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MOTHER'S TIME ALLOCATION, CHILD CARE AND CHILD COGNITIVE DEVELOPMENT

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Abstract. This paper analyzes the effects of maternal employment and non-parental child care on child cognitive development, taking into account the mother's time allocation between leisure and child-care time. I estimate a behavioral model, in which maternal labor supply, non-parental child care and time allocation decisions are considered to be endogenous choices of the mother, and the child cognitive development depends on maternal and non-parental child care. The results show that the mother's child-care time is more productive than non-parental child care, at any age of the child. This implies that a reduction in a mother's child-care time, induced by a higher labor supply, may not be compensated for by the increase in non-parental child care use, and, hence, may lead to a negative effect on the child's cognitive ability. The estimation of a counterfactual model where a mother can only allocate her time between child care and work shows that neglecting the mother's time allocation choice between child care and leisure overestimates the productivity of a mother's time with the child.

JEL classification: D13, J13, J22, C15.

Keywords: mother employment, mother time allocation, non-parental child care, child development, structural estimation.

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1. Introduction

There has been a long-standing interest in the social sciences literature in learning about the production of child cognitive achievement. Psychologists and economists agree that one of the most valuable inputs for child development is the time the child spends with the mother (Cunha et al. 2006). The increase in the maternal employment rate and the associated rise in the use of non-parental forms of child care have raised concerns about the impact they might have on child development, in particular through the decline in maternal child-care time. In the United States, the participation of mothers in the labor market has increased from around 50 percent in the 1970s to more than 70 percent at the end of the 1990s (U.S. Census Bureau 2000), while, in the same period, the fraction of 3 to 5 year-old children enrolled in some forms of non-parental child care programs increased from 7.9 to 51.7 percent for mothers in the labor force (Bianchi 2000). However, recent data from the American Time Use Survey show that, while employed mothers work on average five hours per day, the time spent with their child is only half an hour lower than that of non-employed mothers (U.S. Census Bureau 2013). Moreover, employed mothers are found to spend a substantially lower amount of time in activities, such as socializing, doing sport or watching TV, usually defined as *leisure* (U.S. Census Bureau 2013). This suggests that there might not be a one-to-one corresponding relationship between time spent at work and child-care time, and that mothers not only decide about their labor supply and non-parental child care use, but also about how much of their time out of work should be spent with their child instead of engaging in leisure activities.

This paper analyzes the effects of maternal employment and non-parental child care on children's cognitive development, distinguishing between maternal care and care provided by market services, and taking into account the additional choice between leisure and time with the child.[1] I estimate a behavioral model, in which maternal labor supply and time allocation, as well as non-parental child care, are considered to be the endogenous choices of the mother. The child development process depends on the mother's child-care time and on the amount of time the child spends in non-parental child care. The estimation of such a model makes it possible to deal with the endogeneity and the simultaneity of all the mother's choices, and to identify the contributions of both maternal child-care time and non-parental child care for the cognitive development of the child.

There have been several studies assessing the effects of maternal employment or non-parental child care use on the subsequent cognitive development of children, but only Bernal (2008) evaluates the impact of the two simultaneous choices using a structural approach.[2] Bernal (2008) finds that one year of maternal employment and non-parental child care reduces the child's test scores by 1.8 percent, suggesting a substantial negative effect of both choices. However, the author does not consider the third choice the mother can make regarding her time allocation between time with the child and leisure, and

[1]Non-parental child care includes any type of child-care arrangement provided by people or institutions outside the family, such as child-care centers, babysitters or other types of informal arrangements.

[2]See Ermisch and Francesconi (2005) for a review of studies assessing the effects of maternal employment on children's development, and Bernal and Keane (2011) for a review of studies looking at the impact of non-parental child care services in the U.S.

assumes that a mother's time out of work is entirely spent by the mother with the child.[3] Indeed, employed mothers may allocate their time out of work in such a way as to give priority to the time spent with the child (Bianchi 2000; Hoffert and Sandberg 2001). Recent studies have exploited the information on the actual amount of time spent by the mother with the child, also used for this paper, to assess the effects of maternal time inputs on child development, although they do not consider the role played by non-parental child care. The model presented in this paper builds on Del Boca, Flinn, and Wiswall (2014), who model household choices and investments in child ability from childbirth up to adolescence. They find that the productivity of a mother's time investments declines over a child's age, and that a father's time becomes more productive as the child reaches adolescence. Differently from Del Boca et al. (2014), this paper does not model both parents' labor supply and time allocation decisions, focusing instead on mothers' behavior and on the additional choice of using non-parental forms of care; in other words, instead of considering fathers' time as a substitute for mothers' time with the child, the present study analyzes the role of non-parental child-care time as a substitute for maternal child care.

The contribution of this paper to the literature is threefold. First, I estimate a model incorporating three endogenous choices of mothers' time allocation and investments decisions on the child, namely maternal labor supply, maternal child-care time and non-parental child care use. The model imposes no restrictions on the relationship between a mother's labor supply and a mother's child-care time: it allows a direct estimation of the impact of maternal time on a child's development, accounting for the fact that the mother not only chooses how many hours to work and how much time to use non-parental child care, but also how much time to devote to the child instead of engaging in leisure activities. To this purpose, this paper exploits the actual measure of maternal time instead of using a proxy, hence allowing all the mother's choices to be treated as endogenous. Second, this paper represents the first attempt to estimate the elasticity of a child's ability with respect to both maternal time and non-parental child-care time. To the best of my knowledge, there are no studies that simultaneously evaluate the productivity of both mother and non-parental child care, taking into account the selection of mothers into work and child care use. To this end, the paper is also linking in a novel way data on mother's child-care time with information on non-parental child care use. Third, the paper sheds light on the inter-dependencies between the mother's labor supply decision and the productivity of inputs in the child cognitive development process. In fact, by allowing all the mother's decisions to be endogenous, the model shows how the productivity of a mother's time and of non-parental child care affects the decision to work.

[3]Similarly, Mroz, Liu, and Van der Klaauw (2010) estimate a behavioral model of household migration and maternal employment decisions, and find that part-time employment of the mother reduces the child's score by 3 percent of a standard deviation while the mother's full-time status reduces the score by 5 percent of a standard deviation. Ermish and Francesconi (2013) have instead evaluated the effects of maternal employment on a child's schooling, estimating the parameters of a conditional demand function for the child's education; they find that one year more of a mother's full time employment reduces the probability that the child reaches higher education by 11 percentage points. Both studies assume that the mother's time out of work is entirely spent with the child.

In the model presented in this paper, the mother's utility maximization problem is subject to the mother's time and budget constraints, as well as the child's cognitive ability production function. The mother cares about consumption, leisure and the child's cognitive ability, while child's ability depends on a mother's child-care time and the amount of time the child spends in non-parental child care. In each period, the mother decides her own labor supply and the investments in the child development process. The empirical specification of the model takes into account that mothers who work and use non-parental child care are systematically different from those who do not. The model allows mothers to allocate their time between labor, time with the child and leisure, depending on their preferences, their productivity in the labor market, and their productivity in the child development process.

The model is estimated using U.S. data from the Panel Study of Income Dynamics (PSID), linked to data from the Child Development Supplement (CDS) and the Time Diary (TD) Section. The CDS provides information on all child-care arrangements used from birth until kindergarten and on the arrangement currently used at the time of the survey. The Time Diary (TD) component provides unique data on the amount of time the child spends with the mother, while the main PSID surveys give detailed information on the mother's work history and household income during the child's life cycle. The parameters of the model are retrieved using a Method of Simulated Moments estimator, which minimizes the distance between several data statistics and their model counterparts.

The results show that the productivity of a mother's time with the child is larger than the one of non-parental child care, at any age of the child, though this difference fades out as the child grows up. This implies that an increase in a mother's labor supply induces a reduction in a child's ability through a decrease in a mother's child-care time, which may not be compensated for by the increase in non-parental child care use. Thus, some mothers may have higher gains from substituting their time to non-parental child care, because the productivity of the alternative form of care is much lower than theirs.

The estimated model is used to simulate the effects of policies aimed at increasing the household financial means, at limiting the mother's working time, and at enhancing the quality of non-parental child care. The results confirm that there is not a one-to-one corresponding relationship between a mother's time out of work and child-care time, and that, after a reduction in labor supply, the mother reallocates her time almost equally between child care and leisure. A policy that increases the quality of non-parental child care, by setting its productivity to the level of the mother's time productivity, determines the largest change in the mother's labor supply at the intensive margin: in this case, mothers find it profitable to dedicate more time to work, because the productivity of the alternative form of care is as much as theirs. The estimation of a counterfactual model where the mother can allocate her time only between child care and labor, thus neglecting the additional choice of leisure, gives an estimated productivity of a mother's time with the child which is larger than the one obtained in the baseline analysis. This implies that assuming a one-to-one corresponding relationship between time out of work and child care overestimates the productivity of a mother's time with the child.

The rest of the paper is organized as follows. Section 2 presents key stylized facts in non-parental child care use and maternal time allocation. Section 3 describes the model that is estimated, while Section 4 introduces the data. Section 5 discusses the empirical method used for the identification of parameters, while Section 6 presents the results and the fit of the model. Sections 7 and 8 report the results from, respectively, the policy simulations and the estimation of a counterfactual model. Finally, Section 9 concludes.

2. Background

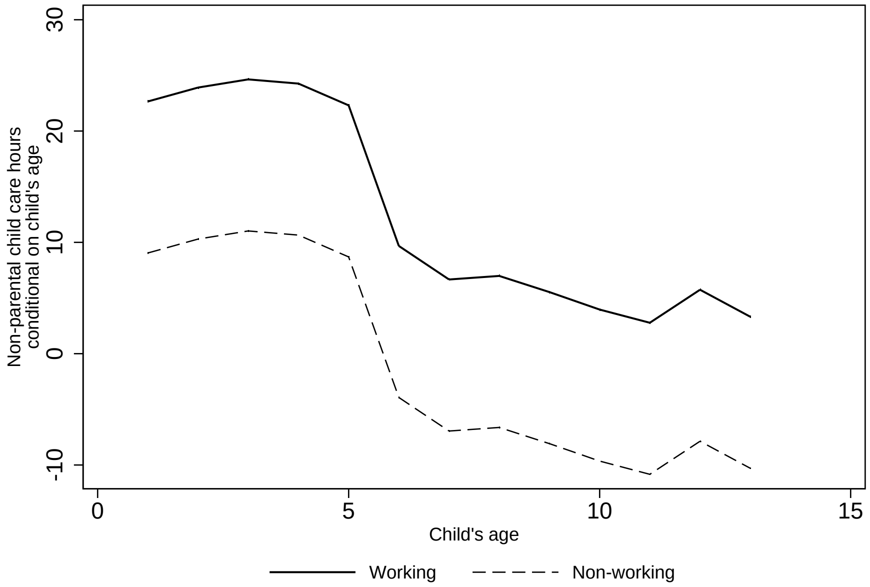
This paper assesses the effects of maternal employment and non-parental child care use on children's cognitive development, by also considering the additional choice the mother makes between child-care time and leisure. This addition makes it possible to account for the fact that the investments made by the mother on the child's ability through her contact time may differ according to how the mother allocates her time between leisure and child-care time.

Even though data on mothers' and children's time use have become available only very recently, there have been some studies suggesting that mothers differ not only in terms of participation decisions but also in terms of the allocation of leisure and child-care time. For instance, Leibowitz (1974, 1977) points out that more skilled mothers may also have a higher propensity to stay with their child, even if working. More recent studies on mothers' time use confirm this point, since they do not find significant differences across employment status in the amount of time mothers spend with their child (Bianchi 2000; Hoffert and Sandberg 2001). Two main reasons may explain the absence of significant differences in maternal time with the child between working and non-working mothers. First, during recent years, non-working mothers have also started using non-parental child care, so that children of non-working mothers may not be always available for maternal investments while attending external child care.[4] Second, working and non-working mothers may allocate their time out of work differently, so that the actual time that they spend with the child does not correspond to the time they spend out of work. According to data from the American Time Use Survey (ATUS) 2005-2009, the amount of time spent by mothers reading and playing with the child does not vary substantially across employment status: while employed mothers work, on average, five hours per day, they spend with their child only 30 minutes less than their non-employed counterpart; in contrast, employed mothers spend, on average, 2.5 hours per day in activities like socializing, doing sports or watching TV, against the 4 hours per day spent by non-employed mothers (U.S. Census Bureau 2013).

Descriptive evidence from the data on non-parental child care use and maternal time with the child, used for this paper, supports the existence of these patterns. Figure 1 reports the fitted values from a regression of non-parental child care hours on a child's age fixed effects and a binary variable indicating whether the mother works in each period, and shows that non-working mothers also use a positive amount of non-parental child care

[4]For instance, Bianchi (2000) shows that from the end of the 1960s to the end of the 1990s, the fraction of 3 to 5 year-old children enrolled in some forms of pre-primary educational programs increased from 4.8 to 44 percent for mothers not in the labor force.

Figure 1
Non-parental child-care time by mothers' employment status.



NOTE. The vertical axis represents the fitted values of the following regression:

$$i_{it} = \eta_0 + \sum_{t=1}^{T} \eta_{1t} t_{it} + \eta_2 d_{it} + \epsilon_{it}$$

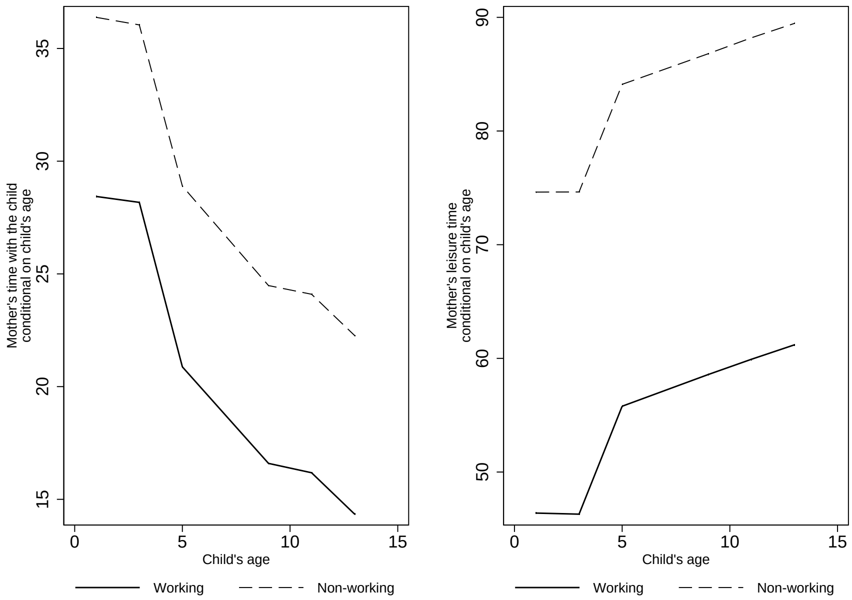
where i_{it} represents (weekly) hours of non-parental child care in each year t, t_{it} are child's age fixed effects (with $t = 1, \ldots, 12$), d_{it} is a dummy variable equal to 1 if the mother of child i works in period t. $\eta_2 = 13.61$ represents the difference in average child care use (conditional on child's age) between working and non-working mothers. Source: own elaboration from PSID-CDS data ($N = 2020$). See Section 4 and Appendix B for a description of the dataset.

for their child. This may happen if, for instance, they value the educational role of the service and choose it as an investment in their child's human capital. However, since the difference in average child-care time between working and non-working mothers is equal to 13 hours per week, the graph also confirms that non-parental child care is needed for its custodial purposes anytime the mother is working.

Figure 2 plots the fitted values from two regressions where the dependent variables are, respectively, maternal child-care time and leisure time, regressed on a child's age fixed effects and a binary variable indicating whether the mother works in each period.[5] The graph on the left (i.e., maternal child-care time) confirms that employed mothers allocate their time out of work in order to spend a positive amount of time with their child. The graph on the right shows that employed mothers spend a lower amount of time out of work in leisure, while the corresponding level for non-working mothers is considerably higher. Notice that while the difference in maternal time with the child between working and non-working mothers is equal to 8 hours per week, the difference in leisure is equal to 28 hours per week. These patterns suggest that working and non-working mothers allocate their time out of work differently and that the choice of devoting time to the child instead of

[5]The leisure time is computed as the difference between the total time endowment, assumed to be 112 hours per week, and the sum between working time and time with the child.

Figure 2
Maternal child-care time and leisure by mothers' employment status.



NOTE. The vertical axis in the graph on the left represents the fitted values of the following regression:

$$\tau_{it} = \eta_0 + \sum_{t=1}^{T} \eta_{1t} t_{it} + \eta_2 d_{it} + \epsilon_{it}$$

while the vertical axis in the graph on the right represents the fitted values of the following regression:

$$l_{it} = \beta_0 + \sum_{t=1}^{T} \beta_{1t} t_{it} + \beta_2 d_{it} + \varepsilon_{it}$$

τ_{it} represents (weekly) maternal time with the child and l_{it} represents leisure time, computed as $l = TT - \tau - h$, where $TT = 112$ is the total time endowment and h represents weekly hours of work. t_{it} are child's age fixed effects (with $t = 1, \ldots, 12$) and d_{it} is a dummy variable equal to 1 if the mother of child i works in period t. $\eta_2 = -7.92$ represents the difference in average maternal time (conditional on child's age) between working and non-working mothers. $\beta_2 = -28.28$ represents the difference in average leisure time (conditional on child's age) between working and non-working mothers. Source: own elaboration from PSID-CDS data ($N = 572$). For these graphs, the information on a mother's employment status available for the year 1996 has been used also for the year 1997, in order to match it with the mother's child-care time and leisure information. See Section 4 and Appendix B for a description of the dataset.

having leisure should be considered endogenous as those of labor supply and non-parental child care use.

3. The model

This section describes the model that is estimated: paragraph 3.1 presents the basic structure, while paragraph 3.2 derives the demand functions for all the choice variables; paragraph 3.3 describes the empirical specification.

3.1. ***Basic structure.*** The model follows a standard framework from Becker and Tomes (1986), where household preferences are described by a unitary utility function, with child's ability as an argument, and subject to a production function for child's ability and budget and time constraints. The functional form assumptions are based on the theoretical model developed in Del Boca et al. (2014).

The model is dynamic and evolves in discrete time. In each period, the mother decides her own labor supply and time allocation, as well as the amount of non-parental child care to use. The choice variables are then: (i) h_t, representing hours of work; (ii) τ_t, the time the mother spends with the child, and (iii) i_t, hours of non-parental child care. The timing is defined as follows: $t = 0$ represents the birth of the child and the mother makes all the decisions at each child's age t until the child reaches T years of age.[6]

The mother is the unique decision maker in the household concerning the investment decisions on the child. This assumption implies that the father's time allocation is exogenous with respect to the mother's choices and to the child development process.[7] The model applies to intact households, where both the mother and the father are present, and only households with one child are considered.[8]

The Mother's Utility Function

The mother's utility in each period is a function of her own leisure time (l_t), i.e, the time the mother spends alone without working, household consumption (c_t), including the father's and the child's consumption, and the child's cognitive ability (A_t). I assume a Cobb-Douglas form for preferences and I restrict the preferences parameters to be stable over time:

$$u(l_t, c_t, A_t) = \alpha_1 ln l_t + \alpha_2 ln c_t + \alpha_3 ln A_t \tag{1}$$

where $\sum_{j=1}^{3} \alpha_j = 1$ and $\alpha_j > 0$, $j = 1, 2, 3$.

The mother maximizes her utility subject to the budget and the time constraints. The budget constraint takes into account household consumption and expenditure for non-parental child care, as well as the total income available in the family (from both parents' labor supply and non-labor income); it is given by:

$$c_t = w_t h_t + I_t - p i_t \tag{2}$$

where w_t is a mother's hourly wage; I_t represents household earnings (including father's labor income and household non-labor income); i_t represents the number of hours that the mother uses non-parental child care and p is the hourly price of child care. The variable i_t includes any type of non-parental child-care arrangements, i.e., all contributions to child development due to the alternative care providers' time. Hence, the model assumes that

[6] $t = 1$ indicates the first 12 months of the child's life, $t = 2$ refers to the next 12 months of the child's life, and so on. $t = T = 13$ represents the terminal period of the model. It may be interpreted as the final period of middle childhood before the child enters adolescence.

[7] The model allows the father to affect child development through his labor income, which influences the mother's choices concerning work, non-parental child care and time with the child. In the sample of intact households that I use for the estimation of the model, all fathers work and the average working time does not change across a mother's employment status. Figure E.1 in Appendix E.1 suggests that a father's time with the child is slightly larger if the mother works; however, the sensitivity analyses performed in Appendix E.1 show that the predictions and the results of the model remain when I focus on the subsample of children with whom fathers are more involved, and when I change the definition of child-care time, in order to include both parents' time.

[8] In the data used to estimate the model, the sample of intact households represents 52.7 percent of the overall sample, while 36.2 percent of families in the sample have only one child. These figures roughly correspond to the official statistics: according to the US Census Bureau data (2012), 68.1 percent of children under 18 live with both parents and 47.8 percent of married women live with their spouse; finally 21.6 percent of married women have only one child. The sample selection may have implications for the estimated parameters; this issue will be further discussed in Section 4.

the mother's decision-making process does not change across types of non-parental child care services, and the same homogeneity is reflected in the price of the service. The model predicts a strictly positive price, implying that services with a potentially zero price in the market are characterized by a shadow price, representing, for instance, the limited availability of informal care or the value of the unpaid care provider's time in alternative activities (Blau and Currie 2006; Ribar 1992). In the empirical analysis, the hourly price of non-parental child care is estimated.[9] Finally, the mother does not make saving decisions, hence household income defined by I_t can be considered exogenous with respect to all the mother's choices.

The time constraint is defined as:

$$TT = l_t + h_t + \tau_t \tag{3}$$

where TT is the mother's total time endowment.[10] Notice that, in each period, the mother can choose to spend her leisure time alone (l_t) or to devote some time to the child (τ_t): hence, the model allows the mother to further choose between leisure and time with the child when she is not at work.

The Child's Cognitive Ability Production Function

The child's cognitive ability production function (hereafter CAPF) is defined using a value-added specification and taking a Cobb-Douglas form:

$$A_{t+1} = \delta_{0t} \times \tau_t^{\delta_{1t}} \times i_t^{\delta_{2t}} \times A_t^{\delta_{3t}} \tag{4}$$

where A_{t+1} is the outcome for a child at time $t+1$; τ_t and i_t are the inputs decided by the mother in each period t, where τ represents the amount of time the mother spends with the child, and i the amount of time in non-parental child care; A_t is the level of child ability at period t. Since current ability influences the child's future ability, equation (4) shows that inputs operate with a lag. Moreover, the structure of the CAPF implies that when deciding the inputs on child development, the mother knows the productivity of each of them and the level of a child's ability in the previous period.

Despite posing some limitations on the substitution pattern across inputs because of the assumed functional form, the model allows the parameters in (4) to vary across the age of the child in order to capture the fact that marginal productivity of inputs varies over the stages of child development (Cunha, Heckman, and Schennach 2010; Heckman 2007). Moreover, δ_{0t} represents a factor productivity shock also varying over time, which is intended to proxy for the time-varying role of missing inputs.[11]

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[9] The actual distribution of non-parental child care price in the data has a large mass toward zero, also for children actually using the service. This may be due to the use of informal child care, that can have a zero market price. Using the direct measure available in the data yields an infinite demand for external child care for those using an arrangement with a zero price, regardless of a mother's labor income and household earnings.

[10] $TT = 112$ hours per week. All choice variables are defined on a weekly basis.

[11] Notice that the introduction of the total factor productivity δ_{0t}, though making it possible to capture the effects of missing inputs on child's ability, does not change the mother's optimal investments decisions. Examples of missing inputs are father's time and schooling, for which Appendix E provides a sensitivity analysis. Another input that is missing in the CAPF specified in Equation (4) is the expenditure in goods bought for the child. This omission is mainly due to the data: information on the goods bought by the

A mother's work is not explicitly included in the CAPF, because it may not have a direct impact on child development *per se*. A mother's employment may indirectly affect child development through a change in her time allocation, together with the use of non-parental child care. This specification makes it possible to test whether, in each period, maternal time is more productive than non-parental child-care time. If this is the case, then, for any period and for an equal amount of maternal time and child-care time used, $\delta_{1t} \geq \delta_{2t}$.[12]

Maximization Problem

In each period, the mother maximizes her expected life time utility, optimally choosing her labor supply, the child care input and the number of hours to devote to the child. In this decision-making process the mother takes into account the level of ability reached by the child in each period, the wage offer that she receives from the market and the level of income in the household. The child's cognitive ability represents an endogenous state variable, while the wage offer the mother receives in each period and the household income are exogenous with respect to the maximization problem but differ for each mother in each period. The initial condition of the problem is given by the value of the state variables in the first period.[13]

The value function for the mother at period t is given by:

$$V_t(S_t) = max_{h_t,\tau_t,i_t} \;\; u(l_t, c_t, A_t) + \beta E_t V_{t+1}(S_{t+1}) \tag{5}$$

$$s.t. \;\; c_t = w_t h_t + I_t - p i_t$$

$$TT = l_t + h_t + \tau_t$$

$$lnA_{t+1} = ln\delta_{0t} + \delta_{1t} ln\tau_t + \delta_{2t} ln i_t + \delta_{3t} lnA_t$$

where the CAPF has been log-linearized for computational convenience, $\beta \in [0, 1]$ and $S_t = \{A_t, w_t, I_t\}$ represents the vector of state variables. The timing of the model implies that after childbirth and during the first 12 months of a child's life the mother observes the initial level of her child's ability and the level of income in the household and receives a wage offer; then she makes her decisions. Similarly, in the following periods, the mother chooses h_t, i_t and τ_t after having observed the corresponding level of A_t and I_t and after having received the wage offer from the labor market.

It should be noticed that the maximization problem of the mother can be solved analytically only if the wage offer is exogenous with respect to the mother's past and current labor supply choices. This implies that the offer the mother receives in period t is not affected by her working decisions in $(t-1)$ and that it does not reflect any depreciation in the mother's productivity as a result of her absence from the labor market after childbirth. The exogeneity of wage is necessary to estimate the model with continuous choice variables

parents for the child is available only in 2002, and missing for a large proportion of children, especially at young ages.

[12]For any period t, the marginal productivity of maternal time (MP_{τ_t}) is larger than the marginal productivity of non-parental child care (MP_{i_t}) if $\frac{\delta_{1t}}{\tau_t} - \frac{\delta_{2t}}{i_t} > 0$. Thus, for $\tau_t = i_t$, $MP_{\tau_t} \geq MP_{i_t}$ if $\delta_{1t} \geq \delta_{2t}$; viceversa, $MP_{\tau_t} \leq MP_{i_t}$ if $\delta_{1t} \leq \delta_{2t}$.

[13]The structure of the initial condition for child's ability and the draws from which the initial values of w_t and I_t are taken will be defined in paragraph 3.3.

and closed-form solutions, which is needed to allow for three choices and, in particular, to take into account the additional choice between leisure and time with the child. However, this assumption may have implications on the estimated parameters and on the fit of the model. In fact, since the definition of the wage process does not take into account the potentially negative effect on wages of leaving the labor market after childbirth, mothers may find it profitable to stay out of the labor market more than they would do in the case of endogenous wages. Thus, the model may overestimate the proportion of mothers not working and underestimate their labor supply, especially during the child's early years of life.

3.2. ***Terminal period value function and solutions of the model.*** The mother makes her decisions (that are relevant for the child development process described by equation (4)) in the first T years of the child's life. After period T, both the mother's optimization problem and the child's ability production function change: the mother may continue to optimally choose labor supply and consumption, but she will no longer consider maternal and non-parental child care choices. The terminal level of a child's cognitive ability is A_{T+1}, i.e., the level of ability reached in $T+1$, that will not be affected by the mother's subsequent decisions. This level of ability may be interpreted as the starting point for the child's future development during adolescence, from $T+1$ on.

The period $T+1$ maximization problem for an infinitely-lived household may be written as:

$$V_{T+1} = \tilde{V}_{T+1} + \sum_{\kappa=0}^{+\infty} \beta^{\kappa} \alpha_3 ln A_{T+1} \tag{6}$$

where

$$\tilde{V}_{T+1} = max_{h_{T+1}} \ \alpha_1 ln l_{T+1} + \alpha_2 ln c_{T+1} + \beta E_{T+1} \tilde{V}_{T+2}(l_{T+2}, c_{T+2})$$

and $\sum_{\kappa=0}^{+\infty} \beta^{\kappa} = \rho$ represents the value given by the mother to the child's ability in the last developmental period.[14] Equation (6) represents the terminal period value function and implies that the mother's maximization problem after period T becomes stationary and does not depend on the choices made by the mother in the previous periods.

The model is solved by backward induction and yields closed-form solutions for all the choice variables. The solution of the model involves the computation of the value function starting from the terminal period and the corresponding optimal solutions in each period. Following a two-stage process, I first derive the optimal solutions for non-parental child care (i_t) and maternal time (τ_t), conditional on h_t, and then compute the solutions for the mother's labor supply h_t. Analytical derivations of the results are in Appendix A.

[14]In the estimation, the discount factor is set at $\beta = 0.95$. In order to increase the flexibility of the model and to allow the discount factor of the mother to differ in the last period of investments with respect to the previous ones, the parameter ρ is estimated.

The demands for maternal child-care time and non-parental child care, conditional on the mother's labor supply, for any period t, are given by:

$$\tau_t^c = \frac{\beta\delta_{1t}D_{t+1}}{(\alpha_1 + \beta\delta_{1t}D_{t+1})}(TT - h_t) \tag{7}$$

$$i_t^c = \frac{\beta\delta_{2t}D_{t+1}}{p(\alpha_2 + \beta\delta_{2t}D_{t+1})}(w_t h_t + I_t) \tag{8}$$

where $D_{t+1} = \frac{\partial V_{t+1}}{\partial lnA_{t+1}}$ represents the marginal utility the mother gets from the child's future cognitive ability, in each period. The sequence of marginal utilities from period $T+1$ to period 1 is given by:

$$\begin{aligned} D_{T+1} &= \rho\alpha_3 \\ D_T &= \alpha_3 + \beta\delta_{3T}D_{T+1} \\ D_{T-1} &= \alpha_3 + \beta\delta_{4T-1}D_T \\ &\vdots \\ D_t &= \alpha_3 + \beta\delta_{3t}D_{t+1} \\ &\vdots \\ D_2 &= \alpha_3 + \beta\delta_{42}D_3 \\ D_1 &= \alpha_3 + \beta\delta_{41}D_2 \end{aligned} \tag{9}$$

Equation (8) shows that the demand for child care may be driven by necessity of custodial care, i.e., if the mother is working and needs someone to look after the child, or by valuing the educational role of the service. In fact, non-working mothers (for which $h_t = 0$) may demand of non-parental child care if they value the child's ability and they think child care may represent an input for the child's development, as long as the household income is strictly positive and sufficiently high.

An implication of the Cobb-Douglas specification used in the mother's utility function and in the child's cognitive ability production function is that all inputs should be strictly positive.[15] However, I do allow for the possibility of corner solutions for the mother's labor supply decisions.

The mother's latent labor supply, conditional on τ_t^c and i_t^c, is given by:

$$h_t^c = \frac{\alpha_2(TT - \tau_t^c)}{\alpha_1 + \alpha_2} - \frac{\alpha_1(I_t - pi_t^c)}{w_t(\alpha_1 + \alpha_2)} \tag{10}$$

Substituting (7) and (8) in equation (10), the latent labor supply becomes:

$$h_t^* = \frac{TT(\alpha_2 + \beta\delta_{2t}D_{t+1})}{(\alpha_1 + \beta\delta_{1t}D_{t+1} + \alpha_2 + \beta\delta_{2t}D_{t+1})} - \frac{I_t(\alpha_1 + \beta\delta_{1t}D_{t+1})}{w_t(\alpha_1 + \beta\delta_{1t}D_{t+1} + \alpha_2 + \beta\delta_{2t}D_{t+1})} \tag{11}$$

The actual labor supply in each period is determined according to the following rule:

$$h_t = \begin{cases} h_t^* & \text{if} \quad h_t^* > 0 \\ 0 & \text{if} \quad h_t^* \leq 0 \end{cases}$$

[15]This means that the model always predicts a positive amount of non-parental child care, regardless of a mother's working status or household income.

According to equation (11), the mother's latent labor supply is negative or zero only if household income is strictly positive and sufficiently high. Notice that a mother's decision to work also depends on the productivity of the alternative forms of care δ_{2t}, because if δ_{2t} increases, the mother may be more willing to substitute her time with the external child care provider's time. Substituting (11) into (7) and (8) yields the unconditional demands for time with the child and non-parental child care.

3.3. ***Empirical specification of the model.*** Unobserved and observed heterogeneity enters any stage of the decision-making process of the mother described in the previous paragraphs. Consider first the mother's utility function, where the parameters, because of the functional form assumptions, should be positive and sum to one. In order to respect these requirements without posing additional constraints to the estimation algorithm, I use a suitable transformation of the original parameters. More precisely, I allow the coefficients in the mother's utility function to vary according to unobserved taste shifters, representing the utility from consumption (γ_2) and the utility from child's ability (γ_3). Thus, the parameters representing the mother's preference for leisure (α_1), consumption (α_2) and child's ability (α_3) are defined as:

$$\alpha_1 = \frac{1}{1 + exp(\gamma_{2k}) + exp(\gamma_{3k})} \tag{12}$$

$$\alpha_2 = \frac{exp(\gamma_{2k})}{1 + exp(\gamma_{2k}) + exp(\gamma_{3k})} \tag{13}$$

$$\alpha_3 = \frac{exp(\gamma_{3k})}{1 + exp(\gamma_{2k}) + exp(\gamma_{3k})} \tag{14}$$

where γ_2 and γ_3 follow a discrete distribution with two points of support ($k = h, l$).

In each period, the mother receives a wage offer and decides whether to enter into the labor market by comparing the value of this offer with her reservation wage. The offer the mother receives is described by the following wage equation:

$$ln(w_t) = \mu_t + \epsilon_t \tag{15}$$

where

$$\epsilon_t \overset{\text{iid}}{\sim} N(0, \sigma^2_\epsilon)$$

is assumed to be uncorrelated over time and represents a transitory shock on wage. The term μ_t is the mean of the log wage draws of the mother at time t and it is defined as follows:

$$\mu_t = \mu_{mk} + \mu_1 Edu + \mu_2 Age_t + \mu_3 Race + \mu_4 Cohort + \mu_5 MacroArea + \mu_6 Cohort \times MacroArea \tag{16}$$

where Edu represents a mother's years of education; $Race$ is a dummy variable equal to one if the mother is white; $Cohort$ indicates the year of birth of the mother, and $MacroArea$ reports the geographical area where the mother lives.[16] The interaction term between

[16]The variable $MacroArea$ takes four values that correspond to the South, Midwest, West and Northeast regions of the US, and are ordered according to the average wage, from the lowest (South) to the highest (Northeast).

$Cohort$ and $MacroArea$ captures differences in the wage opportunities for mothers who belong to the same cohorts but live in different geographical areas.

The component μ_{mk}, where $k = h, l$, represents the mother's unobserved skills in the labor market that are assumed to be correlated with the mother's preferences. The specification of the model assumes that the mother's unobserved productivity and her preferences for child's ability follow a bivariate discrete distribution (Heckman and Singer 1984), with two points of support. This determines four types of mothers, identified by their level of productivity in the labor market and by their level of preference for the child's ability. The probability that a mother belongs to each type should be estimated.

Concerning the child's cognitive ability production function, as stated in Section 3.1, the parameters can vary across a child's age and they are defined as follows:

$$\delta_{0t} = exp(\xi_{0tfp} + \xi_{1tfp} \times t) \tag{17}$$

$$\delta_{1t} = exp(\xi_{0\tau} + \xi_{1\tau} \times t) \tag{18}$$

$$\delta_{2t} = exp(\xi_{0i} + \xi_{2i} \times t) \tag{19}$$

$$\delta_{3t} = exp(\xi_{0A} + \xi_{4A} \times t) \tag{20}$$

Allowing the parameters to vary across a child's age partially compensates for the lack of substitutability implied by the Cobb-Douglas functional form used to define the CAPF. Moreover, it allows me to capture whether the inputs included in the CAPF become less or more productive as the child ages and receives other inputs, such as schooling. The time-varying total factor productivity δ_{0t} captures the contributions of such missing inputs on the level of ability of the child in each period.

As for the wage process, the income process is also exogenous with respect to the mother's input decisions in each period. The household income is assumed to have a lognormal distribution and to depend on the fathers' observable characteristics and a shock:

$$ln(I_t) = \mu_{inc0} + \mu_{inc1} FatherEdu + \mu_{inc2} FatherAge_t + \mu_{inc3} FatherRace + \iota_t \tag{21}$$

where $\iota_t \overset{\text{iid}}{\sim} N(0, \sigma^2_{inc})$.

In order to estimate the model and to take into account the dynamic optimization problem faced by the mother, it is necessary to know the starting level of ability, i.e., the child's cognitive ability the mother observes in the first period before making her investments decisions. The initial ability endowment is assumed to be a function of observed characteristics of the child and the mother at birth. Specifically:

$$A_1 = exp(\eta_0 + \eta_1 BirthWeight + \eta_2 Male + \eta_3 MotherAgeBirth + v) \tag{22}$$

where $BirthWeight$ is a dummy variable indicating if a child has a low birth weight (i.e., lower than 2500 grams), $Male$ is a dummy variable indicating whether the child is a male, $MotherAgeBirth$ indicates the age of the mother at birth, and η_0 is a constant. $v \overset{\text{iid}}{\sim} N(0, \sigma^2_v)$ is a shock representing the variation in initial ability not captured by the observed characteristics.

Recalling the value-added specification of the CAPF, defined in (4), the estimation provides consistent estimates of the productivity parameters for each input if the following conditions hold: (i) A_t is a sufficient statistic for the inputs history received by the child in the previous periods; (ii) the child's initial endowment A_1 (that the mother observes but the researcher does not) is only reflected in the level of ability in the subsequent period and does not affect a child's ability in the future periods (Todd and Wolpin 2003).

Finally, it should be described how the child's true cognitive ability is related to the measure of that given by the test scores. The score measures used in the empirical analysis are the Letter Word and the Applied Problems test scores. Following the approach based on classical test theory (Novick 1966), I define the probability that the child answers correctly each item as a function of the child's true ability:

$$\pi_{score} = \frac{exp(A_t + \lambda LW)}{1 + exp(A_t + \lambda LW)} \tag{23}$$

where LW is a dummy variable indicating whether the test score is the LW raw score, and capture mean differences in the item difficulty between the LW and the AP score. The test score measure is then defined as follows:

$$S_t = \pi_{score} * J_t \tag{24}$$

where J_t is the maximum number of items in the test.[17]

4. Data

This paper uses data from the Panel Study of Income Dynamics (PSID) and its Child Development Supplement (CDS) and Time Diary (TD) component. The PSID is a longitudinal study that began in 1968 with a nationally representative sample of over 18,000 individuals living in 5,000 families in the United States. Starting from 1968, information about each family member was collected, but much greater detail is obtained about the head and the spouse. From 1997, the Child Development Supplement (CDS) has gathered information on children aged 0-12 in PSID families through extensive interviews with their primary caregiver. The CDS has been replicated in 2002 and 2007 for children under 18.

For this analysis, I exploit the child cognitive ability measures and non-parental child care data provided in the Primary Caregiver Interview of the CDS, together with the time use details given in the Time Diary (TD) component of the CDS. To the best of my knowledge, this is the first study linking all the components of the PSID surveys introduced in 1997 and exploiting the rich information on non-parental child care use provided in the CDS.

The CDS asks the primary caregiver about the non-parental child-care arrangements used for the child since childbirth until kindergarten, and at the time of the survey.[18] Using the 1997 and 2002 waves I can recover the complete child-care history (from birth until kindergarten) of the sampled children, as well as information on the arrangement

[17] In the empirical application, $J = 57$.

[18] The CDS questionnaire allows the primary caregiver to indicate more than one arrangement used at each age of the child. If the primary caregiver used simultaneously more than one arrangement in a period, I define the child care variable exploiting the information on the arrangement used more hours per week.

that they use at the time of the survey. The variable of interests is the number of hours the child uses non-parental child care at each age. This variable refers to any type of child-care arrangement provided by people other than the parents.[19]

In 1997 and 2002, the Child Development Supplement includes another instrument to assess the time use of children: the Time Diary (TD). The TD is a unique feature of the CDS and consists in a chronological report filled out by the child or by the child's primary caregiver about the child's activities over a specified 24-hour period.[20] Each participating child completed two time diaries: one for a weekday (Monday-Friday) and one for a weekend day (Sunday or Saturday). The TD additionally collects information on the social context of the activity by specifying with whom the child was doing the activity and who else was present but not engaged. The variable weekly time with the mother is constructed by multiplying the daily hours the child spends with the mother by 5 for the weekday and by 2 for the weekend day, and summing up the total hours in a week.

The CDS supplement provides several measures of child cognitive skills, based on the Woodcock Johnson Achievement Test Revised (WJ-R) (Woodcock and Johnson 1989). The outcome measures considered in this study are the Letter Word (LW) and the Applied Problems (AP) test scores, which are applied to all children older than three and prove, respectively, a child's learning and reading skills, and a child's skills in analyzing practical problems in mathematics (Hoffert et al. 1997). These measures are available in 1997, 2002 and 2007.

The main PSID surveys are used to gather information about the labor supply of mothers and fathers, and the household non-labor income. PSID interviews have been conducted annually until 1997 and, since then, they have been biennial. Since children in 1997 have different ages, ranging from 0 to 13, and in order to identify the necessary information for all of them at every age, CDS data should be matched with family information from PSID surveys in the years 1985-2007.[21] The family information I gather includes each parent's hours of work, wage and non-labor income in each period.[22]

All relevant variables are constructed for each age of the child, defining age one as the first 12 months of child's life, age two as the next 12 months, and so on. For the estimation of the model I consider all children without siblings interviewed in CDS I, living in intact households (where both mother and father are present for the entire period), without

[19]More precisely, non-parental child care includes any type of arrangement used by the mother for the child, excluding parents' (mother's and father's) child care; this can include formal child-care centers, nannies and babysitters, as well as grandparents. When the child reaches school age, this variable indicates any forms of preschool or after school programs, or any other informal arrangement used by the mother in addition to formal schooling.

[20]The primary caregiver completed the time diary for the very young children, while older children and adolescents were expected to complete the time diaries themselves (ISR 2010a,b).

[21]For instance, to identify household information for all relevant periods for a child born in 1996 (1 year old in 1997) I need to use PSID surveys from 1997 to 2007; instead, if a child is born in 1986 (aged 11 years in 1997) I need to use PSID surveys from 1987 to 1999. All PSID surveys in the period 1985-2007 have been exploited, and the children included in the final sample are born between 1984 and 1996. See Appendix B, Tables B.1 and B.2.

[22]Note that all the variables that I use from the main PSID surveys concerning labor and non-labor income of the household members refer to the year before the survey. All monetary variables are deflated into 1997 US$ using the Consumer Price Index (CPI) History for the U.S. See Appendix B for further description of the data sources used for the analysis.

missing data on personal and parents' demographic characteristics. The final sample is made up of 417 observations.

Before presenting the descriptive statistics, it should be discussed what biases might be introduced into the analysis by focusing on the subsample of children in intact households without siblings. This sample selection, in fact, implies that all mothers' investments in child's ability are unrelated with the decision to marry or to cohabit and with fertility. However, if mothers in intact households have more marriage-oriented attitudes, which also influence their time allocation and fertility, they may be more likely to stay at home and to spend more time with their child instead of working. This may lead to an overestimation of the proportion of mothers not working or to an overestimation of the mothers' preferences for a child's ability. Similarly, mothers with only one child may have higher preferences for a child's ability and this may lead to an overestimation of the mother's use of the most productive input. On the other hand, women in long-term relationships and with fewer children may also be more desirable in the labor market; in addition, the fact of having only one child means that the mother has experienced only one work interruption as a result of childbirth, thus making the sample disproportionately represented by highly productive mothers, and leading to an overestimation of a mother's attachment to the labor market. Even though it is difficult to derive a unique direction of the bias induced by the sample selection, the arguments provided above suggest that it may oversample mothers who are more productive either in the labor market or at home with the child.

Table 1 shows the average values of all the variables for the sample. Mothers work, on average, 27 hours per week and use non-parental child care for almost 14 hours; moreover, they spend with their child, on average, 21 hours per week. The mothers' hourly wages are on average 14 US$, while household income amounts to, on average, around 800 US$ per week. In the sample, the average LW score is around 35 out of 57, while the AP score is around 30 out of 52.[23]

Table 2 provides some descriptive statistics on the mothers' work, non-parental child care and time with the child choices, by the child's age. The number of hours mothers work, as well as the proportion of mothers who work, slightly increases over time: mothers work, on average, 24 hours per week when the child is very young, and 29 hours, when the child reaches 11 years of age; when the child is still an infant, 23 percent of mothers do not work, and this percentage decreases to 18 when the child reaches school age. The average number of hours the child is cared for by someone other than the parents decreases as the child ages, ranging from 20 hours per week in the first years of life to almost 3 hours per week when he is 11 years old. The average number of hours the child spends with the mother also decreases as the child grows up: the mother spends with the child around 30 hours per week when the child is younger than five, while the mother's child-care time drops by almost ten hours when the child reaches six years of age.

[23]Table B.4 in Appendix B compares the characteristics of the subsample used for the analysis ($N = 417$) with the ones of the entire PSID-CDS sample ($N = 3243$). It shows that mothers in this subsample are, on average, older and more educated, work more, use more non-parental child care and spend less time with their child, than in the entire sample. However, the wage before childbirth of the mothers in the subsample is not statistically different from the one in the entire sample.

Table 1
Descriptive statistics on all variables for the entire period.

	Mean	SD	Min	Max
Child's LW raw score	35.10	14.46	1	57
Child's AP raw score	29.62	10.53	1	52
Mother's hours of work	27.30	17.53	0	100
Non-working mother	0.19	.39	0	1
Non-parental child care	14.74	18.25	0	70
Mother's time with the child	21.16	17.01	.17	95.75
Child's gender: male	0.51	0.50	0	1
Child's birth weight	3387.16	614.56	907.18	6917.29
Mother's hourly wage	14.37	10.27	5.01	133.93
Mother's age at child's birth	28.20	5.11	16	43
Mother's education	13.27	2.48	2	17
Mother's race: white	0.61	0.48	0	1
Household income	791.36	644.15	0.09	8834.96

NOTE. Monetary variables deflated into 1997 US$. Mother's hours of work, non-parental child care, mother's time with the child and household income are weekly values. Child's birth weight is expressed in grams. Household income includes father's labor income and household non-labor income. Source: own elaboration from PSID-CDS data.

Table 2
Descriptive statistics on maternal employment, non-parental child care and maternal time by child's age.

Child's Age	1-2	3-5	6-10	11-12
Mother's hours of work	24.75	26.46	28.08	29.75
	(17.67)	(17.41)	(17.28)	(17.75)
Non-working mother	0.23	0.20	0.18	0.18
	(0.42)	(0.40)	(0.38)	(0.39)
Non-parental child care	19.54	21.03	3.45	1.77
	(19.23)	(19.06)	(8.69)	(4.19)
Mother's time with the child	28.55	29.05	19.31	16.35
	(18.06)	(20.27)	(14.81)	(15.12)

NOTE. This table shows mean values by a child's age; standard deviations are reported in parenthesis. Source: own elaboration from PSID-CDS data.

5. Estimation

The model parameters are estimated using a Method of Simulated Moments estimator that minimizes the distance between several data statistics and their model counterparts. The data generating process implied by the model described in Section 3 allows to simulate the same statistics for the individuals (mothers and children) in the sample over the child's life cycle. The full list of statistics used to construct the moment functions is reported in Table 3.

The simulation of the data is obtained by taking $N \times R$ random draws from the initial distribution implied by the model, i.e., the shock in the child's initial ability, the mother's skills and type preference distributions, and, for each period, from the wage and income

TABLE 3

Statistics of actual and simulated data used for the estimation of the model.

Mother's choices
mean mother's hours of work, non-parental child care and mother's time with the child by child's age
std deviation mother's hours of work, non-parental child care and mother's time with the child by child's age
proportion of mothers not working by child's age
Test scores
mean test scores by child's age
std deviation test scores by child's age
Correlation between mother's choices and exogenous variables
corr mother's wage and mother's hours of work
corr household income and mother's hours of work
corr mother's wage and mother's time with the child
corr household income and mother's time with the child
corr mother's wage and non-parental child-care time
corr household income and non-parental child-care time
Correlation between mother's choices
corr mother's hours of work and mother's time with the child
corr mother's hours of work and non-parental child-care time
Productivity of inputs
Coefficient of mother's time with the child in a OLS regression of test score in t on a dummy for LW and mother's time with the child in $t-5$
Coefficient of non-parental child care in a OLS regression of test score in t on a dummy for LW and non-parental child care in $t-1$
Coefficient of test score in $t-5$ in a OLS regression of test score in t on a dummy for LW and test score in $t-5$
Child's initial ability and test score specification
Variance of residuals from a child's test score OLS reg on a dummy for LW and child's age fixed effects
Average residuals from a child's test score OLS reg on a dummy for LW and child's age fixed effects for low-birthweight and normal-birthweight children
Average residuals from a child's test score OLS reg on a dummy for LW and child's age fixed effects by child's gender
Average residuals from a child's test score OLS reg on a dummy for LW and child's age fixed effects by mother's age at birth
OLS regression of test score on a dummy for LW (coefficient)
Wage equation and household income
mean and std deviation of mother's wage
average of mother's wage by mother's level of education, race, age, year of birth and area of residence
mean and std deviation of household income
average of household income by father's level of education, race and age
Mother's unobserved productivity and preferences
variance of the residuals from a mother's wage OLS reg on mother's education, age, race, cohort, area of residence and their interaction
OLS reg of residuals from a mother's wage OLS reg on edu, age, race, cohort, area of residence and their interaction in t, on the residuals in $t-1$ (coefficient)
variance of the residuals from a mother's time with the child OLS reg on child's age, mother's wage and household income
variance of the residuals from a non-parental child care OLS reg on child's age, mother's wage and household income
variance of the residuals from a mother's hours of work OLS reg on child's age, mother's wage and household income
10th, 50th and 90th percentiles of a mother's hours of work, a mother's time with the child and non-parental child care
correlation between the residuals from a mother's wage OLS reg on mother's edu, age, race, cohort, area of residence and their interaction with time with the child
correlation between the residuals from a mother's wage OLS reg on mother's edu, age, race, cohort, area of residence and their interaction with non-parental child care
Score transition probabilities
prop of children with score in range p_y in years 1997 or 2002 and p_{y+5} in years 2002 or 2007

NOTE. These statistics are computed using PSID-CDS data on children aged 0-12 in 1997 without siblings, and simulated data according to the model defined in Section 3. Mother's time with the child is measured in 1997 and 2002; child's test scores are measured in 1997, 2002 and 2007, and refer to both the LW and the AP scores; from 1997 on, mother's hours of work, mother's wage and household income are measured every two years and these variables refer to the year before the survey (see Section 4 and Appendix B for a description of the data). Household income includes both father labor income and household non-labor income. Child's age t ranges from 1 to 13. Ranges p_y, with $y = 1997, 2002, 2007$ are defined according to the following ranges of the score distribution: $1st - 25th$ perc, $25th - 50th$ perc, $50th - 75th$ perc, higher than $75th$ perc.

distributions.[24] After having drawn the child's level of ability, the wage offer and the level of income in the first period, the optimal choices of the mother are obtained by exploiting the optimal solutions derived in Section 3.2.[25] This process is repeated for every period, up to the final one T. The simulated data are used to compute the same statistics defined in Table 3. Both actual and simulated statistics are used to construct the objective function

[24] $N = 417$ and $R = 5$.

[25] To test numerically the accuracy of the solutions given by the theoretical model, I also perform a grid search, assuming that the mother's decision to work was actually discrete. In other words, I compute the value of the demands for child care and time with the child, as well as the mother's inter temporal utility, for different levels of the mother's labor supply (with the number of hours of work ranging from 0 up to the total time endowment) and I define as optimal choices those that provide the highest utility. The solutions do not differ from the ones provided by the theoretical model, though the process becomes more time consuming.

to be minimized. The Method of Simulated Moments estimator is then:

$$\hat{\theta} = arg\ min\ \hat{g}(\theta)' W \hat{g}(\theta) \tag{25}$$

where

$$\hat{g}(\theta) = \hat{m} - \hat{M}(\theta) \tag{26}$$

$\hat{m}$ is the vector of statistics defined from the actual data, while $\hat{M}(\theta)$ is the vector of simulated statistics according to the model.[26] Given S number of moments, the weighting matrix is defined as:

$$W = \begin{pmatrix} \hat{V}[\hat{m}_1]^{-1} & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & \hat{V}[\hat{m}_S]^{-1} \end{pmatrix}$$

where $\hat{V}[\hat{m}]$ is estimated with non-parametric bootstrap. The standard errors are also computed with non-parametric bootstrap, by changing the starting values in each bootstrap estimation. Appendix C provides further details.

The estimation requires a unique solution for the minimization of the objective function, which, in practice, depends on the uniqueness of the minimum and on the curvature around it. Figure C.1, in Appendix C, shows that the objective function varies when I perturb each parameter from its estimated value. The identification of the model parameters relies on parametric and functional form assumptions, exclusion restrictions (i.e., variables entering in some parts of the model and not in others), and on the choice of the moment conditions. More precisely, the moments should be informative of the parameter, in such a way that a slight variation in the parameter results in different values of the moments.

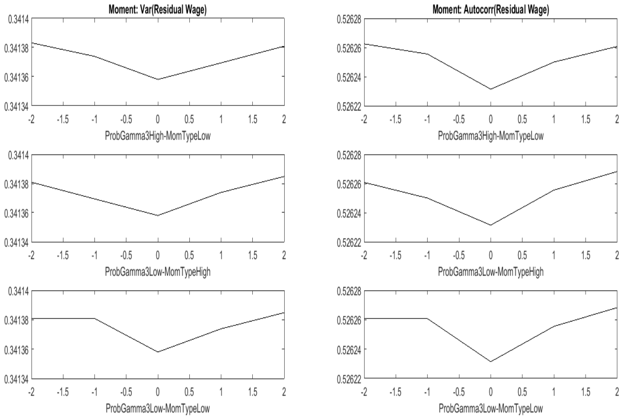
In order to identify the mothers' labor market opportunities, which are proxied by the wage equation, I exploit the variation in wages over the mothers' life cycle (i.e., age profile), over time (i.e., between cohorts), and between geographical areas (i.e., by accounting for the macro-area where a mother lives). The wage offer is also a function of the mother's unobserved productivity in the labor market, and of a transitory shock. To identify these parameters, I use the residuals from a OLS regression of the mother's wage on education, race, cohort, area of residence and their interaction. While the variance of these residuals captures the variation of both the (time-invariant) mother's unobserved productivity and the transitory shock, by regressing the residuals in each period onto their lagged value, I get a moment that depends only on the persistence of types. Furthermore, in order to identify the correlation between the mother's unobserved skills and the mother's preferences, I use the correlation between these residuals and the mother's investment decisions (i.e., time with the child and non-parental child care). Figures 3 and 4 report the variation in these moment conditions, induced by changes in the parameters that represent the proportion of mothers in each skills and preferences category.[27]

The preference parameters are also unobserved, and assumed to be constant over time. Thus, cross-sectional average and standard deviation of choices are used to recover the

[26] The estimation is done using the simplex algorithm, which is robust to non-smooth objective functions.

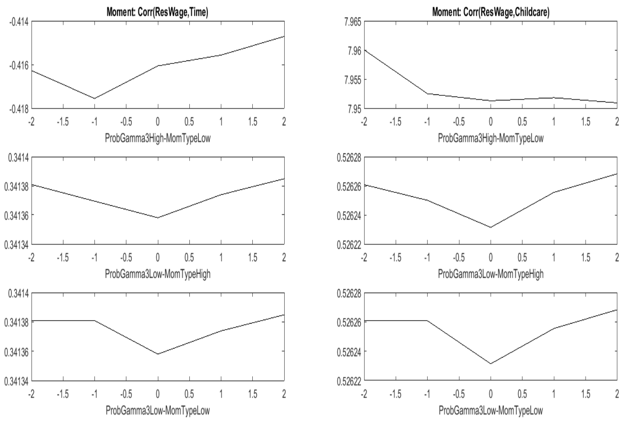
[27] More precisely, as specified in Section 3.3, each group is characterized by a level of unobserved skills in the labor market (*MomTypeLow* and *MomTypeHigh*) and a level of preference for a child's ability (*Gamma3Low* and *Gamma3High*).

FIGURE 3
Variation in the moment conditions used to identify a mother's unobserved productivity, by perturbing the estimated parameters.



NOTE. This graph reports the values of the moment conditions obtained from the variance and serial correlation of the residuals from a OLS regression of a mother's wage on a mother's education, race, age, year of birth, area of residence and their interaction, perturbing the estimated parameters by 2 standard deviations up and down with respect to the estimated value. The parameters represent the proportion of mothers in each group identified by a level of unobserved skills in the labor market (*MomTypeLow* and *MomTypeHigh*) and a level of preference for a child's ability (*Gamma3Low* and *Gamma3High*).

FIGURE 4
Variation in the moment conditions used to identify the correlation between a mother's unobserved productivity and preferences, by perturbing the estimated parameters.

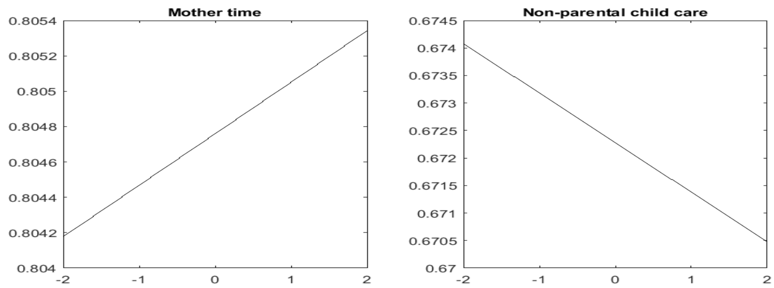


NOTE. This graph reports the values of the moment conditions obtained from the correlation between the residuals from a OLS regression of a mother's wage on a mother's education, race, age, year of birth, area of residence and their interaction, and a mother's choices (e.g., time with the child and non-parental child care), perturbing the estimated parameters by 2 standard deviations up and down with respect to the estimated value. The parameters represent the proportion of mothers in each group identified by a level of unobserved skills in the labor market (*MomTypeLow* and *MomTypeHigh*) and a level of preference for a child's ability (*Gamma3Low* and *Gamma3High*).

mother's preferences parameters. The cross-sectional correlation between choices and exogenous variables, such as the mother's wage and the household income, is also informative of the preference parameters and of the strength of the budget constraint. The correlation between the exogenous variables and non-parental child care is also used to estimate the hourly price of the service, under the identifying assumption that the availability and price of non-parental child care are not correlated with the mother's labor market opportunities in the area.

The productivity parameters in the child's ability production function are identified by the correlation between a mother's choices in t and the child's test scores in $t+1$.[28] Figure 5 shows that these moments vary after perturbing the estimated parameters, and can thus be used for identification purposes. Importantly, the model specification allows only these parameters to vary over a child's age, so that the variation in the child's test scores over time can be used to recover the level of ability in each period, and the temporal variation in the mother's choices reflects the time-varying productivity of inputs. The productivity parameter for the lagged child's ability captures the time dependence of ability, and it is identified by using the transition probabilities from the first score measure available in the data (in 1997 or 2002) to the second score measure (available in 2002 or 2007), as well as the correlation between the first and the second test score observations.

FIGURE 5
Variation in the moment conditions used to identify the productivity of a mother's time with the child and non-parental child care, by perturbing the estimated parameters.



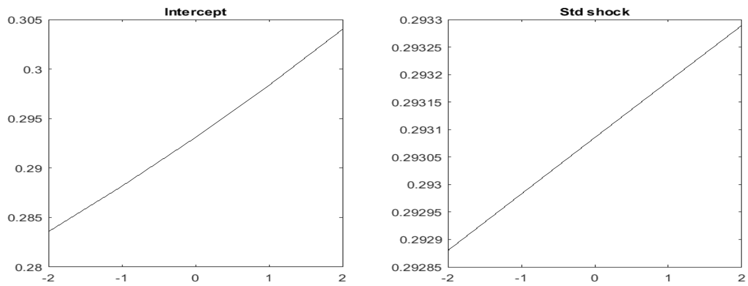
NOTE. This graph reports the values of the moment conditions obtained from the correlation between a mother's choices (a mother's time with the child - Left - and non-parental child care - Right) in t and the child's scores in $t+1$ ($t+5$ in the case of maternal time with the child), conditional on whether the score is LW or AP, by perturbing the estimated parameters by 2 standard deviations up and down with respect to the estimated values. The parameters represent the elasticity of a child's ability with respect to a mother's time with the child (Left), and the elasticity of a child's ability with respect to non-parental child care (Right).

The estimation of the model crucially relies on the identification of the initial condition for the child's level of ability. Since the initial level of child's ability depends on characteristics of the child and the mother at birth (e.g., gender, birth weight, mother's age at birth), I use as moments the correlation between these characteristics and the test scores. The limitation of this strategy is that the moments may not be informative enough of the parameters, especially if derived from test score observations at later ages. In order to partially solve this issue, I define these moments by using only the first test score observed for each child, and I take the residuals from a OLS regression of such scores on child's age fixed effects, in order to partial out any age effects. Figure 6 reports the variation in those moments that is induced by the perturbation of the intercept and shock parameters in the

[28]Due to the structure of the data, when defining this moment for the productivity parameter for a mother's time with the child, I use as outcome the test score observed in the next survey, i.e., after 5 years. For the specification of all moments, the test score refers to both the LW and the AP scores.

initial level of ability, and shows that such moments can be still informative and used for the estimation.

Figure 6
Variation in the moment conditions used to identify the intercept and shock in the child's initial ability, by perturbing the estimated parameters.



NOTE. This graph reports the values of the moment conditions obtained from the variance of the residuals from a OLS regression of a child's first test score observation on a dummy indicating whether the test is LW or AP and a child's age fixed effects, perturbing the estimated parameters by 2 standard deviations up and down with respect to the estimated value. The parameters represent the intercept (Left) and the standard deviation of the shock (Right) in the initial level of ability of the child.

6. Results

This section presents the estimated parameters for the mother's utility function and wage equation, as well as for the child's cognitive ability production function.[29] Table 4 reports the estimated parameters for the mother's utility function and the wage equation, while Table 5 reports the proportions of mothers into each group, characterized by a certain level of preferences and skills in the labor market.

Panel A of Table 4 shows the preference parameters for leisure (α_1), consumption (α_2) and a child's ability (α_3) for each subgroup in the sample, as defined by the levels of preference for consumption (γ_2) and a child's ability (γ_3), according to the specifications in (12), (13) and (14): Type I corresponds to low levels, while Type II corresponds to high levels. The results show that there is not a large difference among the four groups in terms of preferences for leisure; however, as expected, Type II mothers show higher preferences for both consumption and a child's ability. Mothers in the Type I group for consumption have a preference for consumption which is 20 percent lower than the preference level of the Type II group; concerning the preference for a child's ability, the group with the lowest level (belonging to the Type I group for child's ability and Type II group for consumption) have a preference level which is 15 percent lower than the one of the group with the highest (belonging to the Type II group for a child's ability and Type I for consumption). Table 5 shows that 77 percent of mothers in the sample belong to the Type I group for the preference for consumption, while the same proportion for the preference for a child's ability is almost 70 percent. Panel A of Table 4 also reports the estimated parameters for the weight the mother poses on the future child's ability in the

[29]The remaining estimated parameters, namely the untransformed parameters in the mother's utility function, the parameters in the household income function and those in the child's initial ability, are reported in Table D.1, Appendix D.

last period (ρ), which is around 25, and for the hourly price of non-parental child care, which is 5$ per hour.[30]

Table 4
Estimated parameters in the mother's utility function and wage equation.

		Estimate	Std. Errors
Panel A. Utility function			
$\alpha 1_{\gamma_{2low}\,\gamma_{3low}}$	Preference for leisure (Type I consumption. Type I child ability)	0.3782	0.0405
$\alpha 1_{\gamma_{2low}\,\gamma_{3high}}$	Preference for leisure (Type I consumption. Type II child ability)	0.3511	0.0580
$\alpha 1_{\gamma_{2high}\,\gamma_{3low}}$	Preference for leisure (Type II consumption. Type I child ability)	0.3546	0.0346
$\alpha 1_{\gamma_{2high}\,\gamma_{3high}}$	Preference for leisure (Type II consumption. Type II child ability)	0.3307	0.0485
$\alpha 2_{\gamma_{2low}\,\gamma_{3low}}$	Preference for consumption (Type I consumption. Type I child ability)	0.2098	0.0362
$\alpha 2_{\gamma_{2low}\,\gamma_{3high}}$	Preference for consumption (Type I consumption. Type II child ability)	0.1948	0.0357
$\alpha 2_{\gamma_{2high}\,\gamma_{3low}}$	Preference for consumption (Type II consumption. Type I child ability)	0.2591	0.0374
$\alpha 2_{\gamma_{2high}\,\gamma_{3high}}$	Preference for consumption (Type II consumption. Type II child ability)	0.2416	0.0480
$\alpha 3_{\gamma_{2low}\,\gamma_{3low}}$	Preference for child ability (Type I consumption. Type I child ability)	0.4120	0.3814
$\alpha 3_{\gamma_{2low}\,\gamma_{3high}}$	Preference for child ability (Type I consumption. Type II child ability)	0.4541	0.0866
$\alpha 3_{\gamma_{2high}\,\gamma_{3low}}$	Preference for child ability (Type II consumption. Type I child ability)	0.3863	0.0617
$\alpha 3_{\gamma_{2high}\,\gamma_{3high}}$	Preference for child ability (Type II consumption. Type II child ability)	0.4277	0.1083
ρ	Weight on future child's ability in the last period	25.3163	8.1726
p	Hourly price of child care	5.9796	0.8554
Panel B. Wage equation			
μ_{high}	Skill level for High Type mothers	0.1516	0.0238
μ_{low}	Skill level for Low Type mothers	0.0907	0.0147
μ_{edu}	Coefficient of mother's years of education	0.0167	0.0292
μ_{age}	Coefficient of mother's age	0.1147	0.0047
μ_{race}	Coefficient of mother's race	-0.0304	0.2924
μ_{cohort}	Coefficient of mother's year of birth	0.0020	0.0018
μ_{area}	Coefficient of mother's macro-area of residence	-0.0042	0.1135
μ_{inter}	Coefficient of cohort $\times$ macro-area of residence	-0.0121	0.0005
σ_{wage}	Std deviation wage shock	0.5918	0.0916

NOTE. Standard errors are estimated with non-parametric bootstrap, by changing the starting values in each bootstrap estimation. See Appendix C.1 for further details.

In the model, the preference parameters are allowed to be correlated with the unobserved skills of the mother in the labor market (μ_m), which are similarly discrete. The estimated values, reported in Panel B of Table 4, show that the skills level for the Low type mothers is 40 percent lower than the skills level for the High type. Table 5 shows that almost 60 percent of mothers in the sample belong to the Low type. Panel B of Table 4 also reports the other parameters in the mother's wage equation. All parameters in the wage equation have the expected signs and reasonable magnitudes, though the coefficient for a mother's years of education is not statistically significant.

Since the preferences and the unobserved skills of the mother are correlated, it is interesting to look at their degree of correlation in the sample, which is reported in Table 6. Panel A reports the pairwise correlation coefficients between the preference parameters. Interestingly, for any pair of preference parameters the correlation is negative, meaning

[30] The parameter ρ indicates the value the mother poses on the child's level of ability reached in the last developmental period. As Del Boca et al. (2014) point out, having found a discount factor in the last period larger than the one we could get by fixing it to the value assigned to β (i.e., $\beta = 0.95$ so that $\rho = \sum_{\kappa=0}^{+\infty} \beta^{\kappa} = \frac{1}{(1-\beta)} = 20$) implies that the mother gives a large importance to the level of ability that the child reaches in the final period.

that a mother faces a trade-off between all her choices, i.e., between leisure and work, or between time with the child and work, or between leisure and time with the child. Furthermore, the pairwise correlation coefficient between the preference for consumption and the preference for a child's ability is the largest in absolute value: this implies that the strongest trade-off a mother faces is precisely the one between working and using non-parental child care, on the one hand, and spending time with the child, on the other. This may suggest that, according to a mother's preferences, the decision to work and to use non-parental child care is less preferred than spending time with the child herself, though the final decisions in terms of time allocation and labor supply also depend on her productivity in the labor market and with the child. Panel B of the table reports the correlation coefficients between the preference parameters and the mother's skills in the labor market: even though they are smaller than the ones between the preference parameters, they show that mothers who are more productive in the labor market have also higher preferences for consumption, and lower preferences for leisure and their child's ability.

TABLE 5
Estimated proportions of types of mothers.

		Estimate	Std. Errors
$\pi_{\gamma_{2low}}$	Proportion Type I consumption	0.7659	0.0090
$\pi_{\gamma_{2high}}$	Proportion Type II consumption	0.2341	(...)
$\pi_{\gamma_{3low}\ \mu_{high}}$	Proportion Type I child ability & High skilled	0.2909	0.0986
$\pi_{\gamma_{3low}\ \mu_{low}}$	Proportion Type I child ability & Low skilled	0.4146	0.1112
$\pi_{\gamma_{3high}\ \mu_{high}}$	Proportion Type II child ability & High skilled	0.1210	0.0287
$\pi_{\gamma_{3high}\ \mu_{low}}$	Proportion Type II child ability & Low skilled	0.1735	0.0710

NOTE. Standard errors are estimated with non-parametric bootstrap, by changing the starting values in each bootstrap estimation. See Appendix C.1 for further details.

TABLE 6
Pairwise correlation coefficients between the preference parameters and a mother's unobserved skills in the wage equation.

Panel A. Corr between preferences		
$Corr(\alpha_1, \alpha_2)$	Correlation pref. for leisure and pref. for consumption	-0.3253
$Corr(\alpha_1, \alpha_3)$	Correlation pref. for leisure and pref. for child ability	-0.3933
$Corr(\alpha_2, \alpha_3)$	Correlation pref. for consumption and pref. for child ability	-0.7415
Panel B. Corr between skills and preferences		
$Corr(\mu, \alpha_3)$	Correlation labor skills and pref. for child ability	-0.0454
$Corr(\mu, \alpha_2)$	Correlation labor skills and pref. for consumption	0.0956
$Corr(\mu, \alpha_1)$	Correlation labor skills and pref. for leisure	-0.0670

Table 7 presents the estimated (untransformed) parameters in the child's cognitive ability production function, while Figures 7 and 8 show the time-varying elasticities as a function of a child's age, according to the specifications reported in Equations (17), (18), (19) and (20). Figure 7 reports the elasticities of a child's ability with respect to maternal time and non-parental child-care time, while Figure 8 reports the elasticity with respect to the child's ability in the previous period and the time-varying total factor productivity.

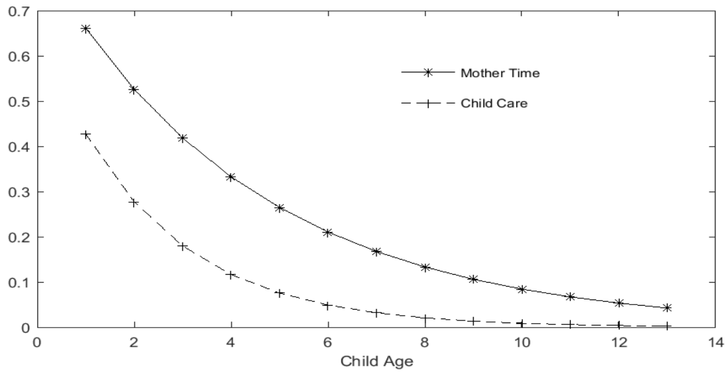
According to Figure 7, maternal time with the child is more productive than non-parental child care, at any age of the child. The elasticity of a child's ability with respect to maternal time ranges between 0.66 when the child is one year old and 0.04 when the

Table 7
Estimated parameters in the cognitive ability production function.

		Estimate	Std. Errors
ξ_{0tfp}	Intercept. Total Factor Productivity	-1.9133	0.2128
ξ_{1tfp}	Slope. Total Factor Productivity	0.1852	0.0166
$\xi_{0\tau}$	Intercept. Productivity of mother's time with the child	-0.1847	0.1774
$\xi_{1\tau}$	Slope. Productivity of mother's time with the child	-0.2290	0.0270
ξ_{0i}	Intercept. Productivity of non-parental child care	-0.4171	0.1699
ξ_{2i}	Slope. Productivity of non-parental child care	-0.4334	0.0484
ξ_{0A}	Intercept. Productivity of child ability in the previous period	0.2139	0.0878
ξ_{4A}	Slope. Productivity of child ability in the previous period	-0.3092	0.0211

NOTE. Standard errors are estimated with non-parametric bootstrap, by changing the starting values in each bootstrap estimation. See Appendix C.1 for further details.

Figure 7
Elasticity of a child's ability with respect to mother's time with the child and non-parental child care.



NOTE. This graph represents the productivity parameters for maternal child-care time (τ_t) and non-parental child care (i_t), as a function of child's age $t = 1, 2, 3, \ldots 13$. The specification of the parameters is reported in Equations (18) and (19).

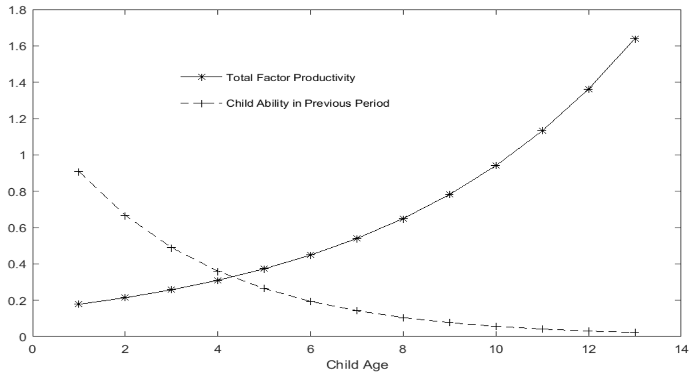
child is 13; similarly, the elasticity of a child's ability with respect to non-parental childcare decreases over time, ranging between 0.43 at one year of age and almost zero when the child grows up. Hence, when the child is one year old, a ten percent increase in the mother's child-care time, corresponding to almost three hours per week, leads to an increase in the level of ability of the child by 6.6 percent; at the same age, an increase in non-parental child care by ten percent, corresponding to almost two hours per week, leads to an increase in the child's ability by only 4.3 percent. It should be noticed that the productivity of non-parental child care reaches a value close to zero already at age six, presumably when the child starts going to primary school. This pattern could be explained by the different purposes that non-parental child care may have from the mother's point of view. In fact, the mother may choose a positive amount of child care if she works and needs someone looking after the child, but also if she thinks it may represent an input for the child's subsequent development. The educational role of child care can be less important when the child starts going to school, because he is receiving educational inputs from other institutions, so that from this age on the custodial role can prevail. As a consequence, the productivity of non-parental child care might decrease over time even if the amount of time that it is used remained constant.

The estimated parameters reported in Figure 7 shed also light on how the different productivity of inputs affects the mothers' decision-making process, especially their decision to work. Indeed, their final decision of whether to join the labor force depends on whether the alternative input (i.e., non-parental child care) can compensate for the reduction in the mother's child-care time, and, if not, whether the loss they would incur in terms of a child's ability could be compensated for by the gains in the labor market. Some mothers, in particular those less productive in the labor market, may find it more profitable to spend time with their child instead of working and using non-parental child care, because of the higher productivity of their time with respect to that of the alternative forms of care.

Figures 7 and 8 also show that the elasticity with respect to all inputs (i.e., maternal time, non-parental child care and previous ability) is higher during the early years and decreases over time, which seems in line with previous studies on human capital accumulation (Carneiro and Heckman 2003; Heckman 2008). However, the increasing trend of the total factor productivity over time, as shown in Figure 8, confirms that other inputs not explicitly included in the model play a more important role as long as the child ages.

I check the sensitivity of these results with respect to two important missing inputs: father's time and schooling. Appendix E.1 shows that the results are qualitatively the same if I re-estimate the model by using only the sample of children for whom the father's time is above the median, or if I include in the definition of a mother's time also the time that both parents share with the child. Appendix E.2 shows that the estimated productivity parameters do not vary if I include in the definition of non-parental child care also the amount of time the child spends at school after age five, even though this estimation leads to a slightly larger estimate for the productivity of non-parental child care.

FIGURE 8
Elasticity of child's ability with respect to the level of ability of the child in the previous period, and estimated total factor productivity.



NOTE. This graph represents the productivity parameters for the level of ability of the child in the previous period (A_t), and the estimated total factor productivity parameter, as a function of child's age $t = 1, 2, 3, \ldots 13$. The specification of the parameters is reported in Equations (17) and (20).

6.1. ***Goodness of fit of the model.*** Table 8 shows the fit of the model for the mother's choice variables, by a child's age. The overall fit of the model for the mother's choices is good, though the model slightly overestimates the proportion of mothers not working, and underestimates the mother's labor supply. As discussed in Section 3.1, this could be due to one of the assumptions needed to solve the model, which is the exogeneity of the wage process with respect to the mother's decisions, implying that the mother does not face any costs associated with her absence from the labor market. Over time, the model well predicts a negative trend in mother's time with the child and non-parental child care, though underestimating the amount of non-parental child care used in pre-school age.

TABLE 8

Goodness of fit for mother's choices by child's age.

	Child age			
	1 – 2	3 – 5	6 – 10	11 – 12
Proportion non-working mothers				
Actual data	0.2291	0.2011	0.1780	0.1830
Simulated data	0.2954	0.2849	0.2409	0.2213
Mother's hours of work				
Actual data	24.7488	26.4614	28.084	29.7518
Simulated data	25.5120	23.8436	25.9052	24.3724
Mother's time with the child				
Actual data	28.5513	29.0493	19.3114	16.3548
Simulated data	47.3609	29.1207	12.4552	21.5686
Non-parental child care				
Actual data	19.5432	21.0344	3.4549	1.7742
Simulated data	14.3982	7.4845	2.2833	2.6359

NOTE. Actual data represent PSID-CDS data on children aged 0-12 in 1997, without siblings. See Section 4 and Appendix B for further details on the data. Simulated data represent the data obtained simulating the model described in Section 3 and setting the parameters at the estimated values.

Table 9 shows how the model performs in fitting the data concerning the wage and the income processes. Specifically, it shows the average and standard deviation of wage and income, observed in the actual and in the simulated data. The model predicts quite well the average wage and income and there are no differences between the actual and simulated data concerning the standard deviation of income.

Figure 9 shows the model fit for the child's score measure.[31] The model predicts quite well the increasing trend of the raw scores in the data, even though it overestimates the level of scores in the first years. This could be due to the functional form assumptions in the CAPF, which imply that any input in any period, including the initial level of ability of the child, should be strictly positive. The model reaches its best fit by simulating a level of initial ability that is close to zero, but still strictly positive, and this leads to higher values of the child's test scores at erly ages.

7. Policy simulations

I use the estimated model to simulate the effects of policies aimed at increasing the household's financial means (through an increase in either household income or a mother's

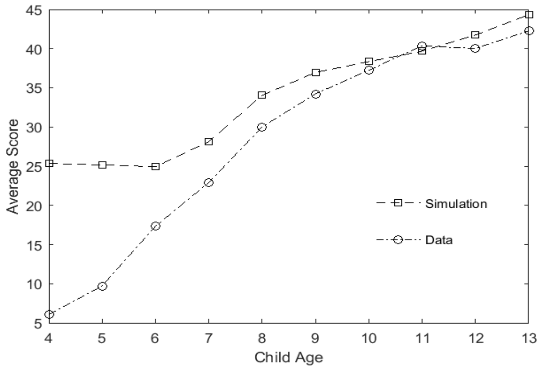
[31]In Figure 9, the child's test score represents the average between the LW and the AP raw scores.

Table 9
Goodness of fit for mother's wage and household income.

	Actual data	**Simulated data**
Mean mother's wage	14.3659	10.3334
Std deviation mother's wage	10.2725	17.1308
Mean household income	7.9136	7.9395
Std deviation household income	6.4406	6.4411

NOTE. Actual data represent PSID-CDS data on children aged 0-12 in 1997, without siblings. See Section 4 and Appendix B for further details on the data. Simulated data represent the data obtained simulating the model described in Section 3 and setting the parameters at the estimated values.

Figure 9
Goodness of fit for child's test score measure by child's age.



NOTE. The test score represents the average between the LW and AP test scores, in both actual and simulated data. Actual data represent PSID-CDS data on children aged 0-12 in 1997, without siblings. See Section 4 and Appendix B for further details on the data. Simulated data represent the data obtained simulating the model described in Section 3 and setting the parameters at the estimated values.

hourly wage), at limiting the mother's working time, and at increasing the quality of non-parental child care. The results of the policy simulations are reported in Table 10: the first column reports the baseline levels of the mother's choices and utility, as well as the child's ability and test score, while the other columns report the percentage changes in those variables with respect to the baseline associated with the implementation of each policy.

The first policy aims at increasing the income available to the household, by subsidizing households with children with 35$ per week. This policy wants to resemble child allowance benefits implemented in several countries, such as Sweden, where families are entitled to about 120$ per month until the child reaches 16 years of age. The results of the simulation of this policy are reported in Column (a) of Table 10. As expected, such policy induces a negative effect on the labor supply of mothers. Furthermore, the reduction in working time is associated with a slightly larger increase in leisure than in child-care time, confirming that the additional choice between child care and leisure plays an important role for the effects of policies aimed at decreasing the mothers' labor supply. The figures in Column (a) also show that mothers spend a large part of the new income for non-parental child care, so the final effect on a child's ability is still positive.

Column (b) of Table 10 reports the effects of a policy explicitly aimed at lowering the amount of time that mothers spend on the labor market. More precisely, this policy sets the maximum amount of time that mothers can work per week to 20, and wants to resemble labor regimes that create disincentives to work for the second earner, i.e., the mother, in order to allow them to spend more time at home with the children. Apart from the obvious negative effect on labor supply, it is interesting to notice that also in this case the additional time that mothers gain after the introduction of the policy is almost equally split between leisure and child-care time. Differently from before, the policy also induces a negative income effect, implying that mothers reduce the expenditure for non-parental child care and their consumption. The final effect on the child's test score and ability is still positive, thanks to the increase in a mother's child-care time that also compensates for the reduction in non-parental child care; however, the mother's utility is reduced, mainly because of the lower consumption.

TABLE 10
Simulation of policies.

	Base	**(a) Child allowance**	**(b) Limit work time**
Score in the last period	46.1067	0.2047	0.2086
Ability in the last period	1.5105	0.7158	0.7283
Mother's hours of work	24.9317	-29.8021	-42.6806
Mother's time with the child	23.8120	8.4031	11.0992
Non-parental child care	5.3595	12.0418	-43.9657
Mother's leisure	63.2562	8.5829	12.6439
Consumption	261.7604	5.8688	-45.0764
Utility	6.51E-06	13.8440	-3.5003
	Base	**(c) Higher mother's wage**	**(d) More productive child care**
Score in the last period	46.1067	0.0029	1.4386
Ability in the last period	1.5105	0.0097	5.7776
Mother's hours of work	24.9317	2.6325	25.5706
Mother's time with the child	23.8120	-0.7169	-9.6696
Non-parental child care	5.3595	19.5514	275.2358
Mother's leisure	63.2562	-0.7677	-6.4384
Consumption	261.7604	19.7606	-10.1213
Utility	6.51E-06	3.3267	3.5695

NOTE. This table reports the baseline values and the variation of the child's test score and ability in the last period, as well as of the average (over the period) mother's choices, consumption and utility, induced by the implementation of the policies listed in each column. The first column reports the baseline values, obtained by simulating the model described in Section 3 and setting the parameters at the estimated values. Policy (a) increases household income by 35$ per week; policy (b) sets the maximum working time for mothers to 20 hours per week; policy (c) increases the mother's wage by 20 percent; policy (d) sets the productivity of non-parental child care at the level of the productivity of mother's time with the child, according to the estimated values reported in Figure 7. Figures in columns (a), (b), (c) and (d) represent percentage changes with respect to the baseline.

Column (c) of the same table reports the effects of a policy that increases the mother's wage offer in each period by 20 percent. Such policy can resemble interventions aimed at lowering labor taxation, and at increasing the participation of mothers in the labor market; similarly to policy (a), such policy can also make the household budget constraint less stringent, even though it has opposite effects on labor supply. Indeed, in this case, the average working time of mothers increases by 2 percent, while both mothers' time with the child and leisure decreases by 0.7 percent. The final effect on the child's ability and test score is positive, but very close to zero, which seems to suggest that the higher amount of non-parental child care used for the child has barely compensated for the reduction in the mother's time with the child, despite having a lower productivity.

Finally, Column (d) of Table 10 reports the effects of a policy aimed at increasing the productivity of non-parental child care. As the solutions of the model suggest, especially Equation (11), a mother's labor supply is affected not only by monetary variables (i.e., wage and income), but also by the productivity of her own time with the child and the productivity of the alternative forms of care available in the market. Under the assumption that the mother knows, or at least perceives, the difference in productivity between her time and non-parental child care, she may find it easier to work if the alternative form of care is of high quality, or at least of the same quality as her time. I thus simulate the effects of a policy that sets the productivity parameter of non-parental child care to the level of a mother's time with the child, according to the estimated values reported in Figure 7. Interestingly, in this case, the change in labor supply is much larger than in policy (c), which may suggest that mothers react more to changes in the environment where the child is taken care of, than to changes in the labor market opportunities. Notice that only in this case, when the alternative form of care available in the market is equally productive, the reduction in maternal time with the child outnumbers the reduction in leisure. The use of non-parental child care increases by 25 percent, which induces a reduction in consumption, but the utility of the mother still increases, due to the higher level of ability of the child in the last period.

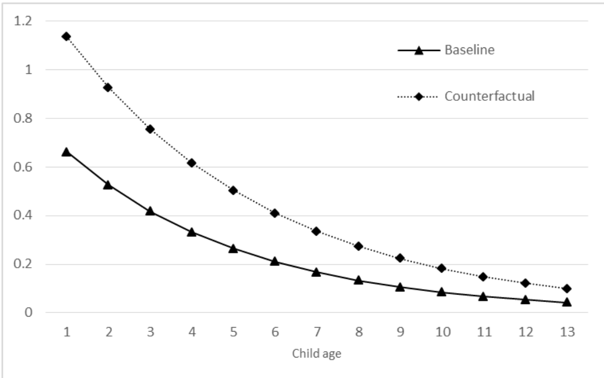
8. Counterfactual model without leisure

This paper estimates a model where mothers can choose how to allocate their time between child care, labor supply and leisure, thus distinguishing between a portion of time out of work that is productive for the child's cognitive development (i.e., time with the child), and a part that is not (i.e., leisure). The introduction of such choice in the model has been motivated by the evidence that there is not a one-to-one corresponding relationship between time out of work and a mother's child care time, and that also the choice of how much time to dedicate to leisure activities can play a role in a mother's time allocation decision. This section discusses the implications of omitting such choice for the estimated productivity of a mother's time with the child.

Previous studies in the literature looking at the effects of non-parental child care and maternal employment on children's outcomes have overlooked the distinction between a mother's time with the child and leisure, and used the total residual time out of work as a measure for maternal child-care time. The use of such proxy may have implications for the effect that is eventually estimated, even though the direction of the bias cannot be clearly anticipated. It could the the case that the use of the proxy may underestimate the productivity of maternal time with the child, because such measure also includes a portion of time with zero productivity (i.e., leisure) and this may bias the estimate toward zero. On the other hand, the measure assumes that the amount of time the mother dedicates to the child is larger than the actual contact time, and this may overestimate the estimated productivity.

In order to shed light on this issue, I re-estimate the model by assuming that the mother allocates all the remaining time out of work to the child, hence without having any leisure.[32] Figure 10 reports the estimated elasticity of a child's ability with respect to a mother's time with the child, in the baseline analysis and in this counterfactual scenario without leisure. The figure shows that the productivity of maternal child-care time estimated in the counterfactual scenario is larger, at any age of the child, and especially during early years, than the one estimated in the baseline analysis. This implies that ignoring the additional choice between leisure and time with the child, and using the residual time out of work as a proxy for the mother's child-care time, overestimates its productivity.

FIGURE 10
Elasticity of child's ability with respect to mother's time with the child, baseline estimate and counterfactual scenario without leisure.



NOTE. This graph represents the productivity parameters for maternal child-care time (τ_t), as a function of child's age $t = 1, 2, 3, \dots 13$, as it is estimated in the baseline model and in a counterfactual model where the mother cannot choose between leisure and time with the child, and all the mother's time is allocated between child care and hours of work.

This result may also explain the strong negative effects of maternal employment found in previous studies. Indeed, the fact that in the baseline analysis the estimated elasticity of a child's ability with respect to a mother's time with the child is larger than the one with respect to non-parental child care suggests that an increase in mother's labor supply may have a negative effect on a child's ability, through a reduction in maternal time in favor of non-parental child care. However, the higher productivity of maternal time estimated in the counterfactual scenario also suggests that such negative effect could be overestimated, if the additional choice between time with the child and leisure is not taken into account.

9. Concluding remarks

This paper estimates a behavioral model where labor supply, non-parental child care, and time allocation choices of the mother are considered endogenous. In contrast to existing studies, I take into account the additional choice the mother makes concerning her time allocation between leisure and time with the child.

In line with previous studies on human capital accumulation reporting diverse productivity of investments over time (Heckman 2008), the results show that the productivity of both maternal child-care time and non-parental child care decreases as the child ages.

[32]The estimation has been done by setting the mother's preference for leisure equal to zero.

Moreover, the elasticity of a child's ability with respect to maternal child-care time is larger than the one with respect to non-parental child care at any age of the child. When the child is one year old, one percent increase in a mother's child-care time leads to an increase in a child's ability by 0.6 percent, while one percent increase in non-parental child care implies an increase in a child's ability by only 0.4 percent. Thus, a mother's employment can be detrimental for the subsequent development of the child if non-parental child care is not productive enough to compensate for the reduction in a mother's time with the child. In this case, some mothers may find it profitable to decrease their labor supply in order to stay home with the child, because they are aware of the lower productivity of the alternative forms of care with respect to theirs.

The estimated model is used to simulate the effects of policies aimed at increasing the household's financial means, at limiting the amount of time mothers spend in the labor market, and at increasing the quality of non-parental child care. The results show that the implementation of all policies induce a change in the mother's labor supply that is not entirely compensated for by the change in a mother's time with the child, confirming that there is not a one-to-one corresponding relationship between time out of work and time with the child. The policy increasing the productivity of non-parental child care to the level of a mother's child-care time induces the largest increase in a mother's labor supply at the intensive margin, which may suggest that mothers react more to changes in the environment where the child is taken care of, than to changes in the labor market opportunities.

The estimation of a counterfactual model where mothers allocate all their time out of work to child care shows that neglecting their choice between child care and leisure leads to an overestimation of the productivity of a mother's child-care time. This may also explain the strong negative effects of maternal employment found in previous studies, that have used a mother's time out of work as a proxy for her total child-care time. Indeed, the higher productivity of maternal time estimated in the counterfactual scenario suggests that the potential negative effect found in the baseline analysis, induced by the higher productivity of a mother's time with respect to the one of non-parental child care, could be overestimated, if the additional choice between time with the child and leisure is not taken into account.

This study provides two relevant insights to the research on the effects of maternal and non-parental child care on the child cognitive development. First, it highlights the importance of considering the mother's time allocation choice between child-care time and leisure. The paper shows that the mothers may not entirely allocate their time out of work to child care and that this has implications for the effects of policies aimed at increasing the amount of time they spend with their child. Second, the paper shows how a mother's labor market participation decisions are affected by the relative productivity of maternal child-care time with respect to non-parental child care.

Nonetheless, the analysis leaves space for further research. For instance, the model does not distinguish between different kinds of child care and assumes that any type of non-parental care has the same productivity for child development. Moreover, little is known about the substitutability or complementarity of mother's child-care time and non-parental child care in the production for cognitive achievement. Future research should better understand how the mother's investment decisions could change, by varying the quality of the alternative forms of care, and how these interact in the production function for child's cognitive ability.

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Appendix A. Analytic solution of the model

In this Appendix I derive analytically the closed-form solutions of the model, for all the choice variables. The process of backward induction involves the solution of the optimization problem in each period, starting from the last one, T. Consider first the choice variables i_t and τ_t. The first step is to find the optimal child care and time input decisions at time T. The value function of the mother at period T can be written as:

$$\begin{aligned} V_T = max_{\tau_T, i_T}\ & \alpha_1 ln(TT - h_T - \tau_T) + \alpha_2 ln(w_T h_T + I_T - pi_T) + \alpha_3 ln(A_T) + \\ & + E_T \beta \{\tilde{V}_{T+1} + \rho \alpha_3 ln A_{T+1}\} \end{aligned} \tag{A.1}$$

where the variables l_T and c_T have been already substituted using the time and budget constraints, the CAPF has been log-linearized for computational convenience, and the braces include the terminal period value function, as specified in (6).

The optimal solutions for τ_T^c and i_T^c at period T, conditional on h_T, are given by the solutions of the following first order conditions (FOCs):

$$\begin{aligned} \tau_T^c &\Rightarrow \frac{\partial V_T}{\partial \tau_T} = 0 \\ i_T^c &\Rightarrow \frac{\partial V_T}{\partial i_T} = 0 \end{aligned} \tag{A.2}$$

Because of the value-added specification of the child cognitive ability production function, as defined by (4), child ability in period $T+1$ is a function of the inputs received by the child at period T. Hence, (A.2) can be rearranged, using total differential, in the following way:

$$\begin{aligned} \tau_T^c &\Rightarrow \frac{\partial \bar{V}_T}{\partial \tau_T} + \frac{\partial V_{T+1}}{\partial ln A_{T+1}} \times \frac{\partial ln A_{T+1}}{\partial \tau_T} = 0 \\ i_T^c &\Rightarrow \frac{\partial \bar{V}_T}{\partial i_T} + \frac{\partial V_{T+1}}{\partial ln A_{T+1}} \times \frac{\partial ln A_{T+1}}{\partial i_T} = 0 \end{aligned} \tag{A.3}$$

where $\bar{V}_T$ is the current utility in period T:

$$\bar{V}_T = \alpha_1 ln(TT - h_T - \tau_T) + \alpha_2 ln(w_T h_T + I_T - pi_T) + \alpha_3 ln(A_T)$$

The corresponding derivatives are given by the following expressions:

$$\frac{\partial \bar{V}_T}{\partial \tau_T} = \frac{-\alpha_1}{TT - h_T - \tau_T} \tag{A.4}$$

$$\frac{\partial V_{T+1}}{\partial lnA_{T+1}} \times \frac{\partial lnA_{T+1}}{\partial \tau_T} = (\beta\rho\alpha_3)\left(\frac{\delta_{1T}}{\tau_T}\right) \tag{A.5}$$

$$\frac{\partial \bar{V}_T}{\partial i_T} = \frac{-p\alpha_2}{w_T h_T + I_T - pi_T} \tag{A.6}$$

$$\frac{\partial V_{T+1}}{\partial lnA_{T+1}} \times \frac{\partial lnA_{T+1}}{\partial i_T} = (\beta\rho\alpha_3)\left(\frac{\delta_{2T}}{i_T}\right) \tag{A.7}$$

and the FOCs become:

$$\tau_T^c \Rightarrow \frac{-\alpha_1}{TT - h_T - \tau_T} + (\beta\rho\alpha_3)\left(\frac{\delta_{1T}}{\tau_T}\right) = 0 \tag{A.8}$$

$$i_T^c \Rightarrow \frac{-p\alpha_2}{w_T h_T + I_T - pi_T} + (\beta\rho\alpha_3)\left(\frac{\delta_{2T}}{i_T}\right) = 0 \tag{A.9}$$

The solutions for the two inputs at period T are given by:

$$\tau_T^c = \frac{\beta\delta_{1T}D_{T+1}}{\alpha_1 + \beta\delta_{1T}D_{T+1}}(TT - h_T) \tag{A.10}$$

$$i_T^c = \frac{\beta\delta_{2T}D_{T+1}}{p(\alpha_2 + \beta\delta_{2T}D_{T+1})}(w_T h_T + I_T) \tag{A.11}$$

where $D_{T+1} = \frac{\partial V_{T+1}}{\partial lnA_{T+1}} = \rho\alpha_3$.

These solutions can be substituted into the value function of the mother at period T, in order to get $V_T(\tau_T^c, i_T^c)$.

Consider now period $T-1$. The value function for this period is:

$$\begin{aligned}
V_{T-1} = max_{\tau_{T-1}, i_{T-1}}\ &\alpha_1 ln(TT - h_{T-1} - \tau_{T-1}) + \alpha_2 ln(w_{T-1}h_{T-1} + I_{T-1} - pi_{T-1}) + \\
&+ \alpha_3 ln(A_{T-1}) + \\
&+ E_{T-1}\beta\{\alpha_1 ln(TT - h_T - \tau_T^C) + \alpha_2 ln(w_T h_T + I_T - pi_T^C) + \alpha_3 lnA_T + \\
&+ \beta\{\tilde{V_{T+1}} + \rho\alpha_3[ln\delta_{0T} + \delta_{1T}ln\tau_T^C + \delta_{2T}lni_T^C + \delta_{3T}lnA_T]\}\}
\end{aligned} \tag{A.12}$$

Applying total differential, the solutions for all inputs in period $T-1$ are given by:

$$\tau_{T-1}^c \Rightarrow \frac{\partial \bar{V}_{T-1}}{\partial \tau_{T-1}} + \frac{\partial V_T}{\partial lnA_T} \times \frac{\partial lnA_T}{\partial \tau_{T-1}} = 0 \tag{A.13}$$

$$i_{T-1}^c \Rightarrow \frac{\partial \bar{V}_{T-1}}{\partial i_{T-1}} + \frac{\partial V_T}{\partial lnA_T} \times \frac{\partial lnA_T}{\partial i_{T-1}} = 0 \tag{A.14}$$

where

$$\bar{V}_{T-1} = \alpha_1 ln(TT - h_{T-1} - \tau_{T-1}) + \alpha_2 ln(w_{T-1}h_{T-1} + I_{T-1} - pi_{T-1}) + \alpha_3 ln(A_{T-1})$$

and

$$\frac{\partial \bar{V}_{T-1}}{\partial \tau_{T-1}} = \frac{-\alpha_1}{TT - h_{T-1} - \tau_{T-1}} \tag{A.15}$$

$$\frac{\partial V_T}{\partial lnA_T} \times \frac{\partial lnA_T}{\partial \tau_{T-1}} = (\beta\rho\alpha_3)\left(\frac{\delta_{1T-1}}{\tau_{T-1}}\right) \tag{A.16}$$

$$\frac{\partial \bar{V}_{T-1}}{\partial i_{T-1}} = \frac{-p\alpha_2}{w_{T-1}h_{T-1} + I_{T-1} - pi_{T-1}} \tag{A.17}$$

$$\frac{\partial V_T}{\partial lnA_T} \times \frac{\partial lnA_T}{\partial i_{T-1}} = (\beta\rho\alpha_3)\left(\frac{\delta_{2T-1}}{i_{T-1}}\right) \tag{A.18}$$

Substituting these expressions, the FOCs for period $T-1$ become:

$$\tau^c_{T-1} \Rightarrow \frac{-\alpha_1}{TT - h_{T-1} - \tau_{T-1}} + (\alpha_3 + \beta\alpha_3)\left(\frac{\delta_{1T-1}}{\tau_{T-1}}\right) = 0 \tag{A.19}$$

$$i^c_{T-1} \Rightarrow \frac{-p\alpha_2}{w_{T-1}h_{T-1} + I_{T-1} - pi_{T-1}} + (\alpha_3 + \beta\alpha_3)\left(\frac{\delta_{2T-1}}{i_{T-1}}\right) = 0 \tag{A.20}$$

The solutions for the choice variables in period $T-1$, conditional on h_{T-1}, are then:

$$\tau^c_{T-1} = \frac{\beta\delta_{1T-1}D_T}{\alpha_1 + \beta\delta_{1T-1}D_T}(TT - h_{T-1}) \tag{A.21}$$

$$i^c_{T-1} = \frac{\beta\delta_{2T-1}D_T}{p(\alpha_2 + \beta\delta_{2T-1}D_T)}(w_{T-1}h_{T-1} + I_{T-1}) \tag{A.22}$$

where

$$D_T = \frac{\partial V_T}{\partial lnA_T} = \alpha_3 + \beta\delta_{3t}\underbrace{(\rho\alpha_3)}_{D_{T+1}}$$

The solutions for period $T-1$ can be substituted in (A.12) to get $V_{T-1}(\tau^c_{T-1}, i^c_{T-1})$. This expression can be used to write down the value function at period $(T-2)$. Using the same process described for periods T and $(T-1)$ and computing the corresponding derivatives yield the solutions for period $(T-2)$. The solutions for all the periods up to period $t = 1$ can be retrieved similarly.

At the end, two sequences of optimal choices can be obtained. The sequence of optimal choices for time with the child, conditional on the mother's labor supply, is given by:

$$\tau_T^c = \frac{\beta\delta_{1T}D_{T+1}}{(\alpha_1 + \beta\delta_{1T}D_{T+1})}(TT - h_T) \tag{A.23}$$

$$\tau_{T-1}^c = \frac{\beta\delta_{1T-1}D_T}{(\alpha_1 + \beta\delta_{1T-1}D_T)}(TT - h_{T-1}) \tag{A.24}$$

$$\tau_{T-2}^c = \frac{\beta\delta_{1T-2}D_{T-1}}{(\alpha_1 + \beta\delta_{1T-2}D_{T-1})}(TT - h_{T-2}) \tag{A.25}$$

$$\vdots$$

$$\tau_t^c = \frac{\beta\delta_{1t}D_{t+1}}{(\alpha_1 + \beta\delta_{1t}D_{t+1})}(TT - h_t) \tag{A.26}$$

$$\vdots$$

$$\tau_2^c = \frac{\beta\delta_{12}D_3}{(\alpha_1 + \beta\delta_{12}D_3)}(TT - h_2) \tag{A.27}$$

$$\tau_1^c = \frac{\beta\delta_{11}D_2}{(\alpha_1 + \beta\delta_{11}D_2)}(TT - h_1) \tag{A.28}$$

Equation (A.26) is equal to equation (7) in the text.

The sequence of the optimal non-parental child care choices, conditional on the mother's labor supply, is given by:

$$i_T^c = \frac{\beta\delta_{2T}D_{T+1}}{p(\alpha_2 + \beta\delta_{2T}D_{T+1})}(w_T h_T + I_T) \tag{A.29}$$

$$i_{T-1}^c = \frac{\beta\delta_{2T-1}D_T}{p(\alpha_2 + \beta\delta_{2T-1}D_T)}(w_{T-1}h_{T-1} + I_{T-1}) \tag{A.30}$$

$$i_{T-2}^c = \frac{\beta\delta_{2T-2}D_{T-1}}{p(\alpha_2 + \beta\delta_{2T-2}D_{T-1})}(w_{T-2}h_{T-2} + I_{T-2}) \tag{A.31}$$

$$\vdots$$

$$i_t^c = \frac{\beta\delta_{2t}D_{t+1}}{p(\alpha_2 + \beta\delta_{2t}D_{t+1})}(w_t h_t + I_t) \tag{A.32}$$

$$\vdots$$

$$i_2^c = \frac{\beta\delta_{22}D_3}{p(\alpha_2 + \beta\delta_{22}D_3)}(w_2 h_2 + I_2) \tag{A.33}$$

$$i_1^c = \frac{\beta\delta_{21}D_2}{p(\alpha_2 + \beta\delta_{21}D_2)}(w_1 h_1 + I_1) \tag{A.34}$$

Equation (A.32) is equal to (8) in the main text, and the sequence of values for D_{t+1} is reported in (9) in the main text.

Having found the solutions for the time allocation and non-parental child care, the solution for the labor supply can be computed using the same backward procedure. Equation (10) represents the optimal labor supply in each period as a function of τ_t and i_t; substituting (7) and (8), it yields the optimal labor supply choice for each period t, as defined by (11).

Appendix B. The PSID data and the CDS-TD supplements

The dataset is composed of different supplements of the Panel Study of Income Dynamics (PSID) gathered in the period 1985-2007. Table B.1 summarizes the main information on availability and sources of data.

Table B.1
Availability and sources of data.

Set of Variables	**Source**	**Survey Years**	**Additional Info**
Non-parental child care	CDS	1997-2002-2007	Retrospective questions on all arrangements used from birth until kindergarten enrollment and questions on the arrangement used at the time of the survey
Child cognitive outcomes	CDS	1997-2002-2007	Only for children older than 3
Child demographic characteristics	CDS	1997-2002	Time-invariant (except *age*)
Maternal time with the child	CDS-TD	1997-2002	Available only for the year of the survey
Parents' hours of work	PSID	1985, 1986, 1987, 1988, 1989, 1990, 1991, 1992, 1993, 1994, 1995, 1996, 1997, 1999, 2001, 2003, 2005, 2007	Referred to the year before the survey
Parents' wages	PSID	1985, 1986, 1987, 1988, 1989, 1990, 1991, 1992, 1993, 1994, 1995, 1996, 1997, 1999, 2001, 2003, 2005, 2007	Referred to the year before the survey
Parents' non-labor income	PSID	1985, 1986, 1987, 1988, 1989, 1990, 1991, 1992, 1993, 1994, 1995, 1996, 1997, 1999, 2001, 2003, 2005, 2007	Referred to the year before the survey
Parents' demographic characteristics	PSID	1997	Time-invariant (except *age*)

To merge PSID and CDS data I exploit the information on the relationship of each CDS child with respect to the head of the household and the primary caregiver. The final sample is made up of all children aged 0-12 in 1997 without siblings and with both parents living in the household, without missing information on child's and parents' characteristics and with at least one test score observation. As summarized in Table B.2, the birth cohorts of children in this sample range from 1984 to 1996, while the terminal period of the model ($T = 13$) corresponds to 1997 for those born in 1984 and to 2009 for those born in 1996.

Table B.3 summarizes the available data for a child born in 1996. This table stresses the existence of a long time-gap of missing data, because of the structure of the surveys and the timing of the interviews. In particular, data on maternal time and child's cognitive outcomes are available only in the years of the TD and CDS supplements, i.e., 1997, 2002 and 2007. Data on non-parental child care suffers from the same issue after kindergarten age, since that information is available only for the year of the CDS survey.

Table B.4 shows the average characteristics of the sample used for the estimation ($N = 417$) and of the total sample of children in CDS, for whom it has been possible to derive information on their parents (3243 observations). This comparison sample includes both families with only one child and families with more children.

Table B.2
Cohorts of children in the final sample.

Year of Birth	Child's Age					
$t = 0$	$t = 1$	$t = 2$	$t = 3$	$\cdots$	$t = 12 = T - 1$	$t = 13 = T$
1984	1985	1986	1987	$\cdots$	1996	1997
1985	1986	1987	1988	$\cdots$	1997	1998
1986	1987	1988	1989	$\cdots$	1998	1999
1987	1988	1989	1990	$\cdots$	1999	2000
1988	1989	1990	1991	$\cdots$	2000	2001
1989	1990	1991	1992	$\cdots$	2001	2002
1990	1991	1992	1993	$\cdots$	2002	2003
1991	1992	1993	1994	$\cdots$	2003	2004
1992	1993	1994	1995	$\cdots$	2004	2005
1993	1994	1995	1996	$\cdots$	2005	2006
1994	1995	1996	1997	$\cdots$	2006	2007
1995	1996	1997	1998	$\cdots$	2007	2008
1996	1997	1998	1999	$\cdots$	2008	2009

Table B.3
Available data for a child born in 1996.

	Child's age (t)													Source	Survey Year
	1	2	3	4	5	6	7	8	9	10	11	12	13		
Non-parental child care	X	X	X	X	X		X					X		CDS	1997, 2002, 2007
Child cognitive outcomes						X					X			CDS	2002, 2007
Child demographic charact.	X					X					X			CDS	1997, 2002, 2007
Maternal time with the child	X					X								TD	1997, 2002
Parents' hours of work		X		X		X		X		X				PSID	1999, 2001, 2003, 2005, 2007
Parents' wages		X		X		X		X		X				PSID	1999, 2001, 2003, 2005, 2007
Parents' non-labor income		X		X		X		X		X				PSID	1999, 2001, 2003, 2005, 2007
Parents' demographic charact.	X		X		X		X		X					PSID	1997, 1999, 2001, 2003, 2005, 2007

Appendix C. Estimation

The estimation is done in two-stages: the parameters of the income process are estimated in the first stage, while all remaining parameters are estimated in the second stage. After computing the statistics defined in Table 3 for the actual data, I proceed with the first-stage estimation of the income parameters. This involves the simulation of the income

Table B.4
Mean characteristics of the sample with respect to PSID-CDS data.

	PSID-CDS	Sample	T-test
Mother's hours of work	23.60	27.30	-10.71^{***}
Non-parental child care	12.34	14.74	-6.75^{***}
Mother's time with the child	25.83	21.16	5.42^{***}
Mother's wage before child's birth	11.01	11.31	-1.25
Mother's education	12.99	13.27	-7.03^{***}
Mother's age at child's birth	26.99	28.20	-14.43^{***}
Mother's race: white	0.62	0.61	0.33
Child's gender: male	0.51	0.51	0.29
Child's birth weight (grams)	3315.53	3387.16	-7.77^{***}
Household income	674.16	791.36	-7.56^{***}

[a] Monetary variables deflated into 1997 US$.
[b] Mother's wage before childbirth refers to the year before the child was born.
[***] Difference statistically significant at the $p < 0.01$ level.

process, after drawning from a standard normal distribution $N \times R$ times, for every period. The statistics used to estimate these parameters are the average and standard deviation of income for all the periods, as well as the average household income by a father's level of education, race and age. I compute these points for both the actual and the simulated income processes. The Method of Simulated Moments estimator for this first stage minimizes an objective function where each moment condition is the distance between the income data moments and their simulated counterparts. Each moment condition is weighted using the inverse of the corresponding statistics in the data.

The second-stage involves the estimation of all remaining parameters using the same estimator. First of all, I simulate the data according to the DGP implied by the model, taking $N \times R \times T$ draws for wage and income and $N \times R$ draws for the child's initial ability shock, the mother's skills, as well as the mother's preferences. Following Keane and Moffitt (1998), I re-draw the errors to simulate the income distribution using the parameters estimated in the first stage. In each period, the values for the mother's labor supply, non-parental child care and maternal time are derived using the optimal solutions implied by the model. Then, after having simulated the data for all the periods, I compute the statistics defined in Table 3 from the simulated data.

The estimator used in this second-stage minimizes an objective function where each moment condition is the distance between the data statistics and the simulated counterparts, as summarized by Table 3:

$$\hat{\theta} = arg\ min\ \hat{g}(\theta)'W\hat{g}(\theta) \tag{C.1}$$

where

$$\hat{g}(\theta) = \hat{m} - \hat{M}(\theta)$$

$\hat{m}$ is the vector of statistics defined from the actual data, while $\hat{M}(\theta)$ is the vector of simulated statistics according to the model that are functions of the structural parameters to be estimated. W is a positive definite diagonal weighting matrix. The most efficient minimum distance estimator uses a weighting matrix whose elements are estimates of the inverse of the covariance matrix of the vector $\hat{m}$; this is the so-called optimal minimum distance (OMD) estimator (Cameron and Trivedi 2005, pag. 203). Since Altonji and Segal (1996) provide evidence of small sample biases in the OMD estimator, I use the diagonally weighted minimum distance estimator proposed by Blundell, Pistaferri, and Preston (2008). Given S number of moments, the weighting matrix is then defined as:

$$W = \begin{pmatrix} \hat{V}[\hat{m}_1]^{-1} & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & \hat{V}[\hat{m}_S]^{-1} \end{pmatrix}$$

where $\hat{V}[\hat{m}]$ is estimated with non-parametric bootstrap and according to the formula (Davidson and MacKinnon 2003, p. 208):

$$\hat{V}\left[\hat{m}\right] = \left[\frac{1}{B}\right] \sum_{b=1}^{B} \left(\hat{m}_b^* - \bar{m}^*\right)\left(\hat{m}_b^* - \bar{m}^*\right)' \tag{C.2}$$

Non-parametric bootstrap (with replacement) is implemented following Wooldridge (2002, p. 379): I use a random number generator to obtain N integers, where $N = 417$ represents the sample size of the actual data, and these integers index the observations drawn from the actual distribution of data. Repeating this process B times, it yields B bootstrap samples on which the statistics defined in Table 3 can be computed: $\hat{m}_b^*$ represents a statistic computed for the sample b, while $\bar{m}^*$ is the average of the statistics across the B samples.[33]

Figure C.1 shows the variation in the objective function (Equation (C.1)) induced by the perturbation of each estimated parameter in the vector $\hat{\theta}$.

C.1. ***Standard errors.*** Non-parametric bootstrap with replacement is also used to compute the standard errors. After having drawn B_{se} samples from the actual data, I repeat the estimation of the parameters for each sample, by using different starting values for each bootstrap iteration.[34] This yields an empirical distribution of the parameters estimates, from which I can recover a bootstrap estimate of the variance, using the formula (Train 2009, pag. 201):

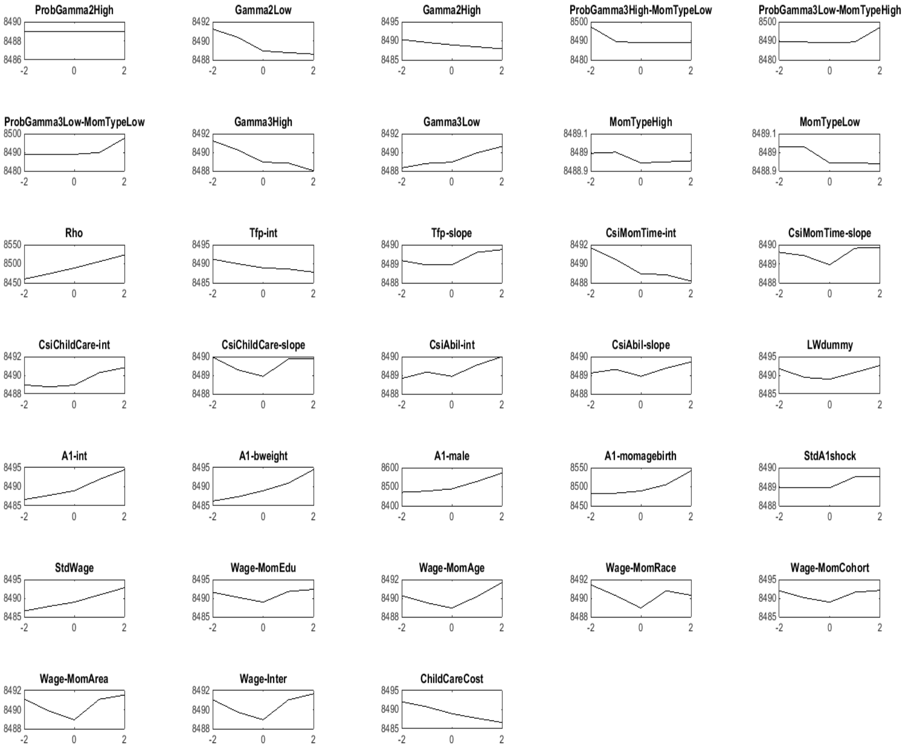
$$\hat{V}\left[\hat{\theta}\right] = \left[\frac{1}{B}\right] \sum_{b=1}^{B} \left(\hat{\theta}_b^* - \bar{\theta}^*\right)\left(\hat{\theta}_b^* - \bar{\theta}^*\right)' \tag{C.3}$$

Taking the square root of (C.3) yields the bootstrap estimate of the standard errors $se_{\hat{\theta}}$.

[33] $B = 200$.
[34] $B_{se} = 50$

Figure C.1

Variation in the objective function around the estimated parameters.



NOTE. This graph reports the values of the objective function perturbing each parameter by 2 standard deviations up and down with respect to the estimated value.

Appendix D. Estimated parameters

Table D.1 reports the estimated untransformed parameters in the mother's utility function (Panel A), the estimated parameters in the household income function (Panel B), and the estimated parameters in the initial level of ability of the child and the score specification (Panel C).

Appendix E. Sensitivity analysis

I check the sensitivity of the results presented in Section 6 with respect to two main dimensions. For the sake of brevity, I report and discuss only the productivity parameters.

E.1. ***Mother's and father's time investments.*** In the baseline analysis, the variable weekly time with the mother is defined considering the time spells when the child is with the mother, either being the mother directly involved in the child's activities or being just around and not participating. This implies that only the mother's time is productive for the child cognitive development, while the father's contribution only comes through his labor income that affects the mother's investment decisions. This specification rules out the possibility that the time the father spends alone with the child, and the amount of time that the mother *and* the father spend with the child together, do not systematically respond to the mother's employment decisions, and do not affect the child development process. Figure E.1 shows the kernel density distribution of both categories of time by

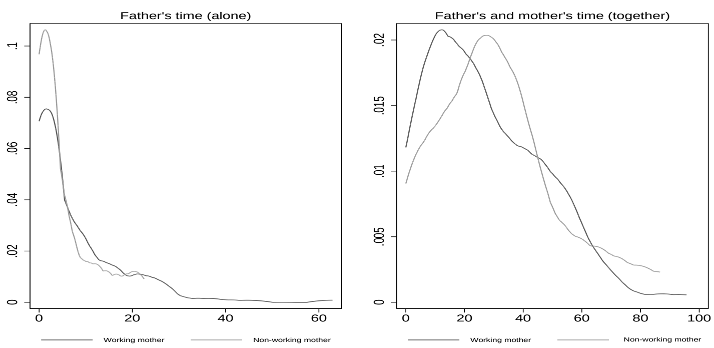
Table D.1
Estimated (untransformed) parameters in the mother's utility function, parameters in the household income function and in the initial level of ability of the child.

		Estimate	Std. Errors
Panel A. Untransformed parameters in the mother utility function			
γ_{2low}	Utility from consumption Type I	-0.5891	0.1594
γ_{2high}	Utility from consumption Type II	-0.3137	0.1201
γ_{3high}	Utility from child ability Type II	0.2574	0.3633
γ_{3low}	Utility from child ability Type I	0.0856	0.2844
Panel B. Parameters in household income function			
σ_{inc}	Std deviation income shock	0.6185	0.0362
μ_{inc0}	Intercept	-0.3759	0.3083
$\mu_{inc\ edu}$	Coefficient for father's years of education	0.1263	0.0148
$\mu_{inc\ race}$	Coefficient for father's race	0.2162	0.0521
$\mu_{inc\ age}$	Coefficient for father's age	0.0102	0.0054
Panel C. Parameters in initial level of ability and score specification			
η_0	Intercept	-72.3416	6.4213
$\eta_{birthweight}$	Coefficient of birth weight	17.2894	17.3953
η_{gender}	Coefficient of gender	59.0441	60.8817
$\eta_{mom\ age}$	Coefficient of mother's age at birth	-8.5614	1.2254
σ_v	Std deviation initial ability shock	20.5048	1.5788
λ	Coefficient for LW test score (vs AP)	-0.5348	0.0674

NOTE. Standard errors are estimated with non-parametric bootstrap, by changing the starting values in each bootstrap estimation. See Appendix C.1 for further details.

a mother's employment status, and suggests that, while there being a slightly higher proportion of fathers spending a positive amount of time with their children if the mother works, the time the parents share with the child does not seem to vary systematically with the mother's decision to work. In order to test whether these issues affect the estimated productivity parameters, I perform two further analyses, aimed at taking into account the differential involvement of both parents with the child.

Figure E.1
Father's time with the child (Left) and both parents' time with the child (Right), by mother's employment status.



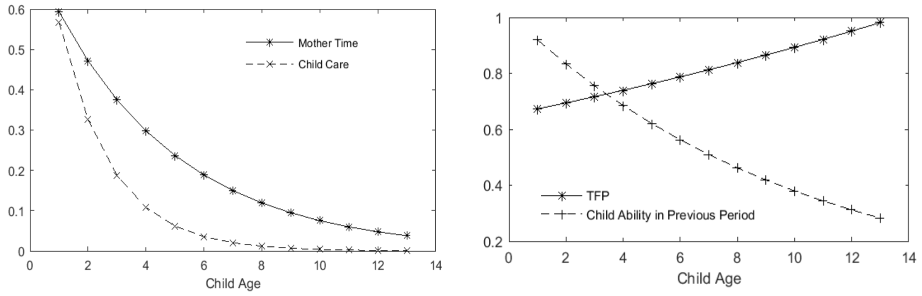
NOTE. This graph represents the Kernel-density distribution of a father's time with the child (Left) and of both parents's time with the child (Right), by a mother's employment status.

The first analysis implies to re-estimate the model, by keeping the baseline definition of a mother's time with the child, but focusing on the subsample of children with whom the father spends more time, i.e. above the median, which is 2.08 hours per week.[35] In other

[35]This results in an estimation sample of 250 observations. The information about the amount of time spent with the father still comes from the Time Diary components of the PSID-CDS, held in 1997 and 2002. For the children for whom such information is available in both surveys, hence at two points in time,

words, this analysis aims at checking whether the results found in the baseline sample hold for a subsample in which the father is more involved with the child. The results are shown in Figure E.2, and confirm that, even in presence of fathers who spend more time with their children, the productivity of the mother's time with the child is still larger than the one of non-parental child care. Interestingly, also the estimated productivity of lagged ability and the estimated total factor productivity are qualitatively similar to the ones estimated in the baseline analysis.

FIGURE E.2
Estimated productivity parameters for the subsample of children with above-median time with the father.



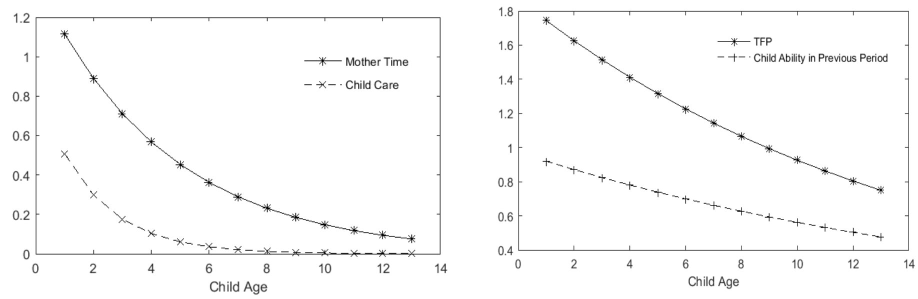
NOTE. This graph represents the productivity parameters for maternal time (τ_t) and non-parental child care (i_t) (Left), as well as the productivity parameter for the lagged level of ability of the child and the estimated total factor productivity (TFP) (Right), as a function of child's age $t = 1, 2, \ldots 13$, for the subsample of children whose father's time is above the median in the sample, i.e. larger than 2.08 hours per week.

The second analysis that I perform involves the re-estimation of the model by changing the time specification. Instead of using only the time the child spends with the mother alone, I add to this measure the amount of time the child spends with both parents (i.e., the measure reported in Figure E.1, Right panel). Results are reported in Figure E.3. The Left panel of the figure shows that the estimated productivities of time and non-parental child care are qualitatively similar to the ones presented in the main analysis, even though the productivity of time is larger than in the baseline. The Right panel shows that with the new time measure the estimated total factor productivity is decreasing over time. This may suggest that the time the child spends with both parents represents a relevant part of the time investments he receives when he grows up, and that, once this component is taken into account, the role of the residual inputs decreases over time.

E.2. ***Child care and schooling.*** The second issue relates to the absence of schooling inputs in the child development process specified in Equation (4). Thus, I check the sensitivity of the estimated productivity parameters to this dimension, by adding to the measure of non-parental child care an amount corresponding to the time the child spends at school. This information comes from the Time Diary component of the PSID, gathered in 1997 and 2002. The main problem with the use of this variable is that it is cross-sectional (i.e., available only for the year of the survey), and assumptions should be made on how to assign these values to the missing ages as well. For this sensitivity analysis,

I take the highest value: this implies that a child is included in the sample if the father's child care time (alone) is larger than the median at least once.

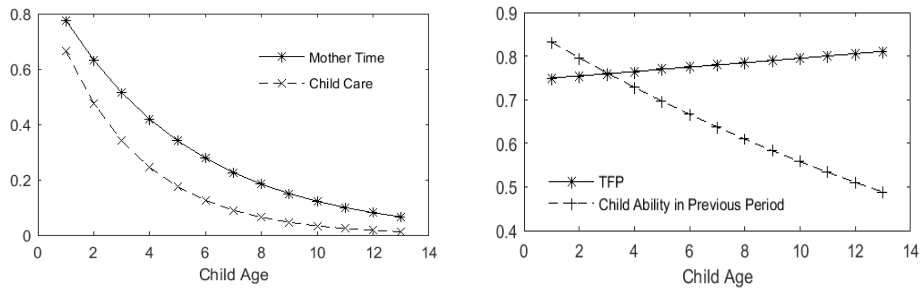
Figure E.3
Estimated productivity parameters, in case maternal time includes also the time the child spends with both parents.



NOTE. This graph represents the productivity parameters for maternal time (τ_t) and non-parental child care (i_t) (Left), as well as the productivity parameter for the lagged level of ability of the child and the estimated total factor productivity (TFP) (Right), as a function of child's age $t = 1, 2, \ldots 13$, in a model where τ includes the time the child spends with both parents.

I recode such variable according to the age of the child. For the children who have a schooling information after age five, I assign that to all ages afterwards. To those, instead, who just have a schooling information before age five, which is equal to zero, I assign the median amount of time in school after age five in the sample. I then re-estimate the model by using a measure of non-parental child care time that incorporates the amount of time the child spends at school after age five. The estimated productivity parameters and total factor productivity are reported in Figure E.4, and confirm the patterns of the parameters estimated in the baseline analysis.

Figure E.4
Estimated productivity parameters, in case non-parental child care also includes time at school.



NOTE. This graph represents the productivity parameters for maternal time (τ_t) and non-parental child care (i_t) (Left), as well as the productivity parameter for the lagged level of ability of the child and the estimated total factor productivity (TFP) (Right), as a function of child's age $t = 1, 2, \ldots 13$, in a model where i_t includes the time spent in non-parental child care and, after age five, at school.