WORKING PAPERS IN ECONOMICS

No 264

Market Imperfections and Wage Inequality

by

Klas Sandén

September 2007

ISSN 1403-2473 (print)
ISSN 1403-2465 (online)
Market Imperfections and Wage Inequality*

Klas Sandén†

Department of Economics
School of Business, Economics and Law
Göteborg University
P.O. Box 640
SE 405 30 Göteborg
Sweden

September 9, 2007

Abstract

This paper investigates the relationship between various market imperfections and the skill premium. The model in this paper assumes perfectly competitive labor markets but distorted product and financial markets. The model predicts that the skill premium is positively correlated with market power, modeled using preference for variety, and shorter product cycles. The effect from financial market distortions or taxes on financial income is ambiguous. Positive external effects among firms developing new goods decrease the skill premium.

JEL: D33, D43, D50, D91, D92, J31, L13, O31
Keywords: Wage Inequality, Monopolistic Competition, Innovation

1 Introduction

After several decades of decreasing wage inequality, most industrialized countries have experienced substantial increases in the dispersion of wages. The U.S. and the U.K. witnessed the change in the early 1980s while many other industrialized countries have seen similar changes during the second half of the

*This work was done at the Department of Economics at Göteborg University. I would like to thank prof. Douglas A. Hibbs, Jr., Włodek Bursztyn, Håkan Locking, and prof. Henry Ohlsson for comments on earlier drafts.
†Correspondence to: klas.sanden@vxu.se
1980s or early in the 1990s (Juhn et al. 1993; Gottschalk 1997; Glyn 2001). The dramatic changes in the wage distribution has generated a large literature that has tried to explain the changes, see for example Acemoglu (2002); Aghion (2002). This literature has increased the understanding of what factors that are important in explaining the distribution of wages. This paper adds to this literature by providing further insight into how market power, the length of the product cycle, and financial market distortions, is related to the return to skill, i.e. the skill premium.

The main result in the paper is that greater market power, modeled by greater preference for variety by consumers, increases the skill premium, shorter product cycles increase the skill premium, capital taxation or capital market distortions have an ambiguous impact on the skill premium, and positive externalities among firms developing new goods decrease the skill premium. A fundamental characteristic of the model in this paper is the division of labor tasks into two distinct categories, production and development, which have different skill requirements. The model postulates that only high-skill workers do development work while only low-skill workers do production work, a crude implementation of the hypothesis that development is human capital intensive.

Further, the model postulates that development must always precede production. Development is costly and financed by households via ownership. Product markets are not perfectly competitive, implying that in equilibrium the profit rate is sufficiently high to motivate households to invest in owner shares. A key insight necessary to understand the predictions of the model is that while production employment increases with competitiveness, development employment decreases because lower profits imply less incentive to develop new products. Therefore the skill premium is closely related to market power.

1.1 Related Literature

The connection between market power, via the ability to pay, and wage premia is well documented by, among others, Blanchflower et al. (1996), Nickell et al. (1994). and Nickell (1999). The discussion generally concerns the distribution of labor market rents among workers and owners via collective bargaining.
between firm and union representatives. This paper on the other hand assumes perfectly competitive labor markets, thereby departing from the assumptions of most labor economists.

This paper has similarities with Mendez (2002), which studies the relation between product life cycles and wage inequality. In Mendez’s dual labor market setting, efficiency wages are paid to workers producing goods in the early stage of the product cycle, while competitive wages are paid to workers producing goods in the later stage of the product cycle. In Mendez setting, shorter product cycles affect wage inequality, but in an ambiguous direction.¹

In Glazer and Ranjan (2001) preference for variety contributes to increased wage differences between high and low-skill workers. However, in Glazer and Ranjan’s paper, the main assumption is that high-skill workers prefer consuming goods produced by high-skill labor, while low-skill workers prefers consuming goods produced by low-skill labor. Preference for variety is a necessary assumption because, in the Dixit and Stiglitz (1977) framework, increasing the number of variations of a good generates a positive externality, increasing the utility of every other variation of the good.

The paper by Dinopoulos and Segerström (1999) is somewhat similar to this paper. Both papers connect the profitability of development, labeled research in Dinopoulos and Segerström, with the demand for high-skill workers. However, in Dinopoulos and Segerström lower tariff rates motivate more development, via higher temporary Schumpeterian profits. In both papers, high-skill workers benefit, relative to low-skill workers, from higher profits. Other studies where high-skill workers do “fixed cost work” and low-skill workers do “production work” are Ranjan (2001), Ekholm and Midelfart (2005), and Burda and Dluhosch (2002). None of those papers investigate the impact of changing the preference for variety, the length of the product cycle, financial market distortions, or externalities among firms developing new goods.

An integral part of the model is preference for variety in consumption, modeled using the same

¹Mendez is primarily concerned with residual wage inequality, i.e. wage inequality between workers with similar observable characteristics, but he also briefly discusses the skill premium, which is shown to be positively correlated with residual wage inequality.
setup as in Dixit and Stiglitz (1977). The preference for variety provides firms with some market power. Without market power firms would not be able mark up prices above marginal cost, which is necessary to recapture development costs.

1.2 Plan of the Paper

The production side of the model is laid out in Section 2. Section 3 gives the various market clearing conditions. In Section 4 the intertemporal choices by households are analyzed, the model is closed, and the results are presented. Section 5 summarizes and discusses the results. Appendix A supplies a record of notation used in the paper. Lengthy derivations are presented in Appendices B and C.

2 Model

The model is dynamic, and solved in three stages. In the first stage (Section 2) the instantaneous choices made by households and firms are analyzed. In the second stage (Section 3) the instantaneous equilibrium conditions are added to the model. Finally, in the third stage (Section 4) the intertemporal choices of households together with the necessary steady state conditions are used to solve the model.

Consider an economy consisting of \( L \) households, each with a single divisible labor unit. A fraction \( \phi L \) of the households supply high-skill labor and \( (1 - \phi)L \) supply low-skill labor. There are two types of goods in the economy, a consumption good and a capital good. There are \( nL \) different variations of the consumption good, where \( n \) denotes the number of variations per household. There are \( \pi L \) different production firms producing variations of the consumption good. Hence, every production firm produces a single variation of the consumption good. New variations are developed by development firms.

Let \( \bar{k} \) denote the amount of capital per household. The amount of capital available for use in production, \( \bar{k} L \), is determined endogenously. Capital depreciates and must constantly be reproduced. Capital is chosen as the numeraire good and its price is normalized to unity. It is assumed that households sup-
ply firms with capital via financial markets, but households are subject to a capital tax or some other distortion.

The consumption good is more attractive to produce because households care for variety, which provide firms with some market power. However, the lifetime of any variation of the consumption good is limited and uncertain. If a variation of the consumption good becomes obsolete, the firm can not sell any output and the firm is shut down. The market for real capital is perfectly competitive with zero profits.

2.1 Demand

Let $y$ denote household net income, $c$ household consumption, $s$ household saving in capital, and $m$ household development saving. Let $\overline{y}, \overline{c}, \overline{s}$ and $\overline{m}$ denote the corresponding averages over all households.

Consider any household in the economy with a net income of $y$ and let consumption be given by $c = y - s - m$. Total saving by a household is $s + m = y - c$. Total saving falls into two different categories, real capital and owner shares (development saving). By purchasing $s$ worth of newly produced capital, households add new capital to its existing stock of capital. By providing development firms with $m$ worth of financial capital households can increase its stock of owner shares in production firms, $n$.

The household devotes $c$ for consumption of the single consumption good. Instantaneous utility is characterized by:

$$u(c) = u[v(c)].$$

The auxiliary $v(c)$ function is defined by optimal allocation of consumption over the different variations
of the consumption good, given the household’s choice of consumption spending, \( c \):

\[
v(c) = \max \left[ \sum_{i=1}^{\pi L} x_i^{1-\beta} \right]^{\frac{1}{1-\beta}}
\]

\[s.t. \quad \sum_{i=1}^{\pi L} p_i x_i = c.\]

The variable \( x_i \) denotes the household’s consumption of the \( i \)th variation of the consumption good. \( \beta \in [0, 1) \) parameterizes household demand for variety, and thereby also contributes to market power of production firms. The solution to this problem (see Appendix B for a derivation) is easily obtained:

\[
x_i(c) = \frac{c}{p_i^{1/\beta} \hat{\beta}}
\]

\[
v(c) = \frac{c \hat{\beta}^{\frac{1}{1-\beta}}}{\hat{\beta}}
\]

\[
\hat{\beta} = \sum_{i=1}^{\pi L} \frac{p_i}{\beta}^{-\frac{1}{1-\beta}}.
\]

Since the demand function is linear in \( c \), aggregate demand is consistently analyzed using a representative agent with average consumption spending. Therefore let \( \bar{c} \) denote the average of all households’ consumption spending:

\[
x_i(\bar{c}) = \frac{\bar{c}}{p_i^{1/\beta} \hat{\beta}}
\]

Relation (4) together with (3c) defines the demand function for any variation of the consumption good.
2.2 Capital Producers

The price of capital is normalized to unity, and the technology for capital production is given by a Cobb-Douglas production function in low-skill labor and capital. Capital producing firms operate on a perfectly competitive market, which is a logical assumption since capital produced by different firms are perfect substitutes in all production activities. The constant return to scale technology and the zero profit condition implies that the number of firms competing is indeterminate, but production of capital can be modeled as if there is a single price taking firm. The firm manager solves the following problem:

\[
\max_{K_k, L_k} a_l K_k^{\alpha_l} L_k^{1-\alpha_l} - rK_k - w_l L_k.
\]

The capital producer hires low-skill labor, \(L_k\), and capital \(K_k\). The \(K_k\) units of capital are rented from households. The wage rate of low-skill workers is denoted \(w_l\) and \(r\) denotes the interest rate for capital. Overall productivity is denoted by \(a_l\), the marginal rate of technical substitution between capital and labor is given by \(\alpha L_k [(1 - \alpha) K_k]^{-1}\).

Each household saves \(s\) in capital and thereby demands \(s\) new capital units. Aggregate demand for new capital therefore equals \(\bar{\sigma} L\). Combing the first order conditions for the problem above with the aggregate demand for capital, i.e. \(\bar{\sigma} L = a_l K_k^{\alpha_l} L_k^{1-\alpha_l}\), yields the factor demand functions for firms producing capital:

\[
K_k(\bar{\sigma}, r) = \frac{\alpha \bar{\sigma} L}{r},
\]

\[
L_k(\bar{\sigma}, w_l) = \frac{(1 - \alpha) \bar{\sigma} L}{w_l}.
\]
2.3 Consumption Good Producers

The production technology used by consumption good producers is the same Cobb-Douglas technology used by capital producers. Let $b_c$ denote the corresponding cost function. A firm producing a variation of the consumption good can alter the employment of low-skill production workers instantly. Therefore, given production during a short interval of time, the firm solves the following problem:

$$\max_{p_i} \quad p_i x_i - b_c(x_i)$$

s.t. \quad \begin{align*}
    x_i &= \frac{rL}{p_i^{1/\beta} \hat{p}} \\
    b_c(x_i) &= \frac{x_i}{a(1-\alpha)} \left[ \frac{wL}{1-\alpha} \right]^{1-\alpha}.
\end{align*}

The firm maximizes revenues minus cost under the demand and technology constraint. The demand constraint is given by relation (4) and the technology constraint is given by the cost function, corresponding to the Cobb-Douglas production function. The firm treats all variables, except the price of the firm’s own variation, $p_i$, as given, i.e. $\partial \hat{p} / \partial p_i = 0$. This is perfectly consistent with rational behavior only if the number of competing firms is infinite, i.e. $nL \to \infty$. Solving the maximization problem (see Appendix C) implies:

$$\begin{align*}
p_i &= \frac{r^\alpha w_i^{1-\alpha}}{a(1-\beta)^{\alpha^\alpha (1-\alpha)^{1-\alpha}}} \quad (7a) \\
x_i(\pi, \bar{\pi}) &= \frac{x_i}{a(1-\beta)^{\alpha^\alpha (1-\alpha)^{1-\alpha}}} \quad (7b) \\
\pi_{ci}(\pi, \bar{\pi}) &= \frac{\beta \pi}{\bar{\pi}}. \quad (7c)
\end{align*}$$
It is immediately clear that zero profits can only occur in two ways; either households do not care for variety and firms have no market power, i.e. $\beta = 0$, or the number of firms producing variations is infinite, i.e. $nL \to \infty$. Labor and capital demand functions conditioned on the quantity produced, $x_i$, materialize in the process of deriving the cost function, $b_c$. Inserting the quantity given by (7b) yields the factor demand functions for firms producing the consumption good:

\[ k_{ci}(\tau, n, r) = \frac{\alpha(1 - \beta)c}{n} \] (8a)

\[ l_{ci}(\tau, n, w_l) = \frac{(1 - \alpha)(1 - \beta)c}{n} \] (8b)

### 2.4 Development Firms

New variations can be developed by combining high-skill labor and capital. More formally, a firm hiring $k_{dj}$ units of capital and $h_{dj}$ units of high-skill labor produces the “development intensity” $z_j$. Depending on the model, $z_j$ can have different interpretations.

In a continuous time setting, it is logical for $z_j$ to represent a firm-specific Poisson process intensity, where a development event implies that the firm succeeds in developing a new variation of the consumption good. During a short period of length $dt$, the probability that a single development event occurs is $z_jdt$.

The logical equivalence in a discrete time setting is that $z_j$ represents the mean of a Poisson distributed random variable, where $z_jdt$ is the expected number of successful developments events during a given time period, $dt$. Alternatively in the discrete time setting $z_j$ can represent some index increasing in the expected number of successful developments.

The technology available for producing the “development intensity” is:

\[ z_j = \left[ a_h h_{dj}^{1 - \gamma} \right]^{\sigma} \tau^{1 - \sigma} \] (9a)
Hence, by hiring more high-skill labor and capital, a development firm increases the probability developing a new variation, or the expected number of new successful developments. If $\sigma$ equals unity, there are no externalities and the production function for development intensity reduces to a standard Cobb-Douglas production function. However as $\sigma$ approaches zero, the incentive to free ride increases as a given firm’s development effort become less important relative to the average effort, denoted by $\bar{z}$.

The parameter $a_h$ parameterizes overall development efficiency and $\gamma$ denotes the relative importance of capital compared to high-skill labor. If a Poisson event occurs, the firm succeeds in developing a new variation of the consumption good. The associated cost function, $b_d$, and factor demand functions for development firms are:

\[
b_d(z_j) = \frac{1}{a_h} \left[ \frac{r}{\gamma} \right]^\gamma \left[ \frac{w_h}{1-\gamma} \right]^{1-\gamma} \left[ \frac{z_j}{\bar{z}^{1-\sigma}} \right]^{1/\sigma} \quad (9b)
\]
\[
k_d(z_j, r, w_h) = \frac{1}{a_h} \left[ \frac{\gamma w_h}{(1-\gamma)r} \right]^{1-\gamma} \left[ \frac{z_j}{\bar{z}^{1-\sigma}} \right]^{1/\sigma} \quad (9c)
\]
\[
h_d(z_j, r, w_h) = \frac{1}{a_h} \left[ \frac{(1-\gamma)r}{\gamma w_h} \right]^\gamma \left[ \frac{z_j}{\bar{z}^{1-\sigma}} \right]^{1/\sigma} \quad (9d)
\]

Those functions are easily derived noting that the production function, see (9a), is a standard Cobb-Douglas function with productivity $a_h z^{1-\sigma}$ and exponents $\gamma \sigma$ and $(1-\gamma)\sigma$.

3 Equilibrium Conditions

The previous section described the overall economy and the behavior of every household and every firm. The following section imposes market clearing conditions. At every moment in time the market for high-skill and low-skill labor must clear; every unit of newly produced capital must be sold and every existing unit of capital must be rented by a development or production firm.
3.1 The Market for Low-Skill Labor

Low-skill workers can be employed either by a firm producing capital or any of $\pi L$ firms producing different variations of the consumption good. Full employment implies:

$$L_k(\bar{x}, w_l) + \pi L c_l(\tau, \pi, w_l) = (1 - \phi)L.$$  

Low-skill labor demand for capital production, $L_k(\bar{x}, w_l)$, can be replaced by the factor demand function in (5b). $\bar{\pi}$ and $l_c(\tau, \pi, w_l)$ are eliminated replacing the factor labor demand function using (8b). Solving for $w_l$:

$$w_l = \frac{(1 - \alpha)(1 - \beta)}{1 - \phi}. \quad (10a)$$

Low-skill workers benefit both from increased consumption and increased capital savings. Both increase aggregate production, to the advantage of low-skill workers. Stronger preference for variety provides production firms with some market power, which decreases supply and thereby the demand for production workers, i.e. low-skill workers.

3.2 The Market for High-Skill Labor

Given that a high-skill or low-skill worker save $m$ by financing development of a new product, it is clear from the cost function (9b) and the factor demand function (9d), that the household employs $m(1 - \gamma)/w_h$ high-skill labor units. Total high-skill labor demand therefore equals:

$$\sum_{i=1}^{\phi L} \frac{1 - \gamma}{w_h} m_{hi} + \sum_{i=1}^{(1 - \phi) L} \frac{1 - \gamma}{w_h} m_{li} = \frac{1 - \gamma}{w_h} mL.$$
Assuming full employment, simplifying, and solving for \( w_h \) yields:

\[
  w_h = \frac{1 - \gamma}{\phi \bar{m}}. \tag{10b}
\]

It is immediately clear that high-skill workers benefit from more development saving. Realizing that consumption, real capital saving, and development saving are rival, high-skill and low-skill workers’ wages are clearly driven by very different underlying forces.

The wage rate of low-skill workers is adversely affected by preference for variety directly via the \( 1 - \beta \) term, as seen by (10a). The effect of preference for variety is likely to be the opposite for high-skill workers. A larger \( \beta \) increases the profit rate of firms producing variations of the consumption good, increasing the incentives to invest in development firms, i.e. increasing \( \bar{m} \).

3.3 The Markets for Capital

Households supply production and development firms with capital. Since there is no alternative usage for capital, aggregate capital supply equals \( \bar{k}L \). Demand for capital by firms producing capital, \( K_k(\bar{r}, r) \), is given by (5a). There are \( \bar{m}L \) production firms. Each production firm’s demand for capital is given by the factor demand function in (8a).

Aggregate capital demand by development firms is obtained by summing over every household’s development investment. By the cost function and factor demand functions in (9b) and (9c), any household investing \( m \) in development hires \( m\gamma/r \) units of capital. Aggregating over all households is straightforward, and equalizing aggregate capital supply with aggregate capital demand implies:

\[
  r = \alpha \left[ \frac{(1 - \beta)(\bar{c} + \bar{s}) + \gamma \bar{m}}{\bar{k}} \right] \tag{10c}
\]

The interest rate increases if consumption, capital investments or development saving increase, since
capital is used in production as well as development.

New capital is produced by capital production firms and bought by households. Aggregate household spending on, and thereby demand for, new capital equals $\pi L$. The aggregate supply is given by the production function of capital producers, i.e. $a K^\alpha k^{1-\alpha}$, together with the factor demand functions in (5a) and (5b). Clearing the market for new capital implies:

$$1 = \left[ \frac{\alpha}{r} \right]^\alpha \left[ \frac{1 - \alpha}{w_l} \right]^{1-\alpha}.$$  \hspace{1cm} (10d)

4 Intertemporal Choices

This section closes the model by analyzing the intertemporal behavior of households. Given the intertemporal choices of households, it is possible to determine average consumption, $\overline{c}$, average capital saving, $\overline{s}$, and average development saving, $\overline{m}$. Several possible configurations are possible. For example, the model can be set in either continuous or discrete time, or populated by infinitely lived households or overlapping generations of households. The configuration used here is a continuous time setting with infinitely lived households.

4.1 A Simple Household Model

The income of any household in the economy can be written as:

$$y = w + (1 - \tau)(rk + \pi n) + \tau (rk + \pi n),$$  \hspace{1cm} (11)

where $w$ denotes the wage rate; $w_l$ for a household that supplies low-skill labor and $w_h$ for a household that supplies high-skill labor. $k$ denotes the amount of capital owned by the household, $n$ denotes the number of production firms, i.e. shares, the household owns, and $\tau$ is a tax on financial income or more
general a capital market distortion. The second term, \((1 - \tau)\left(\tau k + \pi n\right)\) is the financial income from owning capital and production firms. \(\tau k\) captures interest payments by firms renting the household’s capital, and \(\pi\) captures dividend payments.

The parameter \(\tau \in [0, 1]\) has two interpretations. Either it parameterizes a financial market imperfection, i.e. a transaction cost paid by households collected by financial market intermediaries. In this case, the fourth term, \(\tau r(k + \pi)\) distributes the profits earned by financial intermediaries uniformly over all households. Alternatively \(\tau\) can be viewed as a tax on savings, paid by households, where the tax revenues are uniformly distributed as a lump sum to each household.

### 4.1.1 Consumption and Saving

Households maximize the discounted value of lifetime utility of consumption: \(\int_0^\infty e^{-\rho t} u(c)dt\). At every moment in time the household must obey the instantaneous budget constraint \(c = y - s - m\), i.e. divide its income into consumption, \(c\), saving in real capital, \(s\), and saving by financing development, \(m\). Let \(k'\) denote the next period’s capital holding. The law of motion for capital is \(k' = (1 - \delta dt)k + sdt\), given that the price of capital is normalized to unity and the depreciation rate is \(\delta\).

Development saving by some household is \(m\). The probability that the development firm succeeds is \(z(m)dt\). Clearly, development saving is risky. To simplify, it is assumed that households cross-insure their savings in individual development firms, thereby completely eliminating risk. The law of motion for shares in production firms is: \(n' = (1 - qdt)n + z(m)dt\). \(qdt\) parameterizes the probability that the variation produced by a specific production firm becomes obsolete, i.e. that a shut down shock occurs with probability \(qdt\). The decisions of a rational household satisfies:

\[
V(k,n) = \max_{s,m} \left( u(c)dt + \frac{1}{1 + \rho dt} EV(k',n') \right)
\]  

(12)
\[s.t \quad c = y - s - m\]
\[k' = (1 - \delta dt)k + sdt\]
\[n' = (1 - qdt)n + z(m)dt.\]

Differentiating the value function and using the first order conditions result in the following characterization of optimal consumption and development saving:

\[
(1 - \tau)r = \rho + \delta - \frac{cu''(c)}{u'(c)}E\dot{c}/c \tag{13a}
\]
\[
(1 - \tau)z'(m)\pi = \rho + q - \frac{cu''(c)}{u'(c)}E\dot{c}/c + \frac{mz''(m)}{z'(m)}Em/m. \tag{13b}
\]

Those relations form a no-arbitrage relation between the returns from capital and development saving.

### 4.1.2 Steady State

If the economy is in steady state, the change in consumption and development saving is zero, i.e. \(\dot{c} = 0\), and \(\dot{m} = 0\). Further, the household’s holdings of capital and owner shares does not change. From the laws of motion, \(k' = k \rightarrow s = \delta k\) and \(n' = n \rightarrow z(m) = qn\). Therefore in steady state:

\[
r = \frac{\rho + \delta}{1 - \tau} \tag{14a}
\]
\[
\pi = \frac{\rho + q}{(1 - \tau)z'(m)} \tag{14b}
\]
\[
s(k) = \delta k \tag{14c}
\]
\[
z(m) = qn \tag{14d}
\]
\[
c = y - \delta k - b_d(qn). \tag{14e}
\]

Due to the assumption that households are fully insured, development saving is non-stochastic. Com-
Bining (14a) and (14b) gives a steady state no-arbitrage condition:

$$\pi = \frac{\rho + q}{\rho + \delta} \frac{r}{z'(m)}.$$  \hspace{1cm} (15)

The interpretation is straightforward. If the shut down intensity, \(q\), is high relative to the depreciation rate, \(\delta\), the profit rate must be higher to provide households with incentive to save in development. \(z'(m)\) is the marginal “development productivity”. The marginal cost of saving in development is inversely related to marginal productivity, and a high relative marginal development cost naturally makes households demand a higher pay off, i.e. a higher profit rate, for saving in development instead of real capital.

### 4.1.3 Aggregation

Capital saving may differ among different households, but it is linear in capital wealth. Aggregating over every household’s capital saving, given by (14c), implies:

$$\bar{s} = \delta \bar{k}.$$  \hspace{1cm} (16)

The model is most easily solved by assuming that \(\sigma\) is in the interval \((0, 1)\). If \(\sigma \in (0, 1)\) the probability to succeed in developing a new variations is at least partly, but not only, dependent on the development efforts in other developing firms.

Replacing \(\pi\) by use of (7c) and replacing \(z'(m)\) by use of the cost function in (9b), condition (14b) defines a unique optimum for development saving:

$$m(\bar{r}, \bar{w}, w_h, \bar{x}) = \zeta \left[ \frac{\sigma \beta (1 - \tau) \bar{c}}{(\rho + q) \bar{m}} \right]^{\frac{\gamma}{\gamma - 1}} \left[ a_h \left[ \frac{\gamma}{\gamma} \left[ \frac{1 - \gamma}{1 - \gamma} \right] \right] \right]^{\frac{1 - \gamma}{\gamma - 1}}.$$

The most important property of household development saving is that it depends only on aggregate, non-household specific, quantities. Therefore aggregate development saving is distributed uniformly over the
population, and household and average development saving is identical.\textsuperscript{2}

Given household saving in development firms, every household’s steady state wealth in shares can be computed by use of relation (14d). Efficient development firms minimize costs. Using the inverse of the cost function in (9b), and noting that \( b_d = m \), \( n \) is solved for as:

\[
n = \frac{\bar{\gamma}}{q} \left[ \frac{a_h \sigma \beta (1 - \tau)}{(\rho + q) \bar{\pi}} \left[ \frac{\gamma}{r} \right]^{\frac{\gamma - 1}{\gamma}} \left[ \frac{1 - \gamma}{w_h} \right]^{1 - \gamma} \right]^{\frac{\rho}{\sigma}}.
\]

(18)

It is immediately clear that households’ share holdings, \( n \), only depend on aggregate quantities and therefore, every household has the same amount of wealth in shares. This is of course a logical consequence of the previous result that every household’s development saving is equalized. Using that \( \bar{\pi} = n \) and solving for \( \bar{\pi} \) yields:

\[
\bar{\pi}(\bar{\tau}, \bar{r}, w_h, \bar{\pi}) = \left[ \frac{\bar{\gamma}}{q} \right]^{1 - \sigma} \left[ \frac{a_h \sigma \beta (1 - \tau) \bar{\tau}}{\rho + q} \left[ \frac{\gamma}{r} \right]^{\frac{\gamma - 1}{\gamma}} \left[ \frac{1 - \gamma}{w_h} \right]^{1 - \gamma} \right]^{\sigma}.
\]

(19)

Inserting the average households’ share holdings, \( \bar{\pi} \), given by (19), into the expression for household and average development saving, given by (17), reduces average development saving significantly:

\[
\bar{m}(\bar{\tau}) = \frac{\sigma q \beta (1 - \tau) \bar{\tau}}{\rho + q}.
\]

(20)

The market clearing conditions, i.e (10a), (10b) and (10c), provides the basic relations necessary to solve for \( \bar{\tau}, \bar{k} \) and \( w_h \). The remaining endogenous variables can be solved or eliminated. The steady state interest rate \( r \) is pinned down by (14a), and the wage rate for low-skill workers is then given by the capital market clearing condition in (10d). Average capital saving, \( \bar{s} \), is eliminated by (16), and average

\textsuperscript{2}With decreasing individual returns to development investment, \( \sigma < 1 \), efficiency requires uniform investments. This result parallels the inequality growth result that a necessary condition for inequality to affect growth, via human capital investments, is that capital markets are imperfect Aghion and Howitt (1998); Aghion et al. (1999). Capital on the other hand is not subject to individual decreasing returns, and the distribution of capital does not affect efficiency.
development saving, \( \overline{m} \), is eliminated by (20). The resulting system of three equations is:

\[
(1 - \alpha) \left[ (1 - \beta) \overline{c} + \delta \overline{k} \right] - (1 - \phi) w_l = 0 \tag{21a}
\]

\[
(\rho + q) \phi \overline{w}_h - (1 - \gamma) q \sigma \beta (1 - \tau) \overline{c} = 0 \tag{21b}
\]

\[
[\alpha (\rho + q) (1 - \beta) + \gamma q \sigma \beta (1 - \tau)] \overline{c} + (\rho + q) (\alpha \delta - r) \overline{k} = 0. \tag{21c}
\]

Solving this system is straightforward. To simplify the notation, let \( \Delta \) be defined as:

\[
\Delta \equiv \rho \overline{c} - (1 - \beta) (\rho + q) + q \gamma \sigma \beta (1 - \tau) > 0. \tag{22}
\]

The solution to the system is

\[
r = \frac{\rho + \delta}{1 - \tau} \tag{23a}
\]

\[
w_l = (1 - \alpha) a'_l \left[ \frac{\alpha}{r} \right] \frac{\delta}{1 - \alpha} \tag{23b}
\]

\[
w_h = \frac{\sigma q \beta (1 - \gamma) (r - \alpha \delta)}{\Delta} \frac{1 - \phi}{\phi} \frac{w_l}{1 - \alpha} \tag{23c}
\]

\[
k = \frac{\alpha \rho (1 - \beta) + q (\alpha (1 - \beta) + \gamma \sigma \beta (1 - \tau))}{\Delta} \frac{1 - \phi}{\phi} \frac{w_l}{1 - \alpha} \tag{23d}
\]

\[
\omega = \frac{\sigma q \beta (1 - \gamma) (1 - \tau) (r - \alpha \delta)}{(1 - \alpha) \Delta} \frac{1 - \phi}{\phi}, \tag{23e}
\]

where \( \omega = w_h/w_l \) denotes the relative wage of high-skill workers, compared to low-skill workers. The skill premium is defined as \( \ln \omega \).

### 4.2 Comparative Statics

The main concern of this paper is the return to skill. Since the steady state equilibrium conditions provide analytically traceable expressions for all endogenous variables, the skill premium is easily investigated.
To investigate what determines the skill premium, $\ln \omega$ is differentiated with respect to the key parameters of the model.

**Skill Composition**  The standard increased factor supply–decreased factor return logic holds for both kinds of labor, as seen by the negative derivative of $\ln \omega$ with respect to the fraction of high-skill households, i.e. $\phi$:

$$\frac{d \ln \omega}{d \phi} = -\frac{1}{\phi(1-\phi)} < 0. \quad (24a)$$

Hence, increasing the relative supply of high-skill households decreases the skill premium.

**Preference for Variety**  The impact of preference for variety on the skill premium is described by the derivative of $\ln \omega$ with respect to $\beta$. After some algebra:

$$\frac{d \ln \omega}{d \beta} = \frac{(\rho + \delta)(\rho + q)}{\beta(1-\tau)\Delta} > 0. \quad (24b)$$

Increasing $\beta$ makes households more inclined to spread out consumption more evenly over all variations given any fixed set of prices, implying greater market power for the producer of any variation. On the one hand, it follows from (8b) that greater preferences for variety decreases the per firm demand for low-skill labor as the supply of each firm decreases.

On the other hand, it is clear from (7c) that stronger preference for variety increases the value of a firm producing a variation, which in turn increases the incentives to develop new variations. Naturally greater incentives to develop new variations translates into increasing demand for high-skill workers; see (14d). Therefore, in the short run, before the number production firms adjusts, increasing the preference for variety increases the skill premium.

In the long run the number of production firms and development firms, $\bar{n}$ and $\bar{m}$, changes, thereby al-
tering the demand for high-skill and low-skill labor. As seen by the comparative statics and the reasoning above, it is clear that in the short run as well as the long run increased preference for variety increases the skill premium.

**Taxation** Increasing the tax rate on income from capital and owner shares, i.e. increasing $\tau$, has an ambiguous effect on the skill premium.

$$\frac{d \ln \omega}{d \tau} = \frac{\alpha \delta}{\rho + \delta [1 - \alpha (1 - \tau)]} + \frac{2 \gamma \rho \delta \beta}{\Delta} - \frac{1}{1 - \tau}. \quad (24c)$$

The sign is ambiguous and the effect is non-linear. It is easy to see, inspecting (22), that $\Delta$ is bounded and strictly positive as $\tau \to 1$. This implies that for large distortions the derivative is infinitely negative. It follows that if $\tau$ is sufficiently close to unity, improving the financial market increases the skill premium. Hence improving *sufficiently* distorted financial markets increase the skill premium.

It is a bit surprising that the result is ambiguous. The high-skill labor market clearing condition in (10b) implies that the wage rate for high-skill workers is proportional to the average saving in development firms. Saving in development firms is in turn proportional to one minus the tax rate, i.e. $1 - \tau$, as seen by the steady state expression for $\overline{m}$ stated in (20). However, by the same expression, it is clear that average development saving is proportional to average consumption spending, $\overline{c}$. Increasing the tax rate on financial income increases average consumption, and, the effect on development saving is therefore ambiguous, as is the effect on the wage rate of high-skill workers.

The wage rate of low-skill workers clearly decreases as the tax rate increases. Increasing the tax rate increases the steady state interest rate, and that lowers the wage rate of low-skill workers, as seen by (23b). This is a equilibrium result. Capital is the numeraire good and the wage rate of low-skill workers falls out, clearing the market for new capital. As its price is fixed, the wage rate of low-skill workers must adjust to clear the market.
Clearly it is difficult to predict a priori, whether financial market distortions increase or decrease the skill premium. However, excluding capital from the model yields unambiguous results. Letting $\alpha \to 0$ and $\gamma \to 0$ renders capital redundant in the development and production processes. The derivative reduces to:

$$
\frac{d \ln \omega}{d \tau} \bigg|_{\alpha \to 0, \gamma \to 0} = \frac{-1}{1-\tau} < 0.
$$

(24d)

**Shut Down Intensity** The expected lifetime of a variation of the consumption good is $1/q$. Decreasing the expected lifetime of variations of the consumption good, i.e. increasing $q$, increases the skill premium:

$$
\frac{d \ln \omega}{dq} = \frac{\rho(1-\beta)(\rho+\delta)}{q(1-\tau)\Delta} > 0.
$$

(24e)

On the one hand, decreasing the expected life of variations of the consumption good decreases incentives to develop new variations in the short run as lifetime profits decrease. On the other hand, in the long run shorter life spans decrease the number of variations available, and thereby increase the profit rate of each producer, increasing the incentives to save in development firms.

In steady state, increasing the shut down intensity, $q$, implies increasing average development saving over average consumption, see (20). This shift in favor of development saving increases the demand for high-skill workers, which increases the steady state wage rate. Shorter product cycles therefore raise the skill premium.
Development Externalities If development generates strong externalities, i.e. $\sigma$ closer to zero, the skill premium is smaller:

$$\frac{d \ln \omega}{d \sigma} = \frac{(1 - \beta)(\rho + \delta)(\rho + \delta + q)}{\sigma \Delta} > 0.$$  \hspace{1cm} (24f)

A smaller $\sigma$ decreases every households marginal benefit from development saving, since the households are uncoordinated and fail to internalize their positive external effect on every other household. A lower marginal benefit decreases the incentives to save by financing development firms, thereby decreasing the demand for high-skill workers. In the end, the wage rate of high-skill workers must decrease to maintain full employment.

5 Conclusions

The model presented in this paper puts forward the idea that if high-skill workers are mainly used in developing goods and low-skill workers mainly are used in producing existing goods, various market imperfections can alter the skill premium.

All actions by agents in the model are based on rational maximization of lifetime utility and profits in a general equilibrium setting. However to simplify, only steady state results are considered. Therefore all results pertain to the long run.

The model assumes perfectly competitive labor markets, and thereby departs from the existing branch of literature investigating the effect of labor market imperfections on the wage rate. Capital market distortions are also introduced. The paper’s main results are:

- Greater preference for variety in consumption increases the skill premium.
- Shorter product cycles increase the skill premium.
- Financial market distortions, such as taxes, changes the skill premium.
Positive externalities among firms developing new goods decrease the skill premium.

In the short run, preference for variety translates into market power for firms, increasing profits but reducing supply. Reduced supply reduces the demand for production workers, i.e. low-skill workers. Higher profits stimulates development of new variations of consumption good, thereby increasing the demand for development workers, i.e. high-skill workers. However since the supply of low-skill and high-skill workers is fixed, the decreased demand for low-skill workers translates into a lower wage rate, and the increased demand for high-skill workers translates into a higher wage rate for high-skill workers.

Shorter product cycles, all else equal, reduces the profitability of developing new variations. In the long run however, the number of variations decrease, but the income share spent on development relative consumption increases, thereby increasing the skill premium. The model thereby points out shorter product cycles to be a potential explanation for the increasing dispersion in wages during the last 30 years.

The result for taxation on non-labor income, is ambiguous. In a model without capital, a non-labor tax decreases the skill premium. With capital, a necessary condition for a non-labor tax to decrease the skill premium is that the initial tax is sufficiently high. This is a weak prediction, but nevertheless the model hints that there is a connection between financial institutions and the skill premium, pointing towards financial liberalization as a possible explanation of the changes in the skill premium during the 1980s.

If the probability to succeed in developing a new variation depend not only on the effort by the own firm but also on the effort of other firms, then there are positive external effects among development firms. Positive externalities among development firms provide incentives to free ride. Free riding decreases saving in development firms, which in turn decrease both the demand for high-skill workers and the skill premium.

The model presented in this can easily be extended on the household side in order to model, for
example, distorted financial markets and different household characteristics, such as risk aversion or different degrees of precautionary saving. The model, therefore, is rich in future research prospects.

Appendices

A Record of Notation

In general, any variable with an overbar represents an average, usually with respect to households or households supplying either high-skill or low-skill labor. $i$ is used to index a specific firm producing a variation of the consumption good while $j$ is used to index a specific development firm. A complete list of the symbols used in the paper is presented in Table 1.

B Demand for Variations

The aim of this section is to derive the demand function for each variation of the consumption good, given a certain degree of preference for variety. Preference for variety is modeled as in Dixit and Stiglitz (1977). By consuming $\pi L$ different variations of the consumption good, household utility is:

$$\sum_{i=1}^{\pi L} \left[ x_i (1-\beta) \right]^{\frac{1}{1-\beta}}.$$

For $\beta \in (0, 1)$, consuming an extra unit of any of the variation decreases the marginal utility of yet an extra unit of the same variation, and therefore consumers prefer to increase consumption of all variations. Only if prices differ, will a single households consume different quantities of the different variations.

Given a fixed consumption budget $c$, a utility maximizing household must act as if solving the opti-
### Table 1: List of Symbols

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Range</th>
<th>Interpretation</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\alpha)</td>
<td>([0, 1))</td>
<td>Capital’s “weight” in production (Cobb-Douglas).</td>
</tr>
<tr>
<td>(a_h)</td>
<td>(R_+)</td>
<td>High-skill workers’ productivity.</td>
</tr>
<tr>
<td>(a_l)</td>
<td>(R_+)</td>
<td>Low-skill workers’ productivity.</td>
</tr>
<tr>
<td>(\beta)</td>
<td>([0, 1))</td>
<td>Preference for variety.</td>
</tr>
<tr>
<td>(b_c(\cdot))</td>
<td>(R_+)</td>
<td>Cost function for production firms.</td>
</tr>
<tr>
<td>(b_d(\cdot))</td>
<td>(R_+)</td>
<td>Cost function for development firms.</td>
</tr>
<tr>
<td>(c)</td>
<td>(R_+)</td>
<td>An arbitrary household’s consumption spending.</td>
</tr>
<tr>
<td>(C)</td>
<td>(R_+)</td>
<td>Aggregate consumption spending.</td>
</tr>
<tr>
<td>(\delta)</td>
<td>(R_+)</td>
<td>Depreciation rate of capital.</td>
</tr>
<tr>
<td>(\Delta)</td>
<td>(R_+)</td>
<td>Abbreviation, defined in (22).</td>
</tr>
<tr>
<td>(\gamma)</td>
<td>([0, 1))</td>
<td>Capital’s “weight” in development (Cobb-Douglas).</td>
</tr>
<tr>
<td>(h_d)</td>
<td>(R_+)</td>
<td>High-skill labor used by an arbitrary development firm.</td>
</tr>
<tr>
<td>(k)</td>
<td>(R_+)</td>
<td>An arbitrary household’s capital holding.</td>
</tr>
<tr>
<td>(k_d)</td>
<td>(R_+)</td>
<td>Capital used by an arbitrary development firm.</td>
</tr>
<tr>
<td>(K_d)</td>
<td>(R_+)</td>
<td>Aggregate capital used producing capital.</td>
</tr>
<tr>
<td>(L)</td>
<td>(R_+)</td>
<td>Number, i.e. the measure, of households.</td>
</tr>
<tr>
<td>(L_d)</td>
<td>(R_+)</td>
<td>Aggregate low-skill labor used producing capital.</td>
</tr>
<tr>
<td>(m)</td>
<td>(R_+)</td>
<td>An arbitrary household’s investment in development.</td>
</tr>
<tr>
<td>(n)</td>
<td>(R_+)</td>
<td>An arbitrary household’s holding of development shares.</td>
</tr>
<tr>
<td>(\phi)</td>
<td>([0, 1))</td>
<td>Fraction of households supplying high-skill labor.</td>
</tr>
<tr>
<td>(\pi)</td>
<td>(R_+)</td>
<td>An arbitrary consumption firm’s profit rate.</td>
</tr>
<tr>
<td>(p)</td>
<td>(R_+)</td>
<td>Price of an arbitrary variation.</td>
</tr>
<tr>
<td>(q)</td>
<td>(R_+)</td>
<td>Auxiliary price index.</td>
</tr>
<tr>
<td>(q)</td>
<td>(R_+)</td>
<td>Poisson intensity at which variations become obsolete.</td>
</tr>
<tr>
<td>(\rho)</td>
<td>(R_+)</td>
<td>Households’ discount rate.</td>
</tr>
<tr>
<td>(r)</td>
<td>(R_+)</td>
<td>Interest rate.</td>
</tr>
<tr>
<td>(\sigma)</td>
<td>([0, 1))</td>
<td>1 - (\sigma): Free riding possibilities in developing.</td>
</tr>
<tr>
<td>(s)</td>
<td>(R_+)</td>
<td>An arbitrary household’s savings.</td>
</tr>
<tr>
<td>(\tau)</td>
<td>([0, 1))</td>
<td>Tax rate on non-labor income.</td>
</tr>
<tr>
<td>(u(\cdot))</td>
<td>(R_+)</td>
<td>Instantaneous utility function.</td>
</tr>
<tr>
<td>(v(\cdot))</td>
<td>(R_+)</td>
<td>Auxiliary utility function (w.r.t. variations).</td>
</tr>
<tr>
<td>(V(\cdot))</td>
<td>(R_+)</td>
<td>Lifetime utility.</td>
</tr>
<tr>
<td>(\omega)</td>
<td>(R_+)</td>
<td>The skill premium, (w_h/w_l).</td>
</tr>
<tr>
<td>(w_h)</td>
<td>(R_+)</td>
<td>Wage rate for households supplying high-skill labor.</td>
</tr>
<tr>
<td>(w_l)</td>
<td>(R_+)</td>
<td>Wage rate for households supplying low-skill labor.</td>
</tr>
<tr>
<td>(x)</td>
<td>(R_+)</td>
<td>Quantity of an arbitrary variation.</td>
</tr>
<tr>
<td>(y)</td>
<td>(R_+)</td>
<td>An arbitrary household’s income rate.</td>
</tr>
<tr>
<td>(z)</td>
<td>(R_+)</td>
<td>Development intensity of an arbitrary development firm.</td>
</tr>
</tbody>
</table>
mization problem:

\[
v(\vec{x}) = \max_{\vec{x}} \sum_{i=1}^{nL} x_i^{1-\beta} \frac{1}{1-\beta} \quad s.t. \quad \sum_{i=1}^{nL} p_i x_i = c.
\]

Let \( \mu \) denote the Lagrangian multiplier due to the budget constraint. The first order conditions are:

\[
x_i^{-\beta} \left[ \sum_{k=1}^{nL} x_k^{1-\beta} \right]^{\frac{\beta}{1-\beta}} + \mu p_i = 0 \quad \forall i \quad (25a)
\]

\[
\sum_{k=1}^{nL} p_k x_k - c = 0. \quad (25b)
\]

The relative demand of any two variations is easily derived by division of equation (25a) for two distinct \( i \)'s. Using the resulting expression to eliminate \( x_k \) in (25b) and solving for \( x_i \), results in a demand function for any \( x_i \) (\( i \) is arbitrary):

\[
x_i = \frac{c}{p_i^{1/\beta} \hat{p}} \quad \hat{p} = \sum_{k=1}^{nL} p_k^{\frac{\beta-1}{\beta}}.
\]

Substituting back into the utility function and simplifying gives:

\[
v(\vec{x}) = c \hat{p}^{\frac{\beta}{1-\beta}}.
\]

The last three relations verify expressions (3a) – (3c).
C Supply of Variations

Consider the producer of the $i$th variation of the consumption good. The problem of the firm manager is to maximize the instantaneous profit rate. In doing so the manager must obey the first order conditions of the optimization problem:

\[
\pi_i = \max_{x_i, p_i} x_i p_i - b_c(x_i) \tag{26a}
\]

subject to:

\[
x_i = \frac{C}{p_i \hat{b}_c} \tag{26b}
\]

\[
b_c(x_i) = \hat{b}_c x_i. \tag{26c}
\]

Naturally managers must choose a price quantity pair on the demand curve, given by (26b). The demand curve was derived in Appendix B. The only difference is that spending by a single household, $c$, has been replaced by aggregate spending, $C$, obtained by horizontal summation over all households.

By (26c) managers are constrained by the constant return to scale technology defined by the second constrain in problem (6). For a simpler exposition $\hat{b}_c$ is used to denote marginal production cost, accordingly defined as:

\[
\hat{b}_c = \left[ \left( \frac{\alpha}{1-\alpha} \right)^{1-\alpha} + \left( \frac{\alpha}{1-\alpha} \right)^{-\alpha} \right] \frac{r^{\alpha} w_{l}^{1-\alpha}}{a_l}. \tag{27}
\]

As in Dixit and Stiglitz (1977), it is assumed that managers overlook the impact of changing their own price on the index $\hat{p}$, defined by (3c). This is only perfectly rational if there is an infinite number of competing firms. Solving the problem above by inserting the cost function into the objective function, and using $\mu$ to denote the Lagrangian multiplier due to the demand constraint, the first order conditions
are:

\[ p_i + \hat{b}_c + \mu = 0 \]  \hspace{1cm} (28a)

\[ x_i + \mu \frac{1}{\beta} \frac{C}{\hat{p}} \frac{1-\beta}{p_i} = 0 \] \hspace{1cm} (28b)

\[ x_i - \frac{C}{p_i^{1/\beta} \hat{p}} = 0. \] \hspace{1cm} (28c)

The output price of the \( i \)th firm is easily solved for by direct insertion of (28a) and (28c) into (28b) to eliminate \( x_i \) and \( \mu \). The result is a pricing rule with a constant mark up over marginal cost:

\[ p_i = \frac{\hat{b}_c}{1-\beta}. \] \hspace{1cm} (29a)

Clearly if there is no preference for variety, i.e. \( \beta = 0 \), the market becomes perfectly competitive and price equals marginal cost. In order to determine the quantity supplied by each producer, \( x_i \), \( p_i^{1/\beta} \hat{p} \) must be computed. As is clear from the pricing rule in (29a), all producers set the same price, since they all have the same technology. The price index, defined in (3c), reduces to:

\[ \hat{p} = \frac{\hat{b}_c}{1-\beta} \left[ \frac{\hat{b}_c}{1-\beta} \right]^{\frac{\beta-1}{\beta}}. \] \hspace{1cm} (29b)

Given the price index and the demand curve, (28c), the supply of any firm, indexed by \( i \), is:

\[ x_i = \frac{(1-\beta)C}{\hat{p} \hat{b}_c L}. \] \hspace{1cm} (29c)

Inserting \( p_i \), \( \hat{p} \) and \( x_i \) into the profit function defined in (26a) and simplifying gives:

\[ \pi_i = \frac{\beta C}{\hat{p} L}. \] \hspace{1cm} (29d)

28
After replacing $C$ with $\tau L$ and $\hat{b}_c$ by (27), relations (29a) – (29d) verify (7a) – (7c).

References


