UNIVERSITY OF GOTHENBURG
SCHOOL OF BUSINESS, ECONOMICS AND LAW

Bachelor’s Thesis within Bachelors Programme in Financial Economics

Stylized Facts in Financial Markets: A Comparative Study of Gold and Equities

Adam Fransson
Adrian Söör Lafrenz
Supervisor: Gert Sandahl

Spring term 2024
15 ECTS

Abstract

This thesis investigates the applicability of eight stylized facts in financial markets by comparing the behavior of gold and the equities. Using daily, weekly, monthly and annual log-returns data from 2000 to 2023, we apply various statistical tests to analyze properties such as non-stationarity, stationarity, asymmetry, heavy tails, aggregational Gaussianity, lack of autocorrelation, volatility clustering, and the leverage effect. The findings reveal that while most of the stylized facts are applicable on both asset classes, distinct differences exist. For instance, significant autocorrelation in daily returns and the leverage effect are observed in the equities but not in gold, highlighting differences in market dynamics and risk profiles. These results suggest that financial models must be tailored to account for asset-specific characteristics, particularly when managing risks associated with extreme market events. Future research could extend this comparative analysis to other asset classes and explore high-frequency data to capture more granular market behaviors.

Key words: Financial Econometrics, Stylized Facts, Time Series, Statistical Tests, Non-Stationarity, Stationarity, Asymmetry, Heavy Tails, Aggregational Gaussianity, Autocorrelation, Volatility Clustering, Leverage Effect, Efficient Market Hypothesis, Random Walk Theory

JEL Classification: C58, G12, G14, C22, C13

Acknowledgements

We would like to extend our gratitude to our supervisor, Gert Sandahl for his invaluable guidance and support throughout the writing of this thesis. His insightful advice and assistance helped us navigate and overcome numerous challenges, ensuring we stayed on the right path. We also wish to thank our opponents and fellow students for their constructive feedback and suggestions, which added to the quality of our work.

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1. Introduction

1.1. Background

Financial markets, the complex networks that facilitate the exchange of financial securities, are essential for the global economy. They represent the collective behavior of numerous market participants, ranging from individual investors to large institutional entities. The dynamics of these markets reflect the underlying economic conditions, market sentiment, and geopolitical tensions. Within these dynamics, certain regularities are consistently observed across different markets and time periods. In econometrics, these are called stylized facts. Sewell (2011) posits that stylized facts refer to a set of empirical observations about financial return series that hold true across a variety of instruments, markets, and time periods.

Stylized facts in financial markets are important for understanding market dynamics and developing robust financial models. Cont (2001) recognized that these facts serve as benchmarks for financial modeling and risk management. Without a proper understanding of the underlying characteristics behind these stylized facts, models may fail to capture the reality of market behavior.

Among these regularities, the price movements of gold and equity markets stand out for their influential roles. Baur et al. (2010) states that gold has traditionally been perceived as a safe haven for investors due to its low volatility, which often makes it show resilience or even growth in periods of economic uncertainty. At the same time equity markets are more closely tied to the economic cycles, advancing in periods of growth and retracting during economic downturns. This sets the stage for an examination of how the stylized facts manifest in these two different asset classes, each playing a unique role within the financial ecosystem.

1.2. Problem description

Empirical research, such as that conducted by Cont (2001), has highlighted the importance of stylized facts across diverse financial instruments, pointing out their role in enhancing risk management and economic forecasting strategies. Substantial research on stylized facts exists within homogeneous asset classes, for example the investigations by Silvennoinen et al. (2013) on metals and Bollen et al. (2004) on stock indices. However, a comparative analysis across different asset classes remains absent. Baur et al. (2010) states that both gold and equities play important but distinct roles in financial portfolios and react differently to economic events, suggesting that stylized facts might manifest uniquely in each asset class.

Zivot (2021) summarizes the following eight observations in his book: 1. Prices appear to random walk non-stationary; 2. Returns appear to be second order stationary; 3. Returns show negative skewness; 4. Returns show excess kurtosis; 5. Returns exhibit a normal distribution as the data becomes more aggregated; 6. Returns are approximately uncorrelated over time; 7. The volatility of returns appears to be autocorrelated and clustered. 8. There seems to be an increase in volatility during financial crises, a phenomenon that, while not directly attributed to, resonates with Black's (1976) theory of the leverage effect, where financial leverage contributes to heightened volatility following negative price movements.

The absence of studies exploring the characterization of these eight stylized facts between contrasting asset classes leaves unanswered questions about the applicability and reliability of financial theories across broader market scenarios. This oversight restricts the ability to fully explore stylized facts for more comprehensive financial modeling and strategic risk management in diverse economic conditions.

1.3. Purpose

This study aims to examine the applicability of the eight stylized facts in financial markets by comparing gold and the equity market. By examining how these stylized facts manifest in both asset classes, we seek to provide an understanding of their dynamics and implications. By comparing gold, considered one of the safest asset classes, with equities, which are on the opposite side of the investment risk spectrum, the goal is to underscore the broad range of investment behaviors and market reactions.

1.4. Research question

- How does the applicability of the eight stylized facts differ between the gold and equity market?

1.5. Structure of The Thesis

In chapter 2 (Theory) we review relevant theories and previous literature on stylized fact in financial markets. In chapter 3 (Methodology), we outline the methodological approach, including data collection statistical tests, and analysis procedures used to investigate the stylized facts. In chapter 5 (Empirical Result), we present the findings from the statistical analysis, comparing the behavior of gold equities. In chapter 6 (Discussion), the results are discussed in the context of existing theories and previous research, highlighting the implications for financial modeling and risk management. Finally in chapter 7 (Conclusion), we summarize the key findings, address the research question, and suggest directions for future research.

2. Theory

2.1. Theoretical Framework

Stylized fact 1 - Non-Stationarity of Prices

Osborne (1959) posited in The *Random Walk Theory* that stock prices follow a path where each step is random and independent of previous steps, suggesting that prices are inherently unpredictable. This concept is mathematically represented by the equation $X_t = X_{t-1} + Z_t$, where X_t and X_{t-1} are the price at time t and t-1 respectively and Z_t is a white noise error term. The theory highlights the non-stationary behavior of stock prices, as the statistical properties of the series change with each new price realization.

Fama (1970), in his formulation of the Efficient Market Hypothesis (EMH) and particularly its strong form, suggests that all public and private information is fully reflected in stock prices at any given time. According to EMH, changes in stock prices arise only due to new information entering the market, which is inherently random and unpredictable. This theory supports the non-stationarity of prices, implying that price levels can shift significantly upon the arrival of new information, affecting the overall statistical characteristics of the series.

Stylized Fact 2 - Stationarity Of Returns

The *Efficient Market Hypothesis* (EMH) (Fama, 1970) plays an important role in explaining why financial returns tend to exhibit stationarity. Since this new information is random and unpredictable, the resulting price changes (returns) are independent and identically distributed, thus supporting the notion of stationarity in returns.

The *Random Walk Theory* (Fama, 1965) suggests that asset prices evolve stochastically, where price changes are independent of past prices. This independence implies that returns, calculated as the first differences in prices, are stationary because their characteristics depend solely on new information rather than historical data.

Additionally, the *Central Limit Theorem* (Billingsley, 1995; Feller, 1968) provides a theoretical basis for expecting returns to exhibit stationarity over larger time scales. It states that the distribution of aggregated observations tends to approximate a normal distribution, provided the original observations have finite variance.

<u>Stylized Fact 3 - Asymmetry in Returns</u>

Prospect Theory, introduced by Kahneman and Tversky (1979), brings up the psychological aspects of financial decision-making, revealing that investors perceive losses more intensely than equivalent gains. This variation in perception helps explain why negative market news often has a disproportionate impact on market prices, leading to asymmetric responses in financial returns.

Similarly, the *Leverage Effect* theory (Black, 1976) complements this understanding by linking financial leverage to market reactions. This theory suggests that an increase in a firm's leverage, typically following a drop in asset prices, also increases the equity's volatility. This dynamic is particularly evident during market downturns when negative price movements increase volatility more than positive movements would decrease it, thereby contributing to the negative skewness observed in return distributions.

<u>Stylized fact 4 - Heavy Tails in Financial Returns</u>

The exploration of heavy tails in financial returns begins with Mandelbrot's *Stable Paretian Hypothesis* (1963), which posits that financial returns might be more accurately described by stable distributions, known for their heavy tails, rather than normal distributions. These distributions accommodate events with extremely large deviations from the mean, reflecting the significant swings often observed in market prices.

Mandelbrot later expanded his theory (Mandelbrot et al, 1999), suggesting that financial markets exhibit complexities well captured by stable distributions, especially in terms of unpredictability and extremeness of market movements. Additionally, Multifractal Models are introduced to address the multifaceted scaling behaviors in financial markets, accounting for heavy tails and other complex phenomena by considering multiple scaling behaviors or "multifractality" in return distributions.

Stylized Fact 5 - Aggregational Gaussianity

Proposed by Feller (1966), the *Central Limit Theorem* posits that the average of many independent, identically distributed variables with finite means and variances will approximate a normal distribution, regardless of the original distribution. This theorem establishes the expectation of Gaussianity in aggregated financial data, illustrating how random fluctuations in returns tend to cancel out over larger samples, leading to a more predictable normal distribution.

The *Law of Large Numbers* supports this by demonstrating that as more observations are collected, the average of these observations tends to converge towards the expected value, resulting in more stable and predictable patterns that closely resemble a Gaussian distribution (Chow & Teicher, 1997).

Stylized Fact 6 - Lack of Autocorrelation in Returns

Under the *Efficient Market Hypothesis* (Fama, 1970), returns should be uncorrelated because any patterns in past prices would already have been acted upon, leaving no advantage to be exploited.

Complementing the EMH, the *Random Walk Theory* (Malkiel, 2011) suggests each price change is a result of fresh, random and unpredictable information, thereby making past returns poor predictors of future returns.

Stylized Fact 7 - Volatility Clustering

Volatility clustering suggests that large changes in financial markets tend to be followed by large changes, and small changes by small ones. This pattern, well-documented by Mandelbrot (1963) and formalized in Engle's (1982) *ARCH model*, reflects underlying investor behaviors and market reactions.

Shiller's work (2003) discusses how emotional responses to market events can create prolonged periods of volatility, challenging the traditional efficient market hypothesis and highlighting the need for models that accommodate such behaviors.

Stylized fact 8 - Leverage Effect

The *Financial Leverage Hypothesis* (Black, 1976) suggests that as a company's stock price falls, its leverage (debt/equity ratio) increases because the value of equity decreases while the debt remains constant. Higher leverage implies greater risk and consequently increased volatility. This relationship explains why volatility spikes as asset prices plummet.

Volatility Feedback Theory (Campbell & Hentschel, 1992) posits that higher perceived risk (volatility) necessitates a higher risk premium, lowering the current asset price. This price reduction, in turn, escalates volatility, creating a feedback loop where changes in volatility directly influence future returns.

2.2. Previous Literature

Cont's (2001) paper has become a cornerstone in financial econometrics, analyzing the traits of asset returns. The paper organizes empirical findings into a set of stylized facts that remain relevant today. Traits such as fat tails and volatility clustering, identified by Cont, serve as essential benchmarks for this thesis. These benchmarks help assess and compare the behavior of gold and equity markets, aiming to validate these phenomena under different market conditions.

Trejo-Pech et. al.'s study on the Mexican Stock Exchange Index (BMV) provides a benchmark for comparative market studies. This research compares the BMV to indices from developed markets, highlighting subtle deviations in market behavior due to regional economic factors, investor profiles, and regulatory environments. The most important results from their study are the following: Skewness is generally negative, indicating return asymmetry, although this is not the case for the major Mexican and Brazilian exchange indices; all markets exhibit an excess kurtosis[1], suggesting heavy tails; no significant linear autocorrelations after a few lags[2], indicating no autocorrelation in returns; evidence of non-linear autocorrelation, demonstrating volatility clustering.

In challenging the random walk hypothesis, Lo and MacKinlay's (1988) research introduces the possibility of predictability in stock price movements. By identifying autocorrelations in stock returns, they propose that markets may not be entirely efficient. Their methodology and conclusions present a critical contrast to this thesis's examination of the market, questioning

[1] A statistical measure that describes the shape a distribution's tails in relation to its overall shape.
[2] Lag is the number of time periods before the current time t.

whether such patterns of predictability exist beyond equities and how these might influence investment strategies.

Bouchaud et al. (2001) provides a detailed analysis of the leverage effect, showing a negative correlation between future volatility and past returns. In individual stocks, this correlation is moderate and decays over a few months. However, it is much stronger and decays faster in stock indices. This distinction arises from a delayed effect where the reference price for price updates is considered a moving average over recent months, rather than the immediate price. Their model, supported by data from U.S., European, and Japanese stocks, bridges short-term additive and long-term multiplicative stochastic processes. In stock indices, the rapid decay of this correlation suggests a market panic phenomenon, differing from individual stock behavior.

Chevallier & Ielpo (2017) conducted a comprehensive study on the leverage effect within commodity markets using a recursive estimation approach. Their research provided significant insights into the dynamic interactions between past returns and future volatility across various commodities. Unlike the consistent negative correlation seen in equity markets, the leverage effect in commodity markets showed varying degrees and patterns. This variation indicates an interplay between market-specific factors and the broader economic conditions influencing these assets. Their analysis confirmed that the leverage effect exists in commodity markets, even though these markets do not involve companies with equity structures.

Barndorff-Nielsen and Shephard's (2002) research on high-frequency financial data is notable for its approach to analyzing price movements. By examining the microstructure noise and price jumps, they developed advanced tools for examining market behavior. Their insights into price fluctuations are particularly relevant for this thesis, informing the analysis of the gold market's response to macroeconomic news and micro-level trading activities.

2.3. Conclusion Of Theoretical Insights

The theories and previous literature underpinning the eight stylized facts highlight the complexity of financial markets. The Random Walk Theory and Efficient Market Hypothesis emphasize the non-stationarity of prices and the stationarity of returns, driven by new information. Prospect Theory and the Leverage Effect offer insights into the asymmetric responses to market movements and the increased volatility during downturns. The empirical studies by Cont (2001) and Trejo-Pech et al. (2013), confirm the presence of heavy tails, volatility clustering, and the lack of autocorrelation in returns.

4. Methodology

4.1. Method Approach

This chapter outlines the methodological approach to investigate the eight stylized facts of financial returns for gold and equities. The equities' data will be represented by the S&P 500, chosen for its wide coverage as well as the historic data available. We will begin by defining the key statistical properties and theorical foundations necessary for this analysis. We will also apply various statistical tests to thoroughly examine these properties across different time scales. By applying advanced data analysis tools, we aim to provide a comprehensive understanding of the dynamics behind the financial returns of gold and the S&P 500, ensuring a robust comparative analysis of these two distinct asset classes.

To investigate **stylized facts 1 and 2**, we will examine the statistical properties of non-stationarity of prices and stationarity of returns. This will be done for a daily, monthly, and annual frequency. Hamilton (1994) stated that a time-series process X_t, ∀ t ∈ Z (for all t which belongs to the integers Z) is weakly or second order stationary if:

Definition 1

1. $E[X_t] = \mu$ is finite and independent of t ∀ $t \in Z$,
   With $E[X_t]$ being the expected value of the time series process at time t, μ being the sample mean.

2. $V[X_t] = \sigma^2$ is finite and independent of t, ∀ $t \in Z$,
   With $V[X_t]$ being the variance of the time series process, σ^2 being sample variance.

3. $Cov(X_t, X_{t-k}) = \gamma_k$ is finite and independent of t ∀ $t \in Z$,
   $Cov(X_t, X_{t-k})$ being the covariance of the time series process at time t and t-k, γ_k being autocorrelation at lag k. Lag k is the number of time periods before the current time t.

For stylized Fact 1, we will compute and display the adjusted closing prices using the formula $P_t = P_{t-1} + \epsilon_t$, where P_t & P_{t-1} are prices at period t and t-1, and ϵ is white noise (Osborne, 1959). In line with Campbell, Lo, & MacKinlay (1997), a transformation to logarithmic prices will be made to stabilize variance and analyze the properties more precisely: $p_t = \ln(P_t) - \ln(P_{t-1})$. To analyze stylized fact 2, we will use logarithmic returns $r_t = \ln\left(\frac{p_t}{p_{t-1}}\right)$. Logarithmic returns are preferred because they are additive over time and better fit the statistical assumptions of many financial models. This includes normality, as logarithmic returns are $\in (-\infty, \infty)$, whereas simple returns are $\in (-1, \infty)$, thus not satisfying the sample space of a normal distribution (Hamilton, 1994).

To test for autocorrelation, the third requirement of stationarity, we will compute the autocorrelation function as outlined by Chaudhuri and De (2021): $\rho(k) = \frac{E(X_t-\mu)(X_{t-k}-\mu)}{\sigma^2}$, where $\rho(k)$ is the autocorrelation, X_t is the return at time t, μ is the mean of the returns, and σ^2 is the variance of the returns. This will help us determine if there is any significant autocorrelation in the returns.

Stylized fact 3 and 4 describe similar characteristic of financial returns and are best analyzed together. For these two stylized facts, we will compute the skewness to look for asymmetry and the kurtosis for heavy tails as described by Campbell, Lo, & MacKinlay (1997).
The skewness is calculated as:

$$Skewness = E[(\frac{X-\mu}{\sigma})^3]$$

A skewness of 0 means that the distribution is perfectly symmetrical, while a negative skewness value indicates a tendency towards more frequent and severe negative returns than positive ones.
The kurtosis is calculated as:

$$Kurtosis = E[(\frac{X-\mu}{\sigma})^4]$$

The kurtosis is a statistical measure that describes the shape a distribution's tails in relation to its overall shape. A kurtosis value greater than 3, which is the kurtosis of a normal distribution, indicates the presence of heavier tails.
In both equations, X is the return, μ is the mean, σ is the standard deviation, and E denotes the expected value.

We will present the rolling skewness and kurtosis over a trading year (252 days) to show how these measures evolve over time. Additionally, the returns will be plotted using QQ-plots and compared to a normal distribution to visually assess the distribution shape. This will be done for daily, monthly, and annual frequencies.

To support the graphical analysis, we will also provide numerical values of skewness and kurtosis in tables, along with the corresponding D'Agostino p-value in line with D'Agostino (1970). The D'Agostino test evaluates whether the sample data significantly deviates from a normal distribution based on skewness and kurtosis.

The test is calculated as:

$K^2 = Z_1^2 + Z_2^2,$

Where:

$$Z_1^2 = \left(\frac{\sqrt{\left(\frac{n(n+1)}{(n-1)(n-2)-n-3)}\right) \cdot g_1}}{\sqrt{6(n-2)(n+3)(n+5)}} \right),$$

$$Z_2^2 = \left(\frac{3(n-1)^2}{n+1)(n+3)} \cdot \left(g_2 - \frac{3(n-1)}{n+1} \right) \right).$$

In the equations, n is the sample size, g_1 is the sample skewness and g_2 is the sample kurtosis. The null hypothesis states that the data follows a normal distribution.

By analyzing the skewness and kurtosis along with QQ-plots and the D'Agostino test results, we can determine if the returns exhibit asymmetry and heavy tails, as proposed by the stylized facts.

Stylized fact 5 involves testing whether the distribution of returns becomes more normal (Gaussian) as the time scale increases. To investigate this, we will analyze the data at different frequencies and use several statistical tests to determine normality.

To formally test for normality at various frequencies, we will compute the following test statistics:

1. Jarque-Bera test (Jarque & Bera, 1980):

   $JB = \frac{n}{6}(S^2 + \frac{1}{4}(K-3)^2),$

   where n is the sample size, S and K are the measures of skewness and kurtosis of the data respectively. The Jarque Bera test assesses whether the sample data has the skewness and kurtosis matching a normal distribution.

2. Lilliefors test (Lilliefors, 1967):
   $T = \sup|\mathrm{F} * (\mathrm{x}) - \mathrm{S}(\mathrm{x})|$
   Here, sup denotes the supremum, $\mathrm{F} * (\mathrm{x})$ is the empirical cumulative distribution function (CDF) adjusted for the data, and S(x) is the CDF of the normal distribution. The Lilliefors test is used to compare the empirical CDF with the expected CDF of a normal distribution.
3. Kolmogorov-Smirnov test (Kolmogorov, 1933):
   $D_n = sup_x|F_n(x) - F(x)|$
   In this test, sup again denotes the supremum, $F_n(x)$ is the empirical CDF, and $F(x)$ is the CDF of the reference distribution. The Kolmogorov-Smirnov test evaluates the distance between the empirical and theoretical CDFs to assess normality.

We will present the results of these test in a table, where the p-values will be multiplied by 100 for clarity in the table format. When discussing the results in text, the p-values will be presented in their standard format. For example, a p-value of 10 in the table will be discussed as the standard p-value of 0.1.

A rejection of the null hypothesis in any of these tests would indicate that the underlying distribution significantly deviates from a normal distribution. Conducting multiple normality tests provide a comprehensive assessment, as each test uses slightly different methodologies, potentially offering various insights into the distribution characteristics.

Stylized fact 6 involves testing for the absence of autocorrelation in financial returns, which means that past returns do not predict future returns. This property aligns with the efficient market hypothesis, suggesting that returns are random and unpredictable.

To investigate this, we will compute the autocorrelation function (ACF) for different lags. The ACF is defined in line with Chaudhuri and De (2021):

$$\rho(k) = \frac{E(X_t-\mu)(X_{t-k}-\mu)}{\sigma^2}$$

Where $\rho(k)$ is the autocorrelation at lag k, X_t is the return at time t, μ is the mean of the returns, and σ^2 is the variance of the returns.

We will compare the computed autocorrelation values against the Bartlett's confidence intervals defined by (Bartlett, 1937), which are given by:

$1 + 2\sum_{j=1}^{k} \rho_j^2$

Where ρ_j^2 are the autocorrelation values at previous lags. If the autocorrelation values stay within these intervals, it indicates no significant autocorrelation at that specific lag.

Additionally, to quantify and test the significancy of autocorrelations, we will use two statistical tests:

1. Box-Pierce test (Box & Pierce, 1970):

   $$Q = n\sum_{k=1}^{m} \widehat{\rho_k^2}$$

   Where n is the sample size, m is the number of lags being tested, and $\widehat{\rho_k^2}$ is squared the sample autocorrelation at lag k. The Box-Pierce test assesses whether the cumulative autocorrelations are significantly different from zero.
2. Ljung-Box test (Ljung & Box, 1978):

   $$Q = n(n+2)\sum_{h=1}^{H} \frac{\widehat{\rho_h^2}}{n-h}$$

   where n is the sample size, H the number of lags being tested, $\widehat{\rho_k^2}$ is squared the sample autocorrelation at lag h, n-h an adjustment factor for each lag h, and +2 a correction factor to improve the tests approximation. The Ljung-Box test is a refinement of the Box-Pierce test that adjusts for sample size and the number of lags.

A rejection of the null hypothesis would mean a rejection of no autocorrelation. The results of these tests will be presented with the exact figures for both tests in the appendix for daily and monthly frequencies.

Stylized fact 7 involves testing for volatility clustering, which is the phenomenon where large changes in financial markets are often followed by large changes, and small changes are followed by small changes. This indicates that volatility tends to cluster over time.

To investigate this, we will analyze the autocorrelation of the squared returns. Volatility clustering can be detected by examining the autocorrelation function (ACF) of the squared returns. If the autocorrelation of the squared returns is significant and positive for several lags, this indicates volatility clustering.

We will compute the autocorrelation function of the squared returns defined by Hamilton and James (1994):

$$\rho(k) = \frac{E[(r_t^2 - \mu^2)(r_{t-k}^2 - \mu^2)]}{\sigma^4}$$

Where $\rho(k)$ is the autocorrelation at lag k, r_t^2 squared return at time t, μ^2 is the mean of the squared returns, and σ^4 is the variance of the squared returns.

We will plot the autocorrelation function of the squared returns and compare it to the confidence intervals to the determine if the autocorrelations are significant. Significant positive autocorrelations at multiple lags would indicate the presence of volatility clustering.

To visualize volatility clustering, we will also create a time series plot of the squared returns. This plot will help illustrate periods of high and low volatility, demonstrating the clustering effect.

For the autocorrelation function in **stylized fact 6 and 7**, we are going to use daily, weekly, and monthly frequencies instead of the otherwise used daily, monthly and annual. This to get a more robust result, since there are too few observations obtained to analyze the ACF through an annual frequency. At lag 0 the autocorrelation will always equal one since it measures the correlation of the time series with itself.

Stylized fact 8 involves testing for the leverage effect, which describes the phenomenon where negative returns increase future volatility more than positive returns of the same magnitude. This asymmetry is often observing in equity markets and is indicative of a negative correlation between returns and future volatility.

To investigate the leverage effect, we will compute the cross-correlation between past log returns and squared log returns in line with Black (1976):

$Corr(r_{t-j}, r_t^2)$ for $j > 0$

This represents the correlation between the return r_{t-j} at time t-j and the squared return r_t^2 at time t. To showcase the relation properly, this will be demonstrated graphically.

4.2. Data

Utilizing Python, we will compute and analyze each stylized fact through a series of statistical tests and models. The choice of Python is motivated by its extensive libraries and frameworks designed for data analysis, which are ideal for processing financial time series data and conducting complex numerical simulations. To begin with, all the needed data on equities and gold will be downloaded from Yahoo Finance to the Python Jupyter Notebook. This will be the adjusted closing prices of each asset class. The adjusted price is the closing price of the stock after adjustments have been made for any corporate actions that occurred before the next trading session. These adjustments account for events such as dividends, stock splits, rights offerings, and other distributions. The time span of the data begins at 2000-01-01 and ends on 2023-12-31, which adds up to 6037 trading days. This time span is the longest available for gold and therefore the most appropriate one that allows for a comparable analysis. To ensure that the data is not influenced by outliers, we remove any data point that exceeds the mean + 5 standard deviations. This threshold is chosen for several reasons: It ensures that the most extreme values won't disproportionally influence the analysis while maintaining the integrity of the data. A threshold of 5 standard deviations is stringent enough to exclude outliers but not so strict as to remove valid observations. By using this threshold, we can observe the eight stylized facts without distortion. The calculation gets rid of 20, 3, 1 and 1 data points for daily, weekly, monthly, and annual respectively. After this adjustment, the total observations for S&P 500 are 6017, 1249, 287 and 23 for daily, weekly, monthly, and annual respectively. For gold, the total amount of observations after the adjustment are 5842, 1216, 280 and 23 for daily, weekly, monthly, and annual respectively.

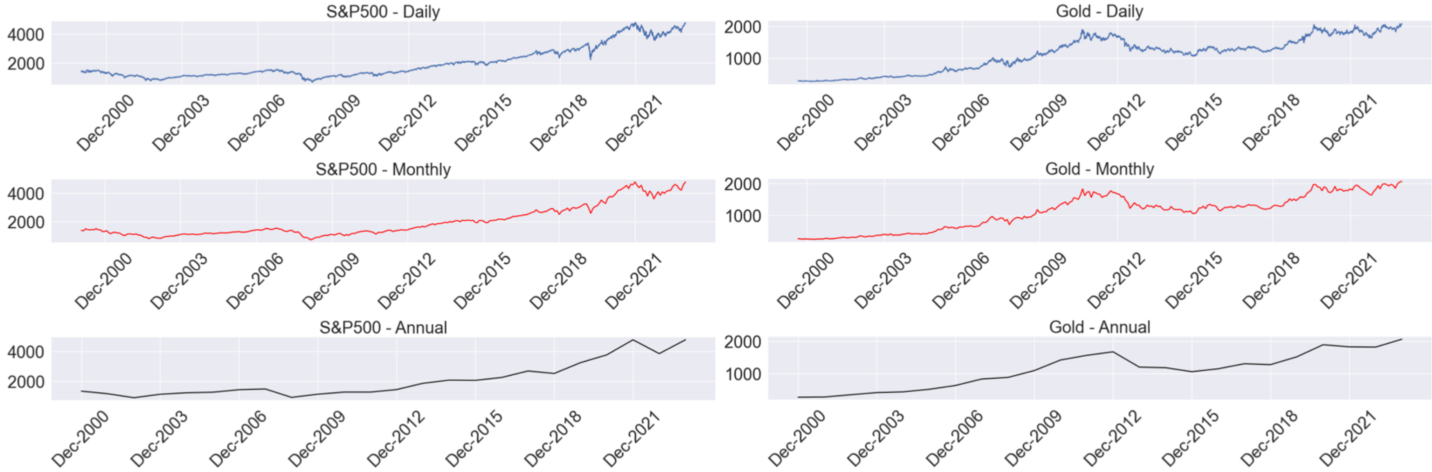
From the Python results, graphs and visualizations are generated directly, providing an understanding of the dynamics behind each market analyzed. This empirical approach allows for a thorough investigation into the properties of both markets, with an emphasis on the differences and similarities that arise when contrasting the two asset classes. To show the numbers behind the graphical results, tables referred to will be provided in the appendix.

5. Empirical Result

Stylized Fact 1: Prices Are Non-Stationary

In figure 1 we see a generally increasing trend in the closing prices for all frequencies across gold as well as the S&P 500. The gradual uptrend suggests that the market on average grows over time, reflecting economic growth and inflation. However, there are also periods of decline that reflect market corrections or economic downturns. Even though it smoothens out as we go to the less frequent monthly and annual data, the trend is still increasing.

Figure 1 – Adjusted Closing Prices For S&P 500 & Gold. Daily, Monthly & Annual (2000-2023)



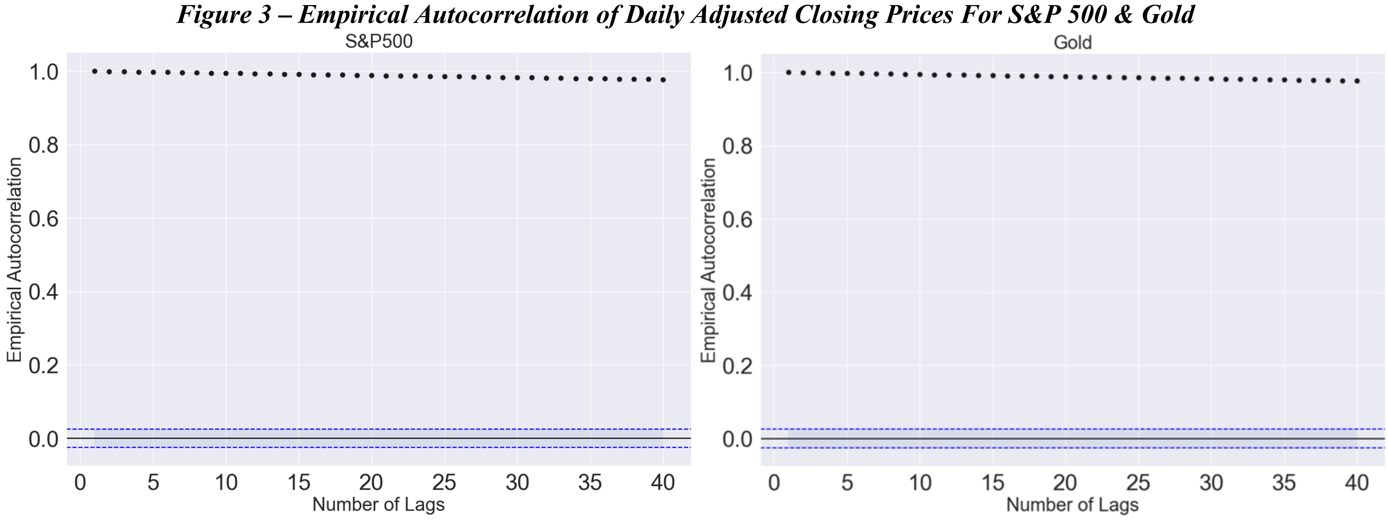
It can be seen from figure 1 that the prices over time resemble a random walk, thus having a time-dependent mean, variance, and covariance. This can be seen by looking at the short-term unpredictability in the time series movements and goes against the definition of weak stationarity. Since the time series resembles a random walk, we can say $E[P_t] = P_0 + t\mu$, $V(P_t) = t\sigma^2$ and $Cov(P_t, P_{t+K}) = t\sigma^2$. Therefore, we can assume that $\mu \rightarrow \infty$ and $\sigma^2 \rightarrow \infty$ as $t \rightarrow \infty$. In other words, as time goes to infinity, we can assume that the sample mean (μ) and variance (σ^2) also do, implying time dependent variables. The presence of the time-dependent parameters gives strong evidence of the non-stationary nature in prices for both gold and the S&P 500.

To confirm the other properties of non-stationarity, figures 2 below shows the logarithmic (log) transformation of closing prices for all frequencies for both gold and S&P 500. This transformation is made to smooth out the fluctuations but retains the overall upward trend for gold as well as the S&P 500. The general growth of the log adjusted closing prices suggest that the random walk is still applicable, further indicating non-stationarity.

Figure 2 – Log Adjusted Close Prices For S&P 500 & Gold, Daily, Monthly & Annual (2000-2023)

Finally, figure 3 illustrates how the daily log prices, of gold and S&P 500 respectively, correlate with their past values at different lags. Each dot represents one lag, and the first lag correlates with itself, meaning it will always equal 1. The peaks at each lag are well outside bartlett intervals represented by the blue dashed lines, indicating that the log prices have a strongly significant positive correlation with their immediate past values. Like in a random walk, the autocorrelation decreases slightly for each lag. This persistence suggests that past values have a significant influence on future values, which is typical of a non-stationary process.

Figure 3 – Empirical Autocorrelation of Daily Adjusted Closing Prices For S&P 500 & Gold

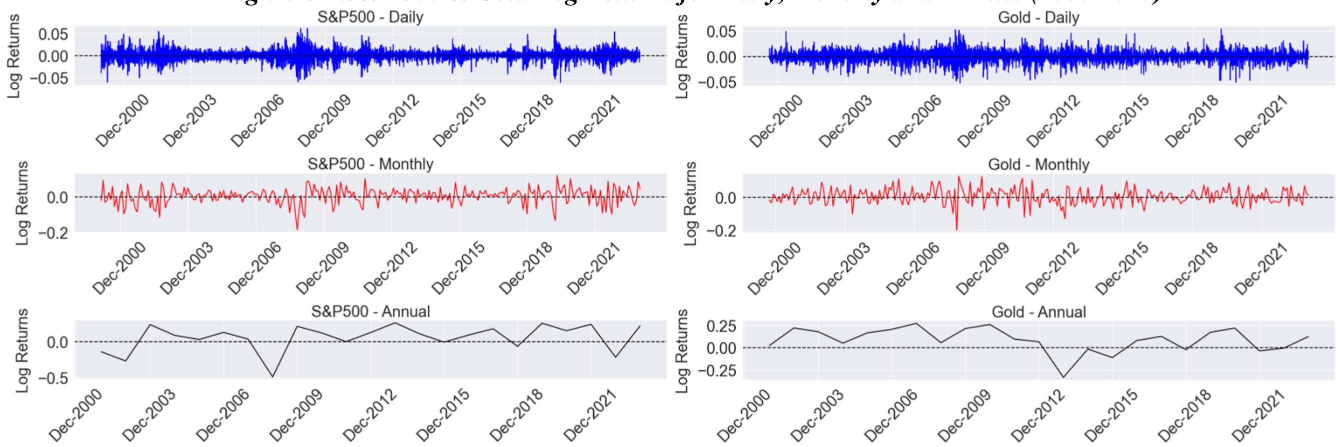


In all graphs, the non-stationary nature is evident in the time series, for both gold and the S&P 500 which shows similar trends. This due to the changing means, variances and covariances over time, which makes them dependent on time, why stylized fact 1 is applicable on both gold and the S&P 500.

Stylized Fact 2: Returns Are Stationary

Figure 4 illustrates the logarithmic returns of the S&P 500 and gold across daily, monthly, and annual data. It shows significant fluctuations, but within a certain range for both asset classes across all frequencies. According to stylized fact 2 returns are stationary, meaning they follow the properties of definition (1) in the methodology chapter.

Figure 4 – S&P 500 & Gold Log Returns for Daily, Monthly and Annual (2000-2023)

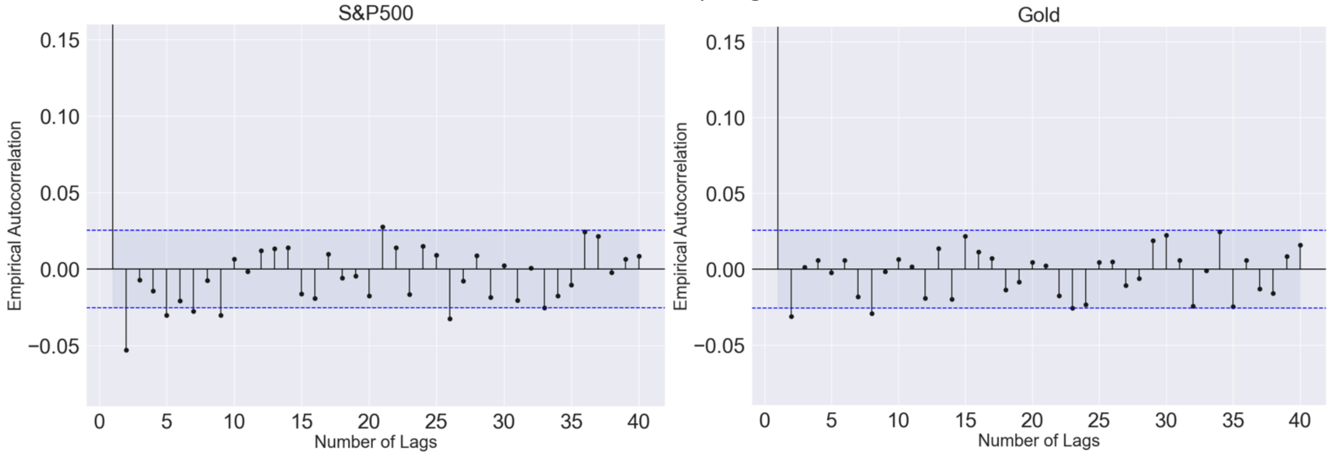


The graphs display a mean logarithmic return close to 0 and remains constant over time, since the observations fluctuate around the black line in the graph throughout the sample. Additionally, the variance seems to cluster in certain periods, but this does not happen in a periodic or systematic way, remaining constant over long subperiods. This is most evident in the daily log returns for both asset classes but can clearly be seen for the monthly and annual frequencies as well, supporting the stationarity of log returns.

In figure 5 below we see the autocorrelation function for S&P 500 and gold. It measures the linear relationship between the log prices of the S&P 500 and their past values over different time periods. The first lag shows an autocorrelation of 1, which is always the case since any dataset is perfectly correlated with itself at lag 0. For subsequent lags, the autocorrelations fall within the blue-dotted Bartlett intervals for almost all lags, indicating that they are not statistically significantly different from zero. Though we observe that S&P 500 display more significant autocorrelations than gold does. Overall, the pattern for both asset classes suggests that there is little to no predictable pattern in the returns from one day to the next.

The behavior aligns with stylized fact 2 regarding returns being stationary. More specifically, it is related to the concept that returns do not typically exhibit significant autocorrelation, meaning today's returns are not a reliable predictor of tomorrow's returns.

Figure 5 – Empirical Autocorrelation of Daily Log Returns For S&P 500 & Gold

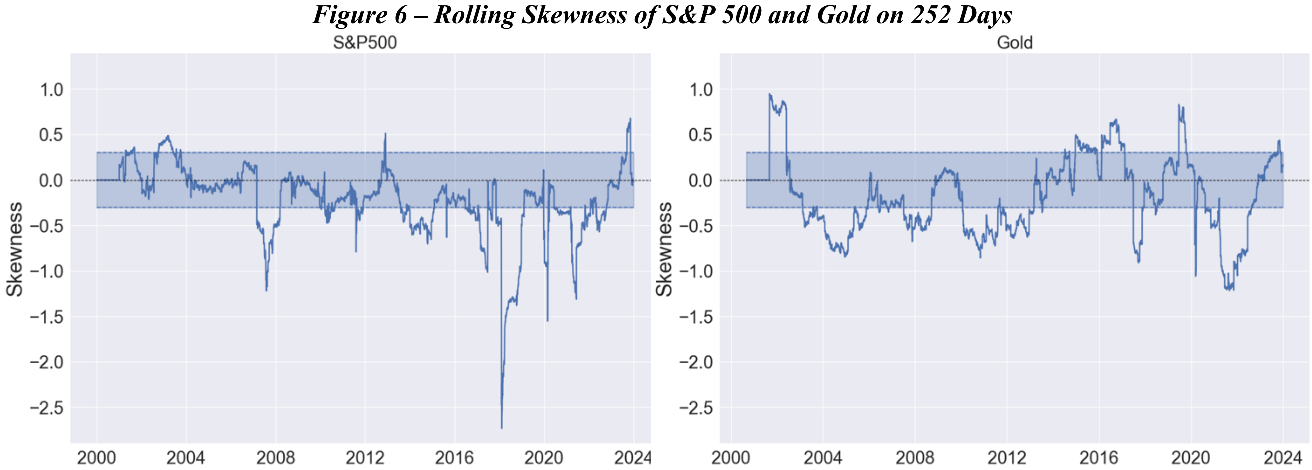


All the graphs show that the data for both gold and S&P 500 meet all the requirements of second order stationarity in definition (1), meaning stylized fact 2 is applicable on both asset classes.

Stylized Fact 3 & 4: Returns Are Asymmetric & Have Heavy Tails

Figure 6 shows the rolling measures of skewness for the S&P 500 and gold returns, with the blue shaded area being the confidence intervals.

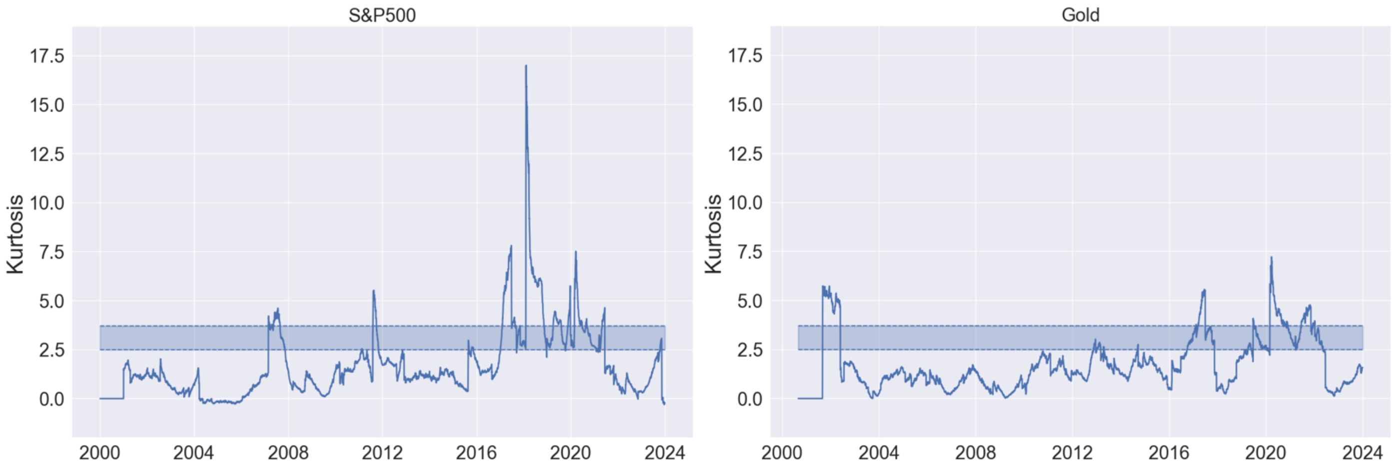
Figure 6 – Rolling Skewness of S&P 500 and Gold on 252 Days



For S&P 500 we see significant fluctuations in skewness. Almost all significant values that are outside of the confidence intervals, are negative. This trend highlights that S&P 500 return distributions tend to be negative, implying the presence of negative skewness and more extreme negative returns relative to positive ones. Such a pattern of negatively distributed returns confirms Stylized Fact 3, which suggests that financial returns are typically asymmetric and negatively skewed. The graph for gold in figure 6 shows variability in skewness, shifting between positive and negative values. The periods of positive skewness suggest that there are times when gold experiences more extreme positive returns than negative ones. Conversely, periods of negative skewness indicate that extreme losses are more frequent than equivalent gains, which align with stylized fact 3. Overall, we observe more periods exhibiting significant negative values, which would confirm stylized fact three for gold. Though, since we still see some periods of a significant positive skewness, we will need to investigate this further.

Figure 7 displays the rolling kurtosis of the S&P 500 and gold returns, with the blue shaded area representing the confidence intervals.

Figure 7 – Rolling Kurtosis of S&P 500 and Gold on 252 Days



The kurtosis values for S&P 500 exhibit substantial variability over the observed period. Notably, there are some kurtosis spikes well above the blue shaded area. These spikes indicate heavy tails in the return distributions, meaning that extreme returns occur more frequently than expected under normal distribution assumptions. For some of the period, the kurtosis values lie within the confidence intervals, suggesting that the tail behavior is within a normal range. Also, the kurtosis can be observed below the blue shaded area for some longer periods, indicating lighter tails and fewer extreme returns.

The kurtosis values for gold also show considerable fluctuations throughout the period. We observe a few spikes where the kurtosis rises above the blue shaded area, though they don't rise as high as for the S&P 500. These elevated values suggest heavy tails in the return distributions, indicating a higher frequency of extreme returns than predicted by a normal distribution. For a portion of the time, the kurtosis values fall within the confidence intervals, suggesting that the tail behavior of gold returns is often within the expected normal range.

There are also longer periods where the kurtosis dips below the confidence intervals, indicating lighter tails and fewer extreme returns, suggesting relatively lower market volatility.

Figure 8 – S&P 500 & Gold Q-Q Plots of Daily, Monthly and Annual Log Returns

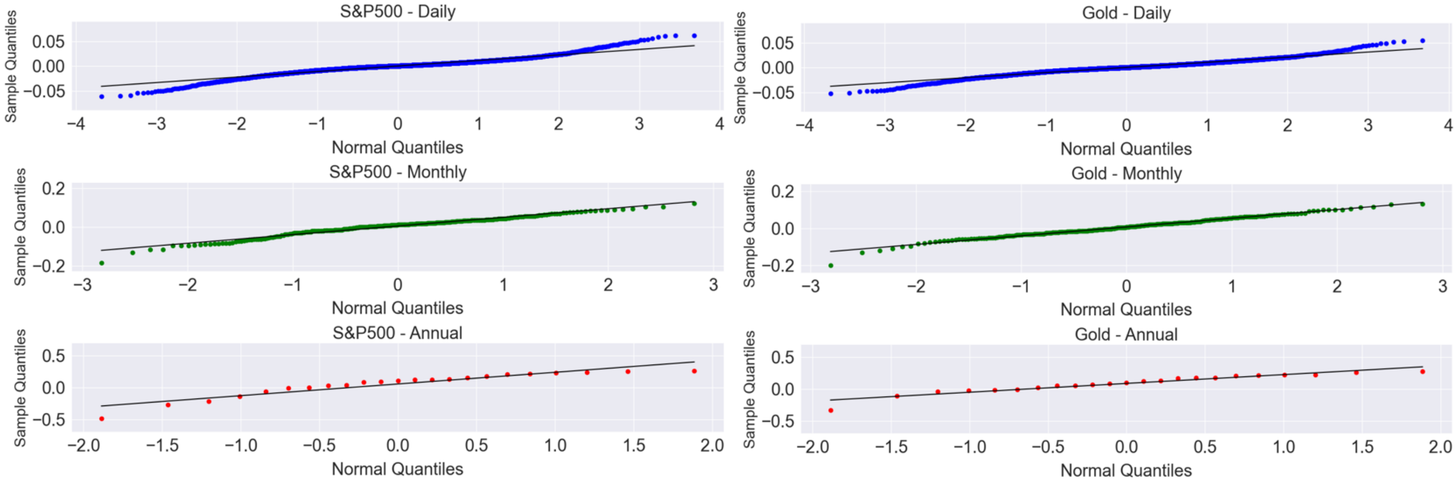


Figure 8 illustrates the distributional characteristics of gold and the S&P 500 log-returns at daily, monthly, and annual frequencies through Q-Q (quantile-quantile) plots. These graphs provide insights into the skewness (stylized fact 3) and kurtosis (stylized fact 4) of the returns.

The Q-Q plot for daily log-returns reveals similar deviations from the normal distribution at the tails for both asset classes, indicating a heavier lower tail. For monthly log-returns, the Q-Q plot shows deviations from the normal line at both ends, reflecting heavier tails. Though, these results show a smaller departure from normality for monthly returns than for daily returns for both asset classes. The Q-Q plots for annual log-returns also show deviations from the normal line in the tails, with the deviation being a little greater for the S&P 500 than for gold.

Since the results from the graphs presented in this chapter are somewhat contrasting and not entirely convincing, some tables of the exact numbers are presented below.

Table 1 displays the sample skewness of S&P 500 and gold returns at different frequencies and the corresponding p-values from the D'Agostino's K-squared test. which is a statistical test used to determine if the skewness of a dataset differs significantly from that of a normal distribution.

Table 1 – Sample Skewness & D'Agostino p-value for S&P500 And Gold

S&P 500

	Sample Skewness	D'Agostino Test p-value
daily	-0.18548	0.00
weekly	-0.30465	0.00
monthly	-0.65290	0.00
annual	-1.29451	0.01

Gold

	Sample Skewness	D'Agostino Test p-value
daily	-0.20555	0.00
weekly	-0.41030	0.00
monthly	-0.24752	0.01
annual	-1.14376	0.01

We observe negative skewness values across all frequencies for both asset classes. This indicates a distribution that leans towards the left, implying that there are more extreme values on the left side of the mean or more frequent extreme negative returns. This aligns with Stylized Fact 3 which states that return distributions are typically negatively skewed. The small p-values suggest that the skewness of these distributions is significantly different from zero, reinforcing the presence of asymmetry.

Table 2 – Kurtosis & Excess Kurtosis for S&P500 (left) and Gold (Right)

S&P 500

	Kurtosis	Excess Kurtosis
daily	6.51474	3.51474
weekly	6.43658	3.43658
monthly	4.07425	1.07425
annual	4.30232	1.30232

Gold

	Kurtosis	Excess Kurtosis
daily	5.44427	2.44427
weekly	4.52766	1.52766
monthly	3.95311	0.95311
annual	4.70039	1.70039

Table 2 presents the kurtosis and excess kurtosis of the returns, which measures the 'tailedness' of the distribution. A kurtosis greater than 3 (which would mean a positive excess kurtosis) indicates a leptokurtic[3] distribution with heavy tails. The daily and weekly returns exhibit high kurtosis values, much larger than 3, confirming the presence of heavy tails as stated in Stylized Fact 4. Monthly and annual returns have lower kurtosis, but they still suggest more extreme outcomes than would be expected from a normal distribution. The D'Agostino test from table 1 and its small p-values suggest that the distribution contrasts that of a normal one significantly different from zero, reinforcing the presence heavy tails for both asset classes.

Together, the graphs and tables presented in this chapter collectively validate stylized fact 3 and 4 for the S&P 500 and gold logarithmic returns, suggesting they are applicable in both

[3] Leptokurtic refers to a type of statistical distribution that exhibits higher kurtosis than the normal distribution, specifically with kurtosis greater than 3

asset classes. The skewness becomes more pronounced and negative as the frequency decreases from daily to annual returns, indicating a higher frequency of extreme negative returns over longer periods. The kurtosis is evident in the deviations from normality, with heavier tails as we move from annual to daily returns, indicating a higher likelihood of extreme returns.

Stylized Fact 5: Aggregational Guassianity in Returns

In table 3, we observe the Jarque Bera, Kolmogorov-Smirnov, and Lilliefors test statistics as well as their p-values which are multiplied with a 100 for the S&P 500. The null hypotheses of the tests are in this case that the underlying data equals a normal distribution. The bigger the test statistic, the further away from a normal distribution the underlying data is.

Table 3 – S&P 500 Jarque Bera, Lilliefors, Kolmogorov-Smirnov stat & p-value (p-values multiplied by 100)

	Jarque.Bera.stat	Jarque.Bera.pvalue	Lillie.test.stat	Lillie.test.pvalue	Kolmogorov-Smirnov.stat	Kolmogorov-Smirnov.pvalue
daily	3131.60215	0.00000	0.08181	0.10000	0.47985	0.00000
weekly	633.93620	0.00000	0.06616	0.10000	0.46636	0.00000
monthly	34.19052	0.00000	0.09226	0.10000	0.45577	0.00000
annual	8.04909	1.78716	0.17295	7.26385	0.39770	0.08565

For S&P 500 the Jarque-Bera test, which assesses whether the sample data matches a normal distribution in terms of skewness and kurtosis, shows a significant deviation from normality at the daily frequency, with a Jarque-Bera statistic of 3131.60215 and a p-value of 0. As we move to weekly returns, the statistic decreases to 633.93620, though the p-value remains at 0, indicating a continued deviation. At the monthly frequency, the Jarque-Bera statistic further drops to 34.19052 with a p-value still at 0, showing some improvement but still significant deviation from normality. Finally, the annual returns exhibit the lowest Jarque-Bera statistic of 8.04909, with a higher p-value of 0.0178716, suggesting a closer fit to normality, although not perfect.

The Lilliefors test also indicates significant deviation from normality at the daily frequency with a statistic of 0.08181 and p-value of 0.001, and weekly frequency with a statistic of 0.06616 and p-value of 0.001. Monthly returns show a statistic of 0.09226 with the same p-value of 0.001. The annual returns, however, show an increased statistic of 0.17295 but with a p-value of approximately 0.0726, saying that we cannot reject normality of the data.

The Kolmogorov-Smirnov test results align with these findings, showing the highest deviation at the daily frequency with a statistic of 0.47985 and p-value of 0, and a gradual decrease in deviation as the time horizon increases. Weekly returns have a statistic of 0.46636, monthly returns 0.45577, both with a p-value of 0. For annual returns, the t-statistic is 0.39770, with a p-value of 0.0008565, indicating the data becomes more normally distributed as the data is more aggregated.

For gold, in table 4, the Jarque-Bera test reveals a significant deviation from normality for daily returns, with a statistic of 1495.41447 and a p-value of 0. For weekly returns, the statistic reduces to 152.36252, though the p-value remains at 0, indicating continued deviation. At the monthly level, the statistic drops significantly to 13.45740, with a p-value of approximately 0.0012, suggesting an alignment closer to normality. The annual returns show the lowest Jarque-Bera statistic of 7.78553, with a p-value of approximately 0.0203, indicating an even better fit to normality.

Table 4 – Gold Jarque Bera, Lilliefors, Kolmogorov-Smirnov stat & p-value (p-values multiplied by 100)

	Jarque.Bera.stat	Jarque.Bera.pvalue	Lillie.test.stat	Lillie.test.pvalue	Kolmogorov-Smirnov.stat	Kolmogorov-Smirnov.pvalue
daily	1495.41447	0.00000	0.06485	0.10000	0.48271	0.00000
weekly	152.36252	0.00000	0.05639	0.10000	0.47048	0.00000
monthly	13.45740	0.11961	0.03018	83.18493	0.44834	0.00000
annual	7.78553	2.03889	0.10717	70.06448	0.41262	0.04664

Similarly, the Lilliefors test indicates significant deviations from normality for daily and weekly frequencies, with a statistic of 0.06485 and 0.05639 respectively, and a p-value of 0.001 for both. However, for monthly returns, the statistic decreases to 0.03018, with a p-value of approximately 0.8318, showing a considerable improvement towards normality. The annual returns have a Lilliefors statistic of 0.10717 and a high p-value of approximately 0.7006, indicating a much closer approximation to normality. These results are clearer for gold than for the S&P 500 and means we cannot reject normality for neither monthly nor annual data, showing evidence of aggregational gaussianity.

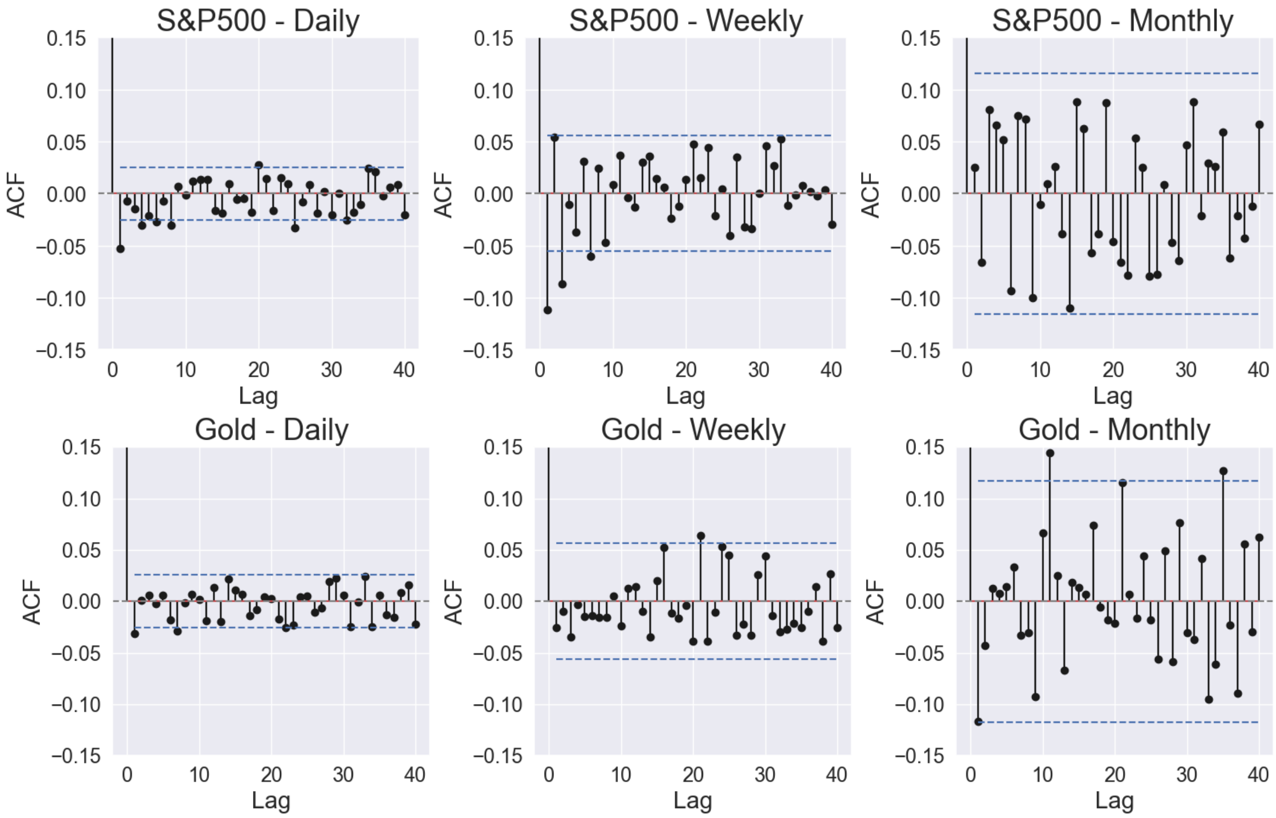
The Kolmogorov-Smirnov test results confirm these findings, showing the highest deviation at the daily frequency with a statistic of 0.48271 and p-value of 0, and a gradual improvement as the time horizon increases. The weekly returns have a statistic of 0.47048, the monthly returns 0.44834, and the annual returns 0.41262. The p-values are 0 for the weekly and monthly frequencies, improving to 0.0005 for the annual frequency, suggesting a closer fit to normality as the data becomes more aggregated.

Overall, the results from these tests collectively support the concept of aggregational gaussianity for the S&P 500 and gold. While daily and weekly returns exhibit significant deviations from normality, the monthly returns show some improvement, and the annual returns demonstrate the greatest approximation towards a normal distribution for both asset classes. This trend is even more clear for gold than it is for the S&P 500. Though, it goes towards normality with increasing aggregation periods for both assets, aligning with stylized fact 5, confirming that as data becomes more aggregated, it becomes more normally distributed.

Stylized Fact 6: Returns Are Not Autocorrelated

Stylized Fact 6 posits that financial returns are generally not autocorrelated. To examine this for gold and the S&P 500, we analyze figure 9 and the autocorrelation functions (ACF) of daily, weekly, and monthly log-returns for both assets. The autocorrelation at lag 0 equals 1 in all graphs since the underlying dataset will always correlate with itself. We also conducted the Box-Pierce and Ljung-box test statistics and p-values to look at the actual numbers behind the graphs. Those can be found in table 5-8 in appendix.

Figure 9 – S&P 500 & Gold AFC for Daily, Weekly and Monthly Log Returns



For the S&P 500, figure 9 reveals most values in the ACF of daily log-returns that most autocorrelations lie within the 95% confidence intervals. This indicates a lack of significant autocorrelation at most lags. Though there are 7 lags with values stretching outside the confidence intervals, opposing the idea behind stylized fact 6. Nonetheless these observations are few when compared to the number of insignificant lags, suggesting minimal and random autocorrelation. To investigate this further, we look at table 5 in the appendix, which shows us the Pierce-Box and Ljung-Box tests. In the table. We find p-values of 0 across all lags, indicating a rejection of the null hypothesis which states that the data is not autocorrelated. This finding suggests that the data is autocorrelated at a daily frequency and therefore not consistent with Stylized Fact 6 for the S&P 500.

In the case of weekly log-returns for the S&P 500, the ACF displays less variability, with fewer lags showing autocorrelation values outside the 95% confidence intervals. This indicates next to nonsignificant autocorrelation overall, though we see somewhat of a deviation from the expectation of no autocorrelation here as well.

Finally, the ACF for monthly log-returns of the S&P 500 shows minimal autocorrelation, with no lags apart from the first one being outside of the confidence intervals. This is consistent with table 6 in the appendix, where we are not able to reject no autocorrelation in the Box-Pierce and Ljung-Box tests, which validates Stylized Fact 6 over longer time periods.

The ACF for daily log-returns of gold demonstrates that most autocorrelation values fall within the 95% confidence intervals, represented by the blue dashed lines. This indicates that there is no significant autocorrelation at most lags. Only a few points lie outside the confidence intervals, suggesting minimal and random autocorrelation for daily returns. This is consistent with the result of the Pierce-Box and Ljung-Box tests in table 7 in the appendix, where we cannot reject the null of no autocorrelation for any lag apart from the first one in the daily returns.

For weekly log-returns in gold, the ACF shows a similar pattern as in the daily data, with only one lag exhibiting an autocorrelation value outside the confidence intervals. This suggests next to no significant autocorrelation for weekly returns in gold, consistent with the result for daily returns and therefore stylized fact 6.

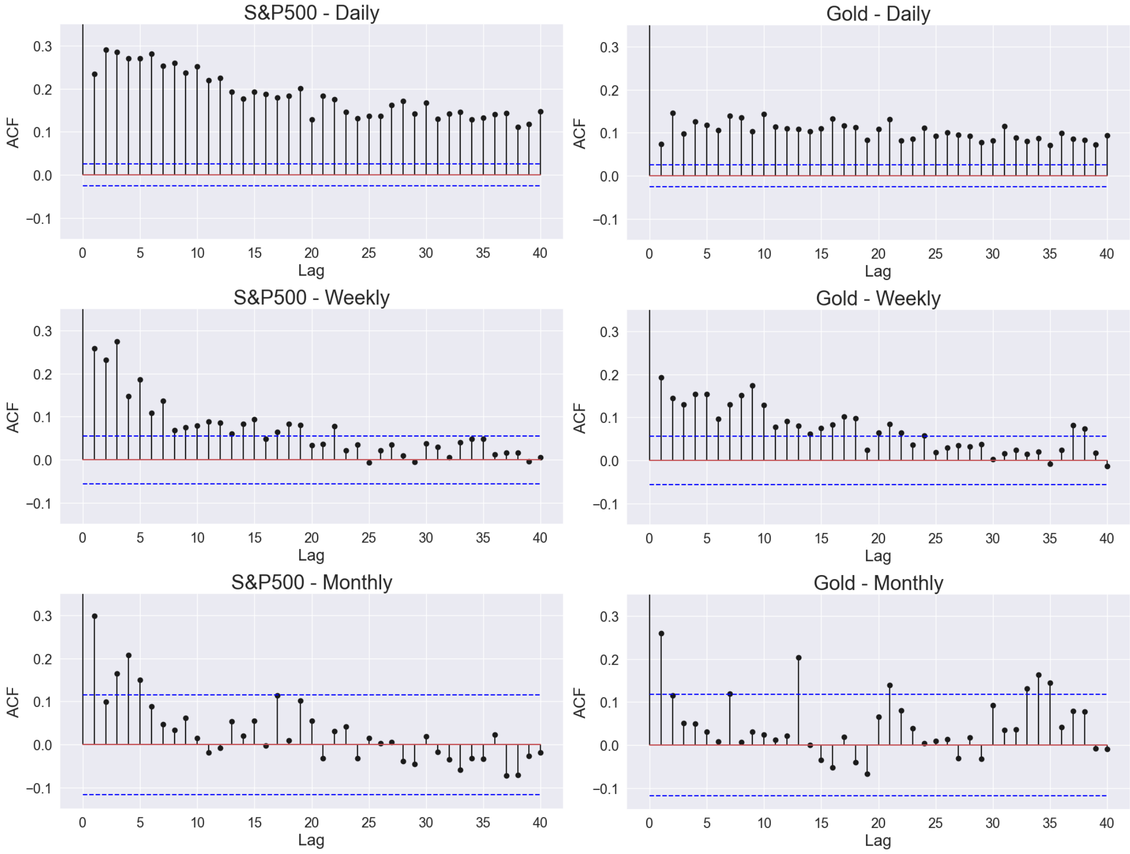
When examining the ACF for monthly log-returns of gold, we capture only the most persistent trends. Here, like in the daily returns, we see significant autocorrelation at a few lags. Though most lags still show next to no autocorrelation in returns for the monthly frequency. We look at table 8 in the appendix to investigate this further but again, from the Box-Pierce and Ljung-Box tests, find that there is no significant autocorrelation at any lags. This finding is in line with stylized fact 6.

In conclusion, The ACF and corresponding Box-Pierce and Ljung-Box tests indicate significant autocorrelation, as evidenced by p-values of 0 across all lags. This suggests that daily returns for the S&P 500 do not align with Stylized Fact 6, showing that autocorrelation is present. This result contrasts the one of gold, which demonstrates that most autocorrelation values fall within the 95% confidence intervals, suggesting no significant autocorrelation at most lags. The Box-Pierce and Ljung-Box tests corroborate this, with no significant autocorrelation detected, aligning with Stylized Fact 6. For the rest the result we observe decreasing autocorrelation as we go to weekly and then annual frequencies for both the S&P 500 and gold. For the S&P 500, this suggests that stylized fact 6 is applicable on a weekly and monthly frequency, but not on daily. For gold, this suggests that stylized fact 6 is applicable across all frequencies.

Stylized Fact 7: Volatility Clustering

To analyze stylized fact 7, volatility clustering in returns, for the S&P 500 and gold, we examine the autocorrelation functions (ACF) of squared log-returns at daily, weekly, and monthly frequencies seen in figure 10. To see the actual numbers behind the graphs of the squared log returns, see table 9 and 10 in the appendix. In the same table, we also computed the Box-Pierce as well as the Ljung-Box test, with the correlated p-value. These tests are statistical tests used to determine whether there is significant autocorrelation in a time series.

Figure 10 – S&P 500 & Gold ACF for Daily, Weekly and Monthly Squared Log Returns



The ACF for daily squared log-returns of the S&P 500 shows significant positive autocorrelation at all lags, with values remaining above the 95% confidence intervals (blue dashed lines). We observe that as the lag increases, the autocorrelation gradually decrease. This strong autocorrelation indicates pronounced volatility clustering, where high volatility periods are followed by more high volatility periods, and low volatility periods follow low volatility periods.

The ACF for weekly squared log-returns also demonstrates significant positive autocorrelation, with the values remaining above the confidence intervals for several lags. The difference is that the pattern starts decaying way faster and more pronounced than it does for the daily data. Overall, there are still many significant autocorrelations, confirming the presence of volatility clustering at a weekly frequency.

Similarly, the ACF for monthly squared log-returns shows significant autocorrelation at various lags, though with lower magnitudes than daily and weekly data. This indicates that volatility clustering persists even at monthly intervals, suggesting long-term periods of high and low volatility.

The ACF for daily squared log-returns of gold shows significant positive autocorrelation at various lags, with values remaining above the 95% confidence intervals for many lags. This strong autocorrelation indicates the presence of volatility clustering

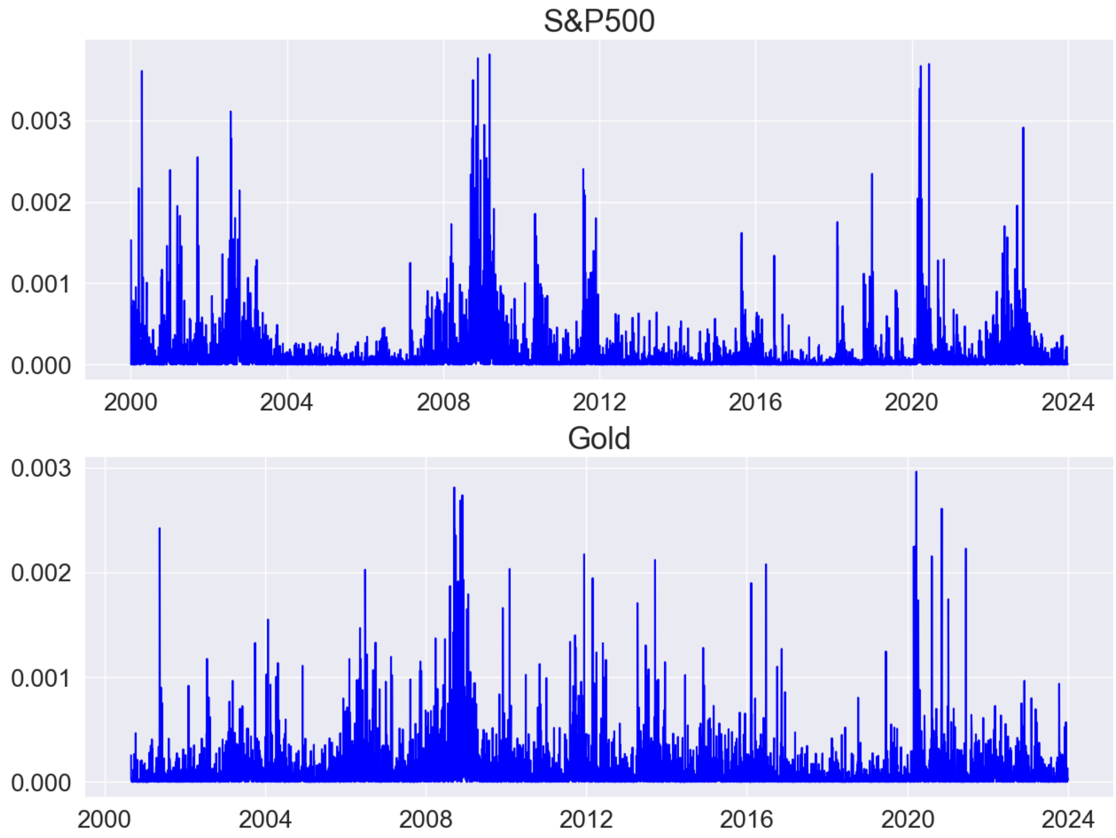
The ACF for weekly squared log-returns also demonstrates significant positive autocorrelation, though the values are generally lower than those observed in the daily data. The autocorrelation values remain above the confidence intervals for several lags, confirming the presence of volatility clustering at a weekly frequency.

The ACF for monthly squared log-returns of gold shows significant autocorrelation at various lags, with values above the 95% confidence intervals, though generally lower than in daily and weekly data. This indicates that volatility clustering persists at the monthly frequency.

For both gold and the S&P 500, significant positive autocorrelation at multiple lags in daily squared returns indicates pronounced volatility clustering. One big distinction is that the autocorrelation is stronger for the S&P 500 than what it is for gold. Weekly squared returns exhibit similar patterns, indicating that volatility clustering extends to a weekly scale but with a reduced intensity. Here the trends look more similar between the two asset classes than they do for the daily frequency. Monthly squared returns also show significant autocorrelation, indicating that volatility clustering persists even over longer periods. Though these observations are fewer for S&P 500 as well as gold, meaning the clustering decays as the frequency decreases. Overall, the volatility clustering is evident across all frequencies. This result is in line with the result from the Box-Pierce and Ljung-Box tests in table 9 and 10 in the appendix, which shows p-values of 0 across all lags, validating stylized fact 7 and suggesting it’s applicable in both asset classes.

The graphs of squared daily S&P 500 and gold log-returns in figure 10 visually demonstrates Stylized Fact 7, showing clear volatility clustering. Periods of high volatility, indicated by spikes, are followed by more high volatility periods, while low volatility periods show smaller fluctuations. Notable clusters of high volatility are observed during significant market events, such as the 2008 financial crisis and the 2020 pandemic, highlighting sustained periods of market turbulence.

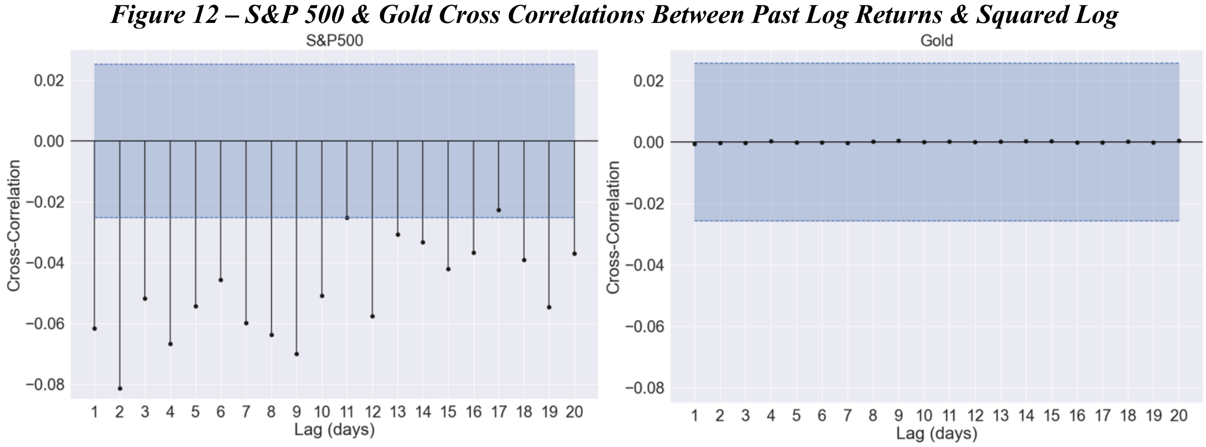
Figure 11 - S&P 500 & Gold Squared Daily Log Returns (2000-2023)



Stylized Fact 8: Leverage Effect

Figure 12 displays the cross-correlation between daily returns and future squared returns over various lags for gold and the S&P 500, with the blue shaded are being the confidence interval.

Figure 12 – S&P 500 & Gold Cross Correlations Between Past Log Returns & Squared Log



For S&P 500, significant negative correlations at multiple lags are observed, with values lying below the 95% confidence intervals (represented by the blue shaded area). This indicates that negative returns today are associated with higher volatility in the future, a hallmark of the leverage effect. Specifically, the cross-correlation values show that negative returns are followed by increased volatility for several days, with the strongest effects observed at shorter lags. This pattern is consistent with the leverage effect, where a decrease in asset prices leads to an increase in financial leverage, resulting in higher future volatility.

For gold, the cross-correlation values for are close to zero and remain within the 95% confidence intervals across all lags. This indicates that there is no significant correlation between current returns and future volatility for gold. The absence of significant negative correlations provides evidence that the leverage effect is not present in gold returns. The lack of significant cross-correlation between current returns and future volatility indicates that negative returns do not lead to higher future volatility in the gold market.

In conclusion, the graphs provide strong evidence of the leverage effect in the S&P 500 returns, with almost all lags extending outside the blue shaded confidence interval. This contrasts the results of gold, where we found no no significant autocorrelations at any lag. Overall, the result shows that stylized fact 8, leverage effect, is applicable on the S&P 500 but not in the gold market.

6. Discussion

Stylized Fact 1

The analysis confirmed that both gold and the S&P 500 exhibit non-stationary price behavior, consistent with Stylized Fact 1 no matter the frequency. Both prices and logarithmic prices showed an increasing trend over time with periods of decline, reflecting market dynamics such as economic growth and downturns. We showed that the all the conditions of second order-stationarity brought up by Hamilton (1994) were contradicted, suggesting stylized fact 1 is applicable in both gold and the S&P 500. This aligns with the Random Walk Theory (Osborne, 1959) and the Efficient Market Hypothesis (Fama, 1970), which suggest that prices are inherently unpredictable and influenced by new information. This finding is also in line with the work of Cont (2001), who emphasized the importance of understanding non-stationary behavior in financial time series. The non-stationarity of prices indicates that financial models must incorporate elements that account for the randomness and unpredictability of price movements. This has implications for the Efficient Market Hypothesis and Random Walk Theory, suggesting that markets are continuously adjusting to new information in a stochastic manner. Investment strategies must account for this unpredictability and make use of models that incorporate stochastic processes to better manage risk and predict price movements.

Stylized Fact 2

Logarithmic returns for both gold and the S&P 500 were found to be stationary across different frequencies, in line with stylized fact 2. The consistent mean and variance over time support the notion of second-order stationarity. Also, we found no significant autocorrelation apart from in the first lag, summing up to meet all requirements of second order stationarity. This result means that stylized fact 2 is applicable in both asset classes discussed in the thesis. This observation is in line with the Random Walk Theory and the Central Limit Theorem, which imply that returns, as opposed to prices, tend to exhibit stationarity. This finding supports Zivot's (2021) observations about the stationary nature of returns. Understanding this can help in the creation of more accurate predictive models for returns, and thereby enhance the precision of financial forecasts and improve portfolio management.

Stylized Fact 3 & 4

The analysis of skewness and kurtosis revealed varying results from the graphs, though there were clear signs of asymmetry with negative skewness as well as an excess kurtosis with heavy tails for both asset classes. After then looking at the actual numbers in table 1 and 2, including the D'Agostino K-squared test used to assess the normality of a distribution, it was clear to see the evidence. The skewness became more pronounced as we went from daily to annual frequency for both asset classes. This was generally the case with the kurtosis as well, except for the monthly frequency, which displayed the lowest kurtosis in both gold and the S&P 500. In summary, this supports the applicability of stylized facts 3 and 4 in both gold and the S&P 500. The presence of negative skewness aligns with Prospect Theory (Kahneman & Tversky, 1979), which suggests that investors react more strongly to losses than gains. Heavy tails are consistent with Mandelbrot's Stable Paretian Hypothesis (1963), indicating that extreme market events are more common than a normal distribution would predict. The findings are also supported by the work of Silvennoinen et al. (2013), who documented similar behaviors in metals. To manage risk, traditional models assuming normality which may underestimate the likelihood of extreme events, could incorporate models that account for skewness and heavy tails, such as the Generalized Pareto

Distribution. This could improve risk assessments and help prevent underestimation of potential losses.

Stylized Fact 5

As the data aggregation increased from daily to annual, the returns distribution for both gold and the S&P 500 became more normal. This was very clear for gold where the Lilliefors test couldn't reject normality of the data for the monthly and annual frequencies with a large p-value for each. For the S&P 500 this wasn't as apparent, though the statistical tests showed that as the frequency went from daily to annual frequency, the data showed more evidence of a distribution like a normal one. In conclusion, this supports Stylized Fact 5 which appeared to be applicable in both asset classes, demonstrating that larger time scales tend to smooth out deviations from normality and thereby aligning with the Central Limit Theorem (Feller, 1971). This trend towards normality with increasing aggregation periods confirms the findings of Bollen et al. (2004) regarding stock indices. These results suggest that for long-term investment horizons, simpler models assuming normality may be sufficient, whereas for short-term trading, models need to account for non-normal behavior to accurately assess risk.

Stylized Fact 6

Returns for both gold and the S&P 500 showed minimal autocorrelation at a weekly and monthly frequency in the graphs as well as the Box-Pierce and Ljung-Box tests, supporting Stylized Fact 6. For gold, this was the case at a daily frequency as well. On the other hand, for S&P 500, this was not the case. When looking at the Box-Pierce and Ljung-Box tests, we rejected the null hypothesis of no autocorrelation in returns for all lags at the daily frequency. The general lack of predictability in returns at all frequencies for gold, but only for weekly and monthly frequencies in the S&P 500, aligns with the Efficient Market Hypothesis and the Random Walk Theory. Though the result of empirical autocorrelation in daily returns for the S&P 500 is consistent with the findings of Lo and MacKinlay (1988), who also identified limited autocorrelation in stock returns. This suggests that in the long term for stock indices and no matter the horizon for gold, past returns are not indicative of future performance, reinforcing the challenge of predicting short-term price movements and supporting the principle for a diversified investment strategy. On the contrary, in the short term past returns evidently could be a good predictor of future ones for the S&P 500.

Stylized Fact 7

Significant positive autocorrelation in squared returns indicated pronounced volatility clustering, confirming Stylized Fact 7 is applicable in both asset classes. The only real difference observed was at the daily frequency, where the clustering seemed to be more pronounced in the S&P 500 than in gold. This phenomenon, where high volatility periods are followed by more high volatility periods, reflects underlying market behaviors and investor reactions, as described by Mandelbrot (1963) and Shiller (2003). The findings align with the volatility patterns observed by Chevallier & Ielpo (2017) in commodity markets. Volatility clustering then suggests that models such as GARCH (Generalized Autoregressive Conditional Heteroskedasticity) are particularly well-suited for forecasting future volatility. Implementing such models can help in developing trading strategies that adjust dynamically to changing market conditions, potentially enhancing returns and reducing risk.

Stylized Fact 8

The S&P 500 exhibited a significant leverage effect, where negative returns led to increased future volatility, with a significant negative correlation between past log returns and squared log returns at almost all lags. This was not observed for gold, with insignificant correlations between past log returns and squared log returns at all lags. This highlights a key difference in the behavior of these asset classes. The leverage effect, as proposed by Black (1976), suggests that market downturns increase volatility due to higher financial leverage. This result is supported by the research of Bouchaud et al. (2001), who documented a similar effect in stock indices. The same goes for Chevallier & Ielpo (2017), who contradicted our result in gold and documented a leverage effect in commodity markets, where gold was one of them. The absence of the leverage effect in gold found in this thesis indicates its role as a safe haven asset, less affected by leverage and past returns, and more by other factors like inflation and currency fluctuations. Furthermore, the absence of a leverage effect in gold could be attributed to the fact that gold is not a leveraged entity like companies in the S&P 500, which have debt and equity structures impacting their volatility. The observed leverage effect in equities but not in gold highlights the differing market dynamics and risk profiles of these asset classes. This finding suggests that during periods of market stress, equity markets may exhibit heightened volatility due to financial leverage, while gold remains relatively stable. These differences should be considered in portfolio diversification and risk management strategies, as they point out the unique roles that different asset classes play in financial markets.

7. Conclusion

This thesis has explored the applicability of 8 stylized facts within financial markets, comparing gold and equities represented by the S&P 500. The analysis utilized daily, weekly, monthly, and annual data from 2000 to 2023. We ensured robust results by excluding data points that exceeded five standard deviations from the mean, thereby minimizing the influence of extreme outliers on the analysis and preserving the integrity of the dataset. We defined the key statistical properties and theoretical foundations necessary for this analysis. We also applied various statistical tests to thoroughly examine these properties across the different time scales. By employing advanced data analysis techniques, we gained a comprehensive understanding of the dynamics underlying the financial returns of gold and equities. Our empirical analysis has led to several key conclusions regarding the behavior of these two distinct asset classes.

The similarities between gold and equities, such as non-stationarity in prices and stationarity in returns, shows that these stylized facts are broadly applicable across different types of financial instruments. This similarity suggests that risk management models can adopt similar strategies to assess and mitigate risks associated with extreme returns across various financial instruments. However, the nuanced differences between asset classes necessitate tailored approaches for accurate risk modeling and management. This difference implies that risk models for equities should incorporate leverage effects, while models for gold should focus on other volatility drivers. Also, we observed a difference in the daily autocorrelation in log returns. For equities, the returns appeared to be autocorrelated, meaning the equity market might not be entirely efficient as opposed to gold.

Future research should focus on developing advanced financial models that specifically incorporate the leverage effect for equities and other volatility factors for gold. These models need to account for the unique risk and return characteristics of each asset class. Designing trading strategies that exploit the unique behaviors of gold and equities could also be beneficial. Strategies could capitalize on leverage effect and volatility clustering in the S&P 500 or the safe-haven properties of gold during market downturns.

For example, one could attempt to create a trading strategy based on the implications of the leverage effect found in the S&P 500. This by using the *volatility feedback hypothesis*, where increased volatility ultimately results in an immediate decrease in the current asset price. In the trading strategy, the research could also use our result from stylized fact 7 and assume that in periods of high volatility which persists, the price decline will continue into the days following the increase in volatility. From there, one could monitor the rolling standard deviation of the S&P 500, calculated over various time scales, and determine when the rolling standard deviation falls within the Bartlett intervals. When the rolling standard deviation drops within the Bartlett intervals, signaling a potential period of sustained high volatility, one could initiate a short sell position on the S&P 500. Hold the short position for 10 days, based on the observation that the cross-correlation between past log returns and squared log returns diminishes after 10 lags. Then buy back the S&P 500 after the 10-day period.

Moreover, extending the comparative analysis to include other asset classes such as bonds, real estate, and cryptocurrencies could provide a more comprehensive understanding of stylized facts across diverse financial instruments. Investigating the behavior of gold and equities using high-frequency data to capture microstructure noise and intraday volatility patterns could reveal additional insights into the dynamics of these markets. On the other hand, extending the analysis over more years than twenty could also provide insights and might affect the result. Also, exploring how significant economic events (e.g. financial crises, pandemics) impact the stylized facts of different asset classes could enhance the predictiveness of financial models during periods of economic stress.

In conclusion, this thesis successfully compares the applicability of stylized facts of financial returns between gold and equities. By empirically examining these asset classes, we provide detailed insights into their unique and shared statistical properties. Specifically, our findings on non-stationarity prices, stationarity of returns, asymmetry, heavy tails, aggregational Gaussianity, autocorrelation, volatility clustering, and the leverage effect offer precise quantitative evidence that can inform better risk management and investment strategies. The distinct leverage effect identified in the equities but absent in gold, for example, highlights the need for equity-specific models that account for financial leverage impacts. Conversely, recognizing gold's consistent behavior without leverage effects suggests it can be more reliably used as a hedge in portfolios. Furthermore, the evidence of volatility clustering and heavy tails in both asset classes highlights the importance for risk models to incorporate these features to accurately predict extreme market movements. These contributions provide a direct support for developing targeted financial instruments and improving existing financial theories, offering precise tools to improve portfolio optimization and risk assessment methodologies. The detailed empirical comparisons also serve as a benchmark for future studies, promoting more nuanced and asset-specific financial research.

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Appendix

Table 5 – Descriptive Statistics of Different Log Returns Sampling Frequencies

S&P 500

	Daily	Weekly	Monthly	Annual
Mean	0.02429	0.12488	0.42850	5.58464
St.Deviation	1.14300	2.40481	4.48710	18.30290
Diameter.C.I.Mean	0.02888	0.13337	0.51914	7.48018
Skewness	-0.18548	-0.30465	-0.65290	-1.29451
Kurtosis	3.51474	3.43658	1.07425	1.30232
Excess.Kurtosis	0.51474	0.43658	-1.92575	-1.69768
Min	-6.07527	-12.33038	-18.56365	-48.59020
Quant5	-1.86894	-3.97543	-8.47165	-26.11363
Quant25	-0.48372	-1.07320	-1.83417	-0.36622
Median	0.05921	0.23785	1.04367	10.78731
Quant75	0.59090	1.43112	3.19866	19.40881
Quant95	1.71763	3.65444	7.35502	25.21441
Max	6.17186	11.42367	11.94208	25.92922
Jarque.Bera.stat	3131.60215	633.93620	34.19052	8.04909
Jarque.Bera.p.value.X100	0.00000	0.00000	0.00000	1.78716
Lillie.test.stat	0.08181	0.06616	0.09226	0.17295
Lillie.test.pvalue.X100	0.10000	0.10000	0.10000	7.26385
N.obs	6017.00000	1249.00000	287.00000	23.00000

Gold

	Daily	Weekly	Monthly	Annual
Mean	0.03680	0.15491	0.71533	8.80793
St.Deviation	1.04816	2.33468	4.68595	13.55477
Diameter.C.I.Mean	0.02688	0.13123	0.54888	5.53967
Skewness	-0.20555	-0.41030	-0.24752	-1.14376
Kurtosis	2.44427	1.52766	0.95311	1.70039
Excess.Kurtosis	-0.55573	-1.47234	-2.04689	-1.29961
Min	-5.23127	-10.13157	-19.85124	-33.17901
Quant5	-1.73287	-3.81023	-6.25570	-10.27626
Quant25	-0.47744	-1.10540	-2.20556	1.00283
Median	0.04440	0.30289	0.61990	9.69657
Quant75	0.60969	1.55749	3.58482	19.23076
Quant95	1.67706	3.58632	7.94047	25.65365
Max	5.44218	9.00895	12.98626	27.33721
Jarque.Bera.stat	1495.41447	152.36252	13.45740	7.78553
Jarque.Bera.p.value.X100	0.00000	0.00000	0.11961	2.03889
Lillie.test.stat	0.06485	0.05639	0.03018	0.10717
Lillie.test.pvalue.X100	0.10000	0.10000	83.18493	70.06448
N.obs	5842.00000	1216.00000	280.00000	23.00000

Stylized Fact 6

Table 7 – ACF Values, Ljung-Box and Box-Pierce Tests for Daily Log Returns of S&P 500

Lag	ACF	ACF Diameter	ACF Test	Box-Pierce Stat.	Box-Pierce p-value	Ljung-Box Stat.	Ljung-Box p-value	Critical Value
1.000	-0.053	0.025	-2.091	16.792	0.000	16.801	0.000	3.841
2.000	-0.007	0.025	-0.278	17.090	0.000	17.099	0.000	5.991
3.000	-0.014	0.025	-0.563	18.315	0.000	18.324	0.000	7.815
4.000	-0.030	0.025	-1.196	23.843	0.000	23.858	0.000	9.488
5.000	-0.021	0.025	-0.824	26.471	0.000	26.489	0.000	11.070
6.000	-0.027	0.025	-1.083	31.015	0.000	31.040	0.000	12.592
7.000	-0.007	0.025	-0.290	31.342	0.000	31.366	0.000	14.067
8.000	-0.030	0.025	-1.193	36.861	0.000	36.895	0.000	15.507
9.000	0.007	0.025	0.260	37.125	0.000	37.160	0.000	16.919
10.000	-0.001	0.025	-0.058	37.138	0.000	37.173	0.000	18.307
11.000	0.012	0.025	0.471	38.001	0.000	38.037	0.000	19.675
12.000	0.013	0.025	0.527	39.080	0.000	39.119	0.000	21.026
13.000	0.014	0.025	0.545	40.234	0.000	40.276	0.000	22.362
14.000	-0.016	0.025	-0.641	41.832	0.000	41.878	0.000	23.685
15.000	-0.019	0.025	-0.751	44.027	0.000	44.080	0.000	24.996
16.000	0.010	0.025	0.378	44.585	0.000	44.639	0.000	26.296
17.000	-0.006	0.025	-0.224	44.781	0.000	44.835	0.000	27.587
18.000	-0.005	0.025	-0.182	44.909	0.000	44.964	0.000	28.869
19.000	-0.018	0.025	-0.691	46.773	0.000	46.834	0.000	30.144
20.000	0.028	0.025	1.085	51.366	0.000	51.444	0.000	31.410
21.000	0.014	0.025	0.551	52.552	0.000	52.635	0.000	32.671
22.000	-0.017	0.025	-0.654	54.226	0.000	54.315	0.000	33.924
23.000	0.015	0.025	0.583	55.555	0.000	55.650	0.000	35.172
24.000	0.009	0.025	0.363	56.070	0.000	56.167	0.000	36.415
25.000	-0.033	0.026	-1.277	62.454	0.000	62.580	0.000	37.652

Table 8 – ACF Values, Ljung-Box and Box-Pierce Tests for Daily Log Returns of Gold

Lag	ACF	ACF Diameter	ACF Test	Box-Pierce Stat.	Box-Pierce p-value	Ljung-Box Stat.	Ljung-Box p-value	Critical Value
1.000	-0.031	0.026	-1.219	5.705	0.017	5.708	0.017	3.841
2.000	0.001	0.026	0.053	5.716	0.057	5.719	0.057	5.991
3.000	0.006	0.026	0.223	5.907	0.116	5.910	0.116	7.815
4.000	-0.002	0.026	-0.083	5.933	0.204	5.936	0.204	9.488
5.000	0.006	0.026	0.223	6.124	0.294	6.127	0.294	11.070
6.000	-0.018	0.026	-0.706	8.045	0.235	8.051	0.234	12.592
7.000	-0.029	0.026	-1.136	13.018	0.072	13.032	0.071	14.067
8.000	-0.002	0.026	-0.065	13.034	0.111	13.048	0.110	15.507
9.000	0.007	0.026	0.254	13.283	0.150	13.297	0.150	16.919
10.000	0.002	0.026	0.068	13.301	0.207	13.315	0.207	18.307
11.000	-0.019	0.026	-0.747	15.454	0.163	15.473	0.162	19.675
12.000	0.014	0.026	0.526	16.523	0.168	16.545	0.168	21.026
13.000	-0.020	0.026	-0.769	18.805	0.129	18.833	0.128	22.362
14.000	0.022	0.026	0.852	21.614	0.087	21.649	0.086	23.685
15.000	0.011	0.026	0.441	22.366	0.099	22.403	0.098	24.996
16.000	0.007	0.026	0.272	22.652	0.123	22.691	0.122	26.296
17.000	-0.014	0.026	-0.526	23.723	0.127	23.765	0.126	27.587
18.000	-0.008	0.026	-0.325	24.132	0.151	24.175	0.149	28.869
19.000	0.005	0.026	0.178	24.255	0.187	24.298	0.185	30.144
20.000	0.002	0.026	0.091	24.287	0.230	24.330	0.228	31.410
21.000	-0.017	0.026	-0.675	26.049	0.205	26.100	0.203	32.671
22.000	-0.026	0.026	-0.993	29.868	0.122	29.934	0.120	33.924
23.000	-0.023	0.026	-0.903	33.032	0.081	33.112	0.079	35.172
24.000	0.004	0.026	0.173	33.148	0.101	33.228	0.099	36.415
25.000	0.005	0.026	0.188	33.285	0.124	33.366	0.122	37.652

Table 9 – ACF Values, Ljung-Box and Box-Pierce Tests for Monthly Log Returns of S&P 500

Lag	ACF	ACF Diameter	ACF Test	Box-Pierce Stat.	Box-Pierce p-value	Ljung-Box Stat.	Ljung-Box p-value	Critical Value
1.000	0.025	0.025	-2.091	0.182	0.669	0.184	0.668	3.841
2.000	-0.066	0.025	-0.278	1.432	0.489	1.451	0.484	5.991
3.000	0.081	0.025	-0.563	3.300	0.348	3.352	0.340	7.815
4.000	0.065	0.025	-1.196	4.529	0.339	4.607	0.330	9.488
5.000	0.052	0.025	-0.824	5.296	0.381	5.393	0.370	11.070
6.000	-0.094	0.025	-1.083	7.815	0.252	7.984	0.239	12.592
7.000	0.075	0.025	-0.290	9.437	0.223	9.658	0.209	14.067
8.000	0.072	0.025	-1.193	10.917	0.206	11.192	0.191	15.507
9.000	-0.100	0.025	0.260	13.775	0.131	14.162	0.117	16.919
10.000	-0.010	0.025	-0.058	13.806	0.182	14.195	0.164	18.307
11.000	0.009	0.025	0.471	13.830	0.243	14.220	0.221	19.675
12.000	0.026	0.025	0.527	14.020	0.299	14.419	0.275	21.026
13.000	-0.039	0.025	0.545	14.450	0.343	14.874	0.315	22.362
14.000	-0.110	0.025	-0.641	17.930	0.210	18.557	0.183	23.685
15.000	0.088	0.025	-0.751	20.156	0.166	20.922	0.139	24.996
16.000	0.063	0.025	0.378	21.281	0.168	22.122	0.139	26.296
17.000	-0.057	0.025	-0.224	22.213	0.177	23.119	0.145	27.587
18.000	-0.039	0.025	-0.182	22.646	0.205	23.585	0.169	28.869
19.000	0.087	0.025	-0.691	24.834	0.166	25.944	0.132	30.144
20.000	-0.046	0.025	1.085	25.441	0.185	26.601	0.147	31.410
21.000	-0.066	0.025	0.551	26.696	0.181	27.964	0.141	32.671
22.000	-0.079	0.025	-0.654	28.471	0.161	29.900	0.121	33.924
23.000	0.053	0.025	0.583	29.276	0.171	30.781	0.128	35.172
24.000	0.025	0.025	0.363	29.453	0.204	30.977	0.154	36.415
25.000	-0.079	0.026	-1.277	31.246	0.181	32.954	0.132	37.652

Table 10 – ACF Values, Ljung-Box and Box-Pierce Tests for Monthly Log Returns of Gold

Lag	ACF	ACF Diameter	ACF Test	Box-Pierce Stat.	Box-Pierce p-value	Ljung-Box Stat.	Ljung-Box p-value	Critical Value
1.000	-0.116	0.026	-1.219	3.794	0.051	3.835	0.050	3.841
2.000	-0.043	0.026	0.053	4.307	0.116	4.355	0.113	5.991
3.000	0.013	0.026	0.223	4.354	0.226	4.403	0.221	7.815
4.000	0.008	0.026	-0.083	4.371	0.358	4.420	0.352	9.488
5.000	0.014	0.026	0.223	4.429	0.489	4.480	0.483	11.070
6.000	0.033	0.026	-0.706	4.736	0.578	4.795	0.570	12.592
7.000	-0.033	0.026	-1.136	5.039	0.655	5.109	0.647	14.067
8.000	-0.031	0.026	-0.065	5.305	0.725	5.385	0.716	15.507
9.000	-0.092	0.026	0.254	7.686	0.566	7.862	0.548	16.919
10.000	0.067	0.026	0.068	8.933	0.538	9.165	0.517	18.307
11.000	0.145	0.026	-0.747	14.789	0.192	15.303	0.169	19.675
12.000	0.025	0.026	0.526	14.966	0.243	15.490	0.216	21.026
13.000	-0.067	0.026	-0.769	16.231	0.237	16.826	0.207	22.362
14.000	0.018	0.026	0.852	16.323	0.294	16.923	0.260	23.685
15.000	0.014	0.026	0.441	16.377	0.357	16.980	0.320	24.996
16.000	0.007	0.026	0.272	16.390	0.426	16.995	0.386	26.296
17.000	0.074	0.026	-0.526	17.940	0.393	18.657	0.349	27.587
18.000	-0.006	0.026	-0.325	17.949	0.459	18.667	0.413	28.869
19.000	-0.018	0.026	0.178	18.037	0.520	18.762	0.472	30.144
20.000	-0.021	0.026	0.091	18.164	0.577	18.900	0.528	31.410
21.000	0.115	0.026	-0.675	21.886	0.406	22.952	0.347	32.671
22.000	0.007	0.026	-0.993	21.898	0.466	22.965	0.404	33.924
23.000	-0.016	0.026	-0.903	21.970	0.522	23.044	0.458	35.172
24.000	0.044	0.026	0.173	22.524	0.548	23.654	0.482	36.415
25.000	-0.018	0.026	0.188	22.617	0.600	23.757	0.533	37.652

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Table 11 – ACF Values, Ljung-Box and Box-Pierce Tests for Squared Log Returns of S&P 500

Lag	ACF	ACF Diameter	ACF Test	Box-Pierce Stat.	Box-Pierce p-value	Ljung-Box Stat.	Ljung-Box p-value	Critical Value
1.000	0.234	0.025	9.265	329.776	0.000	329.940	0.000	3.841
2.000	0.291	0.027	10.929	838.941	0.000	839.444	0.000	5.991
3.000	0.284	0.029	9.949	1325.242	0.000	1326.149	0.000	7.815
4.000	0.271	0.030	8.931	1766.586	0.000	1767.934	0.000	9.488
5.000	0.270	0.032	8.480	2205.044	0.000	2206.902	0.000	11.070
6.000	0.281	0.033	8.461	2681.646	0.000	2684.139	0.000	12.592
7.000	0.253	0.035	7.290	3067.719	0.000	3070.790	0.000	14.067
8.000	0.259	0.036	7.216	3471.705	0.000	3475.448	0.000	15.507
9.000	0.236	0.037	6.375	3807.994	0.000	3812.353	0.000	16.919
10.000	0.251	0.038	6.601	4187.188	0.000	4192.304	0.000	18.307
11.000	0.219	0.039	5.612	4476.580	0.000	4482.322	0.000	19.675
12.000	0.225	0.040	5.644	4781.044	0.000	4787.496	0.000	21.026
13.000	0.193	0.041	4.738	5004.353	0.000	5011.363	0.000	22.362
14.000	0.176	0.041	4.277	5191.519	0.000	5199.029	0.000	23.685
15.000	0.192	0.042	4.614	5414.391	0.000	5422.531	0.000	24.996
16.000	0.188	0.042	4.438	5626.258	0.000	5635.034	0.000	26.296
17.000	0.179	0.043	4.184	5819.265	0.000	5828.652	0.000	27.587
18.000	0.184	0.043	4.241	6022.028	0.000	6032.092	0.000	28.869
19.000	0.200	0.044	4.571	6262.944	0.000	6273.850	0.000	30.144
20.000	0.128	0.044	2.895	6362.176	0.000	6373.446	0.000	31.410
21.000	0.183	0.045	4.112	6564.451	0.000	6576.497	0.000	32.671
22.000	0.176	0.045	3.898	6750.146	0.000	6762.936	0.000	33.924
23.000	0.146	0.046	3.217	6879.116	0.000	6892.444	0.000	35.172
24.000	0.132	0.046	2.872	6983.278	0.000	6997.058	0.000	36.415
25.000	0.137	0.046	2.976	7096.277	0.000	7110.566	0.000	37.652

Table 12 – ACF Values, Ljung-Box and Box-Pierce Tests for Squared Log Returns of Gold

Lag	ACF	ACF Diameter	ACF Test	Box-Pierce Stat.	Box-Pierce p-value	Ljung-Box Stat.	Ljung-Box p-value	Critical Value
1.000	0.073	0.026	2.847	31.144	0.000	31.160	0.000	3.841
2.000	0.145	0.026	5.636	154.467	0.000	154.567	0.000	5.991
3.000	0.098	0.026	3.739	211.013	0.000	211.162	0.000	7.815
4.000	0.126	0.027	4.736	303.392	0.000	303.636	0.000	9.488
5.000	0.118	0.027	4.393	385.243	0.000	385.585	0.000	11.070
6.000	0.105	0.027	3.866	450.236	0.000	450.667	0.000	12.592
7.000	0.140	0.028	5.073	564.350	0.000	564.957	0.000	14.067
8.000	0.136	0.028	4.842	671.799	0.000	672.590	0.000	15.507
9.000	0.103	0.028	3.608	733.303	0.000	734.210	0.000	16.919
10.000	0.144	0.029	5.004	853.662	0.000	854.817	0.000	18.307
11.000	0.113	0.029	3.893	928.901	0.000	930.224	0.000	19.675
12.000	0.109	0.029	3.715	998.768	0.000	1000.258	0.000	21.026
13.000	0.108	0.030	3.644	1067.201	0.000	1068.867	0.000	22.362
14.000	0.103	0.030	3.429	1128.872	0.000	1130.708	0.000	23.685
15.000	0.110	0.030	3.638	1199.368	0.000	1201.410	0.000	24.996
16.000	0.132	0.030	4.349	1301.843	0.000	1304.201	0.000	26.296
17.000	0.117	0.031	3.779	1381.167	0.000	1383.784	0.000	27.587
18.000	0.113	0.031	3.633	1455.853	0.000	1458.726	0.000	28.869
19.000	0.083	0.031	2.641	1496.009	0.000	1499.027	0.000	30.144
20.000	0.108	0.032	3.424	1564.119	0.000	1567.395	0.000	31.410
21.000	0.131	0.032	4.111	1663.814	0.000	1667.484	0.000	32.671
22.000	0.082	0.032	2.538	1702.647	0.000	1706.476	0.000	33.924
23.000	0.086	0.032	2.656	1745.533	0.000	1749.546	0.000	35.172
24.000	0.112	0.032	3.440	1818.175	0.000	1822.514	0.000	36.415
25.000	0.092	0.033	2.831	1868.115	0.000	1872.686	0.000	37.652