Natural Resource use Conflict:
Gold Mining in Tropical Rainforest in Ghana

By

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Abstract

Gold is frequently mined in rainforests that can provide either gold or forest benefits, but not both. This conflict in resource use occurs in Ghana, a developing country in the tropics where the capital needed for mining is obtained from foreign direct investment (FDI). We use a dynamic model to show that an ad valorem severance tax on gross revenue can be used to internalize environmental opportunity costs. The optimal tax must equal the ratio of marginal benefits from forest use to marginal benefits from gold extraction. Over time, this tax must change at a rate equal to the difference between the discount rate and the rate of change in the price of gold. Empirical results suggest that the 3 percent tax rate currently used in Ghana is too low to fully represent the external cost of extraction (i.e., lost forest benefits).

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Summary

The location of gold deposits within valuable natural environments imposes a dilemma that requires an exchange of future benefits from the environments for current benefits from extracted gold. A profit tax – one based on net revenues from extraction – will not usually change the optimal rate of extraction. However, an ad valorem severance tax – one based on gross revenues from extraction – will usually change this rate (e.g., Dasgupta and Heal, 1979, Hanley, Shogren and White 1997). Because severance taxes are widely used in practice, it is fortunate that this distortionary effect can be harnessed to internalize the opportunity costs of environments that are lost or damaged during the gold extraction process. This paper presents the details of an efficient severance tax, and illustrates such a tax using data for gold mining in Ghana’s rainforests.

Our approach must differ in two important ways from classic extraction problems examined by Hotelling (1931) and many subsequent authors. First, gold deposits in Ghana are found in tropical forests that can provide in situ benefits to rural populations if the gold beneath them is not extracted. Second, the capital needed for gold extraction is derived from foreign direct investment (FDI). The former difference will require forest benefits to be considered, while the latter will require that profits from gold extraction be no less than zero.

By extending the literature on sharecropping, we formulate and derive results from a dynamic optimization program for the mining firm (the tenant) and the resource manager of the country (the landlord). The mining firm maximizes a discounted stream of profits from extracting gold. Revenue per unit extracted is equal to the gold
price minus the severance tax, subject to the rate of the gold stock depletion. The resource manager, on the other hand, maximizes the discounted sum of tax revenues and benefits from the forest stock, subject to depletion in the gold and forest stocks, and a profit constraint that requires mining in each period to at least break even.

We find that a severance tax can be used to lead mining firms to choose gold extraction that also is optimal for the manager’s extraction problem, if the tax is set equal to the ratio of marginal forest benefits to marginal benefit from gold extraction. The optimal tax must change at a rate equal to the difference between the discount rate and rate of change in the price of gold. The optimal tax is positively related to the discount rate and negatively related to the price of gold. Empirical simulations suggest that the current 3 percent tax rate is too low to fully represent the external cost of extraction (i.e., lost forest benefits). We conclude that ignoring environmental opportunity costs of extraction when selecting the tax rate may lead to irreversible loss of forest ecosystems. Because similar conflicts are common in other tropical countries, the results from this Ghanaian analysis may cautiously be extended to other natural resources in developing countries.
1. Introduction

Gold, diamonds, and other precious minerals are extracted from rainforests found in numerous developing countries. Resource use conflicts are common, but models of these conflicts are uncommon. Among the exceptions are Ehui et al. (1989, 1990) and Swallow (1994), who study interactions between non-renewable and renewable resource uses. Swallow (1994) examines the relationship between wetland development (i.e. non-renewable resource extraction) and preservation of the wetland for shrimp production (i.e. renewable resource). Ehui et al. (1990) present a theoretical model to determine socially optimum size of tropical forest reserve, when land may be either cleared for agriculture or preserved as forest. The forest in this study is treated as a non-renewable resource, and extraction of it makes land available for agriculture (Hanley et al., 1997).

It has been known for decades that a severance tax decreases per unit revenue, and consequently increases cut-off grade of minerals or decreases optimal extraction of minerals (e.g., Hotelling 1931). The tax has the same effect as an increase in average cost of extraction (Dasgupta and Heal, 1979). It is not surprising that such ad valorem severance taxes are usually opposed by mining firms. Most mining firms in developing but resource-rich countries assert that these taxes increase extraction costs such that a significant portion of the nations’ mineral endowment will never be mined (e.g., Chamber of Mines of South Africa 2002/2003).

To the best of our knowledge, no theoretical model exists on the tradeoff between gold deposits (i.e. non-renewable resource) and rainforests within which the deposits are found (i.e. non-renewable resource) in a country that has foreign capital in mining.
As a share contract, the mining firm provides the inputs required for the mining activities and gives a fixed fraction of the total revenue to the gold-rich country. Following Ehui et al. (1990), we employ dynamic optimization techniques to model the tradeoff between gold deposits and rainforests, with Ghana as a case study. By requiring the firm’s profits each period to be nonnegative, we show that Ghana’s present severance tax may lead to efficient extraction if it is dynamic and includes forest benefits lost due to extraction. The growth rate of the tax is the difference between the rate of interest and rate of change in the price of gold.

The next section gives a brief description of gold extraction in Ghana before and after the national mineral policy, and describes several of the known benefits obtained from the rainforest if gold is not extracted. This is followed by an economic model of extraction in section 3. Section 4 presents an optimal severance tax, and Section 5 describes changes in the optimal tax using comparative statics. Section 6 describes the application of the model with empirical information from Ghana, and Section 7 concludes.

2. Gold Mining and Deforestation in Ghana

Gold mining has been an important source of foreign exchange in Ghana since her independence in 1957. In a bid to provide employment, control the rate of extraction, and generate foreign exchange, the state controlled the mining industry from 1957 to 1986, by owning majority shares of over 55% in the major mining companies (Tsikata, 1997). Inadequate macroeconomic policies – such as an overvalued exchange rate – diminished the funds available to maintain and rehabilitate the mining industry (Aryeetey et al., 2000). The mining industry faced under-capitalization and
low efficiency due to poor management and weak mining skills. Gold extraction was very low, decreasing from 915,317 ounces in 1960 to the lowest level of 287,124 ounces in 1986 as per Figure 1.

![Output in Ounce vs Years](chart.png)

**Figure 1.** Trends in Gold Production in Ghana before the Mineral Sector Reform (1958-1986) and after the Mineral Sector Reform in 1986 (1987-2002).

Beginning in 1986, as part of the Economic Recovery Program sponsored by the International Monetary Fund, there was a shift from state ownership to liberalization, deregulation, and privatization of the mining sector. Mining aspects of this Program were intended to help improve efficiency and raise much needed foreign exchange. A specific requirement of the National Mineral Policy of 1986 was to relax several mining policies. With the revision of the policies, government revenue from the extracted gold was restricted to 3-12% royalty tax, and corporate tax of 35%. In addition, the mining industry was not subject to environmental regulations until 1994, when the Environmental Protection Agency (EPA) Act was passed by Parliament (EPA Act, 1994 (Act 490)). The EPA Act required Environmental Impact Assessments and Environmental Management Plans to be prepared by all new and
existing mining firms (Akabzaa and Darimani, 2001). In practice, lack of resources has limited the enforcement of these provisions.

This drive dramatically increased foreign direct investment (FDI) from $12.8 million in 1986 to $83 million in 1998 (Addy, 1998). Gold production eventually overtook the 1960 peak levels, and reached a record high of 2,481,635 ounces in 1998. By 1994, gold exports generated the highest export earnings (about 45% of total earnings), surpassing cocoa, which had been the leading commodity for export earnings (Akabzaa and Darimani, 2001). Figure 1 shows the general increasing trend of production.

However, this increased production had negative consequences on the environment. The surface mining technologies used to extract rainforest gold led to annual deforestation rates of roughly 2 million acres. By 2001 over 60% of the rainforest in Wassa West District (a typical gold mining district) was lost to gold mining activities (Tockman, 2001). It is estimated that only 12% of the country’s rainforest remains, with surface gold mining the main cause of deforestation (Ismi, 2003).

Ghana’s extremely heterogeneous tropical rainforest provides a wide range of benefits. For example, it is estimated that more than 75% of the protein in West Africa comes from bush meat (Asibey, 1974; Benhin and Barbier, 2004). The bush meat trade supports about 300,000 people in rural areas, out of which 270,000 are self-employed hunters. Annual harvest is estimated at 385,000 tons, worth over $350 million. Of the annual harvest, 225,000 tons, worth $205 million, are locally consumed (Fobi, 2003). In addition, 70% of Ghanaians depend only on traditional
medicine for health care. Traditional medicines are derived from roughly 2000 plants (Zhang, 2001) which are also exported to Europe for the production of medicine (Benhin and Barbier, 2004). Furthermore, many forest products are used as raw materials in household and local production of baskets, furniture, roofing materials, musical instruments, jewelry, hunting tools, traditional drums, and other items. Major rivers such as the Birim, Pra, Ankobra, Bonsa Offin, Densu, and Tano, which provide drinking water to many towns and cities, are fed by rivers and streams that run through all the forest reserves (Anane, 2003).

Regarding biodiversity, the Ghanaian forest is home to several rare species of fauna and flora, the populations of which are declining due to rapid destruction of forest habitats. Some of the rare animal species include giant forest hogs, primates, bongo, small antelopes, small bats and rodents, and birds. In addition, forest elephants disperse seeds of important timber species and create tracks for white-breasted guinea fowls. The International Union for Conservation of Nature and Natural Resources (IUCN) database has noted ten timber species in Ghana to be of conservation concern (Benhin and Barbier, 2004). Unfortunately, these benefits are completely overlooked when concessions are granted to mining companies.

3. The Model

In many non-renewable resource-rich countries, a fraction of the value of the extracted resource is taxed by the state to compensate for the opportunity cost of the extracted resource. In Ghana, all minerals are owned by the state, and the tax for gold extraction is between 3% and 12% of the gross value. The minimum tax of three percent is most commonly charged (Akabzaa and Darimani, 2001). This tax approach
is often preferred, because it guarantees a share of extraction revenues (e.g., Ranck, 1985, Hanley et al., 1997). Because this approach is similar to that of a landlord and tenant farmer, we extend a sharecropping contract model (Cheung 1968), with the mining firm as tenant, and the resource-rich country as the landlord. The basic model must be made dynamic, and extended to include forest opportunity costs, since the mining companies do open-pit or surface mining in the rainforest (Akabzaa and Darimani, 2001).

Several features adapt the model to the Ghanaian empirical context. First, because a small part of the world’s gold is produced in Ghana, we treat the domestic mining market as perfectly competitive. Second, because surface mining involves some of the lowest costs, virtually all firms use this extraction strategy. To reflect this trend, we treat all firms as identical. Third, by the end of 1999, the inflow of FDI to Ghana’s mining exceeded $3 billion (Akabzaa and Darimani, 2001), roughly 147% of that year’s GDP. Consequently, we assume that capital used in mining is from FDI. To streamline the model, the mining firm and the resource manager use the same rate of time preference; mining is done in forest reserves where logging is not permitted; and gold is uniformly distributed beneath the forest cover.

3.1 The Resource Country or Social Planer’s Problem

The surface mining method used by the gold mining firms in Ghana removes the rainforest where the deposits are found, leaving open pits and valleys (Akabzaa and Darimani, 2001). After mining, the land is typically no longer usable for agriculture. As noted earlier, the nation’s rainforest provides infinite stream of direct non-timber forest benefits such as provision of wild fruits, tubers, and cereals for human
consumption; serving as breading ground for mammals that are hunted for animal protein; supporting rivers and streams that provide drinking water, among others. When gold is extracted by a mining firm, the total surplus that accrues to the country consists of total tax revenue (i.e. \( \theta py \)) plus non-timber benefits from the remaining total forest stock (i.e. \( a(f) \)), where \( \theta \) is a tax rate, \( p \) is exogenous world price of gold, \( y \) is quantity of gold extracted by a mining firm within a particular year, \( f \) is the remaining forest stock/cover in the area allocated to the miner and \( a(f) \) is the general functional form of non-timber forest benefits to the society from this forest stock. The country’s social planner therefore chooses a time path that maximizes the stream of surpluses given by equation (1), subject to equations of motion of gold stock depletion (\( \dot{x} \)), forest stock depletion (\( \dot{f} \)), and a non-negative discounted stream of profit constraint of the firm. The gold and forest stock depletion equations are first order differential equations. The linear relationship between the rate of deforestation and the quantity of gold extracted is assumed for simplicity.

\[
\begin{align*}
\text{Max} & \quad \int_0^T \left[ \theta py + a(f) \right] e^{-\alpha t} dt , \\
\text{Subject to:} & \\
\text{a)} & \quad \dot{x} = -y \\
\text{b)} & \quad \dot{f} = -\frac{1}{\alpha} y \\
\text{c)} & \quad \int_0^T \left[ (1-\theta)py - c(y) \right] e^{-\alpha t} dt \geq 0
\end{align*}
\]

\( x \geq 0, y \geq 0, f \geq 0, \ f(0) = f_0 \) and \( x(0) = x_0 \).
Where \( c(y) \) is the cost function of a firm and \( t \) is time, e.g. in years. The cost depends on only the harvest (See Conrad, 1999 for an example). The following partial derivatives: \( c_y > 0 \), \( c_{yy} > 0 \) hold; \( r \) is a positive net benefit discount rate, which we assume to be equal to the social rate of time preference. It is positive because the firm will prefer a given amount of benefit today to the future. \( T \) is the end of the extraction period. We assume that non-timber forest benefits increase in the size of forest stock at a constant rate, hence \( a_f > 0 \) and \( a_{ff} = 0 \). Furthermore \( \alpha \) is the coefficient of gold yield per acre of the deforested land.

Because we assume there is no exploration for gold, the equation of motion defines the rate of depletion, which is the flow without backstop. Also, since tropical rainforest loss is irreversible, we model the forest stock depletion as a non-renewable resource as per the equation of motion (See Ehui et al., 1989; 1990 for a similar presentation). Since the capital comes as FDI, the direct cost of mining has no opportunity cost to the country and is not included in the objective function. Thus, the constraints to equation (1) are the stock depletion equations given by (2a) and (2b), and the additional constraint, which guarantees that the discounted net revenue from mining over the entire period is non-negative (equation (2c)).

The current value Hamiltonian associated with equations (1) and (2a, and b) is

\[
H^C (y, f, \lambda, \mu, \theta, t) = \theta py + a(f) - \frac{H}{\alpha} y - \lambda y
\]  

(3)
Where $\mu$ and $\lambda$ are the user cost associated with total forest and gold stocks, respectively. Since equation (2c) is an additional constraint in isoperimetric form (See Doherty and Posey, 1997; Caputo, 1998, 1999 for some examples of Isoperimetric constraints), we extend the current value Hamiltonian to

$$H^C(y, f, x, \mu, \psi, \theta, t) = \theta py + a(f) - \mu \frac{1}{\alpha} y - \psi [(1 - \theta) py - c(y)]$$

Assuming some quantity of gold is extracted at every point in time (i.e. existence of interior solution), the static efficiency conditions, which are the first order derivatives of the Hamiltonian function with respect to the flow variable $y$ and the choice variable $\theta$ are equations (5) and (6), respectively:

$$p - c_y - \frac{1}{\alpha} \mu - \lambda = 0$$

$$\psi = 1$$

Note that $\psi$ is not a shadow price but a multiplier associated with a constraint that is measured in the unit of price. Further, it takes the value 1 on the optimal path indicating that the additional constraint will hold for the representative firm within the entire mining period. In other words if the firm does not break-even it will relocate or fold up. In Ghana, there is evidence of threat by gold mining firms to relocate to countries with friendlier policies (Ismi, 2003). We derive some important results from the preceding equations.
Since $\lambda$ and $\mu$ are user costs faced by the mining firm, we have a modified non-distortionary static efficiency condition. The rule postulates that under perfect competition the marginal profit from the extracted gold will equate the user cost of the resource. In this particular case the rule is modified because the user cost include both the user costs of the resource on a bare ground ($\lambda$) and that of the gold yield of the forest stock ($\frac{1}{\alpha} \mu$). The equation defines the desired inter-temporal extraction condition of the social planer. Any deviation of the firm from this optimal path condition is undesirable to the planer. Equation (5) can be rewritten as

$$p - c_y = \lambda + \frac{\mu}{\alpha}$$  \hspace{1cm} (7)

From microeconomic theory, if marginal damages are considered, the marginal social cost becomes higher than the private cost leading to an efficient level of output which is lower than otherwise. Consequently, if forest stock effect is neglected, the marginal profit will equate only the user cost of the gold stock and result in over extraction.

The portfolio balance or costate equations are:

$$\dot{\lambda} - r\lambda = 0$$  \hspace{1cm} (8a)

$$\dot{\mu} - r\mu = -a_f$$  \hspace{1cm} (9)

Equation (8a) is the costate equation of the stock of gold associated with the social planer’s problem, which involves only the equation of motion of the stock of gold. Thus, the decision to mine the resource depends on marginal benefit from harvesting
the resources and depositing the revenue at the net benefit discount rate on one hand (i.e. $r\lambda$), and the marginal opportunity cost, which is the marginal benefit from the growth in the rental rate (i.e. $\dot{\lambda}$), on the other hand. Conversely, the return on all other assets in the resource-rich country (i.e. $r$) equals the growth in the shadow price per ounce of gold (i.e. $\frac{\dot{\lambda}}{\lambda}$). Equation (9) stipulates that on the optimal path, the return on all other assets in the economy ($r$) equals the growth in the shadow price per hectare of the forest stock ($\frac{\dot{H}}{\mu}$) plus the value of the loss in marginal benefits of the forest stock adjusted by the shadow price of the forest stock ($\frac{a_f}{\mu}$) (Krautkraemer, 1988).

Since we have stock effect in the objective function, the optimal path condition given by equation (7) could be used to determine the appropriate tax to be levied on the firm.

### 3.2 The Miner’s Problem

The representative miner chooses an extraction path that maximises the net present value of profits (i.e. equation 10) with revenue constituting a fraction of the total proceeds from the sale of gold ($(1-\theta)py$), and cost of production as a function of the harvest of gold (i.e. $c(y)$), subject to the equation of motion of the stock of gold. The discounted stream of net revenue or profit function is:

$$Max_{\{y\}} \int_{0}^{T} ((1-\theta)py - c(y))e^{-rt} dt, \quad (10)$$

---

1 The profit function is concave. From equation (12), $(1-\theta)p - c_y = \dot{\lambda} > 0$ and $-c_{yy} < 0$. Where $c_y$ is partial derivative of the cost function with respect to $y$. 
Subject to equation (2a), $x \geq 0$ and $x(0) = x_0$.

The current value Hamiltonian is:

$$H^c(y, \lambda, t) = (1 - \theta)py - c(y) - \lambda y$$

(11)

The associated Pontryagin maximum principle and the costate equation, which define the static and dynamic efficiency conditions, are equations (12) and (8b), respectively. If some quantity of gold is extracted every year, then:

$$(1 - \theta)p - c_y = \lambda$$

(12)

$$\dot{\lambda} - r\lambda = 0$$

(8b)

From the static efficiency condition, at each point in time the marginal profit from harvesting the gold (i.e. $(1 - \theta)p - c_y$) is equal to the firm’s user cost of the remaining gold stock (i.e. $\lambda$). Equation (8b), just as equation (8a), establishes production decision based on optimal path relationship between the marginal benefits from harvesting the gold today and in the future.

Since the terminal time of the firm’s optimization program is free, equations (4) and (11) must equal to zero at $t = T$ (i.e. $H^c(T) = 0$). Thus, at the end of the planning horizon, the mine shuts down and extraction ceases (Conrad and Clark, 1995). From equation (12), the optimal inter-temporal extraction policy is $(1 - \theta)p - c_y = \lambda(T)e^{r(t-T)}$ for all $t \leq T$. On the other hand, in the absence of the tax, the corresponding inter-temporal extraction policy is $p - c_y = \lambda(T)e^{r(t-T)}$ for all
This implies that for all \( t \leq T \), lower quantity will be extracted if the tax is imposed compared to what will prevail in the absence of the tax, a clear indication of distortionary effect of the tax.

If we compare the static efficiency conditions for the mining firm and the resource-rich country (i.e. equations (5) and (12)), it follows immediately that the firm will not follow the optimum path desired by the gold-rich country if the forest stock depletion is not internalised. The divergence comes from the difference between the tax received \( (\theta p) \) and marginal damage to the rainforest \( (\frac{H}{\alpha}) \).

4. Economic Policy Instrument

If the mining is done on a bare ground, any positive value of \( \theta \) will be distortionary simply because the user cost of gold from the inter-temporal efficiency condition of the social planner cannot equate that of the firm (i.e.\(1 > (1 - \theta)\), since \( p - c_y > (1 - \theta)p - c_y \)). Consequently, the tax is not a desirable economic policy instrument for raising revenue without decreasing the optimal path levels of extraction for all \( t \leq T \): a condition that is well established in the literature (e.g. See Dasgupta and Heal, 1979). Nevertheless, since mining destroys rainforest, the distortionary effect disappears with optimal value of the tax rate.

**Proposition 1:**

The optimal tax equals the ratio of marginal forest benefit to marginal gold benefit. And the current value of the user cost of the forest equals its initial value plus some adjustment for changes in the marginal non-timber forest benefit.
The proof for the above proposition is as follows. If we compare the optimal path of the social planner (i.e. \( \frac{p - c_y - \mu}{\alpha} = \lambda \)) and the firm (i.e. \((1 - \theta) p - c_y = \lambda \)), an expression for a corrective tax can be derived. Following Parks and Bonifaz (1994), the tax expression is the difference between the two equations as

\[
(p - c_y - \frac{\mu}{\alpha}) - ((1 - \theta) p - c_y) = 0 \Rightarrow \frac{\mu}{\alpha} = \theta p
\]  

(13)

Clearly the difference between the two equations is \( \frac{\mu}{\alpha} - \theta p \). Equation (13) simply equates the average tax revenue (\( \theta p \)) to the user cost of the gold yield of the forest stock (\( \frac{\mu}{\alpha} \)) on the optimal path\(^2\). If \( \frac{\mu}{\alpha} - \theta p > 0 \) then the tax rate is too low and as a result, the optimum path of the firm will be higher than what is socially desired. On the other hand if \( \frac{\mu}{\alpha} - \theta p < 0 \), which is the case if the social planner charges the tax for losing the gold and the forest, then the firm’s path will be too low. The optimal tax should therefore equate the marginal damage to the forest. Thus the tax could be used to correct the extraction externality. The appearance of the user cost of the forest stock in the tax equation is consistent with Pigou (1946) and Hanley et al. (1997), among others. Furthermore, the royalty tax is a function of time (See Löfgren, 2003 for an example).

\(^2\) Moreover, the royalty tax is open but bounded between zero and one. From equations (5) and (13):

\[
0 < \theta = \frac{\mu}{\alpha p} = \frac{\mu}{\mu + \alpha \lambda + \alpha c_y} < 1 \text{ for all non-negative values of } \lambda \text{ and } c_y.
\]
From equation (9) since \( a(f) \) is a linear function, the time path of \( \mu(t) \) yields equation (14),

\[
\mu(t) = \mu_0 e^{\mu t} + \left(1 - e^{\mu t}\right) \frac{a_f}{r}
\]

Where \( \mu_0 e^{\mu t} \) is the initial marginal value of the forest stock valued at current price, the last two terms (i.e. \( \left(1 - e^{\mu t}\right) \frac{a_f}{r} \)) is some adjustment for the change in the marginal non-timber forest benefits valued at current price, \( a_f \) and \( \mu_0 \) are positive constants. The assumption of \( \mu_0, a_f > 0 \) is based on the fact that the forest cover in resource-rich countries are highly depleted. Moreover the scarcity value of the forest stock will be increasing over time if its initial value exceeds the infinite stream of marginal non-timber benefits (i.e. \( \mu_0 - \frac{a_f}{r} > 0 \))\(^3\).

In many poor countries where gold is mined, the royalty tax that is presently charged could be designed to take care of the damage. Since this tax is positively related to marginal damages, it will create the incentive for damage reduction. So far many poor but gold-rich countries that have FDI in gold mining have kept the severance tax

\[\frac{\partial \mu(t)}{\partial t} = r \left( \mu_0 - \frac{a_f}{r} \right) e^{\mu t} > 0 \text{ if } \mu_0 - \frac{a_f}{r} > 0 \text{ or } \mu_0 > \frac{a_f}{r} .\]
very low and constant, and basically for the wrong objective of getting some revenue for losing the extracted gold.

**Proposition 2:**

The optimal tax should increase (decrease) when adjusted net return on all other assets in the economy is higher (lower) than the growth in the price of gold.

The preceding proposition addresses the behavior of the tax rate over time. Taking the logarithm of the tax equation (i.e. $\theta = \frac{\mu}{\alpha p}$), we have

$$\log(\theta(t)) = \log(\mu(t)) - \log(p(t)) - \log(\alpha)$$

(15)

The time derivative of equation (15) gives the growth equation of the tax rate as

$$\frac{\dot{\theta}}{\theta} = \frac{\dot{\mu}}{\mu} - \frac{\dot{p}}{p} = r - \frac{a_f}{\mu} - \frac{\dot{p}}{p}$$

(16)

The term $\frac{\dot{\mu}}{\mu}$ of equation (16) denotes adjusted net return on all the other assets in the economy (i.e. $r - \frac{a_f}{\mu}$) from equation (9). Thus, the tax rate will increase if the ratio of the marginal non-timber forest benefits from the remaining forest stock to the scarcity value of the remaining forest stock decreases, given the return on all other assets in the economy and the growth in the exogenous price of gold. As the rate of deforestation increases, the ratio decreases, and given $r$ and $\frac{\dot{p}}{p}$, the tax rate will
increase. Moreover, with the growing commercialization of the enormous non-marketed ecological services that tropical forests provide, such as insurance and information value of biodiversity, amenity values, watershed protection, carbon storage and sequestration and option values, the scarcity value of tropical forest is increasing (Pearce, 2001).

5. Comparative Static Analyses of the Severance Tax

In this section, we investigate the comparative static analyses of the tax rate with respect to the price of gold and the discount rate. Within the 15-year period of 1987-2001, the highest cumulative average price of gold declined from US$446 in 1987 to the lowest of US$271 in 2001 with overall average of US$354.5 and a high standard deviation of 54.9. It will therefore interest the social planner to determine how the optimal tax rate should respond to price volatility.

Furthermore, discount rates in most poor countries are generally low and also volatile. In Ghana, nominal discount rates had been low and unstable even after the IMF sponsored economic recovery program. Within the period between 1987 and 2001, the lowest discount rate of 20% was recorded for 1991 and the highest of 45% was recorded for 1995-1997, with a mean and standard deviation of 32% and 8.1 respectively. Due to the high rate of inflation within this period, real interest rates were generally very low and more volatile.

Proposition 3

The tax is negatively related to the price of gold \((p)\) and positively related to benefit discount rate \((r)\) if \(\mu_0 - \frac{a_r}{r} > 0\).
To determine the comparative static analysis of $\theta$ with respect to $p$, equation (13) is used. The following equation is obtained,

$$\frac{\partial \theta}{\partial p} = -\frac{\mu}{\alpha p^2} < 0$$

(17)

The result from the analysis indicates that the firm should have increased share in per unit price of the resource if the price of the resource increases. The intuition behind the former is that price increment does not stem from increased damage to the rainforest and must therefore benefit the firm. The social planner should therefore charge lower royalty tax rate if the exogenous price of gold increases. Thus, the firm should receive increased after tax per unit price of the resource if the price of the resource increases, given that the increment does not increase the optimal extraction path of the resource.

The relationship between the share and the rate of time preference is not obvious. There are two effects of the increased social rate of time preference: it reduces the firm’s share due to the faster growth of the initial user cost of the forest stock, but increases due to a reduction in the infinite discounted value of the marginal damage to the forest. The comparative static analysis of $\theta$ with respect to $r$ is$^4$

$$\frac{\partial \theta}{\partial r} = \frac{1}{\alpha p} \frac{\partial \mu(t)}{\partial r} = \frac{1}{\alpha p}\left(\mu_0 - \frac{a_f}{r}\right)te^{\alpha r} + \left(e^{\alpha r} - 1\right)\frac{a_f}{r} > 0$$

(18)

$^4$ Equation (18) is positive because the optimal path of the shadow price of the forest is assumed to be non-decreasing (i.e. $\mu_0 > \frac{a_f}{r}$).
Higher discount rates generally indicate scarcity of the resources, hence the optimal path of the shadow price of the resource increases and consequently the path of the tax also increases.

6. Numerical Simulation

In this section, we present numerical illustration of some key results of our model. Due to lack of adequate data on mining activities in Ghana, we calibrated data for \( y(t) \) and also used some specific functional forms of \( c(y) \) and \( a(f) \). It is important that the results from the simulations are interpreted with extreme caution because of the nature of the data used. Emphasis should be on the direction and the relative rather than the absolute values of the estimates. Since the size of the mining industry was stable before the mineral sector liberalization policy in 1986 (Akabzaa and Darimani, 2001), we hypothesise that the data on gold production between 1960 and 1985 describes the slope of the extraction path for 30 years beginning 2002 since there has been very low increments in investment since 1998 (Ismi, 2003). Moreover, the 30 years corresponds to the maximum number of years that concessions are usually exhausted in Ghana (Hilson, 2004). To obtain the slopes, the following OLS regression estimates were obtained from the data:

\[
y(t) = y_0 - 11.17855t - 0.5809315t^2
\]

\( (5.46664) \quad (0.196520) \)

\[ R^2 = 0.9443; \quad F(2, 23) = 212.92; \quad T=26 \]
Where the standard errors are in parentheses, \( t \) is the time trend for the period of 1960 to 1985, and the coefficients of \( t \) and \( t^2 \) are significant at 5% and 1% respectively. Using the last available data on gold production (i.e. 2,023,000 ounces in 2002) as the baseline for \( y_0 \) and the estimated coefficients of \( t \) and \( t^2 \), we generated data for \( y(t) \) shown in Figure 2A.

\[
y(t) = y_0 + t + t^2
\]

**Figure 2A:** The time path of Gold extraction.

Secondly, a total of 37 million ounces of gold exists within a 50km radius (i.e. 7857.14 km\(^2\)) (Mines News Feature Story, 2005). From this, gold yield per acre of deforested land (i.e. \( \alpha \)) is 19.06 ounces, which is used for the simulation. Furthermore, statistics available indicates that Ghana’s remaining forest stock as at 2000 was 15,653,800 acres and annual deforestation is 65,000 acres (FAO, 2003). This puts the forest stock as at 2002 (i.e. \( f_0 \)) at about 15,523,800 acres. Using the discrete time representations of the forest stock dynamic equation (i.e. \( f_t = f_{t-1} - \frac{1}{\alpha} y_t \)), gold stock dynamic equation (i.e. \( x_t = x_{t-1} - y_t \)) and the data generated for \( y(t) \), we generated the time series data for the forest and gold stocks.
Figure 2B shows the time path of the remaining forest stock, if mining is the only activity that leads to deforestation.

\[ \theta = \frac{\mu_0 e^{rt} + \left(1 - e^{rt}\right) a_f / r}{\alpha p} \]. A rough estimate for \( a_f \) is from benefit transfer from earlier studies in some developing countries. The estimate of genetic resources plus forest product collection and environmental benefits from an acre of tropical rainforest per annum is about $170.15. This is made up of estimated potential annual genetic resource value of US$8.51 per acre in Western Ecuador (Simpson et al., 1996) plus annual sustainable non-timber forest product harvest value of US$162 per acre in Cambodia (Bann, 1997). We used the 15-year (1987-2001) average price of gold (i.e. $354.50) for \( p \). Furthermore, to select a value for \( \mu_0 \), we rely on the restrictions that the scarcity value of the forest should be increasing overtime (i.e. \( a_f / r < \mu_0 \)). Since \( a_f / r \approx 3403 \), values of \( \mu_0 = \{3405, 3905, 4405\} \) were chosen for the simulation. Finally, since information on cost of mining is difficult to obtain, we used the specific functional form of the cost function in Fraser.
and chose some values for the parameters in the function. The parameter values were carefully chosen so that the average costs, which is $258.00, for the 30-year simulation period is the same as the forecast for 2005 for a mining firm in Ghana (Russell and Associates, 2004). The cost function is

\[ c(y) = \kappa + \gamma y^2 \]  

(20)

Where \( \kappa, \gamma > 0 \); so that \( c_y > 0 \); and \( c_{yy} > 0 \). For the purpose of the simulation, the following parameter values were chosen: \( k = 200000 \) and \( \gamma = 0.01 \). Due to the high volatility of the domestic real interest rate we used the U.S. government 20-year treasury bills rate of 5% (i.e. \( r = 0.05 \)) such that \( e^{-rt} \approx \rho(t) = \left( \frac{1}{1 - r} \right)^t = 0.9523811 \).

The results obtained from the simulations for the dynamic tax rate, which should be interpreted within the context of the parameter values chosen are shown in Figures 3A through C. From the figures, higher initial values of the scarcity value of the forest (i.e. \( \mu_0 \)), induce higher optimal path of the tax, which may result in a decrease in the terminal period of the gold extraction. Moreover, for each of the three chosen values of \( \mu_0 \), the dynamic tax rate increases overtime with a minimum value of about 50\% for all \( \mu_0 > \frac{a_f}{r} = 3403 \). This implies that the current tax of 3\% that is charged is too low.
Figure 3A: The time path of the tax if $\mu_0 = 3405$. The corresponding $T=29$.

Figure 3B: The time path of the tax if $\mu_0 = 3905$. The corresponding $T=12$.

Figure 3C: The time path of the tax if $\mu_0 = 4405$. The corresponding $T=6$. 
Figure 4: Time path of the shadow price of the rainforest (i.e. $\mu(t)$) for $\mu_0 = 3405$.

The optimal path relationship between the sum of present value (PV) of social benefit or surplus and the initial value of the shadow price of the forest is shown in Figure 4. The social benefit includes the tax revenue from mining and the stream of non-timber benefits from the remaining forest stock. Clearly, higher optimal path of the tax will lead to higher forest conservation but this may not necessarily generate higher stream of social benefits. From Figure 5, the highest social benefit results from the path with the lowest gradient. However, if the rate of increase of the tax path is very low, say for $\mu_0 = 3404$, the stream of benefits to the resource-rich country may be low compared to what is associated with $\mu_0 = 3405$. 
7. Conclusion

The destruction of rainforests for the purpose of mining gold in Ghana is a common problem that many other tropical countries face. Any attempt at ignoring the environmental opportunity costs of extraction when selecting a tax rate may lead to irreversible loss of forest ecosystems.

By examining gold extraction by foreign companies in rainforest in Ghana, we have shown that the ad valorem severance tax on gross revenue from production, which is currently charged, can be used to internalize environmental opportunity cost if it equals the ratio of marginal damage of gold extraction to the marginal benefit from the sale of gold. The tax is dynamic because it is a function of the growing scarcity value of the remaining rainforest stock. Comparative static analyses of the tax with respect to the exogenous price of gold and discount rate show that the tax is positively related to benefit discount rate and negatively related to exogenous price of gold. Furthermore, the growth of the tax rate is equivalent to the net return on all other
assets in the economy and the growth rate of the price of gold. Moreover, empirical results indicate that the 3 percent tax that is currently charged is too low to fully represent the external cost of extraction (i.e. lost forest benefits). Lack of data to estimate the cost and marginal non-timber forest benefits, however, limits the reliance on the absolute values of the estimates from the simulations. Further research on estimating these functions will be useful.

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### Appendix A: Simulated Data.

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Note: \( \pi(t) \) is profit of the firm and \( R(t) \) is total Revenue at time \( t \).