UNIVERSITY OF GOTHENBURG SCHOOL OF BUSINESS, ECONOMICS AND LAW

# WORKING PAPERS IN ECONOMICS 

No 559

## Discounting and Relative Consumption

Olof Johansson-Stenman and Thomas Sterner

March 2013

ISSN 1403-2473 (print)
ISSN 1403-2465 (online)

# Discounting and Relative Consumption 

Olof Johansson-Stenman and Thomas Sterner*<br>Department of Economics, School of Business, Economics and Law, University of Gothenburg, Box 640, S-40530 Gothenburg, Sweden E-mail: Olof.Johansson@economics.gu.se, Thomas.Sterner@economics.gu.se


#### Abstract

We analyze optimal social discount rates when people derive utility from relative consumption. We compare the social, private, and conventional Ramsey rates. Assuming a positive growth rate, we find that 1) the social discount rate exceeds the private discount rate if the importance of relative consumption increases with consumption and that 2) the social discount rate is smaller than the Ramsey rate given quasi-concavity in own and others' consumption and risk aversion with respect to others' consumption. Numerical calculations demonstrate that the latter difference may be substantial and have important implications for long run environmental issues such as global warming.


Keywords: Environmental discounting, global warming, relative consumption, Ramsey rule, positionality.

JEL Classification: D63, D90, H43

[^0]
## 1. Introduction

The theory and practice of discounting is central to economics (e.g., Arrow and Lind 1970; Frederick et al. 2002; and Arrow et al. 1996) and essential for dealing with very long-term phenomena. With the increased attention to environmental issues such as climate change, interest in discounting has experienced a revival (see Gollier 2010; Gollier and Weitzman 2010; and Weitzman 2010 for recent contributions). For example, essentially the entire economics debate in the wake of the Stern Review (Stern 2006) largely focused on the discount rate used-not on climate science or the assessment of costs and benefits of mitigation, for which there are still very large uncertainties. (See, e.g., Brekke and JohanssonStenman 2008; Dasgupta 2007, 2008; Dietz and Stern 2007; Heal 2008; Howarth 2009; Nordhaus 2007a, 2007b; Sterner and Persson 2008; and Weitzman 2007a, 2007b.) The primary reason is, of course, that most of the consequences of climate change will occur far into the future and thus the discount rate has a dramatic effect on their present value.

This paper is, as far as we know, the first to incorporate relative consumption effects into the theory of social discounting. Yet, the idea that humans value consumption in a social context and in relation to others' consumption is far from new. In fact, classical economistssuch as Adam Smith, John Stuart Mill, and Alfred Marshall—emphasized such concepts, and modern research on the subject dates back at least to Duesenberry (1949). There is now a substantial body of empirical evidence suggesting that people not only derive utility from their absolute consumption but are also concerned with their own consumption relative to that of others. ${ }^{1}$ There is also a growing literature dealing with various kinds of optimal policy responses to such relative consumption effects. ${ }^{2}$ For example, Aronson and JohanssonStenman (2008; 2010) show, in a static and a dynamic model, respectively, that optimal marginal income taxes are likely to be substantially larger under relative consumption effects than in a conventional case, where people only care about their absolute consumption level.

[^1]Arrow and Dasgupta's paper (2009) is closest to ours, as it explicitly deals with the implications of relative consumption effects for intertemporal resource allocation. They show that concern for relative consumption does not necessarily lead people to consume more today than is socially optimal, since they are also concerned with relative consumption (and hence produce positional externalities) in the future; this is also shown in slightly different settings by Wendner (2010a, 2011). However, the issue of how discount rates are affected by relative consumption effects has not been analyzed before.

In Section 2, we analyze the question of whether and, if so, how the social discount rate (when positional externalities are taken into account) is smaller or larger than the private discount rate (when people do not take into account that their consumption affects othersalthough they still take into account that they themselves are affected by others' consumption). We show that an important condition derived by Arrow and Dasgupta (2009) for when private and social consumption paths coincide over time translates to the condition for when the private and social discount rates are the same. We then explore the conditions when the social discount rate exceeds the private one, and vice versa. We express these conditions in terms of the degree of positionality, a measure reflecting the extent to which relative consumption matters. For a positive growth rate, we show that if the degree of positionality increases with the consumption levels, consistent with some empirical evidence, then the social discount rate exceeds the private one. We also show that this discrepancy can be internalized by timevarying consumption taxes.

In Section 3, we continue by analyzing a related but distinct issue-relevant from a climate policy perspective-namely whether, and if so how, the conventional optimal social discounting rule, the so-called Ramsey discounting rule (Ramsey 1928), should be modified in the presence of relative consumption effects. ${ }^{3}$ The conventional Ramsey discounting rule says that the optimal discount rate equals the pure rate of time preference plus the product of the individual degree of relative risk aversion multiplied by the growth rate. Hence, this formulation does not take into account that others' consumption will also change in the future.

The formula describing the optimal social discount rate in the presence of relative consumption effects can be written in a form similar to the Ramsey formula. The only difference is that the individual degree of relative risk aversion is replaced by what we denote here as the social degree of relative risk aversion. By this we mean a corresponding measure

[^2]of risk aversion had the individual made a risky choice on behalf of his or her whole generation. Assuming a positive consumption growth rate, we show that the social discount rate is lower than the Ramsey discount rate, if preferences are quasi-concave in own and reference consumption (consisting of others' average consumption) and concave in reference consumption. The latter means that individuals prefer that others have a certain consumption level compared to the case where others' consumption is uncertain with the same expected value. We show moreover that the social discount rate is higher than the private rate if the degree of positionality increases with consumption. Taken together, this implies that the social discount rate is higher than the private rate but lower than the Ramsey rate, if the degree of positionality increases with consumption and preferences reflect risk aversion with respect to reference consumption and are quasi-concave with respect to own and reference consumption.

Finally, we illustrate quantitatively in Section 4 how the overall long-term social discount rates recently suggested by Stern (2006) and Weitzman (2007b) would be modified when taking relative consumption effects into account. We conclude that these modifications may be substantial with commonly used functional forms and reasonable parameter values, and that they are potentially very important for the economics of climate change. Section 5 offers some final remarks and observations.

## 2. Private and Social Discount Rates

In this section, we analyze and compare the private and social discount rates between two arbitrary points in time, when people care about relative consumption. At the end of this section, we also determine the corresponding internalizing consumption taxes.

### 2.1 Preferences and Objectives

Following Arrow and Dasgupta (2009), society consists of identical individuals, with a population size normalized to one, who, at time $t$, consume $c_{t}$ and who, in addition to absolute consumption, also care about relative consumption. The latter depends on own consumption as well as the reference consumption $z_{t}$ (the average of others' consumption), such that $R_{t}=r\left(c_{t}, z_{t}\right)$. The individual instantaneous utility (or felicity) at time $t$ is:

$$
\begin{equation*}
U_{t}=u\left(c_{t}, R_{t}\right)=u\left(c_{t}, r\left(c_{t}, z_{t}\right)\right)=v\left(c_{t}, z_{t}\right), \tag{1}
\end{equation*}
$$

where $u_{1 t}>0, u_{2 t}>0, v_{1 t}>0, v_{2 t}<0, u_{11 t}<0, v_{11 t}<0, r_{1 t}>0, r_{2 t}<0$; where sub-indices reflect partial derivatives $u_{1 t} \equiv \partial u\left(c_{t}, R_{t}\right) / \partial c_{t}$, etc.; and where $u, v$, and $r$ are twice continuously differentiable.

Both the $u$ and $v$ formulations in equation (1) are used extensively in this paper, since they allow us to vary which variable is constant, i.e., either relative consumption $R_{t}$ or others' consumption $z_{t}$. Throughout the paper, we ignore intra-generational issues and assume the same uniform and normalized population in each period. Moreover, we let $r$ satisfy the criterion that it is unaffected if own consumption and others' consumption are changed equally, i.e., $r_{1 t}=-r_{2 t}$. This assumption is fairly innocuous as long as individuals are identical within each generation. For example, it encompasses the most commonly used comparison forms, i.e., the difference comparison form where $R_{t}=c_{t}-z_{t}$ (e.g., Akerlof 1997; Ljungqvist and Uhlig 2000; Bowles and Park 2005; and Carlsson et al. 2007), the ratio comparison form where $R_{t}=c_{t} / z_{t}$, (e.g., Boskin and Sheshinski 1978; Layard 1980; and Wendner and Goulder 2008), and the more flexible functional form suggested by Dupor and Liu (2003), which includes both the difference and the ratio forms as special cases, as long as $z_{t}=c_{t}$ (see below).

In addition, we make the common assumption of (weak) keeping-up-with-the-Joneses property, such that $v_{12 t} \geq 0$ (see, e.g., Gali 1994; Carroll et al. 1997; and Wendner 2010a). ${ }^{4}$ This assumption is made largely for convenience and basically implies that people want to consume more when others consume more, and vice versa. It is easy to see that $v_{12 t} \geq 0$ implies that identical individuals will have the same consumption in each period, further implying that $z_{t}=c_{t}$. In contrast, when $v_{12 t}<0$, it may be privately and socially optimal for identical individuals to have different consumption patterns over time. Other properties of these utility functions will be discussed in subsequent sections.

Each individual maximizes the flow of instantaneous utility over time, subject to a fixed pure rate of time preference-or the utility discount rate $\delta$-implying the following maximization problem:

[^3]\[

$$
\begin{equation*}
\max w^{p} \equiv \int_{0}^{T} u\left(c_{\tau}, r\left(c_{\tau}, z_{\tau}\right)\right) \exp (-\delta \tau) d \tau=\int_{0}^{T} v\left(c_{\tau}, z_{\tau}\right) \exp (-\delta \tau) d \tau \tag{2}
\end{equation*}
$$

\]

Hence, the individual takes others' consumption $z_{t}$ as given in each moment in time, implying that the individual takes into account that a changed consumption path also implies a changed path of relative consumption. The social planner, in contrast, maximizes the flow of instantaneous utility for all individuals (and hence also takes into account the externalities through relative consumption). As a consequence, since all individuals are identical, the social planner takes $R_{t}$ as given. Thus, the social planner solves the following maximization problem:

$$
\begin{equation*}
\max w^{s} \equiv \int_{0}^{T} u\left(c_{\tau}, r\left(c_{\tau}, c_{\tau}\right)\right) \exp (-\delta \tau) d \tau=\int_{0}^{T} v\left(c_{\tau}, c_{\tau}\right) \exp (-\delta \tau) d \tau . \tag{3}
\end{equation*}
$$

### 2.2 Private and Social Discount Rates

Consider now a small project undertaken and paid for in terms of reduced consumption at time zero that results in increased consumption at time $t$. Note that since we are only concerned with given consumption paths, since we deal with small projects as is common in the discounting literature, ${ }^{5}$ our results are independent of any production assumptions, including issues of technical change. If the individual is indifferent between undertaking such a project and not, the private discount rate per time unit between time zero and $t$, given by $\rho^{p}$, is implicitly defined by:

$$
\frac{\partial w^{p} / \partial c_{t}}{\partial w^{p} / \partial c_{0}}=\exp \left(-\rho^{p} t\right)
$$

Note that this is true for any given consumption path, whether optimal or not. Solving for $\rho^{p}$ gives:

$$
\begin{equation*}
\rho^{p}(t)=-\frac{1}{t} \ln \frac{\partial w^{p} / \partial c_{t}}{\partial w^{p} / \partial c_{0}} \tag{4}
\end{equation*}
$$

Similarly, if the social planner is indifferent between undertaking such a project and not, the social discount rate per time unit between time zero and $t$, given by $\rho^{s}$, is implicitly defined by $\partial w^{s} / \partial c_{0}=\exp \left(\rho^{s} t\right) \partial w^{s} / \partial c_{t}$, implying that:

[^4]\[

$$
\begin{equation*}
\rho^{s}(t)=-\frac{1}{t} \ln \frac{\partial w^{s} / \partial c_{t}}{\partial w^{s} / \partial c_{0}} . \tag{5}
\end{equation*}
$$

\]

Next, from equation (2), we have $\partial w^{p} / \partial c_{t}=v_{1 t} \exp (-\delta t)$, which substituted into equation (4) implies:

$$
\begin{equation*}
\rho^{p}(t)=\delta-\frac{1}{t} \ln \frac{v_{1 t}}{v_{10}} . \tag{6}
\end{equation*}
$$

Similarly, we have $\partial w^{s} / \partial c_{t}=\left(v_{1 t}+v_{2 t}\right) \exp (-\delta t)$, such that:

$$
\begin{equation*}
\rho^{s}(t)=\delta-\frac{1}{t} \ln \frac{v_{1 t}+v_{2 t}}{v_{10}+v_{20}} \tag{7}
\end{equation*}
$$

Comparing equations (6) and (7) immediately implies the following result:

Proposition 1. $\rho^{s}(t)=\rho^{p}(t) \forall t$, if and only if $v_{2 t}\left(c_{t}\right)=\beta v_{1 t}\left(c_{t}\right) \forall c_{t}$, where $\beta$ is a constant.

Proposition 1 says that the social and private discount rates are equal between all time periods, if and only if the ratio between the marginal disutility of others' consumption and the marginal utility of own consumption is constant. As such, it resembles Proposition 1 in Arrow and Dasgupta (2009), which says-based on a similar but slightly different model-that the privately and socially optimal consumption paths coincide, if and only if, for all $t, v_{2 t}=\beta v_{1 t}$.

In order to explore the differences between the social and private discount rates in a way that allows for a straightforward economic interpretation, we introduce a measure (following, e.g., Johansson-Stenman et al. 2002 and Aronsson and Johansson-Stenman 2008) that reflects the extent to which relative consumption matters.

Definition 1. The degree of positionality is defined by:

$$
\begin{equation*}
\gamma_{t} \equiv \frac{u_{2 t} r_{1 t}}{u_{1 t}+u_{2 t} r_{1 t}} . \tag{8}
\end{equation*}
$$

Thus, $\gamma_{t} \in(0,1)$ reflects the fraction of the overall utility increase from the last dollar consumed that is due to increased relative consumption. As $\gamma_{t}$ approaches zero, relative consumption at time $t$ does not matter at all at the margin, taking us back to the standard model. In the other extreme case, where $\gamma_{t}$ approaches one, absolute consumption does not matter at all (i.e., all that matters is relative consumption).

From equation (1), we have that $v_{2 t}=u_{2 t} r_{2 t}$ and $v_{1 t}=u_{1 t}+u_{2 t} r_{1 t}$, together implying that:

$$
\begin{equation*}
-\frac{v_{2 t}}{v_{1 t}}=-\frac{r_{2 t}}{r_{1 t}} \gamma_{t}=\gamma_{t} . \tag{9}
\end{equation*}
$$

Substituting equation (9) into equation (7) gives, after some straightforward manipulations,

$$
\begin{equation*}
\rho^{s}(t)=\delta-\frac{1}{t} \ln \left(\frac{v_{1 t}}{v_{10}}-\frac{v_{1 t}}{v_{10}+v_{20}}\left(\gamma_{t}-\gamma_{0}\right)\right)=\delta-\frac{1}{t} \ln \left(\frac{v_{1 t}}{v_{10}} \frac{1-\gamma_{t}}{1-\gamma_{0}}\right), \tag{10}
\end{equation*}
$$

implying that the social discount rate increases in the positionality difference $\left(\gamma_{t}-\gamma_{0}\right)$.
Combining equations (6) and (10), and denoting the instantaneous growth rate at time $t g_{t} \equiv \dot{c}_{t} / c_{t}$, where a dot denotes time derivative, immediately gives us the following result:

Proposition 2. For $g_{t}>0 \forall t, \rho^{s}(t)>(<) \rho^{p}(t) \forall t$, if and only if $\partial \gamma_{t} / \partial c_{t}>(<) 0 \forall t$.

Proof: If $\partial \gamma_{t} / \partial c_{t}>0 \forall t$ and $g_{t}>0 \forall t$, then $\gamma_{t}>\gamma_{0}$, and by comparing equation (6) and equation (10), $\rho^{s}(t)>\rho^{p}(t) \forall t$. Similarly, if $\rho^{s}(t)>\rho^{p}(t) \forall t$, then $\gamma_{t}>\gamma_{0} \forall t$. Since equations (6) and (10) hold, regardless of the initial time $0, \partial \gamma_{t} / \partial c_{t}>0 \forall t$. The proof for the case where $\partial \gamma_{t} / \partial c_{t}<0 \forall t$ is analogous. The case of $\partial \gamma_{t} / \partial c_{t}=0 \forall t$ follows directly from Proposition 1, where $\gamma_{t}=\beta \forall t$.

In other words, if relative consumption becomes more important (compared to absolute consumption) when consumption increases, then the social discount rate is larger than the private one. The intuition is straightforward: When relative consumption becomes more important for higher consumption levels, a larger share of overall consumption increases is "waste" in terms of positional externalities; and since consumption increases over time, this waste share increases over time too. This implies that future consumption becomes less valuable, compared to the present one, from a social point of view, and compared to the case where this waste is not taken into account. Specifically, the social discount rate is higher than the private one.

The empirical evidence, although not conclusive, indeed seems to suggest that $\partial \gamma_{t} / \partial c_{t}>0$, such that relative consumption becomes more important compared to absolute consumption when the individuals get richer (see, e.g., Clark et al. 2008 and Corazzini,

Esposito and Majorano in press and references therein). ${ }^{6}$ This implies that, based on the present model, we may conclude that the social discount rate tends to exceed the private one under relative consumption effects.

### 2.3 Optimal Consumption Taxes

Although not the main purpose of the paper, it is worth mentioning that we can derive the set of consumption taxes that would internalize the time-dependent positional externalities. Let us assume a set of consumption taxes, such that $q_{t}$ is the consumption tax at time $t$, where we normalize such that $q_{0}=0$, and where the tax revenues are distributed back in a lump-sum manner. We can then simply derive $q_{t}$ as follows: If the individual is indifferent between undertaking and not undertaking a small project that will result in increased consumption at time $t$-which is paid for in terms of reduced consumption at time zero, and where consumption at time zero is untaxed and at time $t$ is taxed by $q_{t}$-then the private discount rate per time unit is implicitly defined by:

$$
\frac{\partial w^{p} / \partial c_{t}}{\partial w^{p} / \partial c_{0}}=\left(1+q_{t}\right) \exp \left(-\rho^{p} t\right),
$$

implying that:

$$
\rho^{p}=\delta-\frac{1}{t} \ln \left(\frac{v_{1 t}}{v_{10}} \frac{1}{1+q_{t}}\right),
$$

where $\frac{v_{1 t}}{v_{10}} \frac{1}{1+q_{t}}$ reflects the ratio of the marginal utility of own consumption, per monetary unit spent on consumption, between time $t$ and time zero. If we set this private discount rate equal to the social one, given by equation (10), then:

$$
\begin{equation*}
q_{t}=\frac{\gamma_{t}-\gamma_{0}}{1-\gamma_{t}} . \tag{11}
\end{equation*}
$$

Thus, the tax is directly related to the difference in consumption positionality at time $t$, compared to the untaxed baseline at time zero. The intuition is straightforward, since $\gamma_{t}$ is also a measure of social waste associated with consumption at time $t$. (Cf. with the

[^5]corresponding static optimal tax results in Dupor and Liu 2003 and Aronsson and JohanssonStenman 2008.)

## 3. Comparisons with the Conventional Ramsey Discounting Rule

In the previous section, we analyzed the conditions for when the social discount rate is lower or higher than the private one and concluded that the social discount rate exceeds the private one when the degree of positionality increases with the consumption level. We based this on a simple continuous time model where we discounted over an arbitrary discrete time period, from zero to $t$.

Yet, much of the discounting literature, including recent work on climate change, is based on the Ramsey discount rate, according to which the instantaneous discount rate consists of the pure rate of time preference plus the individual coefficient of relative risk aversion multiplied by the growth rate. It clearly has great policy relevance to compare the optimal social discount rate under relative consumption effects with the Ramsey discounting rule, which is the main purpose of this section.

Since the Ramsey discounting rule is normally derived over an infinitesimally short period of time, ${ }^{7}$ here we solely consider instantaneous discount rates (hereafter discount rates). We will also compare the private and the social discount rates, as well as the private and the Ramsey discount rates, and hence provide conditions for ordering the sizes of the three discount rates. ${ }^{8}$

### 3.1 Three Discount Rates and Three Measures of Relative Risk Aversion

We will undertake our analysis based on equation (1) (the same underlying instantaneous utility function as before), as well as equations (2) and (3) (again the same time-consistent objective functions as before), and we will also keep the assumption of identical individuals with a population size normalized to one. Note first that, when $t$ approaches zero, we have $\partial w^{p} / \partial c_{t}=\partial w^{p} / \partial c_{0}+t \partial\left(\partial w^{p} / \partial c_{t}\right) / \partial t$ and $\partial w^{s} / \partial c_{t}=\partial w^{s} / \partial c_{0}+t \partial\left(\partial w^{s} / \partial c_{t}\right) / \partial t$. Substituting

[^6]these into equations (4) and (5), respectively, and applying l'Hôpital's rule, implies the following private and social instantaneous discount rates:
\[

$$
\begin{align*}
& \rho_{t}^{p}=-\frac{\partial\left(\partial w^{p} / \partial c_{t}\right) / \partial t}{\partial w^{p} / \partial c_{t}}  \tag{12}\\
& \rho_{t}^{s}=-\frac{\partial\left(\partial w^{s} / \partial c_{t}\right) / \partial t}{\partial w^{s} / \partial c_{t}} . \tag{13}
\end{align*}
$$
\]

Thus, $\rho_{t}^{p}$ and $\rho_{t}^{s}$ reflect the private and social discount rates in the time interval $t$ to $t+d t$, whereas $\rho^{p}(t)$ and $\rho^{s}(t)$, analyzed in the previous section, reflect these discount rates in the discrete time interval from 0 to $t .{ }^{9}$ In order to provide an economic interpretation to the comparison between these different instantaneous discount rates, we introduce some useful risk aversion definitions.

Definition 2. The individual coefficient of relative risk aversion is given by:

$$
\begin{equation*}
\sigma_{t} \equiv-c_{t} v_{11 t} / v_{1 t} . \tag{14}
\end{equation*}
$$

The social coefficient of relative risk aversion is given by:

$$
\begin{equation*}
\psi_{t} \equiv-c_{t} u_{11 t} / u_{1 t} . \tag{15}
\end{equation*}
$$

The coefficient of reference consumption relative risk aversion is given by:

$$
\begin{equation*}
\theta_{t} \equiv z_{t} v_{22 t} / v_{2 t} . \tag{16}
\end{equation*}
$$

Thus, whereas $\sigma_{t}$ reflects the conventional measure of relative risk aversion (or the individual's elasticity of marginal utility of increased consumption where others' consumption, $z_{t}$, is held fixed), $\psi_{t}$ is a measure on the curvature of the instantaneous utility function, where relative consumption $R_{t}$ is held fixed. It can therefore be thought of as a measure that reflects risk aversion had the individual made a risky choice on behalf of the whole population. Thus, it is the social coefficient of relative risk aversion. Finally, $\theta_{t}$ is a measure of the concavity, and hence risk aversion, with respect to reference consumption. Its definition is strictly analogous to the conventional measure of individual relative risk aversion, except that it refers to others' consumption. When $\theta_{t}>0$, an individual prefers that others

[^7]have consumption $\hat{z}_{t}$ with certainty over a situation where others' consumption is uncertain (with the expected value $\hat{z}_{t}$ ), whereas the individual is indifferent when $\theta_{t}=0 .{ }^{10} \mathrm{We}$ can now define the conventional instantaneous Ramsey discount rate as follows.

Definition 3. The discount rate according to the Ramsey discounting rule is given by:

$$
\begin{equation*}
\rho_{t}^{R}=\delta+\sigma_{t} g_{t} . \tag{17}
\end{equation*}
$$

Hence, the Ramsey discount rate simply consists of the pure rate of time preference plus the product of the individual coefficient of risk aversion times the growth rate, consistent with existing discounting literature. ${ }^{11}$

### 3.2 The Relationship between the Private and the Ramsey Discount Rate

From equation (2) it follows that $\partial w^{p} / \partial c_{t}=v_{1 t} e^{-\delta t} \quad$ and $\partial\left(\partial w^{p} / \partial c_{t}\right) / \partial t=\left(v_{111} \dot{c}_{t}+v_{12 t} \dot{c}_{t}-\delta v_{1 t}\right) \exp (-\delta t)$, which substituted into equation (12) implies the optimal private discount rate as follows: ${ }^{12}$

$$
\begin{equation*}
\rho_{t}^{p}=\delta-\frac{v_{11 t}}{v_{1 t}} c_{t} g_{t}-\frac{v_{12 t}}{v_{1 t}} c_{t} g_{t} . \tag{18}
\end{equation*}
$$

By combining equations (17) and (18) and using Definition 3, we obtain:

$$
\begin{equation*}
\rho_{t}^{p}=\delta+\sigma_{t} g_{t}-\frac{v_{12 t}}{v_{1 t}} c_{t} g_{t}=\rho_{t}^{R}-\frac{v_{12 t}}{v_{1 t}} c_{t} g_{t} . \tag{19}
\end{equation*}
$$

From equation (19) we immediately get the following result:

Proposition 3. For $g_{t}>0, \rho_{t}^{p}<\rho_{t}^{R}$.

[^8]Thus, if the growth rate is positive and the preferences are characterized by the keeping-up-with-the-Joneses property, then the private discount rate falls short of the Ramsey discount rate. The intuition is that if my preferences are characterized by the keeping-up-with-theJoneses property, then I will perceive future consumption relative to the present one as more valuable if others consume more in the future. Yet, this is the same as saying that my private discount rate will decrease as others' consumption grows.

### 3.3 The Relationship between the Private and the Social Discount Rate

While we have already analyzed this relationship for the case of discounting over a discrete time period, let us for comparison purposes also derive corresponding expressions for the case of instantaneous discounting. From equation (3) follows that $\partial w^{s} / \partial c_{t}=\left(v_{1 t}+v_{2 t}\right) \exp (-\delta t)$, such that

$$
\frac{\partial\left(\partial w^{s} / \partial c_{t}\right)}{\partial t}=\left(v_{11 t} \dot{c}_{t}+v_{12 t} \dot{c}_{t}+v_{21 t} \dot{c}_{t}+v_{22 t} \dot{c}_{t}-\delta\left(v_{1 t}+v_{2 t}\right)\right) \exp (-\delta t),
$$

which substituted into equation (13) implies the following optimal social discount rate:

$$
\begin{equation*}
\rho_{t}^{s}=\delta-\frac{v_{11 t}+2 v_{12 t}+v_{22 t}}{v_{1 t}+v_{2 t}} c_{t} g_{t}=\delta+\sigma_{t} g_{t}-\frac{v_{12 t}}{v_{1 t}} c_{t} g_{t}+\frac{d \gamma_{t} / d c_{t}}{1-\gamma_{t}} c_{t} g_{t}, \tag{20}
\end{equation*}
$$

where, in the second step, we have used $\frac{d \gamma_{t}}{d c_{t}}=-\left(\frac{v_{22 t}}{v_{1 t}}+\frac{v_{21 t}}{v_{1 t}}-\frac{v_{2 t}}{v_{1 t}{ }^{2}} v_{11 t}-\frac{v_{2 t}}{v_{1 t}{ }^{2}} v_{12 t}\right)$. By comparing equations (19) and (20), we obtain: ${ }^{13}$

$$
\begin{equation*}
\rho_{t}^{s}=\rho_{t}^{p}+\frac{d \gamma_{t} / d c_{t}}{1-\gamma_{t}} c_{t} g_{t}, \tag{21}
\end{equation*}
$$

implying that we can again confirm Proposition 2.
In order to simplify this expression further, we can define the degree of nonpositionality as:

$$
\begin{equation*}
\varsigma_{t} \equiv 1-\gamma_{t}=\frac{u_{1 t}}{u_{1 t}+u_{2 t} r_{1 t}}, \tag{22}
\end{equation*}
$$

and the corresponding consumption elasticity of non-positionality as:

$$
\begin{equation*}
\Gamma \equiv \frac{d \varsigma_{t}}{d c_{t}} \frac{c_{t}}{\varsigma_{t}} . \tag{23}
\end{equation*}
$$

[^9]The degree of non-positionality is hence defined by the fraction of the overall utility increase from the last dollar consumed that is due to increased absolute consumption, whereas the consumption elasticity of non-positionality can be interpreted as (approximately) the percentage change in non-positionality that arises from a percentage consumption increase. We can then use equation (23) and rewrite equations (20) and (21) as:

$$
\begin{equation*}
\rho_{t}^{s}=\delta+\sigma_{t} g_{t}-\frac{v_{12 t}}{v_{1 t}} c_{t} g_{t}-\Gamma_{t} g_{t}=\rho_{t}^{p}-\Gamma_{t} g_{t} \tag{24}
\end{equation*}
$$

Thus, the social discount rate is equal to the private one minus the consumption elasticity of non-positionality times the consumption growth rate. Note that $\Gamma_{t} g_{t}$ reflects the growth rate of the fraction of consumption that, from a social point of view, is not waste. If this growth rate is positive, it implies a reason for society to consume more in the future, implying a lower social discount rate. If it is negative, and more in line with the empirical evidence mentioned, then the opposite applies.

### 3.4 The Relationship between the Social and the Ramsey Discount Rate

Let us now turn to the most central part of this section, the comparison between the social and the Ramsey discount rate. So far, in equation (19) we expressed the relationship between the private and the Ramsey discount rate, in terms of a keeping-up-with-the-Joneses measure, and in equation (24) we expressed the relationship between the private and the social discount rate, in terms of how the degree of positionality depends on the consumption level. By combining equations (19) and (24), we can clearly present the relationship between the social and the Ramsey discount rate, in terms of a keeping-up-with-the-Joneses measure and how the degree of positionality depends on the consumption level, as follows:

$$
\begin{equation*}
\rho_{t}^{s}=\rho_{t}^{R}-\frac{v_{12 t}}{v_{1 t}} c_{t} g_{t}+\frac{d \gamma_{t} / d c_{t}}{1-\gamma_{t}} c_{t} g_{t}=\rho_{t}^{R}-\frac{v_{12 t}}{v_{1 t}} c_{t} g_{t}-\Gamma_{t} g_{t} . \tag{25}
\end{equation*}
$$

Thus, according to this formulation, the social discount rate exceeds the Ramsey discount rate if the effects-through increased positionality (or decreased non-positionality) with consumption, and hence over time, given a positive growth rate-exceed the keeping-up-with-the-Joneses effect. Both assumptions are fairly intuitive and, if we accept them, we cannot say anything regarding the relative size of $\rho_{t}^{s}$ and $\rho_{t}^{R}$ without adding information regarding the relative strengths of these mechanisms. Yet, we will indirectly show conditions for when the keeping-up-with-the-Joneses effect dominates the increasing positionality effect.

In doing this, let us use the alternative formulation of equation (3), where $w^{s}$ is expressed in terms of the $u$ function. It then follows that $\partial w / \partial c_{t}=u_{1 t} \exp (-\delta t)$ and $\partial\left(\partial w / \partial c_{t}\right) / \partial t=\left(u_{11 t} \dot{c}_{t}-\delta u_{1 t}\right) \exp (-\delta t)$, which substituted into equation (12) implies the following alternative formulation of the optimal social discount rate:

$$
\begin{equation*}
\rho_{t}^{s}=\delta+\psi_{t} g_{t} . \tag{26}
\end{equation*}
$$

By comparing equations (17) and (26), we immediately have:

Proposition 4. For $g_{t}>0, \rho_{t}^{s}>(<) \rho_{t}^{R}$ if and only if $\psi_{t}>(<) \sigma_{t}$.

In words, given a positive growth rate, the welfare-maximizing social discount rate in the presence of relative consumption effects exceeds the conventional Ramsey discount rate if the social coefficient of relative risk aversion exceeds the individual coefficient of relative risk aversion, and vice versa.

The next question is obvious. Under which conditions does $\psi_{t}$ exceed $\sigma_{t}$, and vice versa? In order to interpret the conditions for this in economic terms, we introduce a measure of the complementarity between own and (reduction of) others' consumption.

Definition 4. The elasticity of substitution between $c_{t}$ and $z_{t}$ is given by:

$$
\begin{equation*}
\Phi_{t} \equiv \frac{-\frac{v_{11 t}}{v_{1 t}{ }^{2}}-\frac{v_{22 t}}{v_{2 t}{ }^{2}}+2 \frac{v_{12 t}}{v_{1 t} v_{2 t}}}{\frac{1}{v_{1 t} c_{t}}+\frac{1}{v_{2 t} z_{t}}}=\left(-\frac{v_{11 t}}{v_{1 t}{ }^{2}}-\frac{v_{22 t}}{v_{2 t}{ }^{2}}+2 \frac{v_{12 t}}{v_{1 t} v_{2 t}}\right) \frac{v_{1 t} v_{2 t}}{v_{1 t}+v_{2 t}} c_{t} . \tag{27}
\end{equation*}
$$

This definition is standard (although it is not often used between one good and one bad in the utility function). It reflects the degree of quasi-concavity of $v$, and hence the degree of convexity of the indifference curves in $c_{t}, z_{t}$ space. When $\Phi_{t}>0$ everywhere, an individual who could hypothetically buy a reduction in others' consumption at a fixed per unit price would then face a unique optimum, whereas $\Phi_{t}=0$ everywhere would give linear indifference curves, such that the increase in consumption necessary to compensate an individual for other people's increase in consumption would be constant. We can now specify the following crucial relation between $\psi_{t}$ and $\sigma_{t}$ :

Lemma 1. The social coefficient of relative risk aversion is given by:

$$
\begin{equation*}
\psi_{t}=\sigma_{t}-\theta_{t}-\Phi_{t} . \tag{28}
\end{equation*}
$$

Proof: Let us first express the social coefficient of relative risk aversion $\psi_{t}$ in terms of the $v$ function. By comparing equations (18) and (23), we have:

$$
\begin{equation*}
\psi_{t}=-\frac{v_{11 t}+2 v_{12 t}+v_{22 t}}{v_{1 t}+v_{2 t}} c_{t} \tag{29}
\end{equation*}
$$

We can now combine equations (14) and (27) and (after some straightforward algebraic manipulations) obtain:

$$
\begin{equation*}
\psi_{t}-\sigma_{t}=-c_{t} \frac{v_{22 t}}{v_{2 t}}-c_{t} \frac{v_{1 t} v_{2 t}}{v_{1 t}+v_{2 t}}\left(-\frac{v_{11 t}}{v_{1 t}{ }^{2}}-\frac{v_{22 t}}{v_{2 t}{ }^{2}}+2 \frac{v_{12 t}}{v_{1 t} v_{2 t}}\right) . \tag{30}
\end{equation*}
$$

Substituting, finally, equations (16) and (27) into equation (30) gives equation (28).

Directly from Lemma 1 and equation (10), we can specify the social discount rate in terms of individual risk aversion as follows:

$$
\begin{equation*}
\rho_{t}^{s}=\delta+\left(\sigma_{t}-\theta_{t}-\Phi_{t}\right) g_{t}=\rho_{t}^{R}-\left(\theta_{t}+\Phi_{t}\right) g_{t} . \tag{31}
\end{equation*}
$$

It is the straightforward to determine the conditions for when the social discount rate exceeds the one based on the conventional Ramsey discounting rule, and vice versa:

Proposition 5. For $g_{t}>0, \rho_{t}^{s}<(>) \rho_{t}^{R}$ if $\theta_{t}+\Phi_{t}>(<) 0$.

Thus, a sufficient, but not necessary, condition for $\rho_{t}^{s}<\rho_{t}^{R}$ is that $v$ is strictly quasi-concave ( $\Phi_{t}>0$ ) and individuals are weakly reference-consumption risk averse ( $v_{22 t} \leq 0, \theta_{t} \geq 0$ ) or, alternatively, that $v$ is weakly quasi-concave $\left(\Phi_{t} \geq 0\right)$ and individuals are strictly referenceconsumption risk averse $\left(v_{22 t}<0, \theta_{t}>0\right)$.

The intuition behind Proposition 5 is that reference-consumption risk aversion and quasi-concavity both contribute to a decrease in the social marginal utility of consumption when consumption increases. Yet, at the same time, they contribute to a decrease in the percentage change in social marginal utility of consumption for a percentage increase in consumption.

In order to gain some further insights, let us consider some special cases where we deal with reference-consumption risk aversion and quasi-concavity separately. Starting with the case of reference-consumption risk aversion (i.e., that $v_{22 t}<0$ ) and perfect substitution (i.e., linear indifference curves, such that $\Phi_{t}=0$ ), we can write instantaneous utility as $f\left(c_{t}-a z_{t}\right)=f\left((1-a) c_{t}+a\left(c_{t}-z_{t}\right)\right)$, where $0<a<1$ reflects the degree of positionality and $f^{\prime}>0$ and $f^{\prime \prime}<0$. Then, $v_{1 t}=f^{\prime}, v_{11 t}=f^{\prime \prime}, u_{1 t}=(1-a) f^{\prime}$, and $u_{11 t}=(1-a)^{2} f^{\prime \prime}$, implying that $\sigma_{t}=-c_{t} f^{\prime \prime} / f^{\prime}$ and $\psi_{t}=-(1-a) c_{t} f^{\prime \prime} / f^{\prime}$, such that $\psi_{t}-\sigma_{t}=a c_{t} f^{\prime \prime} / f^{\prime}<0$, and hence $\rho_{t}^{s}<\rho_{t}^{R}$.

The reason is simply that when $a$ increases, the curvature of $u$ decreases. When $a$ approaches 1 , such that only relative consumption matters, both $u_{1 t}$ and $u_{11 t}$ approach zero, but $u_{11 t}$ approaches zero more rapidly. A 1 percent change in a subset of consumption (consumption minus the effect of reference consumption) will cause a smaller than 1 percent change in consumption, per se. Hence, the corresponding changes in marginal instantaneous social utility will be smaller too.

Consider next the case with quasi-concavity and reference consumption risk neutrality, such that $\Phi_{t}>0$ and $v_{22 t}=\theta_{t}=0$. A simple functional form that fulfills this is $c_{t}^{1-\alpha} /(1-\alpha)-\beta z_{t}=c_{t}^{1-\alpha} /(1-\alpha)+(1-\beta) c_{t}+\beta\left(c_{t}-z_{t}\right)$, where $\alpha, \beta>0$ and $\beta<1+\frac{1}{1+\alpha} c_{t}^{-\alpha}$ to ensure quasi-concavity. $\alpha$ is the individual coefficient of relative risk aversion, such that $\sigma_{t}=\alpha$, and the degree of positionality increases with $\beta$, but is not constant.

It then follows that $u_{1 t}=c_{t}^{-\alpha}+(1-\beta)$ and $u_{11 t}=-\alpha c_{t}^{-\alpha-1}$, implying that $\psi_{t}=\frac{\alpha}{1+(1-\beta) c_{t}^{\alpha}} \quad$ and $\quad \psi_{t}-\sigma_{t}=-\alpha \frac{(1-\beta) c_{t}^{\alpha}}{1+(1-\beta) c_{t}^{\alpha}}=-\frac{\alpha}{1+(1-\beta)^{-1} c_{t}^{-\alpha}}<0$. Hence, again, $\rho_{t}^{s}<\rho_{t}^{R}$. Here there is no direct effect on the curvature from $z$, since we have referenceconsumption risk neutrality, yet $z$ will still affect the curvature of the instantaneous utility function differently depending on whether relative consumption or others' consumption is held fixed.

However, it should be noted that the condition in equation (31) is not independent of equation (25). Indeed, we can directly show that $\frac{d \gamma_{t} / d c_{t}}{1-\gamma_{t}} c_{t}=-\Gamma_{t}=\frac{v_{12 t}}{v_{1 t}} c_{t}-\left(\theta_{t}+\Phi_{t}\right)$, clearly
implying that if $\theta_{t}+\Phi_{t}>0$, then $\frac{d \gamma_{t} / d c_{t}}{1-\gamma_{t}} c_{t} g_{t}<\frac{v_{12 t}}{v_{1 t}} c_{t} g_{t}$. Thus, if $v$ is quasi-concave and individuals are reference-consumption risk averse, then the effects through increased positionality over time in equation (31) cannot exceed the keeping-up-with-the-Joneses effect.

### 3.5 Summary of Main Findings

By combining equation (20) and Proposition 5, we can summarize our main findings in this section regarding the ordering of our three different discount rates as follows: ${ }^{14}$

Corollary 1. For $g_{t}>0$, if $d \gamma_{t} / d c_{t}>0$ and $\theta_{t}+\Phi_{t}>0$, then $\rho_{t}^{p}<\rho_{t}^{s}<\rho_{t}^{R}$.

Spelled out, this means that, for a positive consumption growth rate, the social discount rate is higher than the private rate, but smaller than the Ramsey rate, if the degree of positionality increases with consumption and preferences reflect risk aversion with respect to reference consumption and are quasi-concave with respect to own and reference consumption.

## 4. Numerical Illustration and Orders of Magnitude

So far we have concluded that, under what may seem to be fairly plausible assumptions, the social discount rate tends to exceed the private discount rate (Sections 2 and 3), but fall short of the conventional Ramsey discounting rule (Section 3). The latter finding is particularly important from a policy perspective, for example in the discussion on climate change.

To get an indication of the magnitude of the effects found, we utilize the following simple, albeit quite flexible, functional form characterized by constant elasticity of substitution and constant relative risk aversion, similar to one used by Dupor and Liu (2003) as follows:

$$
\begin{equation*}
U_{t}=\frac{1}{1-\alpha}\left((1-a) c_{t}^{1-\omega}+a\left(c_{t}^{1-\omega}-z_{t}^{1-\omega}\right)\right)^{\frac{1-\alpha}{1-\omega}}=\frac{1}{1-\alpha}\left(c_{t}^{1-\omega}-a z_{t}^{1-\omega}\right)^{\frac{1-\alpha}{1-\omega}} . \tag{32}
\end{equation*}
$$

This functional form has some convenient properties:

- The degree of positionality (see Definition 1) is constant and given by $\gamma_{t}=a$.

[^10]- The elasticity of substitution between own consumption and reference consumption (see Definition 4) is constant and equal to $\omega$.
- The coefficient of reference-consumption relative risk aversion (see Definition 2) is constant and given by $\theta_{t}=\frac{a \alpha-\omega}{1-a}$, implying reference-consumption risk aversion for $\omega<a \alpha$.
- The individual and the social coefficients of relative risk aversion (see Definition 2) are also constant and given by $\sigma_{t}=\frac{\alpha-\omega a}{1-a}$ and $\psi_{t}=\alpha$, respectively, implying that:

$$
\begin{equation*}
\psi_{t}-\sigma_{t}=(\omega-\alpha) \frac{a}{1-a}, \tag{33}
\end{equation*}
$$

which is clearly weakly negative (such that $\rho_{t}^{s} \leq \rho_{t}^{R}$ ) as long as $\omega \leq \alpha$, which is required for the weak keeping-up-with-the-Joneses property assumed.

The constant elasticity of substitution functional form in equation (32) includes as special cases the two most commonly used comparison-consumption functional forms. When $\omega=0$, we obtain the following simple difference comparison form, so that own consumption and (the negative of) others' consumption are perfect substitutes:

$$
\begin{equation*}
U_{t}=\frac{1}{1-\alpha}\left((1-a) c_{t}+a\left(c_{t}-z_{t}\right)\right)^{(1-\alpha)}=\frac{1}{1-\alpha}\left(c_{t}-a z_{t}\right)^{(1-\alpha)} . \tag{34}
\end{equation*}
$$

For this functional form, we find that $\psi_{t}-\sigma_{t}=-\alpha \frac{a}{1-a}<0$. For example, if $a=0.5$ and $\alpha=1$, then $\psi_{t}-\sigma_{t}=-1$, implying that the private coefficient of relative risk aversion $\sigma_{t}=2$, whereas the corresponding social coefficient $\psi_{t}=1$. Hence, it is clear that the effects of relative consumption may be substantial.

Similarly, we obtain the ratio-comparison form by letting $\omega$ approach unity and applying l'Hôpital's rule, implying that equation (32) converges to:

$$
\begin{equation*}
U_{t}=\frac{1}{1-\alpha}\left(c_{t}\left(\frac{c_{t}}{z_{t}}\right)^{a /(1-a)}\right)^{(1-\alpha)}=\frac{1}{1-\alpha} c_{t}^{(1-\alpha) /(1-a)} z_{t}^{-(1-\alpha) a /(1-a)}, \tag{35}
\end{equation*}
$$

such that $\psi_{t}-\sigma_{t}=(1-\alpha) \frac{a}{1-a}$, which is clearly negative if $\alpha>1$ and positive if $\alpha<1$. Note that reference-consumption risk aversion implies that $\alpha>1 / a$, and the keeping-up-with-the-

Joneses assumption implies that $\alpha \geq 1$. Here, too, it is clear that the effects of relative consumption may be substantial as we illustrate further below. In order to assess orders of magnitude of the effects on the discount rates, we substitute equation (33) into equation (25) and obtain that

$$
\begin{equation*}
\rho_{t}^{s}=\delta+\left(\sigma+(\omega-\alpha) \frac{a}{1-a}\right) g . \tag{36}
\end{equation*}
$$

In Figure 1, we plot the optimal social discount rate as a function of the degree of positionality for two commonly discussed sets of assumptions in the economic climate change literature, associated with Stern (2006) and Weitzman (2007b), respectively. Stern (2006) assumed that $\delta=0.1$ (in percentage terms), $\sigma=1$, and $g=1.3$, implying an overall Ramsey discount rate of 1.4 percent annually, ${ }^{15}$ whereas Weitzman (2007b) discusses the case where $\delta=\sigma=g=2,{ }^{16}$ leading to a Ramsey discount rate of 6 percent. Since both Stern and Weitzman conventionally assumed that relative consumption does not matter for utility, the corresponding social discount rates can be found to the left in the diagram where $a=0$. As can be seen, how much relative consumption concerns affect the optimal social discount rate will then depend also on the elasticity of substitution, $\omega$. We only plot the relationships for the case where the weak keeping-up-with-the-Joneses property is fulfilled, i.e. where $\omega \leq \alpha$, implying in the Stern case that $\omega \leq 1$ and in the Weitzman case that $\omega \leq 2$. As can be seen, the optimal social discount rate is always equal to or below the Ramsey discount rate.

We illustrate the parts of the relationships where the individual is referenceconsumption risk averse ${ }^{17}$ with solid lines, and the parts where the individual is referenceconsumption risk-loving with dotted lines. Clearly, for non-negligible levels of $a$, the optimal discount rate tends to be substantially smaller than the Ramsey rate if we assume referenceconsumption risk aversion.

[^11]Figure 1. The Effect of the Degree of Postionality $a$ on the Optimal Social Discount Rates Associated with the Assumptions in Stern (2006) and Weitzman (2007b).


Unfortunately, the empirical literature does not provide any precise estimates of the relevant parameters. The most central one, $\gamma$ (or $a$ ), is obviously difficult to measure, and it is therefore not surprising that the available estimates vary considerably. However, most estimates are substantially above zero. According to the survey-experimental evidence of Solnick and Hemenway (1998; 2005), Johansson-Stenman et al. (2002), Alpízar et al. (2005), and Carlsson et al. (2007), the average degree appears to be in the order of magnitude of 0.5. Wendner and Goulder (2008) argued, based on existing empirical evidence, for a value between 0.2 and 0.4 , whereas evidence from happiness studies, such as Luttmer (2005), suggests a much larger value close to unity. To our knowledge, there are no quantitative estimates of either $\theta$ or $\Phi$-i.e., of either reference-consumption risk aversion or the degree of quasi-concavity between own consumption and reference consumption-beyond what is implied by keeping-up-with-the-Joneses behavior.

In contrast, there are many studies trying to estimate $\sigma$, which is relatively less important for the results here. ${ }^{18}$ Estimates of $\delta$ are highly controversial, in particular for ethical reasons, when dealing with intergenerational issues (Stern 2006). Yet, as shown above, $\delta$ does not affect the difference between the social and the Ramsey discount rates. Future growth rates are of course also difficult to predict.

Overall, the discrepancy between the social and the Ramsey discount rates due to relative consumption effects is clearly difficult to quantify, but may well be substantial and could even exceed 1-2 percentage points in the estimates that are most frequently used in the climate literature.

## 5. Conclusion and Discussion

There are several reasons based on the existing literature why one may argue that social discount rates should in practice be lower than individual ones: individuals are more risk averse than society in the presence of uncertainty, and societal time horizons are longer than individual ones (cf. Arrow and Lind 1970). In the present paper, we show that relative consumption effects do not provide another reason. On the contrary, the social discount rate under positional concern tends to exceed the private one, provided that the degree of positionality increases as we get richer and consumption increases (for which there is some empirical evidence).

Yet, from a climate policy perspective, it is presumably more important whether the optimal social discount rate should be modified compared to the one corresponding to the conventional Ramsey discounting rule. We show that for a positive growth rate, the social discount rate is lower than the Ramsey discount rate if preferences are quasi-concave in own and reference consumption (consisting of others' average consumption) and concave in reference consumption. We also demonstrate numerically that the discrepancies may be substantial, although the underlying parameter estimates are highly uncertain. Since the impacts of the discount interest rates on the economics of long-term phenomena, such as global warming, are so large-even for modest adjustments of the discount rate-it is fair to

[^12]conclude that taking relative consumption effects into account may have a profound effect on the economics of phenomena like global warming.

## Acknowledgments

We are grateful for helpful comments from Thomas Aronsson, Kenneth Arrow, Partha Dasgupta, Martin Dufwenberg, Tore Ellingsen, Reyer Gerlagh, Christian Gollier, Geoff Heal, Michael Hoel, Richard Howarth, Martin Weitzman and Ronald Wendner, as well as seminar participants at the University of Oslo, University of Gothenburg, Resources for the Future in Washington DC, and CESifo (Munich). The research leading to these results has received funding from the European Union Seventh Framework Programme (FP7/2007-2013) under grant agreement no [266992]. Financial support is also gratefully acknowledged from the Swedish Research Council, the Mistra program Indigo and FORMAS COMMONS. .

## References

Abel, A.B. 2005. Optimal Taxation When Consumers Have Endogenous Benchmark Levels of Consumption. Review of Economic Studies 72: 1-19.

Akerlof, G.A. 1997. Social Distance and Social Decisions. Econometrica 65: 1005-1027.
Alpízar, F., F. Carlsson, and O. Johansson-Stenman. 2005. How Much Do We Care about Absolute versus Relative Income and Consumption? Journal of Economic Behavior and Organization 56: 405-21.

Aronsson, T., and O. Johansson-Stenman. 2008. When the Joneses’ Consumption Hurts: Optimal Public Good Provision and Nonlinear Income Taxation. Journal of Public Economics 92: 986-997.
—_ 2010. Positional Concerns in an OLG Model: Optimal Labor and Capital Income Taxation. International Economic Review 51(4): 1071-1095.
Arrow, K.J., and P.S. Dasgupta. 2009. Conspicuous Consumption, Inconspicuous Leisure. Economic Journal 119(541): F497-F516.

Arrow, K.J., W.R. Cline, K.-G. Mäler, M. Munasinghe, R. Squitieri, and J.E. Stiglitz. 1996. Intertemporal Equity, Discounting, and Economic Efficiency. In Climate Change 1995: Economic and Social Dimensions-Contribution of Working Group III to the Second Assessment Report of the Intergovernmental Panel on Climate Change, edited
by J.P. Bruce, H. Lee, and E.F. Haites, 125-44. Cambridge: Cambridge University Press.

Arrow, K.J., and R.C. Lind. 1970. Uncertainty and the Evaluation of Public Investment Decisions. American Economic Review 6: 364-78.

Attanasio, O.P., and M. Browning. 1995. Consumption over the Life Cycle and over the Business Cycle. American Economic Review 85: 1118-37.

Blanchflower, D.G., and A.J. Oswald. 2004. Well-Being over Time in Britain and the USA. Journal of Public Economics 88: 1359-86.

Blundell, R., M. Browning, and C. Meghir. 1994. Consumer Demand and the Life-Cycle Allocation of Household Expenditures. Review of Economic Studies 61: 57-80.
Boskin, M.J., and E. Sheshinski. 1978. Optimal Redistributive Taxation when Individual Welfare Depends upon Relative Income. Quarterly Journal of Economics 92: 589-601.
Bowles, S., and Y.-J. Park. 2005. Inequality, Emulation, and Work Hours: Was Thorsten Veblen Right? Economic Journal 15: F397-F413.

Brekke, K.A., and R. Howarth. 2002. Status Growth and the Environment: Goods as Symbols in Applied Welfare Economics. Ecological Economics 49(3): 414-15.

Brekke, K.A., O. Johansson-Stenman. 2008. The Behavioral Economics of Climate Change. Oxford Review of Economic Policy 24 (2): 280-297.

Carlsson, F., O. Johansson-Stenman, and P. Martinsson. 2007. Do You Enjoy Having More than Others? Survey Evidence of Positional Goods. Economica 74: 586-98.

Carroll, C.D., J. Overland, and D.N. Weil. 1997. Comparison Utility in a Growth Model. Journal of Economic Growth 2: 339-67.

Clark, A.E., P. Frijters, and M.A. Shields. 2008. Relative Income, Happiness, and Utility: An Explanation for the Easterlin Paradox and Other Puzzles. Journal of Economic Literature 46: 95-144.

Clark, A.E., and A.J. Oswald. 1998. Comparison-Concave Utility and Following Behavior in Social and Economic Settings. Journal of Public Economics 70: 133-55.

Corazzini, L., L. Esposito, and F. Majorano. 2011, in press. Reign in Hell or Serve in Heaven? A Cross-Country Journey into the Relative vs. Absolute Perceptions of Wellbeing. Journal of Economic Behavior and Organization.

Dasgupta, P. 2007. Commentary: The Stern Review's Economics of Climate Change. National Institute Economic Review, 199: 47.

Dasgupta, P. 2008. Discounting Climate Change. Journal of Risk and Uncertainty 37: 141169.

Dietz, S., and N. Stern. 2007. Why Economic Analysis Supports Strong Action on Climate Change: A Response to the Stern Review's Critics. Review of Environmental Economics and Policy 2(1): 94-113.
Duesenberry, J.S. 1949. Income, Savings, and the Theory of Consumer Behavior. Cambridge, MA, USA: Harvard University Press.

Dupor, B., and W.F. Liu. 2003. Jealousy and Overconsumption. American Economic Review 93: 423-28.

Ferrer-i-Carbonell, A. 2005. Income and Well-Being: An Empirical Analysis of the Comparison Income Effect. Journal of Public Economics 89: 997-1019.

Fliessbach, K., B. Weber, P. Trautner, T. Dohmen, U. Sunde, C.E. Elger, and A. Falk. 2007. Social Comparison Affects Reward-Related Brain Activity in the Human Ventral Striatum. Science 318: 1305-1308.

Frederick, S., G. Loewenstein, and T. O'Donoghue. 2002. Time Discounting and Time Preference: A Critical Review. Journal of Economic Literature 40( 2): 351-401.

Friend, I., and M.E. Blume. 1975. The Demand for Risky Assets. American Economic Review 65(5): 900-922.

Gali, J. 1994. Keeping Up with the Joneses: Consumption Externalities, Portfolio Choice, and Asset Prices. Journal of Money, Credit, and Banking 26: 1-8.
Gollier, C. 2010. Ecological Discounting. Journal of Economic Theory 145: 812-29.
——. Forthcoming. Pricing the future: The economics of discounting and sustainable development. Princeton University Press.
——. and M. L. Weitzman. 2010. How Should the Distant Future be Discounted when Discount Rates are Uncertain? Economics Letters 107: 350-353.

Halek, M., and J.G. Eisenhauer. 2001. Demography of Risk Aversion. Journal of Risk and Insurance 68: 1-24.

Heal, G. 2008. Climate Change Economics: A Meta-Review and Some Suggestions. New York: Columbia University, School of International and Public Affairs, Faculty Research on Energy and Development. http://www2.gsb.columbia.edu/faculty/gheal/Climate-Change-Review.pdf
Hoel, M., and T. Sterner. 2007,. Discounting and Relative Prices. Climatic Change 84: 26580.

Howarth, R.B. 2009. Discounting, Uncertainty, and Revealed Time Preference. Land Economics 85(1): 24-40.

Johansson-Stenman, O., F. Carlsson, and D. Daruvala. 2002. Measuring Future Grandparents’ Preferences for Equality and Relative Standing. Economic Journal 112: 362-83.
Layard, R. 1980. Human Satisfaction and Public Policy. Economic Journal 90: 737-50.
Ljungqvist, L., and H. Uhlig. 2000. Tax Policy and Aggregate Demand Management under Catching Up with the Joneses. American Economic Review 90: 356-66.

Luttmer, E.F.P. 2005. Neighbors as Negatives: Relative Earnings and Well-Being. Quarterly Journal of Economics 120: 963-1002.

Moav, O. and Z. Neeman (forthcoming), Saving Rates and Poverty: The Role of Conspicuous Consumption and Human Capital, Economic Journal.

Ng, Y.-K. 1987. Relative Income Effects and the Appropriate Level of Public Expenditure. Oxford Economic Papers 39: 293-300.

Nordhaus, W. D. 2007a. A Review of the Stern Review on the Economics of Climate Change. Journal of Economic Literature 45(3): 686-702.

Nordhaus, W. D. 2007b. Critical Assumptions in the Stern Review on Climate Change. Science 317: 201-202.

Oswald, A. 1983. Altruism, Jealousy, and the Theory of Optimal Non-linear Taxation. Journal of Public Economics 20: 77-87.
Ramsey, F. 1928. A Mathematical Theory of Saving. Economic Journal 38: 543-49.
Rayo, L., and G.S. Becker. 2007. Evolutionary Efficiency and Happiness. Journal of Political Economy 115: 302-337.
Samuelson, L. 2004. Information-Based Relative Consumption Effects. Econometrica 72: 93118.

Solnick, S., and D. Hemenway. 2005. Are Positional Concerns Stronger in Some Domains than in Others? American Economic Review 95(2): 147-51.

Stern, N. 2006. The Economics of Climate Change: The Stern Review. Cambridge: Cambridge University Press.

Sterner, T., and U.M. Persson. 2008. An Even Sterner Review: Introducing Relative Prices into the Discounting Debate. Review of Environmental Economics and Policy 2(1): 61-76.

Stevenson, B., and J. Wolfers. 2008. Economic Growth and Subjective Well-Being: Reassessing the Easterlin Paradox. Brookings Papers on Economic Activity (Spring 2008: 1-87.

Vissing-Jørgensen, A. 2002. Limited Asset Market Participation and the Elasticity of Intertemporal Substitution. Journal of Political Economy 4: 825-53.

Weitzman, M. 2007a. Subjective Expectations of Asset Price Puzzles. American Economic Review 97(4): 1102-1130.
——. 2007b. A Review of the Stern Review on the Economics of Climate Change. Journal of Economic Literature 45(3): 703-724.
——_ 2010. Risk-Adjusted Gamma Discounting. Journal of Environmental Economics and Management 60: 1-13.

Wendner, R. 2010a. Growth and Keeping up with the Joneses, Macroeconomic Dynamics 14(S2): 176-199.

Wendner, R. 2010b. Conspicuous Consumption and Generation Replacement in a Model of Perpetual Youth, Journal of Public Economics 94: 1093-1107.

Wendner, R. (2011), Will the Consumption Externalities' Effects in the Ramsey Model Please Stand Up? Economics Letters 111: 210-212.

Wendner, R. and L.H. Goulder. 2008. Status Effects, Public Goods Provision, and the Excess Burden. Journal of Public Economics 92: 1968-85.


[^0]:    * We are grateful for helpful comments from Thomas Aronsson, Kenneth Arrow, Partha Dasgupta, Martin Dufwenberg, Tore Ellingsen, Reyer Gerlagh, Christian Gollier, Geoff Heal, Michael Hoel, Richard Howarth, Martin Weitzman and Ronald Wendner, as well as seminar participants at the University of Oslo, University of Gothenburg, Resources for the Future in Washington DC, and CESifo (Munich). The research leading to these results has received funding from the European Union Seventh Framework Programme (FP7/2007-2013) under grant agreement no [266992]. Financial support is also gratefully acknowledged from the Swedish Research Council, the Mistra program Indigo and FORMAS COMMONS.

[^1]:    ${ }^{1}$ This includes happiness research (e.g., Blanchflower and Oswald 2004; Ferrer-i-Carbonell 2005; and Luttmer 2005), questionnaire-based experiments (e.g., Johansson-Stenman et al. 2002; Solnick and Hemenway 2005; and Carlsson et al. 2007), and brain science (e.g., Fliessbach et al. 2007). There are also recent evolutionary models consistent with relative consumption concerns (Samuelson 2004 and Rayo and Becker 2007). Stevenson and Wolfers (2008) constitute a recent exception in the happiness literature, claiming that the role of relative income is overstated.
    ${ }^{2}$ See, e.g., Boskin and Sheshinski (1978), Layard (1980), Oswald (1983), Ng (1987), Brekke and Howarth (2002), Abel (2005), and Wendner and Goulder (2008). Clark et al. (2008) provide a good overview of both the empirical evidence and economic implications of relative consumption concerns.

[^2]:    ${ }^{3}$ Other papers have dealt with the issue of whether, and if so how, the externalities induced by relative consumption effects affect the consumption pattern over time; see e.g. Wendner (2010b) and references therein.

[^3]:    ${ }^{4}$ Dupor and Liu (2003) use a different definition of Keeping-up-with-the-Joneses preferences in a static model where utility depends on leisure, in addition to absolute and relative consumption. Keeping-up-with-the-Joneses preferences are then naturally defined as preferences implying that an individual's consumption increases (and leisure decreases) when others' consumption increases, which follows if the marginal rate of substitution between consumption and leisure increases with others' consumption.

[^4]:    ${ }^{5}$ See, e.g., Gollier (forthcoming) for a comprehensive state-of-the-art overview of social discounting.

[^5]:    ${ }^{6}$ However, see Moav and Neeman (2012) for theoretical work showing that situations may arise where poor people are particularly positional, supported with some empirical evidence, which may contribute to a so-called poverty trap.

[^6]:    ${ }^{7}$ Alternatively, the formula can be derived over a discrete period of time if one is willing to assume constant relative risk aversion preferences, as well as a constant growth rate and pure rate of time preference. Here we do not want to limit ourselves to particular functional forms.
    ${ }^{8}$ For previous studies that have analyzed modifications of the Ramsey discounting rule, see, e.g., Hoel and Sterner (2007), Howarth (2009) and Gollier (2010).

[^7]:    ${ }^{9}$ Of course, in the special case where $\rho_{t}^{p}$ and $\rho_{t}^{s}$ are both constant in the time interval from 0 to $t$, then $\rho^{p}(t)=\rho_{t}^{p}$ and $\rho^{p}(t)=\rho_{t}^{s}$.

[^8]:    ${ }^{10}$ Note that reference consumption risk aversion $\left(v_{22 t}<0\right)$ is neither necessary nor sufficient for the more frequently discussed concept of comparison concavity ( $u_{2 t}<0$ ). See Clark and Oswald (1998) for some relationships between comparison-concavity and keeping-up-with-the-Joneses behavior.
    ${ }^{11}$ Yet, the measure of relative risk aversion will of course depend on the preferences, which here depend also on relative consumption.
    ${ }^{12}$ It is easy to show that the same rule applies for discounting over a discrete time period (in continuous time), provided that $\psi_{t}$ and $g_{t}$ are constant in the interval considered. Recall also that since $z_{t}=c_{t}$ it follows that $\dot{z}_{t}=\dot{c}_{t}$.

[^9]:    ${ }^{13}$ Alternatively, one could have derived this expression by letting $t$ go to zero in equation (7) and applying l'Hôpital's rule on the second term.

[^10]:    ${ }^{14}$ Note that if the conditions in Corollary 1 are fulfilled, then it also follows that $v_{12 t}>0$, i.e., the keeping-up-with-the-Joneses property is fulfilled.

[^11]:    ${ }^{15}$ It may seem that Stern selected these values partly to compensate for a number of simplifications and omissions. Stern (2006) mentioned combinations of ethical, distributional issues, and deep uncertainty, as well as the effects of different growth rates in different sectors. Relative consumption issues were not mentioned, however.
    ${ }^{16}$ In personal communication, Weitzman recently told us he would personally favor a near-zero rate of pure time preference for climate change, and a coefficient of relative risk aversion of 2.5 or 3 rather than 2. Yet, he still agrees that the 2-2-2 rule, which was originally introduced more as a thought experiment, is still relevant since it is widely used by others in the climate change literature.
    ${ }^{17}$ It is easy to show that the individual is reference consumption risk averse if and only if $\omega<\sigma a /(1+a)$.

[^12]:    ${ }^{18}$ For example, Friend and Blume (1975) concluded that $\sigma$ generally exceeds unity and is probably greater than 2. Blundell et al. (1994) and Attanasio and Browning (1995) found, in most of their estimates, an order of magnitude of 1 or slightly above, whereas Vissing-Jørgensen (2002) found that $\sigma$ differs between stockholders (approximately 2.5-3) and bond holders (approximately 1-1.2). Halek and Eisenbauer (2001) estimated values for a large sample of individuals and found a very skewed distribution with a mean over 3 , but a median of 0.9.

