Credit Default Swaps

-new regulations and conversion problematics

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Abstract:

During the financial crisis, focus on Credit Default Swaps has increased and their contribution to the crisis has been widely discussed. As a result regulators introduced the big bang protocol during 2009 to regulate the CDS market. A major change is that of fixed spreads, which means conversions of running spreads needs to be made. However the methodology of conversion proposed by ISDA might include problems if used outside its proposed context. The purpose of the present study is to underline what changes has been made to the CDS market as well as to compare two methodologies, namely a fixed hazard rate model and a piecewise constant hazard rate model, for conversion.
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1. Introduction
During the recent decade the growth of the credit derivatives market has been tremendous. A big contributor to the growth is the Credit default swap (CDS) contracts, which by the beginning of 2009 had an estimated outstanding notional amount of almost 35 trillion dollars (ISDA; BIS). The increase of the market has to a large extent been possible due to deregulations of the market in 2000 as a result of the PWG report from 1999. CDS contracts have recently widely debated due to its major importance in the recent financial crisis. Due to the nature of the market, which is an over-the-counter market, it’s been difficult to monitor the market of CDS contracts, which indeed made the crisis spread rapidly. In April 2009 new regulations of CDS contracts were introduced with the purpose to increase standardization of CDS’s so that clearinghouses could be introduced in the market and to decrease the notional amount outstanding (Markit 2009a). The introduction of clearing houses and decreasing notional amounts outstanding will make the market easier to monitor. One of the major changes was the introduction of fixed coupon payments, instead of running spreads, with an upfront payment when the contract is entered. Along with the regulatory introduction a conversion method was proposed for conversion of traditionally CDS spreads, or running spreads, to upfront payments. In 2009 problems with the proposed conversion method were brought forward in a report from Beumee et. al. (2009). The report pointed out that the proposed conversion method contained inconsistencies regarding default probabilities, i.e. inconsistent in term structure, as well as it returns ambiguous upfront payment results compared to a consistent method that has a consistent term structure. The importance of this report is major, since many CDS contracts are still quoted as running spreads, instead of upfront payments. Hence usage of the proposed conversion method may lead to continuous inconsistencies, as well as ambiguous upfront payments, if used by investors.

The purpose of this thesis is to present the regulatory changes that have been introduced in the recent regulations, also called the “bang” protocols, as well as replicate the example made by Beumee et. al. (2009) on a larger sample of CDS’s.

The thesis is structure as follows: In section 2 an introduction and the methodology of CDS contracts are given. The third section introduces a CDS model and explains how the price of a CDS contract is set, as well as including a discussion regarding default probabilities. Section 4 explains the impact that CDS’s had in the recent financial crises. The fifth section explains the regulatory changes that have been made through the “bang” protocols and describes the findings of the report by Beumee

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1 International Swaps and Derivatives Association
2 Bank for International Settlements
et. al. (2009) and the problematic of the introduced conversion method. Section 6 describes the methodology approach and section 7 and 8 presents the results and the conclusions.
2. Credit default swaps

In this section CDS contracts are introduced as well as the mechanics of a CDS contract will be explained. Also the size of the market is presented.

Credit Default Swap (CDS) contracts are a way to make it possible to trade credit risk (O’kane 2003). The contract is a derivative contract, traded in the over-the-counter (OTC) market, which works similar to insurance contracts (Duffie 1998). A CDS contract is established by two parties, namely a “holder of contract” (protection buyer) and a seller of contract (protection seller). The contract gives the protection buyer protection on defaultable bonds, if a possible credit event, such as default or restructuring\(^3\), occurs on the company that issued the bond (Hull & White 2000a). The issuing company is also referred to as the reference entity. The seller of the contract has the responsibility to compensate the buyer with the loss made on the face value of the bond issued by the reference entity, if a credit event occurs. This compensation payment is referred to as the default leg. To receive the insurance on the bond given by the CDS contract, the protection buyer has to pay a premium to the protection seller, which is known as the premium leg, and consists of CDS spread payments. CDS contracts are active between the effective date, which is the day after the trade is made, and the maturity date of the contract. CDS payments are normally made on a quarterly basis and since 2003, convention has been to set payment and maturity dates to 20 March, 20 June, 20 September and 20 December due to inspiration by the International Money Market (IMM) dates (O’Kane 2009). However it is important to stress that CDS contracts are made between two parties in the over-the-counter market and therefore has not been standardized, which implies that differences in contracts have existed even though maturity dates has been standardized. An illustration of how the CDS contract works is found in figure 1.

\[\text{CDS BUYER} \quad \text{Owner of the bond issued by the reference entity.}\]

\[\text{CDS SELLER} \quad \text{Premium Leg. Payments of the CDS spread times notional amount on a quarterly basis up to default time or maturity, } T, \text{ whichever occurs first.}\]

\[\text{Reference Entity} \quad \text{The issuer of the bond.}\]

\[\text{Default Leg. Payment of face value - recovery if a credit event occurs before time } T..\]

Figure 1: The mechanics of a CDS contract.

\(^3\) Restructuring are changes in the legal, ownership and other structural changes in a company. In this context restructuring means changes in the debt obligations of the reference entity.
Since the default leg includes payment of the face value of the bond less the recovery⁴ obtained from the reference entity, a problem evolves due to the fact that the recovery is unknown. This is settled in two ways, either by a so called physical settlement or by a cash settlement. The physical settlement simply means that the CDS buyer delivers the bond to the CDS seller, which pays the face value of the delivered bond. Cash settlement means that the CDS seller pays the lost amount of the bond, i.e. face value minus recovery, where the recovery is found out by CDS dealers by polling or an auction process (Lando 2004; O’kane 2009; IMF 2006). In figure 1 it says that the CDS buyer holds the bond issued by the reference entity. This was how CDS contracts worked as they were introduced. However, over time it has been, and still is, possible to enter a CDS contract without actually owning bonds issued by the reference entity. This is also referred to as taking a naked CDS contract position. Hence by taking a naked position it’s possible to speculate in companies’ credit worthiness and default probabilities. (O’kane 2009)

There are several credit events that may trigger a CDS contract. The most common credit events are bankruptcy⁵, restructuring⁶, failure to pay⁷, obligation default⁸, obligation acceleration⁹ and repudiation/moratorium¹⁰ (O’kane 2009). In the case of restructuring there are different clauses that restrict the CDS buyer regarding which bonds that is possible to deliver. There are several different clauses that have been introduced, namely no-restructuring, modified-restructuring, modified-modified restructuring and full-restructuring. In the case of full-restructuring bonds with a remaining maturity of up to 30 years are allowed to be delivered. For the other clauses the maximum allowed remaining maturity on deliverable bonds are: 30 months for the modified-restructuring and 60 months (for restructured bonds) and 30 months (for other bonds) in the case of modified-modified-restructuring. In the case of no-restructuring, restructuring is not included as a credit event. As we will see later on, these clauses have been determined for different regions in the “bang” protocols. (BIS 2005)

Since 2001 International Swaps and Derivatives Association (ISDA) and since the end of 2004 Bank of International Settlement (BIS) has estimated the size of the CDS market. However there exist no good statistics for this market since the contracts are traded OTC. Hence the size of the market reported are only estimates of the market based on surveys. Nevertheless these estimates give an indication about the trend of the CDS market. The estimates show that during the recent years the CDS market

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⁴ Recovery is the amount that the reference entity is able repay when default occurs.
⁵ Bankruptcy means that the issuing firm becomes insolvent or is unable to pay its debts.
⁶ For explanation of restructuring see footnote 3.
⁷ Failure to pay occurs when the issuing firm is unable to pay when debt payment is due.
⁸ Obligation default is when a bond becomes due and payable before default.
⁹ Obligation acceleration means that the bond has been accelerated and become due prior to maturity.
¹⁰ Repudiation/moratorium means that the issuer of the bond rejects the validity of the obligations.
has increased substantially. As can be seen in figure 2 the growth in the CDS market for the past years has increased enormously with a peak of approximately 60 trillion dollars in notional amount outstanding by the end of 2007. However the figure shows the notional amounts outstanding for all types of CDS contracts, both single-name and multi-name CDS contracts, not only those written on single firms, i.e. for instance CDS contracts on iTraxx indices etc. Also important to notice is that the notional amount outstanding is in gross values and not in net notional values. This means that if contracts were netted, i.e. net notional values, the total amounts outstanding would be smaller since there are contracts that offset each other in the market.

Figure 2. Estimated size of the CDS market in gross notional amount outstanding, including both single-name and multi-name CDS contracts. (Source ISDA; BIS)
3. A CDS pricing model

In this section a CDS pricing model will be presented. This includes calculations of the premium leg and the default leg as presented in section 2. This section also includes a presentation of default probabilities and different ways of estimating these for a CDS pricing model. This section also presents the concepts of bootstrapping and upfront payments.

A CDS contract does not have a regular price denominated in a specific amount of currency, such as for instance stocks. Instead the pricing of a CDS contract consists of finding a CDS running spread to be paid by the buyer to the seller during the time period of the contract, i.e. either until a credit event occurs or to the maturity of the CDS contract. The fair CDS spread is the spread which equals the expected discounted value of the premium leg and the expected discounted default leg at the trade date (O’kane 2003; Hull & White 2000b). Hence, to calculate the CDS spread (or price) we need to calculate the discounted values of the premium- and default leg. The pricing model presented here is based on the assumption that interest rates and default time are independent (O’kane 2009).

3.1 Premium leg calculation

As mentioned in section 2 the premium leg is the premium that the CDS buyer has to pay to the seller over the time period of the contract. We assume these payments are made each quarter and that the time zero day is located on one of these coupon payment days. We therefore know that the premium leg is the expected present value of the payments, which is found by discounting all payments from today until a credit event occurs or to maturity of the CDS contract, whichever happens first. Hence we have the following expression, for one currency unit as the notional amount, i.e. one dollar, one Euro etc. (O’kane 2009):

\[ PV_{PL} (T) = S_0 (T) \* \sum_{n=1}^{N} \Delta (t_{n-1}, t_n) \* Surv(0, t_n) \* e^{-\int_0^{t_n} r(s) ds} \]

Equation 1

Where

\( PV_{PL} (T) \) = Present value of premium leg at t=0, i.e. today
\( S_0 (T) \) = CDS spread at time point 0, i.e. today, for a contract with maturity T
\( t_n \) = Premium coupon payment days
\( \Delta (t_{n-1}, t_n) \) = Change between premium payments in years
\( Surv(0, t_n) \) = Survival probability of reference entity up to time \( t_n \) (as seen from time point zero, i.e. today)
\( r \) = Risk free interest rate
T=Maturity date
N=T*4 (Since we sum for each quarter)
Equation 1 tells us to discount, by the risk free interest rate, the premium payments at each payment period. The payments are also weighted by the survival probability of the reference entity, making equation 1 express the expected present value of the premium leg. If the default occurs before the maturity date, T, then the protection buyer pays a so called accrued premium to the protection seller. This means that if the default time, y, happens in payment period n, that is the default time belongs to the interval \([t_{n-1}, t_n]\), then the protection buyer pays the following to the protection seller (O’kane 2009):

\[(y - t_{n-1}) * S_0(T)\]  \hspace{1cm} \text{Equation 2}

which is the accrued premium between the default time and the last premium payment time. The expected discounted value of this is given by the following expression:

\[E \left[ e^{- \int_{0}^{y} r(s) ds} * (y - t_{n-1}) * S_0(T) \right] \]  \hspace{1cm} \text{Equation 3}

If the default time occurs before the maturity, then the default time must lie in one of the periods \([t_{n-1}, t_n]\) so that the total expected discounted value of the accrued premium is:

\[S_0(T) * \sum_{n=1}^{N} E \left[ e^{- \int_{y}^{t_{n}} r(s) ds} * (y - t_{n-1}) * 1(t_{n-1} < r < t_n) \right] \]  \hspace{1cm} \text{Equation 4}

Where \(1(t_{n-1} < r < t_n)\) is an indicator function that takes the value of 1 if \([t_{n-1} < r < t_n]\) is true and a value of 0 otherwise. When we include the expected discounted accrued premium into equation 1 and take the integral over time we receive the following expression for the premium leg present value (O’kane 2009):

\[PV_{PL}(T) = S_0(T) * \sum_{n=1}^{N} \Delta (t_{n-1}, t_n) * Surv(0, t_n) * e^{- \int_{0}^{t_n} r(s) ds} + S_0(T) * \sum_{n=1}^{N} \int_{t_{n-1}}^{t_n} \Delta (t_{n-1}, y) * (-dSurv(0, y)) * e^{- \int_{y}^{t_n} r(s) ds} dy \]  \hspace{1cm} \text{Equation 5}

Here the credit event date is denoted by y. Hence in equation 5 the default is allowed to occur in every time period. We then sum over the time periods and we end up with an expression for the expected present value at time point 0, or the pricing date. Although equation 5 is an accurate expression for the expected present value, if the valuation date occurs on a payment date, it’s fairly complex to evaluate. Hence it could be simplified to the following (O’kane 2009):
\[ PV_{PL}(T) = S_0(T) \times DV01(T) \]  

**Equation 6**

\[
DV01(T) = \\
\sum_{n=1}^{N} \Delta(t_{n-1}, t_n) \times \text{Surv}(0, t_n) \times e^{-\int_0^{t_n} r(s) ds} + \\
\sum_{n=1}^{N} \int_{t_{n-1}}^{t_n} \Delta(t_{n-1}, y) \times (-d\text{Surv}(0, y)) \times e^{-\int_0^{y} r(s) ds} dy
\]

\[ DV01(T) \] is the discounted value of one basis point and one notional amount. The first part in DV01 is the discounted values occurring on payment days, while the second term adds the accrued premium between the last coupon payment and time of the default event. The problematic of the accrued premium is then that the expression still is rather complex and time consuming to evaluate. Hence it could be approximated by assuming that if in-between two payment periods, it occurs in the middle of the payment dates. Hence in the case of quarterly payments we assume default 1.5 months after the last payment is default occurs in-between payment dates. This is a good approximation and it simplifies equation 6 to (O’kane 2003):

\[ PV_{PL}(T) = S_0(T) \times DV01(T) \]  

**Equation 7**

\[
DV01(T) = \\
\sum_{n=1}^{N} \Delta(t_{n-1}, t_n) \times \text{Surv}(0, t_n) \times e^{-\int_0^{t_n} r(s) ds} + \frac{1}{2} \sum_{n=1}^{N} \Delta(t_{n-1}, t_n) \times \left[ \text{Surv}(0, t_{n-1}) - \text{Surv}(0, t_n) \right] \times e^{-\int_0^{t_n} r(s) ds}
\]

Equation 7 simply adds the discounted expected value of premium payments that occurs in the middle of two premium payment days as the accrued premium instead of the fairly complex calculations in equation 6. This approximation can be made since on average a credit event will occur in the middle of the premium payment dates. Hence we have an expression for the premium leg given by equation 7. (O’kane 2009)
3.2 Default leg calculation

In section 2 it was mentioned that the default leg is the payment that the seller makes to the buyer of the CDS contract when default occurs. Hence the expected present value of the default leg is simply the discounted value of the notional amount of the contract less the recovery. However at the initial valuation point there is no knowledge about when the default will occur. Hence we will have to find the following:

\[ DL_{PV}(T) = E \left[ e^{-\int_0^\tau r(s)ds} \ast (1 - R)1_{\tau \leq T} \right] \]  

Equation 8

Where

\[ \tau = \text{Time for credit event (a random variable)} \]
\[ T = \text{Maturity date} \]
\[ 1_{\tau \leq T} = \text{Indicator function that equals 1 if } \tau \leq t \text{ and 0 otherwise} \]
\[ DL_{PV}(T) = \text{Present value of default leg at } t=0, \text{ i.e. today} \]

The expression in equation 8 is simply the expected value of the discounted notional amount less recovery if a credit event occurs. Since we assume that the interest rates and default time are independent, as mentioned in the beginning of section 3, equation 8 can be written as (O’kane 2009):

\[ E \left[ e^{-\int_0^\tau r(s)ds} \ast (1 - R)1_{\tau \leq T} \right] = \int_0^\infty 1_{\tau \leq T} \ast (1 - R) \ast e^{-\int_0^\tau r(s)ds} \ast (-dSurv(0, y))dy = (1 - R) \ast \int_0^T e^{-\int_0^\tau r(s)ds} (-dSurv(0, y))dy \]  

Equation 9

This expression can be approximated by taking the sum over small steps in time instead of the integral over time. Then we get the following expression:

\[ DL_{PV}(T) = (1 - R) \ast \sum_{k=1}^{T+M} e^{-\int_0^{t_k} r(s)ds} \ast [Defprob(t_k) - Defprob(t_{k-1})] \]  

Equation 10

Where

\[ M = \text{Number of intervals each year} \]
\[ Defprob(t) = \text{Default probability up to time } t, \text{ i.e. the probability of that a default will occur in period between today (t=0) and time point } t \]
\[ t_k = k \ast \Delta t \]

By using equation 10 we have an expression for the default leg which is fairly simple to calculate if we know the distribution of the default time of the reference entity, i.e. the default probability as a function of time t for the reference entity.
3.3 Model CDS spread
Since the CDS spread is defined as the spread that equals the expected present value of the default leg and the expected present value of the premium leg at time point zero, we find the CDS spread by the following expression:

\[ S_0(T) \times DV01(T) = (1 - R) \times \sum_{n=1}^{T+M} e^{-\int_{0}^{t_n} r(s) ds} \times [Defprob(t_n) - Defprob(t_{n-1})] \]  

Equation 11

By solving for the CDS time zero spread we get:

\[ S_0(T) = \frac{(1-R) \times \sum_{n=1}^{T+M} e^{-\int_{0}^{t_n} r(s) ds} \times [Defprob(t_n) - Defprob(t_{n-1})]}{DV01(T)} \]  

Equation 12

This is the expression that is used to value a CDS contract. Hence at time zero, or contracting date, the expected discounted premium leg and the expected discounted default leg equals and therefore no payment is made between the parties. Instead the value, or the “price”, determines how much a CDS buyer shall pay in coupon payments over the time period. (O’kane 2009)

3.4 Survival probabilities
As we gave an expression for the CDS spread in the previous section, we did not conclude how the actual default probabilities are determined. The default probability is indeed needed in the premium leg, while the survival probability is needed in the default leg. Hence we need to determine how the default probability will be specified in our model. There are several ways to modeling the default probability, where some of them are:

1. Rating based models
2. Merton model
3. Intensity models

However these are not the only ways of modeling default probabilities. For instance there are models within the intensity models that use a continuous hazard rate\(^{11}\), but these will not be considered due to the advanced mathematics. Once we have determined the default probability, for any time point \(t\), we can easily calculate the survival probability by \(1\)-default probability since a company either survives or defaults.

\(^{11}\) Hazard rate will be explained in section 3.4.3
3.4.1 Rating based models
When rating based models are used, the default probabilities are collected through rating agencies, such as Moody’s or Standard & Poor’s. These agencies look at the ratings of companies and bundles companies with the same credit rating into groups. The agencies then keep track of the bond issuers/companies within the group over time and study if any defaults occur. New groups are formed over time making studies continuous over time. When defaults occur, the default probability is calculated by dividing the number of defaults by the initial number of issuers/companies in the group. Hence, since performed over time, default probabilities could be estimated by looking at the credit rating of the reference entity and the probability of default for the specific time points/maturity for companies with the same credit rating. Important to notice is that the default probabilities from rating agencies are real-world, or physical, default probabilities. This is not the case when we study for instance intensity based models, which are mostly used for valuing CDS’s, since these are so called risk-neutral default probabilities, dependent on expected, discounted values. There are however a few problems with using the rating agency default probabilities. For instance the default probabilities found by rating agencies are historical which means that a user of these probabilities assumes that the historical default probabilities is a good approximation for future default probabilities. However this may not be the case since firm and economical factors that may affect default changes. Also, credit agencies may have different definitions of default than what is considered for a CDS contract, making probabilities from rating agencies inappropriate for finding CDS spreads. (O’kane 2009) The fact that the rating agency measure is a physical measure also means that we need to transform the observed probabilities to be able to use them in the CDS model (Lando 2004).

3.4.2 Merton model
Merton (1974) introduced a model for default that were dependent on the asset values (A) and the face value of debt (F). The model states that the value of a firm’s assets equals the debt and equity (E) of the firm. Merton then stated that default only could occur at the end of the time period and that the payoff to equity holders and debt holders were determined by if the value of assets were larger or smaller than the face value of debt. In cases where the assets were larger than the face value of debt equity holders could retain the difference between the asset value and the face value of debt, after paying debt holders. In cases where the assets were smaller than the face value of debt, debt holders retained the entire value of the firm, leaving zero value for equity holders. By assuming that the asset value followed a lognormal distribution, Merton concluded that the present value of the equity claim and the asset claim could be calculated by the Black-Scholes formulas:
Equation 13

\[ E(t) = A(t) \Phi(d_1) - F * e^{-r(T-t)} * \Phi(d_2) \]

Equation 14

\[ D(t) = F * e^{-r(T-t)} * \Phi(d_2) + A(t)\Phi(-d_1) \]

Where F is the face value of debt, r is risk-free interest rate and \( \Phi \) is the normal distribution. The default probability of the firm can be found by \( \Phi(d_2) \), where we have:

\[ d_2 = d_1 - \sigma_a \sqrt{T-t} \]  \hspace{1cm} \text{Equation 15}

\[ d_1 = \frac{\log\left(\frac{A(t)}{F} + (r + \frac{1}{2} \sigma_a^2)(T-t)\right)}{\sigma_a \sqrt{T-t}} \]  \hspace{1cm} \text{Equation 16}

Although this model could be used for default probabilities, it’s rather complicated in practical applications. For instance it’s complicated to have an updated version of a company’s asset values since company reports are often presented each quarter. Hence finding CDS spreads on an everyday basis is complicated. Also the model is very simplified regarding the capital structure and therefore is rather unrealistic. (O’kane 2009).

### 3.4.3 Intensity based models

Intensity models are used to calculate survival probability in continuous time. Jarrow and Turnbull (1995) proposed that default occurs at some time point \( \tau \), equivalent to \( y \) in section 3, with the probabilities defined as:

\[ Pr[\tau < t + dt | \tau \geq t] = \lambda(t) dt \]  \hspace{1cm} \text{Equation 17}

The probability of default in the small time-period \([t, t+dt]\), conditional that there is no default in the previous time period \([0,t]\), is given by some time dependent function, \( \lambda(t) \) times \( dt \). The function \( \lambda(t) \) is called a hazard rate (or default intensity). One can show that the default time \( \tau \) in equation 17 can be constructed as (Lando 2004):

\[ \tau = \inf[t > 0: \int_0^t \tau(s) ds \geq E] \]  \hspace{1cm} \text{Equation 18}

The definition of \( \tau \) in equation 17 is simply the first time point the integrated hazard rate reaches the level of the exponentially distributed random variable \( E \) with a parameter of 1. From equation 18 it’s possible to show that the survival probability is (Lando 2004):

\[ Pr(\tau > t) = e^{-\int_0^t \lambda(dt)} \]  \hspace{1cm} \text{Equation 19}
Hence we have an expression for the survival probability, which easily could be turned into a default probability since a company either survives or default. Hence the default probability is given by:

$$Pr(\tau \leq t) = 1 - e^{-\int_{t_0}^{t} \lambda(dt)}$$

Equation 20

This model could be used in different ways, either assuming that the hazard rate is constant over time, or that it is piecewise constant. A constant hazard rate model assume that the hazard rate is constant for all time periods while a piecewise constant model lets the hazard rate change for different maturities. Since we now have an expression for the default and survival probability, it’s possible to calculate the CDS spread. In the rest of this thesis the intensity based model will be used to model default probabilities.

### 3.5 Bootstrapping

This section describes how to find the implied hazard rate from the observed market CDS spread.

In the formula for the CDS model spread in section 3.3 we need to know the values of the hazard rates to be able to calculate the default probability and hence the CDS spread. However, often we want to find the implied hazard rates given by the CDS spread observed in the market. For instance it could be interesting if we want to calculate the implied default probability for a firm, estimated by the market under a risk-neutral measurement.

When we find the implied hazard rates we let \( S_{market}(T) \) be the observed market CDS spreads for a CDS contract for a company with maturities \( T=1,2,...,10 \) years. We now assume that the hazard rate \( \lambda(t) \) is piecewise constant according to equation 21 in this thesis.

\[
\lambda(t) = \begin{cases} 
\lambda_1 & \text{if } 0 \leq t \leq 1 \\
\lambda_2 & \text{if } 1 \leq t \leq 2 \\
. & . \\
. & . \\
\lambda_{10} & \text{if } 9 \leq t \leq 10 
\end{cases}
\]

Equation 21

This is also shown graphical in figure 3. As can be seen in figure 3 we assume a constant hazard rate for each time period and at each time period a jump occurs. Hence there is a piecewise constant default intensity model. Important to notice is that the jumps don’t need to be equally big at each time period.
Given $\lambda(t)$ in equation 21, i.e. given the numerical constants of the hazard rates $\lambda_1, \lambda_2, ..., \lambda_{10}$, we can compare the model CDS spreads with the market CDS spreads for the different time periods $T=1,2,..., 10$, by using equation 12 where we use the default probability given in equation 19 and 20 in section 3.4.3, where:

$$P[\tau > t] = \begin{cases} 
    e^{-\lambda_1 \tau} & \text{if } 0 < \tau \leq 1 \\
    e^{-\lambda_1 - \lambda_2 \tau} & \text{if } 1 < \tau \leq 2 \\
    e^{-\lambda_1 - \lambda_2 - \lambda_3 \tau} & \text{if } 2 < \tau \leq 3 \\
    \vdots \\
    e^{-\lambda_1 - \lambda_2 - \cdots - \lambda_{10} \tau} & \text{if } 9 < \tau \leq 10 
\end{cases}$$  

Equation 22

However, as mentioned before, we want to find the constants $\lambda_1, \lambda_2, ..., \lambda_{10}$ given the market spreads, i.e. we want to find the hazard rate constants so that the model CDS spread equals the market CDS spread for each maturity, that is:

$$S_{\text{market}}(T) = S_{\text{model}}(T; \lambda_1, \lambda_2, ..., \lambda_{10})$$  

Equation 23

Where

$$T \in [1,2,...,10]$$
In equation 23 we emphasize that the model CDS spread is a function of the hazard rate constants \( \lambda_1, \lambda_2, ..., \lambda_{10} \). The procedure performed to find the constant hazard rates is referred to as bootstrapping and works in the following way:

1. Find \( \lambda_1 \) so that equation 23 holds, that is so that \( S_{\text{market}} (1) = S_{\text{model}} (1; \lambda_1) \)
2. Given \( \lambda_1 \) from the first step, find \( \lambda_2 \) so that equation 23 holds, i.e. so that \( S_{\text{market}} (2) = S_{\text{model}} (2; \lambda_1, \lambda_2) \). Important to notice is that the model CDS spread does not depend on \( \lambda_3, \lambda_4, ..., \lambda_{10} \) since we only study the time period up to year 2 since the CDS contract matures at this point.
3. Repeat the second step recursively to find the remaining constant hazard rates, i.e. to find \( \lambda_3, \lambda_4, ..., \lambda_{10} \).

In the procedure performed, when we solve for the constant hazard rates\( \lambda_1, \lambda_2, ..., \lambda_{10} \), we each time have to solve a one-dimensional non-linear equation, namely equation 23, where \( \lambda_1, \lambda_2, ..., \lambda_{n-1} \) (n=1,2,...,10) and the market CDS spread for each time period is known and where the model CDS spread is given by equation 12 in section 3.3.

Important to notice is that the bootstrap methodology could either be extended or shorten regarding the number of maturities included, i.e. the number of hazard rates and market CDS spreads could be more or less than 10. However, the presentation regards 10 periods since, as will see in section 6.1, this is the amount of maturities used in this study. The bootstrap methodology presented here will be the one used in the study to find the implied hazard rates and from these calculate the implied default probabilities.

### 3.6 Upfront payments

As will be described later, changes have been made to the CDS market, where changes to fixed coupons are one important change. These changes means that a fixed spread will be paid by the CDS buyer to the seller and the fixed spread may not equal the running spread, i.e. the spread that makes the expected discounted premium- and default leg equal at the trade date. When this occurs a so called upfront payment is made, so that the expected discounted premium- and default leg equals at time point zero. The calculation of an upfront is (O’kane 2009):

\[
Upfr = N \times [DL_{PV}(T) - PL_{PV}(T)] = \text{Equation 24}
\]

\[
= N \times [DL_{PV}(T) - S_{\text{fixed}} \times DV01(T)]
\]

Where \( N \) is the notional amount of the CDS contract and \( S_{\text{fixed}} \) is a spread that is fixed and not necessarily equals the running spread of a CDS contract. Equation 24 simply calculates the difference
between the expected present value of the default leg and the expected present value of the premium leg and multiplies it by the notional amount of the contract. Hence the upfront is used to equal the expected present value of the premium- and default leg at the trade date. If the running spread and the fixed spread equals, no upfront will be paid since the expected present values are equal at time zero. In figure 4 the upfront payment scheme of a 5 year CDS on Bank of America is presented.

Figure 4: An upfront payment scheme for a 5 year CDS spread on Bank of America with a running spread that equals 140.105 basis points.

In cases where the fixed spread is larger than the running spread of a CDS contract, the buyer has to pay a higher premium than in the fair case and hence the seller of the contract has to pay the buyer to equal the expected present values at the trade date (to the lower left of the intersection in figure 4). If the spread is fixed at a lower spread than the running spread the opposite is true, i.e. the buyer pays the seller an upfront at the trade date (to the upper left of the intersection in figure 4). Hence if the upfront is positive the buyer pays and if it’s negative the buyer receives the upfront amount.
4. Credit default swaps and the financial crisis

This section will give a presentation of the involvement of CDS contracts during the recent financial crisis.

The credit default swap contracts have made a big impact recently in the financial crisis. Even though they may not have been the start of the crisis, they contributed to spread the crisis between companies and foremost financial firms. Some of the main reasons to why this could happen have been addressed to the fact that CDS contracts were poorly regulated and that they were traded over-the-counter.

When CDS contracts were firstly introduced, they were foremost used to protect a buyer from potential credit losses on bonds. However, over time, they started to be traded by investors whom actually did not own any underlying bond in the reference entity, i.e. they were taking so called naked positions (Testimony 2008). This simply means that instead of using CDS contracts for protection, they were used to speculate in company defaults. However regulation says that it’s not allowed, for “normal” insurances, to insure something that is not of insurable interest for the insurance taker (FSA 2009). Hence in 2000, the New York Insurance Department decide that naked CDS contract positions were not to be seen as an insurance and thereby allowing companies to take naked positions in CDS contracts (Testimony 2008; Zabel 2008). Also in 2000 a new regulation called “The commodity futures modernization act of 2000” were approved, which made changes to the over-the-counter market. The changes were recommended by the presidential working group on financial markets report called “Over-the Counter derivatives and the commodity exchange act”, which suggested deregulation of several swap contracts, such as CDS contracts (PWG 1999). This meant that CDS contracts were unregulated by the year of 2000 and since that point in time the increase in CDS contracts has been tremendous, as can be seen in figure 2 (section 1).

The over-the-counter market that the CDS contracts were traded in also implies that contracts are made separately between two parties, i.e. the buyer and the seller. This makes the market extremely complex to overview, since there was no central counterparty who knew how many contracts and how big amounts that were outstanding (GAO 2009). This made companies trading with CDS contracts interconnected through the CDS’s (FSA 2009). However, not only connected with the companies that were counterparties to the contract the institution undertook, but also indirect connected to counterparties of other companies they made trades with. The over-the-counter market also made it complicated for parties to know how much risk the counterparty were undertaken, making them exposed to the risk that the counterparty would not be able to fulfill their obligations. (FSA 2009; Testimony 2008)
Another feature of the CDS contracts is that of posting collateral. If the price of a CDS contract rose, i.e. default probabilities rose, this meant that the seller of a CDS contract had to put aside collateral as proportion of the changes in value. Since the CDS price, i.e. the spread, are set on a daily basis, this meant that when large movements in the CDS spread occurred, large amount of collateral were needed to put aside. Also if the credit rating for the CDS seller were decreased they had to post collateral, this collateral works as a protection for the buyer of a CDS contract (FSA 2009). Collateral amounts were further increased since CDS contracts were not netted due to the complex market but were in notional amounts, forcing companies to put more money in collateral than perhaps were needed and freezing unnecessary much capital. (Testimony 2008)

When large amounts need to be put aside for collateral, which can be needed in a short time period, companies need liquid assets. So when the collateral needed to set aside increased, companies need to start selling liquid assets, making prices decrease. Lower asset prices also mean that financial institution might have to make losses to provide the required collateral for CDS contracts. This makes it further needed to put aside collateral due to the safety for the protection buyer. Hence CDS contracts increased the financial crisis, since they made the liquidity in the market to dry up and resulted in a credit crunch.
5. CDS bang protocols

In this section the recent regulatory changes made for the CDS market will be presented. The presentation contains global changes, North American changes, European changes and Asian changes.

Since CDS contracts had such an impact on the financial crisis, discussions about regulations of the CDS market has turned into action. On the 8 of April 2009, ISDA announced that the implementation of the new regulation called “ISDA Credit Derivatives Determinations Committees and Auction Settlement CDS Protocol” was successful (ISDA 2009a). This law is more commonly known as the “big bang” Protocol. This protocol has been developed since the growth of the CDS market has increased tremendously, as presented in figure 2 (section 1), which made the CDS market increasingly hard to overview, leading to a complex market. Hence, by the big bang protocol, regulators want to standardize CDS contracts so that they could be netted in an early stage, the aim is to make it possible to net contracts at time zero. The idea of introducing central clearing is that the market will be less complex as well as to decrease potential domino effects of default if it occurs. (Markit 2009a)

Figure 5 shows how the market was traded before the “bang” protocols and how the market is traded today.

CDS market before big bang protocol

CDS market after big bang protocol

Figure 5: The CDS market before and after the big bang protocol. (Markit 2009a)

By introducing a central clearing house and to net CDS contracts, the notional amount outstanding in the market will be decreased and the market will be easier to overview, as well as the risk undertaken by the companies could easier be monitored. Also if a credit event occurs, then collateral needed to protect the positions will be decreased if contracts are cleared, since the open positions in CDS contracts are known, i.e. only open positions needs collateral to be put aside instead of the notional amount on every position. Hence this will free capital that is not needed to be reserved for collateral, which has an positive effect for investors and may prevent potential liquidity problems, to some extent, which arose during the financial crisis. However, to be able to perform central clearing
the CDS contract has to be more standardized, since the diversity of contracts today makes clearing too inefficient to perform. The big bang protocol introduces several changes to the CDS market to improve the possibility to net contracts at time zero. The changes made in the big bang protocol are to some extent global, while some changes are only implicated in North America. (Markit 2009a)

5.1 Global changes
There are three global changes to the CDS market included in the big bang protocol. These are:

1. Event Determination Committees will be established
2. Auction hardwiring
3. Rolling event effective date

According to the big bang protocol, there will be so called determination committees established for different regions. The regions are determined to be Americas, Asia ex-Japan, Japan, Australia-New Zealand and EMEA (Europe, Middle East and Africa). The event committees’ most important work is to determine whether or not credit events have occurred and when it occurred, as well as determine if there will be one or more auctions for the specific credit events. This means that credit events will be determined by a committee instead of, as it were before the big bang protocol, between the two parties that participated in the CDS contract. This means that credit events occur for all participants in the market and the central clearing house gets one credit event that has occurred for one reference entity. Hence clearing will be easier since all contracts with the same reference entity could be used for clearing, since the default event will be the same for all contracts. (Markit 2009a)

The determination committees will consist of global and regional dealers, buy side members and representatives of ISDA. For a determination committee to consider if a credit event has occurred, an ISDA member has to address that a potential credit event has occurred as well as a member of the committee must agree upon this. This has to be made at least 60 days after the event for credit events and 90 days for succession events. If this is fulfilled the committee votes if the event has occurred or not. (Markit 2009a)

Within the big bang protocol, the auction methodology will be presented in the contracts. Before the big bang protocol firms had to sign up on protocols’ to make an auction take place. Now this will not be necessary and since all participants agree upon the same methodology, contracts could be netted. However there is some information that the event determination committees bring forth when an
Before the big bang protocol was implemented, the effective day of a CDS contract was the day after the trade date. By having this construction for the effective date an exactly offsetting contract bought a few days later will not be truly offsetting, since the effective dates are different and hence a credit event can occur in-between the effective dates. This is a risk that has existed before the big bang protocol. To have the effective dates like this will also implement tremendous problems to net two positions since they might not be equivalent when it comes to effective dates. Consequently the effective dates have been changed in the big bang protocol. Now the effective dates are rolling and determined to be today minus 60 days for credit events and today minus 90 days for succession event. This means that at any given time point, all contracts will have the same effective date, making contracts easier to net. A rolling effective date will also eliminate the risk that arises when offsetting a contract. (Markit 2009a)

5.2 North American changes
The specific North American changes that the big bang protocol regards are:

1. Restructuring clause changes
2. Fixed coupons

The restructuring clauses regard which bonds that are allowed to be delivered to the CDS seller in case of a restructuring event, i.e. how long the maturity of the bond delivered are allowed to be. Before the big bang protocol the so called modified-restructuring were the most traded standard in North America, while in Europe the modified-modified-restructuring were mostly traded. Since the CDS big bang protocol was introduced, the restructuring standard in North America has been changed to no-restructuring, i.e. restructuring does not exist as a credit event. This has been possible to do since many cases of restructuring are included as default in chapter 11. (Markit 2009a)

The most significant change in the CDS market is probably that regarding fixed coupons. As mentioned before, a CDS contract, before the big bang protocol, were priced by a spread that the buyer has to pay to the seller over the time period of the contract. Hence this will mean that if the CDS spread changes every day, no contract will have the same spread to be paid. This makes netting very complex and hence this has been changed in the big bang protocol. Now the CDS spread will be fixed for all contracts. The spread will be either 100 basis points or 500 basis points, making netting at any specific time period easier. The reason to why it’s been decided to be 100 and 500 basis points
is due to that 500 basis points has been used earlier in the market for CDS contracts were reference entities been in distress. The 100 basis point fixed coupon were added to have an extra option for traders as well as it enables to change old contracts into the new standard. However too many coupons will deter from the underlying reason for the changes, i.e. netting will be more complex when amount of different coupons increases. Since contracts will be traded in fixed coupons this means that if the running spread, i.e. the spread as it were before the big bang protocol, not equals 100 or 500 basis points, there will be indifferences in the present values of the default and premium leg. Hence there will be an upfront paid at time zero to make the traders indifferent. (Markit 2009a) At the moment CDS spreads are still quoted in the way they were before the regulation changes, i.e. they are still quoted as running spreads in the market (Markit). However, when an actual trade is performed they are traded to a fixed spread and with an upfront payment.

5.3 European changes
The CDS Big Bang protocol made changes to the global market and the North American market. Hence changes to fixed coupons and restructuring clauses were not applicable to the European market. Instead another “bang” protocol, often referred to as the “small bang” protocol, regards changes on the European market. However the changes made to the European market are not the same as those made to the North American market. (Markit 2009b)

In Europe the restructuring clauses will not be taken away as a credit event. This is mainly because Europe doesn’t have any regulation that is comparable to chapter 11 when it comes to classifying restructuring as default. Hence there will be restrictions regarding which bonds that are acceptable for delivery if a restructuring event occurs. The standard adapted will be the modified-modified-restructuring, i.e. the standard that were mostly traded in the European market before the “bang” protocols. (Markit 2009b)

Another difference in the European market is that there will be more fixed coupons that contracts will be traded at. Instead of only having 100 and 500 basis point fixed coupons there will be 25, 100, 500 and 1000 basis points. The reason for having more coupons is that it’s believed that there are preferences of traders to enter contracts that are closer to the previous running spread. However, including more coupons will decrease the ability to net positions as effectively. (Markit 2009b)
5.4 Asian and Japanese changes
The changes in the Asian market for the CDS contracts are similar to those changes made in North America. The most significant change is that of fixed coupon payments. In Asia the fixed coupons will be set to 100 and 500 basis points, while in Japan the coupons will be set to 25, 100 and 500 basis points. The restructuring convention in the Asian and Japanese market will not be changed and hence a full-restructuring will continue to be the standard. This implies, as mentioned in section 1, that bonds with a remaining maturity of up to 30 years are allowed to be delivered. (Markit 2009c)

5.5 Transformation
Together with the “bang” protocols, a CDS converter were introduced, making it possible for investors to convert traditional or running spreads, into fixed coupon spreads with an upfront. The conversion method proposed uses the information of the running spread, maturity of the contract, the fixed coupon, notional amount of the contract and the expected recovery rate. The expected recovery used in this model is either 40% for senior bonds or 20% for other bonds in North America, Asia and Europe (Markit 2009c; ISDA 2009b).

The conversion mechanism proposed by the “bang” protocols uses the information of the CDS spread for a specific maturity to find an implied hazard rate, i.e. the model is a so called constant intensity model, since it does not consider information from CDS contracts for earlier maturities. The implied hazard rate is then applied to solve for the upfront payment, given by equation 24. Hence a fixed hazard rate is used for the entire time period. Remember from equation 12 that the survival probability, and hence the default probability is found by the hazard rate.

Beumee et. al. (2009) made a warning for using the conversion methodology that has been proposed. They state that there exist two important factors that investors should be aware of, namely that it is 1) inconsistent among maturities and 2) for single CDS deals the upfront payment will not be the same as for a model that is consistent in the term structure.

Beumee et. al. (2009) argued that the consistency problem by the proposed conversion methodology, from now on called flat hazard rate model, is due to the fact that the method implicitly assumes that there exists no other CDS contract with earlier maturities on the reference entity. Hence the model is inconsistent since it doesn’t consider the entire term structure. By only considering one hazard rate for the entire time period of the CDS contract, default probabilities that are implicitly given by earlier maturities market CDS spreads on the reference entity are ignored. This means that there exist information about default probabilities for earlier time periods but these are
ignored instead of taken into consideration, making the proposed conversion method of finding default probabilities inconsistent with those given by the market at each time period. Although if there wouldn’t exist any CDS contracts with earlier maturity, the inconsistent method would in fact be consistent, since it considers all time periods and hence is consistent over the term structure. However, this is not the case for most reference entities since there exist several different CDS contracts with different maturities.

Instead of a flat hazard rate, Beumee et. al. (2009) propose a methodology that is consistent over the term structure. The model proposed were the piecewise constant intensity model. This methodology, from now on called bootstrap method, includes bootstrapping of each individual maturity’s hazard rate, i.e. the hazard rate curve given by the market is found by bootstrapping, and the hazard rate curve is then used to calculate the upfront given the expression in equation 24. This method is consistent with default probabilities at earlier time points in contrary to the proposed inconsistent flat hazard rate model.

A second problem according to Beumee et. al. (2009) is that even for a single deal CDS contract the calculated upfront given by flat hazard rate model is not equal to that of the consistent bootstrap method. Hence they argue that a flat hazard rate method should not be adapted by investors as a modeling tool. If the flat hazard rate model is adapted this will result in continuously inconsistency in default probabilities and it will lead to ambiguous results. Beumee et. al. (2009) therefore points out that the method should only be used as an approximation for calculating upfrons, if used, and that results should not be implemented as the upfront in the market.
6. Methodology
In this section the methodology for the study is presented as well as a presentation of the data. The methods used for calculations are also included as well as a benchmark to the models used by Beumee et. al. (2009).

6.1 Data
When calculation of upfronth with the two methods described in section 5.5, we will consider a CDS contract that matures in 10 years, with earlier CDS contract maturities considered for each year, i.e. for year 1-9. The data for the study consists of CDS spreads from 10 companies located in Europe, North America and Asia. The companies have been chosen from the standard and poor global 100 index, which consists of 100 world companies. From the index 10 companies were randomly chosen to be included in the study. The companies that are included are:

<table>
<thead>
<tr>
<th>Company</th>
<th>Business area</th>
<th>Country</th>
</tr>
</thead>
<tbody>
<tr>
<td>Colgate-Palmolive</td>
<td>Consumer goods</td>
<td>U.S.A.</td>
</tr>
<tr>
<td>Dell</td>
<td>Information technology</td>
<td>U.S.A.</td>
</tr>
<tr>
<td>Johnson &amp; Johnson</td>
<td>Pharmaceuticals</td>
<td>U.S.A.</td>
</tr>
<tr>
<td>Morgan Stanley</td>
<td>Financial services</td>
<td>U.S.A.</td>
</tr>
<tr>
<td>Rio Tinto</td>
<td>Mining</td>
<td>Great Britain</td>
</tr>
<tr>
<td>RWE</td>
<td>Electrical power</td>
<td>Germany</td>
</tr>
<tr>
<td>Samsung</td>
<td>Electronics</td>
<td>Korea</td>
</tr>
<tr>
<td>United Technologies</td>
<td>Conglomerates/technology</td>
<td>U.S.A.</td>
</tr>
<tr>
<td>Volkswagen</td>
<td>Automotive</td>
<td>Germany</td>
</tr>
<tr>
<td>Xstrata</td>
<td>Mining</td>
<td>Switzerland</td>
</tr>
</tbody>
</table>

Table 1: Companies included in the study.

The time period considered begins at same time period as the big bang protocol was implemented in North America, i.e. from April 8, 2009, and is considered until April 30, 2010. The CDS spread data is collected from DataStream and it is the mid quote of the bid and ask spread from different dealers in the market.
6.2 Models and calculations used

All model calculations made in this thesis is performed in MatLab. The models considered in this study are the model proposed by the “bang” protocols, i.e. a constant intensity model, and the model proposed by Beumee et. al. (2009), i.e. a piecewise constant default intensity model. In the case of the flat hazard rate method the hazard rate will be extracted from the 10 year CDS spread, while for the bootstrap method the hazard rate term structure is found by bootstrapping and the market CDS spreads for all maturities as described in detail in section 3.5. When bootstrapping is performed the f-solve function in MatLab is used. This command solves a function so that it equals zero (mathworks 2009), i.e. in our case it solves equation 23.

In this thesis the CDS contracts considered are connected to the senior debt of the reference entities, and therefore a recovery rate of 40% is used for the study as this is the case in accordance to the proposed conversion method (Markit 2009c;ISDA 2009b). The study also considers only 500 basis points fixed spread, however the magnitude of the fixed spread should not make any other impact than scaling the final results. In the calculation an assumption of constant interest rate is made and this is set to zero, since this will make the discount factor equal to 1. This is done since the interest rate term structure has a small implication on the final result and the calculations are simplified. Also the accrued premium is excluded from the models to simplify the calculation.

6.3 Benchmark

Due to the fact that the model used in this thesis is not entirely the same as used in the report by Beumee et. al. (2009), since there is a difference in including the accrued premium, a benchmark using the model in this thesis on the data from the report is made to study if the difference between the models used is significant. In the report a ten year CDS contract for four companies are considered at June 20, 2009. The initial data given in the report is presented in table 2.

<table>
<thead>
<tr>
<th>Company</th>
<th>1 Year</th>
<th>3 Year</th>
<th>5 Year</th>
<th>7 Year</th>
<th>10 Year</th>
<th>Recovery</th>
<th>Fixed Spread (bp)</th>
</tr>
</thead>
<tbody>
<tr>
<td>ArcelorMittal SCA</td>
<td>1287</td>
<td>1109.86</td>
<td>1009.57</td>
<td>938.57</td>
<td>852.57</td>
<td>40%</td>
<td>500</td>
</tr>
<tr>
<td>Continental AG</td>
<td>2168.64</td>
<td>1607.98</td>
<td>1388.67</td>
<td>1245.89</td>
<td>1112.9</td>
<td>40%</td>
<td>500</td>
</tr>
<tr>
<td>AIG Inc</td>
<td>3197.28</td>
<td>2274.91</td>
<td>1913.48</td>
<td>1695.9</td>
<td>1523</td>
<td>35%</td>
<td>500</td>
</tr>
<tr>
<td>Hitachi Ltd</td>
<td>157.92</td>
<td>195.57</td>
<td>217.21</td>
<td>223.87</td>
<td>234.8</td>
<td>35%</td>
<td>100</td>
</tr>
</tbody>
</table>

Table 2: Initial data given in the report of Beumee et. al. (2009)

Hence, as can be noticed is that in the report by Beumee, Brigo, Schiemert and Stoyle (2009), four companies were included and the calculations were made on one single day. It’s also noticeable that the CDS spreads for the companies were very high with Hitachi as an exception. The results of the
calculations of the default and premium leg as well as the upfront payment with the model including the accrued premium and the model specified in this thesis, excluding the accrued premium, is presented in table 3.

### Upfronts at June 20, 2009

<table>
<thead>
<tr>
<th></th>
<th>Beumee, Brigo, Schiemert and Stoyle Result Bootstrap Method ($)</th>
<th>Beumee, Brigo, Schiemert and Stoyle Result Flat Hazard Rate Model ($)</th>
<th>Thesis Model Result Bootstrap Method</th>
<th>Thesis Model Result Flat Hazard Rate Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>ArcelorMittal SCA</td>
<td>Default PV 4,368,123.78, Premium PV 2,561,739.08, Upfront 1,806,384.70</td>
<td>Default PV 4,628,663.44, Premium PV 2,714,535.72, Upfront 1,914,127.72</td>
<td>Default PV 4,280,335.49, Premium PV 2,510,254.57, Upfront 1,770,080.92</td>
<td>Default PV 4,514,923.97, Premium PV 2,647,831.83, Upfront 1,867,092.14</td>
</tr>
<tr>
<td>Continental AG</td>
<td>Default PV 4,709,388.18, Premium PV 2,115,818.22, Upfront 2,593,569.97</td>
<td>Default PV 5,126,323.27, Premium PV 2,303,137.42, Upfront 2,823,185.85</td>
<td>Default PV 4,644,248.06, Premium PV 2,086,552.28, Upfront 2,557,695.78</td>
<td>Default PV 5,021,127.28, Premium PV 2,255,875.31, Upfront 2,765,251.97</td>
</tr>
<tr>
<td>AIG Inc</td>
<td>Default PV 5,424,401.43, Premium PV 1,780,817.29, Upfront 3,643,573.65</td>
<td>Default PV 5,570,652.09, Premium PV 1,828,841.79, Upfront 3,741,810.30</td>
<td>Default PV 5,370,771.41, Premium PV 1,763,221.08, Upfront 3,607,550.33</td>
<td>Default PV 5,833,167.94, Premium PV 1,915,025.59, Upfront 3,918,142.35</td>
</tr>
<tr>
<td>Hitachi Ltd</td>
<td>Default PV 2,055,644.09, Premium PV 875,487.26, Upfront 1,180,156.83</td>
<td>Default PV 2,003,923.55, Premium PV 853,459.77, Upfront 1,150,463.77</td>
<td>Default PV 1,982,908.94, Premium PV 844,509.77, Upfront 1,138,399.17</td>
<td>Default PV 1,963,332.33, Premium PV 836,172.20, Upfront 1,127,160.13</td>
</tr>
</tbody>
</table>

Table 3: The results from Beumee et. al. (2009) and the results using the model in this thesis, i.e. excluding the accrued premium on the same sample.

Where the relative difference between the models are given by:

\[
\text{Relative difference} = \frac{\text{Brigo et al. result} - \text{my model result}}{\text{Brigo et al. result}}
\]

Equation 25

As a result we can conclude that the differences between the two models are relatively small, as it doesn’t exceed five percent. Indeed the differences between the legs and upfronts are in most cases less than 2.5 percent. The difference between the models is due to the exclusion of the accrued premium. Since the difference between the two models is relatively small, it seems to be acceptable to use a model which excludes the accrued premium in this thesis.
7. Results

In this section the results will be presented through graphs as well as tables. To simplify presentation, four companies and a few maturities are reviewed, while the results for the other companies and all maturities are presented in attached appendix.

The companies that will be presented in the text are Morgan Stanley, Rio Tinto, Volkswagen and Johnson & Johnson and the maturities considered will be 1, 5 and 10 years. We begin by looking at how the CDS spread as well as the default probability has changed over the time period. In this case we study the 10 year CDS spread and default probability as an approximation of how the CDS spreads for different maturities and companies has changed over the studied period. The 10 year CDS spreads for the companies are presented in figure 6 as well as in appendix A.

Figure 6: 10 year CDS spread and default probability.

Over the past year, the 10 year CDS spreads has been decreasing for most of the companies and hence the implied default probability given by a bootstrap method has decreased as well. Although the spread has been rather stable for Johnson & Johnson, being in the boundary of about 20 to 65...
basis points, which imply a lower risk of default by the market. On the opposite, the CDS spread of Rio Tinto have had a dramatic decrease over the past year from a spread around 600 to 100 basis points, decreasing the implied default probability from about 60 percent to 20 percent. The spread for Volkswagen and Morgan Stanley has been decreasing, however not as much as Rio Tinto.

Next we study if the implied default probability given by the bootstrap method and the flat hazard rate model differs. The absolute difference between the two methods for different maturities is presented in figure 7 and appendix B.

Figure 7: The absolute difference in default probability between the flat hazard rate model and the bootstrap model

From figure 7 we can see that there exist differences between default probabilities extracted from the bootstrap method and the flat hazard rate model, since the absolute difference not equals zero. In some of the cases the absolute difference between the methods are as much as five percentage points. A positive difference here implies that the default probability from the bootstrap method is
larger than that of the flat hazard rate model. The noticeable difference implies that the flat hazard rate either overestimates (negative values) or underestimate (positive values) the default probability compared to the bootstrap method that is term consistent. The inconsistent method seems to underestimate the default probabilities in the earlier part of the studied time period, while overestimating it in the later parts. It is also interesting to notice that the difference, for some companies, is declining when we study the probabilities for a longer time period. For instance the 10 year default probability difference is smaller than the 1 year default probability difference for Morgan Stanley, Rio Tinto and Volkswagen, while it’s harder to determine this pattern for Johnson & Johnson.

Even though the absolute difference between the methods tells us that there exists a difference in calculating the default probabilities, they don’t tell us by what magnitude they differ. Hence by looking at the relative difference instead of the absolute value we can get further information about by how much they differ. Figure 8 shows the relative difference between the methods, also to be found in appendix C.

Figure 8: The relative difference between the flat hazard rate model and the bootstrap model.
As can be seen the relative difference in default probabilities between the two methods are extremely large for the shorter time period. In some periods the difference is as big as 200 percentages. We also notice in a much more effective way that, that the difference among the two methods is decreasing when the time is increasing, hence the estimates of the flat hazard rate is more accurate in longer time periods. This may be due to that the flat hazard rate is based on the 10 year CDS spread for calculating the hazard rate.

Since the results so far indicates that the default probabilities, and hence the hazard rates, are different for the methods, we should also expect the upfront payments to differ as well. In table 4 the upfront calculations for April 8, 2009 is presented.

<table>
<thead>
<tr>
<th>Upfronts at April 8, 2009</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Bootstrap Method</strong></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>Morgan Stanley</td>
</tr>
<tr>
<td>Rio Tinto</td>
</tr>
<tr>
<td>Volkswagen</td>
</tr>
<tr>
<td>Johnson &amp; Johnson</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th><strong>Flat Hazard Rate Method</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
</tr>
<tr>
<td>Morgan Stanley</td>
</tr>
<tr>
<td>Rio Tinto</td>
</tr>
<tr>
<td>Volkswagen</td>
</tr>
<tr>
<td>Johnson &amp; Johnson</td>
</tr>
</tbody>
</table>

Table 4: Default- & Premium leg results and upfront payment at the April 8, 2009.

As seen in the table the default leg present value given by a 500 basis point CDS contract and a notional amount of the contract on 10 million are not the equal for the two methods, leading to differences in the upfront payment. Also noticeable is that the upfront is negative for Morgan Stanley, Rio Tinto and Johnson & Johnson since the CDS spreads for these companies are less in the market than the fixed spread of 500 basis points (see figure 6). In fact the relative difference in upfront between the methods ranges from 0.7-4.8 percentage disregarding the sign. When we study
the upfront relative difference over the time period we see that the methods generate different upfronds, also found in appendix D, as the relative difference differs from zero. In the case of Rio Tinto the relative difference in upfront payment is extremely large with a difference between the models with about 25% at its maximum point.

Figure 9: The relative difference in upfronds between the flat hazard rate model and the bootstrap model. Also included is the 10 year CDS spread.
8. Analysis
From the results presented in the previous section we can conclude that there exist differences in default probabilities between the flat hazard rate and the bootstrap method in the study. Hence the study strengthens the warning made by Beumee et. al. (2009) about the restricted usage of a flat hazard rate method in cases where there exists CDS contracts for earlier maturities. Investors should therefore be cautious about the usage of the transformation method proposed by the “bang” protocols. Indeed usage of this method may lead to severe inconsistencies and ambiguous result of default probabilities. Therefore should the flat hazard rate method not be used by investors as a modeling tool as Beumee et. al. (2009) points out. It is indeed important to stress that the flat hazard rate model may give large differences in default probabilities compared to a model that is consistent over term structure, such as the bootstrap method, and hence should not be used as an approximation either.

The study can also confirm Beumee et. al. (2009) argument that for specific deals the flat hazard rate model generates different upfront payments than a consistent method does. Hence the proposed method from the “bang” protocols should indeed be used with caution. If used the results should not be implemented as the upfront payment in the market, but rather as an approximation. Even though it is used as an approximation, it should be stressed that these results may differ significantly from the results obtained by a model that is consistent over the term structure.

It is also worth mentioning that the bootstrap model used in this study may not be the mostly preferred model. Indeed models that use a continuous variable hazard rate may be more appropriate. Although in this context the bootstrap model should be considered more preferable than the flat hazard rate model since it is more consistent over term structure and therefore includes more information.
9. Conclusion
The purpose of this thesis were to present the regulatory changes that has been implemented through the “bang” protocols as well as replicate the example presented in the report of Beumee et. al.(2009) on a larger sample. This has been made by calculating the default probabilities and the upfront payment using the proposed conversion method by the “bang” protocols and with a method that is consistent over the term structure.

The results indicate that the proposed conversion method is inconsistent over time when calculating default probabilities compared to a model that is consistent over the term structure. Hence the argument made by Beumee et. al. (2009) regarding that the proposed conversion method is inconsistent is strengthen. The result also shows that the upfront payments calculated by the proposed method are ambiguous compared to a model that is consistent over the term structure. Therefore should the proposed flat hazard rate model not be used by investors as a modeling tool, since this will lead to continuously inconsistent and ambiguous upfront results.
References


Markit (2009a). The CDS big bang: Understanding the changes to the global CDS contract and North American conventions


Appendix A – CDS spreads over studied time period
Here are the 10 year spread and the 10 year default probability (calculated by the bootstrap method) presented for the 10 companies in the sample.
Appendix B – Absolute difference default probabilities
Here the absolute differences, given in basis points, between the default probabilities calculated by the two methods are presented for the 10 companies in the sample.
Appendix C – Relative difference default probabilities

Here the relative difference in default probabilities between the two methods are presented for the 10 companies in the study.
Rio Tinto
Relative Difference Default Probabilities

RWE
Relative Difference Default Probabilities

Volkswagen
Relative Difference Default Probabilities

Xstrata
Relative Difference Default Probabilities

1 Year
2 Year
3 Year
4 Year
5 Year
6 Year
7 Year
8 Year
9 Year
10 Year

Apr09 Jan10 May10
-150
-100
-50
0
50

Time
Difference (%)

Apr09 Jan10 May10
-400
-300
-200
-100
0
100

Time
Difference (%)

Apr09 Jan10 May10
-30
-20
-10
0
10
20

Time
Difference (%)

Apr09 Jan10 May10
-300
-200
-100
0
100
200
300

Time
Difference (%)
Appendix D – Relative difference upfront

Here the relative difference in the upfront payments between the two methods are presented for the 10 companies in the sample.