Master Thesis

Explaining Return Variation in the Frankfurt Stock Exchange:

The Q-Factor Model versus the Fama-French Five Factor Model

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Abstract

This study evaluates the effectiveness of the q-factor model in explaining stock return variation in the German equity market and compares its performance to the CAPM and the Fama and French Five Factor (FF5) model. We base our portfolio construction on the characteristics highlighted by Hou et al. (2015), to create excess returns and factor premiums for the time period from 2004 to 2023. Hou et al.'s q-model includes the market, size, investment, and profitability factor. The methodology consists of testing the model's ability of explaining return variations through regression analysis, aiming for statistically insignificant and zero-alpha intercepts. Our findings suggest that the q-factor model outperforms the CAPM in the German stock market, as indicated by more insignificant alpha values and superior scores in the GRS tests. When comparing the q-factor model to the FF5, results are mixed. While the q-model yields a higher number of insignificant alphas in both portfolio and anomaly approach, it underperforms the FF5 in GRS-test significance. This ambiguity makes it challenging to fully determine whether adding the value factor to the q-factor provides a significant improvement over the q-factor model for explaining stock returns in the German market.

Keywords: CAPM, q-factor model, Fama and French five factor (FF5), Gibbon, Ross and Shanken (GRS), Factor models, German equity market

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1. Introduction

Ever since the introduction of the CAPM in 1964 by William Sharpe, practitioners and researchers have argued about the model's ability to explain stock returns over long consecutive periods of time. When empirical tests did not sufficiently fit the data, theory was used to justify the so-called anomalies. The answer to the question of why abnormal returns could be generated was found in portfolio managers' skill to predict the market. Another argument pointed at the market not being efficient in the first place. In 1993 a groundbreaking paper of Fama and French shed light on the topic of asset pricing. The economists found that a combination of multiple factors can be used, building upon the original work by Sharpe to predict the return variation. Recently a variety of factors have been added to the framework, resulting in the belief that the model is mainly impacted by variable expansion. ETFs with exposure to multiple factors have been published by big issuers like JP Morgan, MSCI and iShares. There are specialized funds as well that make factor investing accessible for private investors.

This proposal builds upon the study from Hou et al (2015), who introduced an extension to older well-renowned factor models. They argue that the q-factors are superior in explaining anomalies in stock returns compared to established factor models such as CAPM, Fama-French 3 factor model, and the Carhart model. The factors introduced in their research are the *market*, *size*, *investment*, and *profitability* factor. Where the profitability factor correlates with the Carhart-momentum factor and the investment factor with the value factor, suggesting that value and momentum are imprecise versions of the q-factors (Hou et al, 2015).

Methodologically, we divide our work into a portfolio approach and an anomaly approach as in the original study. First, we create specific factors by constructing portfolios based on the characteristics introduced by Hou et al. and Fama and French. We regress the excess returns on the factor premiums generated in the period from 2004-2023. Contrary to the CAPM intuition, the alpha coefficients of the model are the objective criterion to be minimized. To limit the pricing error not explained by the model, it must be statistically insignificant and close to zero. Simultaneously the explanatory power of the q-factors will be compared to the Fama French 5 factor model. The second part of the methodology consists of testing our factors from both models against 151 anomalies from Jensen et al. (2023).

With our study we aim to deliver a better understanding of the q-factor model's ability to explain return variation on the German stock market. The different period and the out-of-sample data will contribute to the ongoing academic understanding of asset pricing. Simultaneously, the Fama and French and CAPM model will be tested alongside the q-model as a direct comparison and benchmark, to draw more meaningful conclusions about the model's applicability. The broad anomaly literature provides many possible factors that could generate return premia, but the excessive number of factors obscures the perception of which factors actually matter. Therefore, the anomaly approach rules out a quantity of anomalies that do not explain average returns.

Our research questions are formulated as:

I) Can the q-factor model gain significant results out-of-sample in Germany?
II) How does the model compare to the Fama and French 5 factor model?

Our findings are mostly in line with studies conducted on other developed markets. Firstly, the q-factor outperforms the CAPM model suggesting that it does gain significant results in the German stock market. The q-factor model has both more insignificant alphas in the portfolio approach and the anomaly approach. It does also perform better in the GRS-tests compared to the CAPM model meaning that the alphas are not significantly different from zero. Second, changing the construction of the factors slightly and adding the value factor (FF5) does show ambiguous results of performance. FF5 has a lower number of insignificant alphas in both the portfolio approach and anomaly approach but outperforms the q-factor model in the GRS-test.

Our main conclusion is in line with the original study from Hou et al (2015), where the q-factor model is comparable to or outperforms the CAPM, i.e. the market factor. However, adding the value factor and changing the construction of the profitability factor shows ambiguous results of improving the model. Thus, a conclusion on whether the FF5 is better than the q-factor model at explaining returns on the German market cannot be drawn.

2. Literature Review

Sharpe (1964), Lintner (1965), and Black (1972) are portrayed as the pioneers of factor investing and asset pricing. Their CAPM model is the foundation that led to the discovery of the three-factor model by Fama and French (1993) (*FF3*). Next to the market factor a size and value factor is used to capture the difference in returns between small and big companies in terms of market capital and high versus low book-to-market equity ratios. Carhart (1997) extends the model by adding momentum as a fourth factor. The intuition behind it is that companies that perform well will continue to do so and vice versa, which is reflected in the firm's stock returns.

Chen et al. (2010) proposes an alternative three factor model to capture the later anomalies such as momentum. Next to the market factor they use a variation of the investment and profitability factor to successfully explain average stock returns on the US market. Walkshäusl and Lobe (2011) tested the model out-of-sample in 40 international markets over a period of 27 years. The investment factor carries a considerable risk-premium in Germany but overall, the model does not perform well in the chosen setting outside of Germany. On average the investment and profitability factors have a loading close to zero and thus the model is similar to the CAPM in explaining returns, the FF3 on the other hand does so with higher significance.

Novy-Marx (2013) conducts research on the anomaly strongly related to firm level returns, which is profitability. Titman et al. (2013) observes international evidence connected to investments. They show that investment growth has a negative relationship with a stock's return. Hou et al. (2015) connect these two factors under the Q investment theory and add them together with the size and market factor to their model. The authors conclude that their combination of factors is superior compared to the FF3 and the Carhart model. They additionally study the anomalies in stock returns. Among 80 tested, about half of them are insignificant in earning excess returns.

Simultaneously to Hou et al., Fama and French (2015) incorporate investment and profitability factors to their FF3 model as well, ultimately obtaining the (*FF5*) model. The authors differ slightly in their definition of the firms' characteristics, out of which the factors are constructed. They find that when adding profitability and investment, the value factor becomes irrelevant

which could imply four factors to be sufficient. In (2018) Fama and French add the momentum factor to their five-factor model (*FF6*). They also provide an alternative approach to validating nested factor models, with similar factors compared to those with separate ones. The squared Sharpe-ratio for a model's factor's is maximized to optimally explain the excess returns of the sample period from (1963-2016). They find that the FF6 has the highest value in capturing variation. Nevertheless, the authors acknowledge that adding the momentum factor is relatively random. They postulate that to fight against data mining in factor research, one should concentrate on factors with sound theoretical foundations.

Recently Hou et al. (2021) also augment their factors with expected investment growth to yield the q^5 model. They propose that if expected future investments are high, so must be the present value of future cash flows (holding current investments and profitability constant). The 1-year-ahead-investment-to-asset factor earns an average monthly premium of 0,84% with a high significance (t=10,27). The q^5 beats the FF6 in explaining a set of 150 anomalies. Lin (2021) aims to incorporate the growth factor into the intertemporal CAPM to test its consistency, the results show that the expected growth factor does not explain future expected stock returns positively. The author concludes that the new factor is not a suitable risk factor in Merton's (1973) ICAPM. Hence, the factor can only be intuitively derived from Tobin's investment theory and further empirical research has not yet been conducted. One of the main concerns of this study is therefore to concentrate on differences between the FF5 and the q-model, adding clarity to the question of how many factors are needed to capture most of the cross-sectional variation and what they must consist of.

Out-of-sample Chiah et al. (2016) successfully test the FF5 on the Australian market, contradictory to the original study they find the value factor to have statistical and explanatory significance, justifying further studies. Fama and French (2017) implement the five factors globally, including Europe with 14 countries, among them Germany. The model is appropriate in capturing generic return variation but fails to account for low returns in small stocks with weak profitability and aggressive investment behavior. Grobys and Huhta (2019) connect the value and momentum effect in the Nordic markets. They find significant excess returns generated by the momentum effect while the value effect is decreased when controlled for size, suggesting that the size factor is a better fit in these European markets. Pandey and Joshi (2022) test the FF5 in western European markets, 2007-2018. They conclude that the German market is valid

for the application of FF3 and FF5, however they find the Carhart model not deployable on all the sample-markets. Comparing the FF5 versus the q-model in Germany across a longer time frame is therefore a compelling research direction.

Concerning the q-factor model's external validity on developing markets Asad and Cheema (2017) find significant and negative effects for all high-investment and simultaneously low-profitability portfolios, in line with the original study, on the Pakistani market. Charoenwong et al. (2021) confirm the performance on the Thai market against the FF6. For developed markets, Maeda (2017) on the other hand finds that the q-factors are not relevant on the Japanese market, outperformed by the FF3. The value factor in particular yields significant outcomes, supported by previous research on this market. The Hou et al. investment factor, which is supposed to be correlating with the factor is insignificant, thus suggesting the inadequacy of the q-model in Japan. Özkan (2020) studies the Turkish market with regards to the q-model, it outperforms a variety of different models, among them the FF3, Carhart, Pástor and Stambaugh (2003) model and the FF5, hereby proving the frameworks value and applicability in Europe. Overall research for the q-factors on developed markets is scarce and therefore another out-of-sample study could lead to interesting insights on the capital asset pricing puzzle.

Anomaly research has been gaining academic relevance recently, Hou et al. (2020) study pricing patterns in the US-American setting, they conduct a single hurdle test on 452 anomalies for t-values above 1,96. 65% of these anomalies are insignificant in their difference to the population mean on a 5% significance level, suggesting limited replicability of factors when controlling for microcaps. Conversely, Jensen et al. (2023) postulate a high replicability of factors in scientific applications and the use of multiple factors. In their approach they develop a Bayesian model, measuring the percent of replicated anomalies yielding statistically significant excess returns. The factors from Hou et al. achieve a replication rate of 35%. Additionally, Jensen et al. include 15 anomalies from the literature and remove those from Hou et al. that were not significant in others research, which resulted in a rate of 61%. The reason suggested for this considerable difference is thought to be a more conservative approach in constructing the factors when it comes to the percentile breakpoints and accounting measures. Adding the CAPM-alpha to the model, increases the replication rate to 82%, proving a high replicability and relevance for factor models. With their research Jensen et al. contribute to the global understanding of anomalies and deliver a database we later use to test against our factors.

3. Theoretical Framework

3.1 The Market Factor

In the q-factor model, the market factor (MKT) is directly taken from Fama and French (1996 a) and is calculated by subtracting the risk-free-rate from the market return. First introduced by Sharpe (1964) and Lintner (1965) the CAPM states that in equilibrium, there is a positive linear relationship between expected returns and the standard deviation of returns for efficient combinations of assets. This means that a maximum return can be yielded given a certain amount of risk, found on the efficient frontier. This relationship can be applied to individual assets. Expected returns increase with systematic risk, which explains concepts such as lower expected returns from conservative, defensive investments. A market beta of one is consequently displaying the same risk-return relationship as the representative market portfolio. Ross (1978) questions the robustness and applicability after a decade of CAPM existence, Banz (1981) finds that CAPM is mis specified and Reinganum (1981) and Fama and French (1996 b) confirm its lack of reliability in explaining average returns. Hou et al. (2015) and Fama and French (2015) both find that the market factor captures variation in returns, but still leaves a considerable amount to be explained by other factors.

3.2 The Size Factor

Fama and French (1993) argue that size (SMB) can help as a proxy for common risk factors in stock returns since it relates to the firm's fundamentals. Small stocks have undergone an earnings depression in the 1980s that stemmed from the recession in the beginning of the decade. Big companies were not affected in terms of earnings at that time, which led to the belief that a negative relationship between size and average returns could be caused by expected risk premia for small stocks. Fama and French's (1992) previous research about factors investigates portfolios exclusively formed on size. They find a negative relationship between size and average return, where smaller market capitalization portfolios show higher average returns than larger ones. The analysis shows no clear relation between the CAPM beta and average returns when controlled for size, suggesting that size is an anomaly causing variation in the cross-section. Banz (1981) observed that small NYSE stocks significantly outperformed large caps in risk-adjusted returns over a forty-year period.

Reinganum's (1981) findings about the disappearance of the Price to Earnings (P/E) effect when controlling for size and the significant size effect when controlling for P/E, suggests that P/E is a proxy for size. The fact that this anomaly was observable over longer time periods, recommended against the inefficient market hypothesis, postulated at that time. Heston et al. (1999); Bauer et al. (2010) and Amel-Zadeh (2011) confirm the persistence of the size effect in European markets, among them Germany. Crain (2011) concluded that the size effect lost its influence over time and has ceased to exist beginning from the 1980s in most major markets. The author argues that size could in fact be a proxy for other anomalies related to size just like liquidity, which is mitigated by small-cap funds. Such appeared around that time, offering more capital to small firms, hence diminishing the potential risk premium for investors. Hou et al. (2015) state that the size factor is only secondary in explaining returns and is mainly included to control for other anomalies related to size.

Hypothesis 1: Small firm portfolios earn higher average returns than large firm portfolios.

3.3 The Investment Factor

In their original study Hou et al. (2015) connect profitability and investment under the Q-theory from Tobin (1969). The driving force behind it is that high profit firms take lower investments compared to low profit ones (holding profits fixed). Counterintuitively, it means a high rate of return for the shareholder stems from the firm's lower investments. Vice versa, if the company expands by adding assets, theoretically less dividends can be distributed to investors. Therefore, firms with high investments have lower expected stock returns for the shareholder (Hou et al., 2017).

Regarding the factors robustness Titman et al. (2004) are among the first to report a negative relationship between the capital investment and the stock's return. This abnormal relationship cannot be explained by other specific firm characteristics, risk or e.g., the secondary equity issue anomaly. They argue that companies with high cash flows or low debt ratios have a greater likelihood to over-invest and thus decrease profitability (empire building hypothesis). Conversely, investors react around earnings announcements, suggesting that the negative effect on returns is a surprise reaction, underestimating the ongoing empire building itself. Fama and French (2006) derive an explanation for the negative investment-return relation from the

Gordon-growth model, where growth in book-equity yields a lower expected stock return. They also back-test their theory using asset growth as a proxy for investment, confirming the effect. Liu et al. (2009) test the factor and conclude that companies plan their investment strategy according to their specific cost of capital, thus explaining several anomalies in cross-sectional returns. Cooper et al. (2008) find a highly significant return spread between low and high investment firms of almost 20% on the US market. Our second hypothesis hence postulates:

Hypothesis 2: Low investment firms earn higher average returns compared to high investment firms.

3.4 The Profitability Factor

For the profitability factor Hou et al. (2015) reason in an inverse, dependent relationship to investments. A higher expected profitability is equal to higher expected returns and vice versa, ceteris paribus. Additionally, the momentum, the post earnings-announcement drift and the financial distress effect are described by the factor. The reasoning behind it is that momentum winners have high expected profitability, positive profitability shocks lead to immediate stock price increases and financial distress harms profitability. Fama and French (2006) confirm the profitability effect, based on the use of operating profits as a proxy for profitability. Novy-Marx (2013) deploys gross profits (revenues minus costs of goods sold) in relation to the firm's assets to measure profitability and find similar results compared to Fama and French. The paper also states that gross profits are a "cleaner" measure of true profitability, less sensitive to accounting distortions and investment expenses when compared to earnings and free cash flows.

Artmann et al. (2012) study the performance of the FF3 and the Carhart model in Germany between 1963-2006. While the Carhart model performs well, a modified version that swaps out the size for an earnings-to-price factor is even more precise in explaining returns, suggesting that profitability effects play a substantial role in Germany. A possible explanation for the relevance of earning could be related to the German accounting standards. Hypothesis three states:

Hypothesis 3: High profitability firms generate higher average returns compared to low profitability firms.

3.5 The Value Factor

The fifth factor to the FF5 model is value (HML), based upon the original work from Fama and French, introduced in (1993). It is based on the concept that high book-to-market equity firms earn a value premium compared to low book-to-market equity. Value companies are assumed to have higher relative profitability and lower financial distress. The authors find that HML produces large average return premia which are statistically significant. However, factors like value and size are empirically motivated and without a firm theoretical foundation, more ambiguous than scientifically sound. First researched by Stattman (1980); Titman and Jegadeesh (1993), the value effect has been confirmed on the US market over several periods (see Titman and Jegadeesh (1993); Fama and French (1993); Fama and French (2015)). In their study on the investment and profitability phenomenon Fama and French (2015) point out that size is one of the main drivers of the value effect. They find micro-cap portfolios to gain an average excess return difference between value and growth stocks of 89 basis points, while mega cap portfolios differ with 14 basis points. In the Nordic equity market, Grobys and Huhta (2019) only find significant excess returns for value stocks when including small stocks in their portfolios. The last hypothesis postulates:

Hypothesis 4: The difference between value and growth firms is insignificant in explaining excess returns

4. Data

The stock data is taken from Refinitiv Eikon Datastream. The sample consists of 456 companies acquired from the electronic stock exchange platform Xetra in Frankfurt. The company's total return index is obtained to compute the monthly yield for each stock, adjusted for splits and dividends. Additionally, the firms monthly market capitalization is directly retrieved from Refinitiv Eikon. Total Assets and ROE are not available monthly or quarterly, so instead yearly data is taken. As a proxy for the risk-free rate the 3-month interbank rate for Germany is downloaded from the St. Louis FED. The market portfolio is displayed by the DAX[1] total return index, consisting of the leading German stocks. For the data collection of Fama and French's 5 factors, the monthly book-to-market equity ratio is retrieved from Refinitiv. The market and the size factor are taken from the data for the q-model due to them being equal. The FF5 profitability factor applies a different construction compared to the q-factor model. For simplicity the same data as in Hou et al. is used as a proxy. All the anomalies from JKP factors are downloaded for the research period using monthly value weighted returns[2].

Appendix figure [A.1] depicts the total number of sample companies with available data each month, used for our portfolio construction. Stocks that have a missing value in any of the characteristics are eliminated in the specific month. In the beginning of the time frame many of the current stocks were not yet listed and thus dropped out of the sample, ultimately reducing portfolio size. This could potentially lead to biased portfolios with too little stocks to be considered a diversified portfolio. Our portfolios never contain less than 11 companies from a total minimum of 213 companies, thereby we are in line with the criteria form Jensen et al. (2023) to require a minimum of 5 stocks in each portfolio and 60 complete observations for each month to be included in our entire sample[3]. To fulfill this requirement, we altered our initial window of study of 30 years that was first picked analogously to Fama and French (1993).

[1]The DAX consists of 40 constituents, representing around 80% of listed companies' market capitalization and realizing 90% of the exchange turnover in Germany (Deutsche Börse Group, 2024).

[2]For factor data and details on factor construction see https://jkpfactors.com

[3] Our data only consists of companies still listed today, leaving the possibility of survivorship bias unsolved. Davis (1996) finds an upward pointing overstatement effect in the estimators when controlling for failing companies but not necessarily a spurious regression.

5. Methodology

5.1 Variable Calculation

The monthly returns for the individual stocks are calculated by using the total return index constructed by Refinitive Eikon, the index is adjusted so that dividends are assumed to be re-invested to purchase additional equity at the closing price on the ex-date. The monthly returns are then calculated by the following equation:

$$Return_t^i = \frac{Total\ return\ index_t^i}{Total\ return\ index_{t-1}^i} \tag{1}$$

The market cap variable is calculated by Refinitive monthly by taking the number of shares (NOSH) multiplied with the price (P) of the stock:

$$ME_t^i = NOSH_t^i * P_t^i \tag{2}$$

Investment over assets is simply calculated by dividing total assets at the current year t by total assets at year t-1:

$$\frac{I}{A} = \frac{Total\ assets_t^i}{Total\ assets_{t-1}^i} \tag{3}$$

The yearly return on equity (ROE) from Refinitiv is taken as a profitability proxy for the Compustat variable from Hou et al.. It is calculated by subtracting the bottom line (BI) and preferred dividend requirement (PDR) from net income (NI) and then divide it by the average between last year's and current year's common book equity (CBE):

$$ROE_t^i = \frac{NI_t^i - BI_t^i - PDR_t^i}{\frac{CBE_t^i + CBE_{t-1}^i}{2}} \quad (4)$$

Lastly the fifth characteristic used in the FF5 model is constructed as in Fama and French (1993) by dividing the book equity by the market equity:

$$Book\ to\ market_t^i = \frac{BE_t^i}{ME_t^i} \quad (5)$$

5.2 Factor Construction

The sample data is divided into portfolios based upon the factor characteristics. The rebalancing of the portfolios is done yearly at the first of June, when all the annual reports are published. However, the returns will be calculated monthly for the portfolios. Following the original paper, we separate the data on a 2-by-3-by-3 sorting as illustrated in Figure 1. The first divide is by market equity (ME) where the cutoff point is the 50th percentile. The second divide is on investments over assets (I/A), where we use the NYSE breakpoints at the 30th and 70th percentile. Lastly, the return on equity (ROE) sorting will have the same breakpoints which gives us a total of 18 portfolios as displayed below.

Figure 5.1 Portfolio dividing

		ME					
		50th			50th		
I/A	30th	Portfolio 1	Portfolio 2	Portfolio 3	Portfolio 10	Portfolio 11	Portfolio 12
	30th-70th	Portfolio 4	Portfolio 5	Portfolio 6	Portfolio 13	Portfolio 14	Portfolio 15
	70th	Portfolio 7	Portfolio 8	Portfolio 9	Portfolio 16	Portfolio 17	Portfolio 18
		30th	30th-70th	70th	30th	30th-70th	70th
		ROE			ROE		

Figure 5.1 displays the dividing of the portfolios for the q-factor model, where the first dividing is done on ME, the second on I/A and last dived is based on ROE.

The return for the portfolio (i) is calculated by using value weighted returns. By weighting small and midcaps lower, distorted, and fluctuating returns for the aggregated portfolio are avoided.

$$R_t^i = \sum_{j \in D_{it}} \frac{ME_t^i}{\sum_{j \in D_{it}} ME_t^i} Return_t^i \tag{6}$$

Starting at the RHS of figure [5.1], we see that our sorting yields 9 portfolios with small ME and 9 with big ME on the LHS. Row one displays the 6 portfolios with low I/A and row three shows the 6 with high I/A. Column three and six depict the 6 portfolios with low ROE and column one and four the 6 with high ROE.

First, to create the market factor (r_{MKT}) we subtract the risk-free rate from the market return each month to calculate the market risk premium. To compute the size factor (r_{ME}), we take the simple average of the 9 small ME portfolios minus the simple average of the 9 big ME portfolios (small-minus-big). This is equivalent to taking a long position in the small ME portfolios and a short position in the big ME portfolios. Subtracting the 6 low from the 6 high I/A portfolios (low-minus-high) yields the investment factor ($r_{I/A}$). Conversely, to create the ROE factor (r_{ROE}) the 6 high ROE portfolios average is subtracted from the 6 low ROE portfolios average (high-minus-low). The average return for all factors is updated monthly (Hou et al. 2015).

Concerning the construction of the FF5 factors the value factor is added based on the characteristics by Fama and French (1993). To obtain portfolios like Fama and French's (2015) approach, we construct the factors size, value, profitability and investment with a 2x3 sorting as well[4]. Starting iteratively with first the size at the 50th percentile and then taking divides for the next factor at the 30th and 70th percentile similar to Hou et al. (2015), results in 18 portfolios. The size factor (r_{SMB}) is constructed taking the average of the 9 small size portfolios, minus the average of the 9 big size ones, from the value, profitability and investment factor sorting. The average of these 9 SMB sub-factors results in the main size factor. The value factor (r_{HML}) is obtained by taking the difference between the two high minus the two low book-to-

[4] See Fama and French (2015) p. 6 table 3

market portfolio's average. The profitability factor (r_{RMW}) is a result of the two robust profitability minus the two weak profitability portfolios. The investment factor (r_{CMA}) combines the two conservative portfolios minus the two aggressive investment portfolios average returns.

5.2 Portfolio Approach

Instead of estimating expected stock returns $E[R_i]$, observable historical stock data is used as a proxy for real-world market performance. The model can therefore be tested for empirical validity. Single security choice as a random variable leads to unbiased betas, even under the assumption that returns are not entirely normally distributed. To gain efficiency for the beta estimators on the other hand, the portfolio approach must be used. Sorting securities into groups combines different values of beta, thus yielding higher statistical power and robustness. This approach generally allows for more efficient testing and estimation of multifactor asset pricing models compared to random security selection (Jensen at al., 1972).

Blume (1970) derives that the usage of portfolios can help to avoid idiosyncratic, stock specific risk. Individual negative effects can offset each other if clustered into a group of assets, leading to a diversified risk exposure of portfolios. To find the optimal return given a combined portfolio risk level, an efficient frontier can be obtained. Different sets of risk-return relationships of stocks are maximized with respect to the return, given a specific risk, resulting in an optimal, balanced portfolio.

Fama (1996) extends the classic CAPM mean-variance portfolio with a multi-factor approach to risk, adding different sources that influence stock returns. The paper shows that market portfolios are in fact not multi-factor efficient by default and should be complemented with portfolios that mimic the risk exposure to additional state variables. By doing so, the specific sensitivity to these factors and the risk premia can be estimated.

5.3 Anomaly approach

To contribute to the ongoing empirical studies on factor investing we conduct an analysis on a set of 151 anomalies taken from Jensen et al.'s (2023) website. These monthly excess returns, collected for the German market will be regressed in an identical way as our portfolio returns, against our constructed factors. The return data consists of non-microcaps and is value weighted for maximum comparability. Hou et al. (2015) conclude that the significance of many of the observed anomalies in the US market are due to the wrong weight associated with microcaps and when accounting for those, half of the anomalies disappear in the q-model. The reason for that is the unique characteristics of these companies. The NYSE-AMEX-NASDAQ consists of 60% micro caps but they only account for around 3% on average individually. Transaction costs and low liquidity hinder the investor to benefit from these return patterns.

Jensen et al. (2023) report a variety of anomalies, among them are factors of accruals, debt issuance, low risk, low leverage, momentum, profit growth, quality and seasonality and short-term reversal. Fama and French (1996) define anomalies as patterns in average returns that can't be explained by the CAPM. They argue that many of the anomalies are in fact related to each other and can thus be captured by the FF3. Based on that the data snooping hypothesis is firstly refuted. It states that anomalies are sample specific and should boil down to the normal CAPM market factor if different time periods and countries are tested.

When increasing amounts of research towards the cross section of stock returns was published, questions about its validity emerged. Harvey et al. (2016) postulate that many of these findings are based on data mining and that t-statistics should have to clear a higher hurdle ($t > 3$) to bring order to what Cochrane (2011) describes as: "a zoo of new factors". Harvey et al. propose that the probability of discovering a true new factor is likely decreasing as time moves on. Also, since data on the CRSP and other databases is rather limited, the outcomes are as well, compared to other fields of science where fresh data sets are available daily. Lastly because of modern computing, data mining has become decreasingly time consuming and thus also anomalies without actual theoretical background are tested. Hence the outcomes of our analysis are to be interpreted with caution and attention to detail.

5.4 Data Analysis

To gain knowledge about the statistical properties of the factors constructed from the portfolios in the prior steps, a multivariate regression analysis is firstly conducted using the q-factors.

$$r_t^i - r_t^f = \alpha^i + \beta_{MKT,t}^i \, MKT_t + \beta_{ME,t}^i \, r_{ME,t} + \beta_{\frac{I}{A},t}^i \, r_{\frac{I}{A},t} + \beta_{ROE,t}^i \, r_{ROE,t} + \epsilon^i \quad (7)$$

Equation (7) shows the regression run via Matlab where the exponent (i) denotes the portfolio i's value at month (t). Hence, ($r_t^i - r_t^f$) is the portfolio's return exceeding the risk-free rate, used as the dependent variable. The intercept is displayed by (α) which captures the variation, left unexplained by the regressors. The coefficients ($\beta_{MKT,t}^i$, β_{ME}^i, $\beta_{I/A,t}^i$, $\beta_{ROE,t}^i$) summarize the portfolio's sensitivity to the specific factor return (MKT_t, $r_{ME,t}$, $r_{I/A,t}$, $r_{ROE,t}$) as independent variables. Lastly the error term (ϵ^i) quantifies the residuals (Asad and Cheema, 2017). The methodology undertaken consists of regressing average monthly excess returns from all 18 of the constructed portfolios against the monthly factor returns. The alpha coefficients must be zero and should be statistically insignificant for the model to be well specified and we assume that the error term is equal to zero on average as well, with no heteroskedasticity and serial correlation. Subsequently a Breusch-Pagan test is conducted to rule out portfolios with heteroscedasticity and if found to be the case a robust-error OLS is run. If the Breusch-Godfrey test shows signs of serial correlation in the residuals, a combination of robust and constant error OLS is used.

Regarding the FF5-model a similar approach is taken as seen in equation (8).

$$r_t^i - r_t^f = \alpha^i + \beta_{MKT,t}^i \, MKT_t + \beta_{SMB,t}^i \, SMB_t + \beta_{HML,t}^i \, HML_t + \beta_{RMW,t}^i \, RMW_t + \beta_{CMA,t}^i \, CMA_t + \epsilon^i \quad (8)$$

The dependent variable is measured based on the monthly average excess returns of the specific FF5 portfolio. The alpha, beta-market and the error term have the same meaning as in equation [7], while SMB (small-minus-big), RMW (robust-minus-weak) and CMA (conservative-minus-aggressive) are the Fama-French factors with their respective beta's ($\beta_{SMB,t}^i, \beta_{HML,t}^i, \beta_{RMW,t}^i, \beta_{CMA,t}^i$). The model's precision is again described by insignificant, zero alpha values.

This so-called zero-alpha hypothesis from Fama and French (2015) is based on Merton's (1973) model, according to which up to four additional state variables with risk premia not explained by the market factor could exist. The state variables are unknown, but factors can combine diversified portfolios with exposure to them, thereby forming a multifactor efficient portfolio that captures the expected returns without identifying the underlying state variables. The total magnitude and the p-value of the alphas ultimately explains if the pricing error is equal to zero and displays the model's performance in explaining the diversified portfolio's returns. Therefore, it is the first tool in assessing the specification and validity of the model and hence an essential part of the analysis.

Prior to that the data is tested for heteroscedasticity with a Breush-Pagan test. Having heteroscedasticity present in the OLS regression causes a substantial loss of efficiency in the model and also causes bias in the estimated standard errors (Breusch and Pagan, 1979). We also test the data for autocorrelation by conducting a Breusch-Godfrey test with a lag of one. If there is autocorrelation in the regressions there might be issues with drawing incorrect conclusions (Breusch, 1978). Both tests are using the significance level of a=0,05 meaning if the p-value is below 0,05 we reject the null hypothesis, i.e our regression suffers from heteroscedasticity and autocorrelation. If these assumptions are violated, we will use HAC (heteroskedastic and autocorrelation consistent) errors that overcome these issues by calculating a heteroskedastic and autocorrelation consistent covariance matrix (Newey and West, 1987), this is often used for time series data like this study. The Breusch-Pagan test results (Appendix table [A.1]) show that 8 portfolios of the q-factor model have heteroscedasticity present and 7 of the FF5 portfolios. We therefore account for the violation of the homoscedastic residual OLS assumption by using robust standard errors. From the results of the Breusch-Godfrey (Appendix table A.2) test 8 of the q-factor and 10 of the FF5 portfolios have autocorrelation present.

We conduct a Gibbon, Ross and Shanken GRS test for both the q-factor and FF5 models, to assess if the alphas from the regressions of the models against their respective portfolios are jointly, significantly different from zero. If the null hypothesis is rejected at the 5% level there is evidence for non-zero alphas, resulting in the model's loss of specification (Gibbon, Ross and Shanken, 1989). The GRS test is thereby a summary statistic that evaluates if all our portfolio returns can be captured by the model's factors over the time series of regressions and thus if the factors can generally be applied for our set of data.

6. Results

6.1 Descriptive Statistics

The descriptive results (table [6.1]) for the individual portfolio's mean and volatility shows that portfolios with low ME seem to experience a higher mean return in the q-factor (1-9) and FF5 (19-21, 25-27 and 31-33) model. Portfolios with low investments also show a pattern of a higher mean return compared to the portfolios with higher investments. The q-factor's profitability portfolios have a slight negative pattern with higher profitability which is even clearer in the FF5. The value (19-24) portfolios seem to possess a greater mean return when value is high. The average mean between the two models is similar but the portfolios in the q-factor model have higher volatility.

Table 6.1: Descriptive statistics: monthly mean and standard deviation of portfolios

FF5			Q-factor		
Portfolio	Mean	Standard deviation	Portfolio	Mean	Standard deviation
LL-HML (19)	0,43%	5,95%	LLL (1)	2,56%	7,84%
LM-HML (20)	1,90%	5,08%	LLM (2)	2,13%	5,50%
LH-HML (21)	3,12%	5,64%	LLH (3)	1,73%	6,47%
HL-HML (22)	0,52%	6,95%	LML (4)	1,96%	6,16%
HM-HML (23)	1,03%	5,44%	LMM (5)	1,49%	5,24%
HH-HML (24)	1,68%	5,26%	LMH (6)	1,84%	5,66%
LL-RMW (25)	2,28%	5,92%	LHL (7)	1,90%	7,38%
LM-RMW (26)	1,86%	5,38%	LHM (8)	1,89%	6,12%
LH-RMW (27)	1,70%	4,97%	LHH (9)	1,39%	5,60%
HL-RMW (28)	1,22%	5,78%	HLL (10)	1,35%	6,48%
HM-RMW (29)	1,11%	5,24%	HLM (11)	1,03%	5,76%
HH-RMW (30)	0,90%	5,49%	HLH (12)	1,05%	5,81%
LL-CMA (31)	2,14%	5,54%	HML (13)	1,22%	5,78%
LM-CMA (32)	1,76%	5,12%	HMM (14)	1,11%	5,24%
LH-CMA (33)	1,73%	5,54%	HMH (15)	0,90%	5,49%
HL-CMA (34)	1,14%	5,28%	HHL (16)	0,92%	6,89%
HM-CMA (35)	1,08%	5,21%	HHM (17)	1,05%	7,02%
HH-CMA (36)	0,92%	6,18%	HHH (18)	0,79%	6,82%
Mean	1,47%	5,55%	Mean	1,46%	6,18%

Table 6.2 reports the monthly average of the factor returns in percent. The CAPM market factor earns a premium of 0,68% and the size factor earns a noticeable 0,83%. The two q-factors investment and profitability yield a factor return of 0,32% and -0,37% respectively. The Fama and French size factor earns the same premium as the Hou et al. due to the identical construction. The value factor results in the highest premium with a monthly average of 1,93%. The FF5 investment factor gains a mean return of 0,32% analogously to the q-investment factor. The profitability factor earns a negative average of -0,45%.

Table 6.2: Average monthly factor returns

	mkt	r_{ME}	$r_{I/A}$	r_{ROE}	SMB	CMA	RMW	HML
mean	0,68%	0,83%	0,32%	-0,37%	0,83%	0,32%	-0,45%	1,93%
std	5,50%	3,08%	2,81%	2,53%	3,08%	2,81%	2,79%	3,66%
t-statistic	1,8612	4,0647	1,7196	-2,1910	4,0647	1,7196	-2,4469	7,9775

The correlation matrix (table [6.3]) shows that the market factors (rME and SMB) in the q-factor model and FF5 fully correlate with each other, since they are constructed the same. This is also the case for the investment factors (rI/A and CMA). The profitability factors (rROE and RMW) have a high correlation (0,84), the slight difference stems from the alternative construction using a 2x3 sorting.

Table 6.3: Correlation matrix between the monthly returns of the q-factors and the FF5 factors

	rI/A	rROE	SMB	CMA	RMW	HML
rME	0,1596	-0,1959	1,0000	0,1596	-0,2285	0,1764
rI/A		-0,1975	0,1596	1,0000	-0,3387	-0,0541
rROE			-0,1959	-0,1975	0,8413	0,0521
SMB				0,1596	-0,2285	0,1764
CMA					-0,3387	-0,0541
RMW						0,0708

6.2 Q-factor Portfolio Approach

The results from the time series regression of the q-factors are presented in table 6.4. The alpha values are spanning from 0,21% to 0,64% per month but are significant for half of the portfolios. The market factor is significant and positive for all portfolios with values around a mean of 0,9. The size factor exhibits significance for the small market equity (ME) portfolios (portfolio 1-9), with portfolio (10) as a significant outlier with the expected sign. The investment factor yields negative factor loadings for mid and high I/A portfolios and positive ones for low investments. For the profitability factor (ROE) all low portfolios have a significant negative coefficient. The mid portfolios vary as for the other factors and the high profitability portfolios have positive and significant return sensitivities except portfolio (9). The aggregated measure of fit (adj-R2) has a value of 0,72.

Table 6.4: Regression of q-factor portfolios against the q-factors

	LLL (1)	**LLM (2)**	**LLH (3)**	**LML (4)**	**LMM (5)**	**LMH (6)**	**LHL (7)**	**LHM (8)**	**LHH (9)**
α	0,0037	0,0072*	0,0031	0,0056*	0,0026	0,006*	0,0021	0,0056*	0,0028
β_{MKT}	1,0107*	0,7981*	1,0484*	0,8987*	0,8924*	0,9313*	0,9364*	0,9598*	0,8435*
β_{ME}	1,2435*	0,8674*	0,9597*	0,8439*	0,8363*	0,9388*	1,2269*	1,0817*	0,8754*
$\beta_{I/A}$	0,4732*	0,4102*	0,3961*	-0,1028	-0,1303	-0,1339	-0,9497*	-0,5223*	-0,4365*
β_{ROE}	-0,8787*	-0,0305	0,5816*	-0,3376*	0,0905	0,3583*	-0,9311*	0,1391	0,1567
Adj-R2	0,6634	0,5951	0,6589	0,611	0,757	0,7042	0,7593	0,7352	0,658

	HLL (10)	**HLM (11)**	**HLH (12)**	**HML (13)**	**HMM (14)**	**HMH (15)**	**HHL (16)**	**HHM (17)**	**HHH (18)**
α	0,0052*	0,003	0,0031	0,0041*	0,0053*	0,0031*	0,003	0,0064	0,0054*
β_{MKT}	0,8681*	0,9135*	0,9242*	0,946*	0,9061*	0,9383*	0,9533*	0,9424*	0,9274*
β_{ME}	-0,2246*	0,0154	0,1748	-0,0205	-0,0389	0,1155*	-0,0286	-0,095	-0,0239
$\beta_{I/A}$	0,5263*	0,2111*	0,2466*	0,0048	-0,0045	-0,1529*	-0,623*	-0,6138*	-0,5905*
β_{ROE}	-0,701*	-0,0637	0,3049*	-0,4955*	-0,0036	0,2594*	-0,5146*	-0,1176	0,48*
Adj-R2	0,7182	0,741	0,6844	0,8861	0,923	0,8645	0,7378	0,6831	0,707

Table 6.4 displays the coefficients for the respective factors for all 18 portfolios constructed in the q-factor model. Coefficients assigned with a * are statistically significant on a 5% significance level. The letter L, M and H stand for Low, Mid and High where the first letter is the ME divide, the second is I/A and the third ROE e.g. portfolio LMH is divided into low ME, mid I/A and high ROE.

To assess the precision of the q-factor model in comparison with the benchmark model we regress our portfolios against the CAPM market factor. In figure 6.1 we compare the magnitude of the alphas for the respective portfolios between the CAPM and the q-factor model. We can see that the values for the CAPM model are greater than the q-factor model's and that they are not as stable in the q-factor model. We see higher intercepts in the first 13 portfolios for the CAPM and a trend where the difference decreases when moving towards the RHS. When looking at the significance of the intercepts from both models the CAPM exhibits 4 insignificant alphas (Appendix Table [A.5]) while the q- factor yields 9.

Figure 6.1: Q-factor Portfolio Alphas

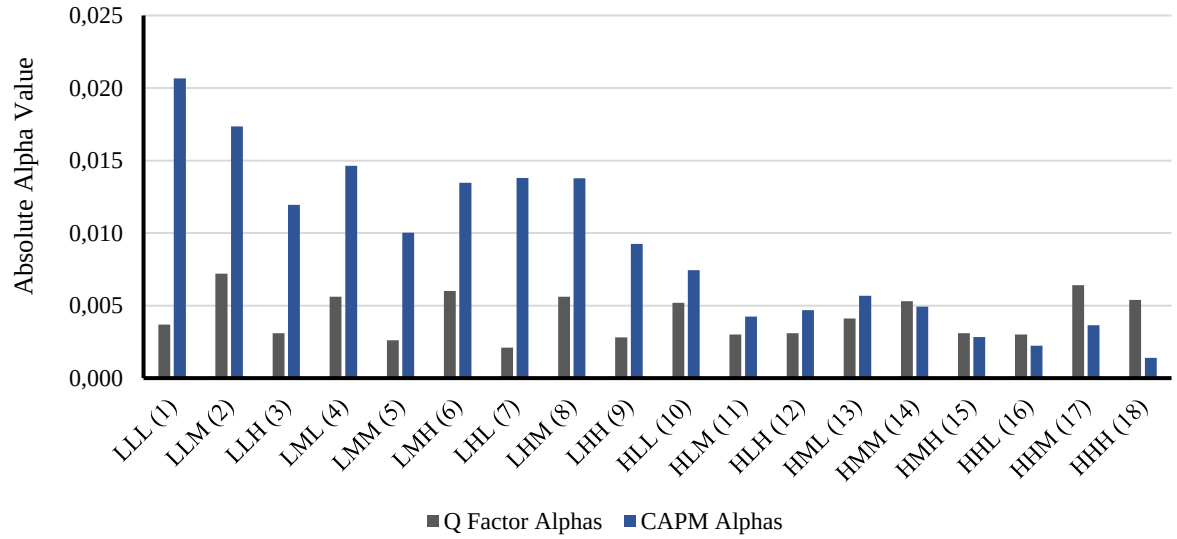


Figure 6.1 1: The time series regression alpha-coefficients are plotted against the CAPM benchmark alphas, yielded from regressing the q-model portfolios against the market factor.

6.3 FF5 Portfolio Approach

The alphas in the FF5 model (see table 6.5) have an absolute average of 0,43% and 11 out of 18 portfolios have significant alphas. The beta for the market factor is significant for all portfolios with a value of around 0,9. The regressions show that the size (SMB) factors loadings are significant and positive around 1 for all the portfolios who have small SMB (with portfolio 23 and 30 as significant outliers despite being big in size). The investment factor (CMA) has significant betas for the portfolios divided on low and high investment (portfolio 31, 33, 34, 36), it also shows that the low investment portfolios have a positive beta, and the high portfolios have a negative beta. The results for the profitability factor (RMW) show that all the

profitability portfolios except portfolio (29) have significant betas with low portfolios having negative beta and vice versa. Most of the CMA portfolios (31-36) have significant betas for RMW with negative coefficients. Lastly, the value factor (HML) is significant for the low and high Book-to-market portfolios with negative betas for low BM and positive ones for high BM. The mean of the adjusted r-squared is 87% which means that the model picks up a considerable amount of variation in the portfolios.

Table 6.5: Regression of FF5 portfolios against the FF5 factors

	LL-HML (19)	**LM-HML (20)**	**LH-HML (21)**	**HL-HML (22)**	**HM-HML (23)**	**HH-HML (24)**	**LL-RMW (25)**	**LM-RMW (26)**	**LH-RMW (27)**
α	-0,001	0,0049*	0,008*	0,0085*	0,0029*	-0,0005	0,0021	0,006*	0,005*
β_{MKT}	0,8919*	0,8739*	1,0288*	1,0205*	0,9755*	0,8836*	0,9288*	0,949*	0,8964*
β_{SMB}	0,8892*	0,9383*	1,0717*	0,0453	0,0697*	-0,1371	1,0576*	0,9952*	0,8759*
β_{CMA}	-0,1098	-0,1601*	-0,1275	-0,1095	-0,0039	-0,0919	-0,1185	-0,0535	-0,1421*
β_{RMW}	0,0388	-0,1404*	-0,0488	-0,0655	-0,0115	0,0221	-0,7813*	0,1282*	0,3134*
β_{HML}	-0,3917*	0,0098	0,3894*	-0,5498*	0,0066	0,6689*	0,1303	-0,0689	0,0241
Adj-R2	0,7667	0,8318	0,8209	0,9153	0,9383	0,7987	0,8743	0,8468	0,8504

	HL-RMW (28)	**HM-RMW (29)**	**HH - RMW (30)**	**LL-CMA (31)**	**LM-CMA (32)**	**LH-CMA (33)**	**HL-CMA (34)**	**HM-CMA (35)**	**HH-CMA (36)**
α	0,004*	0,0033*	0,0011	0,0037	0,0065*	0,0026	0,0044*	0,0028*	0,0056
β_{MKT}	0,9408*	0,9266*	0,9732*	0,9562*	0,8945*	0,9231*	0,8969*	0,9469*	0,93*
β_{SMB}	-0,0447	-0,0531	0,1369*	1,0022*	0,8971*	1,0516*	-0,0063	0,013	-0,0557
β_{CMA}	-0,0906	-0,005	-0,067	0,3908*	-0,1	-0,6666*	0,3104*	-0,0542*	-0,632*
β_{RMW}	-0,553*	-0,0471	0,3521*	-0,1862*	0,1255*	-0,2243*	-0,1202*	-0,0826*	-0,082
β_{HML}	-0,0022	0,0918	0,1039	0,0387	-0,0774	0,041	-0,0299	0,0645*	-0,0322
Adj-R2	0,9028	0,9265	0,8828	0,8508	0,836	0,9125	0,885	0,9728	0,8619

Table 6.5 Displays the coefficients for the respective factors for all 18 portfolios constructed in the FF5 model. Coefficients assigned with a * are statistically significant on a 5% significance level. The letter L, M and H stand for Low, Mid and High. There are 3 separate portfolio constructions depending on the characteristic. The first divide is on ME for L, M & H portfolios and the second is e.g. for HML. This hence means LH-HML is low ME and high book to market.

Figure 6.2 depicts the difference in alpha size between the FF5 and the CAPM model. The overall magnitude of the CAPM regressed against the FF5 portfolios is much higher with extreme values rising above 1,5% per month. The FF5 intercepts have an average of 0,19% with an irregular pattern. Like figure 6.1, the highest CAPM alphas are found in low size sorted portfolios (19-21; 25-27 and 31-33). The maximum factor exposure portfolios (21), (27) and (31) all possess significantly higher alpha values when explained by the CAPM, portfolio (21) and (27) have significant but considerably lower alpha values versus the CAPM ones and the low investment portfolio (31) has an insignificant alpha estimate with only a quarter of the size from that of the CAPM. There is also a pattern where most of the small ME portfolios seem to have higher alphas from the CAPM model.

Figure 6.2: FF5 Portfolio Alphas

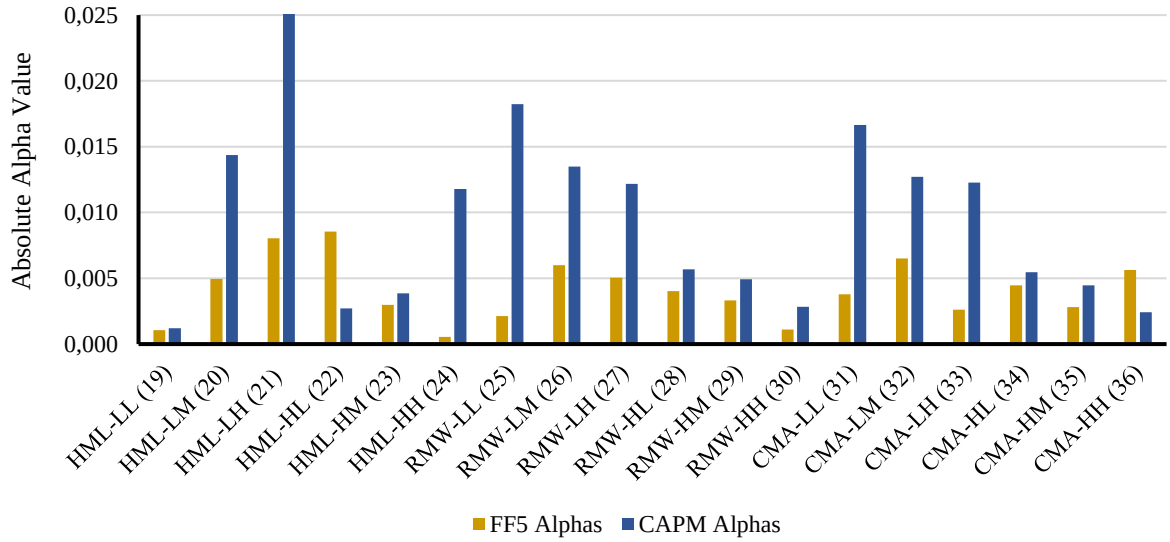


Figure 2: The time series regression alpha values form the FF5 portfolios are plotted against the CAPM regression alphas for each specific portfolio.

The GRS-tests (Appendix Table [A.3]) for the 18 original portfolios from the q and FF5 model show that at least one of the alphas is significantly different from zero, hinting at the fact that the regressions produce pricing errors left unexplained. The mean value of both tests is two bps apart, while the F-statistic is noticeably higher for the q-factor model, which implies more significant alpha values.

6.4 Anomaly Tests

The results from regressing all the JKP factors against the q and FF5 model in Germany shows that out of the 151 anomalies tested, the q-factor model comes out on top with 133 insignificant alphas. The CAPM has 127 insignificant alphas and the FF5 has 126 insignificant alphas. However, when excluding the value factor from the FF5 (*FF4*) the model has more insignificant alphas. (131 insignificant alphas in FF4). The GRS-test result for the anomalies shows that all factor models do not have joint alphas significantly different from zero suggesting that the models capture the returns of the anomalies. The FF5 model shows the best result with the lowest test statistic. Both the q-factor and the FF4 do have high p-values with values very similar to each other. Lastly the CAPM model shows the highest test statistic, which means that it is the closest of the models to having alphas, significantly different from zero.

Table 6.6: Number of insignificant alphas for all 4 models and the respective GRS-test p-value and F- statistic

	q-factor	FF5	FF4	CAPM
F statistic	1,1678	0,9446	1,665	1,244
P-value	0,2372	0,6186	0,2388	0,1513
Insignificant alphas	133	123	131	127

Table 6.6 shows the GRS-test conducted on the 151 alphas in total from the anomaly testing. P-values above 0,05 illustrate that the alphas are not jointly different from zero.

7. Discussion

7.1 Can the q-model gain significant results in Germany?

Portfolio approach

From our time series regressions against the 18 diversified portfolios (table [6.4]) we learn about the q-factor's ability to explain the cross-sectional portfolio returns. Half of the regressions show alphas significantly different from zero suggesting that the model is not well specified. The GRS test conducted for testing if the alphas are jointly different from zero displays a p-value well below 0,05 which further provides evidence for the model's lack of specification. However, comparing the alphas from the q-factor model to the CAPM (considered a benchmark model) where only 4 alphas were zero, one could argue that it's more precise than the CAPM. We generally find deviating results to those of Hou et al. (2015) and other international authors like Asad and Cheema (2017) and Charoenwong et al. (2021). Interestingly, we can partly see the q-model's lack of success in the GRS test as a result that agrees with Maeda (2017) who find that the Hou et al. model is not yielding well specified results in Japan. Possible explanations with regards to our findings could range from the factor construction, over data limitations for the German market, to the applicability of the q-model itself.

Comparing the magnitude of the alphas between the q-factor model and CAPM (figure [6.1]) we see a pattern where the low ME portfolios manifest high alphas for the CAPM model but not for the q-factor model. This suggests that returns generated by low size portfolios are not picked up by the CAPM model. The result from Table 6.4 enhances the evidence of that conclusion since all low ME portfolios have a significant coefficient. The CAPM model seems to pick up the effect of high ME companies suggesting that there is no effect of portfolios containing larger companies. This could mean that the market does not ask for a premium when investing in larger companies. One possible explanation for why the CAPM works better for large companies could be the fact that the market return from the DAX40 is from the largest companies in Germany even if the market return were taken for the entire market these 40 companies stand for 80% of the market cap on the German market. The effect of the small companies on the market return is therefore minimal. Brückner et al. (2012) similarly find that the CAPM works better for big companies as suggested by the size factor and validated in our

results (see table [6.1]). Based on the descriptive statistics there is a higher return for the portfolios with low ME with a slightly higher volatility which *could approve hypothesis 1*. When interpreting the results in table 6.4 we find additional proof for this suggestion. All nine low-size portfolios have highly significant coefficients, while big-size portfolios bear insignificant or low betas. Therefore, the size factor in our results follows previous studies, where smaller companies generate higher returns compared to larger companies. It contradicts the result from Crain (2011) and others who claim that the size factor has lost its influence in most major markets. Hou et al. state that the factor is just a control variable to capture size-related anomalies, the significance of our results either opposes this view or points at a high relevance of these anomalies in Germany. In summary the size factor is the most consistent of our factors and proves to be in line with what the theory suggests.

The investment factor earns a monthly average that is not statistically significant (t-value 1,72). On the other hand, the prevailing literature is approved when considering the regressions on our portfolios, where investment portfolios have a negative investment-return relationship (see table 6.4), creating ambiguity in our findings and leaving hypothesis 3 unanswered. The fact that the factor correlates with the portfolios from which it is constructed can only be counted as a secondary criterion of validity. A possible interpretation of our results could be that the over-investment that causes a firm's profitability to decline, analogous to Titman et al. (2004), is not as present in the German market. Behavioral aspects may be the root of this phenomenon. Interestingly we could not directly replicate the results of Hou et al. and Fama and French and others when it comes to deploying the investment factor in Germany.

The profitability factor earns a monthly average of (-0,37%), with a t-value of -2,19 *rejecting hypothesis 3*. The monthly mean return of all 12 high-versus-low profitability portfolios underlines these findings, the low ROE portfolios have a higher mean return compared to the high ROE portfolios, which explains the negative mean factor return. The time series regressions estimators' conflict with this negative relationship where positive signs are found at high profitability portfolios and vice versa. The contradiction could be caused by the yearly instead of quarterly data we use for the factor. In our construction of the factor, we might miss the pricing effects of when firms release their quarterly earnings. The market has a significant head start in pricing with this information discrepancy since earning calls and pre-reports are published before the financial statements (usually released in Germany from March to May after the last

financial year). Our attempts to use lagged prices did not solve this problem, further research could make use of the method deployed by Fama and French (2015) that utilize operating profitability, which is available more frequently in the German reporting framework. No direct conclusions about the profitability factor in Germany can therefore be drawn from the q-model at this point, we reject Hou et al.'s postulations about profitability for the German stock market.

Anomaly Approach

The results from the anomaly approach portray the q-factor as a well specified model compared to the results from the portfolio approach. 133 alphas for the 151 anomalies were jointly insignificantly different from zero. The q-model has less significant alphas and a higher p-value from the GRS test compared to the CAPM confirming the results from the portfolio approach where the q-factor performs better. These results confirm the findings from Hou et al. (2015); Hou et al. (2020) and Jensen et al. (2023) where the q-model outperforms the CAPM. However, the results only slightly surpass the CAPM, proving the work horse model's general applicability on the German market as found by Brückner et al. (2012). They additionally find that the CAPM works better for big companies as suggested by the size factor and validated in our results (see figure [6.1]). The additional factors in the q-model improve the already good foundation of the CAPM, letting us achieve a noticeably higher explanation rate of anomalies in Germany. Specifically, we yield a rate of 88% surpassing the 65% from Hou et al. (2020) when using a single hurdle test ($t > 1{,}96$).

7.2 How does the q-model and the Fama and French 5 factor model compare?

Portfolio approach

The FF5 model has 7 insignificant alphas in the portfolio approach compared to the q-factor having 9, implying that the q-factor is better specified. However, the results from the GRS-test suggest the q-factor alphas to be further away from zero and thus capture less return variation with that model. These two findings are ambiguous making it difficult to draw conclusions about the model's comparability. Similar or better results are expected for the FF5 time series regressions since the factors are constructed identically (see table [6.3.] for correlation between the factors) and with one added factor. From the descriptive statistics (table [6.1]), we see similar results for the size effect where smaller firms seem to exhibit higher returns in both models. Additionally, when comparing the FF5 alphas to the CAPM alphas we observe low ME portfolios to yield a larger difference in alphas compared to high ME portfolios confirming the importance of the size factor for both models. These findings again confirm the literature like Heston et al. (1999); Bauer et al. (2010) and Amel-Zadeh (2011) about the applicability of the size effect on European, developed markets.

When we assess if the model is better off without the value factor, one can conclude that adding a fifth variable to the model could improve the GRS test statistics, while it decreases the amount of insignificant pricing errors. Adding the value factor also increases the adj-R2 from 72% to 87% suggesting the FF5 model fits the data better however, both would be considered high. Unlike Fama and French (2015) and Grobys and Huhta (2019) we found that size is not the main driver of the value factor. From the descriptive statistics in table 6.1 we can conclude that factor premia gradually increase with HML no matter if size is low or high. We also only find a low (positive) correlation between SMB and HML (table 6.3) further confirming this idea. These ideas are strengthened by the regression coefficients in (table 6.5) with the expected direction for high and low B/M portfolios. Ultimately, *Hypothesis 4 can be rejected from the descriptive parts, while the GRS tests could hint at a possible improvement if the value factor is included in the model*.

Anomaly approach

The anomaly approach seems to confirm the ambiguous result that the portfolio approach implies. The q-factor model has more insignificant alphas (133 compared to the 126 in the FF5 model). However, the GRS-test resulted in a lower test statistic for the FF5 model implying that the alphas in the FF5 are closer to zero than the q-factor model. As a sensitivity analysis we conducted the same anomaly tests for the FF4 model which excludes the value factor. Interestingly, the FF4 regression results in more insignificant alphas, scoring the second place after the q-model. On the other hand, the FF4 has a higher t-statistic in the GRS test than the FF5 (see table [6.6]). Hence, one could argue that the HML factor (the only different factor) penalizes the model because of the less insignificant alphas. The GRS F-test statistic would contradict this statement making it difficult to argue whether to include the fifth factor or not. Again, we find both arguments for findings like Chiah et al. (2016); Pandey and Joshi (2022) and Fama and French (2017). On the other hand we find the lower rate of insignificant alphas for the FF5, ultimately suggesting less precision in capturing return variation. Hence, we could interpret our results similarly to the original FF5 study by Fama and French (2015) who find that adding the value factor together with profitability and investment might be irrelevant, implying that four factors could suffice.

Another interesting observation can be made from the correlation matrix in table [6.3], where Hou et al. (2015) find a correlation of 0,69% between HML and I/A, we only find a small and negative correlation. Consequently, the investment factor does not explain the book-to-market effect in our sample. The FF5 exhibits less insignificant intercepts compared to the q-factor, FF4 but also the CAPM model. These results are somewhat unexpected since the added factors should most likely predict the returns better. However, the GRS-test implies yet again that the FF5 model is better than all three of the other factor models. This implies that the CAPM model seems to work well in capturing anomalies but adding factors can slightly improve the outcome.

Limitations

As we encountered data collecting limitations there are some deviations in the construction of the factors compared to the original study from Hou et al 2015. This study uses yearly data for ROE compared to quarterly as in the original study. This could explain the fact that this specific factor return is negative, future researchers could alternatively use other data resources. Another limitation is the potential survivorship bias stemming from not including companies that have been terminated during the time period. Lastly, regarding the data collection, Refinitiv Eikon sometimes misses values causing stocks to drop out in the factor constructions for certain periods. The q-factor model is mainly an empirical, reduced form model, the results are based on data modelling inference and are not entirely driven by theory. The returns with similar characteristics e.g., profitability and investment are required to move in an inversely correlated way, the q-factor model does not predict this, at least not directly. To improve the model, one would need an investment theoretic approach that accounts for the co-movement behind the model's factors and for cross-sectional differences in the I/A and ROE factor (Hou et al, 2015).

8. Conclusion

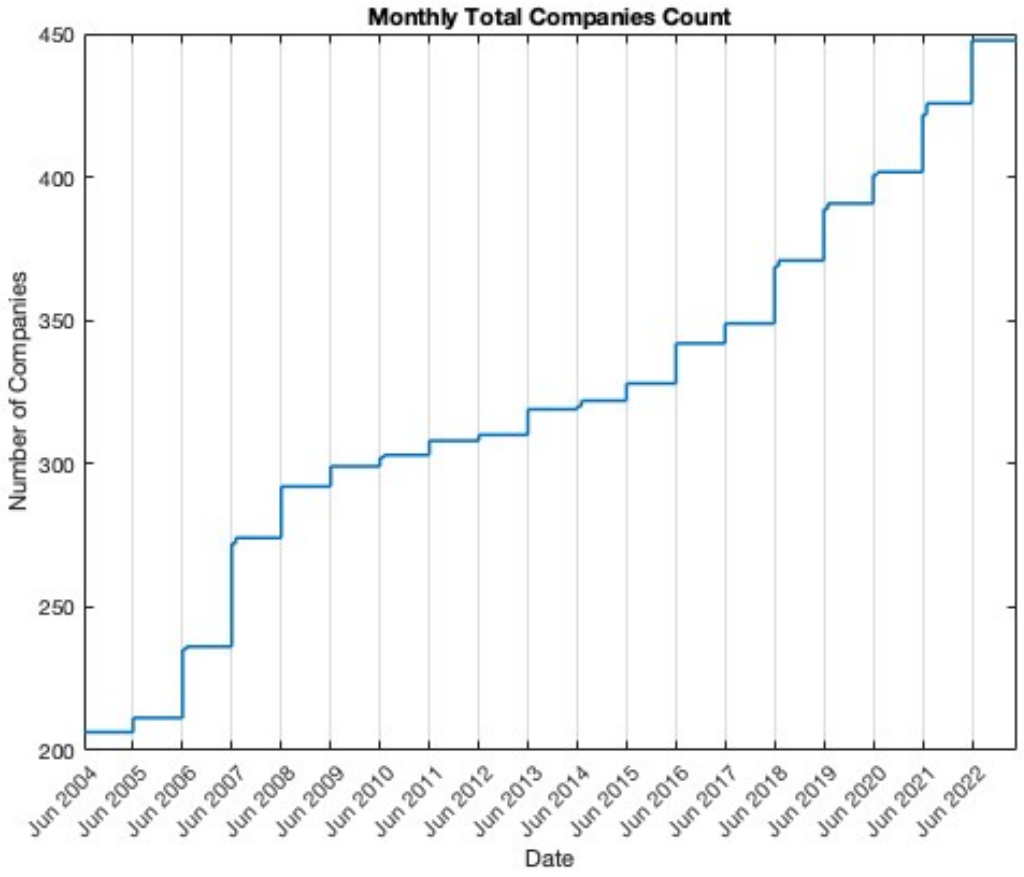
In this paper we study whether the q-factor model can gain significant results in the German market and how it performs against the FF5 model. We implement two strategies, the portfolio and anomaly approach, to get more comprehensive and elaborate results. The results are compared with the CAPM model which is the benchmark for the factor models. Our findings point at the q-factor model's validity in the German market. Comparing it to the CAPM-model it outperforms in the portfolio approach where the q-factor has more insignificant alphas compared to the CAPM model.

The anomaly approach further confirms that the q-factor model outperforms the CAPM with more insignificant alphas and a lower test statistic in the GRS test. However, the q-model's GRS test for the portfolio approach cannot reject the insignificant zero-alpha hypothesis, implying that the model lacks specification, while it beats the benchmark model (CAPM).

When it comes to the specific q-factors we only find the market and size factor to have explicit results. The investment theory proposed in the original model cannot directly be confirmed in our application. The investment and profitability portfolios yield negative or insignificant returns, while the beta estimators from the regressions bear the right signs, indicating the factors to be correct. With this contradiction and the limited meaningfulness of regressing factors against our own portfolios that we constructed, the insights we gain on return variability partly lose its impact.

Lastly, concerning the question about how the q-factor and FF5 model compare, our findings are ambiguous, and we are unable to draw a conclusion to this question. The FF5 has less insignificant alphas in both approaches compared to the q-factor model but at the same time it performs better in the GRS-tests. Therefore, we have results contradicting each other making it difficult to determine whether to include the value factor in factor models.

Appendix



Appendix Figure A1: Depicts the total number of companies available with all the necessary characteristics, considered in our portfolio sorting for the period from June last year to June current year.

Appendix Table A.1

Breusch-Pagan q-factor			Breusch-Pagan FF5		
	Chi-statistic	P-value		Chi-statistic	P-value
LLL (1)	25,9441	0,0000	LL-HML (19)	6,8274	0,2338
LLM (2)	3,3021	0,5086	LM-HML (20)	2,6377	0,7556
LLH (3)	16,8364	0,0021	LH-HML (21)	21,0190	0,0008
LML (4)	6,2424	0,1818	HL-HML (22)	6,9431	0,2249
LMM (5)	3,9317	0,4153	HM-HML (23)	3,8644	0,5691
LMH (6)	7,4362	0,1146	HH-HML (24)	10,7985	0,0555
LHL (7)	14,7208	0,0053	LL-RMW (25)	20,8796	0,0009
LHM (8)	15,9728	0,0031	LM-RMW (26)	11,9187	0,0359
LHH (9)	1,5860	0,8113	LH-RMW (27)	9,1893	0,1017
HLL (10)	12,3740	0,0148	HL-RMW (28)	16,6118	0,0053
HLM (11)	7,3587	0,1181	HM-RMW (29)	14,6081	0,0122
HLH (12)	12,1701	0,0161	HH-RMW (30)	10,0715	0,0732
HML (13)	11,9938	0,0174	LL-CMA (31)	7,0093	0,2200
HMM (14)	16,1838	0,0028	LM-CMA (32)	5,6096	0,3461
HMH (15)	2,4255	0,6580	LH-CMA (33)	11,0434	0,0505
HHL (16)	5,4908	0,2405	HL-CMA (34)	11,2544	0,0466
HHM (17)	1,6039	0,8081	HM-CMA (35)	6,3356	0,2749
HHH (18)	2,6726	0,6140	HH-CMA (36)	14,8360	0,0111

Appendix Table A.1 shows the Breusch-Pagan tests for the q-factor and FF5 portfolios where a p-value below 0,05 means that there is heteroscedasticity present.

Appendix Table A.2

Breusch-Godfrey q-factor			Breusch-Godfrey FF5-factor		
	H0	p-value		H0	p-value
LLL (1)	1	0,0002	LL-HML (19)	1	0,0042
LLM (2)	1	0,0026	LM-HML (20)	1	0,0006
LLH (3)	0	0,2319	LH-HML (21)	1	0,0147
LML (4)	0	0,1576	HL-HML (22)	0	0,1389
LMM (5)	1	0,0212	HM-HML (23)	0	0,7216
LMH (6)	1	0,0461	HH-HML (24)	0	0,3911
LHL (7)	1	0,0000	LL-RMW (25)	1	0,0000
LHM (8)	1	0,0275	LM-RMW (26)	1	0,0086
LHH (9)	1	0,0105	LH-RMW (27)	1	0,0249
HLL (10)	0	0,7660	HL-RMW (28)	0	0,7373
HLM (11)	0	0,3194	HM-RMW (29)	0	0,8274
HLH (12)	0	0,4956	HH-RMW (30)	0	0,4681
HML (13)	0	0,7373	LL-CMA (31)	1	0,0004
HMM (14)	0	0,8274	LM-CMA (32)	1	0,0113
HMH (15)	0	0,4681	LH-CMA (33)	1	0,0000
HHL (16)	0	0,0620	HL-CMA (34)	0	0,7730
HHM (17)	1	0,0107	HM-CMA (35)	0	0,6907
HHH (18)	0	0,3270	HH-CMA (36)	1	0,0234

Appendix Table A.2 shows the Breusch-Godfrey tests conducted on the portfolios where a 1 in the H0 column means that there is auto-correlation present in the data.

Appendix table A.3: GRS-test results for the portfolio approach alphas

	Q-factor	FF5
Mean alpha q	0,0043	0,0041
F statistic	5,1204	3,7077
P-value	0,0000	0,0000

Appendix table A.3 shows the GRS-test conducted on the alphas from the portfolio approach. P-values above 0,05 means that the alphas are not jointly different from zero.

Appendix Table A.4: CAPM regression against the FF5-factor portfolios

	α	β_{MKT}	p-value	p-value
LL-HML (19)	-0,0012	0,8081	0,6472	0,0000
LM-HML (20)	0,0144	0,6801	0,0000	0,0000
LH-HML (21)	0,0265	0,7030	0,0000	0,0000
HL-HML (22)	-0,0027	1,1636	0,1373	0,0000
HM-HML (23)	0,0038	0,9591	0,0000	0,0000
HH-HML (24)	0,0118	0,7451	0,0000	0,0000
LL-RMW (25)	0,0182	0,6820	0,0000	0,0000
LM-RMW (26)	0,0135	0,7523	0,0000	0,0000
LH-RMW (27)	0,0122	0,7057	0,0000	0,0000
HL-RMW (28)	0,0057	0,9646	0,0003	0,0000
HM-RMW (29)	0,0049	0,9151	0,0000	0,0000
HH-RMW (30)	0,0028	0,9170	0,0531	0,0000
LL-CMA (31)	0,0167	0,7004	0,0000	0,0000
LM-CMA (32)	0,0127	0,7249	0,0000	0,0000
LH-CMA (33)	0,0123	0,7351	0,0000	0,0000
HL-CMA (34)	0,0055	0,8845	0,0001	0,0000
HM-CMA (35)	0,0045	0,9322	0,0000	0,0000
HH-CMA (36)	0,0024	0,9987	0,2033	0,0000

Appendix table A.4 displays the coefficients for the CAPM factor model regressed against the portfolios created in the FF5-factor model.

Appendix Table A.5: CAPM regression against the q-factor portfolios

	α	β_{MKT}	p-value	p-value
LLL (1)	0,0207	0,7295	0,0000	0,0000
LLM (2)	0,0174	0,5791	0,0000	0,0000
LLH (3)	0,0119	0,7925	0,0002	0,0000
LML (4)	0,0146	0,7319	0,0000	0,0000
LMM (5)	0,0100	0,7169	0,0000	0,0000
LMH (6)	0,0135	0,7260	0,0000	0,0000
LHL (7)	0,0138	0,7663	0,0008	0,0000
LHM (8)	0,0138	0,7585	0,0000	0,0000
LHH (9)	0,0092	0,6804	0,0011	0,0000
HLL (10)	0,0074	0,8983	0,0089	0,0000
HLM (11)	0,0042	0,8963	0,0348	0,0000
HLH (12)	0,0047	0,8588	0,0396	0,0000
HML (13)	0,0057	0,9646	0,0003	0,0000
HMM (14)	0,0049	0,9151	0,0000	0,0000
HMH (15)	0,0028	0,9170	0,0531	0,0000
HHL (16)	0,0022	1,0210	0,4049	0,0000
HHM (17)	0,0036	1,0123	0,2040	0,0000
HHH (18)	0,0014	0,9627	0,6269	0,0000

Appendix table A.5 displays the coefficients for the CAPM factor model regressed against the portfolios created in the FF5-factor model.

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