Dialetheism and the Law of Non-contradiction

Master's degree thesis, 30 credits

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Abstract To say that the Law of Non-contradiction (LNC) underpins Western philosophy is perhaps not an exaggeration, but the discovery of paradoxes poses a serious challenge to its foundational status. This paper examines and defends one of the most thought-provoking solutions - dialetheism, which proposes that there are indeed true contradictions. With the help of a model explaining how change in logic takes place, I discuss how, despite the LNC being regarded as the hallmark of rational thinking, it is possible to debate the LNC without being circular. I also evaluate arguments from both LNC proponents and opponents. Ultimately I conclude that the dialetheist comes out on top on both establishing a constructive argument and defending his position.

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1) Background

In its most naïve formulation, the Law of Non-contradiction (LNC) states that nothing can have incompatible properties, such as being red and not being red both at the same time. Since it was introduced by Aristotle in Metaphysics, it is usually taken for granted in Western philosophy. As the Arab polymath Avicenna (2005, p.43) once said, 'Anyone who denies the law of non-contradiction should be beaten and burned until he admits that to be beaten is not the same as not to be beaten, and to be burned is not the same as not to be burned.' Indeed, it is difficult to even imagine what it means for there to be a true contradiction.

But the issue gets complicated when paradox is involved. Paradoxes such as the Liar paradox ('This sentence is false') have been troubling philosophers for thousands of years. What makes a paradox perplexing is how a set of true premises and a valid inference can lead to an unacceptable conclusion - in the case of the Liar, one that is seemingly both true and false. There have been countless attempts to resolve the Liar paradox. Tarski's effort is perhaps the most significant: not only did he offer an analysis of why the Liar is possible, but he also proposed a workaround that many contemporary logicians accept.

Tarski was trying to devise a theory of truth, but he was well aware of the problem the Liar paradox poses. Therefore he devoted much of his time analysing it, and concluded that three assumptions are sufficient for the Liar paradox to occur (Tarski, 1944, p.348).

a. The language in which the Liar sentence is stated in is semantically closed, meaning that it has a truth predicate True(x) such that it can refer to its own semantics (where x is the name of the true sentence), and it can refer to its own expressions - for example, a sentence P has the name ⌜P⌝.
b. Laws of logic hold, including the law of identity, the law of excluded middle (LEM) and the LNC.
c. An empirically based premise that allows a sentence to refer to itself can be asserted.

I will elaborate what assumption c) means in a moment. In the meantime, there is one more thing we need for the Liar paradox to occur: the T-schema. Note that when we combine the name of P with the truth predicate we have a way to indicate that P is a true sentence: True(⌜P⌝). The T-schema P ⟷ True(⌜P⌝) then gives us the relation between a sentence and it being true. In natural language, an example would be: Snow is white ⟷ ⌜Snow is white⌝ is true. Clearly, there are infinitely many true sentences in a language, thus this schema has infinitely many instances.

We can now construct the Liar sentence *L* using these assumptions. Suppose we are working with a language that does satisfy all three assumptions. Consider the following sentence:

The sentence on p.4, line 32 of this paper is not true.

This sentence is then assigned the name $\underline{L}$, this means that $\underline{L}$ is identical to the sentence on p.4, line 32 of this paper. By Leibniz's law of identity, we then substitute 'The sentence on p.4, line 32 of this paper' with '$\underline{L}$' and get the following:

$\underline{L}$: $\underline{L}$ *is not true.*

Thus, we have constructed the Liar sentence $\underline{L}$. Notice how we used all three assumptions to construct $\underline{L}$. Assumptions b) (i.e. Leibniz's law) and c) are needed to establish the identity between the sentence and the name we give it. Assumption c) is empirical in the sense that, the fact that the sentence which appears at a specific location in this paper is identical to the referent of the name '$\underline{L}$' can only be established empirically.[1] Furthermore, the way $\underline{L}$ is constructed requires assumption a) in order to refer to its own name (i.e. $\underline{L}$) and assert its own truth value.

The T-schema then tells us that in view of the meaning of $\underline{L}$, $\underline{L}$ is true iff it is not true, i.e. $\underline{L} \leftrightarrow \neg\text{True}(\ulcorner\underline{L}\urcorner)$.

With these ingredients in place, we can see the Liar sentence $\underline{L}$ implies a contradiction. By the LEM, the Liar sentence is either true or not true, i.e. $\text{True}(\ulcorner\underline{L}\urcorner) \vee \neg\text{True}(\ulcorner\underline{L}\urcorner)$. Suppose the left disjunct $\text{True}(\ulcorner\underline{L}\urcorner)$ is true, we then have $\underline{L}$ in virtue of the T-schema. But by the definition of the Liar sentence from $\underline{L}$ we have $\neg\text{True}(\ulcorner\underline{L}\urcorner)$, and couple that with our supposition of the left disjunct we get $\text{True}(\ulcorner\underline{L}\urcorner) \wedge \neg\text{True}(\ulcorner\underline{L}\urcorner)$. The process to obtain the same result from the right disjunct is very similar; thus we get a contradiction either way (Beall et al., 2020). This result is, of course, in direct conflict with the LNC.

Here Tarski faces a dilemma: all three assumptions seem very reasonable, yet it would seem that to avoid the Liar at least one must go. In the end, Tarski (1936a, p.165) chooses to abandon the first assumption, and concludes that since natural language does satisfy this assumption, it is inconsistent. Ultimately, Tarski (ibid, p.165) decides that only formal languages are suitable candidates for the theory because formal languages can be arranged into a hierarchy such that assumption a) is avoided, see criterion i) below.

Having taken the problem the Liar poses into account, Tarski sets three criteria for a theory of truth.

i. A distinction between the object and metalanguage is made. The language under discussion is called the object language (L), while a metalanguage M has a translation of all sentences in L and truth in L is defined in M (a predicate '… is a true sentence in L' is included in M). This is to ensure that while L might be able to refer to its own expressions, it cannot refer to its own semantics, thus preventing L from being semantically closed.[2]

[1] It must be noted, however, that assumption c) is not always necessary to derive a contradiction from Liar-like sentences (Tarski, 1944). In fact, the following example shows that explicit self-reference is not always needed for Liar-like paradoxes: Alex said 'What Boris says is true' and Boris said 'What Alex says is not true'. In this case, there is a circular reference within this set of utterances, but the utterances do not refer to themselves directly like Tarski's example (Beall et al., 2020).

[2] To be precise, fragments of a natural language can indeed be semantically open. For example, if we restrict our attention to the English chemists use, the language does not define its own truth, nor does it invoke/need to invoke

ii. The definition of truth in L should be 'formally correct', i.e. a necessary and sufficient condition should be given in a way that is not circular or presuppose truth to have already been defined.

iii. The definition should be materially adequate. Consider the sentence 'Schnee ist weiß'.[3] Our intuition tells us that this sentence is true iff snow really is white. Material adequacy requires the definition to conform to this intuition, and Tarski's way to satisfy this is with Convention-T, an instance would be like this: 'Schnee ist weiß' is true iff snow is white.[4] This convention takes the general form: True(s) iff p, where s is the name of a sentence in L formed by adding quotation marks around it (e.g. 'Schnee ist weiß'), p is a translation of s in M (e.g. snow is white), and True() is the truth predicate.

Graham Priest (2006a) challenges Tarski's analysis by arguing that there are sentences such that both they and their negations are true. He coined the term 'dialetheia' to refer to such sentences, and this stance is named 'dialetheism'. Priest argues that none of the existing solutions to the Liar paradox, including Tarski's, tells us exactly whether it is one of the assumptions or the inference that is problematic. If we reject an assumption or inference simply because that avoids an undesirable conclusion, then these solutions are ad hoc (Priest, 1979, p.220).

There are several significances to this debate. Dialetheists' belief in dialetheia puts them in direct conflict with the conventional thoughts that support the LNC, meaning the dialetheists opt to reject Tarski's assumption that laws of logic hold instead. Therefore if there emerges a conclusive result from the debate, it will be a significant argument for or against dialetheism. But whether it is possible to conduct the debate in the first place is in itself significant. The LNC is so fundamental as a logical principle that almost all arguments assume it. Is it even possible to debate the LNC without begging the question?[5] For example, an argument by contradiction would be circular, but attempting a direct proof would require an inference with no assumption of the LNC; so do we have enough logic resources to base our debate on?[6] Furthermore, the LNC is often seen as a key component of a minimal set of logical resources (or what's called the laws of thought), without which rational debate is impossible. Thus taking

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the truth predicate. But this fragment of English will not be well defined enough for the task of devising a theory of truth.

[3] 'Schnee ist weiß' means 'Snow is white' in German. It is easier to recognise the difference between object and metalanguage when the two are in different languages, instead of, say, having both in English. Here, the object language is German while the metalanguage is English.

[4] Despite the resemblance in appearance, the T-schema and Convention-T are two different concepts. The former is an inductive definition of truth and describes what a truth predicate looks like, whereas the latter is a requirement Tarski imposes on all theories of truth. If a theory of truth happens to have a truth predicate as described by the T-schema, then it also satisfies Convention-T. However there are languages that do not define truth as per the T-schema, yet still conforms to Convention-T. Intuitionistic logic would be one such example, since its semantics is not truth functional, whereas for the T-schema being truth functional is a key characteristic. In fact, the semantics of intuitionistic logic is usually described as evidence and provability based, instead of truth-valuation based.

[5] It may be suggested that by using a metalanguage with the LNC (and other logic laws) we could debate the LNC in the object language. I do not think this approach is plausible. If the metalanguage assumes the LNC, it seems to me it would be begging the question. For example, assuming such a metalanguage, one can easily conclude in the object language that contradictions cannot be true by using a proof by contradiction in the metalanguage. On the other hand, if the metalanguage does not assume the LNC, then the question of whether the LNC is true in the metalanguage can only be settled in a meta-metalanguage, but this just seems to be an infinite regress.

[6] It is not necessarily the case that a dialetheist must give up reductio ad absurdum; Priest's Logic of Paradox for example, allows reductio ad absurdum as what's called 'quasi-valid inference', which basically means unless one has reasons to suspect that self-reference is at play, one can treat the inference as if it is classical logic. See Priest (1979) for more details.

away the LNC raises the question of whether it is possible to conduct a rational debate. If we are to revise the LNC, we need to explain how rational debate can be conducted in the absence of the LNC.

There are two themes in this paper. The first one discusses whether it is possible to debate about the LNC in the first place, while the second discusses whether we should discard the LNC altogether. Section 2 introduces three common objections against debating about the LNC; it then discusses a logical apriorism motivated objection in detail, and criticises Tarski's substitution requirement which is behind this objection. It also provides justification for logical non-apriorism. Section 3 presents a model that explains how one can rationally debate and execute logic change, and explains how, in combination with logical non-apriorism, this model enables us to tackle Lewis' objection that the LNC is too fundamental as a logical principle to be replaced.

Section 4 moves onto the second theme and discusses the criticism against dialetheists that without the assumption of LNC one is incapable of expressing disagreement, and Priest's replies that are based on implicature and making a distinction between denial and negation. Section 5 turns to perhaps one of the most frequent criticisms, that rejecting the LNC renders one irrational. I discuss how, in view of the evidence available, this criticism is weak. I also discuss Priest's reply based on pragmatics of assertion and denial, and the metatheory of negation. I also offer my own observation on how, in combination with logical non-apriorism, dialetheism seems to imply that what it means to be rational changes as what we are studying changes from ordinary statements to paradoxical ones. Section 6 discusses dialetheists' arguments against the LNC. In particular, why is abandoning the LNC more sensible than another prominent theory regarding truth values - truth value gap. I also discuss Priest's arguments against Aristotle, being perhaps the only one who has offered constructive arguments for the LNC. Section 7 offers an evaluation of all the arguments I have been through, and concludes that be it constructing an argument or refuting the opponent/defending one's own position, the dialetheists are more convincing than the LNC proponents. For readers who are not familiar with paraconsistent logics, there is also a quick overview of Priest's Logic of Paradox at the end of the paper.

2) Is rationally debating the LNC even possible?

First of all let's make it clear what I mean by the LNC. Formulating the LNC such that the dialetheists reject it is actually a delicate task, because there are forms of the LNC that are admissible to dialetheists. In fact, $\neg(P \wedge \neg P)$ is actually valid in the semantics of Priest's Logic of Paradox (2006b, p.78; 1984, p.154).

The LNC that is unacceptable to the dialetheists is as follows: for any sentence P, P and ¬P are never simultaneously true. Dialetheists contend that some sentences are both true and false.

It must be noted however that this is consistent with dialetheists accepting $\neg(P \wedge \neg P)$. To see why, consider two types of sentences: those that are true only/false only, and those that are both true and false. For the former, notice that applying De Morgan's law to $\neg(P \wedge \neg P)$ yields $\neg P \vee \neg\neg P$, thus all it takes for $\neg(P \wedge \neg P)$ to be true is for either P or ¬P to be true. The true only/false only sentences therefore easily qualify. For any sentence P that is both true and false, since P has at least one truth value[7] it also fulfils

[7] Dialetheists do not believe in truth value gaps; in particular, Priest argues that those who assert that there are truth value gaps face difficulty with the strengthened Liar paradox. This will be discussed in section 6 in more detail.

¬(P∧ ¬P). At the same time, since P∧ ¬P is true, ¬(P∧ ¬P) is false. Note that this means for dialetheists, for any sentence P that is both true and false, ¬(P∧ ¬P) is also both true and false.

With the definition of the LNC set, let's talk about whether debating the LNC is possible in the first place. As far as I am aware, there are two common objections against the idea of debating the LNC:[8]

1. Logic should be apriori, in particular logical principles should not be revisable on the basis of extra-logical factors (Tarski, 1936b; Field, 1996).[9]
2. The LNC is too fundamental as a logical principle: Lewis argues that the LNC is one of the few principles that everyone is certain of (except dialethiests, of course) and agree with, that once it has been put into question, there are so few principles which share the same degree of certainty as the LNC that both sides agree with to base the debate on. In his own words, he is being 'dogmatic' about holding onto the LNC (Lewis, 1982).[10]

I will tackle these objections in order.

To discuss the objection from logical apriorists, let us consider the other side of the debate. The motivation behind rejecting the LNC can be categorised as logical non-apriorism. Logical non-apriorism can be defined as the stance that supports extra-logical factors affecting the choice of logic. As per Bueno and Colyvan (2004), extra-logical factor is a catch-all phrase that refers to anything that is not strictly logic-related - it can refer to empirical factor, but is not limited to that.[11] If we consider a logic to encompass logical principles (e.g. classical logic encompasses the LEM, but intuitionistic logic does not), then one important implication of logical non-apriorism is that it is possible to justify or revise logical principles on the basis of extra-logical considerations, which include (but are not limited to) empirical considerations.

Some examples will be helpful here. In the case of quantum logic, distributivity of conjunction over disjunction - a classical logic result - fails when applied to the quantum realm.[12] It is on this extra-logical/empirical basis that logicians such as Putnam (1979) argue that quantum logic is the correct logic to use in the quantum realm, not classical logic. In the case of dialetheism, the extra-logical consideration would be phenomena that appear to violate the LNC such as the Liar. This is crucial to justifying the model I will use to explain how the discussion of LNC is possible, as introduced by Bueno and Colyvan (2004, p.158).

Logical non-apriorism is far from being the conventional thought among logicians and philosophers. This is likely because many giants in the field - in particular Tarski, are advocates of logical

[8] Notice here that this is only talking about whether it is possible to debate about the LNC being revoked/amended, not about whether it should indeed be revoked/amended. The latter part of the paper will address this issue.
[9] This is equivalent to Field's definition of a strongly a priori logic principle, which states that it can be justifiably believed on a basis other than empirical evidence for it, and that it is also empirically indefeasible (Field, 1996).
[10] The two objections may appear to be almost identical, but they are indeed different in a subtle way: the first one is a blanket objection to revising any logical principle, whereas the second one is specific to the LNC.
[11] Examples from Bueno and Colyvan (2004) include: 1) distributivity failing in quantum mechanics, 2) the T-schema, and 3) natural language being semantically closed.
[12] Please refer to Bueno and Colyvan (2004) for a proof of how distributivity fails when applied to quantum mechanics.

apriorism instead. Tarski's idea can best be illustrated by looking at his substitution requirement (1936b, pp.414-417).

Logic has always been understood as a study of forms of arguments. As long as the form of the argument is maintained, the argument should still remain truth-preserving despite changes in the empirical content of the argument. Furthermore, if a conclusion is a logical consequence of a set of premises, it seems intuitive that it cannot be the case that the premises are all true but the conclusion is false. Tarski's substitution requirement[13] captures these two considerations and is as follows. Suppose we have Γ and A, where Γ is a set of sentences and A is a single conclusion. If we are to replace the non-logical constants (such as predicates) in Γ and A while leaving the logical constants (such as conjunction and implication) unchanged, and denote the altered Γ and A as Γ′ and A′ respectively, then A′ must be true provided only that all members of Γ′ are true. This is a necessary condition for A to be a logical consequence of Γ.

But how is this requirement related to logical apriorism? Tarski (ibid, p.414) has been clear that logical consequence should be determined only by the form of the arguments involved: 'Since we are concerned here with the concept of logical, i.e. formal, consequence, and thus with a relation which is to be uniquely determined by the form of the sentences between which it holds, this relation cannot be influenced in any way by empirical knowledge'. Bueno and Colyvan (2004, p.162) interpret this to be Tarski's argument for logical apriorism, since if logical consequence cannot be affected by empirical factor, the only thing that can influence it must be non-empirical, namely - apriori.[14]

But this substitution should not be applicable universally, Bueno and Colyvan (2004, p.161) argue, since classical logical consequence yields an incorrect conclusion when applied to quantum mechanics while upholding the substitution requirement. Specifically, merely preserving the form of argument is not always sufficient to guarantee that the logical consequence can preserve truth because this ignores the context of the domain it is applied to, thus there is no guarantee that the meaning of the constant, while having been fixed, will ensure truth preservation. Instead, they argue that the requirement should be restricted such that after having selected a domain, the requirement is only applied to sentences/objects in that particular domain.

This is not to say that they think logic should not be formal - they still do. In fact Bueno and Colyvan (2004, p.161) argue that one of the merits of only applying this substitution requirement within a certain domain is that it preserves the consequence relation:

By restricting the application of the requirement in this way, we can capture an important aspect of logic: in the context of the particular domain, the systematic substitution of non-logical terms does preserve the consequence relation.

In other words, Bueno and Colyvan are arguing that for a given domain, empirical matters affect our choice of logic for that domain, and there are different forms of truth-preserving logical consequences depending on the domain. In the case of quantum mechanics, the empirical observation that distributivity

[13] It is referred to as the condition F in Tarski's text (1936b, p.415).

[14] I think the quote by Tarski also implies another way that logic must be apriori. I take it to be agreeable that the form of argument is dictated by the logical constants involved, and that logical constants are apriori. If, as Tarski argues, logical consequence is uniquely determined by the form of the sentences, this must mean that ultimately it is the (apriori) logical constants that are responsible. Therefore in the end what determines logical consequence comes down to apriori factors - the logical constants.

fails compels us to use a logic, for example quantum logic, that correctly models logical consequence for that domain. Then having chosen quantum logic, because the logical constants involved would then be specific to this logic, by extension the substitution requirement would only hold within the quantum domain.

If one accepts that the requirement should indeed be restricted in this manner, then this means that empirical factors now affect our consideration of a logic's adequacy to a particular domain. The importance of this point will become clearer when we explore Bueno and Colyvan's model in the next section and its application to explain how we move from a classical research tradition to one of modality.

There is one comment I would like to make. Bueno and Colyvan (2004, p.157) mention that their case is applicable regardless of whether one is a logical monist or logical pluralist. I disagree: I think the restriction they impose on the substitution requirement entails that they are logical pluralists. Suppose that one is a logical monist and believes that classical logic is the one and only 'right' logic. Then no matter what the domain is, the logical constants that can be used will always be the same (classical) set, and thus the notion of logical consequence and forms of arguments that are truth preserving will remain the same across different domains.

Now, suppose that the substitution requirement restriction is in place. But the substitution requirement restriction is not really doing anything; the whole point of the restriction is to have different forms of truth-preserving arguments for different domains, such that in those respective domains even after having substituted the assumptions and conclusions the form will remain truth preserving. But if we only have classical logical consequence, that means we get the same valid form of argument be it in quantum mechanics or constructive mathematics. This is in effect no different from Tarski's original requirement where the substitution requirement applies across the board. This must mean that if we would like to uphold the substitution requirement restriction, it implies we must accept logical pluralism and the idea that there are different logics for different domains, or else the restriction will not do what it is intended to do. Equivalently, we must accept that there are different notions of logical consequences for different domains.

3) <u>Model for theory change</u>

We are now ready to tackle objection 2. In Lewis' own words, his objection is as follows:

No truth does have, and no truth could have, a true negation. Nothing is, and nothing could be, literally both true and false. This we know for certain, and a priori, and without any exception for especially perplexing subject matters. The radical case for relevance should be dismissed just because the hypothesis it requires us to entertain is inconsistent. That may seem dogmatic. And it is; I am affirming the very thesis that Routley and Priest have called into question - contrary to the rules of debate - I decline to defend it. Furthermore, I concede that it is indefensible against their challenge. They have called so much into question that I have no foothold on undisputed ground.[15] So much the worse for the demand that philosophers always must be ready to defend their theses under the rules of debate.

[15] I believe there may be an alternative interpretation of this particular sentence which means that deductive inference presupposes the LNC, which means something slightly different from what I described above. This possible interpretation is dealt with together with one of Aristotle's arguments in the latter part of the paper, please refer to section 6 for details.

(Lewis, 1982, p.101)

To tackle this objection, we need to equip ourselves with a model that explains how logical principle change is possible. Bueno and Colyvan propose a model which explains how we can debate logic change rationally, and how a change in our logical theory can be justified.[16] To understand this model, we recognise that there are three components which all debates about logic range over.

- Aims of logic - what the study of logic tries to achieve, e.g. to model the intuitive notion of consequence as found in natural language
- Metalogical properties - such as a formal system's completeness and decidability
- Logical principles and rules of inferences

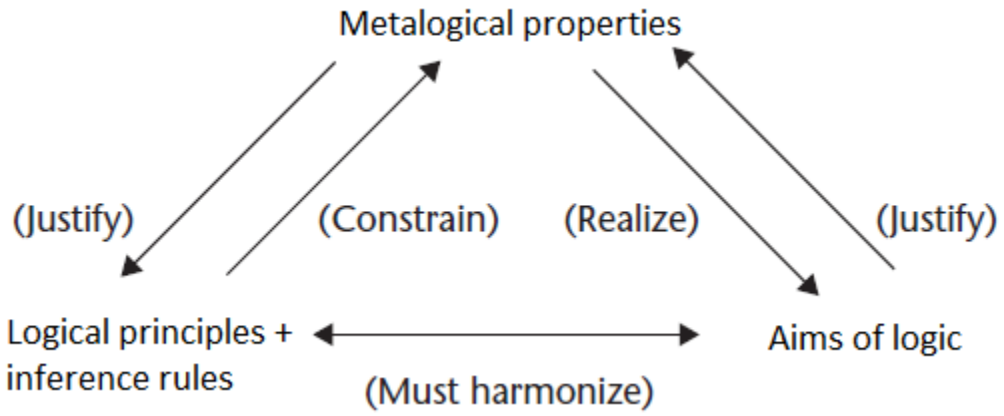

Fig.1: An illustration of Bueno and Colyvan's model

Their point is this: of the three principles mentioned above, often only one of which is being challenged, but never all three at the same time - there is at least one component which both sides of the discussion can agree on, and is therefore fixed at any given time. One can then base one's argument on the fixed component without fear of being circular. And once we have all three components in place, we have a research tradition (e.g. classical). Two examples of applying this model to the debate of the LNC will be illustrated later as a response to Lewis' objection, but for now let's apply it to explain the conception of modal logic. Suppose we begin with classical research tradition and our aim is to capture the consequence relation in natural language. We then develop classical logic with that aim in mind, and with those classical logic principles and inference rules we further develop a set of metalogical results. The three components then enable us to conduct research in logic.

But soon we find instances in natural language which our classical notion of consequence cannot account for. For example, there is no way to express 'It is not necessarily true that Obama is the 44th US president. Therefore it is possible that Obama is not the 44th US president.' in classical logic. Since the inference rules of classical logic are incapable of modelling this newly found problem of modality, we have two choices. Either expand the language and introduce new modal operators to accommodate box and diamond in accordance with the Kripkean semantics, or formulate a new theory and change the

[16] Bueno and Colyvan actually borrowed Laudan's model of scientific theory change and adapted it to model logical theory change instead. But it has been omitted in this paper because it is largely irrelevant other than giving Bueno and Colyvan the basic idea. Interested readers should refer to Laudan 1984 for more details.

domain over which quantifiers range over if one believes in Lewis' modal realism. (Note that in both cases they necessitate changes to the logical principles, and in the former new inference rules must be formulated as well to accommodate the new operators.) But with these changes we must also develop the corresponding metalogical results accordingly, thus leading to changes in metalogical properties as well. Here, despite having the same aim of capturing the notion of consequence in natural language, we have a new set of logical principles/inference rules and metalogical properties, thus moving us from a classical research tradition to one of modal research tradition.

Coming back to Lewis' objection, we need to look at how Bueno and Colyvan's model can be applied to the debate of the LNC. Bueno and Colyvan ask us to consider an argument by a LNC supporter, 'The LNC must be true because if there is true contradiction, then everything will be true'. As they point out, this argument hinges on the validity of ex falso quodlibet, or the principle of explosion - something that a dialetheist rejects precisely because he realises that explosion will lead to trivialism. Here, we can see both sides do share common ground: they both reject that everything is true. Both sides also realise that classical logic and true contradictions conflict with each other.

But as far as agreement goes this is where it ends. Does the objection that there is too little common ground to conduct the debate hold water? Bueno and Colyvan think not. They argue that there is more common ground than Lewis realises, both sides agree that not everything is true, and that classical logic is incompatible with true contradictions. Here one is at a crossroads. The LNC supporters choose to reject true contradiction in order to preserve classical logic, while dialetheists choose to abandon classical logic (of which the LNC is a key principle) in order to reconcile with the fact that seemingly true contradictions such as the Liar do exist, and adopt Priest's Logic of Paradox (LP) instead. While this is as far as agreement goes and where the deadlock begins, they argue that the deadlock is not due to a lack of common ground, but merely a feature which is shared by many other important philosophical questions. In other words, they are able to conduct a debate on the LNC based on the shared common ground as mentioned above - they just disagree on the conclusion and what to do with the LNC.

Furthermore, Bueno and Colyvan point to the response from dialetheists as an example of how progress has indeed been made in this debate. LP is designed specifically to be contradiction tolerating. Bueno and Colyvan argue that it is simply not true to say that no progress has been made in the debate, when, as a reply, the dialetheists have developed LP to show that rational reasoning is possible without presuming the principle of explosion. They argue that this should be considered progress.

Another perspective from a dialetheist is as follows. Recall the three assumptions Tarski has shown to be sufficient for the Liar sentence: a) semantic closure[17], b) laws of logic hold, and c) an empirically based premise that allows a sentence to refer to itself is assertible. From these three assumptions, coupled with the T-schema, we conclude that the Liar is both true and false. The common ground here is that both sides agree that the argument from the three assumptions and the T-schema to the conclusion that the Liar is both true and false is a valid one. What they disagree on is what both sides decide to do with this undesirable conclusion. While the dialetheist acknowledges that the conclusion is

[17] Note that the natural language being semantically closed is an empirical matter, as Bueno and Colyvan point out. It is a matter of empirical fact that 'It is true that the Earth is round' is grammatical and well formed in English. One motivation of dialetheism is to account for logical consequence in natural language, and since natural language is empirically semantically closed, contradictions are bound to happen. Thus establishing logical non-apriorism is crucial to justifying change of our choice of logical theory from classical to dialetheic.

indeed undesirable, in the sense that it goes against our intuition that no sentence can be both true and false, he refuses to reject the conclusion on this ground because there is nothing logically wrong with this argument. Instead, he bites the bullet, rejects the LNC and concedes that there do exist sentences that are both true and false. However the opposite party feels that the dialetheist way is too costly since the status of the LNC is fundamental in logic. In particular, Tarski is forced to concede that a semantic theory of truth is unattainable with semantically closed languages, and works on hierarchical formal language instead. Bueno and Colyvan argue that there is again common ground shared by both parties: they agree on the assumptions (for the non-dialetheists, at least up until they are forced to abandon something) and the inference that leads up to the Liar is valid.

I think both Lewis' and Bueno/Colyvan's argument have merits, but ultimately I am inclined to take the latter's side - though with reservation. If we restrict our focus on simply the debate of the LNC, then it is indeed clear that there is too little common ground between both sides. Bueno and Colyvan's examples above showed that there is indeed common ground, so in that respect I agree with them. But I do not think there is as much common ground as they make out to be. In particular, one of the contentious issues between dialetheists and non-dialetheists is the meaning of negation, and in turn that difference leads them to different conclusions of the status of the LNC. This means not only do they have different logics, they also have different metatheories for logic. Having said that, this does not necessarily mean that a debate is impossible: classicalists and intuitionists have debated on the status of the LEM for a long time despite having the same issue; if the LEM is equally fundamental to be included as one of the laws of thought and a debate of which can happen with no trouble, why is the debate of the LNC any different?

I also don't think this is all there is to the debate, ultimately I think this debate comes down to whether logic is non-apriori and should be revisable. On the one hand, Lewis' self-proclaimed 'dogmatic' attitude of the LNC stems from his belief in its apriori status. In his own words, 'Nothing is, and nothing could be, literally both true and false. This we know for certain, and **a priori**, and without any exception for especially perplexing subject matters' (Lewis, 1982, p.101). On the other hand, the dialetheist believes that logic, in particular logical principles such as the LNC, should be revisable because of extra-logical factors such as the Liar being formulatable in natural language. I think the debate on the LNC is just a part of the bigger discussion of logic's apriority, and the latter is very much a dynamic and ongoing debate which, if fruitful, can be informative on the debate of the LNC itself.

From Lewis' perspective, if a definitive conclusion emerges that suggests logic is indeed apriori, then it effectively denies the dialetheist's extra-logical ground to reject the LNC - at least via the route granted by Bueno and Colyvan's model. From the dialetheist's perspective, if a definite conclusion that suggests the opposite emerges, then by Bueno and Colyvan's model we are now justified to modify our logical principles to accommodate the extra-logical fact that natural language can express paradoxes such as the Liar, since one of the aims of logic is to capture the intuitive notion of consequence found in natural language. This then allows us to revise existing logical principles including the LNC, and devise new inference rules to reflect this revision (and by extension, metalogical properties would have to change as well). We would then move from a classical logic research tradition to one of paraconsistent. In other words, while I think Lewis is correct in saying that there is not enough common ground with regards to the debate of the LNC to make much progress before we have a deadlock, I believe he is being tunnel visioned and too focused on this debate only.

Now that we have seen it is indeed possible to discuss the LNC without begging the question, let's turn to the second theme of the paper: evaluating arguments for and against the LNC. We begin with a defence of the LNC.

4) <u>Inexpressibility of disagreement</u>

The following defence of the LNC is not so much a defence as an attack of the undesirable consequence of abandoning the LNC or accepting of dialetheism. In a nutshell, it argues that someone who accepts that a sentence can be both true and false has difficulty expressing genuine disagreement. As Parsons (1990, p.345) wrote:

> *Suppose that you say 'P' and Priest replies ' ¬P', Under ordinary circumstances you would think that he had disagreed with you. But then you remember that Priest is a dialetheist, and it occurs to you that he might very well agree with you after all - since he might think that P and ¬P are both true. How can he indicate that he genuinely disagrees with you? The natural choice is for him to say 'P is not true.' However, the truth of this assertion is also consistent with P's being true - for a dialetheist, anyway.*

In other words, for a dialetheist to assert ¬P, it can either mean that he believes that P is false and not true (simply false), or P is both true and false. Normally we do not have this problem because the assumption of LNC rules the latter out; but for a dialetheist, since he rejects the LNC we do not have sufficient information to do so.

Priest proposes two replies.

The first is to rely on Gricean implicature. When someone communicates, he is assumed to be cooperating with the other party to convey information by observing a set of maxims. In this case, if Priest asserts ¬P while he believes P∧¬P, the information he provides is incomplete and thus he would have violated the maxim of quantity, which requires him to be as informative as possible. This contradicts the assumption that Priest observes the maxims and tries to be cooperative, thus the other person may infer that Priest does not accept P, and intends to confer exactly that (2006a, p.291).[18]

The second way is to express disagreement of P by denying P. (2006b, p.104) To reject P is a cognitive state which positively refuses to believe P, and denial is a speech act that one can perform to show that one rejects P. One can deny P by asserting ¬P, but Priest argues that the two are not the same, i.e. to deny P one can do so by asserting ¬P, but asserting ¬P does not necessarily entail that one denies P. In other words, Priest can disagree with Parsons by uttering 'It is not the case that P'.

This begs the question: how does asserting ¬P not entail that one denies P? Priest gives several examples to illustrate instances where denial of P is not identical to asserting ¬P. One of the prime examples[19] Priest (2006b, p.105) gave is the Preface Paradox. Imagine that Priest has just written an academic book. He has included many assertions in it and has thoroughly checked every one of them, thus he is justified in believing that all of them are true. At the same time, he knows from experience that there

[18] We cannot outright infer that Priest rejects P because as Shapiro points out, Priest may simply be undecided, to which Priest agrees (Shapiro 2004, p.339; Priest 2006a, p.291).

[19] Another example Priest gives is metalinguistic negation. For example, 'Hitler was not a bad man - he was downright evil.' Despite asserting it is not the case that Hitler was bad, the speaker does not really deny that Hitler was so (Priest, 2006b, p.104).

will be at least one mistake in there. If P means 'every assertion Priest made in the book is true', then this means he has reasons to rationally believe and assert both P and ¬P. Furthermore, this means that someone who asserts ¬P does not reject/refuse to believe P. This distinction is particularly important to the dialetheist because there may be a sentence P (the Liar for example) for which he may have grounds to believe both P and ¬P to be true, yet he may assert ¬P without denying P.

In addition to the two replies, Priest (1995, p.62; 2006a, p.291) argues that what critics of this kind are looking for is consistency; and that is something dialetheists cannot provide. In view of the dialetheists' view on the LNC, I think it is fair to say that indeed they do not have an obligation to do so.

Furthermore, I would like to add that dialetheism is not the same as trivialism. A dialetheist does believe that there are sentences that are both true and false, but this only applies to certain categories of sentences, such as self-referring sentences, vague predicates and certain legal conditions (such as inconsistent bodies of law)[20] (Priest et al., 2018). Therefore unless the sentence P concerns any of these categories, one should not assume that the dialetheist believes in P ∧ ¬P.[21]

5) <u>Rejecting the LNC is irrational</u>

Let's turn to the objection that rejecting the LNC renders a person irrational. Specifically, someone who believes in both P and and ¬P is conventionally viewed as irrational.

The short answer is as follows. Priest has, rightly in my opinion, mentioned that what constitutes rational behavior is in itself contentious. Priest (2006b, p.109) does not want to get bogged down by what can easily span into another book-length discussion, and adopts what he calls the Rationality Principle: If you have good evidence for the truth of P, you ought[22] to accept P.[23] Priest justifies it by describing it as 'natural enough'. From there, Priest simply observes that the existence of the Liar sentence at least suggests that 'there is nothing in principle impossible about the existence of good arguments for true contradictions'. In other words, while a dialetheist does not believe that both 'Earth is round' and 'Earth is not round' to be true because there is no good evidence for the latter, he does believe that both 'The Liar sentence is true' and 'The Liar sentence is not true' to be true because there is indeed good evidence for both.

[20] Although legal conditions appear to be different in nature from the other categories, Priest is still primarily concerned with contradictions here. For example, the Hong Kong government enacted the Prohibition on Face Covering Regulation in October 2019 that prohibits any form of face covering in the hope of curbing pro-democracy demonstrations. However in August 2020 the government also mandated that a face mask must be worn in public at all times in order to control the spread of Covid-19. In other words, I should both wear and not wear a face mask in public in Hong Kong. This would be an example of a true contradiction caused by inconsistent legal conditions.

[21] One may wonder which category does the aforementioned Preface Paradox fall into; it is actually unclear what Priest's take is. As an epistemic paradox, it is not like any of the other categories, thus I suspect he may need to concede that epistemic paradoxes are a distinct category.

[22] Priest made it clear that 'ought' here is not in the moral sense of the word; it means it is rational to believe what is justified by evidence (and likewise it is irrational to believe what is not justified by evidence).

[23] My suspicion is that Priest borrowed this notion from epistemology's justified true belief formulation of knowledge. One can of course go on to question what constitutes good evidence; but that is perhaps out of the scope of this paper. I think Priest would reply that at least in the context of paradoxes, since paradoxes usually refer to themselves as being true and not true, that in itself is good enough as evidence.

An expected rebuttal may be that, if someone accepts both P and ¬P, then since everything follows from a contradiction by logic, he will be reduced to a trivialist. This is of course true in the setting of logics such as classical, but the principle of explosion is invalid in paraconsistent logics - including Priest's LP, so it is indeed possible to reason even if we let contradiction into our system (Priest, 2006b, p.121).

Now let's tackle the objection in detail. Since dialetheists believe that there are sentences that are both true and false (i.e. dialetheias), it is clear that there is a sentence P such that dialetheist will readily assert both P and ¬P. To assert P, one must accept P - i.e. to believe that P is true. Presumably, to assert ¬P entails denying that P is true, and thus rejecting P. How can someone both accept and reject P, and be called rational at the same time?

To answer this question, Priest needs two things: the pragmatics of assertion and denial, and the rationality of accepting contradictions.

It has already been discussed previously (see section 4) that the conventional view that denying P is the same as asserting ¬P is disputed by Priest. In other words, just because a dialetheist is asserting ¬P to be true does not mean he denies that P is true, thus he is not compelled to deny P and is free to assert P at the same time.

Now let's turn our attention to the rationality of accepting contradictions. A major reason to think that it is irrational to accept contradiction is how truth and falsity are conventionally thought to be exclusive - falsity is just the lack of truth, and that if P is false it means ¬P is true. This is certainly the case for classical logic.

But to assume a classical negation (or any logic that assumes truth/falsity as described above), and then argue that from such a negation it entails that a contradiction cannot be true, and thus dialetheism is wrong - that is begging the question, argues Priest. Classical logic and dialetheic logic are two competing logics; if one assumes the former to be the 'correct' logic, then clearly one's arguments against dialetheism will work - because it is already assumed that classical negation is the 'correct' account (Priest, 2006b, p.76, p.122).

Priest (2006b, p.81) argues that theories of negation are really theories of contradictories.[24] In terms of LP, the dialetheic negation is such that P and ¬P are contradictories: ¬P is true iff P is false. This is in stark contrast to classical negation, where ¬P is true iff P is not true. One may wonder, surely P being false is equivalent to P being not true? This is not true if one takes the difference in terms of semantics of both logics into account. Given the semantics of classical logic, P can only be either true or false, but not both. Thus in the classical case ¬P being true ensures that P is false and not true - so in this case P being false is indeed equivalent to P being not true. On the other hand, since dialetheic logic allows a sentence to be both true and false, despite ¬P being true entails that P is not true, that does not rule out P being true as well. The case for such a negation stems, of course, from the motivations of dialetheism - to deal with paradoxes.

[24] Consider 'No S is P' and 'Some S is P', where they cannot both be true and they cannot both be false.

So if we accept the argument for dialetheism and the logic that spawns from it, then just because P is true does not rule out that ¬P is true as well; thus from dialetheic logic's perspective it is rational to accept both P and ¬P to be true.

If we accept Priest's argument regarding negation, then whether rejecting the LNC renders one irrational or not comes down to whether one rejects dialetheism or not. This sounds like stating the obvious, but I think there is an interesting observation to be made here. If we accept:

- Bueno and Colyvan's argument that the substitution requirement only applies once we have chosen the domain we want to study, which further implies that there are different logical consequence for different domains, thus warranting using different logic depending on the domain
- Dialetheist's argument that rejecting the LNC is not irrational

Then it seems to imply that, at least under certain conditions, the definition of rationality changes whenever we are dealing with paradoxes. Let me explain.

Suppose we are trying to study some simple statements, e.g. 'All men are mortal, Socrates is a man, therefore Socrates is mortal'. In this case it seems uncontroversial that classical logic is more than capable of getting the job done, therefore we would be subjecting ourselves to classical logical principles including the LNC. Now suppose we would like to study paradoxes instead, e.g. the Liar paradox. In this case, dialetheic logic would seem like a good choice. But by using dialetheic logic, we would also be subjecting ourselves to the dialetheic perspective and therefore we regard the LNC as false.

When we are using classical logic to study ordinary statements, the LNC is not only a logical principle of classical logic, it is also a law of thought that counts as a criterion of being rational, i.e. a necessary condition for rationality. But when we move onto studying paradoxes and use dialetheic logic instead, the LNC is rejected because the underlying logic is dialetheic. Consequently from the perspective of dialetheic logic it is actually irrational to admit the LNC. This suggests that what counts as rational is relative to the logic one uses, and because we switch logic when we study a different domain, the definition of rationality changes depending on what domain we are working on.

I have emphasised that this conclusion is implied under 'certain conditions', let me now highlight what that is. Dialetheic logic is not the only logic capable of handling paradoxes. Paraconsistent logics are in general contradiction tolerant and allow inferences to remain non-trivial despite having contradictions in the system, yet as far as I am aware only dialetheic logics reject the LNC of the form as defined in section 2. The closest logic that also does this, as far as I am aware, is dual intuitionistic logic which is paraconsistent. It rejects the LNC of the form $\neg(P \wedge \neg P)$, but the logic does not admit a truth value equivalent to LP's 'paradoxical' such that a sentence can both be true and false (Priest, 2009). (i.e. Dual intuitionistic logic does not reject the LNC of the form we are interested in.) Since this paper is ultimately about whether the LNC is true or not, and only dialetheic logics reject the form of LNC we are interested in, here I am assuming that the logic we will use to handle paradoxes is a dialetheic logic, such as Priest's LP.

Could it be that, instead of a domain-dependent definition of rationality, it's really the same definition of rationality throughout, but in different contexts it compels us to use different logics? I don't

think it is. If the LNC is a necessary condition for rationality when we use classical logic, and no longer so when we use dialetheic logic instead, it is clear that whatever the complete formulation of rationality may be, it has changed from including the LNC to excluding it. Therefore I do not see how one can argue that the definition of rationality has remained consistent before and after the switching of logic.

One may also argue that, within the framework of LP, one can use what's called quasi-valid inference[25] to model logical consequence when we do not suspect paradox is involved. If LP alone is entirely capable of modelling logical consequence both in the ordinary case and paradoxical case, why should we use classical logic for the former case? If we are using LP for both cases, then our perspective - in particular, our view on the LNC - would remain consistent throughout the process. Thus there would be no need to change the logic we are using and the observation (that the definition of rationality changes depending on what domain we are working on) would no longer hold true. It is indeed possible to use LP for both, but I would argue that doing so isn't really that different from having two logics for the two cases.

Suppose we are using quasi-valid LP to study ordinary statements where we do not suspect any paradox is involved. Since we are using LP, we will be subjecting ourselves to dialetheic beliefs, i.e. that there are sentences that are both true and false. But the whole point of using quasi-valid LP is to study sentences that are not both true and false. If we suspect or encounter sentences that are of this kind, we switch immediately to the part of LP where quasi-validity is no longer allowed. If we do not, we work with quasi-valid LP, which is essentially what classical logic is.

6) Against the LNC

As far as I know, we have now covered all the common arguments for the LNC. It is now time to review arguments against the LNC. Since dialetheism is the position that rejects the LNC, arguments against the LNC are also in effect arguments for dialetheism. They concern our options when facing paradoxes.

As mentioned at the beginning, the motivation for dialetheism is the existence of paradoxes, be it semantic paradoxes such as the Liar, or set theoretic ones like Russell's. In particular, Priest (1994) argues that all paradoxes share the same structure characterised by what's called the Qualified Russell's Schema (now termed the Inclosure Schema). And since they are of the same kind, this warrants one unified solution for all of them - namely, dialetheism. Obviously dialetheism is not the only proposed solution to paradoxes. Ever since paradoxes were discovered there have been numerous solutions proposed. Yet dialetheism is the only one which advocates doing away with the LNC entirely - why should we favour dialetheism and abandon a seemingly intuitive logical law, instead of opting for other solutions that are less costly?

The first reply is that all other existing solutions are problematic, be it Tarski's which simply avoids the paradox or others that try to solve the problem posed by paradoxes. As mentioned in Section 1,

[25] The basic idea is that LP comes at a cost for taking paradox into account, some inferences that we take for granted, such as modus ponens, are invalid. In view of this, LP allows classical inferences that are invalid in LP to be used as long as all the truth values only involve true or false and not paradoxical. These inferences are called quasi-valid inferences in LP.

Priest argues that Tarski's method is ad hoc, since it is merely a manoeuvre which blocks an undesired conclusion and does not tell us exactly where the inference went wrong like a genuine solution should.

Of course, there are other attempted solutions. It is worthy of another paper in itself to survey all of them, so I will focus on what can be described as the dual of dialetheism - truth value gap theory (henceforth referred to gap theory).[26] Instead of asserting that the Liar sentence is both true and false, gap theorists argue that it is neither true nor false - it has no truth value. In other words, gap theorists reject the LEM. Since the LEM is often considered the dual of the LNC, gap theorists who reject the LEM can also be considered as the dual of dialetheists. The discussion of this approach has enjoyed some renewed interest in recent years due in no small part to Kripke's accompanying discussion of metaphysical grounding, which is why I have chosen to discuss it in detail. However, Priest argues that this approach suffers from a strengthened version of the Liar sentences (SL):

SL: This sentence is not true.

There are three possible scenarios:

- SL is true. Then in virtue of the T-schema, SL is not true.
- SL is not true. Then either it is false, or valueless.
    - If it is false or valueless, then this means SL is not true, thus (again by the T-schema) it is true.

Priest (2006a, p.15) concludes, gap theorists run into problems either way.[27]

There are three responses gap theorists have, but Burge (1979) deems none of them convincing. The first one is to reject that SL is not true - the gap theorists remind us that when they talk about SL being valueless, they really do mean valueless, i.e. it just does not make sense to talk about its truth value as if it is true or false. As Burge points out, this implicitly assumes that both the meta and object languages are gappy. But he further points out that to say that SL is neither true nor false does seem to suggest, intuitively as per the classical conception of negation, that it is not true.

The second is to prohibit the substitutivity of identity. Specifically, the inference from 'This sentence is not true' is not true to SL being not true is deemed invalid according to this argument. In other words, this argument bars the indexical term 'This' from referring to SL. But this also runs into trouble, Burge rebutted, since there are paradoxes that do not rely on substitutivity of identity. Burge did not actually name any, but I suspect what he had in mind was Grelling's paradox.

The last response is adopted not just by gap theorists, but Barwise and Etchemendy as well. It distinguishes a hierarchy of negation. The basic idea is that when we say SL is not true, the 'not' here is

[26] There are roughly five main approaches to the Liar (Dowden, 2021). Tarski's meta/object language hierarchy, gap theory and paraconsistency, of which dialetheism belongs, are among the five. Another one is Bertrand Russell's type theoretic approach: the Liar is meaningless because an attempt to translate the Liar into his infinite hierarchical formal language violates his type theory. But an implication of this approach is that every self-referring sentence is meaningless, including 'This sentence is written in English'. Lastly, there is Barwise and Etchemendy's proposal that depending on the interpretation, the Liar is true only or false only (but not both). The paradoxicality results from an ambiguity of the two interpretations. But this approach is also flawed while trying to deal with the strengthened Liar. See the discussion of the last response of the strengthened Liar.

[27] In fact, Priest (2006b, Chpt 4.7) goes even further by arguing that there is no truth value gap.

limited in the sense that the proposition ('SL is true') fails to be true but also fails to be false (Fitch, 1964). But again as Burge points out, this option is flawed too. There are paradoxes that do not utilise negation, such as Curry's paradox, which this option cannot deal with.

The second reply concerns the justification of the LNC, or lack thereof. According to Priest, Aristotle is the only philosopher who has attempted to put forward a constructive argument for the LNC. But as far as he is concerned, not only are his arguments vague at times, they also fail to justify the LNC. Because the way Aristotle expressed himself is vague, there are multiple interpretations of his arguments. It is impossible to review all of his arguments and interpretations, so I will only pick the most significant one to discuss.

Aristotle has three definitions of the LNC[28]; the one we would consider is that opposite assertions are not simultaneously true (1011^b13). This is consistent with our definition. He then argues that no one can believe $P \wedge \neg P$ (1005^b 22-27). Moreover, he argues that the LNC is 'the principle of all the other axioms' (1005^b 33-35), which Priest interprets to mean that deductive inference presupposes the LNC.[29]

Priest has the following rebuttal. Let's start with the $P \wedge \neg P$ believer. If he does believe so, then he believes P and he also believes ¬P. But from the latter this means he does not believe P. But both believing P and not believing P is a violation of the LNC. The flaw of this argument is of course as clear as day - if the whole point of the argument is to justify the LNC, then the LNC should not be invoked because it is begging the question (Priest, 2006b, p.9).

It is also not necessarily the case that deduction presupposes the LNC. As Priest's LP demonstrates, one can indeed use deduction without assuming the LNC.

Lastly, I would also like to discuss the ramification of embracing logical non-apriorism and dialetheism, and how it may be a point of criticism to some. As discussed, if we are to use Bueno and Colyvan's model to justify our change in logic (without circularity), then it entails that we must also accept that there are different notions of logical consequences for different domains. If we examine Priest's LP, it also seems to confirm this result; recall that modus ponens is a quasi-valid inference in LP when dialetheia is not involved, but not at all valid when dialetheia is indeed involved. This means modus ponens, along with other quasi-valid inferences, embody one notion of logical consequence only when dialetheia is not involved. By extension, if thinking or arguing in accordance with logic - in this case LP - then it would seem that depending on whether we are dealing with dialetheia/quasi-valid inference or not, what it means to think rationally changes as well. The fluidity of the definition of rationality implied from this may be unsettling or even unacceptable to some. But considering we have different notions of logical consequences for different domains and rationality is at least partially dependent on the notion of logical consequence, this is perhaps not that surprising.

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[28] They are: 1) "It is impossible for the same thing to belong and not to belong at the same time to the same thing and in the same respect", 2) "It is impossible to hold (suppose) the same thing to be and not to be", 3) "Opposite assertions cannot be true at the same time" (Metaphysics IV 3 1005b19 - 20, IV 3 1005b24 cf.1005b29 - 30, IV 6 1011b13 - 20).

[29] It is worth mentioning that there are other interpretations possible, but this is the one judged by Priest to be the most probable. And even if this isn't an accurate interpretation of what Aristotle meant, I think it is worth discussing this point in and of itself, because I suspect this is what Lewis meant when he said, '*They have called so much into question that I have no foothold on undisputed ground.'* (see quotes in section 3)

7) How many correct logics are there?

Before I deliver my final verdict, I think an obvious question needs to be addressed: is Priest a logical monist with respect to LP? On the one hand, it is clear that Priest intends LP to be the go-to logic when dealing with paradoxes. On the other hand while modus ponens is an indispensable inference rule in ordinary reasoning, it is only quasi-valid in LP. Essentially, quasi-valid rules are classical rules that are allowed in LP so long as we are dealing with non-paradoxical propositions. So does Priest believe that LP is the only correct logic, or does he think classical logic is sometimes correct, given certain circumstances?

The short answer is: while Priest is a logical monist, it is not clear if he intends LP to be that logic. But he does have an answer as to how a monist can allow something like quasi-validity.

Priest is a logical monist in the sense that he believes that there is only one correct logic that is suited to analyse what makes an argument valid. Analysing validity is obviously not the only application a logic has, other applications may include analysis of electronic circuits or certain grammatical structures. And when it comes to these non-validity related applications, Priest (2006b, pp.195-196) is a pluralist - or what he calls a 'theoretical pluralist', on the grounds that there are many different logics which constitute theories that describe the behaviour of that domain. Note that this appears to differ from the conventional definition of logical pluralism. It seems to me that, instead of asserting that there is more than one correct logic, Priest is merely asserting that there are multiple logics proposed for any given non-validity related application. In other words, theoretical pluralism does not concern the question of the number of correct logics, but merely the number of logics asserted to be correct - this seems to make theoretical pluralism a truism.

However the most important and traditional application of logic, Priest (2006b, p.196) argues, is the analysis of valid reasoning, or what he calls 'canonical application of logic'. And it is here that he is a logical monist. Priest (2006b, p.202) argues, 'It is only truth-preservation over all situations that is, strictly speaking, validity'. Even if there are different forms of truth-preserving arguments depending on the domain, there must be an intersection among all these forms. Since traditionally logic is viewed as a domain-neutral subject, this intersection should be the only correct logic (ibid, p.174). In other words, if a form of argument only preserves truth in one domain but not another, then that form does not do so over all situations, hence it is not valid.

Beall and Restall (2006, p.92) argue that the different situations included in 'all situations' may be so diverse that the intersection may include nothing except the identity inference: from P to infer P. Priest (2006b, pp.202-203) rebuts that we would have more than that, namely inferences in virtue of the meaning of their constituting connectives, e.g. from $P \wedge Q$ we can infer P. But as Russell (2021) points out, there are different theories of meaning of connectives - in fact, as far as I am aware one can find a debate going on regarding almost any of the conventional logical connective, be it negation (e.g. classicalists vs dialetheists), disjunction (e.g. classicalists vs intuitionists), or conditional (e.g. classicalists vs relevant logicians). It is clear enough that Priest asserts that negation is about contradiction; but it is not clear to me, regarding other connectives, which theories of meaning does Priest endorse, or how would he decide which one to endorse. This is not to say Priest's rebuttal does not work, but it does hinge on answers to these questions, which I have yet to see.

Priest did not elaborate if this intersection is indeed the LP. It appears that while Priest believes that such an intersection do exist, he is undecided on what it consists of exactly, because he wrote, 'It could be, I suppose, that this intersection is empty, but I have never heard a plausible argument to this effect' (Priest, 2006b, p.174 footnote 54). Regardless, for those who wonder how Priest can both be a monist and embrace two types of validity (LP validity and quasi-validity), he does have an answer. Take modus ponens as an example. He argues that while modus ponens is not valid in all situations, it is a contingent property of certain domains (namely consistent ones); and so a dialetheist can invoke it while reasoning with consistent propositions without changing logical allegiance (ibid, p.198).

As discussed earlier, if one accepts Bueno and Colyvan's model to justify our change in logic, then it entails that one must also accept logical pluralism. Does Priest's logical monism pose a problem to a dialetheist who wishes to use Bueno and Colyvan's model as a basis to debate the LNC and revise logic? I think this is not a big problem: Priest's monism is independent of dialetheism, and subscribing to dialetheism does not compel one to subscribe to monism. For example, Beall (2004, 2006) is himself a dialetheist with logical pluralistic views.

8) Final verdict

To come up with a fair assessment of which side is more forceful, I think it will serve us well to have some independent criteria to guide us. To this end, I have set two criteria. The first is the persuasiveness of the constructive argument that establishes the position. For the LNC proponent, this would be Aristotle's argument. For the LNC opponent/dialetheist, this would be the observations of paradoxes. The second criterion is how well it can fend off criticisms from the other side or whether it has forceful criticism against the other side. For the LNC proponent, this would be the argument that logic should be apriori, the challenge of whether it is possible to debate about the LNC without circularity, Parson's inexpressibility argument, and the argument that violating the LNC is irrational. For the LNC opponent/dialetheist, it is the charge that all other solutions either fail or are partial and ad hoc.

Let's start by evaluating both sides' arguments in terms of building their cases. Let's start with those who are for the LNC. As remarked earlier, Aristotle has two arguments for the LNC. The first one concerns the impossibility of believing $P \wedge \neg P$ and the second argues that deduction is impossible without the LNC. The former is hopelessly circular, while the latter is clearly not true in the face of paraconsistent logics. It is perhaps a little unfair to look at Aristotle's argument in retrospect with more philosophical and logical developments at our disposal than Aristotle had at the time. But even in present days the LNC is still upheld without question, or even dogmatically so in the case of Lewis. Instead of giving a philosophical argument to support the LNC, LNC proponents often give critical arguments against dialetheists. Even if the arguments are forceful (as I will discuss below, I do not think they are), that does not mean that the LNC should thus be accepted because we still need a constructive argument to establish the position. And even if the alternatives, be it dialetheism or gap theory, are bad, that does not mean we have no choice but to accept the LNC. This is only a viable option if doing so is better than accepting any of the alternatives - because if accepting the LNC is just as bad as accepting any of the rest, why would you do so? Therefore it would seem that the underlying assumption of accepting the LNC is that doing so is marginally better than accepting any of the alternatives - which of course in the context of this debate is circular.

What seems to spook the LNC proponents is how counterintuitive it is to admit that there may be sentences that are both true and false. Intuitive as the LNC may be, considering we almost never encounter dialetheia in daily life, it at most gives us prima facie justification.[30] Yet it is as if the modern day LNC proponents feel they do not need to provide any justification because they have already fulfilled the burden of proof - by conflating this prima facie justification with a constructive argument.

On the other hand, the dialetheists' constructive argument is strong. The existence of paradoxes is an undeniable fact - even non-dialetheists would be hard pressed to deny so. Of course, the existence of paradoxes does not necessitate the rejection of the LNC - there are a variety of proposed solutions. Therefore the dialetheists' task now boils down to arguing why, of all the solutions, rejecting the LNC is the best one. Tarski's attempt is precisely an attempt to deal with paradoxes while remaining faithful to the LNC. However, I think Priest made a forceful point when he observes that Tarski's solution is ad hoc - running away from an inference simply because the conclusion is counter-intuitive is not a good reason. This is akin to cherry picking conclusions that are consistent with our beliefs. If we are to be consistent with a logical methodology, we must be able to tell exactly where the inference logically went wrong.

One may criticise that Priest's solution is actually no less ad hoc than Tarski's. I do not find this criticism convincing. Tarski's solution is ad hoc because it is there only to maintain consistency in the face of paradoxes like the Liar. On the contrary, Priest opts to abandon the LNC not just because this move negates the problem posed by paradoxes, but also because the LNC lacks a convincing constructive argument and it is not justified in the first place - this latter reason serves as an independent justification and therefore Priest's solution is not ad hoc like Tarksi's is.

Priest's strengthened Liar also appears forceful against the gap theorists. None of the gap theorists' three replies is convincing as Burge analysed. It is unclear to me how gap theorists could tackle the criticisms Burge made. For the first criticism, unless the gap theorists have anything other than classical negation in mind, I see no way out. And for the second and third, having other paradoxes that require ad hoc treatment on top of recognising truth value gap seem to suggest that truth value gap theory is on par with Tarski's approach at best.

Next let's discuss how well each side defends their own position/ attacks the other side. The LNC proponents have put forward four arguments: logic should be apriori, trying to discuss the LNC without begging the question is impossible, doing away with the LNC renders one irrational, and dialetheists are incapable of expressing disagreement. I think the first three are perhaps more promising than the inexpressibility argument.

I have to concede that I do not see the significance of the inexpressibility argument, because if being a dialetheist means one is unable to express disagreement in a manner that others understand, then no one would have been able to understand Priest ever since he published his first dialetheic paper and disagreed with the majority of the philosophical community on the LNC. The argument may make somewhat more sense if what Parsons meant was that a dialetheist cannot express genuine disagreement under any circumstance, be it when paradox is suspected to be at play or not, because then we wouldn't have any way of telling whether the dialetheist is being dialetheic about a given subject of discussion. But as mentioned, dialetheists only believe that a sentence may be both true and false under certain

[30] Some argue that intuition does not even provide prima facie justification, see Pust 2019.

circumstances, such as when self-referring paradoxes, vague predicates or legal conditions are involved. When we are under normal circumstances that do not involve those situations, there is simply no reason to suspect that a dialetheist may disagree with you while secretly agreeing with you as well in his mind. I think Priest's response, particularly his use of pragmatics, holds up well. The case is a classic example of implicature; there is nothing to suggest that there is any difference between this case and other usages of implicature, other than the context being one of the debate of dialetheism. But implicature is logic neutral - i.e. it does not presume a particular logic, so it seems that there is nothing to suggest how the context may affect implicature differently, or render implicature inapplicable in this case. Furthermore, as far as I am aware, there is no rebuttal offered against it. Even Shapiro (2004, p.339), who criticised Priest's other responses to Parsons, can offer no criticism against his pragmatics argument.

The argument that logic should be apriori is at least, at first glance, somewhat convincing. If we look at what a logical consequence is: it guarantees that with a set of true premises, in virtue of the meaning of the logical constants involved, the conclusion will be true; then what matters is only the logical constants involved, regardless of the empirical matter involved in the extra-logical constants. But logical consequence is a concept that deals with truth, and truth is as much of an apriori issue as it is an empirical issue. It is apriori when it deals with mathematics[31]; but it is also empirical when it deals with facts in the empirical world, such as the aforementioned case of quantum mechanics in section 2 and semantic closure in natural language - it is just a matter of empirical fact that 'It is true that the Earth is round' is grammatical and well formed in English. And as discussed, logic - at least as far as classical logic is concerned - fails to account for either of those empirical issues. How can we continue to use a piece of tool that purports to guarantee truth, when it clearly fails in certain aspects of the very task it is supposed to do? In view of this overwhelming piece of counterevidence I don't find logical apriorism convincing.

The criticism that we will inevitably be begging the question while discussing the LNC is a genuine threat to the dialetheic project. If we take the goal of a philosophical debate to be producing a convincing argument to support one side instead of the other, we must have some sort of principle to judge the merit of that argument. Reductio ad absurdum is often one such principle which relies on the LNC. Take the LNC away, and we take away reductio as a tool to help us evaluate our arguments. Of course, this does not mean it's impossible to do so, so long as we can find some other principles to do the job. The question is, what other option is there? As Aristotle (1993, 1006a6 - 12) observes, if only there is any doubt about the principle we put forward (to help us evaluate our argument), one can always question that principle and the ones put forward to justify the said principle, then we end up in an infinite regress. Thus we need something that is beyond doubt and is not at all contentious. Two equally fundamental laws of logic come to mind: the law of identity and the LEM. While the law of identity is perhaps the most uncontested law of all, it also does not get us very far in our inference. As for the LEM, it is even more contentious than the LNC. One needs look no further than the arguments put forth by intuitionists.

Fortunately, Bueno and Colyvan's model offers us a way to escape this hopeless infinite regress by providing us a justification outside the loops of a principle justifying another principle/argument. The model also fits in well with how logic change happened before (consider the example of logic change from classical to modal logic in section 3); along with the aforementioned merit of escaping the infinite regress loop, I think their model is a forceful reply to the circularity criticism.

[31] Of course, some may regard geometry as empirical.

The criticism that rejecting the LNC is irrational is perhaps the most damning of them all. Indeed, I suspect the counter-intuitiveness of dialetheism is due in no small part to this very argument. Again, here I think it will be useful to remind ourselves that a dialetheist does not believe that every sentence is true, and with the help of LP, it is also possible to reason without plunging into trivialism despite our tolerance of sentences that are both true and false. I also believe that Priest's rebuttal is a valid one - we are not rejecting the LNC without any justification: the existence of paradoxes is an undeniable fact that we do have sentences that are both true and false. Furthermore, as Priest has shown, alternative options such as value gap theory suffers from other variants of the Liar despite being able to hold its own against the original Liar. Therefore it does seem that our only option is not to fight the reality that the LNC is, at least as far as paradoxes are concerned, false, but to embrace it. Of course, Priest's argument hinges on the Rationality Principle; so for someone who argues that having evidence is not enough to accept dialetheism (or equivalently, having counterevidence is not enough to reject the LNC), the force of Priest's argument would be severely weakened. But I am at a complete loss as to what other conditions may be needed.

Perhaps the one flaw I find in Priest's argument concerns his defence against classical negation. I do agree with Priest that it is circular to assume a classical negation, and then argue because of the exclusivity of truth and falsity entailed, accepting dialetheism/rejecting the LNC is irrational. But to go from that to arguing, because negation in LP behaves in such a way that truth and falsity are not exclusive, it is in fact rational to believe true contradiction - I think Priest is guilty of precisely what he accuses the classical logicians of - circularity. LP is a dialetheic logic, so it is no surprise that as a manifestation of the philosophy, it will be accommodating to the idea of non-exclusivity between truth and falsity. In other words, if one uses LP, one must have already subscribed to dialetheism and rejected the LNC, so naturally one would deem believing true contradiction rational. At most, I think Priest is justified in arguing that since classical and dialetheic negations are competing notions, whether consistency is required for rational thinking is an open question until it is settled which negation is the correct account - which in turn, is settled by the dialetheists vs LNC defendants debate.

But all in all, after having surveyed the arguments presented in this paper, I am inclined to take Priest's dialetheic stance on the debate of the LNC, i.e. there are indeed sentences, such as the Liar, for which the LNC is false. The LNC proponents cannot offer any convincing constructive arguments; and while there are a few minor flaws in the rebuttals dialetheists offered against the LNC proponents, in general I find that the dialetheists' defence hold up.

Needless to say, it is impossible to review all the arguments offered by both sides. But I hope that, even if my discussion has not persuaded the reader, I have shown that at least on the surface, the dialetheist's position is not as silly as one might expect and is worthy of serious consideration.

Reference

Aristotle. (1993). *Metaphysics Books Γ, Δ, and E*, Christopher Kirwan (ed.), 2nd edition, Oxford University Press.

Avicenna. (2005). *The Metaphysics of The Healing*. Michael E. Marmura (ed.), 1st Edition, Brigham Young University.

Beall, J. (2004). "True and False - As If". In Priest, G., Beall, Jc., & Armour-Garb, Bradley P. (eds.). *The Law of Non-Contradiction : New Philosophical Essays*. Oxford University Press.

Beall, J. and G. Restall. (2006). *Logical Pluralism*, Oxford University Press.

Beall, Jc, Michael Glanzberg, and David Ripley, "Liar Paradox", *The Stanford Encyclopedia of Philosophy* (Fall 2020 Edition), Edward N. Zalta (ed.), https://plato.stanford.edu/archives/fall2020/entries/liar-paradox/.

Bueno, O., & Colyvan, M. (2004). "Logical Non-Apriorism and the 'Law' of Non-Contradiction". In Priest, G., Beall, Jc., & Armour-Garb, Bradley P. (eds.). *The Law of Non-Contradiction : New Philosophical Essays*. Oxford University Press.

Burge, Tyler (1979). "Semantical paradox". *Journal of Philosophy*, 76 (4):169-198.

Dowden, B. "Liar Paradox", *The Internet Encyclopedia of Philosophy* (2021 Edition), https://iep.utm.edu/par-liar/.

Field, H. (1996). "The a Prioricity of Logic". *Proceedings of the Aristotelian Society*, *96*, *new series*, 359-379. Retrieved November 30, 2020

Fitch, F. (1964). "Universal Metalanguages for Philosophy". *The Review of Metaphysics*, 17(3), 396-402.

Laudan, L. (1984), *Science and Values: The Aims of Science and Their Role in Scientific Debate* University of California Press, Berkeley.

Lewis, D. (1982), "Logic for Equivocators", *Noûs*, 16: 431 - 41. Repr. in *D. Lewis, Papers in Philosophical Logic* (Cambridge University Press), 1998: 97 - 110.

Parsons, T., (1990), "True Contradictions", *Canadian Journal of Philosophy*, 20: 335 - 53.

Priest, G. (1979). "The logic of paradox". *Journal of Philosophical Logic* 8, 219 - 241.

Priest, G. (1984). "Logic of paradox revisited". *Journal of Philosophical Logic* 13 (2):153 - 179.

Priest, G. (1994). "The Structure of the Paradoxes of Self-Reference". *Mind*, 103(409), new series, 25-34.

Priest, G. (1995). "Gaps and Gluts: Reply to Parsons". *Canadian Journal of Philosophy*, 25(1), 57-66.

Priest, G. (2006a). *In Contradiction: A Study of the Transconsistent*. Oxford University Press.

Priest, G., (2006b). *Doubt Truth to Be a Liar*, Oxford University Press.

Priest, G. (2009). "Dualising intuitionistic negation". *Principia,* Vol. 13 No. 2.

Priest, Graham, Francesco Berto, and Zach Weber, "Dialetheism", *The Stanford Encyclopedia of Philosophy* (Fall 2018 Edition), Edward N. Zalta (ed.), https://plato.stanford.edu/archives/fall2018/entries/dialetheism/.

Pust, Joel, "Intuition", *The Stanford Encyclopedia of Philosophy* (Summer 2019 Edition), Edward N. Zalta (ed.), https://plato.stanford.edu/archives/sum2019/entries/intuition/.

Putnam, H. (1979), "The Logic of Quantum Mechanics", in *Mathematics, Matter and Method: Philosophical Papers*, 2nd edn. (Cambridge University Press) i: 174–97.

Russell, Gillian, "Logical Pluralism", *The Stanford Encyclopedia of Philosophy* (Summer 2021 Edition), Edward N. Zalta (ed.), https://plato.stanford.edu/archives/sum2021/entries/logical-pluralism/.

Shapiro, S. (2004). "Simple Truth, Contradiction, Consistency". In Priest, G., Beall, Jc., & Armour-Garb, Bradley P. (eds.). *The Law of Non-Contradiction : New Philosophical Essays*. Oxford University Press.

Tarski, A. (1936a). "The concept of truth in formalized languages". In A. Tarski (ed.), *Logic, Semantics, Metamathematics.: Papers from 1923 to 1938*, 2nd edn. (Hackett, Indianapolis, 1983), pp. 152 - 278.

Tarski, A. (1936b). "On the Concept of Logical Consequence". In A. Tarski (ed.), *Logic, Semantics, Metamathematics.: Papers from 1923 to 1938*, 2nd edn. (Hackett, Indianapolis, 1983), pp. 409 - 20.

Tarski, A. (1944). "The Semantic Conception of Truth: And the Foundations of Semantics". *Philosophy and Phenomenological Research*, *4(3)*, 341-376. doi:10.2307/2102968

Appendix - overview of Logic of Paradox (LP)*

1 Semantics

As LP is a dialetheic logic, it is no surprise that it admits true contradictions. If a sentence is both true and false, it is assigned the value 'paradoxical' (P). The LP semantics dictates that a sentence is true (T) iff its negation is false (F); therefore if a sentence A is both T and F, $\neg A$ would be both F and T - i.e. P.

This extends to other operators. The values for $\wedge$ when the sub-formulas are T/F are self-explanatory. As for cases involving paradoxical value, consider the case where A is P and B is T. On the one hand A is T and B is T, therefore $A \wedge B$ is T; on the other hand A is also F (because Q is paradoxical) while B is T, therefore $A \wedge B$ is F. Consequently $A \wedge B$ is P.The cases for other operators are very similar, their truth tables are shown below.

$\neg$	
T	F
F	T
P	P

$\wedge$	T	P	F
T	T	P	F
P	P	P	F
F	F	F	F

$\vee$	T	P	F
T	T	T	T
P	T	P	P
F	T	P	F

$\rightarrow$	T	P	F
T	T	P	F
P	T	P	P
F	T	T	T

2 Logical consequence

Let L be a propositional language whose set of propositional variables is S, and Σ be a set of sentences of L. Let $v : S \rightarrow \{\text{T,P,F}\}$, v^+ be the extension of v to all the sentences of L:

$\Sigma \models A$ iff there is no v such that $v^+(A) =$ F but for all $B \in \Sigma$, $v^+(B) =$ T or P.

$\models A$ iff $\varnothing \models A$ (i.e. for all v,$v^+(A) =$ T or P).

*Unless indicated otherwise, all the materials from this appendix come from Priest's 1979 paper *The logic of paradox*. Interested readers should refer to that paper for details.

As a result of the semantics and logical consequence definition, most of the valid deductions from classical logics are carried over to LP despite the differences, some examples are as follows:

$$A \models A \vee B \qquad \neg\neg A \models A \qquad A \models \neg\neg A$$

$$\neg A \wedge \neg B \models \neg(A \vee B) \qquad \neg(A \vee B) \models \neg A \qquad \neg A \rightarrow \neg B \models B \rightarrow A$$

However the followings do not hold:

$$A \wedge \neg A \not\models B \qquad A \rightarrow B, \neg B \not\models \neg A \qquad A, A \rightarrow B \not\models B$$

They are principle of explosion, Modus tollens, and Modus ponens respectively. LP is designed to be a contradiction tolerating deduction system, therefore the fact that it rejects explosion is not surprising. But it is clearly a problem that the latter two are not truth preserving in LP. To avoid this problem, Priest defines what's called 'quasi-validity'.

Priest defines quasi validity as follows: an inference is quasi-valid if it is valid provided all the truth values only involve T/F, even if it is invalid in LP in general. In other words, we can continue to use rules like Modus tollens and Modus ponens so long as we are not working with paradoxical sentences like the Liar.

Priest argues that paradoxes only occur when self-references take place - specifically, under what Priest calls Qualified Russell's Schema. Therefore, in view of how specific and rare these cases are, we are justified to use quasi-valid inferences unless we suspect self reference is involved. Interested readers should refer to Priest's 1994 paper *The Structure of the Paradoxes of Self-Reference* for more details on the Qualified Russell's Schema.