The Secret Life of Fear: Interdependencies Among Implied Volatilities Represented by different Stock Volatility Indices Treated as Assets

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ABSTRACT

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Title
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Abstract
This study focuses on the systemic interdependencies of specified volatility indices, the underlying assets of which are major stock indices of developed financial markets. The volatility indices in question follow the standard VIX specification, and thus give forward-looking 30-day estimates of implied volatilities on each market respectively. Volatility is then considered as an asset. Engle’s Dynamic Conditional Correlation specification of the VAR-MVEGARCH methodology is used to study spillovers in volatilities between different markets, as well as dynamic conditional volatility and correlation structures. Additionally, asymmetric behavior of volatilities is taken into account. The time period from January 2000 to mid-June 2009 includes both times of normal market conditions and crises. The results prove unidirectional spillovers from the US VIX to other indices, and more locally from the VFTSE to the VSMI. The dynamic conditional volatilities include abrupt and large short-term peaks, while the dynamic conditional correlations (DCC) are high and stable. The deviations from DCC means revert back smoothly so that the spillovers between the indices take place over time, and can be interpreted as information transformation. The VDAX and the VFTSE of the main European markets are highly unified, having high correlations but no spillovers between them. All indices contain small but significant volatility asymmetries, and day effects.

Keywords
Dynamic Conditional Correlation, Implied Volatility, MGARCH, Risk Management, Time Series Analysis, Volatility Index, Volatility Spillover

JEL Classification
C32, C53, G12, G13, G14, G15

Additional Information
This study is the 2010 winner of the Richard C. Malmsten Memorial Foundation Award for Best Master's Thesis in Finance at the University of Gothenburg Graduate Business School. The WinRats code written for the modeling by the researcher is also available.
Wonders of life… Volatility indices are absolutely one of them, revealing new things from the very beginning to the very end, during the whole research process. And still, “I can’t conceive the nucleus of all, begins inside a tiny seed, and what we think as insignificant, provides the purest” fear we feel!

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Helsinki, May 12th, 2010
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1. INTRODUCTION

This study continues the research tradition of interrelations between different financial markets – geographically and/or asset wise – and the long and essential line of study of conditional volatilities of historical time series. Volatility indices (VI), the data of the study, are by definition based on implied (i.e. forward-looking) volatilities of derivatives written on underlying indices. Then, the study contains new and unique value by studying implied volatilities of markets through volatility indices. More specifically, the historical time series of this study are VIs for stock indices of the main developed markets in the Western world. Their systemic interdependencies are studied by using a multivariate GARCH (MGARCH) – methodology, which has evolved a great deal recently. A new feature is to treat VIs as assets, an approach not introduced earlier in this research setting as is.

Importance of the study. The study creates new bridges between theory and practice by studying properties and interdependences in this new financial asset group in different markets (Ethridge 2004, 40; Ryan et al. 2002, 50). These results can be utilized in many ways. A better understanding of VIs leads to insights into the use of indices as market timing tools, measures of uncertainty of markets, and as proxies for stock variance swap levels (Carr & Wu 2006). From an asset management perspective, the results will uncover how beneficial it is to diversify inside the volatility asset class, and reveals potential outcomes when a foreign VI is used to hedge investments in a traditional asset class, such as stocks (Grant et al. 2007; Black 2006). Essentially, does investing in volatilities in different markets bring any benefits, or is it enough to invest in volatility in only one developed market without any aspirations for diversification? The question is meaningful right now: developed stock markets have been shown to become increasingly correlated due to on-going international financial integration, but there are still few VIs available (with derivatives written on them) for investing. Thus, investors may not have an opportunity to invest in volatility in their own market, but a great deal of interest in investing in volatility in other developed markets. The time period studied is interesting, as it includes the recent subprime crisis, the effects of which have been unprecedented on a global scale. The study reveals how correlations change when extensive shocks take place in one of the markets.

By and large, the most important contributions serve volatility estimation (e.g. Satchell & Knight 2002), which is an essential part of asset pricing. The increased understanding of volatilities of VIs, and information about their relative relationships, is needed in pricing of derivatives written on these indices. This front is highly active with new VI based derivatives created daily. Moreover, there are on-going attempts to advance existing pricing models for this purpose (e.g. Sepp 2008; Soklakov 2008). Derivatives pricing treats VIs as assets, just as this study assumes.

Main findings. The study reveals that the behavior of VIs is highly correlated, and surprisingly stable, even during the crises. The volatility process is persistent, and changes in dynamic conditional volatilities are extensive and abrupt, as well as very short term. There exist unidirectional relationships between indices, the US VIX being the center and main source of the shocks. The more remote Swiss VSMI is affected locally by the UK’s VFTSE. Spillovers are interpreted as information transformation. The main European Union markets, the German VDAX and the UK’s VFTSE are highly correlated, but without spillovers. The same applies to the German VDAX and the Swiss VSMI. These findings show that proximity and closely related real economies seem to cause volatilities to evolve together, while financial relationships seem to cause spillovers. Volatilities of implied volatilities represented by the indices seem to all have a small but consistent asymmetric element. The distributions of the 1st difference returns are non-normally distributed. The time series also contain day effects.
Structure of the report. The study is reported by loosely following the standard Introduction – Methodology – Results – Discussion (IMRD) – format (Hirsjärvi, Remes & Sajavaara 1997, 232-250). The literature review is separated from the introduction, as well as the methodology from the data chapter. The validity and reliability of the method, and the robustness of the results, are studied separately.
2. LITERATURE REVIEW AND CONCEPTS

Volatility index (VI) related research. Much of the research on VIs has focused on explaining the use of the VIX index—and later derivatives written on it—in applied use, and is needed as such only as a basis for correct data treatment and soundness of the research framework. Probably the most important contribution in the field of VIs thus far is that of Carr & Wu (2006). They explain the mathematical and economic properties, motivate the redefinition of VIX index in 2003, and show how the index is related to the volatility and option theory, as well to variance swaps. Furthermore, they look at historical behavior of the index, and pay attention to the leverage effect, which has been given multiple explanations in related literature (Bekaert & Wu 2000; Black 1976; Campbell & Hentschel 1992; Campbell & Kyle 1993; Haugen et al. 1991). They also find “potential discontinuous index return volatility movements”, relate it to the findings of Eraker, Johannes & Polson (2003), and together with Wu (2005) notice that the variance rate contains a significant jump component, the arrival rate of which is not constant but proportional (also Becker, Clements & McClelland 2009). Carr & Wu (2006) show that the forward-looking VIX as an implicit volatility proxy is alone enough to estimate the volatility of the underlying index, when compared to a historic estimate of GARCH(1,1), which Ahoniemi (2008) finds suitable as a different historic estimation model. There are several sources (e.g. Banerjee, Doran & Peterson 2006; Cipollini & Manzini 2007), which claim that the implied volatility of the VIX can predict the future returns of its underlying index, but numerous sources disagree (Becker, Clements & White 2007; Hsu & Murray 2007). Majmudar & Banerjee (2004) notice that the EGARCH-model is best suited for VIX modeling, among the other GARCH specifications.

Black (2006) presents a positive skew and kurtosis of probability distributions, and pays attention to the mean-reversion property. Gonzalez-Perez & Guerrero (2009) state that the properties of the VIX index are exceptional including non-normality, heteroscedasticity, non-linearity, and most importantly, non-stationarity. They recognize that non-stationarity is not always assumed in research, and consequently, this opinion cannot yet to be considered as a final view of the research community. Moran & Dash (2007; also Siripoulos & Fassas 2008; Cipollini & Manzini 2007) indicate some properties for (spot) VIs, such as high volatility and negative, but asymmetric, correlation with the respective underlying market indices. They also indicate mean-reversion and state that VIs can be used as a) an indicator of market fear (also Whaley 2000, 2009), b) in hedging (through derivatives)\(^1\). Grant, Gregory & Lui (2007), as well as Szado (2009), introduce VIs as c) an asset class. Lately, Whaley (2009) has defined the historical development of the VIX, and described its position in the wider framework of market indicators. Jiang & Tian (2007) maintain that even the current definition of the VIX index might be flawed. This proposition disagrees with most earlier findings.

Volatility co-movements and spillovers. Aboura (2003) conducts the only implied volatility (Day & Lewis 1988, Canina & Figlewski 1993, Christensen & Prabhala 1998) transmission study to date (to the researcher’s knowledge) by using VIs (VX1, VDAX & VIX). He finds implicit processes especially using the mean-reverting jump model and GARCH-GJR—specifications and, advancing the findings of Erb et al. (1994) and Longin & Solnik (1995), maintains that correlations depend on implied volatility and business cycles, and change because of different skew and kurtosis in the true return time series. These differences function as a source for correlation asymmetry (Aboura 2003, 15-16). Earlier research has approached the topic mainly by modeling return and historical variance, most often by using different GARCH—applications (Andrews 1991, Satchell & Knight 2002). This line of research can be divided into sub-sections, studies which research co-movements of returns (e.g. Copeland & Copeland 1998; Andersen & Bollerslev 1997; Susmel & Engle 1994), and those more interested in volatility co-movements. The latter group included Ross (1989), who proved that co-movement studies of volatility are meaningful. Schwert, French &
Stambaugh (1987) noticed volatility persistency and introduced the volatility feedback effect. Theodossiou & Lee (1993) found weak volatility overflows, and that conditional volatility in many markets originated in the USA. They also found strong own-volatility spillovers over time. Koutris & Booth (1995), as well as Booth et al. (1997), studied asymmetric volatility spillovers by using a Multivariate EGARCH concept, and noticed that bad news seemed to have greater effects than positive. There are also a number of studies showing that trading has an essential connection to (especially intra-day) volatility. Some sources do not accept volatility spillovers at all, but consider them as a result of non-synchronous trading (Lin et al. 1994), while the majority (e.g. Stoll & Whaley 1990; Chan, Chan & Karolyi 1991) interprets them as information transmission.

Day of the week effects. There is lots of international evidence that equity, fixed income, derivatives (Gay & Kim 1987) and foreign exchange markets contain seasonal, monthly or weekly anomalies. Osborne (1962), Cross (1973) and French (1980) analyze the so-called Monday effect, which according to Lakonishok & Levi (1982), could be caused by the behavior of market transaction systems. Keim & Stambaugh (1984) speak about the weekend effect, caused by measurement errors in stock prices. Further studies focus on separating the effect of trading and non-trading periods (Rogalski 1984), focus closer on the properties of the effects (Chang et al. 1993), explain roles of informed and liquidity traders (Foster & Viswanathan 1990), as well as interday effects (Foster & Viswanathan 1993). There is some later evidence that the effect has disappeared from some markets. The studies are most often undertaken by using different sorts of OLS and GARCH–specifications (Engle 1982; Kiymaz & Berument 2003; Apolinario & Caro 2006).

2.1. Volatility and Implied Volatility

Volatility of assets is generally considered important in financial modeling, as it has a large effect on asset prices. Hull (2009, 285) defines volatility as a sign of the arrival of new information on the market, but reminds us that volatility is partially caused by trading itself. Volatility is often seen as an outcome of volatility drivers such as inflation, oil and other commodity prices, as summarized by Zimmermann et al. (2003, 121-126). According to them, a special class of volatility drivers is created by currency, interest and market, which function also as value drivers (Fama 1965; French 1980; French & Roll 1986). Hence, there are lots of studies focusing on volatility estimation, the most important results of which can be found as secondary sources (e.g. Satchell & Knight 2002). One common property of volatility is mean reversion (Hull 2009, 482-483), and as the standard GARCH–models follow a mean reverting stochastic process (Engle 1982), they are practical for volatility modeling in practice. Mean reversion can be masked by the fact that the volatility time series may contain structural breaks and jumps. These problems are sometimes taken into account by using jump-diffusion models (Merton 1976) or stochastic volatility models (Hull & White 1987; 1988; Gatheral 2006; Sheppard 2005).

Implied volatility refers to future looking volatility estimated from derivatives markets by using derivatives pricing models such as Black & Scholes (1973; Black 1989; Merton 1973) – most often based on risk-neutral valuation methods (Smith 1976). This is possible, as market prices (and all other variables) for derivatives are known (Hull 2009, 296). Both Carr & Wu (2006), Ahoniemi (2008) and Graham & Harvey (2009) have shown that there are clear relationships between the implied volatility of VIs, and the historical time series based volatility estimations. Still, Siriopoulos & Fassas (2008) claim that the VFTSE index contains information beyond historical volatility.
2.2. Volatility Indices (VI)

*History and definition.* VIs first saw the light as private tools for derivatives traders, the specifications of which used to differ. Soon, they spread among a large audience, and became a standard measure of risk for stock indices (Whaley 2009). The indices chosen for this study are all constructed using an improved VIX – specification, which has become the first globally accepted standard specification for VIs. The general formula for VIX, defined in the so-called White – paper (CBOE 2009; also Carr & Wu 2006, 14; Whaley 2009), is a “kernel-smoothed estimator” and can be presented as

\[
VIX_t = \sqrt{\frac{2}{T} \sum_i \frac{\Delta K_i}{K_i^2} e^{RT} O_i(K_i) - \frac{1}{T} \left[ \frac{F_t}{K_0} - 1 \right]^2} \times 100
\]  

(2.1)

at time \( t \), where \( T \) is time to expiry for options included specified on the minute level of preciseness, and \( F_t \) is a forward index level derived from index options. If the first strike below this forward index level is, \( K_0 \), the strike price for the out-of-the-money option, \( i \), is presented by \( K_i \), which is call, put or both call and put options by following

\[
K_i > K_0: \text{Call} \\
K_i < K_0: \text{Put} \\
K_i = K_0: \text{Both}
\]  

(2.2)

The interval between strike prices, \( \Delta K_i \), is calculated by

\[
\Delta K_i = \frac{K_{i+1} - K_{i-1}}{2},
\]  

(2.3)

the continuously compounded risk-free rate is \( R \), and the middle point in the bid-ask spread for each option with \( K_i \), is presented by \( O_i(K_i) \). Lately, new VIs have been introduced both in the new markets, in India, Hong Kong, and Japan, as well as for new underlying indices such as commodities.

*Properties of volatility index (VI).* A VI, defined by this specification, gauges a measure of the 30-day implied volatilities of the entire volatility smile [sic], is quoted in percentage points, and according to Szado (2009, 9-10) cannot be modeled by using cost of carry model. A common, and heuristically understandable, property of volatility is mean reversion, as the uncertainty measured by volatility cannot increase eternally. There is a general understanding that VIs are mean reverting and thus stationary (Black 2006; Moran & Dash 2007). In practice, VI series may contain jumps and structural breaks, which make confirming this property less clear-cut (Aboura 2003). The improved VIX – specification is considered sound by many sources (Carr & Wu 2006), which is confirmed by the fact that the squared VI value can be considered as the variance swap rate on the underlying index (Carr & Wu 2006, 19-20). Still, Jiang & Tian (2007) have suggested that the VIX – specification may underestimate by 198 or overestimate by 79 basis points the volatility of an underlying asset. These questions still remain open. In terms of other properties, Carr & Wu (2006, 18-19) have revealed that at least the VIX index is affected by the leverage effect, the reasons of which can be numerous (more closely Bekaert & Wu 2000; Black 1976; Campbell & Hentschel 1992; Campbell & Kyle 1993; Haugen et al. 1991; Hull 2009, 395) and contains *The Federal Open Market Committee Meeting Day effect*, which
increases the values of the index roughly 4 trading days before the meeting (ibid, 19-21). Other VI related day effects have not been reported earlier.

Applications. VIs have useful properties that can be utilized for a variety of purposes. They are used 1) to measure uncertainty, as they estimate implied volatilities for the future, and serves then as a practical measure for “fear” (Whaley 2000, Moran & Dash 2007) in the Finance industry. It is also 2) a natural tool for market timing, as it reveals volatility peaks time-wise, by being negatively correlated with stock markets (Black 2006, 12), or 3) to improve skew and kurtosis properties of other investment through derivatives (ibid., 12). In this sense, VIs create a new asset class, as they allow diversifying investments in the implied volatilities (Grant et al. 2007; Szado, 2009). Further, 4) VIs reflect the variance swap levels of their underlying index (Carr & Wu 2006, 19-20). Hence, increased knowledge of these indices, serves not only the highly important front of volatility estimation, but can be utilized in wide variety of purposes from asset allocation and market timing to variance swap pricing and interdependency of markets.

2.3. Volatility as an Asset

One fundamental assumption that has a large impact on the results and reliability of the study is to treat volatility as an asset (Grant et al. 2007). This is a natural direction of development in applied studies (e.g. Black 2006; Szado 2009), as the markets are further and further securitized, and the VIs are based on the prices of derivatives, which are assets themselves. However, considering risk as a separate asset class along with stocks, bonds and commodities is recent and useful in serving asset management and derivatives pricing (e.g. Sepp 2008; Soklakov 2008). It also helps to mitigate potential data-related problems.

There are numerous factors that might bring uncertainty to the data. The VIs might be calculated by using different specifications, or for different time lengths. The number of options and their available maturities, the implied volatilities of which are used to calculate the index, might differ. Additionally, the ways these options are used might differ from market to market, for example, if competing products are readily available in one market for investors to use. Further, the underlying stock indices are fundamentally different from each other. Stock indices consist of a varying number of companies, the types and relative weights of which may be chosen using different criteria – sometimes the index contains only the most actively traded stock series of a company. Efficiency in the markets may differ to a degree, as well as trading and transaction costs. The treatment of volatility as an asset is a clear-cut detour around these problems. When VIs are considered simply as market value of an asset for the risk of the underlying stock index, these market values can be measured as is without the need to evaluate the afore-mentioned problems in the data. Finally, treating volatility as an asset, affects the definition of implied volatility. The VI should not be seen as an unbiased measure of volatility anymore, but as an asset, the price of which implicitly refers to the implied volatility of underlying index and the related market.
3. METHODOLOGY

The rationale of the study is to advance volatility spillover research by observing the systemic behavior of implied volatilities as expressed by VIs. This paper analyzes the implied volatility spillovers between different markets (mean equation) and dynamic conditional volatilities and correlations (variance equation) between these volatility assets in the international context. In other words, the study continues traditional research of volatility and market integration by using a contemporary systemic point of view, and expands newer, more applied research approach, which treats volatility as an asset. This latter approach has become meaningful because of the on-going securitization process of financial markets. There is no generally accepted conceptual framework or methodology (Ethridge 2004, 148) for the study, as the research topic is relatively new.

3.1. Research questions, hypotheses and assumptions

The research questions. The objectives of the study are defined on two levels – as a general perspective, and by narrowing this perspective to specific problems. The research problems are then defined as a general question, which is studied through practical sub-questions (Ethridge 2004, 87). The general objective of the study is to examine:

QG: What kinds of relationships between stock volatility indices of different developed and discrete markets can be observed, when these indices are treated as assets?

This objective is answered through more applied sub-questions:

Q1: Are there implied volatility spillovers between the volatility indices in question?

Q2: How do volatilities and correlations change over time, and what kind of variance-covariance matrix they create?

Q3: Are there asymmetric volatility effects?

The most reasonable approach to answer the sub-questions is to use a Multivariate GARCH- methodology, which allows studying of spillovers between VIs (Q1), by including a mean equation in the Vector Autoregressive fashion, and which also generates a covariance (or correlation) matrix describing relationships between the implied volatilities represented by the indices (Q2). The Dynamic Conditional Correlations (DCC) specification (Engle 2002) takes into account a possibility that volatilities and correlations are changing dynamically over time (Q2), and is capable of describing these changes. The time window includes both times of ordinary market behavior and the latest subprime crisis. The distributions of VIs are often non-normal, and the volatilities of VIs, just like volatilities of different assets, are not always normal and symmetric (Carr & Wu 2006; 18-19) – a phenomenon not thoroughly understood yet. Potential asymmetry is taken into account by using the appropriate EGARCH -specification (Q3). Answering these questions gives new insights about the interdependencies of VIs. As the underlying indices are the main stock indices of discrete developed Western markets, the findings also speak about spillovers, contagion and correlations of uncertainty on the market level (QG).
Hypotheses. The case study setup studies the behavior of the markets in history. Hypotheses are not needed, and pre-set views might lead to faulty analyses (Ethridge 2004, 144). Instead, the main focus lies in describing and illustrating the findings.

The underlying assumptions behind the research questions. By continuing earlier research and following a specified methodology, the study relies on several assumptions. The most essential are assumptions of:

a) Engle's DCC-MGARCH method and assumptions related to it (Engle 2002, 342).

b) The time series of the VIs in question are mean reverting (Ahoniemi 2008; Black 2006, Moran & Dash 2007; Szado 2009, 12), and

c) their implied volatility values, created by using the VIX specification and derivatives on the main market indices, can be considered as sound market prices of risk for the underlying stock indices (Carr & Wu 2006). Still, the fact that investing in VIs takes place through derivatives is not taken into account (Black 2006).

d) The traditional assumptions of financial markets: rational agents, competitive markets, freely available information and no arbitrage, (Ryan et al. 2002, 51-53, see also Hull 2009, 286-287, 289) are implicitly important, as VIs are based on the prices of derivatives. Still, the new VIX specification is free from pricing models used [sic].

e) Finally, for practical conclusions about the relationships of markets, an assumption can be made that a VI has an ability to portray implied volatility of the overall stock market in question, as the VI is based on the respective main stock index.

3.2. Research setting

By following Ethridge the study can be classified as representative of analytical applied research (2004, 20-25), or by following Johnson (1986, 12-13) as disciplinary problem-solving or subject-matter research (Mäki, Gustafsson, Knudsen 1993, 121-128), as the study represents the realm of one single domain. Measured by Laughlin’s classification of theorization (Ryan et al. 2002, 44-46), the article represents a high or medium class of prior theory, and a high level of methodological choice.

Preliminary Research. The pre-research of the data is undertaken as suggested by Gaines (2002). It reveals that many VIs seem to contain a unit root, when traditional unit root tests (KPSS, Augmented Dickey-Fuller, and DFGS-Generalized Least Squares) are used. This is an interesting finding, as volatility should have a theoretical tendency to be mean reverted (Moran & Dash 2007, 98). As stated earlier, there are several potential explanations for this based on general volatility research. Earlier VI related studies explain this phenomenon often as an occurrence of structural break(s), and treat the time series as stable even if the data fulfill this property weakly (Ahoniemi 2008; Black 2006). However, special attention is paid on unit root testing in the study.

General process. The research process is described in appendix 1, which explicitly shows the theoretical and applied segments of the research process. The theoretical framework is created through 2) methodological decisions of the study, which are in turn founded in 1) earlier research. The applied research process begins by 3) choosing appropriate data and studying its statistical properties. All the tests are run for the data both as logarithm and first difference values (equation 3.1). Potential day effects of the data are studied, in order to decide whether the use of daily or weekly data would yield more reliable results, and the final choice of the 4 VI series included in the model are made. 4) The most suitable MGARCH method is then chosen for estimations by relying on residual based diagnostics, summary statistics and information criteria. Hence, these two steps (3-4), as well as methodological orientation (2), create a cyclical, repeated process in practice. 5) The results of this applied process are then reported
and discussed, and the robustness of the results is evaluated. In this context, it is also natural to evaluate the meaning of the findings with respect to theoretical framework, as well as a basis for future study. Still, generalization of the results of the applied study has several limitations and should be handled with care.

3.2.1. Choice of time series and modeling

On one hand, the indices for modeling are chosen (among available ones) to portray differences between different developed markets. On the other hand, this choice is also affected by modeling. The overlapping indices in terms of geographic areas, fiscal and monetary policies are avoided, as well as the same currency. The study treats implied volatility as a tradable asset, which means that the specifications of the indices could vary without jeopardizing the research setting. The estimated models are complex, and the models do not necessarily converge. The process is repetitive, as the number of time series in one model is limited by its convergence. These limitations are also dependent on the properties of each time series and can change case by case.

3.2.2. Testing and residual based diagnostics

General Portmanteau tests are used to reveal possible deviations of data from randomness. Tests of autocorrelation, heteroscedasticity, but also specific tests for volatility, namely tests for ARCH effects and asymmetric volatility are executed three times, 1) for the daily data series in order to choose the appropriate model for the day effect testing, 2) for the weekly data series in order to choose the appropriate MGARCH model, and finally 3) as a residual based diagnostics (Bauwens et al. 2006) for the error terms of the MGARCH model, in order to give further information of the appropriateness of the model. The residual based diagnostics is conducted by following Johansson & Ljungwall (2009), who study MGARCH error terms as separate univariate time series without combined testing (ibid., 101-102). The residual based diagnostics are not a reliable verification tool by themselves, as their distributions for daily and weekly time series may not followed assumed distribution (ibid., 96, 102). As a consequence, ARCH effects are used to bring extra information in addition to LB tests. Information criteria are used where appropriate numbers of lags for the time series are needed.

Summary statistics. The basic properties of the data such as the number of observations, mean, standard deviation, minimum and maximum values, skew, excess kurtosis, Jarque-Bera measure and unconditional raw correlations are reported as natural logarithms and as raw returns calculated by

\[ \Delta y_t = 100 \times \ln \left( \frac{y_t}{y_{t-1}} \right). \]  \hspace{1cm} (3.1)

The 1st difference returns reported in the study are consistently calculated by using the specification in equation (3.1). The used methodology may allow an opportunity to conduct the research by using natural logarithms along the differences of the natural logarithms (e.g. Ahoniemi 2008). In modeling, the previous benefits from containing more information, while the latter has an advantage of being more likely to converge to a solution. The graphical presentations of the time series are also included.

Testing for day effects. The choice between daily and weekly frequency of data must be made. In practice, weekly data is given a priority in order to avoid problems caused by potential non-synchronous trading (chapter 4.1). The use of weekly data may mask the spillovers, however, and it is reasonable to verify the results by using daily values. Hence, day effects are also studied for reference. The standard Ordinary Least Squares (OLS) -method is used
\[ y_t = \alpha_0 + \alpha_1 D_1 + \alpha_2 D_2 + \alpha_3 D_3 + \alpha_4 D_4 + \varepsilon \]  
(3.2)

where \( \alpha \)s are coefficients, \( y \) is the return of the data series at time \( t \), \( D \)s are are dummies for weekdays from Monday to Thursday. Newey-West’s heteroskedasticity-and-autocorrelation-consistent (HAC) standard errors (Newey & West 1987; also e.g. Verbeek 2008, 118-119) are used to mitigate potential autocorrelation and heteroscedasticity of the time series (Gonzalez-Perez & Guerrero 2009, 2). Newey & West establish HAC errors by creating a general case of Eicker’s (1967, according to Verbeek 2008, 94) heteroscedasticity-consistent covariance matrix, the positive (semi)definiteness is confirmed by using Bartlett weights, as an alternative to GLS -method. The lags are defined by information criteria. Eicker-White’s robust error terms (e.g. Verbeek 2008, 94-95), as well as EGARCH –results (Nelson 1991; also e.g. Brooks 2008, 406), including the dummies in the mean equation, are estimated for reference.

Tests of autocorrelation, heteroscedasticity, and ARCH effects. Ljung-Box (LB) test (1978) is used to detect autocorrelation and heteroscedasticity, mainly because it can catch also higher order autocorrelation and has better small sample properties than Box-Pierce equivalent. In the Ljung-Box test statistic

\[
LB = T(T+2) \sum_{k=1}^{m} \frac{\hat{\gamma}_k^2}{T-k} \sim \chi^2(m) \quad \text{where} \quad \hat{\gamma}_k = \frac{\hat{\tau}_k}{\gamma_0}
\]  
(3.3)

and \( T \) refers to sample size, \( m \) to maximal lag length and \( k \) those lag lengths from 1 to \( m \). The distribution of the test follows Chi-squared asymptotically. Tau- values are derived from standard autocovariance function

\[
\gamma_k = E(y_t^2 - E(y_t))(y_{t-k} - E(y_{t-k}))
\]  
(3.4)

where \( y \) refers to values of the return data series at time \( t \) or univariate error terms of VAR-MVEGARCH model. The asymptotic properties of Ljung-Box test are known only for the residuals of ARMA-type regressions. This means that the diagnostics for MGARCH residuals cannot be given any formal asymptotic distribution, and thus must be handled with care. In the beginning, the test is applied to the raw returns time series themselves, as the results are the same as they were for the error terms of the regression including only a constant. This is a common and generally accepted practice. The joint tests are run in 4 groups for lags 1-4, 5-8, 9-12 and 13-16. For each of the group, Null-hypothesis assumes that all 4 taws are equal to zero, as

\[
H_0: \tau_1 = 0, \tau_2 = 0, \tau_3 = 0, \tau_4 = 0 \quad \text{or} \quad H_1: \tau_k \neq 0(\text{any}), \quad \text{where}
\]  
(3.5)

The alternative-hypothesis is true. Heteroscedasticity is tested the same way by using \( y^2 \) terms in the beginning, and MGARCH error terms for the residual diagnostics. Engle’s (1982) classic methodology is used to test ARCH effects in the time series, in order to see whether GARCH -type model specification is needed for the data. The ARCH effects are tested by

\[
\hat{\gamma}_t^2 = \gamma_0 + \sum_{i=1}^{q} \gamma_i \hat{\gamma}_{t-i}^2 + \eta \quad \text{and} \quad TR^2 \sim \chi^2(q)
\]  
(3.6)
where $\gamma$ are coefficients, $\eta$ is an error term, and $q$ is a maximum number of explaining lags, $l$. The variable $y$ is defined as earlier. The test relies on $R^2$-statistics of the regression, which multiplied by sample size, $T$, asymptotically follows Chi-squared distribution, with $q$ degrees of freedom. The joint testing of $\gamma$s, and consequent Null- and Alternative hypotheses are constructed as in equation (3.5). The test is also run on squared values, just as the LB test, and used as a residual diagnostics for error terms as well. The test is theoretically most founded when used for error terms of ARMA-model, but Brooks (2008, 389) maintains that the test can also be run on the raw returns data. The researcher’s own findings reveal that there are infinitely small changes in the results and the statistical significance, when using raw data.

Asymmetries in volatility. The potential need for asymmetric GARCH-models is tested, by using the sign and size bias test of Engle & Ng (1993). The test relies on studying the error terms of the GARCH -model used. The standard symmetric GARCH(1,1) model (e.g. Brooks 2008, 392-395)

$$y_t = \mu + u_t \quad \text{where } u_t \sim N(0, \sigma^2_t) \quad (3.7)$$

$$\sigma_t^2 = \alpha_0 + \alpha_1 u^2_{t-1} + \beta \sigma^2_{t-1} \quad (3.8)$$

is estimated. The terms of the model are mean, $\mu$, return $y$ of the data series at time $t$, and conditional volatility, $\sigma_t^2$, at time $t$, and lagged and squared error terms of the mean equation, $u$, which are normally distributed with zero mean. $\alpha$s and $\beta$ are non-negative coefficients. The model generates the error terms for the OLS-based asymmetry test procedure. Later, VAR-MVEGARCH-model (equation 3.21-3.29) is used for the same purpose, when running for residual diagnostics. Only the joint Engle-Ng-test is run on the error terms as

$$\hat{u}_t^2 = \phi_0 + \phi_1 S^+_{t-1} + \phi_2 S^-_{t-1}u_{t-1} + \phi_3 S^+_{t-1}u_{t-1} + v_t \quad \text{and} \quad TR^2 \sim \chi^2(3), \quad (3.9)$$

which includes these before-mentioned error terms, $u$, coefficients, $\phi$, dummies for negative and positive error terms, $S$ and $S^+$, as well as a new error term for the regression, $v$. Then, the coefficient $\phi_1$ signifies sign bias, where the sign of the shock affects the future volatility the differently, while $\phi_2$ and $\phi_3$ reveal size bias, which means that also magnitudes of the shocks have differing impacts. The test statistics follow asymptotically Chi-squared distribution with 3 degrees of freedom, and is calculated as earlier. The joint Null-hypothesis indicates no asymmetric effects.

Unit Root Tests. The unit root tests are used to reveal potential diversions of the time series from stability. Potential unit roots should be taken into account when constructing the mean equation (3.21) of VAR-MVEGARCH-model. In addition to standard Augmented Dickey-Fuller (ADF) test (e.g. Verbeek 2008, 286-288; Brooks 2008, 329-332), also Zivot & Andrews (1992) and Lee & Strazicich (2004) tests are included to trace a potential structural break (see also Westerlund 2009; Westerlund & Edgerton 2007), which might be caused by the subprime crisis period included in the time series, and to take into account a possibility that the VIs would contain unit roots (Gonzalez-Perez & Guerrero 2009). Zivot & Andrews (1992) test is well-known and advances Perron’s adjusted ADF-testing strategy, by defining the crash model (A), for changes in the intercept,

$$y_t = \hat{\mu}^A + \hat{\theta}^A DU_t(\hat{\lambda}) + \hat{\beta}^A t + \hat{\alpha}^A y_{t-1} + \sum_{j=1}^{k} \hat{c}_j^A \Delta y_{t-j}^+ + \hat{\epsilon}_t \quad (3.10)$$
and the combined model (C), for changes in both intercept and trend,

\[ y_t = \hat{\mu}_C + \hat{\theta}_C U_t(\hat{\lambda}) + \hat{\beta}_C t + \hat{\gamma}_C DT_t(\hat{\lambda}) + \hat{\alpha}_C y_{t-1} + \sum_{j=1}^{k} \hat{c}_j \Delta y_{t-j} + \hat{\epsilon}_t \]  

(3.11)

where the difference raises from the \( T^*_T \), which is the time of exogenous break in the trend function. In both models, the circumflexes refer to estimated parameters, \( \hat{\mu} \) is constant, \( D \) refers to dummy variables, \( k \)–number of parameters \( \Delta y_{t-j} \) are to "eliminate possible nuisance-parameter dependencies in the limit distributions of the test statistics caused by temporal dependencies in the disturbances" (ibid.), and \( \hat{\lambda} \) refers to the break in the data series. Then,

\[ DU_t(\hat{\lambda}) = \begin{cases} 1 & \text{if } t \geq T\lambda \\ 0 & \text{else} \end{cases} \]  

(3.12)

\[ DT_t(\hat{\lambda}) = \begin{cases} t - T\lambda & \text{if } t \geq T\lambda \\ 0 & \text{else} \end{cases} \]  

(3.13)

The unit root is then tested by rejecting Null-hypothesis if

\[ \inf_{\lambda \in \Lambda} t_{\alpha'}(\lambda) < K'_{\alpha',\Lambda} \text{ where } i = \Lambda, C \]  

(3.14)

and in which \( K'_{\alpha',\Lambda} \) is the left-tail critical value \( \alpha \) of t-statistics for \( \alpha' = 1 \), from the asymptotic distribution of \( \inf_{\lambda \in \Lambda} t_{\alpha'}(\lambda) \), the distribution and the test statistics of which Zivot & Andrews establish in their paper (ibid.) with \( \Lambda = [0.001, 0.999] \). As is the case with Lee-Strazicich test, the drift only specification exists also for Zivot-Andrews, but is seldom needed in practice (Lee & Strazicich 2004, 3).

Lee & Strazicich (2004) draw attention to some problems with Zivot-Andrews test, which they classify as "endogenous break unit root test" (p. 1). The test results, according to them, might incorrectly conclude that "a time series is stationary with break when in fact the series is non-stationary with break. As such, 'spurious rejections' might occur and more so as the magnitude of the break increases." They suggest a new minimum Lagrange Multiplier (LM) unit root test to avoid this problem. The concept (ibid.) is a modification of Schmidt-Phillips non-break Unit Root test, the data generating process of which

\[ y_t = \delta Z_t + X_t \text{ where } X_t = \beta X_{t-1} + \epsilon_t \]  

(3.15)

follows Schmidt & Phillips (1992) – where Null-hypothesis of a unit root is \( \beta = 1 \) and vector \( Z_t = [1, t]' \).

Lee & Strazicich’s complex contribution is to define a crash model (for changes in the intercept) with

\[ Z_t = [1, t, D_t]' \text{ where } D_t = \begin{cases} 1 & \text{if } t \geq T_B + 1 \\ 0 & \text{else} \end{cases} \]  

(3.16)
and $T_B$ is the time of the structural break, as well as a combined model (for changes in both intercept and trend) with

$$Z_t = [1, t, D_t, \Delta T_t]$$

where $D_t = \begin{cases} 
    t - T_B & \text{if } t \geq T_B + 1 \\
    0 & \text{if } \text{else}
\end{cases}$

(3.17)

similarly. Vector of coefficients, $\delta_t$, is to be increased accordingly. Then, the test statistics is generated by regression

$$\Delta y_t = \delta_t \Delta Z_t + \phi \tilde{S}_{t-1} + u_t$$

where

$$\tilde{S}_t = y_t - \tilde{\psi}_x - Z_t \delta$$

(3.18)

and in which $\phi = 0$ is the Null-hypothesis, $\delta_t$ refers to the coefficients in the main regression, and $\tilde{\psi}_x$ refers to the restricted Maximum Likelihood Estimator of

$$\psi_x = \psi + X_0.$$  

(3.19)

There are two things to notice. First, differencing removes the first value ($t=1$), and that the test statistics regression (3.18) needs $\Delta Z_t$ instead of $Z_t$, which then become

$$\Delta Z_t = [1, \Delta D_t]$$ and $\Delta Z_t = [1, \Delta D_t, \Delta DT_t]$  

(3.20)

for the crash model and the combined model respectively. Hence, $\Delta D_t$ refers to a change in intercept and $\Delta DT_t$ to trend under Alternative hypothesis, as well as a one time crash and a permanent change in the drift under Null-hypothesis. The drift only specification exists, but is seldom needed in practice (ibid.). The autocorrelated errors can be corrected by including augmented terms as in the basic Augmented Dickey Fuller test (e.g. Verbeek 2008, 286-288; Brooks 2008, 329-332). The location of the break point is not needed for the study and thus not explicitly specified.

Lee & Strazichich (2004) also compare the results of their test with Zivot-Andrews test. They conclude the LM test truly removes the problem by estimating the break point correctly and by being free from size distortions and spurious rejections in the presence of the unit root. The possibility of multiple structural breaks is also studied by following Lee & Strazichich (2003).

**Testing for Cointegration.** The failure in rejecting the unit roots in the time series means that the 1st differences of the time series must be used. Further, Vector Error Correction Model (VECM) –parameters (e.g. Verbeek 2008, 340-341) should be included in the mean equation of the MGARCH –model, if the time series were cointegrated. The cointegration of the simultaneous time series is studied by following normal Johansen methodology (Brooks 2008, 350-355), in which Null-hypothesis suggests that there are $r$ cointegrating relationships. If the hypothesis is rejected, the new Null-hypothesis tests for $r + 1$ cointegrated equations. The procedure is repeated until the maximum rank of cointegrations is found.

**Choice of lags in the tests and models.** The number of time lags must be chosen both for the used tests and for the main model. These decisions are made a) by using predefined number lags (4, 8, 12, 16) for joint testing and reporting all the results, as in Portmanteau tests, or b) by using standard univariate Akaike information criteria (AIC) as in the study of day effects and unit roots. The choice of appropriate number of mean equation lags for MGARCH –model is based on c) the multivariate versions of Akaike, Schwartz’s Bayesian (BIC), and Hannan-Quinn (HQIC) –information criteria (e.g. Verbeek 2008, 61-64, 299-302; Brooks 2008, 232-239, 294-295). The final decisions are
made based on information criteria and residual based diagnostics of VAR-MVEGARCH –model together, in order to compensate known weaknesses in the use of either of them. The Johansen cointegration test uses always the same number of lags as the MGARCH –model used.

3.2.3. Multivariate GARCH –modeling

The main modeling endeavor is undertaken by using the Multivariate version of the Generalized Autoregressive Conditional Heteroscedasticity (MGARCH) Maximum Likelihood estimator. The time series including volatility clustering are traditionally modeled by using GARCH (Engle 2001, Gourieroux & Jasiak 2001, 126-135), and it has become a standard method of analyzing a financial time series including time-varying volatilities (Engle 1982, 2001; Engle & Bollerslev 1986). The model has different variations, and the EGARCH version has been successfully used to model asymmetric behavior of volatility in the earlier stock market study.

The first generation MGARCH models have weaknesses, which make them difficult to use in practice. The simple definition of MGARCH consisting solely of univariate specifications (without correlation part) is barely believable, whereas Bollerslev, Engle & Wooldridge’s (1988) VECH –specification has a large number of parameters, is often unable to yield a positively definite variance-covariance matrix, and it can be only used to estimate a few time series at the time. BEKK –specification (Engle & Kroner 1995), on the other hand, decreases the number of parameters, by assuming a positive definiteness of the variance matrix. Still, it does not converge very well, especially when a larger number of time series is included, and it is difficult to include asymmetry in the model. Hence, second generation models, which are estimated in two steps (Bauwens et al. 2006, 98-99), are more appropriate for practical applications. Silvennoinen & Teräsvirta (2009, 223) also show that more advanced models have an overall tendency to be more reliable.

Engle’s version of the Dynamic Conditional Correlation (DCC) model used for estimation (Engle 2002; about different specifications Bauwens et al. 2006, 91) is successfully used by Silvennoinen & Teräsvirta (2009) in the VIX context without convergence problems. The Vector Autoregressive Exponential Generalized Autoregressive Conditional Volatility (VAR-MVEGARCH) –version estimated by Maximum Likelihood method (Engle 2002, 341-343; Manera et al. 2006, 527; Bauwens et al. 2006, 96) can be specified as follows: The Vector Autoregressive (VAR) mean equation (Johnston & Dinardo 1997, 287-321; Verbeek 2008, 335-338) is

\[
R_{it} = \beta_{i0} + \sum_{j=1}^{J} \sum_{k=1}^{K} \beta_{ij} R_{j,t-k} + \varepsilon_{it}, \tag{3.21}
\]

where \( R_{it} \) is a return vector, each cell of which is defined as in equation 3.21, \( \beta \)'s are coefficients, \( \varepsilon \)'s error terms, and by following standard VAR -methodology, all the variables are endogenous and explaining variables at the same time. The number of lags \( K \) in the model can be estimated by using multivariate information criteria (table 9), as well as residual diagnostics (tables 13 & 14). Then, significant \( \beta_{i,j} \) parameters refer to spillover effects between the time series (if \( i \neq j \)), and to autocorrelation in time (if \( i = j \)). The conditional volatility part in multivariate setting (Engle 2002, 341-343; Manera et al. 2006, 527; Bauwens et al. 2006, 90) is presented by variance-covariance matrix, \( H_t \), which by changing through time describes also the second moment relationships between the time series

\[
E(u_t'u_t' | Q_{t-1}) = H_t = D_t Q_t D_t \tag{3.22}
\]
where \( Q \) is a symmetric positive definite matrix, i.e. unconditional variance matrix \((n \times n)\), which in the second step of the estimation process establishes the relationships between the time series to supplement the first step univariate volatilities

\[
Q_t = (1 - \theta_1 - \theta_2)Q + \theta_1 u_{t-1} u_{t-1}^T + \theta_2 Q_{t-1},
\]

(3.23)

the variable \( \theta_1 \) is positive and \( \theta_2 \) non-negative scalar, \( \theta_1 + \theta_2 < 1 \), catching the DCC effects (Silvennoinen & Teräsvirta 2009, 212-213), and standardized error terms defined as

\[
u_{i,t} = \frac{\varepsilon_{i,t}}{\sqrt{h_{ii,t}}},\]

(3.24)

and \( Q_{t-1} \) is an information set. In case of \( \theta_1 = \theta_2 = 0 \), there are no such effects and the model reduces to Bollerslev’s (1990; Bauwens et al. 2006, 88) Constant Conditional Correlation (CCC) model in which

\[
H_t = D_t Q D_t = \rho_{ij} \sqrt{h_{ii,t} h_{jj,t}}.
\]

(3.25)

\( D_t \) is specified in any case as

\[
D_t = \text{diag}\{h_{ii,t}^{-1/2}\},
\]

(3.26)

which are values generated in the first step of estimation process by using \( n \) number of univariate GARCH models. This first step is to estimate the univariate conditional volatilities of the time series for the second step, in which the relationships between the time series are then created. Engle’s approach is flexible in terms of specifications of the univariate models (Engle 2002, 342; Bauwens et al. 2006, 89), and assumes only that all of them must be specified similarly. This is important as it allows taking into account leverage effects (Bauwens et al. 2006, 93) by choosing appropriate first step model. Hence, E-GARCH –model (Nelson 1991) is used to create the first-step volatility estimates of the univariate time series. The model is specified in univariate form as

\[
y_{i,t} = \mu_t + \varepsilon_{i,t}
\]

(3.27)

\[
\ln(h_{ii,t}) = \ln(\sigma_{ii,t}^2) = \omega_t + \beta_t \ln(\sigma_{ii,t-1}^2) + \gamma_t \frac{\varepsilon_{i,t-1}}{\sqrt{\sigma_{ii,t-1}^2}} + \alpha_t \left[ \frac{\varepsilon_{i,t-1}}{\sqrt{\sigma_{ii,t-1}^2}} - \sqrt{\frac{1}{n}} \right]
\]

(3.28)

where \( \alpha, \beta; \gamma \) are coefficients of the volatility equation, as well as \( \omega \) and \( \mu \) are constants for volatility and mean equations, for the \( n \) univariate EGARCH –estimation, respectively. Error terms \( \varepsilon_{i,t} \) and \( \varepsilon_{i,t-1} \) the first lag of them, are a zero mean normally distributed random variable and \( \sigma_{ii,t}^2 \) is conditional variance. A comfortable property of the model is that its parameters can be negative, as the logarithm specification of exponential GARCH keeps the estimated conditional volatility will be positive. Negative \( \gamma \) refers to negative correlation between volatility and returns, thus allowing asymmetries in the model. The error terms of the model are supposed to follow General Error Distribution (GED), which allows numerous different specifications, and which in case of Normal distribution
follows $\varepsilon_t \sim N(0, \sigma^2_t)$. In this study, the data are assumed to be $t$-distributed, which is taken into account by a specific term, and normalized error terms (equation 3.24) should be normally distributed (i.i.d.) and

$$u_{t,t} \sim N(0,1).$$  

(3.29)

The choice of $t$–distribution takes into account a possibility that real life financial returns data have heavier tails than assumed by Normal distribution. This is a commonly accepted finding and explicitly verified also in the context of Black & Scholes -based derivatives pricing (Ederington et al. 2002; summarized also by Hull 2009, 389-401). Still, Bauwens et al. (2006, 96) explicitly state that normality assumption of innovation terms “is rejected in most applications dealing with daily or weekly data” [sic]. The general conditions for the model then are

$$R_t = \mu_t + H_t^{1/2} \varepsilon_t,$$  

(3.30)

$$\mu_t = E(R_t | \Omega_{t-1}),$$  

(3.31)

$$H_t = Var(Y_t | \Omega_{t-1})$$  

(3.32)

where $\Omega_{t-1}$ is an information set at $t-1$, $\mu_t$ a conditional mean vector, and $H_t$ conditional variance-covariance matrix, which by definition is allowed to change through time, $t$.

3.3. Validity and reliability of the method

The method. The results of the study may suffer from the potential limitations of real life data of social research (Ethridge 2004, 150-154), as an applied inductive method is used. Unlike other data in economics, financial data are standardized and thus relatively reliable (Ethridge 2004, 151). Potential data related problems are more specific by nature. There are claims that – being “data driven” – time-series do not reveal anything about the economic structure, and are of little use. Here the method can be defended by its positivistic procedure, which reveals structures in the existing system (Ethridge 2004, 148). As the inferences are derived from particular instances, the results cannot be used to establish a generally established law or model (Ethridge 2004, 45, 74-77). Thus, the results are meaningful only in the context of the data set.

Data. The choice of data is one of the most vulnerable parts of quantitative study – it relies solely on the researcher’s abilities, knowledge and experience, and the decisions are qualitative in a sense that their soundness cannot always be estimated by any means. As investing in volatility takes place by using derivatives written on a VI, the results do not take into account the fact that the prices on these markets might differ from this theoretical price in practice (Sepp 2008). Finally, some sources claim that currently most sound (and recently improved) VIX- specification for VIs is still flawed and cannot be considered as an unbiased measure of implied volatility (Jiang & Tian 2007).

Model. The validity of the model boils down to the choice of the correct model, and to the soundness of the model itself. In this case, the Multivariate GARCH models are the only reasonable option for older Vector Autoregression specifications (VAR), which only have limited abilities to take into account serial correlation, heteroscedasticity, conditional volatility and asymmetric volatility effects of the data. The more important question will be the choice between the second-generation Multivariate GARCH models. As Engle’s (2002) DCC-GARCH specification
explicitly shows if the CCC-GARCH (Bollerslev 1990) specification is usable, there is little uncertainty about the choice between these models. Another option might be Ling & Aleer’s (2003) VARMA-GARCH model. The study focuses on the situation, in which one of the time series includes an extensive shock, but at the same time the time series may contain asymmetric effects. Ling & Aleer’s model (ibid.) is exceptionally good at mitigating large discrete shocks, but does not converge very well. Then, the choice between these models is also natural. On the other hand, there is some evidence that “constraining the dynamics of the conditional correlation matrix to be the same for all the correlations” as caused by equation 3.23, might not be a reasonable assumption (Bauwens et al. 2006, 90-91 partially based on Billio et al. 2003). Still, this methodology is generally used in the domain, there are few better models for consideration, and Engle’s DCC-specification has been successfully used in the context of the data of the study – Manera et al. (2006) and Silvennoinen & Teräsvirta (2009) compare different DCC–models. The main emphasis should be given to the data and its treatments, as well as sound applications of the used methodology. Finally, there is some room for discussion about the used model (ibid., 223).

Reliability related topics are numerous. With respect to multivariate models, numerical computer methods are used to optimize these complex Maximum Likelihood models at hand. The models may not converge or they may converge to the local maxima, which does not yield the correct results. A special attention should be paid to the choice of estimation algorithm and its used parameters. Bauwens et al. (2006) have studied the current state of multivariate GARCH models. According to them, not only asymptotic properties of Maximum Likelihood Estimation, but also some asymptotic properties of the models themselves are still unknown, and the whole field of multivariate diagnostic testing is still largely open. These problems weaken reliability significantly. Performance and financial value of different specifications of the models have not been compared, which means the consequences of the choices between the models are still not quite understood. Furthermore, there is very little we can say about “unconditional moments of correlations /covariances, marginalization and temporal aggregation” in the DCC setting (Bauwens et al. 2006, 105). This means there is a trade-off between validity and reliability. The VAR -model might potentially improve reliability, at least overall knowledge about (potentially) reliability, but is not satisfactorily specified to model the problem at hand. At the same time, a Multivariate GARCH model, although having some open questions with reliability is better specified, and measures more closely what is needed.

Testing. In terms of used concepts for data series testing, the raw return series as a source for ARCH effects testing causes only minute differences in the test results, and is generally considered as negligible. A larger problem is related to the fact that residual based Portmanteau tests for MGARCH model, do not necessarily follow standard asymptotic properties, as their test statistics do not follow necessarily those of OLS (Bauwens et al. 2006, 102) and more generally their distribution may not be normally distributed (ibid., 96). Information criteria may yield conflicting results, which may be especially problematic, when VAR -lags of the MGARCH –model are to be chosen. Hence, all statistics must be evaluated as a whole.

Reporting. The reporting (Ethridge 2004, 160-168; Ryan et al. 2002, 168-174; Thomson 2001, 1-6) will focus on treatment of the economic (or substantive) significance of measurements (Ziliak & McCloskey 1996; 2008, 1-16; and McCloskey 1998, 112-138). Statistical significance is explained if crucially needed, but reported in order to evaluate “essential economic meaning”. Mathematical models (McCloskey 2002, 9-16) and tests are given explicitly, and the results are reported accordingly (Cochrane 2005; Thomson 2001, 36, 49-60, 103-107,110).
4. DATA

Data used in the study. The table 1 shows 8 different VIs available for the research, which are based on the stock index of developed markets, follow the standard VIX – specifications presented in equation 2.1, and are available since the beginning of 2000. The data in question is fetched from the Thomson Reuters Datastream service. From these, four different indices are chosen for the use of the study.

Table 1: Available Volatility Index (VI) data for developed Stock Markets

<table>
<thead>
<tr>
<th>Name</th>
<th>Underlying</th>
<th>Market</th>
</tr>
</thead>
<tbody>
<tr>
<td>AEX VI</td>
<td>AEX</td>
<td>The Netherlands</td>
</tr>
<tr>
<td>BEL 20 VI</td>
<td>BEL 20</td>
<td>Belgium</td>
</tr>
<tr>
<td>CAC 40 VI</td>
<td>CAC 40</td>
<td>France</td>
</tr>
<tr>
<td>CBOE SPX VIX</td>
<td>S&amp;P 500</td>
<td>USA</td>
</tr>
<tr>
<td>FTSE 100 VI</td>
<td>FTSE 100</td>
<td>UK</td>
</tr>
<tr>
<td>VDAX-NEW VI</td>
<td>DAX 30</td>
<td>Germany</td>
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<td>SMI</td>
<td>Switzerland</td>
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<tr>
<td>VSTOXX VI</td>
<td>EURO STOXX 50</td>
<td>European Area</td>
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</tbody>
</table>

Each index follows VIX – specifications and represents implied volatilities for the next 30 days for the respective developed stock market index. The time series are all available since 2000 or earlier.

4.1. Choice of Data

The indices are chosen to represent the properties of the data in a multifaceted manner, and a qualitative saturation test is used to assure that the data used contains all the relevant information. In other words, new potentially related time series are studied until relevant new information does not emerge anymore. Only 4 indices are finally chosen, as that turns out to be the maximum number of converging time series. The chosen indices (table 2) represent different currencies, as well as Fiscal and Monetary policies. They are well-defined geographically, without overlap, representing globally important markets of the Western world, and bring their own distinctive viewpoints to the topic. The VIX is found to have global effects (Theodossiou & Lee 199) and it contains an immense subprime shock towards the end of the series, and the VDAX is the main market in the Continental Europe. The behavior of the VFTSE of the UK between these two markets is of special interest. The Swiss VSMI behaves as an example of smaller, slightly more remote market, the behavior of which might well differ from those of the larger hubs.

Table 2: Volatility Indices (VI) used in the study

<table>
<thead>
<tr>
<th>Name</th>
<th>Abbreviation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>CBOE SPX VIX</td>
<td>VIX</td>
<td>Main hub, series including a large shock</td>
</tr>
<tr>
<td>FTSE 100 VI</td>
<td>VFTSE</td>
<td>Mediating hub between US and Europe</td>
</tr>
<tr>
<td>VDAX-NEW VI</td>
<td>VDAX</td>
<td>Main continental European market</td>
</tr>
<tr>
<td>VSMI VI</td>
<td>VSMI</td>
<td>Smaller, developed market away from hubs</td>
</tr>
</tbody>
</table>

Time window: The time period is chosen to present both normal market conditions as well as a crisis in the end of the window. This does not refer to event study terminology, in which different parts of the data serve as estimation and event periods. Observations during the crisis can be set in the larger framework by comparing them with the
reference period during normal market conditions. Still, the main benefit of the used advanced DCC -model is an opportunity to analyze changes through time. In this sense, this is not a traditional event study.

Most time series for European VIs are available from the beginning of 2000, which then is a natural starting point. On the other end, the US subprime crisis creates another natural boundary. The problem related to the crisis is that its precise beginning and ending is difficult to define from the standpoint of its effects on the implied volatilities. Many times Lehman Brothers’ Chapter 11 (2008/09/15) is considered as a tangible beginning of the systemic crisis in the US -financial markets, but HSBC reported large subprime related losses already in February 2007, and there was a full-scale panic in the financial markets leading investors to reallocate their investments from stocks and mortgage bonds to commodities. The end of the crisis, although not as critical as the beginning of the crisis, is difficult to pinpoint as well. The rising stock market indices cannot be used as a reliable sign of the end of the crisis, as it might be caused solely by Fiscal and Monetary policies used to remedy the on-going crisis. The most reliable sign available about the end of the crisis is positive GDP, which has two problems. GDP is related to real economy – not as closely to financial markets, and it is crude as a measure and can define only monthly changes at its best. Still, the best estimate about the end of the crisis in the US-markets is June 2009, which was the first positive GDP month of the crisis (Federal Reserve System 2010). The data then runs from the beginning of the 2000 to 2009/06/15. Difficulties in defining the crisis period also affect the modeling. It is theoretically questionable to use dummy variables in the modeling to catch the effects of the crisis period, even if such a model could be created by trial and error, and would yield good results.

Data frequency. Lin et al. (1994) explain seeming volatility spillovers as non-synchronous trading. The effects of potential non-synchronies are mitigated by the use of weekly data, as adding dummies in the model with daily values would not remove the problem. Observed spillovers are then real, in a sense that they can be classified as information transmissions from one market to another (Stoll & Whaley 1990; Chan, Chan & Karolyi 1991).

4.2. Properties of data

Descriptive statistics. The properties of weekly univariate time series are studied for further modeling. The descriptive statistics for the returns of weekly VI data are presented in table 3. The extreme values decrease when moving from the shock prone US- market to the most remote European market. The mean reverting (Black 2006) mean returns are interestingly positive for the period (Grant et al. 2007; Szado 2009) and small. The mean of the VIX is considerably higher (0.071) than the others, and that of the VFTSE is very close to zero (0.001). Volatility of volatility has a tendency to be higher than the volatility of equivalent stock indices (Carr & Wu 2006; Moran & Dash 2007, 97). Here, the volatilities decrease, similar to the extreme values, excluding the VFTSE, which is the highest (12.040). All distributions are positively skewed, the VIX and the VDAX being the lowest and the highest, leptokurtic, the VDAX being the highest and the VSMI the lowest, and non-normal (Black, 2006, 6; Carr & Wu 2006, 17-18). The joint Ljung-Box –tests with different lags reveal autocorrelation and heteroscedasticity for the VIX, and weak evidence of autocorrelation for the VDAX. Similarly calculated ARCH- effects, which can be thought of as autocorrelations for the squared values, exist statistically for the VIX, and only weakly at 5% level for the first four residuals when applied on squared level (3.121). This latter measure can be thought of as a heteroscedasticity of ARCH -effects. No ARCH- effects or heteroscedasticities exist in other time series, the latter finding of which is surprising.
Table 3: Descriptive Statistics of the VI Returns

<table>
<thead>
<tr>
<th></th>
<th>VIX</th>
<th>VFTSE</th>
<th>VDAX</th>
<th>VSMI</th>
</tr>
</thead>
<tbody>
<tr>
<td>Observations</td>
<td>492</td>
<td>492</td>
<td>492</td>
<td>492</td>
</tr>
<tr>
<td>Maximum</td>
<td>56.562</td>
<td>48.247</td>
<td>46.205</td>
<td>39.496</td>
</tr>
<tr>
<td>Mean</td>
<td>0.071</td>
<td>0.001</td>
<td>0.017</td>
<td>0.013</td>
</tr>
<tr>
<td>Skewness</td>
<td>0.472***</td>
<td>0.595***</td>
<td>0.685***</td>
<td>0.496***</td>
</tr>
<tr>
<td>Kurtosis</td>
<td>1.990***</td>
<td>1.507***</td>
<td>2.196***</td>
<td>1.305***</td>
</tr>
<tr>
<td>Jarque-Bera</td>
<td>99.423***</td>
<td>75.534***</td>
<td>137.347***</td>
<td>55.104***</td>
</tr>
<tr>
<td>LB(4)</td>
<td>22.230***</td>
<td>6.953</td>
<td>10.096**</td>
<td>3.707</td>
</tr>
<tr>
<td>LB^2(4)</td>
<td>39.908***</td>
<td>5.161</td>
<td>5.555</td>
<td>5.803</td>
</tr>
<tr>
<td>LB^2(8)</td>
<td>44.857***</td>
<td>12.314</td>
<td>7.673</td>
<td>7.429</td>
</tr>
<tr>
<td>LB^2(12)</td>
<td>46.872***</td>
<td>14.399</td>
<td>8.160</td>
<td>10.701</td>
</tr>
<tr>
<td>ARCH(4)</td>
<td>9.286***</td>
<td>1.175</td>
<td>1.317</td>
<td>1.261</td>
</tr>
<tr>
<td>ARCH(8)</td>
<td>4.861***</td>
<td>1.352</td>
<td>0.924</td>
<td>0.823</td>
</tr>
<tr>
<td>ARCH(12)</td>
<td>3.351***</td>
<td>1.087</td>
<td>0.629</td>
<td>0.801</td>
</tr>
<tr>
<td>ARCH^2(4)</td>
<td>3.121**</td>
<td>0.115</td>
<td>0.057</td>
<td>0.486</td>
</tr>
<tr>
<td>ARCH^2(8)</td>
<td>1.570</td>
<td>0.189</td>
<td>0.080</td>
<td>0.363</td>
</tr>
<tr>
<td>ARCH^2(12)</td>
<td>1.052</td>
<td>0.184</td>
<td>0.100</td>
<td>0.385</td>
</tr>
</tbody>
</table>

The levels of significances are presented with asterisks (*** = 1%, ** = 5%, * = 10%). Kurtosis is calculated as a standard excess measure. Ljung-Box (LB) and the test of ARCH-effects results are expressed as joint tests for different lags, and also for squared values (LB^2, ARCH^2).

The VIs' raw return series are highly correlated, as measured by unconditional correlations (Table 4) – Main European markets (0.865), as well as German-Swiss markets (0.850), being the most correlated. Surprisingly, UK- and US -indices are the least correlated (0.731).

Table 4: Unconditional Correlations of Returns

<table>
<thead>
<tr>
<th></th>
<th>VIX</th>
<th>VFTSE</th>
<th>VDAX</th>
<th>VSMI</th>
</tr>
</thead>
<tbody>
<tr>
<td>VIX</td>
<td>1.000</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>VFTSE</td>
<td>0.731</td>
<td>1.000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>VDAX</td>
<td>0.765</td>
<td>0.865</td>
<td>1.000</td>
<td></td>
</tr>
<tr>
<td>VSMI</td>
<td>0.693</td>
<td>0.818</td>
<td>0.850</td>
<td>1.000</td>
</tr>
</tbody>
</table>

These properties of data can be also seen visually. As a conclusion, appendix 2 shows that the overall contours of the log-level VIs are identical, and VI levels increase during the bear market and crisis, as in the beginning and the end of the time series. This negative correlation with the stock markets is well-diagnosed (Moran & Dash 2007, 97-98). The magnitudes of the changes differ from each other more, which can be seen more explicitly by studying the 1st difference returns of the same data in appendix 3.

Asymmetries of volatilities. Potential asymmetries in volatilities (table 5) affect the choice of MGARCH –specification. There are little but still existent traces of joint asymmetry in the Swiss index (7.279), and traces of negative magnitude
effect in the German index (0.430). Hence, MGARCH estimation might benefit from asymmetric model specification.

Table 5: Engle-Ng (1993) Sign and Size Bias Test

<table>
<thead>
<tr>
<th>Variable</th>
<th>VIX</th>
<th>VFTSE</th>
<th>VDAX</th>
<th>VSMI</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>1.057*** (0.187)</td>
<td>0.994*** (0.186)</td>
<td>1.003*** (0.192)</td>
<td>1.146*** (0.181)</td>
</tr>
<tr>
<td>$S_{t-1}$</td>
<td>-0.080 (0.270)</td>
<td>0.124 (0.276)</td>
<td>0.261 (0.230)</td>
<td>-0.172 (0.260)</td>
</tr>
<tr>
<td>$S_{t-1}^2$</td>
<td>0.151 (0.221)</td>
<td>0.304 (0.232)</td>
<td>0.430* (0.244)</td>
<td>0.238 (0.213)</td>
</tr>
<tr>
<td>$S_{t-1}^3$</td>
<td>0.107 (0.166)</td>
<td>0.136 (0.166)</td>
<td>0.052 (0.174)</td>
<td>0.078 (0.160)</td>
</tr>
<tr>
<td>Joint- $\chi^2$</td>
<td>3.498</td>
<td>3.740</td>
<td>3.403</td>
<td>7.279*</td>
</tr>
</tbody>
</table>

Standard errors are presented in the parentheses and the levels of significances with asterisks (** = 1%, ** = 5%, * = 10%). The coefficients coincide with those in equation (3.9), while constant does not have economic meaning. The joint test statistics follow Chi-squared distribution.

**Unit Root and Cointegration Tests.** Akaike’s Information Criteria (AIC) are used to choose the lags (table 6) for unit root tests.

Table 6: Univariate Akaike Information Criteria (AIC)

<table>
<thead>
<tr>
<th>Variable</th>
<th>Log-level</th>
<th>1st difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>VIX</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>VFTSE</td>
<td>2</td>
<td>0</td>
</tr>
<tr>
<td>VDAX</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>VSMI</td>
<td>1</td>
<td>0</td>
</tr>
</tbody>
</table>

AIC values refer to the optimal numbers of lags for different univariate time series on log and 1st difference return levels.

A volatility time series should be stable by definition (Black 2006; Moran & Dash 2007; Ahoniemi 2008), although Gonzalez- Perez & Guerrero (2009) disagree. Mean reversion of the time series is still confirmed by using unit root tests (table 7), as it has a crucial effect on the robustness of the results.

Table 7: Unit Root and Cointegration Tests

<table>
<thead>
<tr>
<th>Augmented Dickey-Fuller</th>
<th>Zivot-Andrews</th>
<th>Lee-Strazicich</th>
</tr>
</thead>
<tbody>
<tr>
<td>Log-level</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>VIX</td>
<td>-0.157</td>
<td>-2.297</td>
</tr>
<tr>
<td>VFTSE</td>
<td>-0.351</td>
<td>-2.782</td>
</tr>
<tr>
<td>VDAX</td>
<td>-0.340</td>
<td>-2.676</td>
</tr>
<tr>
<td>VSMI</td>
<td>-0.280</td>
<td>-2.871</td>
</tr>
<tr>
<td>1st difference</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Johansen Test for Cointegration

<table>
<thead>
<tr>
<th>Maximum Rank</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Eigenvalue</td>
<td>-</td>
<td>0.30456</td>
<td>0.13128</td>
<td>0.02946</td>
<td>0.02287</td>
</tr>
<tr>
<td>Trace Statistics</td>
<td>291.3035</td>
<td>101.3459</td>
<td>27.7394</td>
<td>12.1024</td>
<td>-</td>
</tr>
<tr>
<td>Crit. Value 5%</td>
<td>47.21</td>
<td>29.68</td>
<td>15.41</td>
<td>7.28</td>
<td>-</td>
</tr>
</tbody>
</table>

Standard errors are presented in the parentheses and the levels of significances with asterisks (** = 1%, ** = 5%). The model specifications are 1: No Intercept or Trend, 2: Intercept, 3: Intercept and Trend. ATTN! The information criteria used for Johansen test is aligned with that of the estimated model, in this case multivariate BIC with 1 lag (chapter 3 and table 9).
Augmented Dickey-Fuller (ADF) test cannot reject unit roots for log level data, which means the time series might not be stable. The potential log level unit roots might be caused by structural breaks in the time series. Zivot-Andrews (1992) and Lee-Strazicich (2004) one-break tests, and Lee-Strazicich multiple-break test (2003) are used for this reason. The Zivot-Andrews does not reveal anything new, but Lee-Strazicich (2004) shows that unit roots for log level data can be rejected on 5% level. The previous is known to be less reliable, and thus there is some, but not conclusive, evidence for rejecting Null-hypotheses of unit roots. All tests confirm that the 1st difference returns are stable, and thus free from unit roots. The results for the Lee-Strazicich (2003) multiple-break test are not relevant, and not reported. The same applies to the drift only specifications of Zivot –Andrews (1992) & Lee-Strazicich one break test (2004), the results of which are not likely to be economically meaningful (Lee & Strazicich 2004, 3). The Johansen test proves that the time series are not cointegrated, as the trace statistics are never smaller than the critical values. As an outcome, the VECM –terms are not needed in modeling.

Day of the week effects. Presence of the day of the week effects is not surprising considering that the underlying assets are stock indices. They are not reported explicitly before (e.g. Carr & Wu 2006) and are included here, although not important to the study. The day effects in all of the time series are verified by Newey-West regression. The results for the 1st difference returns are presented in table 8. Regressions using robust error terms, EGARCH –specification, and log values of data, lead to the same conclusion. Day effects seem to increase when moving from the US markets to Continental European markets – the strongest effects taking place at the most remote Swiss markets. Monday and Wednesday effects are found to be the most common. As a practical outcome it is reasonable to use weekly, instead of daily, time series.

Table 8: Day Effects of the 1st Difference VI Returns

<table>
<thead>
<tr>
<th>Variable</th>
<th>VIX</th>
<th>VFTSE</th>
<th>VDAX</th>
<th>VSMI</th>
</tr>
</thead>
<tbody>
<tr>
<td>Monday</td>
<td>2.280***</td>
<td>2.816***</td>
<td>3.121***</td>
<td>2.991***</td>
</tr>
<tr>
<td></td>
<td>(0.383)</td>
<td>(0.381)</td>
<td>(0.309)</td>
<td>(0.272)</td>
</tr>
<tr>
<td>Tuesday</td>
<td>0.016</td>
<td>0.855**</td>
<td>1.245***</td>
<td>1.038***</td>
</tr>
<tr>
<td></td>
<td>(0.393)</td>
<td>(0.380)</td>
<td>(0.299)</td>
<td>(0.272)</td>
</tr>
<tr>
<td>Wednesday</td>
<td>0.125</td>
<td>1.109***</td>
<td>1.080***</td>
<td>1.002***</td>
</tr>
<tr>
<td></td>
<td>(0.347)</td>
<td>(0.364)</td>
<td>(0.312)</td>
<td>(0.274)</td>
</tr>
<tr>
<td>Thursday</td>
<td>0.322</td>
<td>0.337</td>
<td>0.668**</td>
<td>1.179***</td>
</tr>
<tr>
<td></td>
<td>(0.352)</td>
<td>(0.398)</td>
<td>(0.305)</td>
<td>(0.268)</td>
</tr>
<tr>
<td>Constant</td>
<td>-0.543**</td>
<td>-1.025***</td>
<td>-1.224***</td>
<td>-1.243***</td>
</tr>
<tr>
<td></td>
<td>(0.245)</td>
<td>(0.272)</td>
<td>(0.216)</td>
<td>(0.194)</td>
</tr>
</tbody>
</table>

AIC 10 3 10 11

Standard errors are presented in parentheses, and the levels of significance with asterisks (** = 5%, * = 1%). The constant of the regression, represents the value for Friday, while the coefficients are for dummies (equation 3.2). The lags of Newey–West -specification are chosen using Akaike Information Criteria (AIC).
5. RESULTS

Information criteria. Multivariate Information Criteria (table 9), are used along the residuals diagnostics (tables 13 & 14) to choose an appropriate number of lags of the mean equation of the model. Different criteria speak for a different number of lags – inconsistent AIC is known to estimate too large models and BIC is known to be inefficient and varying case by case, although consistent and asymptotically correct. (Brooks 2008, 232-233, 235-239, 294-295; Verbeek 2008, 61-64, 299-302). The penalty terms of HQ lie between those of AIC and BIC, which means HQ also yields results lying between those two. The results speak for either 1-lag or 3-lag models, the former of which is chosen.

Table 9: Multivariate Information Criteria

<table>
<thead>
<tr>
<th>Lags</th>
<th>AIC</th>
<th>BIC</th>
<th>HQ</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>1</td>
<td>3</td>
<td></td>
</tr>
</tbody>
</table>

The mean equation lags based on Akaike’s (AIC), Schwartz’s Bayesian (BIC), and Hannan-Quinn’s (HQ) information criteria.

MGARCH – modeling. The main research endeavor is undertaken by using the VAR(1)-MVEGARCH –model, as specified in equations 3.21-3.28. Maximum Likelihood Estimation takes place jointly in one step, and includes the search of appropriate degrees of freedom for the used t-distribution of the time series. The model converges only when using 1st difference returns and when carefully choosing the algorithm parameters. The nonlinear, numerical maximization problem is solved by using the derivative-based optimization method of Broyden, Fletcher, Goldfarb & Shanno’s (Press et al. 1988). A derivative-free genetic algorithm, and its differential evolution variant, is used for preliminary estimation to improve the initial parameter values for more probable and faster convergence. Also, the genetic method decreases the probability of convergence into local optima. The number of these subiterations is high due to the use of exact-line search optimization. The inability of genetic estimation method to yield standard errors is not a crucial issue, as the method is only used for pre-estimations.

Results of modeling. The results of the ML estimation are presented in table 10. The coefficients of the lagged variables in the mean equation (research question: Q1) reveal most importantly spillovers between the markets. The statistically significant spillovers are all positive. Lagged values of the VIX have a positive effect on all the other VIIs at least at 5% level, the effect being between 0.117 (VDAX) and 0.153 (VFTSE). The lagged VFTSE does not have any significant effect on the VIX or the VDAX – only on VSMI (0.124). The lagged VDAX and VSMI do not have any significant effects on the other VIIs. Further, there are no bi-directional effects at all. The VIX, while not affected by other VIIs, is a central source of the spillovers. As stated, the VFTSE functions as a local centre by having an impact only on the VSMI. Finally, the VFTSE – VDAX and the VSMI – VDAX –pairs do not seem to have any spillover effects between each other [sic]. The sizes of the coefficients refer to the size of the impact. In this sense, the effects of the VIX on the other VIIs, and the effects of the VFTSE on the VSMI are far higher than the others, which are not only statistically insignificant but also extremely small. Each VI has a negative [sic], highly statistically significant, effect on itself, the VFTSE being the smallest (-0.181) and the VDAX the greatest (-0.271). The self-effects are large by size.
The volatilities of implied volatilities are high. Dynamic Conditional Volatilities (DCV) are presented in estimation. Finally, the optimal figure of degrees of freedom for in all of time series – persistent volatility component, is highly significantly different from zero, and the first one, the US (0.710) to processes contain varying correlations in the data, and the use of DCC-model instead of CCC-specification is founded. There are also modest but highly significant asymmetric effects, \( \gamma_1 \), in all of time series (Q3), and it is reasonable to choose EGARCH -specification for the first-step univariate estimation. Finally, the optimal figure of degrees of freedom for t-distribution for the return data series (equation 3.1) is highly significantly 7.553, which confirms the earlier leptokurtic non-normality assumption.

On the other, the variance equation (Q2-Q3) reveals that, by following Schwert, French & Stambaugh (1987), all the processes contain persistent volatility component, \( \beta_1 \), which have a tendency to decrease logically from the VIX of the US (0.710) to the more remote European VSMI (0.518) (Q2). The second DCC-coefficient, \( \theta_2 \), related to this persistent volatility component, is highly significantly different from zero, and the first one, \( \theta_1 \), related to the ARCH-component is also significant on the 5% level. There are time-varying correlations in the data, and the use of DCC-model instead of CCC-specification is founded. There are also modest but highly significant asymmetric effects, \( \gamma_1 \), in all of time series (Q3), and it is reasonable to choose EGARCH -specification for the first-step univariate estimation. Finally, the optimal figure of degrees of freedom for t-distribution for the return data series (equation 3.1) is highly significantly 7.553, which confirms the earlier leptokurtic non-normality assumption.

### Table 10: VAR(1)-MVEGARCH Results

<table>
<thead>
<tr>
<th></th>
<th>VIX</th>
<th>VFTSE</th>
<th>VDAX</th>
<th>VSMI</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Mean Equation</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Constant</td>
<td>-0.555**</td>
<td>-0.759***</td>
<td>-0.624***</td>
<td>-0.766***</td>
</tr>
<tr>
<td>( VIX(1) )</td>
<td>-0.236***</td>
<td>0.153**</td>
<td>0.117**</td>
<td>0.144***</td>
</tr>
<tr>
<td>( VFTSE(1) )</td>
<td>0.078</td>
<td>-0.181***</td>
<td>0.086</td>
<td>0.124***</td>
</tr>
<tr>
<td>( VDAX(1) )</td>
<td>0.018</td>
<td>-0.028</td>
<td>-0.276***</td>
<td>-0.015</td>
</tr>
<tr>
<td>( VSMI(1) )</td>
<td>-0.036</td>
<td>-0.019</td>
<td>-0.037</td>
<td>-0.262***</td>
</tr>
<tr>
<td><strong>Variance Equation</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Constant, ( \theta_0 )</td>
<td>1.177*</td>
<td>1.932***</td>
<td>1.936***</td>
<td>2.004***</td>
</tr>
<tr>
<td>( ARCH(1), \theta_1 )</td>
<td>0.310***</td>
<td>0.269***</td>
<td>0.184***</td>
<td>0.343***</td>
</tr>
<tr>
<td>( GARCH(1), \beta_1 )</td>
<td>0.710***</td>
<td>0.569***</td>
<td>0.563***</td>
<td>0.518***</td>
</tr>
<tr>
<td>Asymmetry, ( \gamma_1 )</td>
<td>-0.001**</td>
<td>-0.001***</td>
<td>-0.001***</td>
<td>-0.002***</td>
</tr>
<tr>
<td><strong>DCC and Distribution</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( DCC(1), \theta_1 )</td>
<td>0.034**</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( DCC(2), \theta_2 )</td>
<td>0.804***</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( Shape, t )</td>
<td>7.553***</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The levels of significances with asterisks (*** = 1%, ** = 5%, * = 10%).

On the other, the variance equation (Q2-Q3) reveals that, by following Schwert, French & Stambaugh (1987), all the processes contain persistent volatility component, \( \beta_1 \), which have a tendency to decrease logically from the VIX of the US (0.710) to the more remote European VSMI (0.518) (Q2). The second DCC-coefficient, \( \theta_2 \), related to this persistent volatility component, is highly significantly different from zero, and the first one, \( \theta_1 \), related to the ARCH-component is also significant on the 5% level. There are time-varying correlations in the data, and the use of DCC-model instead of CCC-specification is founded. There are also modest but highly significant asymmetric effects, \( \gamma_1 \), in all of time series (Q3), and it is reasonable to choose EGARCH -specification for the first-step univariate estimation. Finally, the optimal figure of degrees of freedom for t-distribution for the return data series (equation 3.1) is highly significantly 7.553, which confirms the earlier leptokurtic non-normality assumption.

### Table 11: Dynamic Conditional Volatilities (DCV)

<table>
<thead>
<tr>
<th></th>
<th>VIX</th>
<th>VFTSE</th>
<th>VDAX</th>
<th>VSMI</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Mean</strong></td>
<td>10.662</td>
<td>11.412</td>
<td>10.099</td>
<td>9.880</td>
</tr>
<tr>
<td><strong>Median</strong></td>
<td>10.349</td>
<td>11.082</td>
<td>9.924</td>
<td>9.500</td>
</tr>
<tr>
<td><strong>Standard Deviation</strong></td>
<td>1.485</td>
<td>1.282</td>
<td>0.903</td>
<td>1.433</td>
</tr>
<tr>
<td><strong>Minimum</strong></td>
<td>8.504</td>
<td>8.619</td>
<td>5.172</td>
<td>5.884</td>
</tr>
<tr>
<td><strong>Maximum</strong></td>
<td>22.970</td>
<td>18.741</td>
<td>14.850</td>
<td>17.867</td>
</tr>
<tr>
<td><strong>1st Difference</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Mean</strong></td>
<td>-0.033</td>
<td>-0.028</td>
<td>-0.020</td>
<td>-0.041</td>
</tr>
<tr>
<td><strong>Median</strong></td>
<td>-1.78</td>
<td>-1.072</td>
<td>-0.717</td>
<td>-1.212</td>
</tr>
<tr>
<td><strong>Skewness</strong></td>
<td>1.492***</td>
<td>1.307***</td>
<td>-0.593***</td>
<td>0.108</td>
</tr>
<tr>
<td><strong>Kurtosis</strong></td>
<td>8.041***</td>
<td>5.707***</td>
<td>11.678***</td>
<td>6.316***</td>
</tr>
<tr>
<td><strong>Jarque-Bera</strong></td>
<td>1501.834***</td>
<td>804.520***</td>
<td>2813.167***</td>
<td>815.351***</td>
</tr>
</tbody>
</table>

The levels of significances are expressed with asterisks (*** = 1%, ** = 5%, * = 10%).

Kurtosis refers to the excess measure.

Dynamic Conditional Volatilities (DCV) are presented in appendix 4, and table 11 shows their main properties (Q2). The volatilities of implied volatilities are high – incidentally, higher than the respective stock index volatilities (Carr &

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Wu 2006, 19-20; Moran & Dash 2007, 97; Szado 2009, 10) – the means and standard deviations are decreasing all the way from the USA to the more remote European markets, and the largest peaks are consistently upwards. These deviations from the mean are abrupt and extreme, but of short term – even during the subprime crisis only the VIX has a longer term elevated DCC levels. This means that the markets are fast and efficient in adjusting to new information. Because of the mean reversion of the time series, the first differences (table 11) are also of interest. All the distributions are leptokurtic, non-normal. Excluding the VSMI and the negatively skewed VDAX, the rest are positively skewed, which means the positive changes are larger in general than the negative ones. Then, the negative means and medians differ quite a bit, and the standard deviations are extensive.

Dynamic Conditional Correlations (DCC) of appendix 5 are high and extremely stable (Q2), as also presented in table 12. The means and medians coincide, and the standard deviations of are small. 1st differences confirm non-normal, leptokurtic behavior for all time series. All statistically significant measures of skew are negative. The changes of the mean in the correlations are time wise longer than those of DCVs, and the ends of those peaks revert back to their means smoothly. New information arriving in a market leads to an abrupt change in the DCC, as it takes some time for the information to spillover. The spillover then gradually mitigates the change in DCC between the markets. These changes in the correlations in the face of the crises are still surprisingly small, although the correlations seem to have a tendency to change in the context of extensive shocks. Also, most correlations have a tendency to fall more than to grow, in the face of subprime crises, excluding the VFTSE-VDAX, which is exceptional by many ways. The Dynamic Conditional Correlations 2 – table of appendix 5 show that regional and local European overall correlations are higher than the global ones with VIX.

Table 12: Dynamic Conditional Correlations (DCC)

<table>
<thead>
<tr>
<th>CORRELATIONS</th>
<th>VIX-VFTSE</th>
<th>VIX-VDAX</th>
<th>VIX-VSMI</th>
<th>VFTSE-VDAX</th>
<th>VFTSE-VSMI</th>
<th>VDAX-VSMI</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>0.734</td>
<td>0.767</td>
<td>0.698</td>
<td>0.866</td>
<td>0.821</td>
<td>0.853</td>
</tr>
<tr>
<td>Median</td>
<td>0.735</td>
<td>0.767</td>
<td>0.696</td>
<td>0.866</td>
<td>0.820</td>
<td>0.853</td>
</tr>
<tr>
<td>Standard Deviation</td>
<td>0.031</td>
<td>0.025</td>
<td>0.034</td>
<td>0.0173</td>
<td>0.0260</td>
<td>0.020</td>
</tr>
<tr>
<td>Minimum</td>
<td>0.573</td>
<td>0.682</td>
<td>0.534</td>
<td>0.813</td>
<td>0.647</td>
<td>0.740</td>
</tr>
<tr>
<td>Maximum</td>
<td>0.839</td>
<td>0.868</td>
<td>0.818</td>
<td>0.924</td>
<td>0.888</td>
<td>0.909</td>
</tr>
</tbody>
</table>

1st Difference

| Mean         | -0.005    | -0.002   | 0.000    | 0.000      | -0.000     | 0.000    |
| Median       | -0.091    | -0.120   | -0.086   | -0.005     | -0.026     | -0.001   |
| Standard Deviation | 2.334   | 1.909    | 2.669    | 1.064      | 1.739      | 1.212    |
| Skewness     | -0.499*** | 0.029    | -0.536***| -0.057     | -4.877***   | -0.927***|
| Jarque-Bera  | 5491.053***| 2059.021***| 4256.6240***| 1020.748***| 109012.585***| 3216.328***|

The levels of significances are expressed with asterisks (** *= 1%, ** *= 5%, * *= 10%). Kurtosis refers to the excess measure.

Residual diagnostics. It is essential to study the properties of the MGARCH – residuals, as they convey information about the fit and robustness of the model. They affect the choice of the final model, as the chase after the most robust, efficient and parsimonious model is cyclical and repetitive by nature. Still, residual testing is not completely reliable as is, because many of the asymptotic properties of the DCC-model used are unknown, and formal critical values may not be available for the test statistics (Bauwens et al. 2006, 102). Thus, the final decisions about the most appropriate model are made based on all information available. The results (table 13) show that all the time series are still positively skewed, leptokurtic, and the Jarque-Bera Null-hypotheses of normality can be rejected, as was predicted by theory (ibid., 96). Autocorrelations and heteroscedasticities, measured by Ljung-Box Portmanteau tests, and different ARCH-effects are non-existent but may suffer from asymptotical impreciseness.
Table 13: Residuals Diagnostics I

<table>
<thead>
<tr>
<th>Variable</th>
<th>VIX</th>
<th>VFTSE</th>
<th>VDAX</th>
<th>VSMI</th>
</tr>
</thead>
<tbody>
<tr>
<td>Observations</td>
<td>491</td>
<td>491</td>
<td>491</td>
<td>491</td>
</tr>
<tr>
<td>Minimum</td>
<td>-2.998</td>
<td>-3.046</td>
<td>-3.597</td>
<td>-3.166</td>
</tr>
<tr>
<td>Maximum</td>
<td>5.470</td>
<td>4.520</td>
<td>4.828</td>
<td>4.207</td>
</tr>
<tr>
<td>Mean</td>
<td>0.071</td>
<td>0.072</td>
<td>0.067</td>
<td>0.080</td>
</tr>
<tr>
<td>Standard Deviation</td>
<td>1.012</td>
<td>1.030</td>
<td>1.039</td>
<td>1.036</td>
</tr>
<tr>
<td>Skewness</td>
<td>0.740***</td>
<td>0.661***</td>
<td>0.734***</td>
<td>0.447***</td>
</tr>
<tr>
<td>Kurtosis</td>
<td>1.955***</td>
<td>1.501***</td>
<td>2.265***</td>
<td>1.095***</td>
</tr>
<tr>
<td>Jarque-Bera</td>
<td>123.041***</td>
<td>81.926***</td>
<td>149.040***</td>
<td>40.862***</td>
</tr>
<tr>
<td>LB(4)</td>
<td>4.333</td>
<td>4.543</td>
<td>3.994</td>
<td>3.863</td>
</tr>
<tr>
<td>LB(8)</td>
<td>12.579</td>
<td>10.487</td>
<td>6.882</td>
<td>10.430</td>
</tr>
<tr>
<td>LB(12)</td>
<td>16.473</td>
<td>12.120</td>
<td>13.566</td>
<td>12.639</td>
</tr>
<tr>
<td>LB^2(4)</td>
<td>0.628</td>
<td>0.804</td>
<td>3.539</td>
<td>1.079</td>
</tr>
<tr>
<td>LB^2(8)</td>
<td>4.396</td>
<td>7.374</td>
<td>5.083</td>
<td>4.266</td>
</tr>
<tr>
<td>ARCH(4)</td>
<td>0.160</td>
<td>0.194</td>
<td>0.866</td>
<td>0.271</td>
</tr>
<tr>
<td>ARCH(8)</td>
<td>0.507</td>
<td>0.831</td>
<td>0.617</td>
<td>0.545</td>
</tr>
<tr>
<td>ARCH(12)</td>
<td>0.484</td>
<td>0.680</td>
<td>0.432</td>
<td>0.572</td>
</tr>
<tr>
<td>ARCH^2(4)</td>
<td>0.026</td>
<td>0.160</td>
<td>0.100</td>
<td>0.134</td>
</tr>
<tr>
<td>ARCH^2(8)</td>
<td>0.042</td>
<td>0.131</td>
<td>0.087</td>
<td>0.133</td>
</tr>
<tr>
<td>ARCH^2(12)</td>
<td>0.047</td>
<td>0.139</td>
<td>0.108</td>
<td>0.153</td>
</tr>
</tbody>
</table>

The levels of significances are presented with asterisks (***, ** = 1%, * = 10%). Kurtosis is calculated as a standard excess measure. Ljung-Box (LB) and the test of ARCH-effects results are expressed as joint tests for different lags, and also for squared values (LB^2 & ARCH^2).

The residuals of one-mean-equation-lag model (suggested by BIC) are more robust compared to those of three-lag model (suggested by AIC and HQ). The Ljung-Box and ARCH-effect results are slightly different but not generally better for either of the models. As an outcome, the one-lag model is chosen that also leads to more meaningful modeling results presented in table 10. Finally, there is no asymmetry left in the residuals tested (table 14), measured by Engle-Ng-test.

Table 14: Residuals Diagnostics II

<table>
<thead>
<tr>
<th>Variable</th>
<th>VIX</th>
<th>VFTSE</th>
<th>VDAX</th>
<th>VSMI</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>1.152***</td>
<td>1.056***</td>
<td>1.193***</td>
<td>1.222***</td>
</tr>
<tr>
<td>S_{t-1}</td>
<td>-0.250</td>
<td>0.251</td>
<td>0.112</td>
<td>-0.134</td>
</tr>
<tr>
<td>S_{t-1}^2</td>
<td>-0.008</td>
<td>0.407</td>
<td>0.401</td>
<td>0.208</td>
</tr>
<tr>
<td>S_{t-1}^2</td>
<td>-0.003</td>
<td>0.067</td>
<td>-0.059</td>
<td>-0.006</td>
</tr>
<tr>
<td>Joint-χ^2</td>
<td>1.655</td>
<td>3.149</td>
<td>2.767</td>
<td>3.484</td>
</tr>
</tbody>
</table>

Standard errors are presented in the parentheses and the levels of significances with asterisks (***, ** = 1%, * = 10%). The coefficients coincide with those in equation (3.9), while constant does not have economic meaning. The joint test statistics follow Chi^2 -distribution.
6. DISCUSSION

The results speak for wider international systemic behavior of the underlying Western stock markets. This increasingly complex phenomenon is often reported as a consequence of increased financial globalization. The results reveal only unidirectional volatility spillovers (Q1) between the VIs, while the American VIX seems to be the central source of the volatility spillovers, while the VFTSE behaves as a local source with respect to the VSMI. These findings, combined with the fact that the VDAX does not have spillover with the VFTSE and VSMI, describe the nature of the spillovers very well. While geographical (proximity), cultural and shared real economy factors seem to contribute higher correlation and shared market behavior, the financial relationships seem to contribute spillovers. By interpreting VIs as indicators of the markets, the UK and Germany have important trade relationships with the USA and it is natural that spillovers exist between these markets. The size and global importance might explain why the spillovers are solely outwards from the US, as also found by Theodossiou & Lee (1993). The lack of spillovers between the highly correlated main European markets can be explained as a sign of the unified nature of regional markets. The markets evolve so closely with each other that there are no follow-ups at the moment t-1. The daily data would be likely to reveal spillovers, but as stated could suffer from the effects of non-synchronous trading. The high correlation between the indices rules out the possibility that the markets were not related at all. The unidirectional spillovers from the VIX confirm the central position of the US market as a source of the new market information globally. In this framework, the Swiss market seems to be a local player closely integrated (correlation) with the German market, but in the UK’s tow (spillovers). As a summary, the results show that volatility spillovers do exist.

The VAR-MVEGARCH specification reveals that the volatilities and correlations include dynamic conditional effects (Q2). The behaviors of the DCV levels are not surprising per se, as the volatilities of VIs, i.e. volatilities of volatilities, have traditionally been high compared to the volatilities of their respective stock indices (Moran & Dash 2007, 97). Still, VIs essentially serve as a fear gauge of the market, not only indicating the stock market plummets, but also indicating the overall confidence of the market (Moran & Dash 2007, 97; Whaley 2000). As a consequence, the changes are short term and abrupt, portraying sentiments of emerging new information related to the underlying index. In terms of hedging, the deviations from the mean reversion target are short-term on the 1st difference return level, and thus appropriate for catastrophe hedging, where large and sudden changes take place on the market. Still, it must be remembered that the indices also portray longer term implied volatility levels, as can be seen by studying the appendix 2. Further, the markets in question are highly correlated. Short-term fluctuations still exist, as the spillovers do not correct the volatility changes immediately, but smoothly over time. Together with earlier spillover findings, this confirms that volatility spillovers are essentially information transmission (Stoll & Whaley 1990; Chan, Chan & Karolyi 1991; French & Roll 1986).

In terms of diversifying the volatility investment, investing even in one single VI may work, as high correlations limit the benefits of diversification – especially if perfect short-term hedging is not important. The problem is that although VIs offer good catastrophe hedging i.e. to compensate for large negative shocks of the underlying stock index (Moran & Dash 2007, 104), even small drops in the correlations in the face of the crisis may become crucial, when the original shock is extensive. Thus, if the local VI truly represents the risk level, it is likely to offer better protection for short-term fluctuations than the foreign VIs. There are exceptions to the rule such as between the VFTSE-VDAX and the VDAX-VSMI. These VIs are part of the same regional market area, highly correlated, and the problem causing spillovers are few. All in all, this might explain why the highest overall correlations are found in the highly integrated European market area, while the formal EU itself is not the essential factor.
There are small but statistically significant volatility asymmetries captured by the model used (Q3) in every single index. This means that the nature of past news is of importance. The preliminary finding for the VIX by Carr & Wu (2006, 18-19) is confirmed and extended. This finding is important in the economic sense as it shows that this asymmetry seems to be a common property of VIs, although explaining this property further lies beyond this study. The asymmetries in the volatilities of implied volatilities could be caused by the leverage effects of the underlying stock indices (Black 1976), or by some other reasons (Bekaert & Wu 2000; Black 1976; Campbell & Hentschel 1992; Campbell & Kyle 1993; Haugen et al. 1991).

Other findings. In the long term, the stability of volatility (and correlations) as a measure is confirmed. In any case, VIs are not free from biases either, and might explain at least some of the bias Jiang & Tian (2007) have found. In the shorter term, the self-effects of the VIs, represented by the 1st lags in the mean equation, are highly significantly negative and relatively large. In other words, the historic values have a geometrically decreasing negative autocorrelation relationship with the new events that is a natural finding. The distributions of the 1st difference returns are leptokurtic with thick tails and are thus non-normal. Finally, the VIs contain day effects, although not traded as is. Besides trading (Carr & Wu 2006), the day effects can most likely be traced back to the underlying stock indices.

6.1. Robustness of the results

There are two issues, related to the robustness of the results, repeatedly raised in the literature, a) the misspecified model (Hoover & Siegler 2008a), and b) the difficulty of strict division between statistical and economic interpretation of the data (Hoover & Siegler 2008b). The latter states that statistical hypotheses are not always either true or false (Leamer 2004) leading to reliability problems.

A misspecified model could lead to completely biased test statistics, which in turn would confirm nonsense results. Feinstein & Thomas (2002) speak about this aspect more closely. They remind us that if there is no symmetry of errors (or the study is based on a biased model without asymptotic properties), the results can be seriously biased without the researcher noticing it. This leads us to the capabilities of the researcher, as the only way around the problem, is the researchers ability to evaluate the appropriateness of the models used (also Horowitz 2004). Hence, the economic reasoning of the data must be observed more widely, and it must depend on the situation. The original figures are made available, so that the reader can make his or her own conclusions. In the context of the model, Engle’s (2002) DCC-model allows only real number specifications for the dynamics of the correlations. It is possible that the model does not reveal, misleadingly specifies, the dynamics of correlations (Bauwens et al. 2006). The results also show a small but statistically significant asymmetry for all the time series, not found in the descriptive statistics (table 7). This finding may reflect the fact that the model does not fit the data perfectly. VARMA-GARCH does converge, so it cannot be used as an alternative. In practice, the most important decision is related to the choice of lags for the mean equation of the MVEGARCH -model.

Difference between statistical and economic interpretation. Errors are indispensable in economic research, while Kirzner (1999, 125-131) pays attention to “correctly calculated imperfection”, which means reaching appropriate results with respect to the relevant search costs. Better results would increase error. Ethridge (2004, 47-49) explains that the error is unavoidable, but non-harmful if its nature and role in the study is correctly understood and tested. Hoover and Siegler (2008a) notice that economic interpretation is subjective by nature to some extent. Berg (2004) offers as a solution, that Null- and Alternative hypotheses, would be augmented with No-decision hypothesis, which would help in those tricky cases, when conclusions can not be drawn (e.g. if economic and statistical statistics do not speak
the same language.) Horowitz (2004) highlights the importance of what “constitutes good practice in handling random sampling errors” and concludes that testing is not always reliable anyway, and that sometimes only the existence of a phenomenon is enough for conclusions, while significance on certain specified level is not an essential property. As long as the problem is poorly specified, the discussion does not lead further. Hence, lots of attention is paid to the problem setting and interpretation of the results, and the ethics behind the interpretation (Wooldridge 2004). The economic and statistical results of the study are largely aligned, but the statistical results presented in the tables and figures are kept separate from the economic reasoning. Hence, the reader can draw his or her own conclusions from the results.

Volatility as a theoretical asset. All conclusions about true volatilities between the markets must be drawn remembering the fact that the VIs are treated here as assets. It is also reasonable to ask, to what degree the results can be applied to practical management of volatility assets? Investing in VIs takes place through derivatives, the time series of which may behave differently from but are related to those of underlying VIs (Nossman & Wilhelmsson 2008), or they are not possible at all. This causes evident problems, not easily answered, highlighting the importance of treating VIs as individual assets. Because of varying properties and poor availability of the data accessible for multivariate studies, it is most reasonable to use VIs themselves, not the derivatives, as data. This approach has been used in the context of VI – research also earlier – most notably by Black (2006). It is still of most importance that the limitations of such results are remembered.

Other issues. The modeling took place by using first difference returns and weekly values. Use of the returns lead inevitably to loss of information in the results. Still, the models do not converge when log-level values are used, and as volatility is treated as an asset, it is reasonable to study returns of the asset instead of VI levels as is. The use of weekly values is founded, in order to avoid the effects of non-synchronous trading, although it may hide some of the short-term spillovers. The sample size is large enough for reliable GARCH estimation, even using weekly values through the time window. Another topical data-related question relates to the questionable mean reversion of the data series, revealed by the preliminary research. It is theoretically unfounded to assume a volatility series containing unit roots – the view supported by the most sophisticated unit root test. On the 1st difference returns level, the hypotheses of unit roots are rejected by all tests used. As the time series are not cointegrated, the VECM-parameters are unnecessary.

6.2. Further Research

The field studied here is developing rapidly, and new assumptions and approaches for modeling are established constantly. These include modeling options on the VIs by using jump-diffusion models (Sepp 2008; Carr & Wu 2006, 18) and introducing a whole new class of information derivatives (Soklakov 2008). Therefore new approaches will be needed in establishing sound research settings. In the context of this study, the same methodology could be applied to study VIs in one specific area, such as the Euro currency area, in order to compare the results with respect to this one. In the future when time series allow, the same methodology could be used to study developing and Asian markets, many of which have recently introduced their own VIs, and which (more often than not) are specified similar to that of the VIX. It would also be interesting to conduct similar studies on derivatives written on different VIs, as soon as data become available. Such studies would shed light on how investing in and trading of implied volatilities through derivatives (e.g. Black 2006; Sepp 2008), is connected between different markets in practice, and also reveal differences between the markets of VIs and their respective derivatives. Finally, the reasons for the day effects could be studied: a) the impact of trading is theoretically interesting, as the derivatives based volatilities are generally considered a relatively stable and reliable measure of risk (e.g. Hull 2009), and b) the impact of underlying stock indices and their relationships to the leverage effects would be worth studying.
6.3. Conclusions

All in all, the volatility indices of Western developed markets, are highly correlated, and carry the unified message of uncertainty in the respective markets in the long run. Even more so on the regional and local level. Theoretical investments in the several volatility indices would yield only limited diversification benefits in the Western world. Depending on the case, it may then be possible to use only one or a few VIs as a hedging tool for an international developed market portfolio, on the condition that the spillover relationships – affected by the financial market relations – between the markets in question are known. In the light of the findings of this study, it is not surprising that the VIX is currently the most popular volatility index among investors. From the variance-covariance (or correlation) matrix standpoint, there are relatively small, but significant dynamic effects over time. In the short run, the dynamic relationships cause temporary differences, and as VI –based hedging strives especially for crash protection, hedging using the local market index would bring the greatest efficiency. Otherwise, the changing (most often decreasing) correlations in the face of crisis (as during the subprime crisis) may jeopardize some of the hedge. Because of the fast changes in the VIs volatility, it is not necessarily so that short delay in liquidizing would fix the problem. This very fact applies also to the use of VIs as the market indicator. Hence, it is crucial to use the local VI signaling the short-term uncertainty of the market, and variance swap levels. Naturally, this is of utmost importance, if VIs are used as market timing tools. Finally, there are information transmissions between the markets – spillovers running unidirectionally from the US VIX to the other VIs.
REFERENCES

Primary and secondary sources are not separated in the list.


APPENDICES

APPENDIX 1: Research Setting

APPENDIX 2: Volatility Indices (VI) – Log-levels
APPENDIX 3: Volatility Indices (VI) – 1st Difference Returns

1st Difference Returns of Volatility Indices

APPENDIX 4: Dynamic Conditional Volatilities (DCV)

Conditional Volatilities
APPENDIX 5: Dynamic Conditional Correlations (DCC)
Although Arak (2006) disagrees.

Moran & Dash (2007) also admit that the derivatives do not completely follow the spot VIX though.

Still, the data series chosen for this study all follow VIX—specification, and yield implied volatilities for the next 30 days.

Alternative hypothesis of Jarque-Bera measure only confirms that the distribution of the time series in question differs from normality. A formal normality test (e.g. Kolmogorov-Smirnov) is not studied, as the normality in the residuals diagnostics is likely to be inessential (Bauwens et al. 2006, 96).


See Westerlund & Edgerton 2007 and Westerlund 2009 for further information.

Notice that all EU-area indices are products of EURONEXT. VDAX follows its new specification.

The beginning date differs slightly from 2000/01/04 for daily data series to 2000/01/10 for weekly analysis.

By January 2010 the Fed had not increased its rates (Federal Reserve System 2010).

One interesting exception being the volatility asymmetry for VIX index, which is statistically significant only at the 5 % level. Still, all the other indices being statistically significant at 1% level, and Carr & Wu (2006, 18-19) having confirmed this the asymmetry for VIX, this finding may carry economic significance.