Abstract

This thesis deals with the mathematical and computational studies of kinetic equations in the presence of an external force field and a Gaussian thermostat. We introduce a stochastic model where particles with one and three-dimensional velocities, in addition to the random collisions, are acted upon by an external force field and in the presence of a Gaussian iso-kinetic thermostat. We derive master equations, which govern the time evolution of the probability distribution of the velocities. Kinetic equations are derived under the assumption of molecular chaos. These equations describe the time evolution of the one-particle distribution function in the limit of infinitely many particles. We also consider the stationary problems.

For the time-dependent thermostatted Kac equation the existence of solutions is established and the time evolution of moments of the solutions are also studied. The non-equilibrium stationary states are treated in further detail. In a particular case, it is shown that, such a state has a singularity. We also discuss a method of constructing asymptotic solutions to the stationary equation.

Numerical simulations using the Monte Carlo method are carried out for both one and three-dimensional velocities. The simulation results, illustrating the main features of the stationary solutions for the respective equations, are presented.

Key words and phrases: Many-particle system, Gaussian iso-kinetic thermostat, Kac's master equation, Kac equation, Boltzmann equation, Non-equilibrium stationary states, Monte Carlo method.

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