or not).

Alternatively, if the *Limit* is a part of the disclosure operation, the lines 12-13 of the algorithm will execute. Similarly, $t_1$ is assigned a guard. It first checks whether the policy and the data corresponds to each other by evaluating their reference. Then, it checks whether the user consents given in the policy of the corresponding personal data include the name of the *ExternalEntity* where the data is being disclosed to (i.e., whether it is allowed for the data to be disclosed to that particular *ExternalEntity* or not).

If none of the aforementioned cases occur, that means the *Limit* is part of a collection, recording or retrieval operation and therefore, the lines 14-15 will execute. The transition $t_1$ is assigned a guard. It first checks whether the policy and the data corresponds to each other by evaluating their reference. Then, it checks whether the purpose of the flow of the data covers the user consents given in the policy for that corresponding data.

In Fig. 3.6, we present an example transformation for “LimG”. On the left-hand side we have the PA-DFD version and on the right-hand side the transformed CPN representation of it. In the guard of the transition a boolean variable checkPurpose is used, which should be replaced by $(\text{intersect } (#2p1) \ [prn])<>[1])$ (when it is part of the usage operation)
or (intersect (#2p1) [een])<>]) (when it is part of the disclosure operation) or (intersect (#2p1) pl(fl(cpd(l1)))<>]) (when it is part of the collection or recording or retrieval operation).

Figure 3.6: An example transformation for sub-component "LimG".

Algorithm 4 Transformation for sub-component “LimE”

1: if selSubComp(i) ≡ "LimE" then
2:    l₁ = irf(i)
3:    l₂ = orf(i)
4:    h₂ = hd(l₂)
5:    l₃ = l₁^l₂ /* "^^" is used to denote concatenation of lists */
6:    t₁ = nt("LimET",l₃)
7:    inst(i,t₁,l₃)
8:    G(t₁) = ((id=(#1p1))
    andalso ((intersect (#2p1) ["erase"])<>]))
9: else
10:    Apply Algorithm 5
11: end if
Algorithm 4 is presented in order to transform the sub-component “LimE”. It is similar in structure to Algorithm 3, except it is dealing with Limit when it is involved in the erasure operation. The transformation of “LimE” is represented in CPN by a single transition \( t_1 \) and a guard assigned to it. The guard checks whether the user consents given in the policy for the corresponding data include the permission for erasing that data. Here, for simplicity check whether the string “erase” is part of the consents. Due to its close resemblance with the transformation of “LimG” We do not give an example for “LimE”.

3.3.3 Transformations for Sub-components of Request

The PA-DFD component Limit is assigned two sub-components: “RG” and “RRE”. “RRE” is present in the model when recording or erasure operation takes place. On the other hand, “RG” is always present in the model when other operations take place.

Algorithm 5 Transformation for sub-component “RG”

```plaintext
1: if selSubComp(i) ≡ "RG" then
2:     l_1 = irf(i)
3:     l_2 = orf(i)
4:     l_3 = l_1^^l_2  /* "^^" is used to denote concatenation of lists */
5:     t_1 = nt("RGT", l_3)
6:     inst(i, t_1, l_3)
7:     Prio(t_1) = P_HIGH
8: else
9:     Apply Algorithm 6
10: end if
```

Algorithm 5 is presented in order to transform “RG” to CPN representation. It is represented as a transition \( t_1 \) which is created in line 5 of the algorithm. A priority \( P_{HIGH} \) is assigned to the transition as Request has a priority assigned to it (Fig. Figure 2.3).

Algorithm 6 Transformation for sub-component “RRE”

```plaintext
1: if selSubComp(i) ≡ "RRE" then
2:     l_1 = irf(i)
3:     l_2 = orf(i)
```

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\begin{align*}
4: & \quad l_3 = l_1 \cap l_2 \quad / \quad "\cap" \text{ is used to denote concatenation of lists} \\
5: & \quad t_1 = nt("RRET", l_3) \\
6: & \quad inst(i, t_1, l_3) \\
7: & \quad Prio(t_1) = P_{\text{HIGH}} \\
8: & \quad \textbf{else} \\
9: & \quad \text{Apply Algorithm 7} \\
10: & \quad \textbf{end if}
\end{align*}

In Algorithm 6, we present the transformation for the sub-component “RRE”. It is almost identical to Algorithm 5, except the naming of the transition is different.

### 3.3.4 Transformations for Sub-components of \textit{Log}

The PA-DFD component \textit{Log} is assigned two sub-components: “LogG” and “LogE”. “LogE” is present in the model when \textit{erasure} operation takes place. On the other hand, “LogG” is always present in the model when other operations take place.

\textbf{Algorithm 7} Transformation for sub-component “LogG”

\begin{algorithm}
\begin{algorithmic}[1]
\State \bf{if} \ \textit{selSubComp}(i) \equiv "LogG" \bf{then}
\State \quad \textbf{if} \\
\State \quad \quad \textit{l}_1 = \textit{irf}(i) \\
\State \quad \quad \textbf{then} \\
\State \quad \quad \quad \textbf{else} \\
\State \quad \quad \quad \quad \textbf{end if} \\
\State \quad \quad \textbf{else} \\
\State \quad \quad \quad \textbf{end if} \\
\State \quad \quad \textbf{end if}
\end{algorithmic}
\end{algorithm}

Algorithm 7 is presented in order to transform “LogG” to CPN representation. It is represented as a transition \( t_1 \) which is created in line 5 of the algorithm.

\textbf{Algorithm 8} Transformation for sub-component “LogE”

\begin{algorithm}
\begin{algorithmic}[1]
\State \bf{if} \ \textit{selSubComp}(i) \equiv "LogE" \bf{then}
\State \quad \textbf{if} \\
\State \quad \quad \textbf{else} \\
\State \quad \quad \quad \textbf{end if}
\end{algorithmic}
\end{algorithm}
Algorithm 8 is applied when the sub-component is “LogE” and it is similar to Algorithm 7, except the transition name is different.

### 3.3.5 Transformations for Sub-components of LogStore

The PA-DFD component LogStore is assigned two sub-components: “LSG” and “LSE”. The latter is not present in the model unless erasure operation happens and the former is always present in the model when other operations take place.

**Algorithm 9** Transformation for sub-component “LSG”

```plaintext
1: if selSubComp(i) ≡ "LSG" then
2:     \( t_1 = nt("LSGT", irf(i)) \)
3:     inst(i, \( t_1, irf(i) \))
4:     \( p_1 = np("LSGP", i, 1) \)
5:     \( C(p_1) = LOG \)
6:     \{ (t_1, p_1) \} \cup A 
7:     \( E((t_1, p_1)) = logInfo(d1, p1) \)
8: else
9:     Apply Algorithm 10
10: end if
```

Algorithm 9 is presented in order to transform the sub-component “LSG” to CPN representation. In line 2 and 4 we create a transition \( t_1 \) and a place \( p_1 \) respectively. The color set \( LOG \) is assigned to \( p_1 \). In line 6, an arc is created and added to the set of all arcs \( A \). The arc
expression for the arc is presented on line 7, where the function \( \text{logInfo} \) is used in order to store relevant information in the place \( p_1 \).

**Algorithm 10** Transformation for sub-component “LSE”

```plaintext
1: if selSubComp(i) \equiv "LSE" then
2: \quad t_1 = nt("LSET", irf(i))
3: \quad inst(i, t_1, irf(i))
4: \quad p_1 = np("LSEP", i, 1)
5: \quad C(p_1) = \text{REFxPOL}
6: \quad \{(t_1, p_1)\} \cup A
7: \quad E((t_1, p_1)) = (id, p_1)
8: \quad else
9: \quad Apply Algorithm 11
10: \quad end if
```

Similar to the sub-component “LSG”, we transform “LSE” to CPN transformation using the Algorithm 10. Algorithms 9 and 10 both are almost similar. However, Algorithm 10 has a place with a color set of \text{REFxPOL} and the arc expression assigned to the arc (at line 7) is different from Algorithm 9. As “LSE” is connected to an incoming flow of type RFlow\_rp, it has only access to the reference of the personal data and the corresponding policy. Therefore, it stores whatever information it gets from the incoming flow to the place \( p_1 \).

### 3.3.6 Transformations for Sub-components of *DataStore*

The component *DataStore* is assigned two sub-components: “DSG” and “DSE”. The former does not correspond to erasure operation, but the latter does. Due to their names, it gives better readability for places and transitions while performing verification and is easier to distinguish between one that deals with erasure operation and one that does not.

**Algorithm 11** Transformation for sub-component “DSG”

```plaintext
1: if selSubComp(i) \equiv "DSG" then
2: \quad l_1 = irf(i)
3: \quad l_2 = idf(i)
```
4: \[ l_3 = \text{orf}(i) \]
5: \[ m_1 = \text{length}(l_1) \]
6: \[ m_2 = \text{length}(l_1) \]
7: \[ n = \text{length}(l_3) \]
8: \[ x = 2 \]
9: \[ p_1 = \text{np}("DSGP",i,1) \]
10: \[ C(p_1) = \text{DATA} \]
11: \[ t_1 = \text{nt}("DSGT",l_2) \]
12: \[ \text{inst}(i,t_1,l_2) \]
13: \[ G(t_1) = (\text{id}=(#1d1)) \]
14: \[ \text{Prio}(t_1) = \text{P_EXTRA_HIGH} \]
15: \[ \{(p_1,t_1)\} \cup A \]
16: \[ E((p_1,t_1)) = d1 \]
17: \textbf{while } (m_1 > 0) \textbf{ do}
18: \[ h_{l_1} = \text{hd}(l_1) \]
19: \[ t_2 = \text{nt}("DSGT",[h_{l_1}]) \quad /* "[ ]" is used to denote list */ \]
20: \[ \text{inst}(i,t_2,[h_{l_1}]) \]
21: \[ \{(t_2,p_1)\} \cup A \]
22: \[ E((t_2,p_1)) = d1 \]
23: \[ l_1 = \text{tl}(l_1) \]
24: \[ m_1 = m_1 - 1 \]
25: \textbf{end while}
26: \textbf{while } (n > 0) \textbf{ do}
27: \[ l_4 = \text{irf}(i) \]
28: \[ h_{l_3} = \text{hd}(l_3) \]
29: \[ p_2 = \text{np}("DSGP",i,x) \]
30: \[ C(p_2) = \text{DATA} \]
31: \[ t_3 = \text{nt}("DSGT",[h_{l_3}]) \]
32: \[ \text{inst}(i,t_3,[h_{l_3}]) \]
33: \[ G(t_3) = ((#1d1)=(#1d2)) \]
For the sub-component “DSG”, the Algorithm 11 is applied. Lines 2-7 of the algorithm deals with storing necessary lists of Connection for each flow (that is connected to the sub-component) in variables. In line 9 and 10 we create the place $p_1$ and assign the color set DATA to it. This place will act as the storage for the DATA token.

The variable $l_2$ contains the list of Connection for the flow that is connecting the component Clean to this DataStore (sub-component “DSG”). Lines 11-16 take care of creating the necessary arc and transition ($t_4$) for the purpose of cleaning data tokens from the place $p_1$. A guard is assigned to $t_4$ in order to check whether the reference sent from the CPN representation of Clean matches with any of the data token stored in $p_1$. If so, the data token is sent from $p_1$ through $t_4$ for deletion. The priority of $t_4$ is set to P_EXTRA_HIGH because the cleaning of the data from a DataStore is of the highest of priorities (as soon as the retention time expires for certain data, that data is deleted). Although we are not capturing the notion of time in the
CPN transformation, we are still presenting a transformation for the component Clean. For that reason, we need to setup the environment for the CPN transformation of Clean to get connected with the CPN transformation of “DSG”.

In lines 17-25 we create the necessary amount of transitions and arcs depending on how many incoming flows are connected with the DataStore.

On the other hand, lines 26-49 deal with setting up the environment (by creating necessary transitions, places, and arcs) for all the outgoing flows that are connected to the DataStore. For each outgoing flow, we create a place \( p_2 \) with the color set \( \text{DATA} \) assigned to it and a transition \( t_3 \). Then we create three arcs \( (p_2, t_3) \), \( (p_1, t_3) \) and \( (t_3, p_1) \). The place \( p_1 \) acts as the storage of all the data until they are cleaned or erased from it. That is the reason two arcs \( (p_1, t_3) \) and \( (t_3, p_1) \) are created instead of creating only \( (p_1, t_3) \). This helps the place \( p_1 \) store the data although it forwards it. From lines 37-44, we access all the transitions that was created earlier in lines 17-25 for all the incoming flows in order to create arcs that connect each of them to the place \( p_2 \).

There is a specific reason for dealing with the outgoing flows connected to the DataStore in such a way (lines 37-44). It is done to avoid the transition \( t_3 \) being always enabled (because of the arcs \( (p_1, t_3) \) and \( (t_3, p_1) \)). If \( p_1 \) has a token, then \( t_3 \) is always enabled. If that happens, the
calculation of state-space does not end as the CPN model then is considered infinite. We avoid that in lines 37-44 by adding a place \( p_2 \). Therefore, when data tokens come, i.e., through the transitions represented here by \( t_2 \), they go to the place \( p_1 \) as well as the places represented here by \( p_2 \). Then, when the data are forwarded, they are forwarded from \( p_1 \) as well as from all the places represented here by \( p_2 \) through the transitions represented here by \( t_3 \) where the guards on them perform a check. It checks whether the data coming from \( p_1 \) and \( p_2 \) are equal. If so, the transitions get enabled and fire the data forward as well as sending them back to the place \( p_1 \) as it acts as the storage for data.

In Fig. 3.7, we show an example transformation for “DSG”. We assume the ID of the DataStore is 5. Also, Connection for each flow is assumed for the sake of the example. One thing to notice here is the use of double-headed arc between the place \( \text{DSGP51} \) and the transition \( \text{DSGT56} \). This arc represents two arcs (one going from the place to the transition and the other going from the transition to the place) with the same arc expression. CPN Tools supports the use of such double headed arcs, which increases readability of the model.

The transitions in Fig. 3.7 will be connected by the CPN representation of the PA-DFD flows. Transitions \( \text{DSGT35}, \text{DSGT45}, \text{DSGT25} \) and \( \text{DSGT56} \) will be respectively connected by the CPN representations of the PA-DFD flows labeled \( d1, d2, ref1 \) and \( d3 \). We are only presenting the transformation of the component here, not the flows. That is covered in section 3.4.

The places in Fig. 3.7 have some basic tasks they fulfil. The place \( \text{DSGP51} \) acts as the storage part of DataStore. On the other hand, the place \( \text{DSGP52} \) is created in order to only forward the data token. When data is deleted from DataStore, we need to make sure the data token is deleted from the place \( \text{DSGP51} \). After that, even if \( \text{DSGP52} \) contains the data token that was deleted, it won’t be able to forward the token making it useless.

Algorithm 12 Transformation for sub-component “DSE”

1: \( \textbf{if} \ \text{selSubComp}(i) \equiv \text{"DSE"} \ \textbf{then} \)
2: \( l_1 = irf(i) \)
3: \( l_2 = ConWith(idf(i), Clean) \)
4: \( l_3 = orf(i) \)
5: \( m_1 = length(l_1) \)
6: \( m_2 = length(l_1) \)
\begin{verbatim}
7:  n = length(l_3)
8:  x = 2
9:  p_1 = np("DSEP",i,1)
10:  C(p_1) = DATA
11:  t_1 = nt("DSET",l_2)
12:  inst(i,t_1,l_2)
13:  G(t_1) = (id=(#1d1))
14:  Prio(t_1) = P_EXTRA_HIGH
15:  {(p_1,t_1)} \cup A
16:  E((p_1,t_1)) = d1
17:  while (m_1 > 0) do
18:      hl_1 = hd(l_1)
19:      t_2 = nt("DSET",[hl_1])  /* "[]" is used to denote list */
20:      inst(i,t_2,[hl_1])
21:      {(t_2,p_1)} \cup A
22:      E((t_2,p_1)) = d1
23:      l_1 = tl(l_1)
24:      m_1 = m_1 - 1
25:  end while
26:  while (n > 0) do
27:      l_4 = irf(i)
28:      hl_3 = hd(l_3)
29:      p_2 = np("DSEP",i,x)
30:      C(p_2) = DATA
31:      t_3 = nt("DSET",[hl_3])
32:      inst(i,t_3,[hl_3])
33:      G(t_3) = (((#1d1)=(#1d2))
34:      {(t_3,p_1),(p_1,t_3),(p_2,t_3)} \cup A
35:      E(a_1) = d1  where, a_1 \in \{(t_3,p_1),(p_1,t_3)\}
36:      E((p_2,t_3)) = d2
\end{verbatim}
We apply the Algorithm 12 for transforming the sub-component “DSE”. The transformation is quite similar to the one presented for “DSG” in Algorithm 11, except we have to deal with the erasure of data (apart from Clean). We take care of this in lines 49-60. In line 49 we get a list
of all the Connection of incoming deletion flows (except for the one which is connected to Clean). Then, in lines 51-60 we perform the task of creating the necessary arcs and transitions \((t_5)\) for the purpose of erasing data from the place \(p_1\). A guard is assigned to the transitions represented here by \(t_5\) in order to check whether the reference sent for erasing a DATA token matches with any of the DATA token stored in the place \(p_1\). If so, the arc \((p_1, t_5)\) carries the DATA token through \(t_5\) and completes the task of erasure.

In Fig. 3.8, we present an example transformation for “DSE”. It is similar to the one presented in Fig. 3.7, except it has a new deletion flow with the label \(ref2\). We assume it is part of the erasure operation and therefore, its source is a Log. On the right-hand side of the figure, we can see the necessary changes that occurred due to this change in PA-DFD. Every place and transition’s name starts with the “DSE” prefix. We also have a new transition \(DSET15\) and an arc connecting the place \(DSEP51\) to it. Everything else remain same and act same as it did in Fig. 3.7.

3.3.7 Transformations for Sub-components of PolicyStore

The component PolicyStore is assigned one sub-component: “PS”. The Algorithm 13 is applied for its transformation.
Algorithm 13 Transformation for sub-component “PS”

1: \textbf{if} \; \text{selSubComp}(i) \equiv "PS" \; \textbf{then}
2: \hspace{1em} l_1 = \text{irf}(i)
3: \hspace{1em} l_2 = \text{ConWith}(\text{orf}(i), \text{Limit})
4: \hspace{1em} l_3 = \text{ConWith}(\text{orf}(i), \text{Clean})
5: \hspace{1em} l_4 = \text{ConWith}(\text{orf}(i), \text{Request})
6: \hspace{1em} m_1 = \text{length}(l_1)
7: \hspace{1em} m_2 = \text{length}(l_1)
8: \hspace{1em} n = \text{length}(l_4)
9: \hspace{1em} x = 3
10: \hspace{1em} p_1 = \text{np}("PSP", i, 1)
11: \hspace{1em} p_2 = \text{np}("PSP", i, 2)
12: \hspace{1em} C(p) = \text{POL} \; \text{where,} \; p \in \{p_1, p_2\}
13: \hspace{1em} \textbf{while} \; (m_1 > 0) \; \textbf{do}
14: \hspace{2em} h_l_1 = \text{hd}(l_1)
15: \hspace{2em} h_l_2 = \text{hd}(l_2)
16: \hspace{2em} p_3 = \text{np}("PSP", i, x)
17: \hspace{2em} C(p_3) = \text{POL}
18: \hspace{2em} t_1 = \text{nt}("PST", [h_l_1]) \quad /* "[]" \; \text{is used to denote list} */
19: \hspace{2em} \text{inst}(i, t_1, [h_l_1])
20: \hspace{2em} t_2 = \text{nt}("PST", [h_l_2])
21: \hspace{2em} \text{inst}(i, t_2, [h_l_2])
22: \hspace{2em} G(t_2) = \{ (\#1p1) = (\#1p2) \}
23: \hspace{2em} \{ (t_1, p_1), (t_1, p_3), (t_1, p), (t_2, p_1), (p_1, t_2), (p_3, t_2) \} \cup A
24: \hspace{2em} E(a_1) = p_1 \; \text{where,} \; a_1 \in \{ (t_1, p_1), (t_1, p_3), (t_1, p), (t_2, p_1), (p_1, t_2) \}
25: \hspace{2em} E((p_3, t_2)) = p_2
26: \hspace{2em} l_1 = \text{tl}(l_1)
27: \hspace{2em} l_2 = \text{tl}(l_2)
28: \hspace{2em} m_1 = m_1 - 1
29: \hspace{2em} x = x + 1
30:       end while
31:     \[ t_3 = nt(PST', l_3) \]
32:     \[ \text{inst}(i, t_2, l_3) \]
33:     \[ G(t_3) = ((\#1p1) = (\#1p2) \text{ and also } (\text{nil} \not< \text{nil})) \]
34:     \[ \text{Prio}(t_3) = \_\_\_\text{EXTRA\_HIGH} \]
35:     \[ \{(t_3, p_1), (p_1, t_3), (p_2, t_3)\} \cup A \]
36:     \[ E(a_2) = p_1 \text{ where } a_2 \in \{(t_3, p_1), (p_1, t_3)\} \]
37:     \[ E(p_2, t_3) = p_2 \]
38:     while \( (n > 0) \) do
39:       \[ l_5 = irf(i) \]
40:       \[ hl_4 = hd(l_4) \]
41:       \[ p_4 = np(PSP', i, x) \]
42:       \[ C(p_4) = \text{POL} \]
43:       \[ t_4 = nt(PST', [hl_4]) \]
44:       \[ \text{inst}(i, t_4, [hl_4]) \]
45:       \[ G(t_4) = ((\#1p1) = (\#1p2)) \]
46:       \[ \{(t_4, p_1), (p_1, t_4), (p_4, t_4)\} \cup A \]
47:       \[ E(a_2) = p_1 \text{ where } a_2 \in \{(t_4, p_1), (p_1, t_4)\} \]
48:       \[ E(p_4, t_4) = p_2 \]
49: while \( (m_2 > 0) \) do
50:       \[ hl_5 = hd(l_5) \]
51:       \[ t_5 = selTrans(hl_5, i) \]
52:       \[ \{(t_5, p_4)\} \cup A \]
53:       \[ E((t_5, p_4)) = p_1 \]
54:       \[ l_5 = tl(l_5) \]
55:       \[ m_2 = m_2 - 1 \]
56: end while
57: \[ l_4 = tl(l_4) \]
58: \[ n = n - 1 \]
59: \[ x = x + 1 \]
In lines 2-5 we store different lists of Connection for different incoming and outgoing flows in different variables. We create a place $p_1$ which acts as the storage for the policy tokens. All the other places in the algorithm are created in order to forward policy tokens for outgoing flows (just like the transformation Algorithms (11,12) for sub-components of DataStore).

In lines 13-30 we create necessary places, arcs, and transitions for incoming flows as well as outgoing flows (going to the component Limit) connected to the PolicyStore (sub-component “PS”).

In lines 31-37 we create the required transition ($t_3$) and arcs to setup the environment for connecting to the CPN representation of the component Clean. As we are not capturing the notion of time for PA-DFDs to CPN transformation and Clean only performs when retention time of data expires, we do not have the use of Clean. However, we will still give a CPN transformation for the component Clean (without the notion of time) and therefore we need to prepare for connecting to it. To make sure the CPN transformation of Clean is not used (i.e., no token passes through it) we assign a guard to the transition $t_3$ (in line 33). The guard will always evaluate to false, making it always disabled.

For the rest of the Algorithm 13 (lines 38-61), we create necessary transitions and places for connecting to the CPN representations for the outgoing flows connected to the PolicyStore (sub-component “PS”).

In Fig. 3.9 an example transformation for “PS” is presented. We assume the Connection for each flow and the ID of the component PolicyStore for the sake of the example. Any incoming flows to the component should come from Request. In this case, the flow labeled $p1$ is that one. We assume the outgoing flows labeled $p2$, $p3$ and $p4$ are respectively going to Clean, Request and Limit. See Fig. **Figure 2.3** for a better understanding of how PolicyStore is connected with different components. See Fig. **Figure 2.3** for a better understanding of how PolicyStore can get connected with other components in a PA-DFD model.
The transitions in Fig. 3.9 will be connected by the CPN representation of the PA-DFD flows. Transitions PST35, PST57, PST56 and PST58 will be respectively connected by the CPN representations of the PA-DFD flows labeled p1, p2, p3 and p4. We are only presenting the transformation of the component here, not the flows. That is covered in section 3.4.

The places in Fig 3.9 have some basic tasks they fulfil. The place PSP51 acts as the storage part of PolicyStore. On the other hand, the other places are created in order to only forward the policy token. When a policy is deleted from PolicyStore, we need to make sure the policy token is deleted from the place PSP51. After that, even if the other places contain the policy token that was deleted, they won’t be able to forward the token making it useless.

3.3.8 Transformations for Sub-components of Process and Reason

The PA-DFD component Process is assigned one sub-component: “Pr”. It can get connected to different types of incoming and outgoing flows. While dealing with personal data, it can get connected to one or more incoming and outgoing flows of type RFlowdp. On the other hand, when it deals with non-personal data, it can get connected to one or more incoming and outgoing flows of type RFlowd. Apart from these, when the erasure operation takes place, it
can get connected with outgoing flows of type RFlowₜ (that carries the reference of the personal data to be erased) and RFlowₚ (that carries the corresponding policy of that personal data). We assume that, in order to carry out the erasure operation, the Process needs to be provided with the reference of the personal data as well as the corresponding policy by an incoming flow of type RFlowᵣₚ. It was not strictly mentioned in [4] what type of incoming flow is connected to the Process for the erasure operation when the author of the thesis was working on this.

Algorithm 14 is applied for transforming the sub-component “Pr”.

Algorithm 14 Transformation for sub-component “Pr”

```plaintext
1: if selSubComp(i) ≡ "Pr" then
2:     l₁ = inflow(i, RFlowᵣᵈₚ)
3:     l₂ = outflow(i, RFlowᵣᵈₚ)
4:     l₃ = outflow(i, RFlowₚ)
5:     l₄ = outflow(i, RFlowₜ)
6:     l₅ = l₃^l₄
7:     l₆ = inflow(i, RFlowᵣᵣₚ)
8:     l₇ = inflow(i, RFlowᵣₜ)
9:     l₈ = outflow(i, RFlowᵣᵣₜ)
10:    if length(l₁) > 0 then
11:        t₁ = nt("PrT", l₁)
12:        inst(i, t₁, l₁)
13:    end if
14:    if length(l₂) > 0 then
15:        p₁ = np("PrP", i, 1)
16:        C(p₁) = DATAxPOL
17:        if length(l₃) > 0 then
18:            {(t₁, p₁)} ∪ A
19:            E((t₁, p₁)) = (d₁, p₁)
20:        end if
21:        t₂ = nt("PrT", l₂)
22:        inst(i, t₂, l₂)
```
\begin{verbatim}
23:   \{(p_1,t_2)\} \cup A
24:   E((p_1,t_2)) = (d1,p1)
25:   \textbf{end if}
26:   \textbf{if} length(l_6) > 0 \textbf{then}
27:      t_3 = nt("PrT",l_6)
28:      inst(i,t_3,l_6)
29:   \textbf{end if}
30:   \textbf{if} length(l_5) > 0 \textbf{then}
31:      p_2 = np("PrP",i,2)
32:      C(p_2) = REFxPOL
33:      \textbf{if} length(l_1) > 0 \textbf{then}
34:         \{(t_3,p_2)\} \cup A
35:         E((t_3,p_2)) = (id,p1)
36:      \textbf{end if}
37:      t_4 = nt("PrT",l_5)
38:      inst(i,t_4,l_5)
39:      \{(p_2,t_4)\} \cup A
40:      E((p_2,t_4)) = (id,p1)
41:   \textbf{end if}
42:   \textbf{if} length(l_7) > 0 \textbf{then}
43:      \textbf{while} (length(l_7) > 0) \textbf{do}
44:         hl_7 = hd(l_7)
45:         t_5 = nt("PrT",[hl_7])
46:         inst(i,t_5,[hl_7])
47:         l_7 = tl(l_7)
48:      \textbf{end while}
49:   \textbf{end if}
50:   \textbf{if} length(l_8) > 0 \textbf{then}
51:      p_3 = np("PrP",i,3)
52:      C(p_3) = DATA
53:      l_9 = inflow(i,RFlow)d
\end{verbatim}
while (length(l9) > 0) do
    hl9 = hd(l9)
    t7 = selTrans(hl9, i)
    {(t7, p3)} ∪ A
    E((t7, p3)) = d1
    E(a1) = empty where, a1 ∈ {(t7, p1), (t7, p2)}
    l9 = tl(l9)
end while

    t6 = nt("PrT", l8)
    inst(i, t6, l8)
    {(p3, t6)} ∪ A
    E((p3, t6)) = d1
end if

if length(l2) > 0 then
    E(a2) = empty where, a2 ∈ {(t1, p2), (t1, p3)}
end if

if length(l5) > 0 then
    E(a3) = empty where, a3 ∈ {(t3, p1), (t1, p3)}
end if
else
    Apply Algorithm 15
end if

We store necessary lists of Connection for all the different types of flows that are connected to Process in lines 2-9. Lines 10-13 is executed if there is an incoming flow of type RFlow_{dp} connected to Process. Similarly, for outgoing flows of type RFlow_{dp} connected to Process the lines 14-25 is executed. This transformation works when there is only one incoming flow of type RFlow_{dp} connected to the Process. However, there is no such restriction for the outgoing flows of type RFlow_{dp}. It is the case because when the author of this thesis was working on this transformation, it was not yet defined how to handle multiple incoming flows of such type.
connected to the Process. This along with few other limitations of this transformation is discussed in more details in Chap. 6 (discussion).

**Algorithm 15** Transformation for sub-component “Rs”

1: if selSubComp(i) \equiv "Rs" then  
2: \quad p_1 = np("RsP", i, 1)  
3: \quad C(p_1) = POL  
4: \quad t_1 = nt("RsT", irf(i))  
5: \quad inst(i, t_1, irf(i))  
6: \quad Prio(t_1) = P\_HIGH  
7: \quad \{(t_1, p_1)\} \cup A  
8: \quad E((t_1, p_1)) = p_1  
9: \quad t_2 = nt("RsT", orf(i))  
10: \quad inst(i, t_2, orf(i))  
11: \quad Prio(t_2) = P\_HIGH  
12: \quad \{(p_1, t_2)\} \cup A  
13: \quad E((p_1, t_2)) = p_1  
14: else  
15: \quad Apply Algorithm 16  
16: end if

Lines 26-29 in the algorithm are executed when there is an incoming flow of type RFlow\_rp connected to Process. Similarly, lines 30-41 are executed when outgoing flows of types RFlow\_r and RFlow\_rp are connected to Process. This happens, when the erasure operation takes place.

The rest of the Algorithm 14 deals with the incoming and outgoing flows that carry non-personal data and is connected to Process. Lines 42-49 are executed when there is an incoming flow carrying non-personal data. On the other hand, lines 50-65 are executed when there is an outgoing flow carrying non-personal data.
Lines 59, 67-72 are executed in order to connect some transitions to places with arcs having the expression *empty* (similar to *null*). This is done to make sure everything is connected and no transition or place is isolated.

The PA-DFD component *Reason* has one sub-component: “Rs”. Algorithm 15 is applied when the sub-component is “Rs”. As we are going through each row of *ComponentTable* and applying the appropriate transformation, let the value of the column *IdColumn* be *i* for that row. The transitions created (in line 6 and 11) are assigned the priority P_HIGH as *Reason* has a priority assigned to it (Fig. Figure 2.3) on the PA-DFD level too. As *Reason* deals with incoming and outgoing flows of type RFlow only, the place (*p₁*) created is assigned the color set POL.

3.3.9 Transformations for Sub-components of *Clean*

The PA-DFD component *Clean* has one sub-component: “Cl”. Algorithm 16 is presented in order to transform the sub-component “Cl” into CPN representation. It is represented as a transition *t₁* with the priority P_EXTRA_HIGH assigned to it.

```
Algorithm 16 Transformation for sub-component “Cl”

1: if selSubComp(i) ≡ "Cl" then
2:    l₁ = irf(i)
3:    l₂ = orf(i)
4:    l₃ = l₁^*l₂ /* "^^" is used to denote concatenation of lists */
5:    t₁ = nt("Cl", l₃)
6:    inst(i, t₁, l₃)
7:    Prio(t₁) = P_EXTRA_HIGH
8:    end if
```

3.4 Transformations for Flows

We follow a similar approach for transforming each flow of the parsed PA-DFD model. The table *FlowTable* stores necessary information for all the flows. Each row of the table represents and stores information for a distinct flow. Let us go through each row of the table and apply appropriate CPN transformation for it. By doing so, we will ensure all the flows of the PA-
DFD model are transformed into corresponding CPN representations which in turn will guarantee the completion of the transformation for the whole PA-DFD model to a CPN model.

Let us first define some helper functions that we will use along with some of the already defined ones from previous sections for the transformation of the flows. They are as follows:

- The function \( \text{selFlowType}(s, d) \), given \( s, d \in ID \), returns \( \text{FlowTable.TypeOfFlow} \) where \( \text{FlowTable.SourceID} \equiv s \) and \( \text{FlowTable.DestID} \equiv d. \)
- The function \( dl((x, y)) \), given a \( \text{Connection} \) as an argument, returns label of the data from \( \text{FlowTable.FlowLabel} \) where \( \text{FlowTable.SourceID} \equiv x \) and \( \text{FlowTable.DestID} \equiv y. \) For example, \( \text{FlowTable.FlowLabel} \) can store data and policy together as a label of the flow of type \( \text{RFlow}_{dp}. \) From that it extracts the data label and returns it.

As we are going through each row of the table \( \text{FlowTable} \), first we have to identify the type of the flow for that row. Let \( s \) and \( d \) be the value of \( \text{FlowTable.SourceID} \) and \( \text{FlowTable.DestID} \) for the row that we are currently in. With the help of the function \( \text{selFlowType}(s, d) \) we are able to identify the type of the flow. Each of the Algorithm 17, 18, 19, 20 and 21 starts with the identification of the type of the flow in line 1.

In Algorithm 17, the transformation for flows of type \( \text{RFlow}_{d} \) is presented. After identification of the flow, we identify the transitions to which the flow will be connected after it is transformed into a CPN representation. In the rest of the algorithm (lines 4-13), the flow is transformed into corresponding CPN representation. In lines 4-6, we define the place \( p_1 \) which has the color set \( \text{DATA} \) assigned to it (because the flow carries data) and the two arcs connected to the place. In line 7, it is checked whether the incoming arc for the place \( p_1 \) originates from a CPN representation of \( \text{Process} \) or a \( \text{DataStore} \). If it does, then the expression for that arc will be as stated in line 8. In this expression the label of the data \( d_1 \) is being changed to \( fl((s, d)) \), which is the label of the flow (the \( \text{Connection} \) of which is \( (s, d) \)). Otherwise, it will be as stated in line 10. The arc expression for the outgoing arc from the place \( p_1 \) always remains the same and is stated at line 12.

**Algorithm 17** Transformation for flows of type \( \text{RFlow}_{d} \)

\[
\begin{align*}
1: \quad & \text{if } \text{selFlowType}(s, d) \equiv \text{RFlow}_{d} \text{ then} \\
2: \quad & t_1 = \text{selTrans}((s, d), s) \\
3: \quad & t_2 = \text{selTrans}((s, d), d)
\end{align*}
\]
\begin{algorithm}
\begin{align*}
4: & \quad p_1 = np(\text{"DP"}, s, d) \quad / * \text{"DP" for ‘D’ata ‘P’lace. The place for tokens of color set DATA} \quad */ \\
5: & \quad C(p_1) = \text{DATA} \\
6: & \quad \{(t_1, p_1), (p_1, t_2)\} \cup A \\
7: & \quad \textbf{if} \ \text{selSubComp}(s) \equiv (\text{"Pr" or "DSG" or "DSE"}) \ \textbf{then} \\
8: & \quad \quad E((t_1, p_1)) = \text{dataLab}(d_1, fl((s, d))) \\
9: & \quad \quad \textbf{else} \\
10: & \quad \quad \quad E((t_1, p_1)) = d_1 \\
11: & \quad \quad \textbf{end if} \\
12: & \quad \quad E((p_1, t_2)) = d_1 \\
13: & \quad \quad \textbf{else} \\
14: & \quad \quad \quad \text{Apply Algorithm 18} \\
15: & \quad \quad \textbf{end if}
\end{align*}
\end{algorithm}

Algorithm 18 states the transformation for flows of type RFlow_p. It is almost similar to Algorithm 17. The only differences are, the color set assigned to the place \( p_1 \) is POL (because the flow carries policy) and both the incoming and outgoing arcs for the place have the same expression (line 7).

\textbf{Algorithm 18} Transformation for flows of type RFlow_p
\begin{algorithm}
\begin{align*}
1: & \quad \textbf{if} \ \text{selFlowType}(s, d) \equiv \text{RFlow}_p \ \textbf{then} \\
2: & \quad \quad t_1 = \text{setTrans}((s, d), s) \\
3: & \quad \quad t_2 = \text{setTrans}((s, d), d) \\
4: & \quad \quad p_1 = np(\text{"PP"}, s, d) \quad / * \text{“PP” for ‘P’olicy ‘P’lace. The place for tokens of color set POL} \quad */ \\
5: & \quad \quad C(p_1) = \text{POL} \\
6: & \quad \quad \{(t_1, p_1), (p_1, t_2)\} \cup A \\
7: & \quad \quad E(a) = p_1 \ \text{where,} \ a \in \{(t_1, p_1), (p_1, t_2)\} \\
8: & \quad \quad \textbf{else} \\
9: & \quad \quad \quad \text{Apply Algorithm 19} \\
10: & \quad \quad \textbf{end if}
\end{align*}
\end{algorithm}
Algorithm 19 is almost identical to Algorithm 17, except it is for the flows who have type RFlow\textsubscript{dp} (i.e., flows that carry personal data and policy together as a tuple). Understandably, the place \( p_1 \), in this case, needs to have the color set DATAxPOL assigned to it in order to accommodate the tokens of the same color set. The incoming arc to the place can have one of two different expressions. In line 7 it is checked whether the incoming arc to the place \( p_1 \) originates from the CPN representation of a Process. If that is the case, then the expression will be as it is stated in line 7 (where, after changing the label of the data \( d_1 \) to the label of the flow, the function processData is applied, which adds to the PR\_HISTORY of \( d_1 \) the name of the Process the arc originates from). Otherwise, the expression for the incoming arc to \( p_1 \) will be as stated in line 10. The outgoing arc from \( p_1 \) always has the same expression and is stated in line 12.

\begin{algorithm}
\begin{algorithmic}[1]
  \State \textbf{if} selFlowType\((s, d)\) \texttt{\equiv} RFlow\textsubscript{dp} \textbf{then}
  \State \quad \( t_1 = \text{selTrans}\((s, d), s\)\)
  \State \quad \( t_2 = \text{selTrans}\((s, d), d\)\)
  \State \quad \( p_1 = \text{np}\("DPP", s, d\) \quad /\ast \text{ "DPP" for 'D'ata and 'P'olicy 'P'lace. The place for tokens of color set DATAxPOL } \ast/\)
  \State \quad \( C(p_1) = \text{DATAxPOL}\)
  \State \quad \{\(t_1, p_1\), \((p_1, t_2)\)\} \cup A\)
  \State \quad \textbf{if} selSubComp\((s)\) \texttt{\equiv} "Pr" \textbf{then}
  \State \quad \quad \( E((t_1, p_1)) = (\text{processData}\left(\text{dataLab}\((d_1, dl((s, d)))\right),\)
  \State \quad \quad \quad \quad ConTo(d, (s, d), Process)), p1)\)
  \State \quad \quad \textbf{else}
  \State \quad \quad \quad \( E((t_1, p_1)) = (d_1, p1)\)
  \State \quad \quad \textbf{end if}
  \State \quad \textbf{end if}
  \State \\textbf{else}
  \State \quad \( E((p_1, t_2)) = (d_1, p1)\)
  \State \textbf{end if}
\end{algorithmic}
\caption{Transformation for flows of type RFlow\textsubscript{dp}}
\end{algorithm}
Algorithm 20 states the transformation for flows of type RFlow_r and DFlow_r. It is almost identical to Algorithm 18. Few differences include, the color set assigned to the place $p_1$ is REF (because the flow carries reference to certain data) and both the incoming and outgoing arcs for the place have the same expression, id (except when the incoming arc’s source is Clean).

**Algorithm 20** Transformation for flows of type RFlow_r and DFlow_r

1: if $selFlowType(s, d) \equiv (RFlow_r \text{ or } DFlow_r)$ then
2: $t_1 = selTrans((s, d), s)$
3: $t_2 = selTrans((s, d), d)$
4: $p_1 = np(\text{"RP"}, s, d)$ /* “RP” for ‘R’efERENCE ‘P’lace. The place for tokens of color set REF
5: $C(p_1) = \text{REF}$
6: $\{(t_1, p_1), (p_1, t_2)\} \cup A$
7: if $conWith([[(s, d)], \text{Clean}) \equiv [(s, d)]$ then
8: $E(t_1, p_1) = \#1p1$
9: else
10: $E(t_1, p_1) = \text{id}$
11: end if
12: $E((p_1, t_2)) = \text{id}$
13: else
14: Apply Algorithm 21
15: end if

In Algorithm 21 we present the transformation for flows of type RFlow_{rp}. Again, it is almost identical to Algorithm 18. The only differences are, the color set assigned to the place $p_1$ is REFxPOL (because the flow carries reference to certain data and a policy together as a tuple) and both the incoming and outgoing arcs for the place $p_1$ have the same expression (line 7).

**Algorithm 21** Transformation for flows of type RFlow_{rp}

1: if $selFlowType(s, d) \equiv \text{RFlow}_{rp}$ then
\begin{verbatim}
2: \quad t_1 = selTrans((s,d), s)
3: \quad t_2 = selTrans((s,d), d)
4: \quad p_1 = np("RPP", s, d) /* "RPP" for 'R'efrence 'P'olicy 'P'lace. The place
\quad for tokens of color set REFxPOL
5: \quad C(p_1) = REFxPOL
6: \quad \{(t_1,p_1),(p_1,t_2)\} \cup A
7: \quad E((t_1,p_1),(p_1,t_2)) = (id,p)
8: \quad \textbf{end if}
\end{verbatim}

After all the components and flows of the PA-DFD model are transformed, we get a Colored Petri Net represented by the nine-tuple \((P, T, A, \Sigma, V, C, G, E, I)\). However, we do not yet use the \emph{initialization function} \(I\) (to initialize places with tokens) in the transformation. As tokens will be used to represent information, we need to initialize them (in places of the CPN model) at the time of verification.
4 Applying the Transformation on a Case Study

In this chapter, we will apply the transformation presented in Chap. 3 on an example PA-DFD model. We divide this chapter into three sections. In section 4.1, we will present the DFD model and explain it. Then, in section 4.2, we will present the corresponding PA-DFD model and explain that also. Finally, in section 4.3, we will transform the PA-DFD model to the corresponding CPN model and explain the details of the transformation.

4.1 DFD Model for the Case Study

In this section, an example DFD model is presented. The model is provided in Fig. Figure 4.1. It can be considered a subset of a healthcare information system because such complete system can be much more extensive. We are presenting a part of such system where we encounter more personal data.

We capture all kinds of privacy hotspots in this model (Fig. Figure 4.1) which we presented earlier in Fig. Figure 2.3. This model has five Process components. They are named as “Provide
Symptoms”. “Provide Treatment”, “Use for Research”, “Use for Advertising” and “Manage”. It also has three ExternalEntity components. They are named as “Patient”, “Health Insurance Company” and “Admin”. We have one DataStore in the model named “Patient History”.

The model consists of different kinds of flows. We provide shorthands to all the labels of the flows in order to be able to refer to them with more ease. Also, in the obtained PA-DFD model from this DFD model, we will use these shorthands instead of the full labels due to the lack of space and will provide more readability. The flows in the model are as follows (with assigned shorthands):

"Credentials" = c1
"Symptoms" = s1, s2
"Treatment" = t1
"History" = h1, h2, h3, h4
"Reference to History" = ref1
"Admin ID" = ai

Let us now discuss the flow of data through the model. Patients give their credentials and provide symptoms. The symptoms are then used to provide necessary treatments back to the patients. On the other hand, the symptoms are also stored in the database of the system as histories of the patients, which can be further used for research purposes or for advertisement purposes. After such use, those can also be shared with health insurance companies. Apart from that, there is an admin who manages the system. In this part of the model, he can delete patients’ histories.

As mentioned earlier in this section, we consider the model as a subset of a complete system. We do not present here how the patient registered into the system or how they are logging into the system. There can be much more aspects to take into account when such systems are designed. However, we are mostly interested in the flow of personal data where we can make use of the concept of the PA-DFD and later transform it to a suitable CPN model.

4.2 PA-DFD Model for the Case Study

In this section, we present the PA-DFD model\(^1\) that is obtained from the DFD model (in Fig. Figure 4.1) after applying appropriate transformations stated in [4]. The obtained PA-DFD

\(^1\) https://www.dropbox.com/s/kqmz74b0w04xt11/HealthCareSystem_PA-DFD.pdf is the PA-DFD model.
model is too big to properly fit into this report as a single figure. However, we provide different parts of the model as separate individual figures.

Table 4.1: Personal data flow classification for the DFD model in Fig. Figure 4.1.

<table>
<thead>
<tr>
<th>FlowLabel</th>
<th>DataSub</th>
<th>PurpOfFlow</th>
<th>RetentionTime</th>
<th>PolList</th>
</tr>
</thead>
<tbody>
<tr>
<td>“credentials”</td>
<td>“patient”</td>
<td>purp1</td>
<td>0</td>
<td>[“cp1”]</td>
</tr>
<tr>
<td>“symptoms”</td>
<td>“patient”</td>
<td>purp2</td>
<td>0</td>
<td>[“sp1”, “sp2”]</td>
</tr>
<tr>
<td>“treatment”</td>
<td>“patient”</td>
<td>purp3</td>
<td>0</td>
<td>[“tp1”]</td>
</tr>
<tr>
<td>“history”</td>
<td>“patient”</td>
<td>Purp2</td>
<td>0</td>
<td>[“hp1”, “hp1”, “hp1”, “hp1”]</td>
</tr>
</tbody>
</table>

The Table 4.1 (DFClass) is the personal data flow classification. Before the transformation to PA-DFD, this table is provided by the DFD designer. With the help of this table, privacy hotspots in the DFD model can be identified and consequently it gets transformed into PA-DFD model.

![DFD and PA-DFD versions of the Process "Provide Symptoms".](image)

For the Column PurpOfFlow, we are using some shorthands so that the table can be more readable. The shorthands with their values are in details as follows (for simplicity, we keep all the string values as lowercase):

purp1 = ["provide symptoms"]
purp2 = ["provide treatment", "use for research", "use for advertising"]
"health insurance company"]

purp3 = ["patient"]

The column RetentionTime is given the value 0 for all of its rows because we are not considering the notion of time for the transformation. Here, 0 can be interpreted as infinite retention time.

The table has another column named PolList. We add this column and its values after the PA-DFD transformation is made. This column stores the list of labels of the policy flows in the PA-DFD model for corresponding personal data in the row. This column is particularly important at the time of the transformation of the PA-DFD model to CPN model.

Two different parts of PA-DFD model are presented as separate figures with their corresponding DFD parts. In Fig. 4.2, we present the Process “Provide Symptoms”. Here, on the PA-DFD level, the Limit and Request (written in short as “Lim” and “Req”) receive the data c1 and its corresponding policy cp1 respectively. The process “provide symptoms” takes as an input the tuple (c1, cp1). Then, the data s1 and s2 are produced and forwarded to the rest of the model with their corresponding policies sp1 and sp2 after necessary information regarding the data c1 is logged.

Similarly, In Fig. 4.3, we present the DataStore “Patient History” (with only the recording part on the PA-DFD level). Here, Limit and Request receive the data s1 and its corresponding policy sp1. Then, necessary information about them are logged before being stored in the
DataStore and PolicyStore. From DataStore data can later be forwarded as h1 and h2. On the other hand, from the PolicyStore policies corresponding to data can later be forwarded as hp1 and hp2. Moreover, flows labeled ref1 and ref2 (each carrying a reference for data) are connected to the DataStore for deleting data from it.

4.3 CPN Model for the Case Study

In this section, we present the CPN model obtained from the PA-DFD model. Moreover, we include all the different tables with relevant information stored in them after parsing the PA-DFD model.

As explained in section 3.1, the PA-DFD model is parsed and each component of the model is uniquely identified and required information regarding them are stored in different tables. We present the PA-DFD model1 where each component is uniquely identified. It will be easier for the reader to relate when explaining the information stored in different tables against each unique ID.

Table 4.2: ComponentTable for storing information about each uniquely identified components in the PA-DFD model.

<table>
<thead>
<tr>
<th>IdColumn</th>
<th>CompColumn</th>
<th>CompName</th>
<th>SubCompColumn</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>ExternalEntity</td>
<td>“Patient”</td>
<td>“EE”</td>
</tr>
<tr>
<td>2</td>
<td>Request</td>
<td>“RG”</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>Limit</td>
<td>“LimG”</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>Log</td>
<td>“LogG”</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>LogStore</td>
<td>“LSG”</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>Limit</td>
<td>“LimG”</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>Request</td>
<td>“RG”</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>Process</td>
<td>“provide symptoms”</td>
<td>“Pr”</td>
</tr>
<tr>
<td>9</td>
<td>Reason</td>
<td>“Rs”</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>Log</td>
<td>“LogG”</td>
<td></td>
</tr>
<tr>
<td>11</td>
<td>LogStore</td>
<td>“LSG”</td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>Log</td>
<td>“LogG”</td>
<td></td>
</tr>
<tr>
<td>13</td>
<td>LogStore</td>
<td>“LSG”</td>
<td></td>
</tr>
<tr>
<td>14</td>
<td>Limit</td>
<td>“LimG”</td>
<td></td>
</tr>
<tr>
<td>15</td>
<td>Request</td>
<td>“RRE”</td>
<td></td>
</tr>
<tr>
<td>16</td>
<td>Log</td>
<td>“LogG”</td>
<td></td>
</tr>
<tr>
<td>17</td>
<td>LogStore</td>
<td>“LSG”</td>
<td></td>
</tr>
<tr>
<td>18</td>
<td>DataStore</td>
<td>“Patient History”</td>
<td>“DSE”</td>
</tr>
</tbody>
</table>

---

1. https://www.dropbox.com/s/lohukwhzvw34lvk/HealthCareSystem_PA-DFD_with_ID.pdf is the PA-DFD model where each of its components are uniquely identified.
<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>19</td>
<td>Clean</td>
<td>“Cl”</td>
</tr>
<tr>
<td>20</td>
<td>PolicyStore</td>
<td>“PS”</td>
</tr>
<tr>
<td>21</td>
<td>Request</td>
<td>“RG”</td>
</tr>
<tr>
<td>22</td>
<td>Limit</td>
<td>“LimG”</td>
</tr>
<tr>
<td>23</td>
<td>Log</td>
<td>“LogG”</td>
</tr>
<tr>
<td>24</td>
<td>LogStore</td>
<td>“LSG”</td>
</tr>
<tr>
<td>25</td>
<td>Limit</td>
<td>“LimG”</td>
</tr>
<tr>
<td>26</td>
<td>Request</td>
<td>“RG”</td>
</tr>
<tr>
<td>27</td>
<td>Process</td>
<td>“use for advertising”</td>
</tr>
<tr>
<td>28</td>
<td>Reason</td>
<td>“Rs”</td>
</tr>
<tr>
<td>29</td>
<td>Log</td>
<td>“LogG”</td>
</tr>
<tr>
<td>30</td>
<td>LogStore</td>
<td>“LSG”</td>
</tr>
<tr>
<td>31</td>
<td>Request</td>
<td>“RG”</td>
</tr>
<tr>
<td>32</td>
<td>Limit</td>
<td>“LimG”</td>
</tr>
<tr>
<td>33</td>
<td>LogStore</td>
<td>“LSG”</td>
</tr>
<tr>
<td>34</td>
<td>Log</td>
<td>“LogG”</td>
</tr>
<tr>
<td>35</td>
<td>ExternalEntity</td>
<td>“health insurance company”</td>
</tr>
<tr>
<td>36</td>
<td>Limit</td>
<td>“LimG”</td>
</tr>
<tr>
<td>37</td>
<td>Request</td>
<td>“RG”</td>
</tr>
<tr>
<td>38</td>
<td>Log</td>
<td>“LogG”</td>
</tr>
<tr>
<td>39</td>
<td>LogStore</td>
<td>“LSG”</td>
</tr>
<tr>
<td>40</td>
<td>Limit</td>
<td>“LimG”</td>
</tr>
<tr>
<td>41</td>
<td>Request</td>
<td>“RG”</td>
</tr>
<tr>
<td>42</td>
<td>Process</td>
<td>“use for research”</td>
</tr>
<tr>
<td>43</td>
<td>Reason</td>
<td>“Rs”</td>
</tr>
<tr>
<td>44</td>
<td>Log</td>
<td>“LogG”</td>
</tr>
<tr>
<td>45</td>
<td>LogStore</td>
<td>“LSG”</td>
</tr>
<tr>
<td>46</td>
<td>Request</td>
<td>“RG”</td>
</tr>
<tr>
<td>47</td>
<td>Limit</td>
<td>“LimG”</td>
</tr>
<tr>
<td>48</td>
<td>Limit</td>
<td>“LimE”</td>
</tr>
<tr>
<td>49</td>
<td>Log</td>
<td>“LogE”</td>
</tr>
<tr>
<td>50</td>
<td>LogStore</td>
<td>“LSE”</td>
</tr>
<tr>
<td>51</td>
<td>Request</td>
<td>“RRE”</td>
</tr>
<tr>
<td>52</td>
<td>Process</td>
<td>“manage”</td>
</tr>
<tr>
<td>53</td>
<td>Request</td>
<td>“RG”</td>
</tr>
<tr>
<td>54</td>
<td>Reason</td>
<td>“Rs”</td>
</tr>
<tr>
<td>55</td>
<td>Limit</td>
<td>“LimG”</td>
</tr>
<tr>
<td>56</td>
<td>Process</td>
<td>“provide treatment”</td>
</tr>
<tr>
<td>57</td>
<td>Log</td>
<td>“LogG”</td>
</tr>
<tr>
<td>58</td>
<td>LogStore</td>
<td>“LSG”</td>
</tr>
<tr>
<td>59</td>
<td>Limit</td>
<td>“LimG”</td>
</tr>
</tbody>
</table>
After parsing the PA-DFD model, we store relevant information in the tables *ComponentTable* and *FlowTable*, which are respectively provided here as Tables 4.2 and 4.3. The empty rows under the column *CompName* in Table 4.2 are considered as empty strings or null. Under the column *FlowLabel* (Table 4.3), we have used the shorthands (introduced in section 4.1) for the label of some flows in order to be consistent with the PA-DFD model.

### Table 4.3: *FlowTable* for storing information about each flow in the PA-DFD model.

<table>
<thead>
<tr>
<th>SourceID</th>
<th>DestID</th>
<th>TypeOfFlow</th>
<th>FlowLabel</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>RFlow_p</td>
<td>“cp1”</td>
</tr>
<tr>
<td>1</td>
<td>3</td>
<td>RFlow_d</td>
<td>c1</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
<td>RFlow_p</td>
<td>“cp1”</td>
</tr>
<tr>
<td>2</td>
<td>7</td>
<td>RFlow_p</td>
<td>“cp1”</td>
</tr>
<tr>
<td>3</td>
<td>4</td>
<td>RFlow_dp</td>
<td>“credentials, cp1”</td>
</tr>
<tr>
<td>4</td>
<td>5</td>
<td>RFlow_dp</td>
<td>“credentials, cp1”</td>
</tr>
<tr>
<td>4</td>
<td>6</td>
<td>RFlow_d</td>
<td>c1</td>
</tr>
<tr>
<td>6</td>
<td>8</td>
<td>RFlow_dp</td>
<td>“credentials, cp1”</td>
</tr>
<tr>
<td>7</td>
<td>6</td>
<td>RFlow_p</td>
<td>“cp1”</td>
</tr>
<tr>
<td>7</td>
<td>9</td>
<td>RFlow_p</td>
<td>“cp1”</td>
</tr>
<tr>
<td>8</td>
<td>10</td>
<td>RFlow_dp</td>
<td>“symptoms, cp1”</td>
</tr>
<tr>
<td>8</td>
<td>12</td>
<td>RFlow_dp</td>
<td>“symptoms, cp1”</td>
</tr>
<tr>
<td>9</td>
<td>15</td>
<td>RFlow_p</td>
<td>“sp1”</td>
</tr>
<tr>
<td>9</td>
<td>53</td>
<td>RFlow_p</td>
<td>“sp2”</td>
</tr>
<tr>
<td>10</td>
<td>11</td>
<td>RFlow_dp</td>
<td>“symptoms, cp1”</td>
</tr>
<tr>
<td>10</td>
<td>14</td>
<td>RFlow_d</td>
<td>s1</td>
</tr>
<tr>
<td>12</td>
<td>13</td>
<td>RFlow_dp</td>
<td>“symptoms, cp1”</td>
</tr>
<tr>
<td>12</td>
<td>55</td>
<td>RFlow_d</td>
<td>s2</td>
</tr>
<tr>
<td>14</td>
<td>16</td>
<td>RFlow_dp</td>
<td>“symptoms, sp1”</td>
</tr>
<tr>
<td>15</td>
<td>20</td>
<td>RFlow_p</td>
<td>“sp1”</td>
</tr>
<tr>
<td>16</td>
<td>17</td>
<td>RFlow_dp</td>
<td>“symptoms, sp1”</td>
</tr>
<tr>
<td>16</td>
<td>18</td>
<td>RFlow_d</td>
<td>s1</td>
</tr>
<tr>
<td>18</td>
<td>22</td>
<td>RFlow_d</td>
<td>hl</td>
</tr>
<tr>
<td>18</td>
<td>36</td>
<td>RFlow_d</td>
<td>h2</td>
</tr>
<tr>
<td>19</td>
<td>18</td>
<td>DFlow_r</td>
<td>“ref2”</td>
</tr>
<tr>
<td>Row</td>
<td>Column</td>
<td>Breakpoint</td>
<td>Type</td>
</tr>
<tr>
<td>-----</td>
<td>--------</td>
<td>------------</td>
<td>------</td>
</tr>
<tr>
<td>20</td>
<td>21</td>
<td>RFlow_p</td>
<td>“hp1”</td>
</tr>
<tr>
<td>20</td>
<td>37</td>
<td>RFlow_p</td>
<td>“hp2”</td>
</tr>
<tr>
<td>21</td>
<td>22</td>
<td>RFlow_p</td>
<td>“hp1”</td>
</tr>
<tr>
<td>21</td>
<td>26</td>
<td>RFlow_p</td>
<td>“hp1”</td>
</tr>
<tr>
<td>22</td>
<td>23</td>
<td>RFlow_dp</td>
<td>“history, hp1”</td>
</tr>
<tr>
<td>23</td>
<td>24</td>
<td>RFlow_dp</td>
<td>“history, hp1”</td>
</tr>
<tr>
<td>23</td>
<td>25</td>
<td>RFlow_d</td>
<td>h1</td>
</tr>
<tr>
<td>25</td>
<td>27</td>
<td>RFlow_dp</td>
<td>“history, hp1”</td>
</tr>
<tr>
<td>26</td>
<td>25</td>
<td>RFlow_p</td>
<td>“hp1”</td>
</tr>
<tr>
<td>26</td>
<td>28</td>
<td>RFlow_p</td>
<td>“hp1”</td>
</tr>
<tr>
<td>27</td>
<td>29</td>
<td>RFlow_dp</td>
<td>“history, hp1”</td>
</tr>
<tr>
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<td>31</td>
<td>RFlow_p</td>
<td>“hp4”</td>
</tr>
<tr>
<td>29</td>
<td>30</td>
<td>RFlow_dp</td>
<td>“history, hp1”</td>
</tr>
<tr>
<td>29</td>
<td>32</td>
<td>RFlow_d</td>
<td>h4</td>
</tr>
<tr>
<td>31</td>
<td>32</td>
<td>RFlow_p</td>
<td>“hp4”</td>
</tr>
<tr>
<td>31</td>
<td>35</td>
<td>RFlow_p</td>
<td>“hp4”</td>
</tr>
<tr>
<td>32</td>
<td>34</td>
<td>RFlow_dp</td>
<td>“history, hp4”</td>
</tr>
<tr>
<td>34</td>
<td>33</td>
<td>RFlow_dp</td>
<td>“history, hp4”</td>
</tr>
<tr>
<td>34</td>
<td>35</td>
<td>RFlow_d</td>
<td>h4</td>
</tr>
<tr>
<td>36</td>
<td>38</td>
<td>RFlow_dp</td>
<td>“history, hp2”</td>
</tr>
<tr>
<td>37</td>
<td>36</td>
<td>RFlow_p</td>
<td>“hp2”</td>
</tr>
<tr>
<td>37</td>
<td>41</td>
<td>RFlow_p</td>
<td>“hp2”</td>
</tr>
<tr>
<td>38</td>
<td>39</td>
<td>RFlow_dp</td>
<td>“history, hp2”</td>
</tr>
<tr>
<td>38</td>
<td>40</td>
<td>RFlow_d</td>
<td>h2</td>
</tr>
<tr>
<td>40</td>
<td>42</td>
<td>RFlow_dp</td>
<td>“history, hp2”</td>
</tr>
<tr>
<td>41</td>
<td>40</td>
<td>RFlow_p</td>
<td>“hp2”</td>
</tr>
<tr>
<td>41</td>
<td>43</td>
<td>RFlow_p</td>
<td>“hp2”</td>
</tr>
<tr>
<td>42</td>
<td>44</td>
<td>RFlow_dp</td>
<td>“history, hp2”</td>
</tr>
<tr>
<td>43</td>
<td>46</td>
<td>RFlow_p</td>
<td>“hp3”</td>
</tr>
<tr>
<td>44</td>
<td>45</td>
<td>RFlow_dp</td>
<td>“history, hp2”</td>
</tr>
<tr>
<td>44</td>
<td>47</td>
<td>RFlow_d</td>
<td>h3</td>
</tr>
<tr>
<td>46</td>
<td>35</td>
<td>RFlow_p</td>
<td>“hp3”</td>
</tr>
<tr>
<td>46</td>
<td>47</td>
<td>RFlow_p</td>
<td>“hp3”</td>
</tr>
<tr>
<td>47</td>
<td>63</td>
<td>RFlow_dp</td>
<td>“history, hp3”</td>
</tr>
<tr>
<td>63</td>
<td>64</td>
<td>RFlow_dp</td>
<td>“history, hp3”</td>
</tr>
<tr>
<td>63</td>
<td>35</td>
<td>RFlow_d</td>
<td>h3</td>
</tr>
<tr>
<td>48</td>
<td>49</td>
<td>RFlow_dp</td>
<td>“reference to history, refp1”</td>
</tr>
<tr>
<td>49</td>
<td>18</td>
<td>DFlow_r</td>
<td>ref1</td>
</tr>
<tr>
<td>49</td>
<td>50</td>
<td>RFlow_dp</td>
<td>“reference to history, refp1”</td>
</tr>
<tr>
<td>51</td>
<td>48</td>
<td>RFlow_p</td>
<td>“refp1”</td>
</tr>
<tr>
<td>52</td>
<td>51</td>
<td>RFlow_p</td>
<td>“refp1”</td>
</tr>
<tr>
<td>52</td>
<td>48</td>
<td>RFlow_r</td>
<td>ref1</td>
</tr>
</tbody>
</table>
With the help of these two tables (Tables 4.2 and 4.3) and the transformation algorithms stated in sections 3.3 and 3.4, we obtain the CPN model\(^1\) from the PA-DFD model. During the application of the algorithms, we also stored information in the table *TransTable* regarding the transitions in the CPN model in order to ensure transformations for all PA-DFD components and flows get connected to each other accurately. We present that here as Table 4.4.

### Table 4.4: TransTable for storing information regarding transitions.

<table>
<thead>
<tr>
<th>IdSubTrans</th>
<th>Transition</th>
<th>ConnectionList</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>EET1213</td>
<td>[(1,2), (1,3)]</td>
</tr>
<tr>
<td>1</td>
<td>EET601611</td>
<td>[(60,1), (61,1)]</td>
</tr>
<tr>
<td>2</td>
<td>RGT122327</td>
<td>[(1,2), (2,3), (2,7)]</td>
</tr>
<tr>
<td>3</td>
<td>LimGT231334</td>
<td>[(2,3), (1,3), (3,4)]</td>
</tr>
<tr>
<td>4</td>
<td>LogGT344546</td>
<td>[(3,4), (4,5), (4,6)]</td>
</tr>
<tr>
<td>5</td>
<td>LSGT45</td>
<td>[(4,5)]</td>
</tr>
<tr>
<td>6</td>
<td>LimGT</td>
<td>[(4,6), (7,6), (6,8)]</td>
</tr>
<tr>
<td>7</td>
<td>RGT277679</td>
<td>[(2,7), (7,6), (7,9)]</td>
</tr>
<tr>
<td>8</td>
<td>PrT68</td>
<td>[(6,8)]</td>
</tr>
<tr>
<td>8</td>
<td>PrT810812</td>
<td>[(8,10), (8,12)]</td>
</tr>
<tr>
<td>9</td>
<td>RsT79</td>
<td>[(7,9)]</td>
</tr>
<tr>
<td>9</td>
<td>RsT915953</td>
<td>[(9,15), (9,53)]</td>
</tr>
<tr>
<td>10</td>
<td>LogGT81010111014</td>
<td>[(8,10), (10,11), (10,14)]</td>
</tr>
<tr>
<td>11</td>
<td>LSGT</td>
<td>[(10,11)]</td>
</tr>
<tr>
<td>12</td>
<td>LogGT81212131255</td>
<td>[(8,12), (12,13), (12,55)]</td>
</tr>
<tr>
<td>13</td>
<td>LSGT1213</td>
<td>[(12,13)]</td>
</tr>
<tr>
<td>14</td>
<td>LimGT101420141416</td>
<td>[(10,14), (20,14), (14,16)]</td>
</tr>
</tbody>
</table>

\(^1\)https://www.dropbox.com/s/zch60w8ibk0nqls/HealthCareSystem_Model.cpn is the CPN model obtained after the transformation of the PA-DFD model.
<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>15</td>
<td>RRET9151520</td>
<td>[(9,15), (15,20)]</td>
</tr>
<tr>
<td>16</td>
<td>LogGT</td>
<td>[(14,16), (16,17), (16,18)]</td>
</tr>
<tr>
<td>17</td>
<td>LSGT1617</td>
<td>[(16,17)]</td>
</tr>
<tr>
<td>18</td>
<td>DSET1618</td>
<td>[(16,18)]</td>
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<td>DSET1837</td>
<td>[(18,37)]</td>
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<td>18</td>
<td>DSET1918</td>
<td>[(19,18)]</td>
</tr>
<tr>
<td>18</td>
<td>DSET4918</td>
<td>[(49,18)]</td>
</tr>
<tr>
<td>19</td>
<td>CI20191918</td>
<td>[(20,19), (19,18)]</td>
</tr>
<tr>
<td>20</td>
<td>PST1520</td>
<td>[(15,20)]</td>
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<td>[(20,14)]</td>
</tr>
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<td>[(20,21)]</td>
</tr>
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<td>20</td>
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<td>20</td>
<td>PST2019</td>
<td>[(20,19)]</td>
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<td>RGT20212122126</td>
<td>[(20,21), (21,22), (21,26)]</td>
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<tr>
<td>22</td>
<td>LimGT182221222223</td>
<td>[(18,22), (21,22), (22,23)]</td>
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<tr>
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<td>[(22,23), (23,24), (23,25)]</td>
</tr>
<tr>
<td>24</td>
<td>LSGT2324</td>
<td>[(23,24)]</td>
</tr>
<tr>
<td>25</td>
<td>LimGT232526252527</td>
<td>[(23,25), (26,25), (25,27)]</td>
</tr>
<tr>
<td>26</td>
<td>RGT21262652628</td>
<td>[(21,26), (26,25), (26,28)]</td>
</tr>
<tr>
<td>27</td>
<td>PrT2527</td>
<td>[(25,27)]</td>
</tr>
<tr>
<td>27</td>
<td>PrT2729</td>
<td>[(27,29)]</td>
</tr>
<tr>
<td>28</td>
<td>RsT2628</td>
<td>[(26,28)]</td>
</tr>
<tr>
<td>28</td>
<td>RsT2831</td>
<td>[(28,31)]</td>
</tr>
<tr>
<td>29</td>
<td>LogGT272929302932</td>
<td>[(27,29), (29,30), (29,32)]</td>
</tr>
<tr>
<td>30</td>
<td>LSGT2930</td>
<td>[(29,30)]</td>
</tr>
<tr>
<td>31</td>
<td>RGT28313132323135</td>
<td>[(28,31), (31,32), (31,35)]</td>
</tr>
<tr>
<td>32</td>
<td>LimGT3132293223234</td>
<td>[(31,32), (29,32), (32,34)]</td>
</tr>
<tr>
<td>33</td>
<td>LSGT3433</td>
<td>[(34,33)]</td>
</tr>
<tr>
<td>34</td>
<td>LogGT3223434333435</td>
<td>[(32,34), (34,33), (34,35)]</td>
</tr>
<tr>
<td>35</td>
<td>EET34353135</td>
<td>[(34,35), (31,35)]</td>
</tr>
<tr>
<td>36</td>
<td>LimGT183637363638</td>
<td>[(18,36), (37,36), (36,38)]</td>
</tr>
<tr>
<td>37</td>
<td>RGT203737363741</td>
<td>[(20,37), (37,36), (37,41)]</td>
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<tr>
<td>38</td>
<td>LogGT363838393840</td>
<td>[(36,38), (38,39), (38,40)]</td>
</tr>
<tr>
<td>39</td>
<td>LSGT3839</td>
<td>[(38,39)]</td>
</tr>
<tr>
<td>40</td>
<td>LimGT384041404042</td>
<td>[(38,40), (41,40), (40,42)]</td>
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<tr>
<td>41</td>
<td>RGT374141404143</td>
<td>[(37,41), (41,40), (41,43)]</td>
</tr>
<tr>
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<td>PrT4042</td>
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<td>PrT4244</td>
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<td>43</td>
<td>RsT4143</td>
<td>[(41,43)]</td>
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<tr>
<td>43</td>
<td>RsT4346</td>
<td>[(43,46)]</td>
</tr>
<tr>
<td>44</td>
<td>LogGT</td>
<td>[(42,44), (44,45), (44,47)]</td>
</tr>
</tbody>
</table>
Due to the size of the CPN model, it is not feasible to include the full model in the report. However, we give parts of the model for some corresponding parts of the PA-DFD model. In Fig. 4.2 and 4.3, we presented something similar.

In Fig. 4.2, we presented a snippet of the Process “provide symptoms” from both the DFD and PA-DFD models. In Fig. 4.4, we show the snippet of the same Process from the CPN model. Here, the tuple of data (“credentials”) and policy is incoming to the process “provide symptoms” via the transition PrT68. Then, the data gets processed and changed into “symptoms” (in the meantime adding the current process name in the process history of the data) while it is outputted via two arcs (in the DFD and PA-DFD figures outputted s two flows) from the transition PrT810812. Afterward, they are forwarded to the CPN representations of Log and LogStore.
Figure 4.4: Snippet from the CPN model that corresponds to the Process “provide symptoms”.

Similarly, in Fig. 4.3, we show snippets for the DataStore “patient history”. In Fig. 4.5, we provide, for the same component, the part of the CPN model that represents it. Here, the place PSP201 is acting as the storage in PolicyStore. All the other places whose name start with the prefix “PSP” act as mediums for forwarding policy tokens from the storage. Policy tokens are forwarded via the transitions whose name starts with the prefix “PST”. On the other hand, the place DSEP181 acting as the storage in DataStore. All the other places whose name start with the prefix “DSEP” act as mediums for forwarding data tokens from the storage. Data tokens are forwarded via the transitions whose name starts with the prefix “DSET”.

It is important to mention that in the obtained CPN model after the transformation, we do not have any tokens in any places of the model, i.e., we are yet to use the initialization function $I: P \rightarrow EXPR_0$. We will do it at the time of performing verification. Some example tokens will be used in order to check some properties.
Figure 4.5: Snippet from the CPN model that corresponds to the **DataStore** "patient history".
5 Verification on the Obtained CPN Model from the Case Study

In this chapter, we discuss the verification done in the CPN model from section 4.3. Properties are checked in the model by means of CPN ML queries and using CPN Tools. Table 5.1 includes the privacy properties enforced at each hotspot (as mentioned in section 2.2) when a DFD model is transformed to a PA-DFD model [4]. Taking these as a base, we check properties in the CPN model obtained from the case study.

Table 5.1: Privacy properties applied to each hotspot [4].

<table>
<thead>
<tr>
<th>Collection</th>
<th>Purpose limitation. The collection of personal data is only allowed if the consent for that data given by the data subject (external entity) covers the purpose of this collection.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Accountability. The collection of personal data is allowed only if it the collection is logged.</td>
</tr>
<tr>
<td></td>
<td>Right to change. Request can be made by the data subject at any given time to change their current consent for what concerns the purpose of collection of their personal data.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Disclosure</th>
<th>Purpose limitation. The disclosure of personal data is only allowed if the consent for that data given by the data subject (external entity) covers the purpose of the disclosure.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Accountability. The disclosure of personal data is allowed only if it this disclosure is logged.</td>
</tr>
<tr>
<td></td>
<td>Policy propagation. Disclosure of personal data can only be made if the purpose mentioned in the current consent (and retention time if applicable) for collection, disclosure, usage, recording, retrieval, and erasure is propagated.</td>
</tr>
<tr>
<td></td>
<td>Right to change. Request can be made by the data subject at any given time to change their current consent for what concerns the purpose of disclosure of their personal data.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Usage</th>
<th>Purpose limitation. The usage of personal data is only allowed if the consent for that data given by the data subject (external entity) covers the purpose of the usage.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Accountability. The usage of personal data is allowed only if it this usage is logged.</td>
</tr>
<tr>
<td></td>
<td>Right to change. Request can be made by the data subject at any given time to change their current consent for what concerns the purpose of usage of their personal data.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Recording</th>
<th>Purpose limitation. The recording of personal data is only allowed if the consent for that data given by the data subject (external entity) covers the purpose of the recording.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Time retention. The recorded personal data can be in the system as long as the current retention time given by the data subject has not expired.</td>
</tr>
</tbody>
</table>
### Accountability
The recording of personal data is allowed only if it this recording is logged.

### Right to change
Request can be made by the data subject at any given time to change their current consent for what concerns the purpose of recording of their personal data.

### Right to erasure
Data subject can request to erase their personal data at any given time.

### Purpose limitation
The retrieval of personal data is only allowed if the consent for that data given by the data subject (external entity) covers the purpose of the retrieval.

### Accountability
The retrieval of personal data is allowed only if this retrieval is logged.

### Right to change
Request can be made by the data subject at any given time to change their current consent for what concerns the purpose of retrieval of their personal data.

### Purpose limitation
The erasure of personal data is only allowed if the consent for that data given by the data subject (external entity) covers the purpose of the erasure.

### Accountability
The erasure of personal data is allowed only if this erasure is logged.

### Right to change
Request can be made by the data subject at any given time to change their current consent for what concerns the purpose of erasure of their personal data.

We are performing verification on the CPN model from the case study (presented in section 4.3). As tokens will be used to represent information, they need to be initialized in the model (as previously mentioned in section 3.4). For verification purposes, we need tokens in the model. We will initialize the place EEP11 with different tokens of color set DATAxPOL, where the policy for each data will be different from one another. We will also initialize the place PrP522 with tokens for erasing certain data.

Due to size of the model, when initialized with multiple tokens the state space calculation is not fully completed. Instead CPN tools generates a partial model. We need the full calculation of state space to check properties for the model. We discuss more about this in detail in Chap. 6 (discussion).

However, when initializing the model with only one token, CPN Tools is able to create the complete state space. Therefore, we initialize the model with only one token (in the place EEP11), e.g., token \( t_1 \) and calculate the state space for it. Then, we check properties for \( t_1 \) in the calculated state space. After that, we remove \( t_1 \) from the place (EEP11) and initialize it with a different token \( t_2 \). We calculate the state space of the model again, this time for \( t_2 \). Then,
we check properties for $t_2$ in the calculated state space. We do this for five different tokens whose policies are different from each other’s (allowing them to travel different parts of the model). These tokens are listed as follows:

- $t_1 = 1`((1, "patient", "credentials", nil), (1, ["provide symptoms", "provide treatment", "patient"]))$
- $t_2 = 1`((2, "patient", "credentials", nil), (2, ["provide symptoms", "provide treatment", "patient", "use for research"]))$
- $t_3 = 1`((3, "patient", "credentials", nil), (3, ["provide symptoms", "provide treatment", "patient", "use for research", "erase"]))$
- $t_4 = 1`((4, "patient", "credentials", nil), (4, ["provide symptoms", "provide treatment", "patient", "use for advertising"]))$
- $t_5 = 1`((5, "patient", "credentials", nil), (5, ["provide symptoms", "provide treatment", "patient", "use for advertising", "health insurance company"]))$

Logging of events are performed regarding the data that gets collected, disclosed, used, recorded, retrieved or erased by means of LogStore. The obtained CPN model after the transformation of the PA-DFD model represents each LogStore as a transition and a place that are connected by an arc. Tokens in that place are relevant information of the data that are kept in order to indicate logging of events. Therefore, for the different tokens stated above, places that correspond to LogStore in the CPN model will store different information. By checking these information in these places, we can ensure accountability. Furthermore, as logging of events always takes place after the data had gone through Limit first, we can, therefore, ensure successful purpose limitation.

For the purpose of checking properties with state space queries, we additionally define a color set and two helper functions. They are provided as follows:

colset LOGs = list LOG;

fun LoggedOrNot(a1:LOGs, a2:REF)
  = if (a1=nil) then false
  else (if (($1(hd a1))=a2) then true
else (LoggedOrNot((tl(a1)),a2)));   

fun LoggedOk(a1:LOGs, a2:REF, a3:PR_HISTORY)  
=    if (a1=nil) then false 
    else (if ( (#1(hd a1))=a2) then (contains (#4(hd a1)) a3) 
    else (LoggedOk((tl(a1)),a2,a3))); 

The state space queries will make use of these aforementioned definitions in order to investigate the state space. The color set LOGs is defined in order to access the multiset of a color set (tokens in the places that has the color set LOG and correspond to LogStore). The function LoggedOrNot, given a list of LOG (synonymous to multiset of LOG), and a2:REF, goes through each element of the list and checks whether information regarding certain data with the reference a2 was logged or not. The function LoggedOK, given a list of LOG, a2:REF, and a3:PR_HISTORY, goes through each element of the list and checks whether information regarding certain data with the reference a2 includes the desired process history a3. If it does, that confirms the data in question was processed by the processes whose names are in a3.

The places in the CPN model that correspond to LogStore are as follows:

- LSGP51 corresponds to the LogStore with the ID 5 in the PA-DFD model. This performs logging when the collection operation takes place.
- LSGP621, LSGP331, and LSGP641 correspond to the LogStores with the ID 62, 33 and 64 respectively in the PA-DFD model. The first one performs logging when disclosure operation happens to the ExternalEntity named “patient” and the last two perform logging when disclosure operation happens to the ExternalEntity named “health insurance company”.
- LSGP131, LSGP111, LSGP581, LSGP301, and LSGP451 correspond to the LogStores with the ID 13, 11, 58, 30 and 45 respectively in the PA-DFD model. The first two perform logging when usage operation happens with the Process named “provide symptoms”. The last three perform logging when usage operation happens with the Processes named “provide treatment”, “use for advertising”, and “use for research” respectively.
• LSGP161 corresponds to the LogStore with the ID 16 in the PA-DFD model. It performs logging when recording operation happens with the DataStore named “patient history”.

• LSGP391 and LSGP241 correspond to the LogStores with the ID 39 and 24 in the PA-DFD model. Both of these perform logging when retrieval operation happens with the DataStore named “patient history”.

• LSGP501 corresponds to the LogStore with the ID 50 in the PA-DFD model. This performs logging when the erasure operation takes place.

We do not check properties regarding erasure for the token $t_1, t_2, t_4$ and $t_5$ because we also need to initialize a token in the place $PrP522$ in order to perform erasure operation (if the policy corresponding the data allows). However, we do check it for the token $t_3$ because when we initialize the model with this token in the place $EEP11$, we will also add a token (which we present later) to the place $PrP522$. For now, let us first initialize the model with the token $t_1$ only, which can be denoted formally as follows:

$$I(EEP11) = t_1$$

Figure 5.1: State space queries for the model when it is initialized with token $t_1$ only.

According to the policy provided in the token, its corresponding data can be collected from the ExternalEntity “patient”, processed by the Processes “provide symptoms” and “provide treatment”, and disclosed to the ExternalEntity “patient”. The policy does not cover necessary consents for its corresponding data to get recorded in the DataStore “patient history” or get
retrieved from it. After calculation of the state space for the model initialized with token $t_1$, we check properties with the state space queries provided in Fig. 5.1.

Query number 1 (in Fig. 5.1) corresponds to the **collection** of the data referenced 1. It states there exists a case where starting from the initial node we can reach a node in the state space where logging is performed in the place LSGP51 for the data. After evaluating the query it returns true, which is the desired result.

Query number 2 (in Fig. 5.1) corresponds to the **disclosure** of the data referenced 1. This query is divided into three parts. The first part states there exists a case where starting from the initial node we can reach a node in the state space where logging is performed in the place LSGP51 for the data where it has gone through the Processes “provide symptoms” and “provide treatment” earlier before being logged. The second part of the query states, it is never the case starting from the initial node in the state space we reach a node where logging is performed in LSGP331. Finally, the third part of the query states, it is never the case starting from the initial node in the state space we reach a node where logging is performed in LSGP641 (because according to the policy provided in the token $t_1$, the data should never be disclosed to the external entity “health insurance company”). Each of these three parts of the query needs to be separately true in order for the query to be true. After evaluating the query, it returns true, which is the desired outcome.

**Figure 5.2:** State space queries for the model when it is initialized with token $t_2$ only.
Query number 3 (in Fig. 5.1) corresponds to the **usage** of the data referenced 1. The query is divided into five parts where each of these parts needs to be true in order for the query to return true. The first part states there exists a case starting from the initial node we can reach a node in the state space where logging is performed in the place LSGP131 for the data where it has gone through the Process “provide symptoms” before being logged. The second part is similar to the first, except the logging happens in the place LSGP111. The third part states there exists a case starting from the initial node we can reach a node in the state space where logging is performed in the place LSGP581 for the data where it has gone through the Process “provide treatment” before being logged. The fourth part of the query states, it is never the case starting from the initial node in the state space we reach a node where logging is performed in LSGP301 (because according to the policy provided in the token \(t_1\), the data should never be used by the process “use for advertising”). Finally, the fifth part of the query states, it is never the case starting from the initial node in the state space we reach a node where logging is performed in LSGP451 (because according to the policy provided in the token \(t_1\), the data should never be used by the process “use for research”). The query returns true when evaluated, which is the desired outcome.

![Figure 5.3](image_url)

**Figure 5.3:** State space queries for the model when it is initialized with token \(t_4\) only.

Query number 4 (in Fig. 5.1) corresponds to the **recording** of the data referenced 1. The query states, it is never the case starting from the initial node in the state space we reach a node where logging is performed in LSGP161. The query returns true when evaluated, which is the desired result.
Query number 5 (in Fig. 5.1) corresponds to the **retrieval** of the data referenced 1. It has two parts. The first part states, it is never the case starting from the initial node in the state space we reach a node where logging is performed in LSGP391. The second part states, it is never the case starting from the initial node in the state space we reach a node where logging is performed in LSGP241. The query returns true when evaluated, which is the desired outcome.

We now initialize the model with the token $t_2$ in place EEP11 as follows:

$$I(\text{EEP11}) = t_2$$

According to the policy provided in the token, its corresponding data should be collected from the *ExternalEntity* “patient”, processed by the *Processes* “provide symptoms”, “provide treatment”, and “use for research”. It should also be disclosed to the *ExternalEntity* “patient”. In addition, it should be recorded to the *DataStore* “patient history” and retrieved from it. After calculation of the state space for the model initialized with token $t_2$, we check properties with the state space queries provided in Fig. 5.2. All of the 5 queries evaluate to true, which is the desired outcome. As these are similar to the ones presented in Fig. 5.1, the reader should be able to understand them.

![Figure 5.4: State space queries for the model when it is initialized with token $t_5$ only.](image)

After checking the properties for $t_2$, we now initialize the model with the token $t_4$ in place EEP11 as follows:

$$I(\text{EEP11}) = t_4$$
The only thing different in \( t_4 \) from \( t_2 \) is the policy for the corresponding data includes the consent of it to go through the Process “use for advertising” instead of “use for research”. Therefore, the properties checked for \( t_4 \) will be almost similar to the ones checked for \( t_2 \) with slight changes. The state space for the model with this token is calculated and properties are checked by means of the state space queries. They are presented in Fig. 5.3. All of them return true as expected.

Thus far, we have checked properties for the tokens \( t_1, t_2 \) and \( t_4 \). We now initialize the model with the token \( t_5 \) in place EEP11 as follows:

\[
I(EEP11) = t_5
\]

There is a difference between token \( t_4 \) and \( t_5 \). The latter includes an extra consent in the policy for the corresponding data: “health insurance company”, which allows the data to be disclosed to the ExternalEntity of that name. After initializing \( t_5 \), we calculate the state space of the model for it. Then we check the properties for it using the state space queries and they are presented in Fig. 5.4.

We have checked properties for all of the tokens we intend to, except \( t_3 \). The consent given in the policy for its corresponding data in \( t_3 \) includes a consent “erase”. Our aim is to check whether erasure of personal data from the DataStore happens or not. For that purpose, we need to initialize another token in the place PrP522, which will include the reference and policy for the data in \( t_3 \). We initialize the model with the token \( t_3 \) as follows:

\[
I(EEP11) = t_3
\]

![Figure 5.5: State space queries for the model when it is initialized with token \( t_3 \) along with another token for erasure.](image)

We mentioned earlier in the previous chapter (when explaining the DFD for the case study), the model is a subset of a larger model where it can be even more comprehensive and can have much more components and flows. We assume, the place PrP522 (Process “manage”) was
provided with the necessary reference and policies of data to perform erasure of such data by some other component or flow that is not part of the case study model. Keeping that in mind, we initialize the token as follows:

\[ \text{PrP522}(3, \text{"provide symptoms", "provide treatment", "use for research", "erase"}) ]

After the initialization of the two tokens, we calculate the state space of the model. Token \( t_2 \) and \( t_3 \) are similar, except \( t_3 \) has an extra consent “erase” included in the policy of its corresponding data. Therefore, the properties we checked for \( t_2 \) using the five queries (in Fig. 5.2) will remain true for \( t_3 \) also. We only have to make sure to give the second argument for \text{LoggedOrNot} \text{ and LoggedOk} \ the reference of the data for \( t_3 \) (i.e., 3). In Fig. 5.5, we only present the part where the property related to \text{erasure} \ was checked.

The place \text{DSEP181} \ corresponds to the \text{DataStore} “patient history”. On the other hand, the place \text{RP4918} corresponds to the flow that is connecting the \text{Log} (with ID 49) to the \text{DataStore}. This flow carries the reference to the data to be deleted from the \text{DataStore}. Therefore, it makes sense that before erasure occurs, both the places \text{DSEP181} and \text{RP4918} will be non-empty (ready for the erasure). After the erasure, both the places should become empty. That way, we know erasure of the data happened. From the initial node, we get a list of all reachable nodes in the state space where both \text{DSEP181} and \text{RP4918} are non-empty at the same time. We save this list of nodes in \text{e1}. Then, from the initial node, we get a list of all reachable nodes in the state space where both \text{DSEP181} and \text{RP4918} are empty at the same time. We save this list of nodes in \text{e2}. After that, with the help of the function \text{reachList} (which takes two lists on nodes as arguments) we search for a node \( m \) in \text{e1}, for which there exists a node \( n \) in \text{e2} such that \( n \) is reachable from \( m \). If we find such nodes, we can say that there exists a reachable node from the initial node of the state space where erasure of the data occurs. The query \text{reachList(e1,e2)} returns true, as expected (Fig. 5.5).

All the state space queries presented here are provided with the CPN model itself. However, it is important to follow the aforementioned instructions on how and when to initialize certain tokens in order to evaluate certain state space queries.
6 Discussion

The Petri nets semantics presented in this thesis captures most of the behaviors of PA-DFDs. There are also some other behaviors that it fails to capture. PA-DFDs deal with lot more diverse components and flows than DFDs. Therefore, an appropriate classification of those is made, which in turn is convenient for the CPN transformation. Using CPN and its modeling language, the core elements of PA-DFDs (data and policies) are implemented. Manipulating the connections between transitions and places allows to model different components of PA-DFDs and their behaviors. Apart from that, verification is performed using CPN Tools on the obtained CPN model from a given PA-DFD model. The verification done and presented in this thesis, are far from exhaustive. However, the resources, albeit inconsistent and somewhat outdated, are there to be exploited further to gain comprehensive knowledge to performing verification more elegantly.

One behavior of the PA-DFDs that the proposed Petri nets semantics is unable to capture, is the notion of time. In PA-DFDs, it is called retention time of personal data. Retention time is included along with the consents given in the policy of a corresponding data. When the retention time expires for certain personal data, it is deleted from the system by means of *Clean*. This can be modeled as a ticking clock or a countdown timer where it is kept in check whether or not the time has expired for the data. If this approach was taken to model it in CPN, then the transition representing *Clean* and the transitions representing the connections between *Clean* and other components should have higher priority than any other transitions in the model (as immediately the retention time expires, *Clean* should execute the removal of the data). Priority wise, this is how *Clean* is represented in the proposed Petri nets semantics. However, there is no notion of time used in it. Therefore, the use of *Clean* does not occur in this semantics. In timed CPN and CPN Tools, the tokens are timed and there is a global clock for the model. It is possible to add time to the token’s time via the arc expressions or when it goes through a transition. On the other hand, the global clock neither acts as a ticking clock, nor as a countdown timer. The time of the global clock passes the amount that is added to the token while the token goes through a transition. Moreover, time of a token can only be increased, but not decreased. Also, it is not possible to access time of a token directly in order to compare with another time or the global clock’s time. For such reasons, implementing the notion of time in PA-DFDs to CPN models becomes cumbersome.
Another notable behavior that the Petri nets semantics presented in this thesis is unable to capture, is when there are more than one incoming personal data flow to a Process. When a single incoming personal data goes through a Process, its corresponding policy is forwarded through Reason beforehand. However, when there are more than one incoming personal data $d_1$ (having policy $p_1$) and $d_2$ (having policy $p_2$) to a Process and an outgoing data $d_3$ from it, it is not defined for Reason how to produce a policy for $d_3$ from $p_1$ and $p_2$. One way to address this could be to take the most restrictive policy out of the two, but it is still to be decided what approach will suit best in this case. Taking this into account, the proposed CPN semantics of Process is kept simple capturing the case when it is given a single personal data as input, rather than multiple ones.

Other than the uncaptured behaviors from PA-DFDs, verification performed in the CPN model from the case study was far from exhaustive. Due to some notable shortcomings from CPN Tools, performing verification task was troublesome. As can be seen from section 4.3, the CPN model obtained from the case study is extremely large. Therefore, the state space for it becomes even larger and takes a huge amount of time to generate. The initial idea was to initialize the place EEP11 with 10 different tokens and the place PrP522 with few more at once. However, with that amount of tokens, the calculation of state space becomes understandably even larger. The calculation took a lot of time and eventually did not get finished as CPN Tools randomly crashed during the process. After several tries, the amount of tokens was lessened by half. Still, that had no effect on the calculation. Later, it was decided to initialize the place EEP11 with two tokens at once. Then, calculation of the state space was performed, which surprisingly enough, did not get finished after half an hour. It is worth mentioning, a university machine (desktop computer) was used in order to perform these tasks. The author’s personal computer (laptop) was worse in terms of performance. In order to make sure at least some verification can be performed, eventually it was decided to use one token at a time and calculate the state space separately for each of them. This approach also took some time for tokens that covered the model more. Following this approach, it was possible to check some properties by means of state space queries and defining some CPN ML functions. It would be convenient if there was an option to save the calculated state space. With such approach, even if the state space takes a lot of time to generate, we only have to generate it once and load it every time we perform verification on it. However, as there is no way to do such thing, with a limited time
frame for the thesis work, it is not feasible to work with such a huge state space. Moreover, when managing models of this size, the tool tend to crash unexpectedly.

6.1 Future Work

There are a number of potential ways future research can improve the Petri nets semantics for the PA-DFDs presented in this thesis along with the verification performed on it. We list some significant ones below:

- As mentioned earlier, the Petri nets semantics presented in the thesis does not capture the notion of time from PA-DFDs. Therefore, a next step would be to find an effective way to extend the semantics with time.

- The CPN transformation for the component Process presented in the thesis has some limitations. One reason for that is the definition for the component Reason is still a work in progress. After that is resolved, a more efficient transformation is needed for this component.

- In this thesis, we perform the transformation and implement the CPN model manually in CPN Tools. However, an automatic implementation is the ideal goal for implementing the model. Therefore, it is a natural candidate to be a future work.

- The size of a CPN model for its corresponding PA-DFD model is quite large. We cannot help but address the redundancy of components in a PA-DFD model. If there is a way to decrease the size of the PA-DFD model keeping the behavior as well as the relationship between components same, then the transformation to CPN will also result in a smaller model.

- Verification of the CPN model obtained from a corresponding PA-DFD model can be improved and done in a more comprehensive manner. As mentioned earlier, due to some shortcomings of CPN Tools as well as the time frame of the thesis work, it was harder to perform a comprehensive verification of the model in this thesis. Furthermore, a good amount of knowledge regarding verification techniques in CPN Tools ([21], [32], [33]) is also needed, which will take time considering the instructions available are quite inconsistent. Although we have progressed with CPN Tools for implementation as well as the verification of the model in this thesis, it is not absolutely mandatory to use the tool. The transformation provided for the PA-DFDs can also be implemented in some other better Petri nets tool provided that supports necessary
concepts used for the transformation. This area needs further exploring in future research too.

- Finally, due to the lack of formal semantics for PA-DFDs, we leave proving the correctness of the transformation as future work.
7 Conclusion

New information systems are built rapidly making collection of personal data an ever increasing factor. This raises the importance of privacy of personal data. Privacy by Design (PbD) is an effective way to address this. Keeping that in focus along with important privacy principles stated in GDPR, PA-DFDs are proposed to tackle privacy of personal data from the earliest of stages of information system design. However, it is not possible to perform formal verification on PA-DFDs because it lacks concrete semantics.

In this thesis, we explored the DFDs, PA-DFDs and different variants of Petri nets. The aim was to give a Petri nets semantics by defining appropriate transformations for PA-DFDs, demonstrate the effectiveness of the transformations on a case study, and to perform verification tasks on the Petri nets model obtained from the case study.

Firstly, we use Colored Petri Nets (CPN) as the suitable Petri nets variant to give a semantics for PA-DFDs. We do that by giving transformation algorithms (which uses formal definition of CPN) for different PA-DFD components and flows. In order to do that, we classify various PA-DFD components and flows beforehand. Then, we follow a modular approach for the transformations, where for a given PA-DFD, we first transform each component to suitable CPN representation. When we finish doing that for all the components of the model, we start transforming the flows to their suitable CPN representations. Upon finishing these transformations, we obtain a complete CPN model that corresponds to the PA-DFD provided prior to the transformation.

Secondly, we conduct a case study for a subset of a health care information system design where we apply the transformations defined earlier. We start with a DFD of the system. The DFD was designed in such a way, so that all the privacy hotspots are included. After applying appropriate transformations on the identified hotspots in the DFD, we get a PA-DFD. Finally, we obtained a CPN model by means of applying the transformations defined in this thesis on the PA-DFD. We implemented the CPN model manually in CPN Tools. The case study was helpful in pointing out some of the missing cases in the transformation algorithms (defined in this thesis), which led to fixing of them so that they can capture such cases.

Finally, we perform verification tasks on the obtained CPN model from the case study. CPN Tools was used for this purpose. With the use of CPN ML and state space queries, we were
able to check some privacy properties such as logging, erasure and purpose limitation for the model. The properties checked in this thesis is trivial, but with more time and comprehensive knowledge of the verification techniques in CPN Tools, it is possible to check more interesting privacy properties. Apart from that, other Petri nets tool can also be used for implementation and verification purpose.

The Petri nets semantics for PA-DFDs presented in this thesis are concrete, making it possible for verification tasks to be carried out. It is, however, beckons further research in future for improvement with the evolution of PA-DFDs.

Above all, the Petri nets semantics presented along with the rest of the supporting work done in this thesis establish a step forward when it comes to privacy of personal data in information systems.
References


