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Financial Economics

Key Determinants of Gold Price and Testing for the Optimal S&P 500/Gold Allocation

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Abstract

In the light of an ever more intense discussion regarding the potential benefits of investing in physical gold, this study aims to provide investors with a deeper understanding for the fundamental drivers of gold price development, as well as give investors a fact-based approach on how gold investments can be incorporated in a stock market portfolio most optimally. By processing data gathered from a range of selected professionals with expertise within the field of gold, the result of this study reveals that the US Monetary Supply, Geopolitical risks and US real interest rates are deemed to be three key determinants of gold prices, giving unique insights regarding the expected characteristics of the market. Moreover, a portfolio analysis conducted on S&P500 (SPX) and gold price movements between 2008 and 2022, proved that the highest Sharpe Ratio obtainable for this gold/equity portfolio were to be reached if 30% of the portfolio would be allocated towards gold, greatly exceeding the suggested gold allocation of similar studies during other time periods.

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Tables, Diagrammes and Equations

1. Background Description

Since ancient times, gold has served as an attractive commodity for people around the globe. In the article "*Gold: The Most Precious of Metals"* Brian Dowd provides an interesting description of how people from different parts of the world, which often had very little in common, shared a desire to own gold in some form (Dowd, 2016). It is clear that since its discovery, gold has always been seen as a valuable asset, even though it first came to use in jewellery and other decorations rather than coins. Dowd describes that gold was first used as a medium of exchange in Egypt in approximately 1500 BC, where traders found that the non-eroding property of gold made it particularly useful for keeping value. Since then, more and more uses for gold has been discovered by different civilisations. Today, gold is used for a large number of purposes, including jewellery, electronics, culinary, medicine and, most relevant for this study, as a way to store wealth.

In times of uncertainty and worries for inflation and rising interest rates, precious metals are often being promoted as attractive investment alternatives to more conventional forms of savings, such as stocks and bonds (Fortune India, 2022). Supporters of gold investments often point to its long history of being a value keeping commodity, as well as its naturally limited supply, which in comparison with monetary assets might provide a higher degree of certainty regarding its underlying value. At the same time, the area is highly disputed, as other prominent investors claim that gold does not provide any substantial value creation, making investments in gold a less preferable option than exposing one's wealth to the stock market (Business Insider, 2022).

2. Aim of the Study

To provide new insights to the discussion related to gold investments, we aim to investigate which factors are estimated to have the highest relative impact on the development on gold price. As a complementary, second part of this thesis, we are also giving examples of how investors can incorporate gold into their portfolios. Since gold is one of the most frequently traded commodities, a better understanding of the key drivers for gold price could help a wide range of investors gain a deeper knowledge for the influencing factors when making investment decisions related to gold. Meanwhile, the second part of the study aims to find the optimal allocation of gold in a gold/equity portfolio, based on historical numbers during a recent, 14-year time period.

2.1. Aim of the Analytical Hierarchy Process and Pairwise Comparison

By hosting interviews and gathering input from individuals with a record of expertise within the field of gold, an Analytical Hierarchy Process is conducted, allowing us to create a ranking of the factors believed to be the most influential in the determination of gold prices. Although the comparison methodology is subjective by nature, the input gathered from a wide range of gold market actors will help shape an understanding for the expectations and beliefs of the market, which itself is the determining factor of gold prices. Hence, the qualitive way of gathering data aims to provide valuable and unique insights on the speculative aspects upon which the gold market is based.

The research question for this part of the study could be summarised in the following way:

"Which factors are relevant to monitor among a set of factors previously proved significant in explaining gold price drivers?"

2.2. Aim of the Portfolio Optimisation

The study also aims to give a clear example of how investors can optimise their exposure towards gold in their portfolios, based on the performance of gold and stocks during the time period between January 2008 and March 2022. With gold often being viewed as an alternative investment tool that can be combined with investments in the stock market in order to reduce risks, such a study can be performed by locating an optimal portfolio based on the portfolio opportunity set. Thus, the historical covariances and returns of the stock market and gold prices can be used to derive the highest possible Sharpe ratio, indicating how gold can be combined with investments in the stock market most efficiently.

The research question for this part of the study could be summarised in the following way:

"What would have been the optimal allocation of gold in an equity/gold portfolio between January 2008 and March 2022?"

3. Problem Description and Problem Analysis

Despite undoubtedly being a common source of investments, the discussion related to gold diversification is broadly surrounded by vague, often personal interpretations on why investors should or should not own physical gold. Although often being promoted as an efficient tool of "hedging" against the movements of the stock market, there seems to be limited consensus on the more precise situations in which gold shall be held, as well as what the true factors, determining gold prices actually are. In an April 2022 monthly comment, Blackrock CFA and portfolio manager Russ Koesterich summarises the situation accordingly:

"One of the many challenges of defending gold: Even its advocates can't agree on why to own it. Many consider gold a hedge against inflation. Others suggest it is a long-term hedge against a depreciating dollar and/or stock market risk" (Blackrock, 2022)

Koesterich's statement captures a problematic aspect of investments in gold, as even knowledgeable experts struggle to gain a comprehensive understanding for the determinants. This implies that the concept of gold, as a diversifying component of a portfolio, is even harder to grasp for those with less expertise since those providing advice often have confusing, sometimes contradictory perceptions on what aspects to consider. Meanwhile, like all assets, the price of gold is determined by the actions and analysis provided by human beings, highlighting the importance of subjective perceptions, expectations and beliefs. Therefore, a comprehensive study of experts' estimates on key factors would strengthen the understanding for movements on the gold market, reducing the complexity for potential retail gold investors.

While conducting research on the topic of price drivers of gold, we discovered that most previous studies within the field have pursued the approach of hypothesis testing, focusing on the statistical significance of potential key drivers. Based on the insights gained from such research, a wide range of factors have, on different occasions, been proved to be statistically significant. Due to the already high coverage of these kinds of studies, this study takes a different approach, as we will be investigating the scale of impact from each factor. This is due to the fact that few other researchers have done this, as the main focus of previous research has been to determine whether certain potential price drivers are relevant or not, hence not focusing on the quantitative impact of each driver. With support from the extensive research previously conducted within the area, and with information gathered from individuals with expertise knowledge on gold, we therefore decided to pursue a different path

than most former research, allowing us to provide meaningful knowledge to this field of studies.

The first part of this thesis aims to conclude what factors gold market participants believe play the biggest role in the determination of gold prices, while the second part aims to give investors a fact-based approach on how to practically construct a portfolio of gold combined with equity. By including the latter aspect, the research of this paper not only examines the factors subjectively believed to impact gold prices, but also investigates the practical implications of historical data on such factors. Hence, the inclusion of a portfolio analysis was deemed to be a relevant complement to the results developed by the Analytical Hierarchy Process method. Especially since two of the ten factors examined are directly connected to the returns and volatility of the stock market, in terms of its impact on gold prices. As opposed to the limited usage of Analytical Hierarchy Process in earlier academic literature that concerns gold price, different applications of portfolio analysis and Modern Portfolio Theory have during numerous occasions been applied in the context of constructing efficient equity/gold portfolios. However, many of such research papers have been based on data stretching over many decades and has generally not included the drastic market movements of the last couple of years. Meanwhile, both the public interest for gold investments as well as prices on gold has risen, making it highly relevant to shed light upon the volatile developments of the last couple of years. For that reason, the portfolio analysis conducted in this thesis spans over the time period between 2008 and 2022, hence capturing major modern-day events that has shaped the global economy and the underlying financial markets.

4. Theoretical Framework and Results of the Literature Review

4.1 Factors Relevant for Determining the Price of Gold

In this study, ten factors will be evaluated and compared, in terms of each factors' relative impact on the development of gold prices. The aspects being covered consists of a range of macroeconomic, financial, and societal factors that in earlier studies have been proved to be statistically significant in its roles as determinants of gold prices. The selection of determinants has also partly been based on the frequency of these factors occurring in previous academic literature, as frequent coverage on these factors suggests a high degree of relevance, as well as provides us with suitable and well researched background knowledge on the topics investigated.

4.1.1. Real Interest Rate – Factor 1

Since gold offers zero interest, financial logic suggests gold prices should attract investors in times of negative interest rates. In the opposite scenario, high interest rates should in theory decrease willingness to invest in gold, implying an inverse relationship between movements in interest rates and development of gold prices. With interest rates playing a key role in the overall performance of financial- and asset markets, interest rates have therefore often been portrayed as a key metric of investigation. In *The Application of Gold Price, Interest Rates and Inflation* (Abdullah & Bakar, 2015) the gold prices were deemed "inversely related to real rates", a conclusion similar to the findings of numerous other pieces of research, such as *Do gold prices respond to real interest rates? Evidence from the Bayesian Markov Switching VECM model* (2014).

4.1.2. Monetary Supply – Factor 2

One of the key supporting arguments for pursuing gold investments is the claim that gold serves a role as a hedge against inflation, as its value-bearing characteristics provides a higher degree of certainty in times of expanding monetary supply. Therefore, the change in monetary supply in the economy (indicating underlying inflation) has been a suggested core factor in a vast number of studies determining the relevant factors. The gathered consensus of many such reports is that fluctuations in US monetary supply provides a statistically significant correlation to the development of gold prices, (Sipkova. H, Sipko. J, 2014 and Yanhui & Jon, 2017)

4.1.3. Exchange rate – Factor 3

With transactions on the global gold market most commonly being denominated in US dollar, the value and development of the USD in relation to other currencies have often been cited as one of, if not the main key driver of gold prices. Although certain recent studies have questioned this approach, the influence of the US exchange rates have repeatedly been emphasised by researchers, claiming that holding gold offers a significant hedge against US exchange rates, since an appreciating USD should hamper USD price development of gold (Güris & Kiran, 2014 and Shen, 2014).

4.1.4. Stock Market Returns – Factor 4

As gold is often being viewed as an attractive tool of wealth diversification during weaker business cycles (often measured by the returns of the broader stock market), estimating its correlation to the stock market would give a hint on such a connection. More precisely, in- or outflows of capital towards gold purchases are often financed by stock market sell-offs, potentially creating a causality. In "What Are the Real Drivers of Gold Prices", the quantitative analysis conducted pointed this out, claiming that the influence of S&P 500 returns was statistically significant to the movements of gold prices. Similarly, *"Economic drivers of volatility and correlation in precious metal"* (2021), also highlights the significant, influential long-term relationship between stock market indices (including S&P 500) and USD spot prices of gold.

4.1.5. Stock Market Volatility – Factor 5

While returns on the stock market has been proved to play a significant role in the determination of gold prices, a 2013 study published in Journal of Applied Finance (Boscaljon. J, 2013) also revealed that the volatility of the stock market itself (measured by VIX index) had a correlation to the returns of gold producing mining firms. This conclusion was supported by the thesis that periods of uncertainty on equity markets benefits investments in gold, offering investors a so called "Flight-to-Safety".

4.1.6. Fiscal Deficits – Factor 6

Numerous metrics related to the monetary policies of a Central Bank (Money supply, interest rate) have, as mentioned, been proved to influence gold prices. However, ratios related to the fiscal policies of governments can also impact gold prices (Bapna, 2012), as periods of increasing global fiscal deficits tends to trigger appreciations in gold prices. This is, according to Bapna, due to the fact that parts of the money generated from changes in fiscal deficits tend

to be invested in gold asset. In situations of growing fiscal deficits, the development of the overall economic standing of countries might also be questioned, further strengthening gold's status as a "safe haven" during periods of economic turmoil, hence appreciating gold prices. Using both multivariate regression and stepwise regression, the findings of another publishing: *Economic Variables Affecting the Gold Price* (2020), also showed statistical significance between movements of US fiscal deficits and gold prices.

4.1.7. Price of Cryptocurrency – Factor 7

As cryptocurrencies, and primarily Bitcoin has been emerging as an alternative source of investment to more conventional ways of pooling wealth, many describe cryptocurrencies in terms of "digital gold", as both gold and crypto investments are often labelled as ways of protecting the buyer against inflation. With a somewhat similar investment thesis between the two, the correlation between prices on gold and cryptocurrencies have been an area which has seen growing attention in academic research (for example *Cryptocurrency: determining the relationship between bitcoin currency and gold prices* (Bagci, H. & Keskin Köylü, M, 2019)). With cryptocurrencies being emerging, disruptive form of investment that gains growing attention both among the broader public as well as from researchers, an inclusion of such a factor therefore felt highly relevant and feasible to pursue for this study. Moreover, numerous studies have concluded that the development of Bitcoin and gold prices does have a significant correlation. Although this significance has shifted greatly during different time periods, such studies includes *Cryptocurrency: determining the relationship between bitcoin currency and gold prices,* but was also highlighted in *Bitcoin Correlation to Gold Jumps in 2020* (Vaneck, 2020).

4.1.8. Price of Silver – Factor 8

Sharing many of the physical attributes of gold, the two commodities have a somewhat similar history, as silver also has been a medium for trade and value storage for thousands of years. For that reason, demand for silver can be assumed to have a high degree of correlation to the demand for gold. The relationship between movements of gold and silver prices therefore helps us gain an understanding for whether demand for gold can be seen as relatively exogenous, or if gold price development rather is an indication of general inflows in the market for value storing commodities overall. In *what are the real drivers of gold prices? (2014),* Hana Sipkova and Juraj Sipko argues that the correlation between silver and gold prices have grown significantly during recent years, a conclusion they attribute to the fact that

the market for combined investments in silver and gold has seen a rise in later years. The phenomenon of correlating gold and silver prices was also proved significant by Cetin Ciner (2001), who performed a long-term investigation based on silver and gold traded on Tokyo Commodity Exchange during the 1990s.

4.1.9. Price of Crude Oil – Factor 9

Similar to gold in many ways, but very different in others, crude oil is commonly traded commodity for investors around the world. Previous research suggests a strong correlation between the price of gold and the price of crude oil between the years of 2000 – 2008 (Zhang & Wei, 2010). While the relationship between oil prices and gold often is being explained by oil prices effects on inflation, as well as gold investments as hedge against inflation, researchers have also suggested more direct connections between crude oil prices and demand for gold. In *Oil price shocks and gold returns*, written in 2012 by Thai-Ha Le and Youngho Chan, the authors state the following: "In order to disperse market risk and maintain commodity value, dominant oil-exporting countries use high revenues from oil sales to invest in gold. Since several countries including oil producers retain gold as an asset in their international reserve portfolios, rising oil prices (and hence oil revenues) may have implications for increases in gold prices".

4.1.10. Geopolitical Risks – Factor 10

Since gold is often used as a way to secure wealth when other assets are deemed too risky, it is highly relevant to analyse the correlation between a measure of geopolitical risk and the price of gold. Previous research on this topic indicates a positive relationship between geopolitical risk and the price of gold. One example of this is the study *The Gold market as a safe haven against the stock market uncertainty: Evidence from geopolitical risk* (Triki & Ben Maatoug, 2021). One of their study's highlights says that gold seem to display safe-haven characteristics in times of stress. This means that gold prices tend to appreciate in times of rising geopolitical tensions, such as wars and conflicts, a conclusion also made by Lee A. Smales and Dirk G. Baur in *Hedging Geopolitical Risk with Precious Metals (*2020).

4.2 Testing for Optimal Gold-equity Allocation

4.2.1. Is Gold a Hedge or a Safe Haven? An Analysis of Stocks, Bonds and Gold

In the paper "Is Gold a Hedge or a Safe Haven. An analysis of Stocks, Bonds and Gold", Brian M. Lucey and Dirk G. Baur examines whether gold, on average, provides either a hedge or a safe haven towards stocks and bonds in times of violent market turmoil. With a Hedge being defined as an asset that is "uncorrelated or negatively correlated with another asset or portfolio on average" (Lucey, Baur, 2010), the study covers the time period between November 30, 1995, until November 30, 2005, from which data on MSCI stock indices and spot prices of gold was gathered.

The results of the study concluded that for stocks, gold partially provides a substantial hedge against the movements of the stock market. Based on the results from a regression analysis, Lucey and Baur argue that gold offers a significant hedge for stocks during bear markets, but that there are no significant estimates regarding gold as a hedge during periods characterized as bull markets. Meanwhile, the term "safe haven", can be seen as a somewhat extended measure of hedge, by the authors described as the following: "A safe haven is defined as an asset that is uncorrelated or negatively correlated with another asset or portfolio in times of market stress or turmoil". According to the findings from Lucey and Baur, holding gold functions as a safe haven against stocks during a limited period of time of up to around 15 trading days. In the longer term, however, the asset loses its negative correlation with the stock market. Hence, the study suggests that the results show a large difference as to if investors should hold gold at all times, or only purchase such assets in the aftermath of extreme negative stock market shocks.

4.2.2. Do Precious Metals Shine? An Investment Perspective

In "Do Precious Metals Shine? An Investment Perspective", David Hillier, Paul Draper & Robert Faff investigates the potential diversification benefits of holding a set of precious metals, combined with a stock portfolio based on S&P500. The paper examines a measure of portfolio efficiency in terms of relative reward-to-risk ratio, and exemplifies a range of investment portfolios containing equities, gold, platinum, and silver.

The study, published in 2006, is based on a sample period of the years between 1976 and 2004, and gathers data from S&P 500 Price Index, MSCI EAF Index, as well as spot prices on equities, gold, platinum and silver. The test was conducted by applying a standardised

GARCH (generalized autoregressive conditional heteroskedasticity) process, an econometric model optimized to derive volatility on financial markets (Bollerslev, 1986).

During the study, each commodity and its contribution to the equity portfolio was evaluated separately and independent from each other. Hence, conclusions could be made about all three commodities and its corresponding historic records as diversifiers/hedge tools against the stock market. The research concluded that gold, platinum, and silver all possess potential to take diversifying roles in such investment portfolios, reducing the overall volatility that would be obtained if merely holding equities. More specifically, portfolios with gold were proved to enable a higher relative reward-to-risk ratio, if weighted moderately. The absolute highest reward-to-risk was found to be achieved if the portfolio is allocated towards 9,5 percent physical gold, reaching a relative reward-to-risk ratio, of 1.1850

5. Methods and Choice of Methodology

5.1. Analytical Hierarchy Process and Pairwise Comparison

This part of the study will be based around certain parts of an Analytical Hierarchy Process (AHP), a method optimised for the purpose of comparing the impact of different factors and using that result to make an optimal choice. An important part of the AHP, and a major part in our study, is to make a Pairwise Comparison of each individual factor. Thomas L Saaty explains the uses of an Analytical Hierarchy Process in his article "*Relative Measurement and Its Generalization in Decision Making: Why Pairwise Comparisons are Central in Mathematics for the Measurement of Intangible Factors - the Analytic Hierarchy/Network Process*" (Saaty, 2008). Saaty gives examples of when the AHP has been used to decide between different alternatives, where some of the most prominent examples are described on page 253 in his text. To be precise, applying an AHP, where the main focus in our study lie in the Pairwise Comparison part, will allow us to make a conclusion regarding how influential each of our chosen factors are for the development of gold prices. The factors used are based on findings from previous academic research, where a few key significant factors have been identified.

As a key part of the process of applying the Pairwise Comparison methodology, the input will be based on the knowledge of individuals with expertise within the field of commodities and finance. Such experts include individuals holding highly influential positions within the Swedish gold trade sphere, and their input will be gathered by hosting personal interviews or letting the respondents answer a survey through Google Forms. The survey has been created by us and allows the expert to rank the importance of each factor in relation to the other included factors. Our definition of an expert is someone that is, or has been, professionally involved in decision-making or research related to both gold price and its connection to our examined factors. Our aim has been to find experts with different backgrounds and professional roles, with the common denominator that they are highly knowledgeable about gold price and economic factors connected to the determination of said price. Comparing their answers by using the Pairwise Comparison method enables the qualitative input of the gathered data to be converted into an easily interpreted matrix. With the area of application and the problem description in mind, a Pairwise Comparison makes a good fit for the aim of this study, which is to quantify and measure the insights gathered from those with extensive

knowledge about the gold market. The method applied in this paper follows the applicable steps described in page 603 – 611 in “An Introduction to Management Science” (2016).

A key aspect of the Pairwise Comparison methodology lays within the name itself, as usage of the model relies on the comparison of factors in pairs of two. The area of use is broad since the target of the comparison is to be determined by the user. Hence, the Pairwise Comparison framework can be viewed as any process that compares a set of entities in pairs, and determines which entity has a greater amount of a certain quantitative property (Ross. M, 2020). In the case of this study, the different entities used in the comparison will be the ten gold price determinants previously mentioned, and the Pairwise Comparison will measure the relative impact of one determinant in relation to that of another.

5.1.1 Questionnaire and Initial Setup

The first step of the Analytical Hierarchy Process is to gather input data that later will be subject to the mathematical calculations leading to the final output of the study. Based on the Pairwise Comparison methodology, which suggests entities to be compared on a scale from 9 – 1/9, the participants of the study will be asked rank the relative importance of each factor with regard to another factor with a number between 9 – 1/9. The numbers used in this method can be interpreted by the statements in the table below.

1. Pairwise Comparison – Factor Degree of Importance

The table below shows how to interpret the numbers that the respondents assign to the factors included in this study.

9	**Extreme importance**
8	
7	**Very strong importance**
6	
5	**Strong importance**
4	
3	**Moderate importance**
2	
1	**Equal importance**
1/2	
1/3	**Moderately weak importance**
1/4	
1/5	**Weak importance**
1/6	
1/7	**Very weak importance**
1/8	
1/9	**Extremely weak importance**

The degree of importance refers to the comparison of two factors. For example: a given number of 3 suggests that, according to the respondent, one factor is moderately more important than another in terms of its influence on gold price development. The complete

questionnaire can be summarised by the following table, although display related adjustments have been made in order to construct a more user-friendly outlook.

2. Pairwise Comparison – Questionnaire

The matrix below is filled with the respondents' answers as they are collected. There is one individual matrix per respondent.

	US Monetary supply (M2)	US Real Interest rates	USD Exchange rate	Fiscal deficits	Stock market returns (S&P 500)	Stock market volatility (VIX)	Price of Bitcoin USD	Price of silver USD	Oil price USD	Geopolitical risk
US Monetary supply (M2)	1,00									
US Real Interest rate	########	1,00								
USD Exchange rate	########	########	1,00							
Fiscal deficits	########	########	########	1,00						
Stock market returns	########	########	########	########	1,00					
Stock market volatility (VIX)	########	########	########	########	########	1,00				
Price of Bitcoin USD	########	########	########	########	########	########	1,00			
Price of silver USD	########	########	########	########	########	########	########	1,00		
Oil price USD	########	########	########	########	########	########	########	########	1,00	
Geopolitical risk	########	########	########	########	########	########	########	########	########	1,00

Even though there are a total of 100 comparisons to be made, half of the adjustable cells are fully determined by the input of one corresponding cell, making them derivable from already existing input. Therefore, the respondents need to fill in 40 columns to complete the data gathering. Despite the possibilities to share an identical Excel sheet with the chosen respondents, we estimated that the chances of getting suitable answers would be higher by providing a more simple, modified sheet still giving the same results. For that reason: a form containing the same information structured as a multiple-choice questionnaire was sent to experts that we estimated could have sufficient knowledge to answer the questions.

5.1.2. Data Transformation Process

When the process of gathering input from the chosen respondents has been finalised, the data must be summarised and converted to a format applicable to what the final matrix is aimed to look like. This procedure follows a pre-determined, standardised path, which works the same for most AHP applications of Pairwise Comparisons, with the full-length description of the steps being provided in the appendix.

When the input data has been gathered and inserted into the form, the given numbers on relative impact can be summarised by an added row in the sheet, referred to as "sum".

3. Pairwise Comparison – Row for the Sum of Relative Factor Importance

The matrix is identical to the one above, with an added row and column for the sum and criteria weights respectively.

	US Monetary supply (M2)	US Real Interest rates	USD Exchange rate	Fiscal deficits	Stock market returns (S&P 500)	Stock market volatility (VIX)	Price of Bitcoin USD	Price of silver USD	Oil price USD	Geopolitical risk	Criteria Weights
US Monetary supply (M2)	1,00										
US Real Interest rate	########	1,00									
USD Exchange rate	########	########	1,00								
Fiscal deficits	########	########	########	1,00							
Stock market returns	########	########	########	########	1,00						
Stock market volatility (VIX)	########	########	########	########	########	1,00					
Price of Bitcoin USD	########	########	########	########	########	########	1,00				
Price of silver USD	########	########	########	########	########	########	########	1,00			
Oil price USD	########	########	########	########	########	########	########	########	1,00		
Geopolitical risk	########	########	########	########	########	########	########	########	########	1,00	
Sum											

4. Pairwise Comparison – Factor Weight Calculation

The matrix below shows how to calculate the normalised values for each data entry.

	US Monetary supply (M2)	US Real Interest rates	USD Exchange rate	Fiscal deficits	Stock market returns (S&P 500)	Stock market volatility (VIX)	Price of Bitcoin USD	Price of silver USD	Oil price USD	Geopolitical risk
US Monetary supply (M2)	1/B12	Given Value/C12	Given Value/D12	Given Value/E12	Given Value/F12	Given Value/G12	Given Value/H12	Given Value/I12	Given Value/J12	Given Value/K12
US Real Interest rate	Given Value/B12	1/C12	Given Value/D12	Given Value/E12	Given Value/F12	Given Value/G12	Given Value/H12	Given Value/I12	Given Value/J12	Given Value/K12
USD Exchange rate	Given Value/B12	Given Value/C12	1/D12	Given Value/E12	Given Value/F12	Given Value/G12	Given Value/H12	Given Value/I12	Given Value/J12	Given Value/K12
Fiscal deficits	Given Value/B12	Given Value/C12	Given Value/D12	1/E12	Given Value/F12	Given Value/G12	Given Value/H12	Given Value/I12	Given Value/J12	Given Value/K12
Stock market returns	Given Value/B12	Given Value/C12	Given Value/D12	Given Value/E12	1/F12	Given Value/G12	Given Value/H12	Given Value/I12	Given Value/J12	Given Value/K12
Stock market volatility (VIX)	Given Value/B12	Given Value/C12	Given Value/D12	Given Value/E12	Given Value/F12	1/G12	Given Value/H12	Given Value/I12	Given Value/J12	Given Value/K12
Price of Bitcoin USD	Given Value/B12	Given Value/C12	Given Value/D12	Given Value/E12	Given Value/F12	Given Value/G12	1/H12	Given Value/I12	Given Value/J12	Given Value/K12
Price of silver USD	Given Value/B12	Given Value/C12	Given Value/D12	Given Value/E12	Given Value/F12	Given Value/G12	Given Value/H12	1/I12	Given Value/J12	Given Value/K12
Oil price USD	Given Value/B12	Given Value/C12	Given Value/D12	Given Value/E12	Given Value/F12	Given Value/G12	Given Value/H12	Given Value/I12	I/J12	Given Value/K12
Geopolitical risk	Given Value/B12	Given Value/C12	Given Value/D12	Given Value/E12	Given Value/F12	Given Value/G12	Given Value/H12	Given Value/I12	Given Value/J12	1/K12
Sum	**B12**	**C12**	**D12**	**E12**	**F12**	**G12**	**H12**	**I12**	**J12**	**K12**

5. Pairwise Comparison – Criteria Weights Calculation

The matrix below shows how to calculate the criteria weights for each factor.

	US Monetary supply (M2)	US Real Interest rates	USD Exchange rate	Fiscal deficits	Stock market returns (S&P 500)	Stock market volatility (VIX)	Price of Bitcoin USD	Price of silver USD	Oil price USD	Geopolitical risk	Criteria Weights
US Monetary supply (M2)	1/B12	Given Value/C12	Given Value/D12	Given Value/E12	Given Value/F12	Given Value/G12	Given Value/H12	Given Value/I12	Given Value/J12	Given Value/K12	
US Real Interest rate	Given Value/B12	1/C12	Given Value/D12	Given Value/E12	Given Value/F12	Given Value/G12	Given Value/H12	Given Value/I12	Given Value/J12	Given Value/K12	
USD Exchange rate	Given Value/B12	Given Value/C12	1/D12	Given Value/E12	Given Value/F12	Given Value/G12	Given Value/H12	Given Value/I12	Given Value/J12	Given Value/K12	
Fiscal deficits	Given Value/B12	Given Value/C12	Given Value/D12	1/E12	Given Value/F12	Given Value/G12	Given Value/H12	Given Value/I12	Given Value/J12	Given Value/K12	
Stock market returns	Given Value/B12	Given Value/C12	Given Value/D12	Given Value/E12	1/F12	Given Value/G12	Given Value/H12	Given Value/I12	Given Value/J12	Given Value/K12	
Stock market volatility (VIX)	Given Value/B12	Given Value/C12	Given Value/D12	Given Value/E12	Given Value/F12	1/G12	Given Value/H12	Given Value/I12	Given Value/J12	Given Value/K12	
Price of Bitcoin USD	Given Value/B12	Given Value/C12	Given Value/D12	Given Value/E12	Given Value/F12	Given Value/G12	1/H12	Given Value/I12	Given Value/J12	Given Value/K12	
Price of silver USD	Given Value/B12	Given Value/C12	Given Value/D12	Given Value/E12	Given Value/F12	Given Value/G12	Given Value/H12	1/I12	Given Value/J12	Given Value/K12	
Oil price USD	Given Value/B12	Given Value/C12	Given Value/D12	Given Value/E12	Given Value/F12	Given Value/G12	Given Value/H12	Given Value/I12	I/J12	Given Value/K12	
Geopolitical risk	Given Value/B12	Given Value/C12	Given Value/D12	Given Value/E12	Given Value/F12	Given Value/G12	Given Value/H12	Given Value/I12	Given Value/J12	1/K12	
Sum	**B12**	**C12**	**D12**	**E12**	**F12**	**G12**	**H12**	**I12**	**J12**	**K12**	

$$\text{Criteria Weights} = \frac{\text{Cumulative value of all cells on the US monetary supply row}}{\text{Number of factors compared (10)}}$$

The next step in the data transformation process is to assign each row a factor/criteria weight, later allowing us to approach a conclusion on the magnitude of each factors impact. Note that the sum of all ten criteria weights must equal one, if calculated correctly.

As the Pairwise Comparison methodology is fully dependent on the "given values" inserted by the respondents, the method is highly reliant on the validity of the data. Hence, inconsistent or randomized answers must be detected and considered, since such inputs risks causing unreliable and misleading results. To check whether the calculated values are correct or not, a consistency test will be conducted. While a full-length description of all steps in the consistency check process being provided in the appendix, the process can be summarised by the calculations of the so called "consistency ratio", which for each data set must be below 0.10.

6. Pairwise Comparison – Consistency Index Calculation

$$Consistency\ index = \frac{(\lambda max\ -\ \mathrm{n})}{(\mathrm{n}-1)}$$

7. Pairwise Comparison – Consistency Ratio Calculation

$$Consistency\ ratio = \frac{Consistency\ index}{Random\ index}$$

A consistency ratio value under 0.10 is deemed acceptable, since it is not always certain that all respondents are able to perfectly rank all factors in relation to all other factors included in the study. If the consistency ratio value exceeds 0.10 however, the answers provided by the expert need to be revised in order to track the origin and cause of the inconsistencies.

5.1.2. Weaknesses of the Analytical Hierarchy Process Method

Even though an Analytical Hierarchy Process was deemed the most suitable method available for achieving the goals of this study, there are certain aspects of the model that needs to be considered when evaluating the results. Firstly, even though the respondents that participate in this study possess a deeper knowledge of our chosen topic than the vast majority of people, the answers they provide are based on their own experiences and estimations. This means that the answers could potentially be biased by subjective opinions or, perhaps more likely, recent events that affects one or more of the factors included in this study. For example, the current war between Russia and Ukraine could potentially influence an expert into giving the factor Geopolitical Risks a higher importance than what it would have under normal circumstances. This could be a consequence of the sharp increase in gold price that followed the war between Russia and Ukraine, causing the experts to overestimate the importance of the Geopolitical Risk factor since the data-gathering phase of this study was conducted at roughly the same time as the outbreak of the war. Due to the nature of the gold market, which is driven by human perceptions, expectations and estimates, the subjective style of research provided by an AHP was however not viewed as a shortcoming of the model in this specific case, rather, a good fit for the purpose.

Another scenario that could lessen this study's reliability is if the experts fail to provide consistent answers. Even if the answers achieve a consistency ratio under the threshold of 0.10, it is still possible that they do not submit answers that are consistent with their actual estimations of the importance of one or several factors. Reasons for inconsistencies among the answers could be something as simple as a misinterpretation of a stated question or ticking a different box than intended when filling in the survey. Luckily, these kinds of errors are usually easy to both detect and fix since we have created an Excel-sheet that only needs the answers from the respondents to perform all the necessary calculations. Therefore, if it is obvious that an expert has provided an answer mistakenly when filling the provided form, it is easy to correct when feeding the answers to our Excel-sheet.

5.2. Method for Constructing the Optimal Gold/equity Portfolio

5.2.1. Modern Portfolio Theory

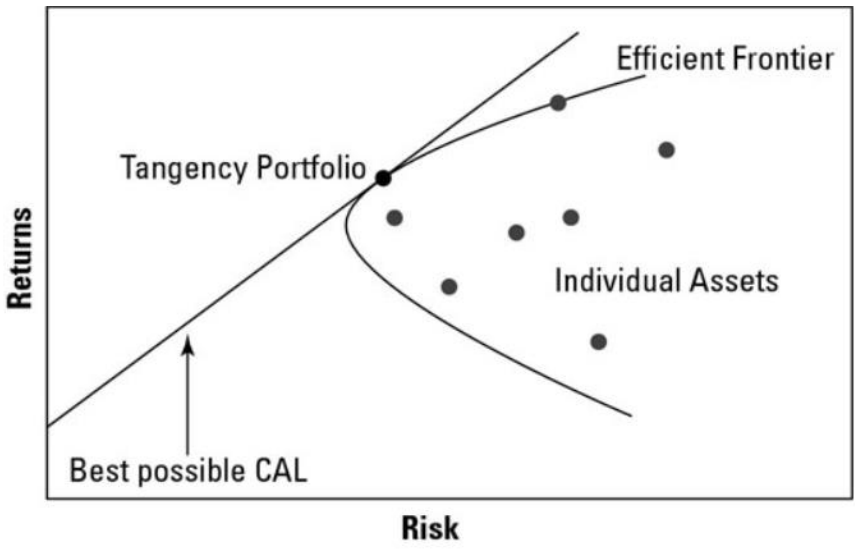
Launched by American economist Harry Markowitz in "Portfolio Selection", published in the Journal of Finance in 1952, Modern Portfolio Theory (MPT) describes a practical method of optimising the return of an investment portfolio, in exchange for an acceptable degree of risk.

The key takeaway of the theory is the emphasis of diversification benefits, as investments either offers high risk and high return, or low risk and low return. Modern Portfolio Theory is based on the idea that investors are risk averse, and that the general goal of managing financial assets is to achieve superior investments returns. Taking the benefits of diversification into account, Markowitz argues that the best possible results from this point of view can be reached by combining asset classes of different return and risk profiles. Hence, according to Markowitz, the optimal allocation of assets should therefore not only be based on the characteristics of each asset, rather also be evaluated based on its impact on the correlation with the other elements of the portfolio (Investopedia, 2021)

A cornerstone component of the MPT methodology is the construction of the efficient frontier, which is a graphic illustration of the method used.

8. Portfolio Optimisation – Efficient Frontier

The graph shows how the best capital allocation line tangent the efficient frontier at the optimal portfolio.



Source: Analystnotes 2022.

The graph plots the expected return of each individual asset on the Y-axis, with the standard deviation (volatility) of the assets being plotted along the right hand-side X-axis. Combining the characteristics of the different asset classes, a portfolio opportunity set (sometimes referred to as a "Markowitz Bullet") can be derived. The set illustrates the opportunities to shape a portfolio according to one's risk preferences, and plots the expected return related to such an allocation. The leftmost point on the portfolio opportunity set depicts the lowest achievable volatility, known as the point of Global Minimum Variance (GMV), while the curve above this point form the "efficient frontier". With higher expected return than the corresponding standard deviations of the points below the GMV, a rational investor should therefore aim to reach the allocation associated with the efficient frontier. By including the opportunity to allocate resources towards risk free investments, the portfolio opportunity set can also be combined with the best possible Capital Allocation Line. The Capital Allocation Line is an illustration of the trade-off made between excess return and risk and represents the steepest possible tangency line of the efficient frontier.

5.2.2. Capital Allocation Line and Sharpe Ratio

The Sharpe ratio, a concept first described by William Sharpe in 1966 (Sharpe, W. F., 1966), complements Markowitz theory and makes an addition to the model that enables the asset manager to find the most optimal diversification obtainable. Derived from the slope of the CAL, the so-called Sharpe Ratio is a standardised measure of comparing different portfolios, measured by the excess return (difference between return and risk-free rate) achieved adjusted to the corresponding risk (standard deviation).

The equation below shows how the ratio is calculated. In the numerator, the return of an investment portfolio is subtracted by the risk-free rate, generally measured as the yield of US Treasury Bills. Hence, the nominator equals the risk premium, which is then divided by the standard deviation of the very same portfolio (σ). A high ratio signals a high risk-adjusted return, which should be what a rational investor desires (Lioudis, 2021).

9. Portfolio Optimisation – Sharpe Ratio

$$\frac{Rp - Rf}{\sigma} = Sharpe\ ratio$$

5.2.3 Case Specific Application, Portfolio Optimisation

The calculations on the stock market portfolio return will be based on the S&P 500 index (SPX), a widely used price index tracking S&P 500 (consisting of 500 large cap US stocks). The rationale for applying the S&P 500 as a proxy for the stock market as a whole is simply explained by the sheer width of the index, as it covers far more companies and sectors than, for example, the commonly used Dow Jones Industrial Average index. S&P 500 has also been more widely applied as a benchmark during similar portfolio analysis related studies, making the upcoming results of this study more comparable to former research. The SPX index used is a price index, and the return of the stock portfolio examined does therefore not consider the yield gained from dividends being reinvested. Hence, S&P 500 index is treated in a similar manner as the non-interest yielding physical gold and does therefore not rely on speculations regarding reinvestments being made. Furthermore, dividends are heavily subjected to shifting tax policies, limiting the individual investor's ability to invest according to the assumptions made by the total return index. The rationale of applying a price index was also partly inspired by a 2014 report from Lyxor Asset Management called "Which is the Best Index-tracking solution?", which concludes that the total return of a stock portfolio shifts greatly depending on the tax regime applied. When later deriving the optimal allocation of the portfolio, the Sharpe Ratio, upon which the allocation will based on, will be denominated as a monthly figure. To increase comparability and relevance in relation to other financial research and findings, the monthly Sharpe Ratio is most optimally converted to an annual rate, which can be done by multiplying the obtained value by $\sqrt{12}$.

6. Expected Results

6.1. Expected Result of the Analytical Hierarchy Process and Pairwise Comparison

The authors expect all the factors to have at least some level of importance, since all of the investigated factors have been proven to have an impact to the price of gold. Other than that, we have a general assumption that factors which currently experience turbulence or attention in news and other sources will have a higher degree of importance compared to less visible factors. This means that the factors geopolitical risks, monetary supply and stock market volatility would have a higher degree of importance than, for example, the price of silver.

Since the factors examined in this study have been chosen based on their proved significance during hypothesis tests in earlier research, the possibilities of achieving criteria weights of zero has been fully excluded. Hence, an underlying assumption of our expected results is the reliance on former research, stating that none of the chosen factors are fully irrelevant. However, a wide span of criteria weights is expected to be achieved, with certain factors expected to show very low importance in the determination of gold prices. The core hypothesis/excepted results of the study could be derived from the following notions and intuitive ideas:

- The importance of the monetary supply in the US economy will be a majorly important factor, as the inflation hedging attributes of gold is often portrayed as a main reason to expose one's wealth towards this asset.
- Being one of the most common sources of easily accessible, risky investments, equity stands for a substantial part of both private and institutional wealth pooling. Hence, the perceived attractiveness of stocks in terms of both return and volatility is assumed to also play a big role in the price development of safer assets, such as gold. For that reason, both S&P 500 return and VIX volatility index is estimated to have a key influential impact.
- In the wake of geopolitical turmoil during the beginning of 2022, gold prices have seen a significant rise in price. With major geopolitical happenings such as wars gaining tremendous attention in financial media, as well as causing violent movements on the financial markets, such risks are by us believed to have the absolute highest general impact on gold prices. Even a higher degree of impact than that of purely market related factors.

- Although probably not being a key determinant, the rise of cryptocurrencies during recent years has introduced investors to emerging investment alternatives, often being described as sharing the inflation protecting attributes of gold. Being the dominant form of cryptocurrency, Bitcoin development might therefore provide an indication of market sentiment driving gold prices. Such similarities between gold and Bitcoin can therefore create spillover effects between the two asset classes, which strengthens the role of Bitcoin as a gold price determinant. The very same logic could, to an even higher extent, be applied to the relationship between silver and gold.
- Crude Oil prices, as well as fiscal deficits, is expected to finish further down the list, as those factors are believed to have a more far-fetched, speculative connection to gold prices, in comparison to the other determinants investigated.

The arguments stated above leads to the following expected results of the Pairwise Comparison study, where each factor is assessed based on its expected scale of influence on gold prices.

10. Pairwise Comparison – Expected Ranking of Factor Importance

The table below shows the expected rankings of the experts' combined estimations.

Relative Importance	Factor
1	Geopolitical risk
2	US Monetary supply (M2)
3	Stock market returns
4	Stock market volatility (VIX)
5	US real interest rate
6	Price of silver in USD
7	USD exchange rate
8	Price of Bitcoin in USD
9	Fiscal deficits
10	Oil price USD

6.2 Expected Results of the Portfolio optimisation

Since the sample period of the portfolio analysis conducted is based on a vastly different time span than that of the majority of former research within the field, the results are expected to shift greatly compared to what has been previously found. More precisely, we believe that the inclusion of gold in a portfolio will turn out to have been more beneficial for the risk adjusted return (measured as Sharpe ratio) between 2008 and 2022 than what other research has suggested during other time horizons. Hence, we believe that the optimal weight of gold in a gold/equity portfolio during our period applied will be higher than suggested in, for example, "*Do Precious Metals Shine? An Investment Perspective*". This is due to two major intuitive perceptions:

- Between 2008 and 2022, gold prices have outperformed its historical return, increasing the expected future return derived from the study.
- During two of the major stock market sell-offs of the period (2008 financial crisis and 2020 Covid drop), gold has performed relatively well, strengthening the "hedge" attributes of the commodity and decreasing the overall volatility of an S&P500/gold portfolio.

For the reasons mentioned above, we anticipate the optimal weight of gold in the portfolio to reach roughly 20 percent.

7. Data

7.1 – Pairwise Comparison

After a process of evaluating the presumed expertise and accessibility of a wide range of individuals with professional experience within both gold trade and financial issues, five complete answers were obtained. The results were gathered both by hosting digital interviews as well as from a form sent out by email, with the choice of method being decided by the responding expert. Regardless of choice, the input was summarised identically after each session, according to the Pairwise Comparison matrix stated in the methodology.

This section will show how the respondents chose to rank each factor. If a respondent has not given a complete answer to the survey or during an interview, this will also be disclosed under the corresponding respondent.

7.1.1. Respondent 1 – Commodity Market Expert

Respondent 1 gave a complete set of answers to nine of the ten chosen factors on the survey. For the factor regarding interest rate, the respondent chose not to provide any answers. For that reason, we exclude that factor in the tables concerning respondent 1.

Respondent: Commodity Market Expert, former Head of Commodities at SEB and Handelsbanken, as well as portfolio manager at Skandia.

11. Pairwise Comparison – Respondent 1 Answer

The matrix below shows how respondent 1 answered the form.

	US Monetary supply (M2)	USD Exchange rate	Fiscal deficits	Stock market returns (S&P 500)	Stock market volatility (VIX)	Price of Bitcoin USD	Price of silver USD	Oil price USD	Geopolitical risk
US Monetary supply (M2)	1,00	7,00	7,00	9,00	6,00	9,00	0,50	1,00	1,00
USD Exchange rate	0,14	1,00	1,00	3,00	1,00	9,00	1,00	1,00	1,00
Fiscal deficits	0,14	1,00	1,00	3,00	1,00	9,00	0,33	0,33	0,50
Stock market returns	0,11	0,33	0,33	1,00	0,25	1,00	0,20	0,20	0,20
Stock market volatility (VIX)	0,17	1,00	1,00	4,00	1,00	9,00	1,00	1,00	1,00
Price of Bitcoin USD	0,11	0,11	0,11	1,00	0,11	1,00	0,11	0,11	0,11
Price of silver USD	2,00	1,00	3,00	5,00	1,00	9,00	1,00	5,00	1,00
Oil price USD	1,00	1,00	3,00	5,00	1,00	9,00	0,20	1,00	0,50
Geopolitical risk	1,00	1,00	2,00	5,00	1,00	9,00	1,00	2,00	1,00
Sum	**5,67**	**13,44**	**18,44**	**36,00**	**12,36**	**65,00**	**5,34**	**11,64**	**6,31**

The consistency ratio for respondent 1 is: 0.093.

7.1.2. Respondent 2 – Financial Author and Researcher

Table 2. Respondent: Financial Author/Researcher. Founder of Swedish Investment Platform Börslabbet, has published numerous articles regarding the diversification benefits of gold allocation in investments. The respondent gave their answers through the digital survey.

12. Pairwise Comparison – Respondent 2 Answer

The matrix below shows how respondent 2 answered the form.

	US Monetary supply (M2)	US Real Interest rates	USD Exchange rate	Fiscal deficits	Stock market returns (S&P 500)	Stock market volatility (VIX)	Price of Bitcoin USD	Price of silver USD	Oil price USD	Geopolitical risk
US Monetary supply (M2)	1,00	2,00	7,00	1,00	3,00	3,00	9,00	3,00	3,00	3,00
US Real Interest rate	0,50	1,00	2,00	0,33	1,00	2,00	5,00	4,00	2,00	2,00
USD Exchange rate	0,14	0,50	1,00	0,14	0,14	0,14	4,00	1,00	0,25	0,25
Fiscal deficits	1,00	3,00	7,00	1,00	5,00	5,00	9,00	5,00	5,00	5,00
Stock market returns	0,33	1,00	7,00	0,20	1,00	3,00	9,00	1,00	1,00	1,00
Stock market volatility (VIX)	0,33	0,50	7,00	0,20	0,33	1,00	5,00	0,33	0,33	1,00
Price of Bitcoin USD	0,11	0,20	0,25	0,11	0,11	0,20	1,00	0,20	0,20	0,20
Price of silver USD	0,33	0,25	1,00	0,20	1,00	3,00	5,00	1,00	1,00	3,00
Oil price USD	0,33	0,50	4,00	0,20	1,00	3,00	5,00	1,00	1,00	4,00
Geopolitical risk	0,33	0,50	4,00	0,20	1,00	1,00	5,00	0,33	0,25	1,00
Sum	**4,42**	**9,45**	**40,25**	**3,59**	**13,59**	**21,34**	**57,00**	**16,87**	**14,03**	**20,45**

The consistency ratio for respondent 2 is: 0.094.

7.1.3. Respondent 3 – Gold Trader and Entrepreneur

Table 3. Respondent: Gold trader/entrepreneur. CEO of Nordic Gold Trade. This respondent gave their answer through an interview.

13. Pairwise Comparison – Respondent 3 Answer

The matrix below shows how respondent 3 answered the form.

	US Monetary supply (M2)	US Real Interest rates	USD Exchange rate	Fiscal deficits	Stock market returns (S&P 500)	Stock market volatility (VIX)	Price of Bitcoin USD	Price of silver USD	Oil price USD	Geopolitical risk
US Monetary supply (M2)	1,00	7,00	7,00	7,00	5,00	3,00	8,00	2,00	2,00	0,20
US Real Interest rate	0,14	1,00	2,00	1,00	1,00	3,00	7,00	1,00	1,00	0,11
USD Exchange rate	0,14	0,50	1,00	1,00	2,00	1,00	9,00	1,00	1,00	0,13
Fiscal deficits	0,14	1,00	1,00	1,00	2,00	1,00	9,00	0,33	0,33	0,13
Stock market returns	0,20	1,00	0,50	0,50	1,00	0,25	4,00	0,25	0,50	0,13
Stock market volatility (VIX)	0,33	0,33	1,00	1,00	4,00	1,00	5,00	0,33	0,33	0,13
Price of Bitcoin USD	0,13	0,14	0,11	0,11	0,25	0,20	1,00	0,11	0,11	0,11
Price of silver USD	0,50	1,00	1,00	3,00	4,00	3,00	9,00	1,00	2,00	0,13
Oil price USD	0,50	1,00	1,00	3,00	2,00	3,00	9,00	0,50	1,00	0,13
Geopolitical risk	5,00	9,00	8,00	8,00	8,00	8,00	9,00	8,00	8,00	1,00
Sum	**8,09**	**21,98**	**22,61**	**25,61**	**29,25**	**23,45**	**70,00**	**14,53**	**16,28**	**2,17**

The consistency ratio for respondent 3 is: 0.095.

7.1.4 Respondent 4 – Associate at the European Parliament

Table 4. Associate at the European Parliament, Committee on Economic and Monetary Affairs. Previously B.Sc. of Finance, with a thesis researching the statistical significance of gold price drivers. This respondent gave their answers through the digital survey.

14. Pairwise Comparison – Respondent 4 Answer

The matrix below shows how respondent 4 answered the form.

	US Monetary supply (M2)	US Real Interest rates	USD Exchange rate	Fiscal deficits	Stock market returns (S&P 500)	Stock market volatility (VIX)	Price of Bitcoin USD	Price of silver USD	Oil price USD	Geopolitical risk
US Monetary supply (M2)	1,00	0,50	5,00	4,00	4,00	3,00	4,00	6,00	6,00	2,00
US Real Interest rate	2,00	1,00	8,00	8,00	8,00	7,00	8,00	9,00	9,00	3,00
USD Exchange rate	0,20	0,13	1,00	1,00	2,00	0,25	1,00	5,00	5,00	0,13
Fiscal deficits	0,25	0,13	1,00	1,00	3,00	0,33	0,33	5,00	3,00	0,13
Stock market returns	0,25	0,13	0,50	0,33	1,00	0,20	0,20	3,00	1,00	0,14
Stock market volatility (VIX)	0,33	0,14	4,00	3,00	5,00	1,00	5,00	7,00	7,00	0,17
Price of Bitcoin USD	0,25	0,13	1,00	3,00	5,00	0,20	1,00	4,00	3,00	0,14
Price of silver USD	0,17	0,11	0,20	0,20	0,33	0,14	0,25	1,00	1,00	0,11
Oil price USD	0,17	0,11	0,20	0,33	1,00	0,14	0,33	1,00	1,00	0,11
Geopolitical risk	0,50	0,33	8,00	8,00	7,00	6,00	7,00	9,00	9,00	1,00
Sum	**5,12**	**2,70**	**28,90**	**28,87**	**36,33**	**18,27**	**27,12**	**50,00**	**45,00**	**6,92**

The consistency ratio for respondent 4 is: 0.097.

7.1.5 Respondent 5 – Head of Sales

Table 5. Respondent: Head of Sales, KA Rasmussen (Swedish provider of Precious Metals). This respondent gave their answers through the digital survey.

15. Pairwise Comparison – Respondent 5 Answer

The matrix below shows how respondent 5 answered the form.

	US Monetary supply (M2)	US Real Interest rates	USD Exchange rate	Fiscal deficits	Stock market returns (S&P 500)	Stock market volatility (VIX)	Price of Bitcoin USD	Price of silver USD	Oil price USD	Geopolitical risk
US Monetary supply (M2)	1,00	5,00	6,00	3,00	3,00	4,00	2,00	5,00	3,00	5,00
US Real Interest rate	0,20	1,00	5,00	1,00	1,00	3,00	3,00	7,00	1,00	1,00
USD Exchange rate	0,17	0,20	1,00	0,50	0,50	1,00	5,00	6,00	0,33	0,25
Fiscal deficits	0,33	1,00	2,00	1,00	1,00	1,00	1,00	5,00	1,00	0,33
Stock market returns	0,33	1,00	2,00	1,00	1,00	3,00	3,00	5,00	1,00	0,50
Stock market volatility (VIX)	0,25	0,33	1,00	1,00	0,33	1,00	1,00	1,00	0,33	0,20
Price of Bitcoin USD	0,50	0,33	0,20	1,00	0,33	1,00	1,00	1,00	0,33	0,14
Price of silver USD	0,20	0,14	0,17	0,20	0,20	1,00	1,00	1,00	0,33	0,11
Oil price USD	0,33	1,00	3,00	1,00	1,00	3,00	3,00	3,00	1,00	0,11
Geopolitical risk	0,20	1,00	4,00	3,00	2,00	5,00	7,00	9,00	9,00	1,00
Sum	**3,52**	**11,01**	**24,37**	**12,70**	**10,37**	**23,00**	**27,00**	**43,00**	**17,33**	**8,65**

The consistency ratio for respondent 5 is: 0.096.

7.2. – Portfolio Optimisation

The data on both gold prices and S&P 500 index was gathered by applying constraints in Yahoo Finance related to the time period between January 2008 and March 2022, on which numbers for closing prices were accumulated. From these prices (146 months in total), average monthly returns on both asset classes could be derived, creating the total expected return used in the portfolio analysis. Regarding the risk-free rate applied, data was gathered in a similar manner as that of gold and S&P 500, with the risk-free rate being determined by the average 13-week US Treasury Bill coupon equivalent, based on data from US Department of the Treasury.

8. Results

8.1. Pairwise Comparison – Key Determinants

The table below shows the criteria weights calculated by the model derived from the average of the five experts' combined estimations. The calculations have been performed as described in section 5.1.2.

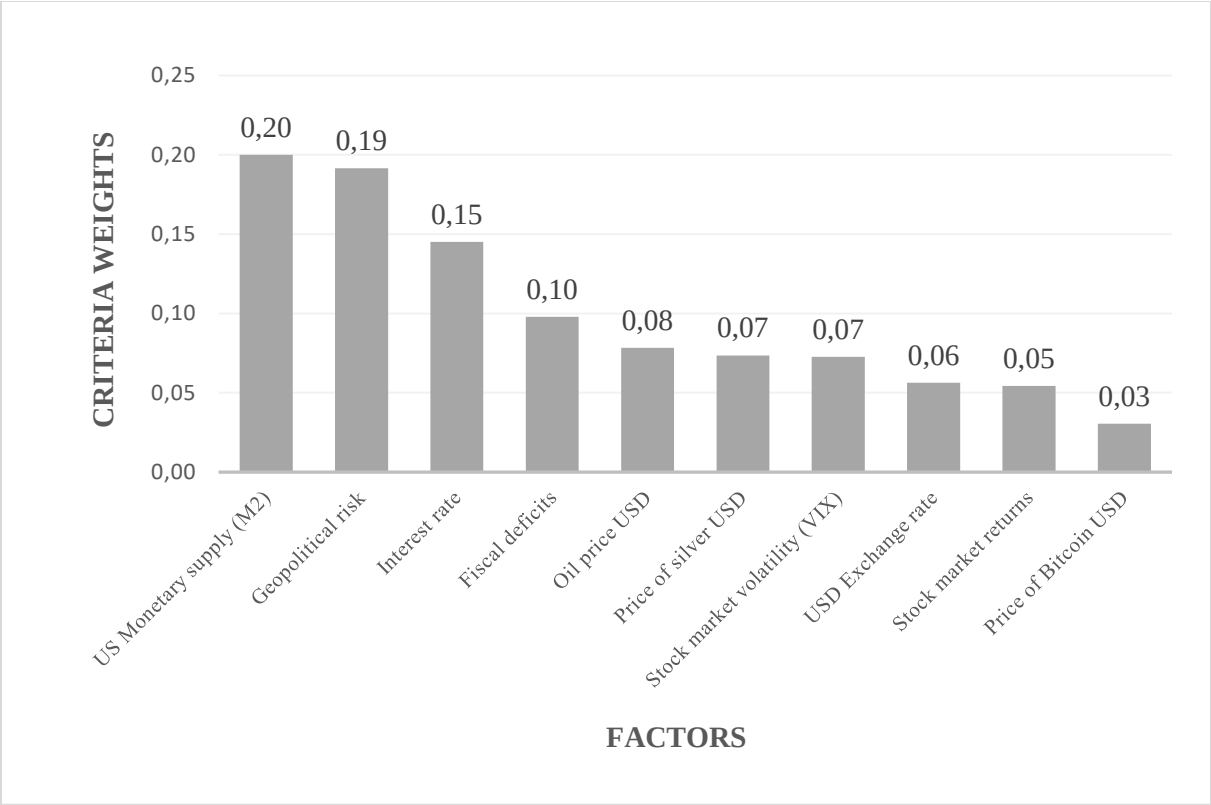
16. Pairwise Comparison – Experts' Estimations of Factor Importance – Table

The table below shows the experts' combined estimations in valuing the importance of our researched factors.

Factor	Criteria Weight
US monetary supply (M2)	0.20
Geopolitical risk	0.19
Interest rate	0.15
Fiscal deficits	0.10
Oil price in USD	0.08
Price of silver in USD	0.07
Stock market volatility (VIX)	0.07
USD exchange rate	0.06
Stock market returns	0.05
Price of Bitcoin in USD	0.03

17. Pairwise Comparison – Experts' Estimations of Factor Importance – Histogram

The histogram below shows the experts' combined estimations in valuing the importance of our researched factors.



As shown in the table and histogram, the monetary supply of the US economy was estimated as being the most important factor, having the highest relative impact on the development of gold prices. Since the sum of the criteria weights equal 1, a criteria weight of 0.20 means that the factor US Monetary Supply (M2) explains 20 percent of the price of gold among the included factors. The criteria weights can also be compared on the ratio-scale, which means that it is possible to compare the different criteria weights by dividing them with each other. The resulting ratio shows the relationship between different factors, for example, the ratio:

18. Pairwise Comparison – Calculation of the Relative Importance of a given factor – Example

$$\frac{US\ monetary\ supply}{Fiscal\ deficits} = \frac{0.2}{0.1} = 2$$

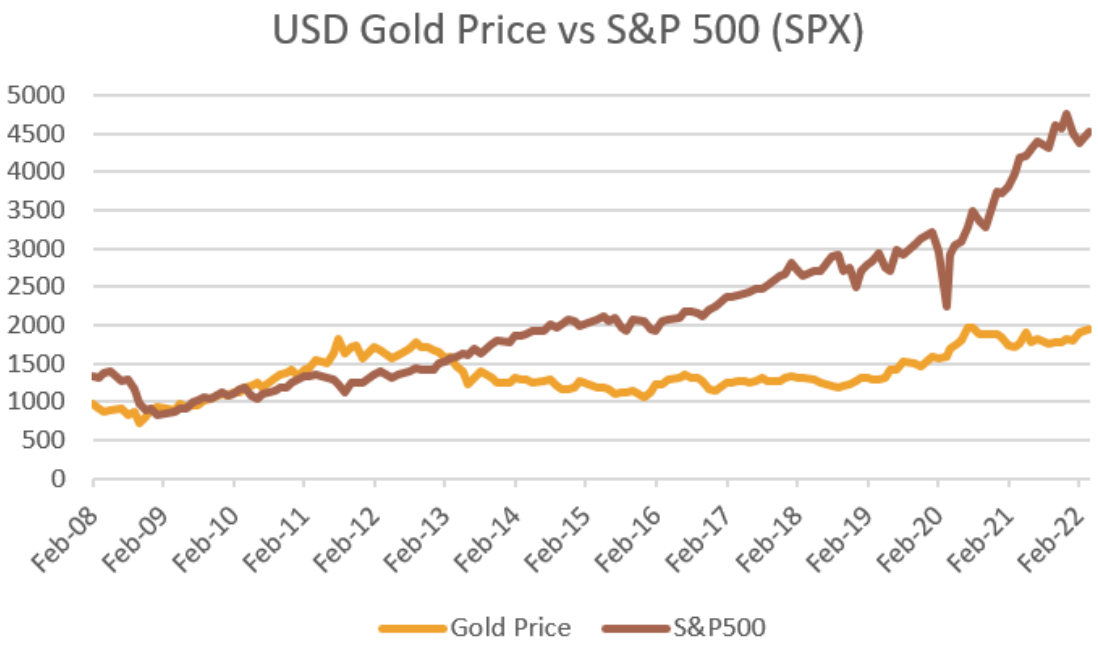
shows that the US monetary supply is twice as important as fiscal deficits in explaining the price of gold.

8.2. Modern portfolio theory – Optimal Portfolio Allocation

Since the Modern Portfolio Theory Methodology is based on different combinations of return and risk for a portfolio, the values of those two metrics have to be calculated. Therefore, historical data (from January 2008 to March 2022) on both USD spot prices of gold, as well as of S&P 500 index was gathered, giving the following output

19. Portfolio Optimisation – Historical Data of gold prices and the S&P 500 Index

The diagram below shows the development of the gold price and the S&P 500 index between January 2008 and March 2022.



From the historical data on the monthly returns of the S&P 500 and USD spot prices of gold, the covariance of the two was estimated (through the Excel command: covariance.s) and summarised accordingly. In addition to the estimates of return on the two asset classes, the risk-free rate was then derived by averaging the coupon equivalent rate on 13-week US Treasury Bills, based on the same time period applied for Gold Prices and S&P 500.

20. Portfolio Optimisation – Average Monthly Return of Asset Classes (Expected)

The table below shows the average monthly return of the S&P 500, gold price and the risk-free rate between January 2008 and March 2022.

Index/asset class	**Average monthly return of each asset class**
S&P 500	0.960%
Gold price (USD)	0.621%
Risk free rate (T-bill)	0.0456%

21. Portfolio Optimisation - Bordered Covariance Matrix

The table below shows the covariance between returns on asset classes between the time period January 2008 and March 2022.

	S&P 500	**Gold price**
S&P 500	0.002110	0.000222
Gold price	0.000222	0.002725

Based on the data on return and variance of each asset, as well as the covariance between the two, different sets of portfolios could be constructed by applying Excel's solver function to the matrix (Benninga, 2011, p. 360). By setting constraints related to the measures covered below (monthly return, standard deviation, Sharpe Ratio), the portfolio weight cells change in an order to fulfil the criteria set. The test does not allow the possibility of shorting assets, nor borrowing funds. Hence, the portfolio weight of any asset cannot exceed one or fall below zero, and the value of the allocation towards the two assets must add up to the value of one (100 percent exposure).

22. Portfolio Optimisation – Excel Solver Portfolio Setup

The image below shows the values being used in the Excel solver function to construct the portfolio opportunity set.

Portfolio weights		0,0000	0,0000
		S&P500	Gold
0,0000	S&P500	0,002110	0,000222
0,0000	Gold	0,000222	0,002725
0,0000		0,0000	0,0000

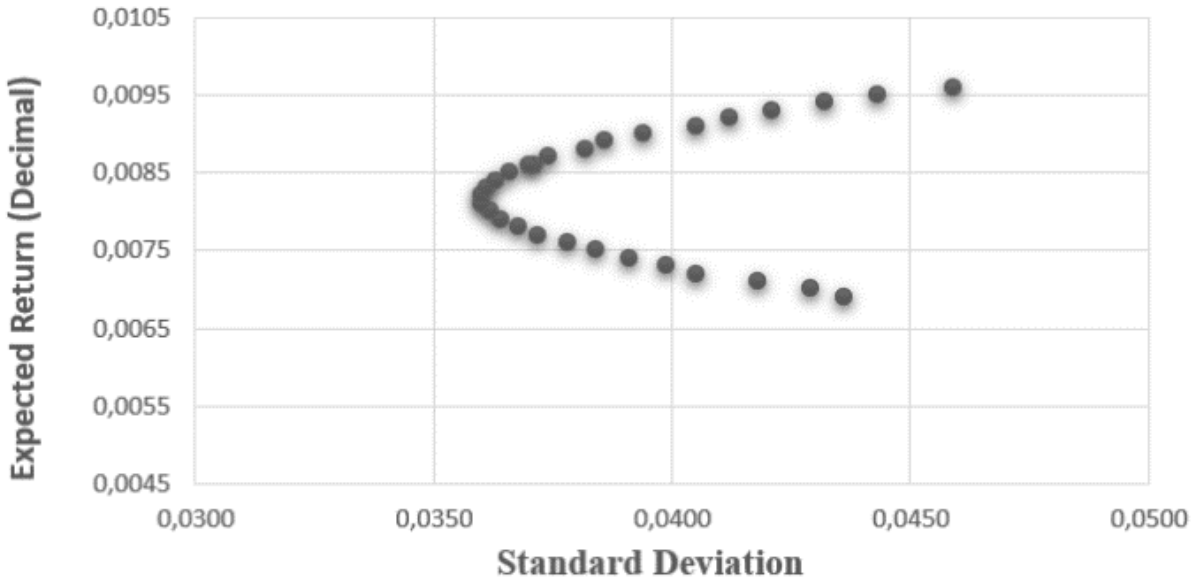
Monthly return	0,0000
Std Dev	0,0000
Sharpe ratio (Monthly)	

Weighted variance contribution of each asset, determined by the assigned portfolio weights

By manually inputting a range of portfolio weights, stretching from 100% gold to 100% S&P 500, the following portfolio set output was realized:

23. Portfolio Optimisation – Portfolio Opportunity Set

The image below shows the portfolio opportunity set derived from the Excel solver function.



From the shape of the Markowitz bullet, a slope of the hypothetical Capital Allocation Line (Sharpe ratio) can be derived, if combined with the formula for the Sharpe Ratio. Setting a maximal Sharpe-Ratio as a constraint in the solver then automatically adjusts the portfolio to the weights deemed to provide the optimal allocation.

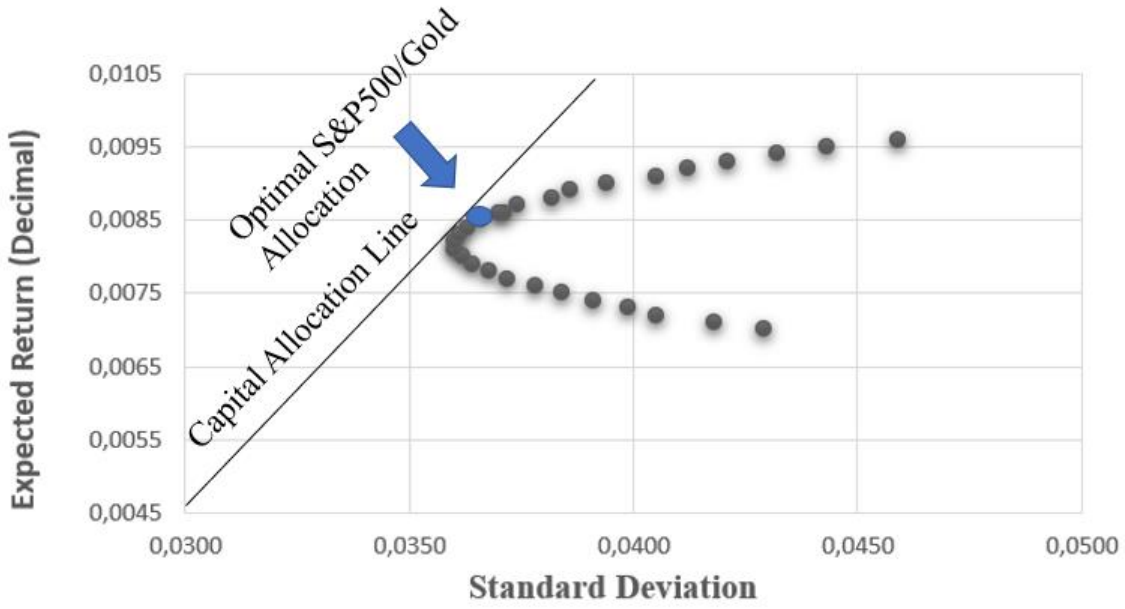
24. Portfolio Optimisation – Optimal Portfolio

The table below shows portfolio weights and performance metrics for the optimal portfolio.

Portfolio weights		**Monthly returns**	0.0086
S&P 500	0.7007	**Standard deviation**	0.0371
Gold	0.2993	**Sharpe ratio (monthly)**	0.2194

25. Portfolio Optimisation – Position of the Optimal portfolio in the Portfolio Opportunity Set

The diagram below shows the position of the optimal portfolio in the portfolio opportunity set.



As the results show, the portfolio analysis concludes that during the time period between January 2008 and March 2022, the highest Sharpe Ratio obtainable would have been reached if roughly 70 percent of the portfolio would be allocated in S&P 500, while the remaining 30 percent gets invested in physical gold. Such portfolio diversification leads to an expected monthly return of 0,86 percent and a standard deviation of 0,0371. Converting the Sharpe

Ratio from being monthly to annually denominated, a value of 0,76 is reached (0,2194 * $\sqrt{12.}$)

The figure above shows that the optimal portfolio in the chosen time period would contain 70% S&P 500 and 30% gold.

9. Discussion

9.1. Analytical Hierarchy Process and Pairwise Comparison

As expected, the US monetary supply and geopolitical risks play a highly important role in explaining the price of gold. Together with interest rate, these three most important factors stand for more than 50 percent of the total explanation of gold price that this model provides. It is also interesting to compare the most influential factor to the least influential one. Our result says that the US monetary supply is nearly 7 times more important than Bitcoin in regards of explaining the price of gold. Even though we were quite certain of the large impact of monetary supply and geopolitical risk, the same cannot be said regarding the interest rate. We did not expect the interest rate to play such a large role as the experts' aggregated answers suggest. The same is true for fiscal deficits, a factor which the experts value a lot higher than we originally thought. We have also underestimated the importance that the experts place on the oil price. While we placed the oil price at the bottom, our experts' combined opinion places the price of oil as number five out of the ten factors. Though the differences are small, the importance of oil exceeds both the stock market volatility and stock market returns, which deviates greatly form our expected results. Overall, except for the top two factors, our predictions did not reflect the experts' opinions to a large extent. However, since our own assumptions regarding the most important factors are in harmony with the experts' assessments, one could still say that we have successfully managed to make correct predictions where it counts the most.

It is also worth noting that the authors have not performed any in-depth research regarding each and every investigated factor. Rather, we have developed our expected result by using our own intuition, just like the participating experts have. We believe that this improves comparability between the expected result and the final result.

We believe that our result is reasonable, since there are many examples of previous research that supports the result we have gathered. For example, the articles previously referred to under the theoretical framework section all find a relationship between the factor in question and price of gold. We can also see that the factors shown to be important in our result are also discussed in financial news media. As an example, the weekly magazine Barron's published an article discussing the effects that war and inflation has on gold price (Saefong, 2022). In this article, it is stated that the gold price is close to its all-time high as investors rush to place their money in an asset deemed safe in times of uncertainty. This uncertainty is caused by

rising inflation and increased geopolitical instability. Saefong also includes statements of several people with relevant knowledge about this issue, among them the head of research at Summerhead Investment Management Geetesh Bhardwaj. Bhardwaj states that "Nothing stokes the gold bugs like the risk posed by a war", further emphasising the huge impact that geopolitical tension has on gold price. The CEO of Sprott, Peter Grosskopf, is also featured in Saefong's article. He, like Bhardwaj, says that geopolitical tensions affect the gold price. But he seems to believe that the main drivers of gold price are the deeply negative real rates and the high inflation, and that geopolitical tensions, in combination with the surge in oil price, are only secondary in comparison. Grosskopf states that investors are drawn to gold to "seek a counterbalance to inflation and corrections in other markets". Regardless of which factor is the most impactful, the main point is that uncertainty drives investors to buy something that seem safe in comparison. Our result for the Analytical Hierarchy Process part of this study quantifies how impactful this uncertainty is on the price of gold.

9.2. Portfolio Optimisation

As suggested in the expected results, the portfolio analysis resulted in an optimal portfolio with a gold allocation far higher than that derived from similar studies in earlier time periods. While we believe that there are a wide range of logical, financially oriented reasons as to why the gold/equity portfolio in this study was found optimal with a relatively high exposure towards gold (30 percent), it is also worth addressing a few, purely technical factors that also affect the results obtained. Such factors include the impact of the assumptions made, the decisions taken during the data gathering process as well as the choice of method applied.

Firstly, the data on asset class returns and standard deviation was derived from price indices, which in the case of stock market returns contrasts the total returns gained if dividends were to be included. As of March 2022, the mean dividend yield of S&P500 companies was 1.37 percent (Ycharts, 2022), which if reinvested does create a total return exceeding that of the price index development, taking the effects of interest on interest over time into account. While both the advantages and disadvantages of price indices versus total return indices are being discussed in section 5.2.3, the differences make an aspect worth considering when making comparisons to other academic literature. In a similar manner, the different metrics of defining optimal portfolios (Sharpe Ratio/Relative Reward-to-Risk ratio etc) need to be considered when drawing conclusions related to comparisons between different studies.

Setting differences regarding research methods and assumptions aside, the implications of high allocation towards gold in a gold/equity portfolio are many. One possible reason to why gold has been a more value adding component in such a portfolio during recent years, is that the overall volatility of the stock market seems to have grown (Seeking Alpha, 2018), contributing to gold playing a bigger role as a risk reducing asset than before. While a wide range of factors can lead to a rise of stock market volatility (economic turmoil being the most obvious), other researchers have stated that new, systematic sources of volatility has emerged during recent years as a result of financial innovation on the fund market. For example, the rise of passive capital flows in the form of, for example, index funds have been suggested as emerging sources of volatility as the mandates and actions of such funds offset irrational spikes in returns and volumes. This contributes to overall increased standard deviation (Gabaix, et al., 2006). Hence, the portfolio addition of less volatile assets such as gold strengthens the reduction of portfolio volatility, benefitting the Sharpe ratio when adding gold.

While one key component of the Sharpe ratio is the volatility of a portfolio, the risk premium is the other one (portfolio return minus the risk-free rate). Therefore, an optimal allocation consisting of a large share of gold, might indicate that the expected return of gold is relatively high in relation to that of the stock market. With a sample period stretching from 2008 to 2022, a quick analysis does suggest that this is the case. Based on the very same return data on which the portfolio analysis is based upon, the compounded annual growth rate (CAGR) of gold price between January 2008 and March 2022 was 5,5 percent. For reference, comparable numbers between the period of 1976 to 2004, which was subject to analysis in “Do Precious Metals Shine? An Investment Perspective” gave a CAGR of 4,4 percent. Therefore, gold overperforming its historical performance might provide an additional explanation to why gold allocation during more recent time periods adds risk adjusted return, when incorporated to an equity portfolio.

10. Conclusion

With this report, we hope to make investments in gold more interesting and accessible for inexperienced investors by providing a basic understanding of how different factors affect the price of gold. This study gives the reader a hands-on approach as well as key background information regarding which factors are deemed relevant to analyse by experts. This is done by determining important factors and deriving an optimal portfolio allocation, based on a recent time period. The findings of our research suggests that a few core factors are determined to play crucial roles in the development of gold prices, with the US monetary supply, geopolitical risk and US Real Interest rates accounting for more than 50 percent of the total impact the chosen, statistically significant factors have on the price of gold. Furthermore, the portfolio analysis conducted on a gold/US equity portfolio during the time period between 2008 and 2022 indicates an optimal gold allocation of 30 percent. Hence, gold has the potential to play a substantial, value adding role as a diversifying complement to equity investors seeking maximal risk adjusted returns.

During the process of creating this thesis, a number of suggestions and thoughts have been considered. While some of them did not make it into the final version of this thesis, we still believe that we have identified a few potential topics for future research. For example, by making several Pairwise Comparisons about the same factors, but with data from completely different types of stakeholders, the results could then be compared to interpret how different groups value different factors. The purpose for such a study could be to assess and compare the knowledge of one group versus another, more knowledgeable group. Since there are no limits on either the topic being investigated or factors being compared, an Analytical Hierarchy Process is a very flexible model that can be tailored to fit the researchers needs. Researchers also have the opportunity to give different weights to different respondents' opinions. For example, if one respondent is deemed to have a higher level of knowledge, that respondent's answer could be given a higher weight when finalising the Pairwise Comparison matrix. The data would then be skewed in such a way that the more knowledgeable a respondent is, the higher the impact on the final result. We decided against this approach for this study as we could not see any obvious difference in the respondent's knowledge in our area of research. We believe that weighting the answers become more important if the span of knowledge that the respondents have widens.

11. References

Abdullah, A. & Bakar, M. J. A., 2015. The Application of Gold Price, Interest Rates and Inflation. *International Journal of Economics and Finance,* Volume Vol. 7, p. 300.

Anderson, D. et al., 2016. *An Introduction to Management Sciences.* Third Edition ed. Hampshire: Cengage Learning EMEA.

Apergis, N., Cooray, A., Kheraief, N. & Apergis, H., 2014. Do gold prices respond to real interest rates? Evidence from the Bayesian Markov Switching VECM model. *Journal of International Financial Markets, Institutions and Money,* Volume 60.

Bagci, H. & Keskin Köylü, M., 2019. Cryptocurrency: determining the correlation between bitcoin cryptocurrency and gold prices. *Cryptocurrencies in all aspects,* pp. 193-206.

Bapna, I., 2012. Dynamics of Macroeconomic Variables Affecting Price Innovation in Gold: A Relationship.

Baur, D. G. & Smales, L. A., 2020. Hedging Geopolitical Risk with Precious Metals. *Journal of Banking & Finance,* pp. 1-24.

Baur, D. J. & Lucey, B. M., 2010. Is Gold a Hedge or a Safe Haven? An Analysis of Stocks, Bonds and Gold. *The Financial Review,* 16 April, p. 12.

Benninga, S., 2011. *Principles of Finance with Excel - second edition.* New York: Oxford University .

Bilgin, M. H., Gozgor, G., Lau, M. & Sheng, X., 2018. The effects of uncertainty measures on the price of gold. *International Review of Financial Analysis,* Volume 58, pp. 1-7.

BlackRock, 2022. *BlackRock.com.* [Online]
Available at: https://www.blackrock.com/us/individual/insights/gold-still-hedges-chaos
[Accessed 29 05 2022].

Bollerslev, T., 1986. Generalized Autoregressive Conditional Heteroskedasticity. *Journal of Econometrics,* Volume 31, p. 21.

Boscaljon, B. & Clark, J., 2013. Do Large Shocks in VIX Signal a Flight-to-Safety in the Gold Market. *Journal of Applied Finance,* p. 10.

Business Insider, 2022. *markets.businessinsider.com.* [Online]
Available at: https://markets.businessinsider.com/news/stocks/warren-buffett-berkshire-hathaway-russia-ukraine-stocks-gold-bitcoin-dollars-2022-2
[Accessed 28 03 2022].

Cetin, C., 2001. On the long run relationship between gold and silver prices. *Global Finance Journal,* 12(2), p. 299–303.

Csató, L. & Ágoston, K. C., 2022. Inconsistency thresholds for incomplete pairwise comparison matrices. *Omega,* Volume 108.

Dowd, B., 2016. *Gold: The Most Precious of Metals.* [Online]
Available at: https://www.focus-economics.com/blog/gold-the-most-precious-of-metals
[Accessed 17 March 2022].

Fortune India, 2022. *fortuneindia.com.* [Online]
Available at: https://www.fortuneindia.com/people/have-a-greater-allocation-to-gold-ray-dalio/106877
[Accessed 30 3 2022].

Gabaix, X., Gopikrishnan, P., Plerou, V. & Eugene, S. H., 2006. Institutional Investors and Stock Market Volatility. *Quarterly Journal of Economics,* Volume 121, p. 44.

Gkillas, K., Gupta, R. & Pierdzioch, C., 2020. Forecasting realized gold volatility: Is there a role of geopolitical risks?. *Finance Research Letters,* Volume 35.

Güris, B. & Kiran, B., 2014. The Price of Gold and the Exchange Rate: Evidence From Threshold. *Acta Oeconomica,* 64(1), p. 100.

Hillier, D., Draper, P. & Faff, R., 2019. Do Precious Metals Shine? An Investment Perspective. *Financial Analysts Journal,* p. 10.

Investopedia, 2021. *Modern Portfolio Theory.* [Online]
Available at: https://www.investopedia.com/terms/m/modernportfoliotheory.asp
[Accessed 17 04 2022].

Investopedia, 2021. *Understanding the Sharpe Ratio.* [Online]
Available at: https://www.investopedia.com/articles/07/sharpe_ratio.asp
[Accessed 15 04 2022].

Jiang, T., 2020. Economic Variables Affecting the Gold Price. *2nd International Conference on Economic Management and Model Engineering (ICEMME)*, 2020, pp. 530-537.

Le, T.-h. & Chang, Y., 2012. Oil price shocks and gold returns. *Économie Internationale*, 3(131), pp. 71-103.

Llinas, A., 2014. *Which is the best Index-tracking Solution?*, Paris: Lyxor Asset Management.

Markowitz, H., 1952. Portfolio Selection. *The Journal of Finance*, 7(1), pp. 77-91.

Notes, A., 2022. *Subject 7. Optimal Portfolio.* [Online]
Available at: https://analystnotes.com/cfa-study-notes-discuss-the-selection-of-an-optimal-portfolio-given-an-investors-utility-or-risk-aversion-and-the-capital-allocation-line.html
[Accessed 7 June 2022].

Ross, M., 2020. *Pairwise Comparison – An objective review*, Oxfordshire: Commerce Decisions Limited.

Saaty, T. L., 2008. Relative Measurement and Its Generalization in Decision Making: Why Pairwise are Central in Mathematics for the Measurement of Intangible Factors - The Analytic Hierarchy/Network Process. *Review of the Royal Academy of Exact, Physical and Natural Scienes, Series A: Mathematics (RACSAM)*, 102(2), pp. 251 - 318.

Saefong, M. P., 2022. *War and Inflation Will Keep Gold Prices Volatile. What Investors Need to Know.*. [Online]
Available at: https://www.barrons.com/articles/wheat-prices-rally-51652981251
[Accessed 20 05 2022].

SeekingAlpha, 2018. *Are Stock Markets Becoming More Volatile?.* [Online]
Available at: https://seekingalpha.com/article/4230501-are-stock-markets-becoming-volatile
[Accessed 19 05 2022].

Sharpe, W. F., 1966. Mutual Fund Performance. *Journal of Business*, p. 119–138.

Shen, Z., 2014. *How the US Dollar Index Affects Gold Prices*, Saint Mary's University: s.n.

Sipkova, H. & Sipko, J., 2014. What Are the Real Drivers of Gold Prices?. *International Journal of Business and Social Science* , Volume 5, p. 71.

Theu, D., Stephane, G., Duc Khoung, N. & Thomas, W., 2021. Economic drivers of volatility and correlation in precious metal. *Journal of Commodity Markets*, pp. 1-20.

Triki, M. B. & Ben Maatoug, A., 2021. The GOLD market as a safe haven against the stock market uncertainty: Evidence from geopolitical risk. *Resources Policy*, march.Volume 70.

US Deparment of the Treasury, 2022. *Resource Center | US Deparment of the Treasury.* [Online]
Available at: https://home.treasury.gov/resource-center/data-chart-center/interest-rates/TextView?type=daily_treasury_bill_rates&field_tdr_date_value_month=202205
[Accessed 01 05 2022].

VanEck, 2020. *vaneck.com.* [Online]
Available at: https://www.vaneck.com/us/en/blogs/digital-assets/bitcoin-correlation-to-gold-jumps-in-2020/
[Accessed 15 05 2022].

Yahoo Finance, 2022. *Historical Data | GOLD 1 OZ (XAUUSD=X).* [Online]
Available at: https://finance.yahoo.com/quote/XAUUSD%3DX/history?p=XAUUSD%3DX
[Accessed 24 04 2022].

Yahoo Finance, 2022. *Historical Data | S&P 500 (^GSPC).* [Online]
Available at: https://finance.yahoo.com/quote/%5EGSPC/history?p=%5EGSPC
[Accessed 29 04 2022].

Yanhui, Z. J. F. & Jon, T., 2017. The impact of monetary policy on gold price dynamics. *Research in International Business and Finance.*

Ycharts, 2022. *ycharts.com.* [Online]
Available at: https://ycharts.com/indicators/sp_500_dividend_yield
[Accessed 18 05 2022].

Zhang, Y.-J. & Wei, Y.-M., 2010. The crude oil market and the gold market: Evidence for cointegration, causality and price discovery. *Resources Policy,* 35(3), pp. 168-177.

12. Appendix

12.1 Calculation of the consistency ratio for the Analytical Hierarchy Process

The calculations of the consistency ratio are initially based on the non-normalized values assigned to each cell by the respondents, with the "Given Value" stretching from 1/9 to 9. The criteria weights remain the same as those derived in section 5.1.2.

26. Pairwise Comparison – Matrix with Given Values

The matrix below shows a matrix filled with answers from a given respondent.

Criteria Weights	A1	B1	C1	D1	E1	F1	G1	H1	I1	J1
	US Monetary supply (M2)	US Real Interest rates	USD Exchange rate	Fiscal deficits	Stock market returns (S&P 500)	Stock market volatility (VIX)	Price of Bitcoin USD	Price of silver USD	Oil price USD	Geopolitical risk
US Monetary supply (M2)	1,00	Given Value	Given Value	Given Value	Given Value	Given Value	Given Value	Given Value	Given Value	Given Value
US Real Interest rate	Given Value	1,00	Given Value	Given Value	Given Value	Given Value	Given Value	Given Value	Given Value	Given Value
USD Exchange rate	Given Value	Given Value	1,00	Given Value	Given Value	Given Value	Given Value	Given Value	Given Value	Given Value
Fiscal deficits	Given Value	Given Value	Given Value	1,00	Given Value	Given Value	Given Value	Given Value	Given Value	Given Value
Stock market returns	Given Value	Given Value	Given Value	Given Value	1,00	Given Value	Given Value	Given Value	Given Value	Given Value
Stock market volatility (VIX)	Given Value	Given Value	Given Value	Given Value	Given Value	1,00	Given Value	Given Value	Given Value	Given Value
Price of Bitcoin USD	Given Value	Given Value	Given Value	Given Value	Given Value	Given Value	1,00	Given Value	Given Value	Given Value
Price of silver USD	Given Value	Given Value	Given Value	Given Value	Given Value	Given Value	Given Value	1,00	Given Value	Given Value
Oil price USD	Given Value	Given Value	Given Value	Given Value	Given Value	Given Value	Given Value	Given Value	1,00	Given Value
Geopolitical risk	Given Value	Given Value	Given Value	Given Value	Given Value	Given Value	Given Value	Given Value	Given Value	1,00

From the products obtained by multiplying the given value with the corresponding criteria weight, the next step in the consistency test process is to derive the weighted sum value of each row.

27. Pairwise Comparison – Calculation of the Weighted Sum Value

The matrix below shows how to calculate the weighted sum value of a row in the matrix.

Criteria Weights	B1	C1	D1	E1	F1	G1	H1	I1	J1	K1	
	US Monetary supply (M2)	US Real Interest rates	USD Exchange rate	Fiscal deficits	Stock market returns (S&P 500)	Stock market volatility (VIX)	Price of Bitcoin USD	Price of silver USD	Oil price USD	Geopolitical risk	Weighted Sum Values
US Monetary supply (M2)	1*B1	Given Value*C1	Given Value*D1	Given Value*E1	Given Value*F1	Given Value*G1	Given Value*H1	Given Value*I1	Given Value*J1	Given Value*K1	⬭
US Real Interest rate	Given Value*B1	1*C1	Given Value*D1	Given Value*E1	Given Value*F1	Given Value*G1	Given Value*H1	Given Value*I1	Given Value*J1	Given Value*K1	
USD Exchange rate	Given Value*B1	Given Value*C1	1*D1	Given Value*E1	Given Value*F1	Given Value*G1	Given Value*H1	Given Value*I1	Given Value*J1	Given Value*K1	
Fiscal deficits	Given Value*B1	Given Value*C1	Given Value*D1	1*E1	Given Value*F1	Given Value*G1	Given Value*H1	Given Value*I1	Given Value*J1	Given Value*K1	
Stock market returns	Given Value*B1	Given Value*C1	Given Value*D1	Given Value*E1	1*F1	Given Value*G1	Given Value*H1	Given Value*I1	Given Value*J1	Given Value*K1	
Stock market volatility (VIX)	Given Value*B1	Given Value*C1	Given Value*D1	Given Value*E1	Given Value*F1	1*G1	Given Value*H1	Given Value*I1	Given Value*J1	Given Value*K1	
Price of Bitcoin USD	Given Value*B1	Given Value*C1	Given Value*D1	Given Value*E1	Given Value*F1	Given Value*G1	1*H1	Given Value*I1	Given Value*J1	Given Value*K1	
Price of silver USD	Given Value*B1	Given Value*C1	Given Value*D1	Given Value*E1	Given Value*F1	Given Value*G1	Given Value*H1	1*I1	Given Value*J1	Given Value*K1	
Oil price USD	Given Value*B1	Given Value*C1	Given Value*D1	Given Value*E1	Given Value*F1	Given Value*G1	Given Value*H1	Given Value*I1	1*J1	Given Value*K1	
Geopolitical risk	Given Value*B1	Given Value*C1	Given Value*D1	Given Value*E1	Given Value*F1	Given Value*G1	Given Value*H1	Given Value*I1	Given Value*J1	1*K1	

⬭ = Cumulative value of all cells on US monetary supply row (given value *B5) + ... + (given value*K5)

With the weighted sum values of each factor being created, these newly derived values should be divided by the criteria weights, giving an output quota that must be further applied in the process of consistency testing. To illustrate this, the criteria weights data (same values as before) is transferred to the right of the "weighted sum value" column. The quota between the weighted sum value and the criteria weights gets inserted in the newly added "quota" column.

28. Pairwise Comparison – Ratio Between Weighted Sum Value and the Criteria Weights

The matrix below shows how to calculate the quota between the weighted sum value and the criteria weights.

Criteria Weights	B1	C1	C1	E1	F1	G1	H1	I1	J1	K1		⬇	⬇
	US Monetary supply (M2)	US Real Interest rates	USD Exchange rate	Fiscal deficits	Stock market returns (S&P 500)	Stock market volatility (VIX)	Price of Bitcoin USD	Price of silver USD	Oil price USD	Geopolitical risk	Weighted Sum Values	Criteria Weights	Quota
US Monetary supply (M2)	1*B1	Given Value*C1	Given Value*D1	Given Value*E1	Given Value*F1	Given Value*G1	Given Value*H1	Given Value*I1	Given Value*J1	Given Value*K1	⬭	B1	
US Real Interest rate	Given Value*B1	1*C1	Given Value*D1	Given Value*E1	Given Value*F1	Given Value*G1	Given Value*H1	Given Value*I1	Given Value*J1	Given Value*K1		C1	
USD Exchange rate	Given Value*B1	Given Value*C1	1*D1	Given Value*E1	Given Value*F1	Given Value*G1	Given Value*H1	Given Value*I1	Given Value*J1	Given Value*K1		D1	
Fiscal deficits	Given Value*B1	Given Value*C1	Given Value*D1	1*E1	Given Value*F1	Given Value*G1	Given Value*H1	Given Value*I1	Given Value*J1	Given Value*K1		E1	
Stock market returns	Given Value*B1	Given Value*C1	Given Value*D1	Given Value*E1	1*F1	Given Value*G1	Given Value*H1	Given Value*I1	Given Value*J1	Given Value*K1		F1	
Stock market volatility (VIX)	Given Value*B1	Given Value*C1	Given Value*D1	Given Value*E1	Given Value*F1	1*G1	Given Value*H1	Given Value*I1	Given Value*J1	Given Value*K1		G1	
Price of Bitcoin USD	Given Value*B1	Given Value*C1	Given Value*D1	Given Value*E1	Given Value*F1	Given Value*G1	1*H1	Given Value*I1	Given Value*J1	Given Value*K1		H1	
Price of silver USD	Given Value*B1	Given Value*C1	Given Value*D1	Given Value*E1	Given Value*F1	Given Value*G1	Given Value*H1	1*I1	Given Value*J1	Given Value*K1		I1	
Oil price USD	Given Value*B1	Given Value*C1	Given Value*D1	Given Value*E1	Given Value*F1	Given Value*G1	Given Value*H1	Given Value*I1	1*J1	Given Value*K1		J1	
Geopolitical risk	Given Value*B1	Given Value*C1	Given Value*D1	Given Value*E1	Given Value*F1	Given Value*G1	Given Value*H1	Given Value*I1	Given Value*J1	1*K1		K1	

A key component being used in the consistency testing is the *lambda max value (λmax)* which is the average of all ten weighted sum value/criteria weights ratios (quota).

29. Pairwise Comparison – Calculation of the λ-max Value

$$\lambda \max = \frac{Sum\ of\ the\ ten\ ratios\ from\ each\ factor}{Number\ of\ ratios}$$

To determine the validity and consistency of the input from a respondent, the last step is to calculate the consistency index and consistency ratio of the data.

The process of creating the random index number is explained by Csató and Ágoston in their article *Inconsistency thresholds for incomplete Pairwise Comparison matrices* (Csató & Ágoston, 2022). In short, the Random Index for a Pairwise Comparison model is generated by calculating the consistency index from a large value of Pairwise Comparison matrices, and then averaging the different values of the consistency indices. This has been done by Csató & Ágoston, and the values they calculated for Pairwise Comparison matrices with 9 and 10 factors respectively have been used in our own calculations of the consistency ratio for our answers.

n = number of factors

30. Pairwise Comparison – Calculation of the Consistency Index

$$Consistency\ index = \frac{(\lambda max\ -\ \mathrm{n})}{(\mathrm{n}-1)}$$

31. Pairwise Comparison – Calculation of the Consistency Ratio

$$Consistency\ ratio = \frac{Consistency\ index}{Random\ index}$$