UNIVERSITY OF GOTHENBURG
SCHOOL OF BUSINESS, ECONOMICS AND LAW

Graduate School, School of Business, Economics and Law, University of Gothenburg, Sweden

Master of Science in Finance

Portfolio Performance Analysis: Combining Cryptocurrencies with Traditional Assets

Supervisor: **Candidate:**

Oben K. Bayrak **Mihail Huzun**

Academic Year: 2021/2022

Abstract

This paper investigates the role of cryptocurrencies in enhancing the performance of portfolios constructed with traditional assets. Therefore, my thesis wants to ascertain if investors should consider adding cryptocurrencies to their investment portfolios. The sample period covers almost seven years of daily data. I consider four cryptocurrencies: Bitcoin, Ethereum, Ripple, and Litecoin. Using the mean-variance framework, I document that these cryptocurrencies can improve the risk-adjusted returns of different allocation strategies. Both the in-sample and out-of-sample analyses suggest that the diversification benefits of cryptocurrencies are present before and after the onset of the Covid-19 pandemic. The contribution and weight of cryptocurrencies within portfolios vary according to investors' preferences. The Maximum Utility portfolio suggests that risk-taking investors should give more weight to cryptocurrencies in their portfolios. As for an investor inclined to minimize the volatility, the Minimum Variance portfolio recommends weights close to zero.

Keywords: Cryptocurrencies, Bitcoin, Ethereum, Ripple, Litecoin, Portfolio Strategies, Diversification Benefits, Markowitz.

Table of Contents

List of Figures

List of Tables

Chapter 1: Introduction

Cryptocurrencies are a growing and evolving asset class, and mass adoption seems to be more a question of *when* rather than *if*. In September 2021, El Salvador became the first country to accept Bitcoin as legal tender, followed by the Central African Republic in April 2022. Instead, the Swiss city of Lugano aims to become the center of blockchain technology in Europe after it announced Bitcoin and two other cryptocurrencies as legal tender in March of this year (Cointelegraph, 2022).

In January 2021, cryptocurrencies reached US$1 trillion in market capitalization for the first time in their history. Only a few months later, in April 2021, the capitalization doubled and then approached US$3 trillion in November of the same year. As of May 1st, 2022, the cryptocurrency market capitalization is around US$1.7 trillion (CoinMarketCap, 2022).

The blockchain technology with which most cryptocurrencies have strong links is getting more and more into institutional and corporate operations mainstream. According to blockchain research firm Blockdata, 81 out of the world's top 100 public companies by market capitalization use blockchain technology, 27 of which have a fully functioning live product (Blockdata, 2021).

Although cryptocurrencies were not created as investment assets, many investors use them as such. This huge interest in the world of cryptocurrencies has also attracted the interest of institutional investors. In August 2021, for example, JPMorgan Chase launched its bitcoin fund (CoinDesk, 2021). At the end of the third quarter of 2021, Morgan Stanley had a US$300 million exposure to Bitcoin through the Grayscale Bitcoin Trust (Cointelegraph, 2021). In October 2021, the first Exchange Traded Fund (ETF), which tracks Bitcoin futures in the United States, was approved (CNBC, 2021). Even regulators have finally noticed this and started a massive campaign to regulate the markets, albeit in a scattered and poorly coordinated order. With all these continuing developments, it should be safe to assume that cryptocurrencies will play an important role in investment portfolios.

Cryptocurrencies are a relatively new asset class to the financial world, and as a result, not much research has been done on the effects of including them in a portfolio. Therefore, this empirical study aims to investigate the role of cryptocurrencies in enhancing the performance of portfolios constructed with traditional assets. In particular, the paper at hand tries to answer the following research question:

Should investors consider adding cryptocurrencies to their investment portfolios?

This thesis further develops the research of Inci and Lagasse (2019), who also investigate if and how cryptocurrencies can be used to diversify an investment portfolio. Their analysis, however, only considers the tangency portfolio, thus ignoring the possible different risk preferences of investors. Furthermore, due to the lack of statistically sufficient data, they also exclude from the research one of the most popular and essential cryptocurrencies, Ethereum. On the other hand, this study introduces Ethereum into the analysis and, to address the various risk preferences of investors, uses five different portfolio models: Minimum Variance portfolio, Tangency portfolio, Risk-Parity portfolio, Equally Weighted portfolio, and Maximum Utility portfolio. Moreover, the availability of a larger dataset, and a period of global financial distress, allow for a more detailed and complete analysis.

The sample period covers almost seven years of daily data. I consider four cryptocurrencies: Bitcoin, Ethereum, Ripple, and Litecoin. Using the mean-variance framework, I document that these cryptocurrencies can improve the risk-adjusted returns of all the considered allocation strategies. Both the in-sample and out-of-sample analyses suggest that the diversification benefits of cryptocurrencies are present before and after the Covid-19 pandemic.

The contribution of cryptocurrencies to the final performance varies depending on the allocation strategy considered. As suggested by the Maximum Utility portfolio, a risk-taking investor should give more weight to cryptocurrencies. For an investor seeking to minimize the risk, instead, the benefit of diversification into cryptocurrencies is still present but to a lesser extent, highlighting the volatile nature of these assets. Investors should therefore consider including cryptocurrencies in their investment portfolios.

Current literature offers preliminary and fragmented evidence on the portfolio effects of multiple cryptocurrency investments. Hence, this study aims at closing the research gaps on the risk-return impact of cryptocurrencies in multi-asset portfolios. First, the paper contributes to the literature on cryptocurrency markets and their role in portfolio management decisions. Second, the sample period is the largest that has been used in this strand of the literature and allows for the comparison of different portfolio strategies during the pre-/post-pandemic period. Third, the research tries to be as complete and exhaustive as possible, considering various scenarios, constraints, sub-periods, and data selection methods. To the author's knowledge, this is the first study to address the portfolio effects of multiple cryptocurrency investments so extensively.

The remainder of this study is organized as follows: Chapter 2 presents the literature review with an overview of cryptocurrencies. Chapter 3 describes the theoretical background. Chapter 4 is about data and methodology. Chapter 5 covers the analysis of the results. Chapter 6 concludes.

Chapter 2: Literature review

2.1 Cryptocurrencies Overview

Cryptocurrencies can be defined as assets or value items that exist digitally and are created by the software. As such, they have no issuer. No person, company, or entity backs them, and there are no guarantees associated with them. Like physical gold, cryptocurrencies simply exist and are created or destroyed according to the rules articulated in the codes that govern them. As these digital assets move from one account to another, they are recorded on their respective transaction databases, known as blockchain: a public ledger that records all transactions ever made of a particular cryptocurrency, from its creation to the most recent transfer or payment. (Lewis, 2018)

Bitcoin and Ethereum are two of the best-known cryptocurrencies and together currently account for more than 60% of the total cryptocurrencies' market capitalization. Since the birth of Bitcoin, many other blockchains with their respective cryptocurrencies have been created. As of April 24th, 2022, the number of cryptocurrencies in circulation was around 19,000 (CoinMarketCap, 2022).

Bitcoin was created in the aftermath of the 2008 financial crisis by a pseudonymous Satoshi Nakamoto. The reason for its existence is to be found in the whitepaper published by Satoshi in October 2008: "*A purely peer-to-peer version of electronic cash would allow online payments to be sent directly from one party to another without going through a financial institution*" (Nakamoto, 2008). The idea was to create a secure low-cost payment system without using specific third-party intermediaries. This paper launched the basis for developing decentralized payment systems and applications.

Bitcoins are digital assets whose ownership is recorded on their blockchain, which is simultaneously updated on many independently managed computers worldwide. Transactions in Bitcoin are created and validated according to a protocol: a set of rules that defines and characterizes Bitcoin itself. The protocol is articulated in computer codes and is implemented by software that participants run on their computers. Transactions are created and grouped into blocks to form Bitcoin's blockchain when the software is executed. Bitcoin uses a proof-of-work mechanism to ensure that anyone can add blocks to the blockchain at a specific cadence (approximately every 10 minutes) without a central counterpart providing permission. Different cryptocurrencies have different target block creation times. Anyone can buy, own, and send Bitcoins. (Lewis, 2018)

Ethereum is currently the second-largest cryptocurrency by market capitalization, and although Vitalik Buterin founded it in 2013, its network with its respective underlying blockchain was only released on 30 July 2015. Ethereum's vision is to create a worldwide computer where globally accessible and uncensored applications can be developed. Indeed, unlike Bitcoin, which is mainly used to send monetary value between people, Ethereum could be used to send information between programs. Ethereum's innovation is, in fact, to offer itself as a platform to give the possibility to anyone to create decentralized applications (dApps) that give life to decentralized companies (DAO), animated by an idea of freedom and security. These dApps can be thought of as complex smart contracts activated by sending them Ether, Ethereum's virtual currency. (Burniske and Tatar, 2017)

A third cryptocurrency considered in this study is Litecoin. In the first two years of Bitcoin's life, among the many new projects and cryptocurrencies that have been released, Litecoin is the first that has maintained a significant value to this day. The cryptocurrency was developed by Charlie Lee, a software engineer at Google. Litecoin's goal was to improve Bitcoin by increasing the settlement times for transactions (to about one block every 2.5 minutes) and lowering the fees. Other than that, much of Litecoin remained similar to Bitcoin (Burniske and Tatar, 2017). As of April 2022, Litecoin is no longer among the top ten cryptocurrencies by market capitalization (CoinMarketCap, 2022).

The last cryptocurrency to take part in this research is Ripple. Initially designed to be a currency competing with the traditional payments system used by banks, it has since moved on to be used by banks themselves to improve foreign exchange and international payments. It was created in 2012 by OpenCoin, then rebranded Ripple Inc in 2015 (Lewis, 2018). As of April 2022, Ripple ranks among the top 10 cryptocurrencies by market capitalization, capitalizing around $32 billion (1.77% of the total cryptocurrencies’ market capitalization) (CoinMarketCap, 2022).

2.2 Cryptocurrencies as Financial Assets

Early debates on cryptocurrencies have focused on whether they can be associated with traditional currencies or a new type of asset class (Glaser et al., 2014; Baek and Elbeck, 2015). Lewis (2018), for example, points out that as a unit of account Bitcoin fails miserably, mainly due to its volatility. Moreover, other studies have found important stylized facts about cryptocurrencies that are common to traditional financial assets, such as heteroscedasticity (Gkillas and Katsiampa, 2018), leptokurtosis (Chan et al., 2017), and long-memory (Phillip et al., 2018).

As public interest in this new asset class has increased over the years, the literature has started looking into cryptocurrencies, with a particular focus on Bitcoin, from a portfolio management perspective. Briere et al. (2015) use weekly data on Bitcoin over the 2010-2013 period and conclude that even a tiny proportion of Bitcoins may dramatically improve the risk-return of well-diversified portfolios. However, they also point out that the results should be taken with caution as the data may only reflect the early-stage behaviour of Bitcoin, which may not last in the medium or long term. More recent literature confirms the findings of Briere et al. (2015). The results of Bouri et al. (2017) and Klein et al. (2018), for example, suggest that Bitcoin does not serve as a safe haven, but it’s suitable for diversification purposes only. Klabbers (2017) approaches the performance of Bitcoin from a global investment point of view and, by using six years of data, shows that Bitcoin is an effective diversifier. Furthermore, Kajtazi and Moro (2019) conclude that by adding bitcoin, the portfolio performance improves; but this is due more to the increase in returns than to the reduction of volatility.

Other studies analyze the role of diversification among cryptocurrencies. Using empirical data on ten cryptocurrencies, Liu (2019) evaluates the out-of-sample performance of different asset allocation models across cryptocurrencies and concludes that diversification can significantly improve investment results. Brauneis and Mestel (2019), instead, use the Markowitz mean-variance framework and find that combining cryptocurrencies enriches the set of low-risk cryptocurrency investment opportunities.

There are relatively fewer studies, instead, that focus on the contribution of multiple cryptocurrencies to the performance of portfolios constructed with traditional assets. Corbet et al. (2018) look into investing in traditional assets, along with three cryptocurrencies (Bitcoin, Ripple, and Ethereum), and conclude that cryptocurrencies may offer short-term diversification benefits. Inci and Lagasse (2019) consider a more extensive sample period and examine the role of the same three cryptocurrencies in an optimal portfolio, following the mean-variance optimization technique of Merton (1990). They find that the portfolio with the traditional asset classes in equity, fixed income, real estate, and the volatility index representing the derivatives market for sophisticated investors is further enhanced with the cryptocurrencies, Bitcoin, Ripple, and Litecoin. Finally, Colombo et al. (2021) analyze the effect of adding cryptocurrencies on a sample of developing and developed economies. Their results suggest that even a global portfolio, which already enjoys international diversification, may benefit from investing marginally in cryptocurrencies. These diversification benefits occur both before and during the early stage of the Covid-19 pandemic.

The last two studies mentioned above are the closest to mine. However, there are several points where my research differs from theirs. First, I consider a larger number of investment strategies and performance measures to make the results more robust. Second, this paper also investigates the impact of transaction costs on the overall portfolio performances, which is closer to real-life investments. Third, I use the mean-variance efficient frontier to offer a visual representation of how cryptocurrencies can expand the choice of efficient portfolio allocations towards a region with higher risk-adjusted returns.

Chapter 3: Theoretical models

The paper at hand uses the mean-variance framework to analyze the performance of different portfolio models with and without cryptocurrencies. Before getting in the hearth of the empirical study, this chapter briefly sets out the theoretical background used for the analysis. Section 3.1 traces the key steps in portfolio construction, while section 3.2 reviews the fundamental concepts of the mean-variance theory.

3.1 Portfolio Construction

Portfolio construction can be defined as the process of understanding how different asset classes and weightings impact each other, their performance and risk and how decisions are made according to the investor's objectives (BlackRock, 2022). According to Markowitz (1959), a good portfolio should be more than a long list of good stocks and bonds. It should be able to provide "*the investor with protections and opportunities with respect to a wide range of contingencies*".

Broadly, the process of creating an investment strategy can be divided into three steps (Pomante, 2008):

1. Identification of investor preferences. In portfolio selection, the essential variables to consider are the expected return of the portfolio and the variability of returns. A rational investor will try to maximize the former, as the higher the expected return and the greater the likelihood of achieving higher values of the final return. At the same time, he will try to minimize the latter; as the latter increases, so does the likelihood that future returns will deviate from the expected value, assuming thus negative values.
2. Estimation of the future performance of asset classes. The asset manager is called upon to make the forecasts that will act as input in the final optimization phase. This step is closely linked to the previous one, as it is the investor who defines the time horizon, the asset classes, and the statistical parameters to be estimated. The task of the asset manager is, therefore, to produce reliable estimates to be used in the optimization process.

3. Optimization. Once the investor's preferences have been outlined, and the future distribution of the market returns has been estimated, the portfolio that maximizes the investor's preferences is determined. If a non-rigorous approach prevails, the optimization phase can only occur in qualitative terms, giving rise to the so-called *naive* solution. In a mathematical framework, instead, the portfolio weights are given by maximizing the investor's preference function in compliance with the constraints (V) imposed by the composition of the portfolio, such as the budget and the short sales constraints:

$$\underset{w}{Max}\, F(p) \qquad \text{with respect to: V}$$

where w identifies the portfolio asset weights vector and $F(p)$ is the investor's preference function.

3.2 Mean-Variance Framework

The theory of portfolio selection based on the mean-variance framework is a cornerstone of modern asset management. The underlying assumption is that investors are rational, and they make choices among risky assets based only on expected return and risk, where risk is measured in terms of variance. Two primary sources of risk affect the investor: systematic risk and non-systematic risk. The former refers to the risk of investing in the market itself, whereas the latter corresponds to the specific firm or asset risk.

The systematic risk is subject to macroeconomic factors, such as economic cycles and interest rates. The Covid-19 pandemic, for example, has negatively impacted most asset classes, from stocks and commodities to currency and cryptocurrency markets. By no means the market risk can be reduced by any amount through diversification (Klabbers, 2017; Mangram, 2013; Ross, Westerfield, and Jaffe, 2002).

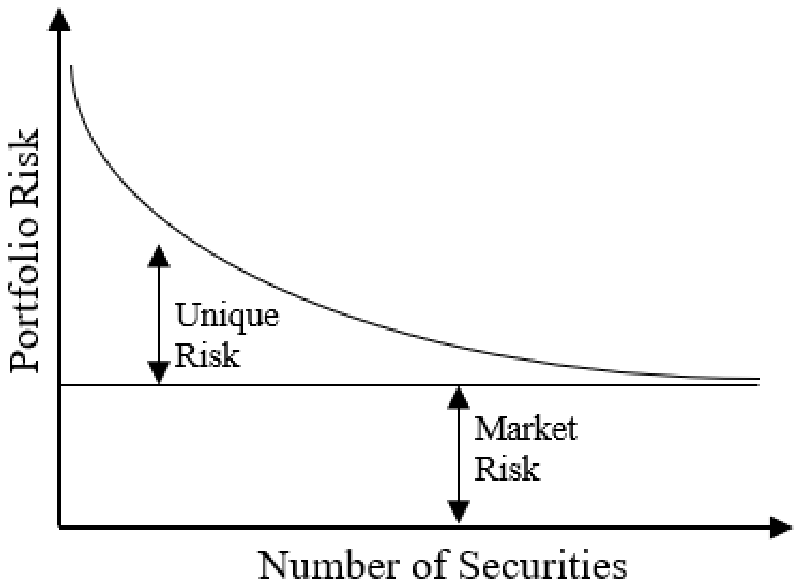
On the other hand, the specific asset risk is determined by microeconomic factors. As an example of non-systematic risk, one can consider a problem regarding blockchain technology. Such a problem will primarily and perhaps only affect the cryptocurrency market, not other traditional markets. This risk can be eliminated through appropriate diversification.

The difference between the firm-specific risk and the market risk can be seen in Figure 1, where a convex line represents the specific risk. In contrast, the market risk is indicated with a horizontal straight line. As we can see, the former decreases as the number of assets in the portfolio increases while the latter remains constant. In other words, "Diversification cannot eliminate all variance" (Markowitz, 1952, p.79).

Figure 1

Unique Risk and Market Risk

The image plots the relationship between the portfolio risk and the number of securities in the portfolio. The convex line represents the unique risk, as it can be diversified away. The market risk, instead, cannot be reduced. The overall risk decreases as the number of securities increases, but it cannot be eliminated. Source: based on the author's calculation



Investing the wealth in a single asset could generate significant returns. However, it also bears the risk of losing the entire investment. According to Markowitz (1952, 1959), the risk-return ratio can be increased through optimal diversification. An investor should construct a portfolio with assets that are little or not correlated at all, thus spreading the individual risk of each asset to reduce the overall risk. By leveraging the interaction of assets, losses related to one asset are offset by gains from another one. Diversification allows investors to increase the expected return while reducing the specific risk of each asset. Thus, the correlation among asset returns can represent the critical factor for optimal diversification.

3.2.1 Only two assets in the portfolio

To better illustrate the concept of proper diversification, consider the simplest case where we only have two assets: Bitcoin and S&P 500. The sample period ranges from September 2015 to January 2022. During this time, Bitcoin recorded an average daily return (μ_B) of 0.33% and a daily standard deviation (σ_B) of 4.23%. In the same period, the S&P 500 recorded an average return (μ_S) of 0.06%, with a standard deviation (σ_B) equal to 1.15%. The expected return of a portfolio composed exclusively of these two assets is therefore given by:

$$\mu_P = w_B \mu_B + (1 - w_B)\mu_S \tag{1}$$

where w_B represents the weight imposed on Bitcoin. Since the investment consists of only two assets and the sum of the weights must be equal to 1, the investment in the S&P 500 index is given by $(1 - w_B)$. The variance, instead, is given by:

$$\sigma_P^2 = w_B \sigma_B^2 + (1 - w_B)\sigma_S^2 + 2w_B(1 - w_B)\sigma_B \sigma_S \rho_{BS} \tag{2}$$

where ρ_{BS} indicates the correlation between Bitcoin and S&P 500.

As one can see from the equations above, the correlation between assets enters positively into the formula of portfolio variance. A higher correlation leads to greater variability in portfolio returns, thus increasing its risk. Instead, a low or negative correlation decreases the specific risk linked to individual assets. A risk-free return could be achieved in the extreme case of a perfect negative correlation. One can visualize these concepts in Figure 2.

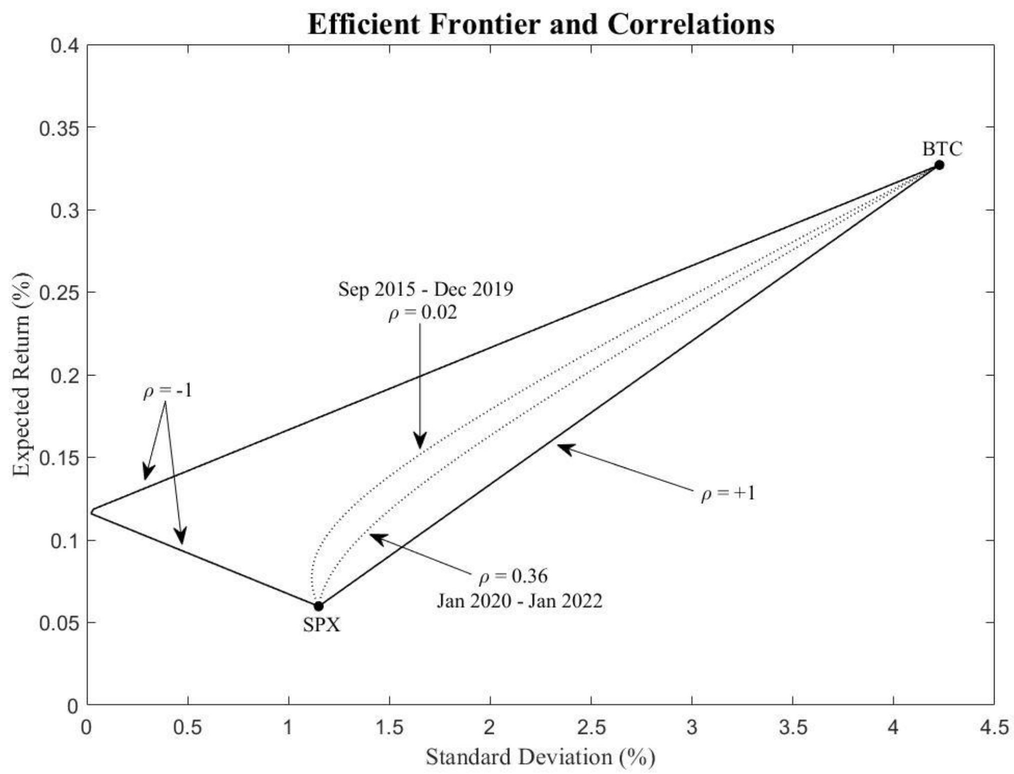
Figure 2 shows all the possible portfolios that can be obtained by combining the two assets as a function of the correlation between the two assets. Assuming the correlation between Bitcoin and S&P 500 returns is perfectly positive, the line of feasible portfolios takes the form of a segment. In this case, there is no diversification benefit, as the standard deviation of the portfolio, like the expected return, is the weighted average of the risks of the two assets. As the value of the correlation coefficient decreases, the curve of feasible portfolios takes a concave shape. The lower the correlation coefficient, the greater the benefits of diversification.

To highlight the relevance of the correlation between assets in the portfolio construction, Figure 2 shows two intermediate correlation coefficients between Bitcoin and S&P 500. One refers to the period Sep 2015 – Dec 2019, while the second refers to Jan 2020 – Jan 2022. In the first case, the correlation between the two assets equals 0.02, while in the second case, this value rises to 0.36. This substantial difference can be attributed to the turbulent period financial markets have been going through since the beginning of the pandemic. In this regard, Forbes and Rigobon (2002) show how in times of crisis, stock market correlations increase. This increase in stock market correlations lowers the efficient diversification of the portfolio. One can see how the correlation between Bitcoin and S&P 500 was relatively low until 2020, with a correlation coefficient of 0.02. This characteristic of Bitcoin, and cryptocurrencies in general, gained the attention of investors for their ability to offer greater

Figure 2

Efficient Frontier and Correlations

The image plots the set of all possible portfolios that can be obtained by combining two assets (Bitcoin (BTC) and the Standard and Poor's 500 stock market index (SPX)) as a function of the correlation between the two assets. Short sales are not allowed. Daily data for Bitcoin are obtained from Coin Metrics, while for S&P 500 are obtained from Yahoo Finance. Source: based on author’s calculation.



diversification in a portfolio. Several studies (Wu and Pandey, 2014; Andrianto and Diputra, 2017; Klabbers, 2017) have emphasized the usefulness of cryptocurrencies in improving the efficiency of an investment portfolio. However, cryptocurrencies had never yet faced a period of general stress in the markets. Instead, the last two years have been evidence of how the cryptocurrency market also suffers the negative consequences of macroeconomic factors. Thus, the correlation between the S&P 500 and Bitcoin has risen to 0.36.

In Figure 2, one can see how the curve of possible portfolios between the two assets in the pre-pandemic period is above the one which describes the last two years. Therefore, the figure shows that a lower correlation between the assets in the portfolio brings greater benefits deriving from diversification. For a fixed value of the standard deviation, it was possible to obtain a higher return in the first period than in the second. Similarly, for a fixed value of the expected return, the variability of these returns was lower before the 2020 crisis.

3.2.2 The general case of n assets in the portfolio

If one assumes that the number of assets (n) is higher than three and construct the portfolios using all the combinations of weights, we can still get the set of feasible portfolios. However, unlike the case of two assets, the different combinations of mean (μ_P) and standard deviation $(\sigma_P,)$ are located in a region and no longer on a line. Figure 3 offers a graphical representation of the cloud of risk-return combinations of portfolios built with three assets: Bitcoin, Ethereum, and S&P 500.

In this case, a portfolio will be considered mean-variance efficient if it minimizes variance for a given expected average return or maximizes the expected average return for a given variance. The efficient frontier is thus the set of mean-variance efficient portfolios.

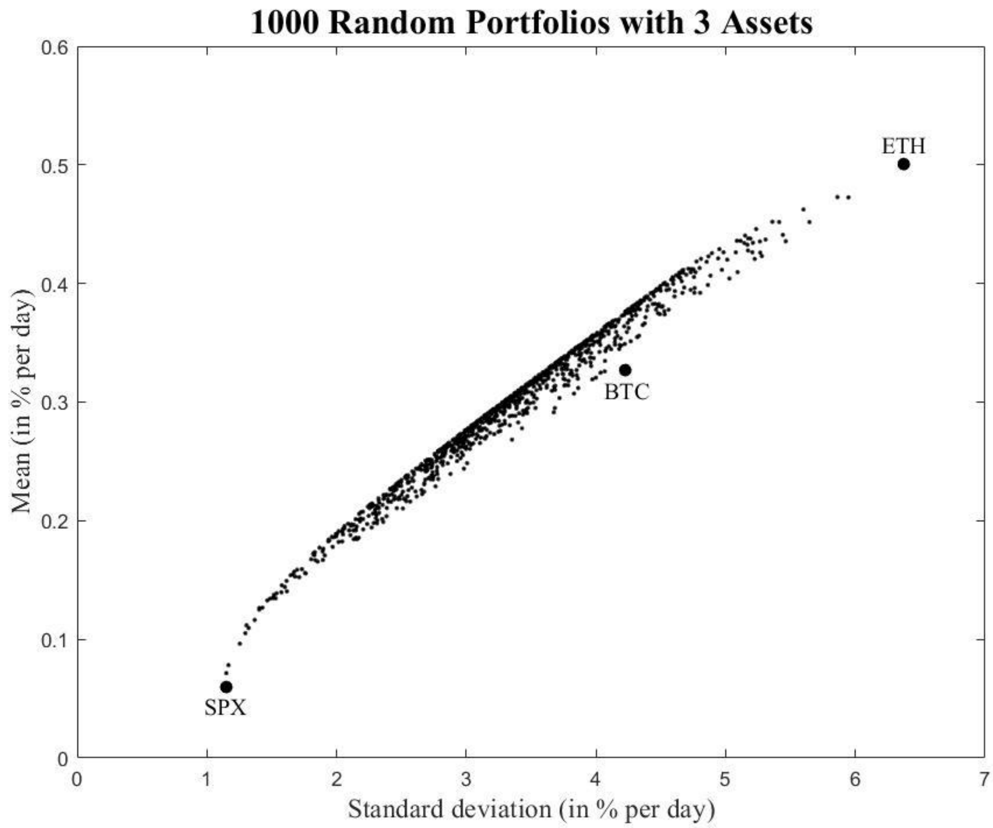
The mean-variance approach is widely used in optimizing portfolio allocation. However, the model is not free from criticism, especially regarding the stringent set of hypotheses it needs to work well (Michaud and Michaud, 2008; Pomante, 2008). It requires, for example, returns to be normally distributed (Klabbers, 2017), whereas cryptocurrencies do not have this feature (Chan et al., 2017). Notwithstanding, several studies like Levy and Markowitz (1979) or Kroll et al. (1984) found almost equivalence of the mean-variance approach to expected utility maximization under non-normality.

I have decided to follow Inci and Lagasse (2019) and use the mean-variance approach in this study as I believe it represents a good benchmark. Furthermore, it incorporates two decisive aspects for every asset manager: the contribution of diversification and the trade-off between risk and expected return (Pomante, 2008). The model also allows quickly reaching an optimal solution without the aid of powerful processors (Pomante, 2008).

Figure 3

Simulation of 1000 Random Portfolios with 3 Assets

The image plots 1000 random portfolios obtained by combining three assets: Bitcoin (BTC), Ethereum (ETH), and Standard and Poor's 500 stock market index (SPX). Short sales are not allowed. Daily data for the two cryptocurrencies are obtained from Coin Metrics, while for S&P 500 are obtained from Yahoo Finance. Source: based on author’s calculation.



Chapter 4: Data and Methodology

4.1 Data

Daily price data for eight different asset classes are used in the analysis. These are four cryptocurrencies: Bitcoin, Ethereum, Ripple, Litecoin; equity represented by the S&P 500; real estate expressed by the Vanguard Real Estate Index; fixed income securities represented by the Vanguard Total Bond Market Index; and a derivatives proxy utilized by sophisticated investors represented by the CBOE Volatility Index (VIX, hereafter). For the four cryptocurrencies, the data are obtained from Coin Metrics. The price data for the other four asset classes are collected from Yahoo Finance. I focus on Bitcoin, Ripple, and Litecoin because they have been in existence the longest, and Ethereum because it has an essential role in the whole blockchain infrastructure. Bitcoin and Ethereum together account for 60% of the total market capitalization of cryptocurrencies. As Ethereum started to be traded in the Summer of 2015, I have decided to consider a sample period with daily frequency going from September 1st, 2015, to January 31st, 2022.

Once the price data of all the asset classes were collected from the two different sources, daily returns were calculated using the following formula:

$$r_j = \frac{P_t - P_{t-1}}{P_{t-1}} \equiv \frac{P_t}{P_{t-1}} - 1 \tag{3}$$

where:

r_j = return on stock j

P_t = Price of the stock j at time t

P_{t-1} = Price of the stock j at time $t-1$

Since cryptocurrencies are traded non-stop seven days a week while traditional markets are closed on weekends and public holidays, there is a difference in the number of observations considered.

Traditionally, researchers have employed different techniques to deal with missing values. The two most common solutions include deletion and single imputation approaches (Baraldi and Enders, 2010; Peugh and Enders, 2004). In this case, the former would imply dropping weekend returns for cryptocurrencies. In contrast, the latter would mean imputing the weekend missing data of the traditional markets with seemingly suitable replacement values. The first would alter cryptocurrencies' moments and sample size, while the second would change traditional market moments and sample size.

Following the relatively more recent literature (Inci and Lagasse, 2019; Klein et al., 2018; Gil-Alana et al., 2020), this study uses the deletion technique, considering only returns during the week to synchronize the dataset.[1]

Once the returns were computed, and the adjustments were made, one final cohesive dataset was created. The final sample contains 1615 daily returns for eight different asset classes.

4.2 Descriptive Analysis

Before delving into the methodology, it is helpful first to look at the performance of the different asset classes and how they relate to each other. Table 1 reports summary statistics for the eight securities considered in the study, while Table 2 reports the correlation matrix.

One can notice that cryptocurrencies have had significantly higher average daily returns than the other types of asset classes reported in the table. Ripple is the asset with the highest average return during the period considered, while the Bond Market Index recorded the lowest one. However, the higher returns of cryptocurrencies are offset by greater risk, as evidenced by the standard deviation. The second column of Table 1 highlights how the average daily volatility of Ripple is equal to 7.65%, against 1.15% of the S&P 500 or 0.30% of the Total Bond Market Index. The risk related to the cryptocurrency market is also visible from column 3 in Table 1, representing the maximum drawdown the assets had to face during the period under review. The range between the considered cryptocurrencies goes from 83.78% of Bitcoin to 94.90% of Ripple. Arguably few investors would be willing to face such a high drawdown.

[1] Note that I replicated the study by using a single imputation approach and the main results remained unchanged.

The Sharpe Ratio (for details, see Section 4.5.1) indicates that the best asset in terms of the risk-return relationship is Ethereum, followed by Bitcoin and Ripple. The worst performance, instead, is given by the Bond Market Index.

Skewness and kurtosis point out that the returns of these assets are far from being normally distributed. The normal distribution of returns is one of the assumptions underlying the mean-variance approach used in this study. Nevertheless, as already mentioned in the previous chapter, I have decided to use this approach as it turns out to be a good benchmark. More importantly, the aim of this empirical study is to investigate the role of cryptocurrencies in enhancing the performance of portfolios constructed with traditional assets. Therefore, the mean-variance approach has the advantage of incorporating the contribution of diversification and the trade-off between risk and expected return, two key factors for the portfolio analysis (Pomante, 2008).

Table 1

Descriptive Statistics

ETH is Ethereum, BTC is Bitcoin, XRP is Ripple, LTC is Litecoin, SPX is the Standard and Poor's 500 stock market Index, VIX is the CBOE Volatility Index, BND is the Vanguard Total Bond Market Index, and VNQ is the Vanguard Real Estate Index. Std is the Standard Deviation, MAX DD is the Maximum Drawdown, SR is the Sharpe Ratio, Skew is the Skewness, and Kurt is the Kurtosis. Mean, Std and Max DD are expressed in percentage terms. Daily raw price values and raw return values are used for the statistical calculations in the table. The sample period is from September 1, 2015, to January 31, 2022. Values rounded to the 3rd decimal place. Source: own elaboration.

	Mean	Std	Max DD	SR	Skew	Kurt
ETH	0,501	6,373	93,998	0,079	0,365	7,507
BTC	0,327	4,226	83,783	0,077	-0,140	9,844
XRP	0,503	7,652	94,905	0,066	2,207	20,891
LTC	0,263	6,263	93,635	0,042	2,127	24,955
SPX	0,060	1,150	33,925	0,052	-0,685	22,790
VIX	0,352	9,048	81,848	0,039	2,667	24,485
BND	0,002	0,301	8,717	0,005	-2,299	99,497
VNQ	0,033	1,340	42,844	0,024	-1,632	30,973

As it can be seen from Table 2, all the assets are negatively correlated with the VIX. This is because volatility-based investment opportunities such as VIX naturally have high negative correlations with equity markets. In particular, the VIX is based on the implied volatility of S&P 500 Index options, and therefore this explains the high negative correlation between S&P 500 and VIX.

The highest positive correlation coefficient is between the S&P 500 and the Vanguard Real Estate Index. On the other hand, among cryptocurrencies, the highest positive correlation is between Bitcoin and Litecoin, with a value equal to 0.67. This number suggests that the major cryptocurrencies generally tend to be highly correlated. A possible explanation could be that Bitcoin represents the leading cryptocurrency in the sector, in some periods even accounting for over 60% of the total market capitalization. Therefore, the price movements in the cryptocurrency market are often dictated by Bitcoin's performance.

The correlations between cryptocurrencies and the other investment opportunities range between -0.17 (Ethereum - VIX) and 0.18 (Bitcoin – S&P 500). These values are relatively low, considering that U.S. stock correlations are often within a 0.3 to 0.5 range (Michaud and Michaud, 2008). Therefore, the correlation matrix highlights that we could improve the performance of portfolios by including cryptocurrencies. This might be one of the reasons behind the recent rise in the popularity of this new asset class.

Table 2

Correlation Matrix

ETH is Ethereum, BTC is Bitcoin, XRP is Ripple, LTC is Litecoin, SPX is the Standard and Poor's 500 stock market Index, VIX is the CBOE Volatility Index, BND is the Vanguard Total Bond Market Index, and VNQ is the Vanguard Real Estate Index. Daily returns are used for the Pearson correlation values in the table. The sample period is from September 1, 2015, to January 31, 2022. Values rounded to the 2nd decimal place. Source: own elaboration.

	ETH	BTC	XRP	LTC	SPX	VIX	BND	VNQ
ETH	1,00							
BTC	0,56	1,00						
XRP	0,43	0,42	1,00					
LTC	0,56	0,67	0,49	1,00				
SPX	0,18	0,18	0,13	0,16	1,00			
VIX	-0,17	-0,16	-0,11	-0,15	-0,71	1,00		
BND	0,10	0,13	0,08	0,09	0,04	0,03	1,00	
VNQ	0,11	0,13	0,09	0,13	0,75	-0,45	0,19	1,00

The fact that correlations between cryptocurrencies and traditional financial assets were low in the past does not mean that they will be low in the future as well. In the previous chapter, we have seen how the correlation between Bitcoin and the S&P 500 has risen to 0.36 in the last two years. Therefore, it is interesting to analyze if cryptocurrencies can truly offer diversification benefits and improve the overall performance of a portfolio made up of traditional assets.

4.3 Portfolio Models

I have decided to consider five different portfolio models to analyze the impact of cryptocurrencies in a portfolio with traditional assets and see if they add value in terms of the risk-return relationship. First, I consider the naively diversified $1/N$ strategy, which may also be seen as a benchmark portfolio (Brauneis and Mestel, 2019; DeMiguel et al., 2009). DeMiguel et al. (2009) argue, for example, that the naïve rule could be used as a benchmark as it is easy to implement. Moreover, despite the multiple alternative sophisticated methods developed in the past, investors still rely on such simple allocation rules (DeMiguel et al. 2009).

Next, I consider four other data-driven optimized portfolios: Minimum Variance (MV, hereafter) portfolio, Tangency portfolio, Risk-Parity (RP, hereafter) portfolio, and Maximum Utility (MU, hereafter) portfolio. These portfolios have been chosen in such a way as to consider different investor categories. Some investors may be more risk-averse and therefore consider portfolios such as MV and RP more, while others may be more risk-seeking and therefore prefer portfolios such as Tangency and MU.

The general steps in the portfolio optimization technique (Inci and Lagasse, 2019) are:

1. Creation of the return vector (μ), where each element is the average of the daily returns of the financial securities included in the portfolio;
2. Creation of the variance-covariance matrix, $(\sum)$, using the entire data set of returns of all the investments in the portfolio; and
3. Determination of the optimal weights vector, (w), according to the specific objective function of each portfolio.

This section discusses the characteristics of the five risk-based portfolio construction methodologies. Consider the following terminology for the next paragraphs:

- $w -\ N \times 1$ vector of asset weights
- $\sum -\ N \times N$ variance-covariance matrix of asset returns
- $\mu - N \times 1$ vector of expected returns
- $\iota\ -\ N \times 1$ vector of ones

4.3.1 Equally Weighted Portfolio

An equally weighted (EW, hereafter) portfolio represents one of the simplest approaches to constructing a portfolio and attempting to achieve diversification. In an EW portfolio, all assets are given the same weight. If we consider that we have N assets in the portfolio, then the weight of each asset is given by:

$$w_{EW} = \frac{1}{N} \tag{4}$$

This strategy does not involve any optimization or estimation problem and completely ignores the characteristics of the assets in a portfolio.

4.3.2 Minimum Variance Portfolio

The MV portfolio allocation strategy aims at building an optimal portfolio without making assumptions about returns but simply minimizing the volatility of the portfolio returns using only historical data, such as the variances of the securities and the correlations existing between them. In this way, one of the major problems concerning asset allocation decisions is avoided, namely the choice to be based on returns (historical, current, expected, estimated, or other): a choice that inevitably brings subjective hypotheses that are difficult to sustain. For example, Chopra and Ziemba (1993) pointed out that the cash equivalent loss for errors in expected values is greater than that for errors in variances and covariances. The MV portfolio is the only portfolio on the efficient frontier without expected returns as inputs.

The MV portfolio solves the following minimization problem:

$$\min_{w} \quad w'\sum w \qquad\qquad s.t. \qquad w'\iota = 1 \tag{5}$$

The solution to the above maximization problem leads to a closed form equation for the asset weights. These are given by:

$$w_{MV} = \frac{\sum^{-1}\iota}{\iota'\sum^{-1}\iota} \tag{6}$$

where w_{MV} are the MV portfolio weights, and $\sum^{-1}$ is the inverse of the variance-covariance matrix.

4.3.3 Tangency Portfolio

The Tangency portfolio is that portfolio on the efficient frontier that maximizes the Sharpe Ratio, defined as:

$$\text{Sharpe Ratio} = \frac{R_P - R_f}{\sigma_p} \tag{7}$$

where: R_P is the portfolio return, R_f is the risk-free rate, and σ_p is the standard deviation of the portfolio's excess return.

Considering a zero risk-free rate and given that the portfolio return (R_P) and standard deviation (σ_p) in matrix format are given by:

$$R_P = w'\mu \qquad\qquad \sigma_p = \sqrt{w'\Sigma w} \tag{8}$$

we then have that the Tangency portfolio maximizes the following objective function:

$$\max_{w} \frac{w'\mu}{\sqrt{w'\Sigma w}} \qquad\qquad s.t. \qquad w'\iota = 1 \tag{9}$$

By solving the unconstrained maximization problem (considering, however, the constraint that the weights add to 1), we get a closed-form analytical solution for the Tangency portfolio weights (w_T), which, for portfolios with no risk-free assets, are given by:

$$w_T = \frac{\Sigma^{-1}\mu}{\mu'\Sigma^{-1}\iota} \tag{10}$$

Instead, the constrained maximization problem, where short sales are not allowed, for example, does not have a simple closed-form analytical solution.

4.3.4 Risk-Parity Portfolio

The RP approach to portfolio construction wights the asset classes by their risk contribution to the portfolio. In other words, each asset should contribute equally to the portfolio's volatility.

Depending on how we measure or define the risk, there are several ways to construct an RP portfolio. However, following Brzenk, Cheng and Liu (2020), I have decided to use the standard deviation of returns to measure risk, as in practice is the default risk measure adopted by investors.

We can then define the marginal contribution of asset i to the total value of portfolio risk as:

$$MC_i = w_i * \sigma_p * \beta_i \tag{11}$$

where w_i is the weight of asset i in the portfolio, σ_p is the standard deviation of the portfolio's returns, and β_i is defined as:

$$\beta_i = \frac{\sigma_{ip}}{\sigma_p^2} \tag{12}$$

with σ_{ip} being the covariance between asset i and portfolio p while σ_p^2 is the variance of the portfolio.

A portfolio is therefore said to be a RP portfolio if it satisfies:

$$w_i\beta_i = w_j\beta_j = \frac{1}{N} \tag{13}$$

Finding a solution for the vector of weights is not a simple task. Consider, for example, the case where all volatilities are equal, $\sigma_i = \sigma$ for all i, while the correlations between assets differ. In this case, we have that the weight attributed to each component i is inversely proportional to its beta (Maillard et al., 2010):

$$w_i = \frac{\beta_i^{-1}}{\sum_{i=1}^{n} \beta_i^{-1}} \tag{14}$$

Since β_i depends on portfolio p, which by definition includes w_i, the solution is endogenous, and consequently, it is necessary to resort to numerical solutions.

Maillard et al. (2010) propose a solution by using a Sequential Quadratic Programming algorithm:

$$w = \arg\min f(x) \qquad s.t. \qquad w'\iota = 1 \; and \; 0 \leq w_i \leq 1 \tag{15}$$

where:

$$f(x) = \sum_{i=1}^{n} \sum_{j=1}^{n} \left(w_i (\Sigma w)_i - w_j (\Sigma w)_j \right)^2 \tag{16}$$

A simpler approach to construct an RP portfolio, instead, is to set each asset's weight to be proportional to the inverse of its standard deviation (Brzenk, Cheng and Liu, 2020):

$$w_i = \frac{1/\sigma_i}{\sum_{i=1}^{n}\left(\frac{1}{\sigma_i}\right)} \tag{17}$$

The result is that lower volatile assets will have a higher weight than higher volatile assets.

The simpler version allows us to get a closed-form solution, but it assumes that we have equal correlations for every couple of assets in the portfolio. Nevertheless, Bridgewater Associates, one of the biggest investment management firms globally, promotes a version of RP portfolio that only focuses on volatility and ignores the correlation information. Their motivation is based on the fact that correlations are unstable and unpredictable over time. Asset risk is therefore difficult to predict (Bridgewater, 2022).

However, regardless of the approach used, the RP portfolio is generally fixed-income heavy, implying lower portfolio volatility and returns (Chaves et al., 2011).

4.3.5 Maximum Utility Portfolio

Following the steps of Liu (2019), I use the utility (U) of an asset defined by the quadratic form:

$$U_i = \mu_i - \frac{\gamma}{2}\sigma_i^2 \tag{18}$$

where γ represents the risk aversion parameter.

The risk aversion parameter plays a critical role in defining the utility of an asset. As one can see from the utility function above, it enters the equation with a negative sign, meaning that the greater the degree of risk aversion of the person, the lower the utility. This factor has implications for the weights to be attributed to the assets in the portfolio. The higher the degree of risk aversion, the lower the weights attributed to the higher volatile assets or, the greater the weights assigned to the less risky assets in the portfolio. I follow Liu (2019) and report all the results considering $\gamma = 1$.

The MU portfolio is defined by the weights that maximize the following objective function:

$$\max_{w} \quad w'\mu - \frac{\gamma}{2} w'\Sigma w \qquad s.t. \qquad w'\iota = 1 \tag{19}$$

This maximization problem does not offer a closed-form solution where weights sum to 1. In my study, I therefore perform a constrained optimization.

4.3.6 Policy Constraints

One of the most significant advantages of using the mean-variance approach is incorporating policy constraints (Klabbers, 2017). The unique policy constraints can be found by altering the weights of the assets in the portfolio.

In this study, I initially limit the weights of the assets in the portfolio to be non-negative, $w_i \geq 0$. This means that short selling is not allowed. Short sales are often difficult to perform and very risky. To compute the weights of the assets for an optimized portfolio, I rely on numerical algorithms, as there is no closed-form solution.

Subsequently, to obtain a more complete and robust analysis, I allow the weights to assume negative values, meaning that short sales are allowed. Thus, if an asset's weight at any given point in time is negative, that asset is sold short in that time frame. Here I compute the weights by using the closed-form solution whenever possible, as it is simpler and quicker to obtain.

However, in both cases, the budget constraint is kept, $w'\iota = 1$, meaning that all the wealth is invested. In the first case, I introduce the constraint manually in the optimization problem. In the second case, the closed-form formulas are by construction such that $w'\iota = 1$.

Table 3 summarizes the various asset allocation models.

Table 3

Asset Allocation Models

The first column shows the names of the asset allocation models. The second column shows the optimization problem of each model under the no short sales constraint ($w_i \geq 0$) and the budget constraint ($w'\iota = 1$). The third column shows the closed-form formulas used for calculation when short sales are allowed. The budget constraint here is respected by construction. Source: own elaboration.

Model	Constrained Optimization ($w'\iota = 1,\ w_i \geq 0$)	Closed-Form Solution ($w'\iota = 1$)
1/N Rule (EW)		$w_{EW} = \frac{1}{N}$
Minimum Variance (MV)	$\min_{w} \ w'\Sigma w$	$w_{MV} = \frac{\Sigma^{-1}\iota}{\iota'\Sigma^{-1}\iota}$
Tangency Portfolio (TP)	$\max_{w} \ w'\mu/\sqrt{w'\Sigma w}$	$w_{TP} = \frac{\Sigma^{-1}\mu}{\mu'\Sigma^{-1}\iota}$
Risk-Parity (RP)	$\min_{w} \ \sum_{i=1}^{n}\sum_{j=1}^{n}\left(w_i(\Sigma w)_i - w_j(\Sigma w)_j\right)^2$	$w_i = \frac{1/\sigma_i}{\sum_{i=1}^{n}\left(\frac{1}{\sigma_i}\right)}$
Maximum Utility (MU)	$\max_{w} \ w'\mu - \frac{\gamma}{2} w'\Sigma w$	

4.4 In- and Out-of-Sample Analysis

To have a more robust and complete analysis, this thesis addresses the research question both in- and out-of-sample.

The in-sample approach uses the entire sample of observations to estimate the parameters for the optimization process, such as the mean and the variance-covariance matrix. This approach, however, is infeasible for actual investments. For example, if our starting date is September 1st, 2015, one can obtain the data no later than the starting date. Instead, the in-sample analysis uses data before and after that date, which is unrealistic in real investments (Liu, 2019). Therefore, the performance under the in-sample method is viewed more as an ideal theoretical case than a real one.

In the out-of-sample analysis, on the other hand, the weights are computed in a rolling window procedure. I follow the Liu (2019) methodology to structure the out-of-sample analysis. The steps in this process are:

1. Use the nearest $M = 252$ days' before and on the rebalancing time t to estimate the optimization process parameters, namely the expected value and the variance-covariance matrix. Liu (2019) uses $M = 360$ (1 year) in his sample as it includes only cryptocurrencies, and therefore, he has a full year of daily observations. Since in this study only days when the stock market was open are considered, one year of observations corresponds approximately to 252 trading days;
2. Solve the corresponding optimization problem according to the portfolio strategies and obtain the weights at time t;
3. Rebalance the weights according to the first two steps after the holding period τ.

For example, the dataset for this study ranges from September 1st, 2015, to January 31st, 2022, and I consider a rebalancing period of $\tau = 22$ days, which represents approximately the number of trading days in one month. Liu (2019) also considers one month, but as that study is focused only on cryptocurrencies, the rebalancing period is chosen to be $\tau = 30$ days. The first 252 daily returns are used to estimate the parameters of the specific portfolio strategy and solve the weights on the first rebalancing date, which is 31-Aug-2016. These weights last for the following $\tau = 22$ trading days. The second rebalancing date is 03-Oct-2016, and the nearest 252 days' returns are used to estimate the parameters of the different portfolio models and solve the weights on 03-Oct-2016, which will then last for the next $\tau = 22$ days. This routine is repeated until the end of the sample period.

4.5 Evaluating portfolio performance

Several standard investment performance measures are used to evaluate the effect of adding cryptocurrencies to portfolios with traditional assets: Sharpe Ratio, Sortino Ratio, Omega Ratio, Value at Risk (hereafter, VaR), Expected Shortfall (hereafter, ES), Turnover, and Transaction Costs.

4.5.1 Sharpe Ratio

The Sharpe Ratio originated in 1966 and was introduced by William Sharpe (Sharpe, 1966). It is the most popular measure of risk-adjusted performance, and it measures the performance of an investment compared to a risk-free asset after adjusting for its risk. It tells us how well the return of an asset can compensate for the risk associated with the asset itself. The higher the SR, the more the investor is compensated for the risk he has borne.

The ratio makes use of the first two moments of the return distribution and is computed in the following way:

$$\text{Sharpe Ratio} = \frac{R_P - R_f}{\sigma_p} \tag{20}$$

where: R_P is the portfolio return, R_f is the risk-free rate, and σ_p is the standard deviation of the portfolio's excess return.

For simplicity, I follow Brauneis and Mestel (2019) and assume a zero risk-free rate for this study.

4.5.2 Sortino Ratio

A direct evolution of the Sharpe Ratio is the Sortino Ratio, developed by the economist Frank Sortino. He proposed to use as a measure of risk the standard deviation only of the distribution of negative returns, called Downside Risk (σ_D):

$$\sigma_D = \sqrt{Var(R_P | R_P < 0)} \tag{21}$$

Thus, the Sortino Ratio is given by:

$$\text{Sortino Ratio} = \frac{R_P - R_f}{\sigma_D} \tag{22}$$

The formula shows that the greater the downward deviation of the portfolio returns with respect to the expected return of the investor, the greater the risk and, therefore, the lower the Sortino Ratio.

4.5.3 Omega Ratio

It is a generally accepted fact of empirical finance that returns are not normally distributed. This is especially true when considering cryptocurrencies (Chan et al., 2017). Thus, higher moments should be considered in addition to mean and variance. Keating and Shadwick

(2002) propose a ratio that accomplishes the task of considering all of the higher moments of the distribution of returns.

The Omega Ratio represents the ratio between the average gain and the average loss and is defined as:

$$\text{Omega Ratio} = \frac{\sum_{t=1}^{T} \max(0, +R_{pt})}{\sum_{t=1}^{T} \max(0, -R_{pt})} \tag{23}$$

Therefore, the advantage of the Omega ratio is that it does not rely on any assumption regarding the distribution of returns.

4.5.4 Value at Risk and Expected Shortfall

VaR and ES are two risk measures that seek to summarize portfolio total risk through a single number (Hull, 2015).

VaR was pioneered by JPMorgan, and it is a standard risk measure in the financial industry today. It answers the following question:

- *"What loss level is such that we are X% confident it will not be exceeded in N business days?"*

Below I report a general definition of the VaR, which holds for any loss L:

Definition 1: Given a loss L and a confidence level $\alpha \in (0,1)$, then $VaR_\alpha(L)$ is given by the smallest number x such that the probability that the loss L exceeds x is not larger than $1 - \alpha$, that is

$$VaR_\alpha(L) = inf\{x \in \mathbb{R}: F_L(x) \geq \alpha\} \tag{24}$$

where $F_L(x) = P(L \leq x)$ is the distribution of L.

Therefore, the VaR is a function of two parameters: the time horizon N, usually one day, a week, or one month, and the confidence level α, usually equal to 0.95 or 0.99.

There are several ways to compute the VaR, and the most common are presented below (Philippe, 2001):

1. Delta-method. This is an analytical method as the VaR is obtained through a closed-form solution. The biggest drawback of this approach is related to the assumption of normally distributed returns.
2. Historical simulation. It is a nonparametric method and does not require distributional assumptions. It is relatively simple to implement, but its biggest drawback is that it tries to predict immediate future price movements based on the analysis of past market events.
3. Monte Carlo simulation. This approach is a parametric method that generates random movements in risk factors from estimated parametric distributions. Its most significant advantages are that it is very general to implement, and it works for any probabilistic model from which you can simulate random variables. The biggest drawback, instead, is its computational time.

As Monte Carlo simulation is by far the most powerful method to compute VaR (Philippe, 2001), I have decided to use this methodology in this study.

VaR is an attractive measure because it is easy to understand, and it can summarize risk in a single number. However, by definition, the VaR does not give any information about the severity of the loss L which may occur with probability $1 - \alpha$. The ES, instead, tries to solve this drawback.

The ES is a measure of how bad things can go, given that we know that the loss is greater than the VaR. It answers to the following question:

- *"If things do get bad, what is the expected loss?"*

The rigorous definition is reported below:

Definition 2: Let L be a loss which is a continuous random variable with distribution function $F_L(x)$. Then, for any confidence level $\alpha \in (0,1)$, the expected shortfall (ES) is

$$ES_\alpha = E[L|L \geq VaR_\alpha(L)] \tag{25}$$

where E is the expected value.

The ES, compared to the VaR, has better theoretical properties, as it satisfies the subadditivity condition, namely that the risk measure for two portfolios after they have been merged should be no greater than the sum of their risk measures before they were merged (Hull, 2015).

4.5.5 Turnover and Transaction Costs

Portfolio weights constantly change even without active trading. For example, if at the beginning of month $t-1$ we start with weights $w_{i,t-1}^{(j)}$, where $i = 1, \dots, N$ and indicates the weight put on asset i, by the beginning of the following month (t) the portfolio weights change because of individual stock returns during month $t-1$. The portfolio turnover (PT), thus, tells us what the average fraction of the portfolio that has been changed over the period considered is. Therefore, it measures the degree of the amount of trading required to implement a specific strategy.

For any portfolio strategy j, considering a monthly rebalancing frequency, the PT at the beginning of the month t is defined in the following way (Champagne et al., 2018):

$$PT_t^{(j)} = \frac{1}{2}\sum_{i=1}^{N}\left(\left|w_{i,t}^{(j)} - \widetilde{w}_{i,t}^{(j)}\right|\right) \tag{26}$$

where $w_{i,t}^{(j)}$ is the optimal weight of asset i under strategy j at the beginning of month t while $\widetilde{w}_{i,t}^{(j)}$ represents the buy-and-hold weight of asset i at the start of month t and is defined as:

$$\widetilde{w}_{i,t}^{(j)} = w_{i,t-1}^{(j)} \frac{1+R_{i,t-1}}{1+R_{p,t-1}^{(j)}} \tag{27}$$

with $w_{i,t-1}^{(j)}$ being the optimal weight of asset i under strategy j at time $t-1$, $R_{i,t-1}$ the return of asset i during month $t-1$, and $R_{p,t-1}^{(j)}$ the buy-and-hold portfolio return under strategy j in month $t-1$.

A high turnover rate implies a large number of transactions necessary to rebalance the portfolio weights, and because of transaction costs, this translates into more expenses. Therefore, transaction costs are an important variable to consider when evaluating the performance of a particular strategy. For example, Chalmers et al. (2001) found a strong negative relation between fund returns and trading expenses, while Grinold and Kahn (1999) highlighted how, on average, active U.S. equity managers underperformed the S&P 500 mainly because of transaction costs.

The transaction cost adjusted portfolio return $(\tilde{R})$ for month t can be then calculated as:

$$\tilde{R}_{p,t}^{(j)} = \sum_{i=1}^{N} \left(w_{i,t}^{(j)} R_{i,t} - c * \left| w_{i,t}^{(j)} - \widetilde{w}_{i,t}^{(j)} \right| \right) \tag{28}$$

where c denotes the one-way proportional transaction costs.

Following Anyfantaki et al. (2018), I consider 35 bps for the transaction costs of stocks and bonds. Instead, transaction costs usually occur in two ways in the cryptocurrency market: trading fees and bid-ask spread. Anyfantaki et al. (2018) argue that the bid-ask spread is a minor issue compared to other fees, such as trading and deposit/withdrawal/fees. Therefore, they decided to use higher transaction costs for cryptocurrencies than for stocks and bonds, i.e., 50 bps, which is also consistent with Platanakis et al. (2018). I follow the literature in my study and consider 50 bps for the transaction costs of cryptocurrencies included in this study.

4.6 Performance Analysis Before and During Covid-19 Pandemic

As already mentioned in Chapter 3, Forbes and Rigobon (2002) have shown how in times of crisis, stock market correlations increase. This increase in stock market correlations could lower the benefit of efficient portfolio diversification. Therefore, it is interesting to analyze if the Covid-19 emergency has lowered the benefit of introducing cryptocurrencies in a portfolio with traditional assets.

We have already seen how the correlation between Bitcoin and the S&P 500 Index has risen since the start of the pandemic. Table 4, instead, reports the correlation matrix between all the eight assets before (panel 4a) and during (panel 4b) the Covid-19 emergency. Correlations between the eight assets have increased during the last two years, confirming the findings of Forbes and Rigobon (2002). Bitcoin, for example, had a correlation with the S&P 500 Index of 0.02 before 2020, 0.18 considering the entire sample of observations, and 0.36 if we consider only the Covid-19 period. This is also true for Ethereum and the other cryptocurrencies considered in this analysis. Bitcoin and Ethereum, in particular, were negatively correlated with the bond market and the real estate indexes before the pandemic. Then the correlations became positive with the start of the Covid-19 emergency.

The global crisis has not increased only the correlations between cryptocurrencies and the other traditional assets. It increased the linear relationship between all the asset classes considered. For example, the correlation between the two major cryptocurrencies, Bitcoin and Ethereum, is equal to 0.83 if we consider only the last two years, compared to 0.45 in the pre-pandemic case. At the same time, the correlation between the S&P 500 Index and the Vanguard Real Estate Index has increased from 0.53 to 0.86.

Table 4

Correlation Matrix Before and During Covid-19

Panel A depicts the correlation matrix of the eight assets before the start of the pandemic (September 1st, 2015 - December 31st, 2019), while panel B reports the correlation matrix during the pandemic period (January 1st, 2020 - January 31st, 2022). ETH is Ethereum, BTC is Bitcoin, XRP is Ripple, LTC is Litecoin, SPX is the Standard and Poor's 500 stock market Index, VIX is the CBOE Volatility Index, BND is the Vanguard Total Bond Market Index, and VNQ is the Vanguard Real Estate Index. Daily returns are used for the Pearson correlation values in the table. Values rounded to the 2nd decimal place. Source: own elaboration.

	ETH	BTC	XRP	LTC	SPX	VIX	BND	VNQ
a) Correlation Matrix Before Covid-19								
ETH	1,00							
BTC	0,45	1,00						
XRP	0,36	0,33	1,00					
LTC	0,45	0,59	0,41	1,00				
SPX	0,05	0,02	0,05	0,04	1,00			
VIX	-0,08	-0,05	-0,05	-0,05	-0,78	1,00		
BND	-0,02	-0,01	0,02	-0,02	-0,23	0,18	1,00	
VNQ	-0,03	-0,01	0,02	0,01	0,53	-0,42	0,19	1,00
b) Correlation Matrix During Covid-19								
ETH	1,00							
BTC	0,83	1,00						
XRP	0,65	0,61	1,00					
LTC	0,85	0,82	0,68	1,00				
SPX	0,38	0,36	0,25	0,33	1,00			
VIX	-0,37	-0,37	-0,25	-0,36	-0,69	1,00		
BND	0,27	0,26	0,17	0,22	0,18	-0,09	1,00	
VNQ	0,29	0,27	0,18	0,27	0,86	-0,53	0,20	1,00

All these considerations allow to ask ourselves and then investigate whether cryptocurrencies represent an appropriate choice in the composition of a portfolio during a period of financial and economic distress.

Chapter 5: Results and Analysis

This chapter presents the main results of this study. The first part is devoted to in-sample results, while the second part of the chapter focuses on out-of-sample results. To be parsimonious, I only report and comment on the results obtained under the no-short sales constraint. However, the general conclusions of this chapter also apply to the case where short selling is allowed, and the results are reported in Appendix 1.

5.1 In-Sample Results

To obtain a complete analysis, this section is divided into three parts. First, the results are analyzed and presented for the whole sample period. Then the sample period is divided into two subsamples: before and during the Covid-19 pandemic. In this way, one can get a general overview of the contribution of cryptocurrencies to portfolios built with traditional assets. Still, it will also allow us to understand how this contribution has changed during a period of global economic and financial markets stress.

5.1.1 Performance Analysis Considering the Whole Sample Period

Figure 4 draws the mean-variance efficient frontier for portfolios with (in blue) and without (in red) cryptocurrencies during the entire sample period, which lasts from September 1st, 2015, to January 31st, 2022, under the no short sales constraint.

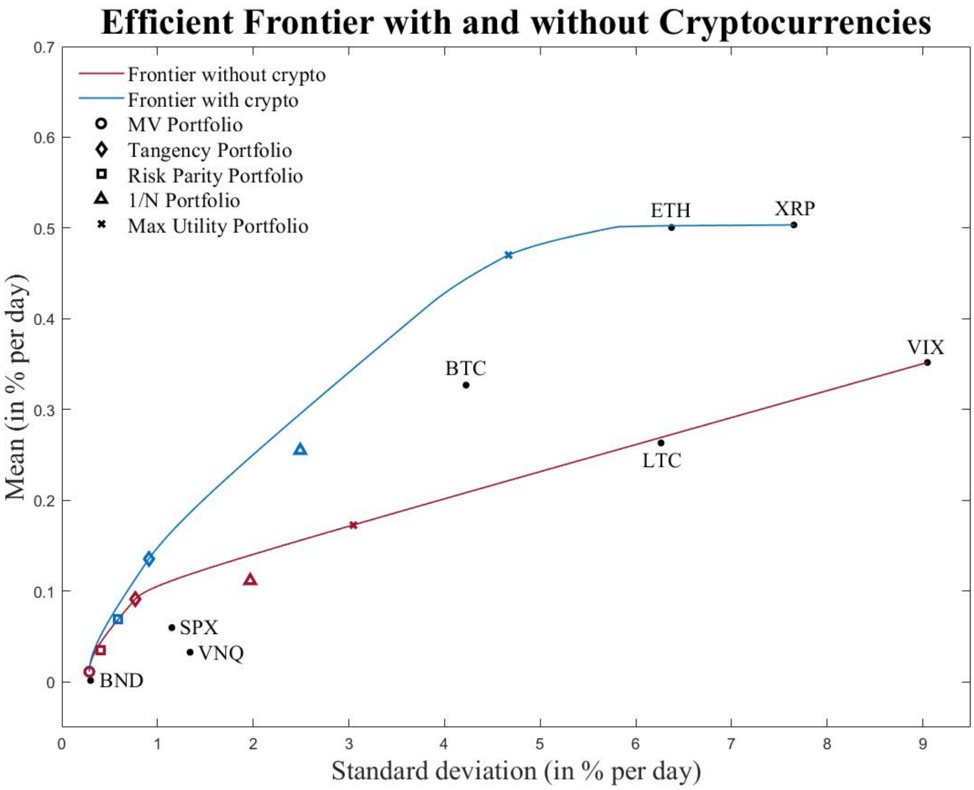
The result is obvious; cryptocurrencies offer diversification benefits. By allowing investments in cryptocurrencies during the period considered, one can expand portfolio allocations to the northwest region (higher risk-adjusted returns). The blue curve is much steeper and is also shifted to the left, meaning that for a given level of risk (standard deviation), portfolios including cryptocurrencies outperform those which include only traditional assets.

This outcome is not surprising. We have already seen in Table 1 how cryptocurrencies offer much higher average returns than traditional assets. Table 2 instead pointed out the low values of the correlation coefficients between these asset classes. The combination of these two results suggested that cryptocurrencies may be effective diversifiers, and Figure 4 provides initial evidence.

Figure 4

Efficient Frontier with and without Cryptocurrencies

In red are the efficient frontier and portfolios excluding cryptocurrencies. In blue are the efficient frontier and portfolio models including cryptocurrencies. Short sales are not allowed. ETH is Ethereum, BTC is Bitcoin, XRP is Ripple, LTC is Litecoin, SPX is the Standard and Poor's 500 stock market Index, VIX is the CBOE Volatility Index, BND is the Vanguard Total Bond Market Index, and VNQ is the Vanguard Real Estate Index. Daily data for the cryptocurrencies are obtained from Coin Metrics, while for S&P 500 are obtained from Yahoo Finance. The sample period is from September 1st, 2015, to January 31st, 2022. Source: based on author's calculation.



One can now analyze the performance of the specific portfolio strategies and their composition. Table 5 reports the empirical performance of the portfolio models during the entire sample period. Panel 5a depicts portfolios without cryptocurrencies, while panel 5b reports the results for portfolios with all assets. Table 6, instead, provides the optimal weights of the portfolios as percentages. Panel 6a illustrates the weights for the whole sample period, panel 6b for the pre-pandemic period, and panel 6c describes the composition of the different strategies during the last two years.

First, one can observe in Table 5 that portfolios with all assets report higher means and standard deviations. It can be noticed that, although riskier, cryptocurrencies significantly increased the risk-adjusted returns for all the portfolio strategies considered in this study. This evidence holds for the Sharpe, Sortino, and Omega ratios, as highlighted by panel 5b.

The only exception to this rule is given by the MV portfolio, where the Sharpe ratio remains unchanged while the Sortino and Omega ratios only slightly increase. By looking at the weights of the MV portfolio, one can understand why the results are not so pronounced. The MV portfolio for this sample period, in fact, primarily includes traditional assets. The second row of panel 6a highlights how the cumulative share of portfolio weights given by the four cryptocurrencies equals 0.04%, very close to 0. This implies that the performance of the MV portfolio does not change much when cryptocurrencies are included.

The other optimal portfolios include a larger share of cryptocurrencies. The Tangency portfolio, for example, holds 12,32% of its weight on cryptocurrencies, with Bitcoin and Ethereum having a higher contribution to the portfolio's performance. In comparison, Litecoin receives 0% weight. This portion increases to 79,12% in the case of the MU portfolio, with Ethereum being the leader with 51,97%, followed by Ripple with 26,97%.

The last two columns of Table 5 describe the riskiness of these five allocation strategies in terms of VaR and ES. Including cryptocurrencies generally leads to an increased risk, which is not surprising as they are highly volatile.

There are, however, some exceptions. For example, the EW portfolio with all assets reports a 1-day VaR at a 95% confidence level equal to 2,941%, compared to the 2,953% of the portfolio without cryptocurrencies. One can also see how including 0.04% of cryptocurrencies in the MV portfolio helps lower the risk in terms of VaR, from 0.340% to 0.339%.

Table 5

Empirical Performance of Portfolios

Empirical performance of portfolio models (in % for mean, standard deviation (Std), value at risk (VaR), and expected shortfall (ES)) during the all-sample period. Panel 4a depicts portfolios without cryptocurrencies, while panel 4b reports the results for portfolios with all assets. 1-day VaR and 1-day ES are considered, with a 95% confidence level ($\alpha = 0.95$). Short sales are not allowed. Daily returns are used for the calculations. The sample period is from September 1st, 2015, to January 31st , 2022. Source: own elaboration.

	Mean	Std	Sharpe R.	Sortino R.	Omega R.	VaR	ES
a) No Crypto							
TP	0,091	0,768	0,119	0,224	1,507	1,134	1,446
MV	0,011	0,287	0,039	0,056	1,149	0,340	0,430
RP	0,035	0,409	0,085	0,141	1,345	0,580	0,737
EW	0,112	1,969	0,057	0,129	1,190	2,953	3,724
MU	0,173	3,045	0,057	0,130	1,191	4,638	5,849
b) All Assets							
TP	0,135	0,909	0,149	0,278	1,563	1,224	1,569
MV	0,011	0,287	0,039	0,057	1,151	0,339	0,430
RP	0,069	0,588	0,117	0,199	1,427	0,764	0,974
EW	0,255	2,492	0,102	0,185	1,340	2,941	3,729
MU	0,470	4,666	0,100	0,189	1,327	5,277	6,684

Table 6

Portfolio Optimal Weights

The table shows the optimal weights (in %) for each strategy in three different periods: Whole sample period (September 1st, 2015 – January 31st, 2022), Before Covid-19 pandemic (September 1st, 2015 – December 31st, 2019), and During Covid-19 pandemic (January 1st, 2020 – January 31st, 2022). All assets are considered. TP is the Tangency Portfolio, MV is the Minimum Variance Portfolio, RP is the Risk-Parity Portfolio, EW is the Equally Weighted Portfolio, and MU is the Maximum Utility Portfolio. Short sales are not allowed. Daily returns are used for the calculations. Source: own elaboration.

	ETH	BTC	XRP	LTC	SPX	VIX	BND	VNQ
a) Whole Sample Period								
TP	4,18	5,76	2,38	0,00	76,99	10,69	0,00	0,00
MV	0,01	0,01	0,01	0,01	10,62	0,97	88,36	0,02
RP	2,04	2,93	1,81	1,95	22,06	4,74	51,69	12,78
EW	12,50	12,50	12,50	12,50	12,50	12,50	12,50	12,50
MU	51,97	0,15	26,97	0,03	0,02	20,84	0,02	0,02
b) Before Covid-19								
TP	1,37	2,08	1,03	0,00	57,81	5,01	32,71	0,00
MV	0,05	0,16	0,01	0,04	17,04	0,98	81,71	0,02
RP	1,27	1,93	1,06	1,18	27,90	3,18	51,31	12,16
EW	12,50	12,50	12,50	12,50	12,50	12,50	12,50	12,50
MU	46,08	0,19	42,35	0,03	0,03	11,29	0,02	0,02
c) During Covid-19								
TP	8,15	11,43	0,00	0,00	64,61	15,82	0,00	0,00
MV	0,00	0,01	0,01	0,00	9,28	1,68	89,00	0,02
RP	2,93	3,87	2,64	2,82	18,83	7,51	48,42	12,99
EW	12,50	12,50	12,50	12,50	12,50	12,50	12,50	12,50
MU	66,79	0,00	0,00	0,00	0,00	33,20	0,00	0,00

The following two sections are dedicated to the performance analysis of the different allocation strategies before and during the Covid-19 pandemic.

5.1.2 Performance Analysis Before Covid-19

Yarovaya et al. (2020) recall that the Covid-19 pandemic is the first global stress period cryptocurrencies have faced since they first started circulating. I use December 31st, 2019, as the cut-off to identify the periods before and after the start of the COVID-19 pandemic, consistent with Corbet et al. (2020) and Colombo et al. (2021).

Figure 5 plots in blue the mean-variance efficient frontier for portfolios with all assets, while in red, one can see the efficient frontier without cryptocurrencies. The sample period is from September 1st, 2015, to December 31st, 2019.

The evidence suggests again that cryptocurrencies enhance the performance of the portfolio allocation strategies by allowing us to move towards higher risk-adjusted returns. The blue line is shifted outward with respect to the red one. However, it can be noticed that, with respect to the previous case, most of the optimal portfolios are concentrated in one point in the southwest area, suggesting a lower average return for these portfolios compared to the entire sample period.

Figure 5

Efficient Frontier Before Covid-19

In red are the efficient frontier and portfolios excluding cryptocurrencies. In blue are the efficient frontier and portfolio models, including cryptocurrencies. Short sales are not allowed. ETH is Ethereum, BTC is Bitcoin, XRP is Ripple, LTC is Litecoin, SPX is the Standard and Poor's 500 stock market Index, VIX is the CBOE Volatility Index, BND is the Vanguard Total Bond Market Index, and VNQ is the Vanguard Real Estate Index. Daily data for the cryptocurrencies are obtained from Coin Metrics, while S&P 500 are obtained from Yahoo Finance. The sample period is from September 1st, 2015, to December 31st, 2019. Source: based on author’s calculation.

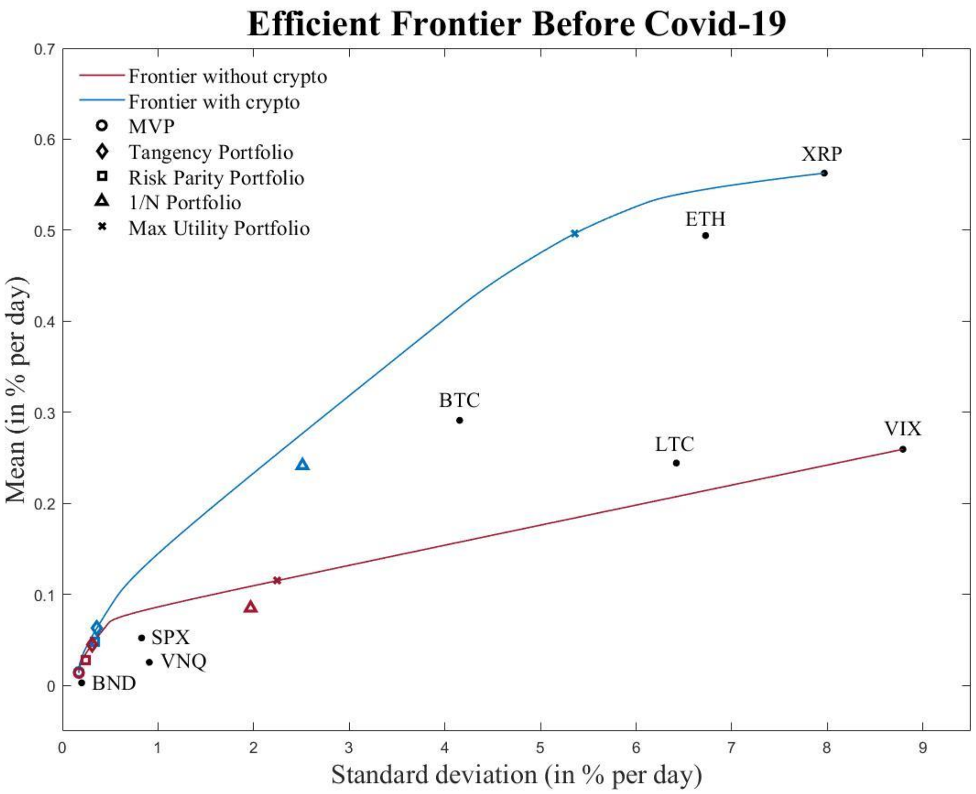


Table 7 reports the empirical performance of the portfolio models before the start of the pandemic. Panel 7a depicts portfolios without cryptocurrencies, while panel 7b reports the results for portfolios with all assets.

All portfolios show higher risk-adjusted returns when including cryptocurrencies, meaning that the higher risk brought by cryptocurrencies is more than well compensated by the higher returns. The magnitude of the increase is similar to the previous case when the entire sample period was considered.

By comparing the two periods, it turns out that the first three portfolio models (Tangency, MV, and RP portfolios) have performed better in the pre-pandemic case than if one considers the whole sample period. As the MV and RP portfolios focus more on risk, this could be due to the lower correlation between cryptocurrencies and traditional assets before the Covid-19 pandemic, as highlighted in the previous chapter. Instead, the other two strategies (EW and MU) have performed better considering the entire sample period. An explanation here could be that cryptocurrencies experienced a bear market in 2018, as Colombo et al. (2021) also pointed out. Consequently, portfolios with higher distributed weights on cryptocurrencies (EW and MU) suffered more.

Panel b of Table 6 (see above) shows the optimal weights of the portfolio as percentages before the Covid-19 pandemic. Here some interesting results can be observed.

First, one can notice that compared to the full-sample case, the percentage of cryptocurrencies' weight in the MV portfolio has increased from 0.04% to 0.26%. This confirms that in the pre-pandemic case, cryptocurrencies were poorly related to the market's returns and therefore offered higher benefits in reducing the risk of a portfolio.

The Tangency and the RP portfolios, on the other hand, saw a decrease in cryptocurrencies' weights. This result may partially explain why these two portfolios recorded lower average returns in the pre-covid period than the whole sample period.

Table 7

Empirical Performance of Portfolios Before Covid-19

Empirical performance of portfolio models (in % for mean, standard deviation (Std), value at risk (VaR), and expected shortfall (ES)) before the start of the pandemic. Panel 6a depicts portfolios without cryptocurrencies, while panel 6b reports the results for portfolios with all assets. 1-day VaR and 1-day ES are considered, with a 95% confidence level ($\alpha = 0.95$). Daily returns are used for the calculations. The sample period is from September 1st, 2015, to December 31st, 2019. Source: own elaboration.

	Mean	Std	Sharpe R.	Sortino R.	Omega R.	VaR	ES
a) No Crypto							
TP	0,045	0,312	0,146	0,285	1,537	0,345	0,446
MV	0,014	0,172	0,080	0,136	1,234	0,256	0,325
RP	0,028	0,243	0,116	0,212	1,374	0,327	0,417
EW	0,085	1,969	0,043	0,098	1,143	3,409	4,286
MU	0,115	2,244	0,052	0,117	1,174	3,667	4,616
b) All Assets							
TP	0,063	0,359	0,175	0,345	1,620	0,401	0,520
MV	0,015	0,172	0,086	0,145	1,252	0,255	0,324
RP	0,048	0,337	0,143	0,272	1,471	0,470	0,601
EW	0,242	2,511	0,096	0,185	1,317	3,514	4,444
MU	0,496	5,353	0,093	0,186	1,316	6,079	7,637

5.1.3 Performance Analysis During Covid-19

The recent empirical literature (Conlon and McGee, 2020; Corbet et al., 2020) focuses on the pandemic's early days, with Colombo et al. (2021) considering the sample period till the last month of 2020. However, 2021 will most likely be remembered as when the adoption of digital currencies took off globally. According to the composite index prepared by the company Chainalysis, between the end of 2019 and mid-2021, the use of digital currencies increased 23 times (+2300%), with an explosion starting from January 2021 (Chainalysis, 2021). The availability of an additional year's worth of data could therefore provide greater clarity on how the pandemic has impacted traditional markets and cryptocurrencies.

Figure 6 plots in blue the mean-variance efficient frontier for portfolios with all assets, while in red, we can see the efficient frontier without cryptocurrencies. The sample period is from January 1st, 2020, to January 31st, 2022.

First, one can notice that the asset with the highest return registered during this period is no more a cryptocurrency, but the VIX. This should not come as a surprise. The last two years have been a period of global macroeconomic uncertainty due to the Covid-19 pandemic and its consequences on the economic and financial world. Arnold et al. (2008) provided empirical evidence on the link between stock market volatility and macroeconomic uncertainty. As the VIX measures the implied volatility of S&P 500 Index options, it should be no surprise that it registered a higher average return.

Second, the efficient frontier with all assets highlights how cryptocurrencies offered a significant advantage for portfolio construction even in times of market turmoil. The cryptocurrencies allowed us to obtain higher risk-adjusted returns by shifting the efficient frontier to the northwest region, confirming thus the diversification benefits of the previous two sections.

Figure 6

Efficient Frontier During Covid-19

In red are the efficient frontier and portfolios excluding cryptocurrencies. In blue are the efficient frontier and portfolio models, including cryptocurrencies. Short sales are not allowed. ETH is Ethereum, BTC is Bitcoin, XRP is Ripple, LTC is Litecoin, SPX is the Standard and Poor's 500 stock market Index, VIX is the CBOE Volatility Index, BND is the Vanguard Total Bond Market Index, and VNQ is the Vanguard Real Estate Index. Daily data for the cryptocurrencies are obtained from Coin Metrics, while S&P 500 are obtained from Yahoo Finance. The sample period is from January 1st, 2020, to January 31st, 2022. Source: based on the author's calculation

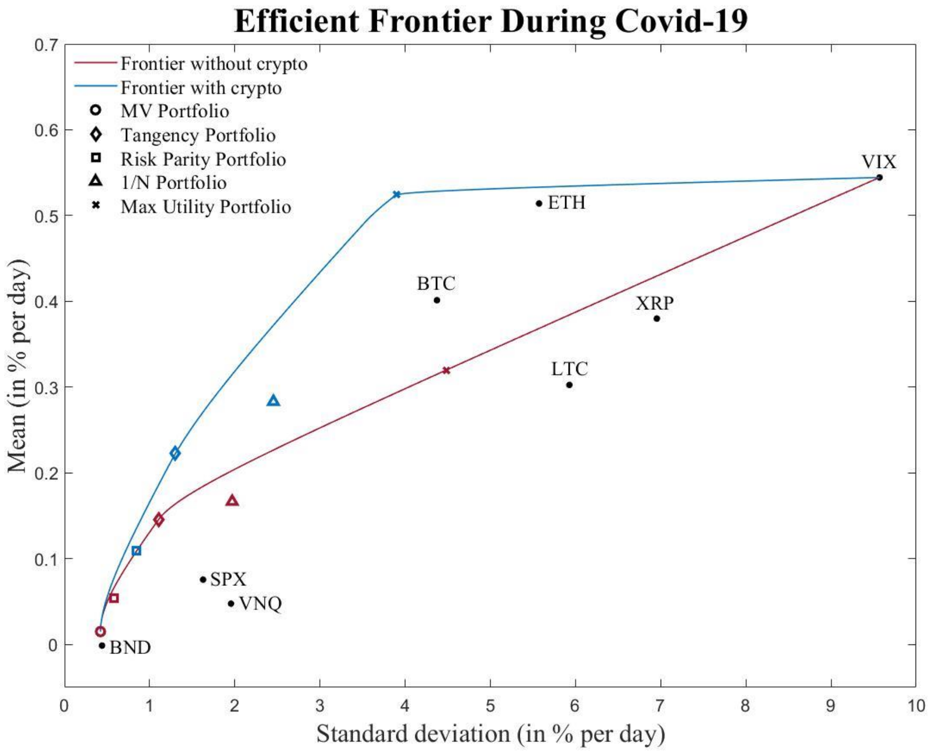


Table 8 reports the empirical performance of the portfolio models from the start of the pandemic until January 31st, 2022. Panel 8a depicts portfolios without cryptocurrencies, while panel 8b reports the results for portfolios with all assets.

Some significant results can be highlighted here. Average returns have increased, except for the MV portfolio, where the return remained unchanged. All investment strategies registered higher risk-adjusted returns when cryptocurrencies were included. This evidence holds for the Sharpe, Sortino, and Omega ratios. Interestingly, the MU portfolio with all assets has achieved a higher average return. At the same time, it also saw a decrease in the portfolio risk, as highlighted by the standard deviation in column 2 of panel 8b. The VaR and ES support this result. In the last two columns of Table 8, the 1-day VaR at a 95% confidence level is equal to 4,583% for the portfolio with all assets, compared to 6,921% of the one without cryptocurrencies.

Table 8

Empirical Performance of Portfolios During Covid-19

Empirical performance of portfolio models (in % for mean, standard deviation (Std), value at risk (VaR), and expected shortfall (ES)) during the Covid-19 pandemic. Panel 8a depicts portfolios without cryptocurrencies, while panel 8b reports the results for portfolios with all assets. 1-day VaR and 1-day ES are considered, with a 95% confidence level ($\alpha = 0.95$). Daily returns are used for the calculations. The sample period is from January 1st, 2020, to January 31st, 2022. Source: own elaboration.

	Mean	Std	Sharpe R.	Sortino R.	Omega R.	VaR	ES
a) No Crypto							
TP	0,146	1,106	0,132	0,265	1,543	1,147	1,484
MV	0,015	0,422	0,036	0,051	1,151	0,293	0,373
RP	0,054	0,581	0,093	0,158	1,382	0,625	0,798
EW	0,167	1,969	0,085	0,195	1,291	2,974	3,771
MU	0,320	4,481	0,071	0,169	1,242	6,921	8,754
b) All Assets							
TP	0,223	1,303	0,171	0,325	1,644	1,419	1,837
MV	0,015	0,422	0,036	0,052	1,152	0,349	0,443
RP	0,110	0,841	0,131	0,214	1,489	0,770	0,996
EW	0,283	2,453	0,115	0,187	1,390	2,865	3,639
MU	0,524	3,897	0,134	0,247	1,446	4,583	5,870

In this subsample, the contributions of the cryptocurrencies become more selective. Panel c of Table 6 shows the optimal weights of the different allocation strategies as percentages since the start of the Covid-19 emergency.   Bitcoin and Ethereum increased their weights in the Tangency portfolio, while Ripple and Litecoin observed a 0% weight. Despite this, the cumulative share of portfolio weights has risen to 19,58%, highlighting a more critical role for cryptocurrencies in this portfolio. Instead, the MV portfolio decreased cryptocurrency weights to only 0,02%. Finally, the RP portfolio experienced a higher share of cryptocurrencies' weights compared to the pre-pandemic case. At the same time, only Ethereum had a role in the MU portfolio, with 66,79% of the total weight.

One final consideration could be made on the MV portfolio. This portfolio exhibits a nearly zero allocation in cryptocurrencies over all the sample periods considered. Despite their diversification benefits, cryptocurrencies remain too volatile to be included in the lowest-risk portfolio. These results are consistent with Colombo et al. (2021) and Kajtazi and Moro (2019). Kajtazi and Moro (2019), in particular, pointed out that adding bitcoin to a well-diversified portfolio improves its performance, but this is due more to the increase in returns than to the reduction in volatility.

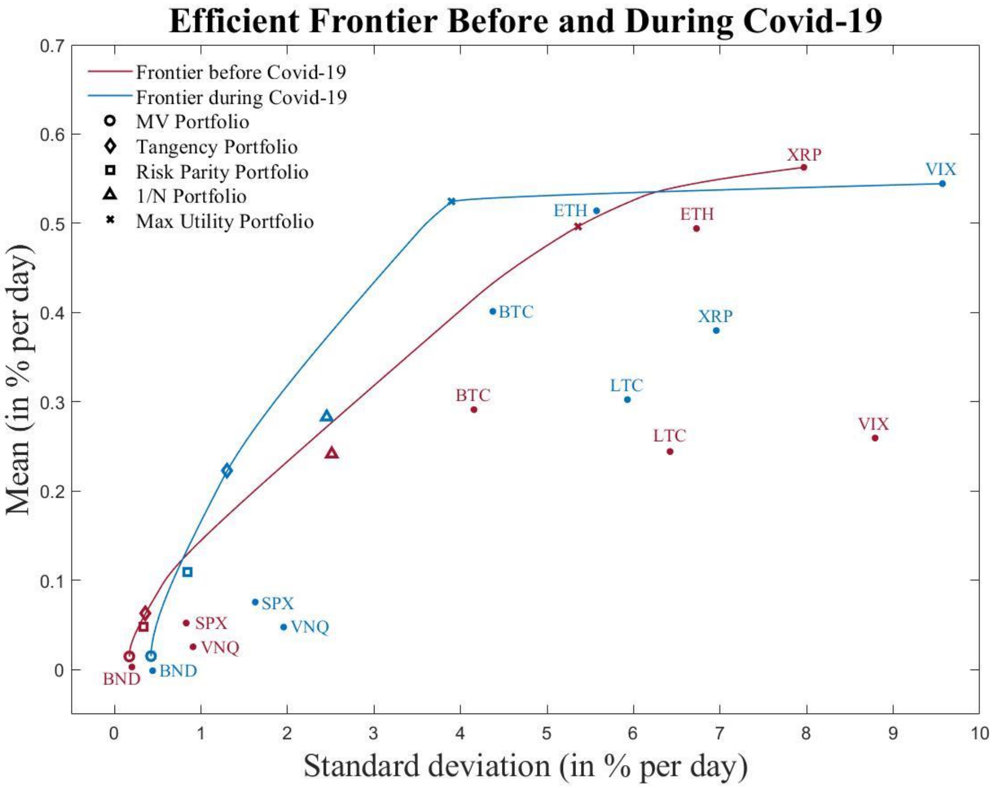
Before delving into the analysis of out-of-sample results, it is helpful to analyze the effect of the 2020 crisis on portfolios comprising all assets. Figure 7 plots in blue the mean-variance efficient frontier for portfolios with all assets since the start of the pandemic, while in red, one can see the efficient frontier for the same portfolios before the beginning of the Covid-19 emergency.

It is not clear which of the two periods reported higher risk-adjusted returns. For portfolios aiming at minimizing the risk, the pre-pandemic period offered a better solution, with lower variability of returns. This could be because cryptocurrencies were negatively or very low correlated to the traditional markets before the Covid-19. Instead, the pandemic has increased the correlation between all the assets, which shifted the MV portfolio to the right in Figure 7.

Figure 7

Efficient Frontier Before and During Covid-19

In red are the efficient frontier and portfolios models before the start of the pandemic. In blue are the efficient frontier and portfolio models since the pandemic's beginning. Short sales are not allowed. The cut-off date is the December 31st, 2019. ETH is Ethereum, BTC is Bitcoin, XRP is Ripple, LTC is Litecoin, SPX is the Standard and Poor's 500 stock market Index, VIX is the CBOE Volatility Index, BND is the Vanguard Total Bond Market Index, and VNQ is the Vanguard Real Estate Index. Daily data for the cryptocurrencies are obtained from Coin Metrics, while S&P 500 are obtained from Yahoo Finance. Source: based on the author's calculation



For portfolios with a daily variability between 1% and 6%, the Covid-19 allowed to obtain better results. The last two years offered higher returns for a specific level of standard deviation in this range than the pre-pandemic case. However, by comparing the risk-adjusted measure in Table 8 with those in Table 7, it is still not possible to determine whether the Covid-19 pandemic has improved or worsened the performances of the different portfolios. Depending on the investor's preferences, it could have done better or worse. An extremely risk-averse person probably suffered more during the last two years than a risk-taking one.

5.2 Out-of-Sample Results

It is important to note that the weights obtained by the in-sample analysis use information concerning the whole sample period. However, in practice, portfolio managers can only use past data to decide on their allocation strategy, as future returns are unknown (Colombo et al., 2021). Consequently, in this section, I will analyze the out-of-sample performance of the different portfolios.

Table 9 shows the out-of-sample performances of the different allocation strategies for portfolios without (panel 9a) and including cryptocurrencies (panel 9b). The values shown are monthly, and the rebalance frequency is $\tau = 22$ days. Results for further rebalancing periods are reported in Appendix 2.

First of all, it can be seen that the mean and standard deviations of all portfolios are higher when including cryptocurrencies. The MU portfolio is the one that reports the highest values for both mean and standard deviation, while the MV portfolio is the one that reports the lowest values. However, following the results of the in-sample analysis, all three measures of risk-adjusted returns report significantly higher values for portfolios that include cryptocurrencies. In particular, the Tangency portfolio has the highest values of the three measures considered (Sharpe, Sortino, and Omega ratios). In contrast, the MV portfolio has the lowest values, although better than when cryptocurrencies are excluded.

Since the portfolios are rebalanced every month, an essential factor to consider in the out-of-sample analysis is the turnover of the portfolios. The last column of Table 9 presents the turnover values for the different strategies. What stands out is that, on average, the turnover is lower for portfolios without cryptocurrencies. In both cases, the MV portfolio has the lowest turnover, probably because the weight of this portfolio is almost entirely distributed over the Bond Market Index. This is an important consideration to keep in mind, as a higher turnover translates into higher transaction costs, thus eroding part of a portfolio's performance. For example, the MU portfolio gives the highest turnover among the various strategies considered. Transaction costs are therefore expected to have a more significant impact here than in the case of the MV portfolio.

Table 9

Out-of-Sample Performances

Out-of-sample performance of portfolio models (in % for mean, standard deviation (Std), value at risk (VaR), expected shortfall (ES), and turnover). Panel 9a depicts portfolios without cryptocurrencies, while panel 9b reports the results for portfolios with all assets. Monthly rebalancing frequency ($\tau = 22$ days). Monthly results are reported for mean and standard deviation. 1-month VaR and 1-month ES are considered, with a 95% confidence level ($\alpha = 0.95$). The sample period is from September 1, 2016, to January 31, 2022. Source: own elaboration.

	Mean	Std	Sharpe R.	Sortino R.	Omega R.	VaR	ES	Turnover
a) No Crypto								
TP	0,955	1,985	0,481	1,322	3,631	2,209	3,031	9,314
MV	0,290	1,558	0,186	0,358	1,927	1,345	1,737	2,361
RP	0,601	1,690	0,356	0,869	2,964	1,664	2,226	2,570
EW	2,340	11,640	0,201	1,186	2,799	18,223	23,435	4,572
MU	1,279	15,698	0,081	0,189	1,371	23,072	29,363	22,793
b) All Assets								
TP	2,060	3,701	0,557	2,228	5,875	3,965	5,425	11,893
MV	0,314	1,577	0,199	0,385	1,996	1,373	1,781	2,466
RP	1,571	3,014	0,521	1,843	4,940	3,287	4,369	4,032
EW	7,232	21,786	0,332	1,560	3,621	22,711	28,733	8,507
MU	11,179	52,487	0,213	1,162	2,691	34,502	43,243	23,681

As transaction costs play an essential role in defining the final performance, Table 10 shows the out-of-sample performances of the different allocation strategies for portfolios with (panel 10b) and without (panel 10a) cryptocurrencies, net of transaction costs.

The result is that average returns declined for all portfolios. However, the risk-adjusted measures are still in favour of portfolios that include cryptocurrencies. Despite the higher turnover and transaction costs introduced by cryptocurrencies, portfolios that include cryptocurrencies still perform better, once again implying that the higher risk is more than well compensated by the higher returns.

Table 10

Out-of-Sample Performances Including Transaction Costs

Out-of-sample performance of portfolio models (in % for mean, standard deviation (Std), value at risk (VaR), expected shortfall (ES), and turnover) including transaction costs (35bps for traditional assets and 50 bps for cryptocurrencies). Panel 10a depicts portfolios without cryptocurrencies, while panel 10b reports the results for portfolios with all assets. Monthly rebalancing frequency ($\tau = 22$ days). Monthly results are reported for mean and standard deviation. 1-month VaR and 1-month ES are considered, with a 95% confidence level ($\alpha = 0.95$). The sample period is from September 1, 2016, to January 31, 2022. Source: own elaboration.

	Mean	Std	Sharpe R.	Sortino R.	Omega R.	VaR	ES	Turnover
a) No Crypto								
TP	0,889	1,984	0,448	1,203	3,339	2,285	3,112	9,314
MV	0,273	1,556	0,176	0,336	1,855	1,341	1,731	2,361
RP	0,584	1,684	0,347	0,835	2,867	1,654	2,211	2,570
EW	2,308	11,646	0,198	1,160	2,747	18,246	23,459	4,572
MU	1,120	15,693	0,071	0,165	1,319	23,213	29,505	22,793
b) All Assets								
TP	1,969	3,699	0,532	2,033	5,420	4,058	5,518	11,893
MV	0,297	1,575	0,188	0,361	1,918	1,368	1,773	2,466
RP	1,540	3,005	0,512	1,787	4,783	3,293	4,372	4,032
EW	7,159	21,780	0,329	1,536	3,572	22,760	28,779	8,507
MU	10,967	52,538	0,209	1,130	2,624	34,952	43,727	23,681

Intuitively, given that the average turnover is lower for portfolios without cryptocurrencies, one would expect the latter to have suffered more after the inclusion of transaction costs. I report then in Table 11 the change in performance measures after the introduction of transaction costs. Panel 11a reports those for portfolios without cryptocurrencies, while panel 11b reports the ones for portfolios with all assets included.

What the study shows is precisely the opposite. The Tangency portfolio, for example, suffered a reduction in the average monthly return of -6.83% when only traditional assets were considered, compared to -4.38% when all assets were included. Moreover, this is true for all other portfolios, right up to the extreme case of the MU portfolio, where the reduction in average return due to transaction costs is -12.47% for the possibility of only traditional assets versus -1.90% when cryptocurrencies are included. The same pattern is also observed in all three risk-adjusted return measures. It can be observed in columns 3-5 of Table 11 that the decrease in the values in panel *a* is more significant than in panel *b*.

One possible explanation for the above results could be that introducing cryptocurrencies into the various portfolios increased the average return of these portfolios considerably more than the transaction costs managed to worsen their performances.

Table 11

Impact of Transaction Costs on Allocation Strategies

The table shows the change in performance measures after introducing transaction costs. Transaction costs are equal to 35bps for traditional assets and 50 bps for cryptocurrencies. Monthly rebalancing frequency ($\tau = 22$ days). Monthly results are reported. 1-month VaR and 1-month ES are considered, with a 95% confidence level ($\alpha = 0.95$). The sample period is from September 1, 2016, to January 31, 2022. Source: own elaboration.

	Mean	Std	Sharpe R.	Sortino R.	Omega R.	VaR	ES
a) No Crypto							
TP	-6,83%	-0,06%	-6,77%	-8,98%	-8,05%	3,45%	2,66%
MV	-5,70%	-0,16%	-5,55%	-6,17%	-3,75%	-0,30%	-0,37%
RP	-2,99%	-0,38%	-2,61%	-3,90%	-3,26%	-0,56%	-0,67%
EW	-1,37%	0,05%	-1,42%	-2,21%	-1,88%	0,13%	0,10%
MU	-12,47%	-0,03%	-12,44%	-12,79%	-3,77%	0,61%	0,48%
b) All Assets							
TP	-4,38%	-0,05%	-4,34%	-8,73%	-7,75%	2,34%	1,71%
MV	-5,58%	-0,15%	-5,44%	-6,05%	-3,92%	-0,36%	-0,45%
RP	-2,00%	-0,31%	-1,70%	-3,05%	-3,17%	0,16%	0,08%
EW	-1,01%	-0,03%	-0,98%	-1,57%	-1,35%	0,22%	0,16%
MU	-1,90%	0,10%	-1,99%	-2,71%	-2,47%	1,30%	1,12%

Finally, in Table 12, I report the out-of-sample average weights of the different allocation strategies as percentages before (panel 12a) and since (panel 12b) the start of the Covid-19 pandemic.

The overall result is that cryptocurrencies have contributed to the allocation strategies considered. This contribution has increased since the start of the pandemic. Indeed, Table 12 shows that the total weight attributed to cryptocurrencies in the various portfolios has increased during Covid-19. The only exception is the MV portfolio. However, as pointed out in the previous sections, this portfolio only focuses on risk. Therefore, this exception is likely due to increased market correlations during the pandemic.

The MU portfolio has the highest concentration of cryptocurrencies, with 72.02% in the pre-pandemic period and 76.47% in the subsequent period. Instead, the MV portfolio has the lowest concentration. The overall weight of cryptocurrencies does not exceed 0.25%, both before and after the start of the pandemic. Ethereum and Bitcoin have taken up the most significant portion of the cryptocurrency weight on average. Litecoin, on the other hand, took a minor role. Indeed, considering only the optimization strategies, Litecoin achieved a greater weight of 0.5% only in the RP strategy, both before and after the start of the pandemic.

Table 12

Out-of-sample average weights

The table shows the out-of-sample average weights (in %) for each strategy before and during the Covid-19 pandemic. Monthly rebalancing frequency ($\tau = 22$ days). TP is the Tangency Portfolio, MV is the Minimum Variance Portfolio, RP is the Risk-Parity Portfolio, EW is the Equally Weighted Portfolio, and MU is the Maximum Utility Portfolio. Short sales are not allowed. Daily returns are used for the calculations. Source: own elaboration.

	ETH	BTC	XRP	LTC	SPX	VIX	BND	VNQ
a) Before Covid-19								
TP	1,36	2,11	1,08	0,13	64,21	5,22	22,74	3,15
MV	0,03	0,14	0,02	0,04	19,35	1,07	79,33	0,02
RP	1,22	1,92	1,13	1,26	30,09	3,06	49,70	11,60
EW	12,50	12,50	12,50	12,50	12,50	12,50	12,50	12,50
MU	35,21	10,87	25,46	0,48	1,75	16,93	0,02	9,27
b) During Covid-19								
TP	3,21	11,03	0,05	0,00	56,95	12,05	12,97	3,74
MV	0,02	0,03	0,01	0,05	12,43	1,99	84,59	0,89
RP	2,50	3,16	2,65	2,48	20,61	6,64	49,63	12,33
EW	12,50	12,50	12,50	12,50	12,50	12,50	12,50	12,50
MU	29,29	43,75	3,42	0,01	0,01	23,48	0,01	0,01

Chapter 6: Conclusions

This paper investigates whether cryptocurrencies can be used to diversify and improve the risk-return relationship of portfolios constructed with traditional assets.

The sample period covers almost seven years of daily data, and four cryptocurrencies are considered: Bitcoin, Ethereum, Litecoin, and Ripple. Their contribution to an investment portfolio is analyzed through various investment strategies to characterize investors' different risk preferences.

The results suggest that cryptocurrencies contribute positively to a portfolio built with traditional assets. This effect is consistent across portfolio construction techniques and is valid both before and after the onset of the Covid-19 pandemic. By allowing investments in cryptocurrencies, one can expand portfolio allocation choices and reach higher risk-adjusted returns. For a given level of risk, portfolios including cryptocurrencies outperform those which exclude them.

Both in- and out-of-sample analyses report better performances for portfolios including the four cryptocurrencies. The Sharpe, Sortino, and Omega ratios improve for all portfolios and all sample periods considered. This result is robust to the inclusion of transaction costs.

The contribution and weight of cryptocurrencies within portfolios vary according to investors' preferences. The MU portfolio suggests that risk-taking investors should give more weight to cryptocurrencies in their portfolios. As for an investor inclined to minimize the volatility, the MV portfolio recommends weights close to zero.

6.1 Discussion

Although the results suggest that cryptocurrencies are suitable assets to be included in an investment portfolio, there are still some reasons for concern. Past studies have associated diversification benefits with low correlations between cryptocurrencies and traditional financial markets. However, the fact that cryptocurrencies have performed so well in the past does not mean that they will do the same in the future. We have already seen, for example, how the past two years have sharply increased the correlation between these assets. What if correlations remain high as adoption increases?

Another reason of concern is that cryptocurrencies are exposed to technology risk and market manipulation (Arsi et al., 2021). For example, Luna, one of the most capitalized cryptocurrencies at the beginning of May 2022, lost more than 99% of its value in less than a week due to a problem with the underlying technology (CNBC, 2022).

As for the potential issues in the data analysis, the author is aware of possible limitations to this study. In particular, even if the four cryptocurrencies examined account for more than 60% of the cryptocurrencies' market capitalization, this may not be a sufficiently representative sample. However, this is still a relatively new and fast-growing market. New projects using blockchain technology are created every day. Fifteen of the top twenty projects by market capitalization were created after 2017. Cryptocurrencies that were very popular three years ago have now lost much interest as more attractive projects take their place (see Litecoin case). As a result, I had to trade-off between market capitalization and years of statistically sufficient data.

While there have been substantial strides in adoption and regulation over the past two years, this remains a niche and highly volatile market. As of May 8th, 2022, Bitcoin and Ethereum are experiencing a 50% drawdown from their historical highs a few months ago. For many other cryptocurrencies, it is even worse. More market knowledge and future clearer regulation could benefit developments in this market. With increased adoption and regulation, the volatility and the expected values of cryptocurrencies may decrease, and their contribution to an investment portfolio may become more informative. Therefore, future research could investigate the impact of the increased adoption and regulation of cryptocurrencies on their investment performances.

Chapter 7: Appendices

7.1 Appendix 1

This section shows the results of the various investment strategies after removing the short-sales constraint.

The results support the fact that cryptocurrencies could positively contribute to portfolio management. However, some minor exceptions need to be mentioned.

Appendix Table 1.1 shows the performance of the various portfolios and their composition considering the in-sample analysis. Panel I) and III) of Table 1.1 point out that the MV portfolio offers better risk-adjusted returns if cryptocurrencies are excluded. This evidence is supported by all three ratios considered. Once again, it confirms that if an investor's goal is to minimize risk, perhaps including cryptocurrencies in his investment portfolio may not be a good idea. Despite this, the MV portfolio with all assets achieved a lower standard deviation during the periods considered.

Appendix Table 1.1: In-Sample Performances

Empirical performance of portfolio models (in % for mean, standard deviation (Std), value at risk (VaR), and expected shortfall (ES)). Short sales are allowed. 1-day VaR and 1-day ES are considered, with a 95% confidence level ($\alpha = 0.95$). Daily returns are used for the calculations. Cut-off day is December 31st, 2019. Source: own elaboration.

	Mean	Std	Sharpe R.	Sortino R.	Omega R.	VaR	ES
I) Whole Sample Period							
a) No Crypto							
TP	0,094	0,750	0,125	0,247	1,493	0,980	1,255
MV	0,013	0,280	0,047	0,069	1,182	0,314	0,397
RP	0,024	0,381	0,063	0,094	1,250	0,576	0,736
EW	0,112	1,969	0,057	0,129	1,190	2,953	3,725
MU	1,364	11,626	0,117	0,224	1,440	16,407	20,941
b) All Assets							
TP	0,185	1,171	0,158	0,307	1,575	1,556	1,999
MV	0,010	0,278	0,037	0,055	1,141	0,331	0,418
RP	0,067	0,692	0,097	0,156	1,341	1,002	1,273
EW	0,255	2,492	0,102	0,185	1,340	2,942	3,729
MU	2,352	15,305	0,154	0,299	1,540	21,878	28,031

II) Before Covid-19							
a) No Crypto							
TP	0,043	0,297	0,147	0,287	1,541	0,333	0,431
MV	0,014	0,166	0,082	0,140	1,238	0,277	0,352
RP	0,018	0,231	0,079	0,131	1,233	0,305	0,389
EW	0,085	1,969	0,043	0,098	1,143	3,408	4,285
MU	1,488	12,144	0,123	0,236	1,415	14,304	18,342
b) All Assets							
TP	0,063	0,353	0,178	0,355	1,632	0,374	0,487
MV	0,014	0,166	0,084	0,143	1,242	0,277	0,351
RP	0,048	0,429	0,112	0,205	1,357	0,583	0,743
EW	0,242	2,511	0,096	0,185	1,317	3,513	4,441
MU	2,478	15,700	0,158	0,305	1,531	19,980	25,699
III) During Covid-19							
a) No Crypto							
TP	0,199	1,398	0,142	0,297	1,571	1,364	1,777
MV	0,018	0,418	0,042	0,062	1,180	0,326	0,414
RP	0,036	0,559	0,064	0,094	1,281	0,617	0,783
EW	0,167	1,969	0,085	0,195	1,291	2,975	3,773
MU	1,858	13,573	0,137	0,270	1,547	13,732	17,838
b) All Assets							
TP	0,813	4,276	0,190	0,369	1,692	5,483	7,075
MV	0,007	0,410	0,018	0,027	1,069	0,351	0,442
RP	0,098	1,074	0,092	0,138	1,326	1,261	1,611
EW	0,283	2,453	0,115	0,187	1,390	2,865	3,638
MU	3,586	18,922	0,190	0,368	1,686	24,751	31,929

In the appendix Table 1.2, it is possible to observe the composition of the optimal portfolios during the three periods considered. Litecoin is the only cryptocurrency that assumes a positive weight in the MV portfolio during all three periods considered. At the same time, it is the only cryptocurrency that assumes negative weights in the Tangency and MU portfolios. Other than this, Bitcoin, Ethereum, and Ripple saw increased weights in the investment portfolios if short selling is allowed. Therefore, this result confirms the general positive contribution of cryptocurrencies to an investment portfolio.

Appendix Table 1.2: In-Sample Portfolio Optimal Weights

The table shows the optimal weights (in %) for each strategy in three different periods: Whole sample period (September 1st, 2015 – January 31st, 2022), Before Covid-19 pandemic (September 1st, 2015 – December 31st, 2019), and During Covid-19 pandemic (January 1st, 2020 – January 31st, 2022). All assets are considered. TP is the Tangency Portfolio, MV is the Minimum Variance Portfolio, RP is the Risk-Parity Portfolio, EW is the Equally Weighted Portfolio, and MU is the Maximum Utility Portfolio. Short sales are allowed. Daily returns are used for the calculations. Source: own elaboration.

	ETH	BTC	XRP	LTC	SPX	VIX	BND	VNQ
a) Whole Sample Period								
TP	6,14	11,26	3,91	-5,78	123,88	13,73	-24,06	-29,08
MV	-0,25	-0,36	-0,07	0,04	17,56	1,02	89,53	-7,47
RP	2,74	4,13	2,28	2,78	15,17	1,93	57,98	13,01
EW	12,50	12,50	12,50	12,50	12,50	12,50	12,50	12,50
MU	85,68	155,82	53,50	-78,30	1447,30	171,98	-1437,90	-298,08
b) Before Covid-19								
TP	1,50	2,85	1,19	-1,20	58,21	4,86	35,74	-3,16
MV	0,00	0,17	-0,07	0,06	19,81	0,89	85,27	-6,12
RP	1,85	2,99	1,56	1,93	14,99	1,41	61,61	13,66
EW	12,50	12,50	12,50	12,50	12,50	12,50	12,50	12,50
MU	75,80	135,41	63,57	-63,48	1953,80	200,91	-2411,80	145,85
c) During Covid-19								
TP	51,54	57,31	1,92	-45,34	293,34	48,58	-220,12	-87,24
MV	-1,77	-0,68	0,08	0,64	17,04	1,54	89,61	-6,45
RP	4,25	5,42	3,41	4,00	14,57	2,48	53,75	12,12
EW	12,50	12,50	12,50	12,50	12,50	12,50	12,50	12,50
MU	235,19	257,08	8,24	-203,72	1245,10	210,65	-1287,00	-365,52

The out-of-sample analysis has the same implications as the in-sample one. In addition, however, some characteristics of the Tangency portfolio must be stressed. Appendix Table 1.3 presents the out-of-sample results, allowing for short selling.

The Tangency portfolio without cryptocurrencies performs better in Sharpe and Sortino ratios but not in the Omega ratio. If, however, transaction costs are included, the performance is better only in terms of the Sharpe ratio.

In this regard, it should be noted that an essential difference between the three ratios considered in this study is that the Sharpe and Sortino ratios consider only the first two moments of the distribution of returns. In contrast, the Omega ratio considers all moments of a distribution. Benhamou et al. (2019) compute the Omega ratio for normal distribution and show that under some distribution symmetry assumptions, the Omega ratio does not provide any additional information compared to the Sharpe ratio. However, Rambo and Vuuren (2017) provide evidence of the superiority of the Omega ratio when returns are not normally distributed. The short-selling context and some stylized facts of cryptocurrencies, such as heavy tails, skewness, and leverage effects (Zhang et al., 2018), bring the returns of portfolios with cryptocurrencies to be far from normally distributed. Therefore, these results should be taken with caution as the Omega ratio could better measure performance.

Appendix Table 1.3: Out-of-Sample Performances

Out-of-sample performance of portfolio models (in % for mean, standard deviation (Std), value at risk (VaR), expected shortfall (ES), and turnover). Short sales are allowed. Monthly rebalancing frequency ($\tau = 22$ days). Monthly results are reported for mean and standard deviation. 1-month VaR and 1-month ES are considered, with a 95% confidence level ($\alpha = 0.95$). The sample period is from September 1, 2016, to January 31, 2022. Source: own elaboration.

	Mean	Std	Sharpe R.	Sortino R.	Omega R.	VaR	ES	Turnover
I) No Transaction Costs								
a) No Crypto								
TP	1,056	2,714	0,389	0,908	2,917	3,542	4,727	41,189
MV	0,347	1,480	0,235	0,495	2,218	1,284	1,676	2,634
RP	0,400	1,992	0,201	0,325	1,961	2,029	2,598	1,706
EW	2,340	11,640	0,201	1,186	2,799	18,207	23,411	3,957
MU	41,419	94,204	0,440	1,421	4,751	143,59	186,88	2424,2
b) All Assets								
TP	2,803	8,784	0,319	0,811	3,265	6,569	8,896	75,329
MV	0,267	1,562	0,171	0,342	1,742	1,367	1,770	3,083
RP	1,749	4,364	0,401	1,050	3,318	4,982	6,321	3,458
EW	7,232	21,786	0,332	1,560	3,621	22,719	28,728	7,615
MU	85,063	193,52	0,440	2,101	5,218	220,26	295,31	4039,7
II) Including Transaction Costs								
a) No Crypto								
TP	0,768	2,824	0,272	0,535	2,104	3,640	4,806	41,189
MV	0,329	1,474	0,223	0,465	2,130	1,306	1,700	2,634
RP	0,388	1,988	0,195	0,315	1,925	2,040	2,610	1,706
EW	2,313	11,645	0,199	1,164	2,756	18,340	23,451	3,957
MU	24,450	99,760	0,245	0,539	2,240	169,34	217,12	2424,2

b) All Assets								
TP	2,224	8,939	0,249	0,552	2,421	6,875	9,239	75,329
MV	0,245	1,559	0,157	0,310	1,662	1,394	1,798	3,083
RP	1,721	4,358	0,395	1,027	3,250	4,996	6,339	3,458
EW	7,166	21,778	0,329	1,537	3,575	22,765	28,796	7,615
MU	55,085	204,00	0,270	0,772	2,508	258,96	341,83	4039,7

Appendix Table 1.4: Out-of-Sample Average Weights

The table shows the out-of-sample average weights (in %) for each strategy before and during the Covid-19 pandemic. Short sales are allowed. Monthly rebalancing frequency ($\tau = 22$ days). TP is the Tangency Portfolio, MV is the Minimum Variance Portfolio, RP is the Risk-Parity Portfolio, EW is the Equally Weighted Portfolio, and MU is the Maximum Utility Portfolio. Daily returns are used for the calculations. Source: own elaboration.

	ETH	BTC	XRP	LTC	SPX	VIX	BND	VNQ
a) Before Covid-19								
TP	0,17	1,63	0,10	-0,09	13,48	0,57	88,14	-3,98
MV	-0,08	0,19	-0,05	0,02	21,80	0,96	83,32	-6,17
RP	1,86	2,97	1,66	2,03	16,08	1,37	60,85	13,19
EW	12,50	12,50	12,50	12,50	12,50	12,50	12,50	12,50
MU	-55,40	267,48	84,00	-34,66	2805,2	219,66	-3170,1	-16,16
b) During Covid-19								
TP	22,45	39,03	0,03	-33,77	146,90	20,35	-68,15	-26,84
MV	-2,08	-0,32	0,85	0,00	19,73	1,87	85,43	-5,48
RP	3,91	4,78	3,81	3,79	14,07	2,22	55,33	12,10
EW	12,50	12,50	12,50	12,50	12,50	12,50	12,50	12,50
MU	304,20	385,27	-32,82	-372,26	2092,0	258,15	-2477,8	-56,81

7.2 Appendix 2

Appendix 2 reports out-of-sample results for additional rebalancing periods. In particular, four further rebalancing periods are proposed: two weeks ($\tau = 10$ trading days), two months ($\tau = 44$ trading days), three months ($\tau = 66$ trading days), and six months ($\tau = 132$ trading days). Appendix Table 2.1 depicts the out-of-sample results for the different rebalancing frequencies, while appendix Table 2.2 reports the out-of-sample average weights.

The results perfectly confirm the conclusions obtained with a rebalancing frequency of one month for the first two periods. All the risk-adjusted performance measures (Sharpe, Sortino, and Omega ratios) improve when cryptocurrencies are included.

For more extended periods of rebalancing frequency (three and six months), some portfolios (Tangency and EW portfolios, in particular) report higher Sharpe ratios if cryptocurrencies are excluded. However, this is not true for the Sortino and Omega ratios. Therefore, the discussion on the difference between the ratios in Appendix 1 also applies here.

Appendix Table 2.1: Out-of-Sample Performances for different rebalancing frequencies

Out-of-sample performance of portfolio models (in % for mean, standard deviation (Std), value at risk (VaR), expected shortfall (ES), and turnover) for different rebalancing frequencies (two weeks ($\tau = 10$ trading days), two months ($\tau = 44$ trading days), three months ($\tau = 66$ trading days), and six months ($\tau = 132$ trading days)). Short sales are NOT allowed. The performance measures refer to the specific rebalancing frequencies (when $\tau = 10$ it means that mean, std, VaR, ES and turnover refers to a period of 10 days). A 95% confidence level ($\alpha = 0.95$) is used for VaR and ES. The sample period is from September 1, 2016, to January 31, 2022. Source: own elaboration.

	Mean	Std	Sharpe R.	Sortino R.	Omega R.	VaR	ES	Turnover
I) τ = 10 days								
a) No Crypto								
TP	0,411	1,576	0,261	0,502	2,122	2,088	2,725	6,348
MV	0,111	0,711	0,156	0,239	1,523	0,853	1,096	1,157
RP	0,238	1,018	0,234	0,452	1,954	1,309	1,704	1,442
EW	0,919	5,597	0,164	0,466	1,685	8,254	10,584	3,178
MU	0,848	10,336	0,082	0,167	1,325	14,392	18,329	14,178
b) All Assets								
TP	0,995	2,497	0,399	1,084	3,546	2,070	2,714	7,056
MV	0,117	0,717	0,163	0,251	1,548	0,876	1,126	1,186
RP	0,631	1,640	0,385	0,988	3,102	2,056	2,689	2,263

EW	2,812	10,666	0,264	0,789	2,318	13,382	17,294	4,903
MU	4,163	22,063	0,189	0,546	1,934	24,373	30,928	14,234

II) τ = 44 days

a) No Crypto

TP	1,955	3,126	0,625	1,707	5,835	3,085	4,367	14,668
MV	0,616	2,289	0,269	0,747	2,455	4,827	6,207	3,508
RP	1,294	2,502	0,517	1,653	4,706	2,724	3,745	4,315
EW	5,123	19,835	0,258	2,126	4,646	26,966	35,121	5,787
MU	-1,438	14,973	-0,096	-0,136	0,750	22,187	27,704	22,438

b) All Assets

TP	4,239	6,328	0,670	3,234	10,106	6,380	8,992	18,539
MV	0,677	2,322	0,292	0,821	2,596	4,976	6,413	3,621
RP	3,498	6,372	0,549	3,859	9,856	3,574	5,300	6,545
EW	15,000	36,512	0,411	2,637	5,570	16,239	22,755	10,726
MU	21,120	73,447	0,288	1,995	3,813	21,934	27,301	34,381

III) τ = 66 days

a) No Crypto

TP	2,302	2,854	0,807	2,742	6,884	2,255	3,411	15,681
MV	0,507	1,748	0,290	0,612	2,009	2,269	2,973	4,770
RP	1,175	2,187	0,537	1,233	3,720	3,137	4,260	5,103
EW	2,842	5,803	0,490	1,428	4,075	4,513	6,426	4,353
MU	3,274	10,117	0,324	0,801	2,288	9,245	12,520	28,998

b) All Assets

TP	6,117	10,903	0,561	3,767	8,342	10,222	14,334	21,078
MV	0,616	1,848	0,333	0,738	2,212	2,307	3,048	4,885
RP	4,485	7,615	0,589	4,181	7,887	3,354	5,135	8,297
EW	20,811	48,723	0,427	2,755	5,461	24,696	32,850	11,801
MU	43,769	123,308	0,355	2,871	5,212	22,521	27,916	46,624

IV) τ = 132 days

a) No Crypto

TP	4,500	3,631	1,239	7,368	25,437	3,212	5,226	21,580
MV	1,117	2,483	0,450	1,250	2,971	3,172	4,264	5,890
RP	2,438	2,463	0,990	3,367	11,162	1,974	3,091	7,415
EW	5,581	9,473	0,589	3,107	7,252	7,257	10,553	6,785
MU	2,340	10,141	0,231	0,498	1,765	13,871	17,984	33,093

b) All Assets								
TP	10,195	10,643	0,958	13,032	44,222	6,502	10,744	26,497
MV	1,274	2,568	0,496	1,396	3,360	3,088	4,202	5,981
RP	8,471	12,567	0,674	7,946	20,919	5,348	8,543	12,527
EW	34,897	67,419	0,518	6,617	9,118	21,995	31,638	13,767
MU	65,448	118,712	0,551	8,099	12,002	39,150	47,620	32,270

Appendix Table 2.2: Out-of-Sample Average Weights for Different Rebalancing Frequencies

The table shows the out-of-sample average weights (in %) for each strategy before and during the Covid-19 pandemic for different rebalancing frequencies (two weeks ($\tau = 10$ trading days), two months ($\tau = 44$ trading days), three months ($\tau = 66$ trading days), and six months ($\tau = 132$ trading days)). Short sales are NOT allowed. TP is the Tangency Portfolio, MV is the Minimum Variance Portfolio, RP is the Risk-Parity Portfolio, EW is the Equally Weighted Portfolio, and MU is the Maximum Utility Portfolio. Daily returns are used for the calculations. Source: own elaboration.

	ETH	BTC	XRP	LTC	SPX	VIX	BND	VNQ
I) τ = 10 days								
a) Before Covid-19								
TP	1,45	2,25	1,13	0,14	65,17	5,32	21,39	3,15
MV	0,03	0,15	0,02	0,04	19,22	1,05	79,48	0,02
RP	1,23	1,93	1,13	1,28	30,03	3,05	49,75	11,60
EW	12,50	12,50	12,50	12,50	12,50	12,50	12,50	12,50
MU	34,84	11,80	26,31	0,29	1,38	17,46	0,02	7,91
b) During Covid-19								
TP	3,41	10,89	0,14	0,00	58,56	12,27	10,83	3,91
MV	0,02	0,03	0,01	0,05	12,95	2,00	84,08	0,86
RP	2,46	3,11	2,61	2,43	21,17	6,60	49,16	12,44
EW	12,50	12,50	12,50	12,50	12,50	12,50	12,50	12,50
MU	33,14	39,07	3,06	0,18	0,38	24,13	0,01	0,03
II) τ = 44 days								
a) Before Covid-19								
TP	1,29	1,96	1,11	0,12	63,95	5,13	22,58	3,86
MV	0,03	0,14	0,02	0,04	19,33	1,06	79,36	0,02
RP	1,21	1,92	1,14	1,28	30,01	3,06	49,79	11,59
EW	12,50	12,50	12,50	12,50	12,50	12,50	12,50	12,50
MU	36,62	5,94	27,01	0,80	0,08	16,15	0,02	13,38

b) During Covid-19								
TP	2,79	9,68	0,00	0,00	53,90	10,84	20,07	2,72
MV	0,02	0,03	0,01	0,05	13,28	1,93	83,94	0,75
RP	2,40	3,03	2,54	2,36	21,30	6,33	49,89	12,14
EW	12,50	12,50	12,50	12,50	12,50	12,50	12,50	12,50
MU	30,91	42,68	1,27	0,02	0,41	24,69	0,01	0,03
III) τ = 66 days								
a) Before Covid-19								
TP	1,42	2,64	1,00	0,09	65,44	5,45	20,94	3,02
MV	0,03	0,13	0,02	0,05	19,20	1,04	79,52	0,02
RP	1,22	1,91	1,16	1,32	29,99	3,04	49,68	11,67
EW	12,50	12,50	12,50	12,50	12,50	12,50	12,50	12,50
MU	36,45	7,48	24,10	1,14	3,30	17,15	0,02	10,35
b) During Covid-19								
TP	3,11	11,84	0,03	0,00	57,04	12,07	13,01	2,90
MV	0,02	0,04	0,01	0,03	13,21	2,00	83,80	0,89
RP	2,51	3,17	2,68	2,50	21,80	6,67	48,18	12,48
EW	12,50	12,50	12,50	12,50	12,50	12,50	12,50	12,50
MU	36,52	40,29	2,07	0,02	0,01	21,06	0,01	0,01
IV) τ = 132 days								
a) Before Covid-19								
TP	1,04	2,93	0,90	0,09	70,00	5,37	16,11	3,57
MV	0,03	0,12	0,02	0,07	19,81	1,09	78,84	0,02
RP	1,19	1,95	1,21	1,36	30,68	3,04	48,99	11,57
EW	12,50	12,50	12,50	12,50	12,50	12,50	12,50	12,50
MU	43,20	0,39	29,12	2,46	0,18	13,95	0,02	10,67
b) During Covid-19								
TP	2,31	8,87	0,00	0,00	52,90	10,29	22,65	2,98
MV	0,03	0,03	0,01	0,03	12,65	1,79	84,66	0,80
RP	2,27	2,83	2,48	2,23	21,45	6,02	50,07	12,66
EW	12,50	12,50	12,50	12,50	12,50	12,50	12,50	12,50
MU	46,17	13,13	0,51	0,02	0,03	26,07	0,02	14,05

Bibliography

Andrianto, Y. and Diputra, Y. (2017), “The effect of cryptocurrency on investment portfolio effectiveness”, *Journal of Finance and Accounting*, Vol. 5 No. 6, pp. 229-238.

Anyfantaki, S., Arvanitis, S. and Topaloglou, N. (2018), “Diversification, integration and cryptocurrency market”, *Working paper,* Bank of Greece, Athens, March 1.

Arnold, I. and Vrugt, E. (2008), “Fundamental uncertainty and stock market volatility” *Applied Financial Economics*, 18(17), 1425-1440.

Arsi, S., Ben Khelifa, S., Ghabri, Y. and Mzoughi, H. (2021), “Cryptocurrencies: Key Risks and Challenges”, *World Scientific*.

Baek, C. and Elbeck, M. (2015), “Bitcoins as an investment or speculative vehicle? A first look”, *Applied Economics Letters*, 22(1), 30–34.

Baraldi, A. N. and Enders, C. K. (2010), “An introduction to modern missing data analyses”, *Journal of School Psychology,* 48, 5-37.

Benhamou, E., Guez, B. and Paris, N. (2019), “Omega and sharpe ratio”, *ArXiv*.

Bouri, E., Molnár, P., Azzi, G., Roubaud, D. and Hagfors, L. I. (2017), ‘‘On the hedge and safe haven properties of Bitcoin: Is it really more than a diversifier?’’ *Finance Research Letters*, vol. 20, pp. 192–198.

Brauneis, A. and Mestel, R. (2019), “Cryptocurrency-portfolios in a mean-variance framework”, *Finance Research Letters*, Vol. 28 No. C, pp. 259-265.

Briere, M., Oosterlinck, K. and Szafarz, A. (2015), “Virtual currency, tangible return: portfolio diversification with Bitcoin”, *Journal of Asset Management*, Vol. 16 No. 6, pp. 365-373.

Brzenk, P., Cheng, T. and Liu, B. (2020), “Indexing risk parity strategies”, *S&P Dow Jones Indices*.

Burniske, C. and Tatar, J. (2017), “Cryptoassets: The innovative investor’s guide to Bitcoin and beyond”, *McGraw-Hill Education,* New York.

Chalmers, J., Edelen, R. and Kadlec, G. (2001), “Fund Returns and Trading Expenses: Evidence on the Value of Active Fund Management”, *Working Paper*, University of Pennsylvania.

Champagne, C., Karoui, A. and Patel, S. (2018), "Portfolio turnover activity and mutual fund performance", *Managerial Finance*, 44(3):326-356.

Chan, S., Chu, J., Nadarajah, S. and Osterrieder, J. (2017), “A statistical analysis of cryptocurrencies”, *Journal of Risk and Financial Management*, 10(2), 12.

Chaves, D., Hsu, J., Li, F. and Shakernia, O. (2011), “Risk parity portfolio vs. other asset allocation heuristicportfolios”, *Journal of Investing*, Vol. 20, No. 1, pp. 108–18.

Chopra, V., and Ziemba, W. (1993), “The Effect of Error in Means, Variances and Covariances on Optimal Portfolio Choice,” *Journal of Portfolio Management*, 19, 6–11.

Colombo, J., Cruz, F. I. L., Paese, L. H. Z. and Cortes, R. X. (2021), “The diversification benefits of cryptocurrencies in multi-asset portfolios: cross-country evidence”, *Technical report*, SSRN.

Conlon, T. and McGee, R. (2020), “Safe haven or risky hazard? bitcoin during the covid-19 bear market”, *Finance Research Letters*, page 101607.

Corbet, S., Meegan, A., Larkin, C., Lucey, B. and Yarovaya, L. (2018), “Exploring the dynamic relationships between cryptocurrencies and other financial assets”, *Economic Letters*, Vol. 165 No. C, pp. 28-34.

Corbet, S., Larkin, C. and Lucey, B. (2020), “The contagion effects of the COVID-19 pandemic: Evidence from gold and cryptocurrencies”, *Finance Research Letters*, 101554.

DeMiguel, V., Garlappi, L. and Uppal, R. (2009), “Optimal versus naive diversification: How inefficient is the 1/n portfolio strategy?”, *Review of Financial Studies*, 22, 1915-1953

Forbes, K. and Rigobon, R. (2002), “No Contagion, Only Interdependence: Measuring Stock Market Comovements”, *Journal of Finance*, vol. 57, no. 5: 2223–2261.

Gkillas, K. and Katsiampa, P. (2018), “An application of extreme value theory to cryptocurrencies”, *Economics Letters*, 164: 109–11.

Glaser, F., Zimmermann, K., Haferkorn, M., Weber, M.C. and Siering, M. (2014), "Bitcoin—asset or currency? Revealing users' hidden intentions", In *22nd European conference on information systems*, Tel Aviv

Gil-Alana, L., Abakah, E. and Rojo, M. (2020), "Criptocurrencies and Stock Market Indices, are They Related?", *Research in International Business and Finance*, 51.

Grinold, R. C. and Kahn, R. N. (1999), "Active Portfolio Management", *McGraw-Hill*.

Hull, J. (2015), "Risk Management and Financial Institutions", 5th edition, *Wiley Finance*.

Inci, A.C., and Lagasse, R. (2019), "Cryptocurrencies: applications and investment opportunities", *Journal of Capital Markets Studies*, 3(2).

Kajtazi, A. and Moro, A. (2019), "The role of Bitcoin in well diversified portfolios: A comparative global study", *International Review of Financial Analysis*, 61, 143–157.

Keating, C. and Shadwick, W. F. (2002), "A universal performance measure", *Journal of Performance Measurement*, *6*(3), 59–84.

Klabbers, S. (2017), "Bitcoin as an investment asset", *Working paper,* Radboud Universiteit Nijmegen, Nijmegen, June 2.

Klein, T., Thu, H.P. and Walther, T. (2018), "Bitcoin is not the new gold – a comparison of volatility, correlation, and portfolio performance", *International Review of Financial Analysis*, Vol. 59 No. C, pp. 105-116.

Kroll, Y., Levy, H. and Markowitz, H.M. (1984), "Mean-variance versus direct utility maximization", *Journal of Finance*, Vol. 39 No. 1, pp. 47-61.

Levy, H. and Markowitz, H.M. (1979), "Approximating expected utility by a function of mean and variance", *American Economic Review*, Vol. 69 No. 3, pp. 308-317.

Lewis, A. (2018), "The basics of bitcoins and blockchains: an introduction to cryptocurrencies and the technology that powers them", *Mango Media Inc*.

Liu, W. (2019), "Portfolio diversification across cryptocurrencies", *Finance Research Letters*, Vol. 29 No. C, pp. 200-205.

Mangram, E.M. (2013), “A Simplified Perspective of the Markowitz Portfolio Theory” *Global Journal of Management and Business Research*, 7, 59–70.

Markowitz, H. (1952), “Portfolio selection”, *Journal of Finance*, Vol. 7 No. 1, pp. 77-91.

Markowitz, H. (1959), “Portfolio Selection. Efficient Diversification of Investments”, *John Wiley & Sons*, New York.

Merton, R.C. (1990), *Continuous-Time Finance*, Blackwell, Oxford

Michaud, R. O. and Michaud, R. O. (2008) “Efficient Asset Management: A Practical Guide to Stock Portfolio Optimization and Asset Allocation”, *Oxford University Press*, New York.

Maillard, S., Roncalli, T. and Teiletche, J. (2010), “On the properties of equally weighted risk contributions portfolios”, *Journal of Portfolio Management*, 60–70.

Nakamoto, S. (2008), “Bitcoin – a peer-to-peer electronic cash system”, available at https://bitcoin.org/ bitcoin.pdf (accessed May 16th, 2022).

Peugh, J., and Enders, C. (2004), “Missing data in educational research: A review of reporting practices and suggestions for improvement”, *Review of Educational Research*, 74(4), 525–556.

Philippe, J. (2001), “Value at Risk: The New Benchmark for Managing Financial Risk”, *McGraw-Hill Professional*, New York.

Phillip, A., Chan, J. and Peiris, S. (2018), “On long memory effects in the volatility measure of Cryptocurrencies”, *Finance Research Letters*, 28.

Platanakis, E., Sutcliffe, C. and Urquhart, A. (2018), “Optimal Vs Naïve Diversification in Cryptocurrencies”, *Economics Letters*, 171: 93–96.

Pomante, U. (2008), “Asset allocation razionale”, *Bancaria editrice*, Roma.

Rambo, J. and Vuuren, G. (2017), “An omega ratio analysis of global hedge fund returns”, *Journal of Applied Business Research*, (JABR), 33:575.

Ross, S., Westerfield, R. and Jaffe, J. (2002), “Capital market theory: An overview” *Corporate finance*, 6, 226-247.

Sharpe, W. (1966), “Mutual fund performance”, *Journal of Business*, 39(S1), 119–138.

Wu, C.Y. and Pandey, V.K. (2014), “The value of Bitcoin in enhancing the efficiency of an investor’s portfolio”, *Journal of Financial Planning*, Vol. 27 No. 9, pp. 44-52.

Yarovaya, L., Matkovskyy, R. and Jalan, A. (2020), “The effectsof a “black swan” event (COVID-19) on herding behavior in cryptocurrency markets: evidence from cryptocurrency USD, EUR, JPY and KRW markets”, *EUR, JPY and KRW Markets*.

Zhang, W., Wang, P., Li, X., and Shen, D. (2018), “Some stylized facts of the cryptocurrency market”, *Applied Economics*, 50 (55), pp. 5950–5965.

Website References

BlackRock (2022), "Understanding portfolio construction": https://www.blackrock.com/americas-offshore/en/education/portfolio-construction/understanding-portfolio-construction .

Bridgewater (2022), "Risk Parity Is About Balance": https://www.bridgewater.com/research-and-insights/risk-parity-is-about-balance .

Blockdata (2021), "81 of the Top 100 Public Companies are using blockchain technology": https://www.blockdata.tech/blog/general/81-of-the-top-100-public-companies-are-using-blockchain-technology .

Chainalysis (2021), "The 2021 Global Crypto Adoption Index: Worldwide Adoption Jumps Over 880% With P2P Platforms Driving Cryptocurrency Usage in Emerging Markets": https://blog.chainalysis.com/reports/2021-global-crypto-adoption-index/

CNBC (2021), "The first bitcoin futures ETF in the U.S. is set to begin trading in the week ahead": https://www.cnbc.com/2021/10/15/the-first-bitcoin-futures-etf-in-the-us-is-set-to-begin-trading-next-week.html .

CNBC (2022), "Cryptocurrency luna now almost worthless after controversial stablecoin it is linked to loses peg": https://www.cnbc.com/2022/05/12/cryptocurrency-luna-now-almost-worthless-after-ust-falls-below-peg.html

Coindesk (2021), "JPMorgan Launches In-House Bitcoin Fund for Private Bank Clients": https://www.coindesk.com/markets/2021/08/05/jpmorgan-launches-in-house-bitcoin-fund-for-private-bank-clients/ .

CoinMarketCap (2022): https://coinmarketcap.com/charts/ .

Cointelegraph (2021), "Morgan Stanley increased exposure to Bitcoin, held $300M in Grayscale shares": https://cointelegraph.com/news/morgan-stanley-increased-exposure-to-bitcoin-held-300m-in-grayscale-shares .

Cointelegraph (2022), “The city of Lugano will accept Bitcoin, Tether and LVGA tokens as 'de facto' legal tender”: https://cointelegraph.com/news/the-city-of-lugano-will-accept-bitcoin-tether-and-lvga-token-as-de-facto-legal-tender .