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## **Introduction to financial surveillance**

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# **Introduction to financial surveillance**

Marianne Frisé

## **1 What is financial surveillance?**

In financial surveillance the aim is to signal at the optimal trading time. A systematic decision strategy is used. The information available at each possible decision time is evaluated in order to judge whether or not there is enough information for a decision about an action or if more information is necessary so that the decision should be postponed. Financial surveillance gives timely decisions.

Financial decision strategies are based, in one way or another, on continuous observation and analysis of information. This is financial surveillance. Statistical surveillance uses decision theory and statistical inference in order to derive timely decision strategies. Hopefully, this report will serve as a bridge between finance and statistical surveillance. For further details on the subject see (Frisé 2008).

Textbooks describing financial problems and statistical methods are for example (Föllmer and Schied 2002), (Härdle, Kleinow and Stahl 2002), (Gourieroux and Jasiak 2002), (Franke, Härdle and Hafner 2004), (Cizek, Härdle and Weron 2005) and (Scherer and Martin 2005). Many and various statistical techniques are described in these books.

In Section 2 statistical methods which are useful for financial decisions will be discussed. In Section 2.2 the area of statistical surveillance is described, and the characteristics of surveillance are compared to other areas in statistics. Evaluations in surveillance are described in Section 2.3. This is an important area, since the choice of evaluation measures will decide which methods are considered appropriate. General methods for aggregating information over time are described in Section 2.4. Special aspects of surveillance for financial decisions are discussed in Section 2.5.

## **2 Statistical methods for financial decision strategies**

Statistical methods use observations of financial data to give information about the financial process, which produces the data. This is in contrast to probability theory, where assumptions about the financial process are used to derive which observations will be generated.

### **2.1 Transaction strategies based on financial data**

In finance, the relation between observations and decisions is often informal. Statisticians have taken on the role of presenting statistical summaries of quantitative data. In many areas, including finance, this means providing point and interval estimates for the quantities of interest. Methods for providing such summaries are highly formalised and constantly evolving. The discipline of statistics uses observations to make deductions about the real world. It has its own set of axioms and theorems besides those of probability theory. While decision making is the incentive for much statistical analysis, the process that transforms statistical summaries into decisions usually remains informal and ad hoc.

In finance, the timeliness of transactions is important to yield a large return and a low risk. The concept of an efficient arbitrage-free market is of great interest. One central question is whether the history of the price of an asset contains information, which can be used to increase the future return. A natural aim is to maximise the return. The theory of stochastic finance has been based on an assumption of an efficient market where the financial markets are arbitrage-free and there is no point in trying to increase the return. Even though this view is generally accepted today, there are some doubts that it is generally applicable. When the information about the process is incomplete, as for example when a change point could occur, there may be an arbitrage opportunity, as demonstrated by (Shiryayev 2002). In (Bock, Andersson and Frisén 2008) it was discussed how technical analysis relies on the possibility of using history to increase future returns. The support for the efficient market hypothesis depends on the knowledge about the model, as is discussed below in Section 2.1.1.

#### **2.1.1 Modelling**

In finance, advanced stochastic models are necessary to capture all empirical features. The expected value could depend on time in a complicated non-linear way. Parameters other than the expected value are often of great interest, and the risk (measured by variance) is often of great concern. Complicated dependencies are common, which means that complicated measures of variance are necessary. Multivariate data streams are of interest for example when choosing portfolio. The models may be described in

continuous or discrete time. The use of the models should be robust to errors in the model specification.

#### 2.1.1.1 Stochastic model assumed known

When the stochastic model is assumed to be completely known there is no expected return to be gained. We will have an arbitrage-free market. We can use probability theory to calculate the optimal transaction conditions. Important contributions are found in the book by (Shiryaev 1999) or in articles in the scientific journal *Finance and Stochastics*. Also the proceedings of the conference *Stochastic Finance* in 2004 and 2007 are informative on how to handle financial decisions when the model is completely known.

#### 2.1.1.2 Incomplete knowledge about the stochastic model

When the model is not completely known, the efficient and arbitrage-free market assumptions are violated. Changes at unknown times are possible. One has to evaluate the information continuously to decide whether a transaction at that time is profitable. Statistical inference is needed for the decision (Shiryaev 2002).

### 2.1.2 Evaluation of information

Statistical inference theory gives guidelines on how to draw conclusions about the real world from data. Statistical hypothesis testing is suitable for testing a single hypothesis but not a decision strategy including repeated decisions, as will be further described in Section 2.3.1. Statistical surveillance is an important branch of inference. The relatively new area of statistical surveillance deals with the sequential evaluation of the amount of information at hand. It provides a theory for deciding at what time the amount of information is enough to make a decision and take action. This bridges the gap between statistical analysis and decisions.

Here,, we concentrate on the methodology of statistical surveillance. This methodology is of special interest for financial decision strategies, but it is also relatively new in finance. The ambition here is to give a comprehensive description of such aspects of statistical surveillance that may be of interest in finance. Thus the next sections will give a short review on statistical surveillance.

## 2.2 What is statistical surveillance?

### 2.2.1 General description

Statistical surveillance means that a time series is observed with the aim of detecting an important change in the underlying process as soon as possible after the change has occurred. Statistical methods are necessary to separate important changes in the process from stochastic variation. The inferential

problems involved are important for the applications and interesting from a theoretical viewpoint, since they bring different areas of statistical theory together.

Broad surveys and bibliographies on statistical surveillance are given by Lai (1995), who concentrates on the minimax properties of stopping rules, by Woodall and Montgomery (1999) and (Ryan 2000), who concentrate on control charts, and by (Frisén 2003), who concentrates on the optimality properties of various methods.

The theory of statistical surveillance has developed independently in different statistical subcultures. Thus, the terminology is diverse. Different terms are used to refer to “statistical surveillance” as described here. However, there are some differences in how the terms are used. “Optimal stopping rules” is most often used in probability theory, especially in connection with financial problems. However, this does not always include the statistical inference from the observations to the model. Literature on “change-point problems” does not always treat the case of continuous observations but often considers the case of a retrospective analysis of a fixed number of observations. The term “early warning system” is most often used in the economic literature. “Monitoring” is most often used in medical literature and as a non-specific term. Timeliness, which is important in surveillance, is considered in the vast literature on quality control charts, and here also the simplicity of procedures is stressed. The notations “statistical process control” and “quality control” are used in the literature on industrial production and sometimes also include other aspects than the statistical ones.

The statistical methods suitable for surveillance differ from the standard hypothesis testing methods. In the prospective surveillance situation, data accumulated over time is analysed repeatedly. A decision concerning whether, for example, the variance of the price of a stock has increased or not has to be made sequentially, based on the data collected so far. Each new possibility demands a new decision. Thus, there is no fixed data set but an increasing number of observations. In sequential analysis we have repeated decisions, but the hypotheses are fixed. In contrast, there are no fixed hypotheses in surveillance. The statistics derived for a fixed sample may be of great value also in the case of surveillance, but there are great differences between the systems for decision. The difference between hypotheses and on-line surveillance is best seen by studying the difference in evaluation measures (see Section 2.3.1).

In complicated surveillance problems, a stepwise reduction of the problem may be useful. Then, the statistics derived to be optimal for the fixed sample problem can be a component in the construction of the prospective surveillance system. This applies, for example, to the time series problems described in Section 2.5.3 and the multivariate problems described in Section 2.5.7.

### 2.2.2 History

The first modern control charts were developed in the 1920s, by Walter A. Shewhart and co-workers at Bell Telephone Laboratories. In 1931 the famous book “Economic Control of Quality of Manufactured Product” (Shewhart 1931) was published. The same year Shewhart gave a presentation of the new technique to the Royal Statistical Society. This stimulated interest in the UK. The technique was used extensively during World War II both in the UK and in the US. In the 1950s, W. E. Deming introduced the technique in Japan. The success in Japan spurred the interest in the West, and further development started.

In the Shewhart method each observation is judged separately. The next important step was taken when Page (1954) suggested the CUSUM method for aggregating information over time. Shortly afterwards, (Roberts 1959) suggested another method for aggregating information – the EWMA method. A method based on likelihood which fulfils important optimality conditions was suggested by (Shiryayev 1963).

In recent years there have been a growing number of papers in economics, medicine, environmental control and other areas, dealing with the need of methods for surveillance. The threat of bioterrorism and new contagious diseases has been an important reason behind the increased research activity in the theory of surveillance. Hopefully, the time is now ripe for finance to benefit from all these results.

### 2.2.3 Specifications of the statistical surveillance problem

The general situation of a change in distribution at a certain change-point time  $\tau$  will now be specified. The variable under surveillance could be the observation itself or an estimator of a variance or some other derived statistic, depending on the specific situation. We denote the process by  $X = \{X(t): t = 1, 2, \dots\}$ , where  $X(t)$  is the observation made at time  $t$ . The purpose of the monitoring is to detect a possible change. The time for the change is denoted by  $\tau$ . This can be regarded either as a random variable or as a deterministic but unknown value, depending on what is most suitable for the application.

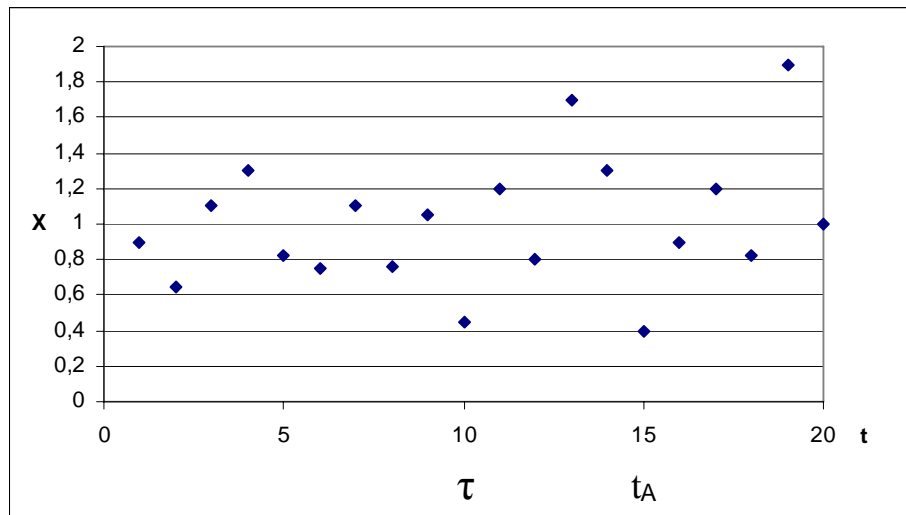


Figure 1.1 The first  $\tau-1$  observations  $X_{\tau-1} = \{X(t); t \leq \tau-1\}$  are “in-control” with a small variance. The subsequent observations (from  $t=\tau$  (here 10) and onwards) have a larger variance. The alarm time is  $t_A$ , which happens to be 15. Thus the delay is  $t_A - \tau = 5$ .

The properties of the process change at time  $\tau$ . In many cases we can describe this as

$$X(t) = \begin{cases} Y(t), & t < \tau \\ Y(t) + \Delta, & t \geq \tau \end{cases} \quad (1.1)$$

where  $Y$  is the “in-control” or “target” process and  $\Delta$  denotes the change. More generally we can denote the “in-control” state by  $D$  and the state which we want to detect by  $C$ . The (possibly random) process that determines the state of the system is here denoted by  $\mu(t)$ . This could be an expected value, a variance or some other time-dependent characteristic of the distribution. Different types of states between which the process changes are of interest for different applications.

The change to be detected differs depending on the application. Most studies in literature concern a step change, where a parameter changes from one constant level, say,  $\mu(t) = \mu^0$  to another constant level,  $\mu(t) = \mu^1$ . The case  $\mu > 0$  is described here. We have  $\mu(t) = \mu^0$  for  $t = 1, \dots, \tau-1$  and  $\mu(t) = \mu^1$  for  $t = \tau, \tau+1, \dots$

Even though autocorrelated time series are studied for example by (Schmid and Schöne 1997), (Petzold, Sonesson, Bergman and Kieler 2004), processes which are independent given  $\tau$  are the most studied. This simple situation will be used to introduce general concepts of evaluations, optimality and standard methods.

Some cases of special interest in financial surveillance are discussed in Section 2.5.

## 2.3 Evaluations

Quick detection and few false alarms are desired properties of methods for surveillance. Knowledge about the properties of the method in question is important. If a method calls an alarm, it is important to know whether this alarm is a strong indication of a change or just a weak indication. The same methods can be derived by Bayesian or frequentistic inference. However, evaluations differ. Here we present measures suitable for frequentistic inference.

### 2.3.1 The difference between evaluations for hypothesis testing and on-line surveillance

Measures for a fixed sample situation can be adopted for surveillance, but some important differences will be pointed out. In Table 1.1 the measures conventionally used in hypothesis testing and some measures for surveillance are given. These measures will be described and discussed below.

	<i>Test</i>	<i>Surveillance</i>
<i>False alarms</i>	Size $\alpha$ , Specificity	ARL <sup>0</sup> , MRL <sup>0</sup> , PFA
<i>Detection ability</i>	Power, Sensitivity	ARL <sup>1</sup> , MRL <sup>1</sup> , CED, ED, maxCED, PSD, SADT

Table 1.1. Evaluation measures for hypothesis testing and the corresponding measures for on-line surveillance.

Different error rates and their implications for a decision system were discussed by Frisén and de Maré (1991). Using a constant probability of exceeding the alarm limit for each decision time means that we have a system of repeated significance tests. This may work well also as a system of surveillance and is often used. The Shewhart method described in Section 2.4.2 has this property. This is probably also the motive for using the limits with the exact variance in the EWMA method described in Section [2.4.4](#).

Evaluation by significance level, power, specificity and sensitivity, which is useful for a fixed sample, is not appropriate without modification in a surveillance situation since these measures do not have unique values in a surveillance system. One problem with evaluation measures originally suggested for the study of a fixed sample of, say,  $n$  observations is that the measures depend on  $n$ . For example, the specificity will tend to zero for most methods and the size of the test will tend to one when  $n$  increases.

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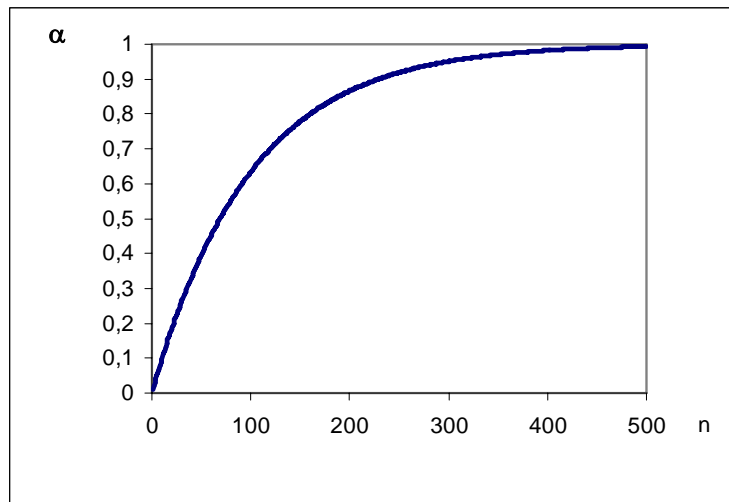


Figure 1.2. The size,  $\alpha$ , of a surveillance system which is pursued for  $n$  time units, when the probability of a false alarm is 1% at each time point.

(Chu, Stinchcombe and White 1996) and others have suggested methods with a size less than one

$$\lim_{n \rightarrow \infty} P(t_A \leq n | D) < 1.$$

This is convenient since ordinary statements of hypothesis testing can be made. However, (Frisén 2003) demonstrated that the detection ability of methods with this property declines rapidly with the value of time  $\tau$  of the change. Important consequences were illustrated by (Bock 2007).

The performance of a method for surveillance depends on the time  $\tau$  of the change. Generally, the sensitivity will not be the same for early changes as for late ones. It also depends on the length of time for which the evaluation is made. Thus, there is not one unique sensitivity value in surveillance, but other measures may be more useful. Accordingly, conventional measures for fixed samples should be supplemented by other measures designed for statistical surveillance, as will be discussed in the following.

### 2.3.2 Measures of the false alarm rate

The false alarm tendency is more complicated to control in surveillance than in hypothesis testing, as was seen above (for example in Figure 1.2). There are special measures of the false alarm properties which are suitable for surveillance. The most commonly used measure is the Average Run Length when there is no change in the system under surveillance,  $ARL^0 = E(t_A | D)$ . A variant of the ARL is the Median Run Length, MRL.

A measure commonly used in theoretical work is the false alarm probability,  $PFA = P(t_A < \tau)$ . This is the probability that the alarm occurs before the change.

### 2.3.3 Delay of the alarm

The delay time of the detection of a change should be as short as possible. The most commonly used measure of the delay is the Average Run Length until the detection of a true change (that occurred at the same time as the surveillance started), which is denoted by  $ARL^1$ . The part of the definition within parentheses is seldom spelled out but generally used in the literature (see for example (Page 1954) and (Ryan 2000)). Instead of the average, (Gan 1993) advocates that the median run length should be used on the grounds that it may be more easily interpreted. However, also here only a change occurring at the same time as the surveillance started is considered.

In most practical situations it is important to minimise the expected delay of detection whenever the change occurs. (Shiryayev 1963) suggested measures of the expected value of the delay. The expected delay from the time of change,  $\tau=t$ , to the time of alarm,  $t_A$ , is denoted by

$$ED(t) = E[\max(0, t_A - t) \mid \tau=t].$$

Note that  $ARL^1 = ED(1) + 1$ . The  $ED(t)$  will typically tend to zero as  $t$  increases. Thus, it is easier to evaluate the conditional expected delay

$$CED(t) = E[t_A - \tau \mid t_A \geq \tau = t] = ED(t) / P(t_A \geq t).$$

$CED(\tau)$  is the expected delay for a specific change point  $\tau$ . The expected delay is generally not the same for early changes as for late ones. For most methods, the CED will converge to a constant value. This value is sometimes named the “steady state average delay time” or SADT. It is, in a sense, the opposite to  $ARL^1$  since only a very large value of  $\tau$  is considered. SADT has been advocated for example by (Srivastava and Wu 1993), (Srivastava 1994) and (Knoth 2006).

For some situations and methods the properties are about the same regardless of when the change occurs. However, this is not always true, as illustrated by Frisén and Wessman (1999). Then, it is important to consider more and other cases than just  $\tau=1$ . The values of CED can be summarised in different ways. One is the maximal value over  $\tau$ . Another approach is to regard  $\tau$  as a random variable with the probabilities  $\pi(t) = P(\tau=t)$ . These probabilities can also be regarded as priors. The intensity of a change is defined as  $\nu(t) = P(\tau=t \mid \tau \geq t)$ , which is usually assumed to be constant over time. (Shiryayev 1963) suggested a summarised measure of the expected delay

$$ED = E[ED(\tau)].$$

Sometimes the time available for action is limited. The Probability of Successful Detection suggested by (Frisén 1992) measures the probability of detection with a delay time no longer than  $d$

$$PSD(d, t) = P(t_A - \tau \leq d \mid t_A \geq \tau = t).$$

This measure is a function of both the time of the change and the length of the interval in which the detection is defined as successful. Also when there is no absolute limit to the detection time it is often useful to describe the ability to detect the change within a certain time. In such cases it may be useful to calculate the PSD for different time limits  $d$ . This has been done by (Marshall,

Best, Bottle and Aylin 2004). The ability to make a very quick detection (small  $d$ ) is important in surveillance of sudden major changes, while the long-term detection ability (large  $d$ ) is more important for ongoing surveillance where smaller changes are expected.

### 2.3.4 Predictive value

When an alarm is called, one needs to know whether to act as if the change is certain or just plausible. To obtain this, both the risk of false alarms and the risk of delay must be considered. If  $\tau$  is regarded as a random variable this can be done by one summarising measure. The probability that a change has occurred when the surveillance method signals was suggested by (Frisén 1992) as a time-dependent predictive value

$$PV(t) = P(t_A \leq \tau \mid t_A = t).$$

When there is an alarm ( $t_A = t$ ), PV indicates whether there is a large probability or not that the change has occurred ( $t_A \leq \tau$ ). Some methods have a constant PV. Others have a low PV at early alarms but a higher one later. In such cases, the early alarms will not prompt the same serious action as later ones.

### 2.3.5 Optimality

#### 2.3.5.1 Minimal expected delay

(Shiryayev 1963) suggested a highly general utility function, in which the expected delay of an alarm plays an important role. Shiryayev treated the case where the gain of an alarm is a linear function of the value of the delay,  $t_A - \tau$ , and the intensity of the change is constant. The loss associated with a false alarm is a function of the same difference. This utility can be expressed as  $U = E\{u(\tau, t_A)\}$ , where

$$u(\tau, t_A) = \begin{cases} h(t_A - \tau) & \text{if } t_A < \tau \\ a_1(t_A - \tau) + a_2 & \text{else} \end{cases}$$

The function  $h(t_A - \tau)$  is usually a constant (say,  $b$ ), since the false alarm causes the same cost of alerts and investigations irrespectively of how early the false alarm is given. In this case, we have

$$U = b P(t_A < \tau) + a_1 ED + a_2.$$

We would have a maximal utility if there is a minimal ( $a_1$  is typically negative) expected delay from the change point for a fixed probability of a false alarm (see Section 4.3). This is termed the ED criterion. Variants of the utility function leading to different optimal weighting of the observations are suggested for example by (Poor 1998) and (Beibel 2000).

#### 2.3.5.2 Minimax optimality

The minimum of the maximal expected delay after a change considers several possible change times, just like the ED criterion. However, instead of an expected value, which requires a distribution of the time of change, the least favourable value of CED( $t$ ) is used.

Moustakides (1986) uses an even more pessimistic criterion, the “worst possible case”, by using not only the least favourable value of the change time, but also the least favourable outcome of  $X_{\tau-1}$  before the change occurs. This criterion is very pessimistic. The CUSUM method, described in Section 2.4.3, provides a solution to the criterion proposed by Moustakides. The merits of the studies of this criterion have been thoroughly discussed for example by Yashchin (1993) and Lai (1995). Much theoretical research is based on this criterion.

### 2.3.5.3 ARL optimality

Optimality is often stated as a minimal  $ARL^1$  for a fixed  $ARL^0$ .  $ARL^1$  is the expected value under the assumption that all observations belong to the “out-of-control” distribution, whereas  $ARL^0$  is the expected value given that all observations belong to the “in-control” distribution. Efficient methods for surveillance (see Section 2.4) will put most weight on the most recent observations. Statistical inference with the aim of discriminating between the two alternatives that all observations come from either of the two specified distributions should, by the ancillarity principle, put the same weight on all observations. To use efficient methods and evaluate them by the ARL criterion is thus in conflict with this inference principle.

(Pollak and Siegmund 1985) argue that for many methods, the maximal value of  $CED(t)$  is equal to  $CED(1)$ , and with a minimax perspective this can be an argument for using  $ARL^1$  since  $CED(1)=ARL^1-1$ . However, this argument is not relevant for all methods. In particular, it is demonstrated by (Frisén and Sonesson 2006) that the maximal  $CED$ -value is not  $CED(1)$  for the EWMA method in Section 2.4.5. In the case of this method, there is no similarity between the optimal parameter values according to the ARL criterion and the minimax criterion, while the optimal parameter values by the criterion of expected delay and the minimax criterion agree well.

The dominating position of the ARL criterion was questioned by (Frisén 2003), since methods useless in practice are ARL optimal. The ARL can be used as a descriptive measure and gives a rough impression, but it is questionable as a formal optimality criterion.

### 2.3.6 Comments on evaluation measures

Computer illustrations of the interpretation of some of the measures mentioned below are made by (Frisén and Gottlow 2003). Formulas for numerical approximations of some of the measures are available in literature.

## 2.4 General methods for aggregating information

In surveillance it is important to aggregate the available information in order to benefit from all information. This aggregation can be carried out in

accordance with some general inference principles. Specific methods are then derived from the general ones. Different principles of aggregation have different properties and are thus suitable for different problems.

Some methods are highly flexible and have several parameters. The parameters can be chosen to make the method optimal for the specific conditions of the application (for example the size of the change or the intensity of changes). Many methods for surveillance are based, in one way or another, on likelihood ratios. Thus, we will start by describing the likelihood ratio component. The likelihood ratio for a fixed value of  $\tau$  is

$$L(s, t) = f_{X_s}(x_s | \tau=t) / f_{X_s}(x_s | D).$$

Most commonly used methods can be described as different combinations of these components.

#### 2.4.1 The Shiryaev-Roberts method

The simplest way to aggregate the likelihood components is just to add them. This means that all possible times for the change up to the decision time  $s$  are given equal weight. (Shiryaev 1963) and (Roberts 1966) suggested the method, now called the Shiryaev-Roberts method, in which an alarm is triggered at the first time  $s$ , so that

$$\sum_{t=1}^s L(s, t) > G,$$

where  $G$  is a constant alarm limit. This method can also be given a natural interpretation if the time of the change  $\tau$  is regarded as a random variable. This method can in that case be regarded as a special case of the full likelihood ratio method. This will be further discussed in Section 2.4.6.

#### 2.4.2 The Shewhart method

The Shewhart method, (Shewhart 1931) (Ryan 2000), is simple and certainly the most commonly used method for surveillance. It can be regarded as performing repeated significance tests. An alarm is triggered as soon as an observation deviates too much from the target. Thus, only the last observation is considered in the Shewhart method. An alarm is triggered at

$$t_A = \min\{s; X(s) > L\},$$

where  $L$  is a constant. The alarm criterion for independent observations can be expressed by the condition  $L(s, s) > G$  where  $G$  is a constant. The alarm statistic of the LR method reduces to that of the Shewhart method when  $C(s) = \{\tau = s\}$  and  $D(s) = \{\tau > s\}$ . This is the case when we want to discriminate between a change at the current time point and the case that no change has happened yet. In this situation, we are only interested to see whether something has happened “now” or not. Thus, the Shewhart method has optimal error probabilities for these alternatives for each decision time  $s$ . For large shifts, the LR method of Section 2.4.6 and the CUSUM method of Section 2.4.3 converge to the Shewhart method (Frisén and Wessman 1999).

By several criteria, the Shewhart method performs poorly for small and moderate shifts. By the minimax criterion, however, it works nearly as well as the LR method for some situations.

### 2.4.3 The CUSUM method

The CUSUM method, first suggested by (Page 1954), is closely related to the minimax criterion. (Yashchin 1993), (Siegmund and Venkatraman 1995) and (Hawkins and Olwell 1998) give reviews of the CUSUM method. The alarm condition of the method can be expressed by the partial likelihood ratios as

$$t_A = \min\{s; \max(L(s, t); t=1, 2, \dots, s) > G\}$$

where  $G$  is a constant. The method is sometimes called *the* likelihood ratio method, but this combination of likelihood ratios should not be confused with the full likelihood ratio method, LR.

The most commonly described application of the CUSUM method concerns the case of independent normally distributed variables. In this case, the CUSUM statistic reduces to a function of the cumulative sums

$$C_r = \sum_{t=1}^r (X(t) - \mu(t))$$

There is an alarm for the first time  $s$  for which

$$C_s - C_{s-i} > h + ki \text{ for some } i=1, 2, \dots, s,$$

where  $C_0 = 0$  and  $h$  and  $k$  are chosen constants. In the case of a step change, the value of the parameter  $k$  is usually  $k = (\mu^0 + \mu^1)/2$ .

Closely related to the CUSUM method are the Generalised Likelihood Ratio (GLR) and Mixture Likelihood Ratio (MLR) methods. For the MLR method suggested by (Pollak and Siegmund 1975), a prior for the shift size is used in the CUSUM method. For the GLR method, the alarm statistic is formed by maximising over possible values of the shift (besides the maximum over possible times of the shift). (Lai 1998) describes both GLR and MLR and proves a minimax result for a variant of GRL suitable for autocorrelated data.

The CUSUM method satisfies the minimax criterion of optimality described in Section 2.3.5.2. Other good qualities of the method have been confirmed for example by (Srivastava, et al. 1993) and (Frisén, et al. 1999). With respect to the expected delay, the CUSUM method works almost as well as the LR and Shiryaev-Roberts methods.

### 2.4.4 Moving average and window-based methods

The Moving average method can be expressed by the likelihood ratios as

$$L(s, s-d) > G$$

where  $G$  is a constant and  $d$  is a fixed window width. In the standard case of normally distributed variables this will be a moving average. It will have the optimal error probabilities of the LR method when we want to detect a change

which occurred at time  $s-d$  (i.e. for  $C=\{\tau=s-d\}$ ) and will thus have optimal detection abilities for changes which occurred  $d$  time points earlier.

Sometimes, as in (Lai 1998), advanced methods such as the GLR method are combined with a window technique in order to ease the computational burden.

#### 2.4.5 Exponentially weighted moving average methods

The EWMA method is a variant of a moving average method which does utilise all information. The alarm statistic is based on exponentially weighted moving averages,

$$Z_s = (1-\lambda)Z_{s-1} + \lambda Y(s), \quad s=1, 2, \dots$$

where  $0 < \lambda < 1$  and  $Z_0$  is the target value, which is normalised to zero. The EWMA statistic gives the largest weight to the most recent observation and geometrically decreasing weights to all previous ones. If  $\lambda$  is near zero, all observations have approximately the same weight. Note that if  $\lambda=1$  is used, the EWMA method reduces to the Shewhart method. The asymptotic variant, EWMAa, will give an alarm at

$$t_A = \min \{s: Z_s > L\sigma_Z\},$$

where  $L$  is a constant. In another variant of the method, EWMAe, the exact standard deviation (which is increasing with  $s$ ) is used instead of the asymptotic one in the alarm limit. (Sonesson 2003) found that the EWMAa version is preferable for most cases.

The EWMA method was described by Roberts (1959). Positive reports of the quality of the method are given for example by (Crowder 1989), (Lucas and Saccucci 1990), (Domangue and Patch 1991) and (Knoth and Schmid 2002). The choice of  $\lambda$  is important, and the search for the optimal value of  $\lambda$  has been of great interest in literature. Small values of  $\lambda$  result in a good ability to detect early changes while larger values are necessary for changes that occur later.

Most reports on optimal values of the parameter  $\lambda$  refer to the ARL criterion. Frisén (2003) demonstrated that by this criterion,  $\lambda$  should approach zero. Methods which allocate the power to the first time points will have good ARL properties but less ability to detect a change that happens later. In fact, and wisely enough, no one seems to have suggested that  $\lambda$  should be chosen to zero, even though this should fulfill the ARL criterion.

The EWMA method can be seen as a linear approximation of the full LR method (see Section 2.4.6). When a change from  $N(0, \sigma)$  to  $N(\mu, \sigma)$  occurs with the intensity  $v$ , the parameter  $\lambda$  that gives the optimality properties of the full LR method is

$$\lambda^* = 1 - \exp(-\mu^2/2)/(1-v),$$

This was shown by (Frisén 2003) and confirmed by large-scale simulation studies by (Frisén, et al. 2006).

#### 2.4.6 The full likelihood ratio method

When the time of the shift is regarded as a random variable, we can utilise this property. The full likelihood ratio method (LR) is optimal with respect to the criterion of minimal expected delay and also to a wider class of utility functions (Frisén and de Maré 1991). The full likelihood is a weighted sum of the partial likelihoods

$$L(s, t) = f_{X_s}(x_s | \tau=t) / f_{X_s}(x_s | D(s)).$$

The alarm set consists of those values of  $X$  for which the full likelihood ratio exceeds a limit. The following notation can be used: At decision time  $s$  we want to discriminate between the event  $C(s)=\{\tau<s\}$  and the event  $D(s)=\{\tau>s\}$ . The time of an alarm for the LR method is

$$t_A = \min \left\{ s; \frac{f_{X_s}(x_s | C(s))}{f_{X_s}(x_s | D(s))} > \frac{P(\tau>s)}{P(\tau \leq s)} \cdot \frac{K}{1-K} \right\} = \min \left\{ s; \sum_{t=1}^s w(s,t) \cdot L(s,t) > G(s) \right\}$$

where  $K$  is a constant and  $G(s)$  is the alarm limit. The time of an alarm can equivalently be written as the first time the posterior probability of a change into state  $C$  exceeds a fixed level

$$t_A = \min \{ s; P(C(s) | X_s = x_s) > K \}.$$

The posterior probability of a change has been suggested as an alarm criterion for example by (Smith and West 1983). When there are only two states,  $C$  and  $D$ , this criterion leads to the LR method (Frisén, et al. 1991). In cases where several changes may follow after each other, the process may be characterised as a hidden Markov chain and the posterior probability for a certain state may be determined (for example (Harrison and Stevens 1976) and (Hamilton 1989)). Sometimes the use of the posterior distribution, or equivalently the likelihood ratio, is named “the Bayes method”. However, it depends on the situation whether the distribution of  $\tau$  should be considered as a “prior” or as an observed frequency-distribution or if it just reflects the situation for which optimality is desired.

When the intensity,  $v$ , of a change tends to zero, the weights  $w(s, t)$  of the partial likelihoods do not depend on  $t$ , and the limit  $G(s)$  of the LR method does not depend on  $s$ . (Shiryayev 1963) and (Roberts 1966) suggested the Shiryayev-Roberts method (mentioned in Section 1.5.1), for which an alarm is triggered at the first time  $s$ , such that

$$\sum_{t=1}^s L(s, t) > G$$

where  $G$  is a constant. The method can be seen as the limit of the LR method when  $v$  tends to zero. The Shiryayev-Roberts method can also be derived as the LR method with a non-informative prior for the distribution of  $\tau$ . Both the LR method and the Shiryayev-Roberts method can be expressed recursively. One valuable property of these methods is an approximately constant predictive value (Frisén, et al. 1999), which allows the same interpretation of early and late alarms.



The LR method is optimised for the values of the change size and for the change intensity. In the case of a normal distribution, the LR method gives an alarm at

$$t_A = \min\left\{s; \sum_{t=1}^s P(\tau=t) \exp\{t\mu^2/2\} \exp\left\{\mu \sum_{u=t}^s Y(u)\right\} > \exp\{(s+1)\mu^2/2\} P(\tau>s) \frac{K}{1-K}\right\}$$

where the constant  $K$  determines the false alarm probability.

As mentioned before, several methods can be described by approximations or combinations of likelihood ratios (Frisén 2003). Linear approximations of the LR method are of interest for two reasons – first, for obtaining a method which is easier to use and analyse but whose properties are as good as those of the LR method, and second, for getting a tool for the analysis of the approximate optimality of other methods as in (Frisén 2003).

## 2.5 Special aspects of surveillance for financial decisions

### 2.5.1 General approaches which can be used in complex situations

Situations in finance are often complex. Thus, some general approaches for surveillance in more complicated situations than those of the earlier sections are of interest. When the models are completely specified both before and after the change, the likelihood components  $L(s,t)$  can usually be derived or approximated. Then, these components can be combined by any of the general information aggregation methods mentioned in Section 2.4. (Lai 1995), (Lai 1998) and (Lai and Shan 1999) argue that the good minimax properties of generalisations of the CUSUM method make the CUSUM suitable for complicated problems. The likelihood ratio method, LR, with its good optimality properties can also be used. (Pollak, et al. 1985) argue that the martingale property (for continuous time) of the Shiryaev-Roberts method makes this more suitable for complicated problems than the CUSUM method. The LR method also has this property, but the CUSUM method does not.

### 2.5.2 Evaluation by return

The return from buying an asset at  $t = 0$  and selling at time  $t$  is  $r(t) = x(t) - x(0)$ , where  $X$  is a monotonic function of the price.

The expected return  $E[r(t_A)]$  of selling at the alarm time  $t_A$  is maximal when  $E[X(t_A)]$  is maximal. Thus, a sell signal should ideally come at time  $\tau$  which corresponds to a peak of the price. In (Bock, et al. 2008) on technical analysis this surveillance problem is analysed.

### 2.5.3 Surveillance of dependent data

Financial time series often have complicated time dependencies. The theory for surveillance of dependent data is not simple. The general approaches in Section 2.5.1 can be applied to obtain methods with known optimality properties. This was made for example by (Petzold, et al. 2004).

The most common approach to surveillance in the case of models with dependencies is to monitor the process of residuals. (Pettersson 1998) demonstrated that for an autoregressive process, this is an approximation of the LR method. Another common approach (also used by (Pettersson 1998)) is to adjust the alarm limit in order to adjust the false alarm risk resulting from ignoring the dependency. (Okhrin and Schmid 2008a), and the references in this, contain important contributions to this very sparsely discussed area.

#### **2.5.4 Surveillance of discrete distributions**

Most of the theory of surveillance is derived for normal distributions, but a bibliography of surveillance of attribute data is given by (Woodall 1997).

#### **2.5.5 Gradual changes**

Most of the literature on surveillance treats the case of an abrupt change. In many cases in finance, however, the change is gradual. The change is thus more complicated than the standard situation of a sudden shift in a parameter from one value to another. Important characteristics should be captured by the statistic under surveillance. In the presence of a nuisance parameter, a general approach is to use a pivot statistic. (Krieger, Pollak and Yakir 2003) suggest the CUSUM and Shiryaev-Roberts methods based on a statistic, which does not depend on the unknown parameters in a case of an unknown pre-change regression. (Arteaga and Ledolter 1997) compare several procedures with respect to ARL properties. One of the suggestions in the paper is a window method based on the likelihood ratio and isotonic regression techniques. In general, window methods (see Section 2.4.4) are inefficient for detecting gradual changes (Järpe 2000). (Yashchin 1993) discusses generalisations of the CUSUM and EWMA methods to detect both sudden and gradual changes.

It may be hard to model the shape of a gradual change exactly or even to estimate the baseline accurately. Then, the timely detection of a change in monotonicity is of interest. The start of an increase is of course of special interest, but also the decline may be of interest to get timely sell and buy signals.

When the knowledge on the shape of the curve is uncertain, non-parametric methods are of interest. (Frisén 2000) suggested surveillance that is not based on any parametric model but only on monotonicity restrictions. This surveillance method was described and evaluated by (Andersson 2002) and is further described in (Bock, et al. 2008).

#### **2.5.6 Changes between unknown levels**

After a change, the level of the statistic under surveillance (for example the variance) is seldom known. However, this is not a serious problem. The false alarm properties will remain the same even if the level after the change is not known. The method could be designed to be optimal for a change of a specific size, but this is not required. The unknown parameters can be handled within

different frameworks corresponding to different restrictions on possible optimality.

To control false alarms is usually more important than to optimise the detection ability. Knowledge of the pre-change conditions is important. The baseline is often estimated and used as a plug-in value in the method. The estimated baseline value will affect the performance of the method. In the situation where we want to detect an increase, we will get more false alarms if the baseline is underestimated than if the true value had been used. The opposite is true if the baseline is overestimated.

One way to avoid the problem of unknown parameters is to transform the data to invariant statistics. (Frisén 1992) and (Sullivan and Jones 2002) use the deviation of each observation from the average of all previous ones. (Gordon and Pollak 1997) use invariant statistics combined by the Shiryaev-Roberts method to handle the case of an unknown pre-change mean of a normal distribution. (Krieger, et al. 2003) use invariant statistics combined by the CUSUM and Shiryaev-Roberts methods for surveillance of a change in regression.

When both the baseline and the change are unknown, the aim of the surveillance could be to detect a change in a stochastically larger distribution. (Bell, Gordon and Pollak 1994) suggested a non-parametric method geared to the exponential distribution. The non-parametric method of (Bock, et al. 2008), designed for the detection of a change in monotonicity, also avoids the problem of unknown values of the baseline and the change.

The use of the maximum difference (measured for example by the likelihood ratio) between the baseline and the changed level is a useful approach. The GLR method ((Lai 1995) and (Lai 1998)) uses the maximum likelihood estimator of the value after the change. (Kulldorff 2001) used the same technique for the detection of clustering in spatial patterns.

Another general approach for unknown levels is the Bayesian one. The MLR method suggested by (Pollak, et al. 1975) uses priors for the unknown parameters in the CUSUM method. (Lawson 2004) used priors for the unknown parameters to calculate the posterior means in a Bayesian space-time interaction model.

### **2.5.7 Multivariate surveillance**

We may have several data streams containing information. This is the case for example in portfolio optimisation. We may also have several statistics, such as both the mean and the variance, to monitor. In (Okhrin and Schmid 2008b) multivariate techniques for financial problems are extensively discussed.

If the model can be completely specified both before and after the change, then it is possible to derive the likelihood components  $L(s,t)$  and aggregate them by a method which guarantees optimality. In complicated problems, however, this is seldom realistic. Instead, a reduction of the multivariate surveillance problem is common (Sonesson and Frisé 2005).

A reduction of the dimensionality of the problem is a natural first approach. Principal components could be used to reduce the dimensionality, but (Lowry and Montgomery 1995) argued that unless the principal components can be interpreted, a surveillance method based on them may be difficult to interpret. In (Rosolowski and Schmid 2003) and (Golosnoy, Schmid and Yatsyshynets 2008) the Mahalanobis distance is used to reduce the dimensionality of the statistic, thus expressing the distance from the target of the mean and the autocorrelation in a multivariate time series.

The most common way to handle multivariate surveillance is to reduce the information to one statistic and then monitor this statistic in time. (Wessman 1998) proved that this is a sufficient reduction when changes occur simultaneously in all variables.

Another commonly used approach is to make parallel surveillance for each variable and make a general alarm when there is an alarm for any of the components, see (Stoumbos, Reynolds Jr, Ryan and Woodall 2000). Any univariate surveillance method could be used. Parallel CUSUM methods were used by (Marshall, et al. 2004). The false alarms were controlled by using the False Discover Rate (FDR) from (Benjamini and Hochberg 1995). For evaluating the detection ability, the probability of successful detection (see Section 2.3.3) was used.

The more advanced approach of vector accumulation is an in-between of the reduction by time and the reduction by variable. Here the accumulated information on each component is used to transform the vector of component-wise alarm statistics into a scalar alarm statistic and make an alarm if this statistic exceeds a limit, see for example (Rogerson and Yamada 2004). It is also possible to construct the multivariate method while aiming at satisfying some global optimality criterion. (Järpe 1999) suggested an ED optimal surveillance method of clustering in a spatial log-linear model. (Lowry, Woodall, Champ and Rigdon 1992) proposed a multivariate extension of the univariate EWMA method, which is referred to as MEWMA. This can be described as the Hotelling  $T^2$  control chart applied to univariate EWMA statistics instead of the original data from only the current time point and is thus a vector accumulation method. (Crosier 1988) suggested the MCUSUM method, where a statistic consisting of univariate CUSUMs for each component is used. This is similar to the MEWMA statistic, which corresponds to a vector accumulation method. However, the way in which the components are used is not the same. An alternative way to construct a vector accumulating multivariate CUSUM is given by (Pignatiello and Runger 1990). The methods use different weighting of the variables. One important feature of these two methods is that the characteristic zero-return of the CUSUM technique is constructed in a way that is suitable when all the components change at the same time point.

Different aspects on approaches for multivariate surveillance were given by (Frisén 2003) and (Sonesson, et al. 2005). The multivariate methods can be evaluated by the measures and criteria described above or by generalised

measures. (Wessman 1999) suggested a generalisation of the ARL measure to allow for the possibility of different change times for different variables. Controlling the false discovery rate is of interest when making conclusions about several variables and is used for example by (Wong, Moore, Cooper and Wagner 2003). However, the question of optimality is always complex in multi-dimensional cases.

## 2.6 References

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## Research Report

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