UNIVERSITY OF GOTHENBURG

DEPARTMENT OF EDUCATION, COMMUNICATION & LEARNING

SCAFFOLDING MATHEMATICAL CONVERSATIONS:

How interactive exhibits and parents support children's mathematics learning at a science center

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Thesis:	30 higher education credits
Program and/or course:	International Master's Programme in IT & Learning
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Supervisor:	Lena Pareto
Examiner:	Mona Lundin
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Abstract

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Purpose: This study aims to address the current research gap on interactive mathematics exhibits by investigating parents' role in supporting their children's mathematics learning using such displays. In addition to this, the study examines the design of an interactive exhibit and explores whether it can facilitate mathematics learning among parents and children. Lastly, the study also explores the way mathematics learning occurs in informal settings.

Theory: The sociocultural and constructionist theories of learning serve as the guiding theoretical framework for the study. While the sociocultural theory offers insights into the parent-child learning dynamic, the constructionist theory helps examine the interactive exhibit as a potential "object-to-think-with".

Method: This study examines a mathematics interactive exhibit located in the science center Universeum, in Gothenburg, Sweden. Adopting a mixed-methods research methodology, the study examines 10 parent-child groups whose interaction with the exhibit was audio and video recorded. The coding scheme used was inspired by Tscholl and Lindgren's (2016) work on learning talk around interactive exhibits and Schnieder and Schuh's (2022) research on parent scaffolding behaviours for mathematics learning using technology-based games.

Results: The findings highlight parents' important role in guiding their children's mathematical learning. Parents were found to provide significant conceptual support through explanations, inquiry, demonstrations, prompts, and corrections. However, the study also indicated that parents encountered challenges in connecting the interactive experience with the underlying mathematical concepts. Additionally, the study revealed that the exhibit's interactive features contributed to the co-construction of knowledge between the participants by facilitating conversations, hands-on manipulation, exploration, and problem-solving strategies. Moreover, the exhibit encouraged both parents and children to participate in the interactive activity. This finding contrasts with previous studies on interactive technologies showing that such technologies inhibit social interaction.

Foreword

First and foremost, I would like to express my heartfelt gratitude to my supervisor, Lena Pareto, for her guidance and encouragement throughout the entire writing process of completing this thesis. Your constant support and invaluable insights have been instrumental in finishing this work. I am truly fortunate to have had such a dedicated supervisor.

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Table of content

1. Introduction

Mathematics is recognised worldwide as the foundation for scientific and technological knowledge, as well as for the progress of political and socio-economic change (Hwa, 2018). Furthermore, it functions as an essential tool in several fields, such as engineering, medicine, natural sciences, and finance (Hwa, 2018). In addition to this, mathematical competence plays a fundamental role in active citizenship, social inclusion, and employability in the modern technology-oriented society (Berlinski & Busso, 2013). Hence, a lack of mathematical skills can significantly impact one's ability to make crucially significant decisions related to education, career, and life in general (Sherman et al., 2015).

While mathematics is undoubtedly acknowledged for its significant role, it is also generally regarded as a difficult subject to master. Furthermore, mathematics has also been previously argued to bear the label of an ambiguous discipline that confuses novice learners (Barwell, 2005). This confusion has been claimed to stem from several factors, some of them being represented by the mathematical register with the specific language which differs from everyday language (Barwell, 2005; Schleppegrell, 2007), the use of different signs and symbols (Riccomini et al., 2015; Xu et al., 2022), and the difficulty of understanding the connection to day-to-day life events (Cooper, 2011; Martin & Gourley-Delaney, 2014). These factors have been argued to contribute to confusion and ambiguity in children’s learning and consequently leading to poor performance in the subject and math anxiety (Hwa, 2018; Schleppegrell, 2007).

Formal schooling stands as one of the primary means for children to learn mathematics, yet it has also faced criticism for its limitations on deep mathematics learning. While the conventional classroom environment is undoubtedly essential for gaining a foundational knowledge of mathematical theories and equations, it has also been scrutinised for restricting children’s view of the subject (Hwa, 2018). This restricting nature has been argued to lie in the standardised curricula and assessments, which leave little room for personalised exploration and engagement. This may result in rote memorization, with limited opportunities for developing problem-solving skills and applying the acquired knowledge in everyday contexts. Furthermore, the standardised curricula do not accommodate the diverse learning styles of individual students, potentially leaving some struggling to keep up while others are advancing (Duncan et al., 2007). In this manner, the structure meant to facilitate learning might inadvertently hinder a deep understanding of mathematics, underlining the need for more flexible approaches.

Considering the negative reputation surrounding mathematics and the challenges of formal mathematics learning, initiatives were taken to design more enjoyable and accommodating activities for people to learn and deepen their knowledge in the subject. One of the most recent initiatives is the implementation of mathematics-oriented exhibits in science centers and museums. Since these spaces are typically considered fun, engaging places where learning is considered a leisure activity, they hold considerable potential for changing visitors' perceptions of the subject (Allen, 2004; Cooper, 2011). Furthermore, since such informal spaces are normally designed to encourage social learning through collaboration defined by conversations and hands-on manipulation (Allen, 2004; Tscholl & Lindgren, 2016) there is very limited research on the learning dynamic around these exhibits, especially about how parents and children use and learn with help of these exhibits. However, the focus on mathematics interactive exhibits has been quite limited compared to science, art, and history exhibits (Falk & Dierking, 2000; Leinhardt et al., 2002; Cooper, 2011). The lack of research in this area can be attributed to several factors, with one of the primary challenges being the relatively limited number of interactive mathematics exhibits. Guberman et al. (1999) address this issue in their paper, pointing out that the abstract nature of mathematics poses considerable obstacles in designing exhibits that are accessible to the public, foster mathematics learning and are also entertaining. Another factor contributing to the lack of research in the area is the difficulty of measuring learning in informal environments. Unlike formal education settings, science centers and museums are social environments that lack standardized assessments and allow free-choice learning (Yoon et al., 2012). As a result, measuring learning in such settings normally proves to be quite challenging.

Parent-child groups constitute a considerable share of the overall visitor demographics in museums and science centers, underscoring the essential role parents play in cultivating their children's curiosity and expanding their knowledge. Moreover, it underlines the importance of informal learning environments complementing formal education, leading to a deeper understanding of specific subjects and phenomena. Given the limited existing research concerning parents' support of their children's learning of mathematics using mathematics exhibits, combined with the important role parents play in their children's learning, it becomes imperative to analyse parent-child interactions using these exhibits. Thus, this analysis is fundamental for several reasons. Firstly, many studies emphasize that children need to develop a solid mathematical foundation starting in the very early years (Cooper, 2011). Parents engage their children with mathematics in various contexts, from home activities to science center visits. Therefore, an analysis of the nature of their support would help comprehend how these initiatives help towards developing this solid mathematical foundation. Secondly, how parents and children interact with each other in joint museum activities may affect what children learn from these experiences. This aspect was also pointed out by comparison studies on how parents scaffold their children's learning with technology-based and traditional board game formats (Schnieders & Schuh, 2022). As highlighted by Schnieders and Schuh (2022), the tutorial interaction between an adult and a child involves various scaffolding processes which have different implications for children's learning. To understand parent-child interactions in museum and science center activities, a focus on parent scaffolding behaviours would be valuable. Lastly, since interactive mathematics exhibits have been recently implemented, research on such technologies would contribute significantly to a better understanding of their contribution to mathematics learning.

1.1 Purpose and Aims

Taking into account the abovementioned considerations, this study investigates how parents [1]support their children's mathematics learning using an interactive equations-focused exhibit called "*Vågade Ekvationer*" ("Bold Equations" in English), located in the Universeum science center in Gothenburg, Sweden. Additionally, this research explores whether the design of the exhibit helps scaffold visitors' learning of mathematics. Therefore, the study aims to address the following research questions:

> RQ1: *How do parents support their children's learning of mathematics using an interactive exhibit in a science center?*
>
> RQ2: *Can the design of the interactive exhibit promote learning through conversations, hands-on manipulation, and problem-solving among parents and children, and if so, how?*

To answer the research questions, this study is designed as exploratory research, that analyses visitor conversations and gestures to investigate the way parents scaffold their children's learning. These scaffolds will be analysed in relation to learning talk categories through which participants' actions are structured. Given the study's focus on parent-child interactions and the use of interactive technologies for learning, the sociocultural and constructionist theories of learning will be used as the basis for examining these aspects.

All in all, the present study serves a dual purpose. Firstly, the research aspires to contribute valuable insights to the current gap in research on parents' role in supporting their children's mathematics

[1]Throughout this study, the term "*parent*" is primarily used, although "*caregiver*" is a more inclusive term since it attends to various forms of "parenting". This choice is made to maintain consistency with prevalent terminology widely used in the research field and to facilitate comparisons with previous studies.

learning using interactive mathematics exhibits in science centers and museums. Secondly, it aims to inform future exhibit design, by analysing whether its design may promote exploration, conversation, problem-solving skills, and hands-on manipulation.

1.2 Overview of the Thesis Work

The thesis is organised into 7 sections. Following the first introductory section, the second section is represented by the theoretical framework with the two theories that influenced this study, sociocultural and constructionist. The purpose of having the theoretical framework before the background literature is to give a conceptual foundation for understanding the relevant literature related to the topic. Having this in mind, the third section is represented by the background literature where three main aspects will be discussed: informal learning and mathematics, learning with interactive exhibits in science centers, and parent scaffolding for mathematics learning. The fourth section presents the methodology of the study with information about the research setting and the observed interactive exhibit, the research procedure, eligibility criteria, participants, and ethical considerations. In the fifth section, the findings of the study will be presented both quantitatively and qualitatively. The sixth section discusses the findings concerning the research question of the study. Lastly, the seventh section contains the conclusions and implications of the study.

2. Theoretical Framework

The overarching approach that was chosen for guiding the design of the study is the sociocultural view on learning. This approach was chosen based on several theoretical considerations which will be further discussed in this section. In addition to the sociocultural theory, Papert’s constructionist theory is also used, focusing on how mathematics exhibits can be used as objects-to-think-with. Firstly, the sociocultural perspective will be introduced by presenting its relevance to the study. Then, the key concepts of this theory will be discussed in relation to the wider scope of the study. This serves the purpose of laying the groundwork for understanding the potential role parents may fulfil in guiding children’s learning of mathematics using the equation exhibit. Lastly, the constructionist theory will be explored, emphasizing its focus on learning-by-doing.

2.1 Sociocultural Theory & Science Centers

Originating in the field of psychology, the sociocultural theory has gained prominence across educational sciences as well, providing valuable insights into how society may contribute to individual development. The sociocultural theory has been widely adopted as the guiding framework for studying learning in informal learning settings such as science centers and museums (Phipps, 2010; Rennie & Johnston, 2004). This theory emerged from the work of the Soviet psychologist Lev Vygotsky and was later developed by numerous scholars, researchers, and educators.

To begin with, the sociocultural perspective is well-known for focusing on the relationship between human action and the cultural and institutional context in which it occurs (Schnieders & Schuh, 2022). Regarding this, Nadelson (2013) theorizes that, the key principle of the sociocultural model in informal science centers is based on the idea that knowledge is socially constructed and culturally dependent. This aspect plays an essential role in science centers where visitors’ learning is conveyed through their interaction with the exhibits and each other to reach a deeper understanding of the displayed phenomena.

Furthermore, adopting a sociocultural approach to examine family interactions has been demonstrated to provide invaluable insights into parent-child collaborations and understand how learning is supported through these interactions (Phipps, 2010; Nadelson, 2013; Tscholl & Lindgren, 2016). Nadelson (2013) points out one of the advantages stating that, taking a sociocultural approach to parent-child interactions helps with understanding how parents scaffold their children to help them gain conceptual understanding:

> The application of the sociocultural model theorizes that parent and- child interactions in the form of conversations or shared problem-solving result in the transfer of knowledge from the parent to the child through the social interaction leading to the development of conceptual understanding. (Nadelson, 2013, p.479)

In addition to the social and cultural focus on learning, inherent in this theory is the focus on language and tools as mediators of learning (Anderson, 1997; Ash, 2003; Tscholl & Lindgren, 2016; Vygotsky, 1978). Science centers are typically designed as social environments that foster communication and interaction. This is usually achieved by adopting interactive exhibits that encourage collaboration and communication, as well as learning through hands-on activities.

Taking into consideration all the aforementioned aspects, the sociocultural theory provides a suitable frame for the present research on family conversation elicited by an interactive exhibit in an informal science center. In the following section, key concepts belonging to this theory will be introduced and discussed in relation to the current study and how they may help answer the research questions.

2.1.1 Scaffolding

One of the most central ideas within the sociocultural theory is represented by the interaction with more knowledgeable partners and how this interaction shapes the development of less knowledgeable peers. This concept was theorized by Lev Vygotsky in his Zone of Proximal Development (ZPD) theory and it examines the difference between what a learner can do independently and what they can do guided by a more knowledgeable other (Vygotsky, 1978). According to this theory, effective learning occurs by including "activities slightly beyond the child's current abilities but not outside a realistic zone of his or her potential" (Bjorklund et al., 2004, p.348). These activities are also defined as scaffolds which refer to "any action or statement intended to provide guidance, actively help, or motivate the child to complete the tasks in the game" (Schnieders & Schuh, 2022, p.47). In this view, a parent's role is to engage the children in activities that get progressively more challenging till the less competent person becomes independently proficient. Highly related to this view is Rogoff's (1990) concept of "apprenticeship in thinking" which relates to how children become participants in society by observation, practice, assistance, and collaboration.

According to Chaiklin (2003), the collaboration process does not follow any systematic principles or sequential order in ZPD. Instead, the support offered by the parent is constantly adapted to the cognitive and social demands of the child (Bjorklund et al., 2004). As an example, Vygotsky suggests the following interventions for supporting the cognitive development of the child:

> we show the child how such a problem must be solved and watch to see if he can do the problem by imitating the demonstration. Or we begin to solve the problem and ask the child to finish it. Or we propose that the child solve the problem that is beyond his mental age by cooperating with another, more developed child or, finally, we explain to the child the principle of solving the problem, ask leading questions, analyze the problem for him, etc. (Vygotsky, 1998, p.202)

One additional aspect that is also important to mention is that the sociocultural theory doesn't benefit only the less experienced learner, but it can benefit the more experienced one as well. For example, in the process of scaffolding a child's learning, parents can test and further consolidate their knowledge. Furthermore, there might be instances when the parent is not able to provide help to the child, but by collaborating they can both solve the problem and learn from their experience. Wells (1999) highlights this aspect as well stating that, people who prove to be more knowledgeable in one task may themselves need assistance with another since most activities include a variety of constituent tasks. Additionally, it can also be the case that none of the individuals have more expertise, but by working collaboratively, they can achieve similar results as they would have by working with a more knowledgeable partner. Wells (1999) stresses this idea claiming that:

> although no member has expertise beyond his or her peers, the group as a whole, by working on the problem together, is able to construct a solution that none could have achieved alone. (p.324)

From this perspective, ZPD applies to any situation where individuals work collaboratively to achieve mastery of practice or gain knowledge about a topic (Wells, 1999). The aspects discussed may prove to be highly relevant in the context of this study, since the parent may not always play the role of the more knowledgeable other, but by working collaboratively with their children, they may both complete the tasks displayed by the exhibit and gain conceptual understanding of the underlying mathematical concepts.

In conclusion, the sociocultural theory sees learning as a transaction that mutually benefits the parent and the child, and which focuses primarily on social interaction as the driving mechanism (Phipps, 2010). By examining the nature of this collaboration, we may not only better understand how children learn mathematics guided by their parents, but we may also discover how parents as well learn by teaching or by being guided by their children.

2.1.1 Language & Tools as Learning Mediators

Central to Vygotsky's sociocultural theory is also the notion of social mediation through language and tools. Since much of the social interaction observed within science centers and museums is conversation and hands-on manipulation, analysing these two aspects are highly relevant for understanding family learning in informal settings (Falk & Dierking, 2000).

To begin with, the significant role that language plays in learning has been extensively discussed by scholars, as evidenced by the numerous studies in the field. Vygotsky stresses the fundamental role of social communication in children's development, calling language "the tool of the tools" (Falk & Dierking, 2000; Vygotsky, 1978). In connection to this, Halliday (1993) considers that language reflects the process of experience becoming knowledge. Also, according to Tscholl and Lindgren (2016), conversations are scaffolds that not only explore and support children's understanding, but also help parents challenge their own. Having this in mind, dialogue can be argued to serve as a "negotiating medium for teaching and learning" (Tscholl & Lindgren, 2016, p. 139).

Tools have been undoubtedly argued to play an important role in learning. Previous research on embodied cognition supports the essential role physical action and representation play in knowledge acquisition (Jant et al., 2014; Nadelson, 2013). Science center and museum exhibits are designed to encourage hands-on learning, supporting the assumption that visitors learn best through direct interaction with objects (Falk & Dierking, 2000; Jant et al., 2014).

2.2 Constructionist Theory & Science Centers

In addition to the sociocultural theory, the constructionist theory was also used to further understand the role the interactive exhibit may play in participants' learning. The focus of this theory in relation to the study is analysing exhibits as "objects-to-think-with", an aspect which is central to the constructionist theory.

To start with, one of the pioneers of the constructionist theory is Seymour Papert who promoted the concept of "learning-by-doing" through his work on the "Logo" programming language and tool "Logo Turtle". Papert believed that education institutions focus more on teaching children what to learn, instead of how to learn, creating a gap in children's development of problem-solving skills (Morado et al., 2021). With this in mind, Papert points out that, children need to develop their own cognitive skills through active participation in their learning, this being achieved by using tools that allow them to build their own intellectual structures (Papert, 1980). Having this in mind, the constructionist theory highlights the importance of tools as "objects-to-think-with", which are defined by three main characteristics: they allow children to explore complex ideas through bodily engagement, they are used in disciplinary domains, and are part of children's socio-material environment (Morado et al., 2021; Papert, 1980). The underlying idea of using these tools is that learners have the opportunity to master complex and abstract concepts (Morado et al., 2021).

Learning in constructionist environments has been regarded as self-directed, with learners taking initiative in their learning, being stimulated by the externalization of their actions with the tools, most of the times represented by a range of media (Morado et al., 2021). Furthermore, constructionist learning sees knowledge as defined in situ and "determined by the state of being immersed in the context, connected and sensitive to the changes in milieu" (Morado et al., 2021, p.1096). Thus, the constructionist theory emphasizes active hands-on learning through the process of creating and constructing knowledge by engaging with the learning material at hand.

Taking into consideration the aforementioned aspects, interactive mathematics exhibits can be argued to serve as "objects-to-think-with" in several ways. Firstly, they allow visitors' exploration of complex and abstract mathematical concepts through hands-on manipulation. This exploration is commonly designed to lead to improved mathematical knowledge, critical thinking, and making connections to real-life phenomena. Secondly, learners are engaged in the learning activity, being stimulated by the outcome of their interactions with the exhibits, such as images or numbers being displayed on the screen, or making motors move. Also, visitors have the opportunity to self-direct their learning, focusing on specific points of interest and building on their previous knowledge.

While both the sociocultural and constructionist theories share common principles in terms of learners' active participation, collaboration, and use of tools for learning, they present different perspectives on how learning occurs and how these aspects potentially contribute to learning. Constructionist learning emphasizes the active process of constructing knowledge, with tools serving as means to facilitate individual problem-solving, creativity, and exploration. In this case, learners engage directly with tools for creating tangible learning experiences or to engage in activities which lead to the construction of new knowledge. This view is different from the sociocultural theory which sees tools more as cultural artifacts that mediate and facilitate learning and communication through social interaction. Furthermore, the sociocultural perspective sees tools not only just as physical objects, but also include language and symbols. Moreover, the tools are used rather socially and in a collaborative context compared to constructionism which focuses rather on individual experiences.

To conclude, in the context of this study, using both the sociocultural and the constructionist theory will help shed light not only on how parents support their children's learning with the mathematics interactive exhibit, but also to see the role the exhibit plays in this learning process.

3. Background Literature

In addition to the theoretical framework of the study, it is also important to locate the current study within related research in informal mathematics learning using interactive exhibits. This chapter presents a comprehensive review of relevant literature on family conversations in science centers and museums around interactive hands-on exhibits. Firstly, an overview of informal learning will be provided, focusing on its characteristics and potential for mathematics learning. Then, an analysis of interactive technologies in science centers and museums will be conducted to understand their benefits and limitations. Since the focus on interactive mathematics exhibits is very limited due to their recent implementation, the research will be mainly focused on how science exhibits foster learning. Lastly, the nature of family conversations will be explored by looking at how they are shaped by interactive exhibits. These aspects have the purpose of getting a broader perspective of previous research and explaining the relevance of the research questions in relation to the identified research gaps.

3.1 Informal Learning & Mathematics

Mathematics learning typically occurs in formal classroom settings, following a curriculum that outlines predetermined learning objectives. This formal approach often includes individual assessments through standardized tests to evaluate students' progress and understanding of the subject. While formal mathematics education can bring benefits in terms of building a foundational understanding of mathematical principles and theories, there are also perceived limitations that need to be considered. These limitations have been found to contribute to poor performance in mathematics and math anxiety in students (Hwa, 2018; Sherman et al., 2015; Felder and Brent, 2005). Firstly, one limitation of formal mathematics learning is the promotion of rote memorization, more specifically, learning mathematical concepts through repetition, without necessarily gaining a conceptual understanding of the rationale behind the steps taken for solving mathematical problems (Sherman et al., 2015). Secondly, research into formal mathematics learning shows that formal school settings promote limited contextualization of mathematics, restricting the connection to real-world contexts. This point is highlighted by Gravemeijer (1997) who is of the opinion that, "in general, the classroom climate is one that endorses this separation between school mathematics and everyday-life reality" (p.389) Lastly, formal learning was contested to not support diverse learning styles (Hwa, 2018). The underlying problem for this is the prevalence of standardized approaches which disregards the individual cognitive modalities through which children learn and understand mathematical concepts (Hwa, 2018). As pointed out by Felder and Brent (2005), "students have different levels of motivation, different attitudes about teaching and learning, and different responses to specific classroom environments and instructional practices" (p.57). Thus, a standardised approach to mathematics may impede a holistic development of mathematical understanding and skills.

Informal learning has been previously argued to play an essential role in complementing formal classroom experiences (Bonotto, 2005; Cooper, 2011). As most formal mathematics education focuses on isolated instruction which prepares students for standardized tests, there are very limited opportunities for learners to apply their knowledge in real-life situations (Cooper, 2011). As Cooper (2011) points out, children need to be guided to see mathematics as more than just what they learn in school, allowing them to see the impact it has on the world and their everyday life:

> It is important for young learners to realize that mathematics is more than counting and number facts, or recognition of geometric shapes, and the application of mathematical procedures […]. They also need to see the mathematics that makes up the world they live in. (p.51)

In contrast to formal learning, informal learning environments promote learning through self-exploration and experimentation. This type of learning is defined as social, interactive, and nondidactic (Rogoff et al., 2016; Yoon et al., 2012, Hurst et al., 2019). The social and interactive aspects highlight the importance of social interaction both as a means for receiving help from peers, but also as a way of encouraging learners to explain their thinking and problem-solving processes (Hurst et al., 2019). The nondidactic aspect implies that learners can freely choose how and what to learn, allowing them to fulfill their own interests and curiosities (Hurst et al., 2019). Moreover, informal learning settings have no teachers forcing learners to complete specific tasks or to test them. Without restrictions, visitors can explore freely, fostering a sense of ownership over their learning and encouraging a more immersive and personally meaningful experience. All these aspects bring significant benefits to mathematics learning by supporting a more inviting environment where children can learn mathematics in a more engaging manner and make connections to real-life contexts. Previous research highlights these benefits, showing that promoting mathematics in informal learning settings can help young learners considerably by predicting academic learning, fostering positive attitudes and interest in the subject, and developing critical thinking skills (Cabello & Ferk Savec, 2018; Hurst et al., 2019).

Informal mathematics has been observed and researched across various settings, from science centers and museums (Gyllenhaal, 2007; Guberman et al., 1999) to home activities such as playing board games (Ramani & Siegler, 2008; Schnieders & Schuh, 2021; Bjorklund et al., 2004; Le Fevre et al., 2009), counting numbers (Anderson, 1997) and outdoor activities (Cabello & Ferk Savec, 2018). When it comes to science centers, the focus on mathematics-focused exhibits has been quite limited compared to science, art, and history (Falk & Dierking, 2000; Leinhardt et al., 2002; Cooper, 2011). One primary reason for this is represented by the difficulty of translating mathematical ideas into exhibits that are accessible to the public. Guberman et al. (1999) discuss this challenge based on their evaluation of two prototypes of interactive mathematics exhibits. Their findings convey that, the inherently abstract nature of mathematics poses considerable obstacles in designing exhibits that are accessible to a wide range of visitors, foster mathematics learning, and are also entertaining (Guberman et al, 1999).

On the other hand, although informal learning environments offer numerous advantages for mathematics learning, they also pose several challenges. One primary concern is represented by the difficulty of measuring learning itself. First of all, informal learning inherently lacks assessments which normally play an important role in evaluating learners' work and progress. In connection with this, the flexibility and variability of learning bring challenges to identifying and measuring specific learning outcomes. Also, activities in science centers are often experienced in single-visit episodes with little follow-up or reflection on the content of the exhibits (Yoon et al., 2012). Therefore, assessing the depth of understanding becomes difficult. Lastly, an additional drawback of informal learning contested by previous studies is the lack of teachers to guide, enforce concentration, and assess learner’s progress (Allen, 2004). As pointed out by Allen (2004) in her evaluation of a museum setting, “if an exhibit has a boring or effortful or confusing component, visitors have no way of knowing whether the reward for persisting will be worth the effort; and in an environment full of interesting alternatives, they are very likely to simply leave the exhibit and move on” (Allen, 2004, p.18).

3.2 Learning with Interactive Exhibits in Museums & Science Centers

Over the years, museums and science centers encountered a conceptual shift in terms of how their exhibits should be designed and organized to attract visitors and promote learning. While they initially served as repositories for object displays that visitors could only observe (Schauble et al., 1997), during the 1960s a more interactive approach was discovered. This shift in exhibit design encouraged visitors to switch from being passive observers, as in the case of static exhibits, to active participants in their learning. Such exhibits are often accompanied by descriptors such as “interactive”, “participatory” and “hands-on” to accentuate visitors’ active involvement (Tuckey, 1992). Throughout this study, the term “interactive” will be used as an umbrella term for all exhibits that involve physical manipulation.

Interactive exhibits refer to displays or installations, designed to encourage visitors to engage with the displayed content, often through direct hands-on manipulation and dialogue. Some examples of interactive features that these displays use are digitally enhanced objects, touchscreens, buttons and switches, sensors, and more recently augmented reality and virtual reality components. While physical manipulation plays an important role in these exhibits, Tuckey (1992) highlights that, such displays need to also allow manipulation of ideas:

> This is the ideal, that children will be motivated to explore scientific phenomena by becoming engaged at an intellectual level with an exhibit that is sufficiently flexible to accommodate the demands of an inquiring mind. (p.273)

Having this in mind, interactive exhibits do not only serve as displays to interact with, but also to think with, further supporting the constructionist belief in tools supporting cognitive development.

Interactive exhibits are designed with the purpose of communicating information to the public in novel ways, as well as facilitating new forms of participation (Heath et al., 2005). Furthermore, several studies emphasize their stimulating, multisensory, and fun features which enrich the learning experience (Allen, 2004). At the core of these exhibits lies social interaction and collaboration between visitors, with a growing body of research emphasizing the fundamental role hands-on manipulation and conversations play in one’s museum experience (Crowley, 2001; Ash, 2003; Heath et al., 2005; Tscholl & Lindgren, 2016). This conceptual shift towards interactive exhibits is argued to have been influenced by the emergence of the sociocultural theory where “objects, tools and artifacts feature in and mediate learning in visitor groups” (Heath et al, 2005, p.92). As Säljö (2010) highlights, interactive technologies are increasingly beginning to focus on the performative and social aspects of learning. In this view, the goal of these exhibits is to foster an environment where children can satisfy their inquisitive nature and bolster their knowledge through free exploration, collaboration, conversation, and hands-on manipulation.

While interactive exhibits are becoming more prevalent in science centers and museums, research on their educational impact has been quite limited. Yoon et al. (2012) address this issue in their study about learning with augmented reality exhibits, pointing out that much of the research focuses on factors such as engagement and usability, rather than on what and how visitors actually learn by using them. While some studies indicate that interactive technologies can yield significant benefits owing to their engaging and innovative features, alternative viewpoints posit that they may actually distract individuals from learning.

To start with, interactive exhibits have been found to present significant learning potential. Interactive exhibits have been previously discovered to be extremely appealing to visitors, encouraging them to engage more and for a longer time with the displayed content (Heath, 2005). For instance, Tscholl and Lindgren’s (2016) explorative study on an interactive science exhibit reveals that, children’s playful and engaging interactions promoted by the exhibit enhanced their conceptual understanding of science-related concepts. In addition to this, the exhibit’s interactive features encouraged participants to collaborate, empowering parents to guide their children’s learning of complex scientific concepts. Another study carried out by Sommerauer and Müller (2014) on an augmented mathematics exhibit shows that, participants who used the interactive exhibit had significantly better results in terms of knowledge acquisition and retention, compared to participants who used non-augmented exhibits. In addition to these benefits, a noteworthy advantage of interactive exhibits is represented by their scaffolding features which give real-time feedback. These scaffoldings refer to the supportive structures embedded within the design of the exhibit for facilitating and enhancing learning. Examples of such scaffolding features are real-time visualizations of visitors' gestures and interaction with the exhibit or prompts given after each step the user takes towards reaching a goal. The research carried out by Yoon et al. (2012) provides an example of the positive effects of digital augmentation scaffolds in science museums. They discovered that children who used the digital augmentation scaffolds demonstrated greater cognitive gains and conceptual knowledge of science, compared to control groups who were not given these scaffolds. In addition to the scaffolding features, interactive exhibits were discovered to foster problem-solving skills, by involving visitors in creating mathematical strategies (Pareto, et al., 2023). Lastly, it was found that interactive technologies promote collaboration and conceptual understanding. In the study carried out by Pareto et al., (2012) about an interactive teachable game it was discovered that interactive technologies can positively affect math comprehension and collaboration, with competition serving as the motivational force for completing the tasks.

On the other hand, interactive technologies were also found to present downsides concerning learning. For example, Narayanan and Hegarty (2000) are of the opinion that, the benefits of interactive devices tend to be overstated in the current era of multimedia technologies. In their study, it was discovered that interaction with a device may not necessarily imply a deep understanding of the underlying ideas. Relating to this, Heath et al. (2005) discuss in their study the complex nature of interactivity, addressing the issues and difficulties that arise when designing interactive exhibits. They discovered that, interactive technologies may actually inhibit, rather than promote social interaction between visitors when engaging with the exhibit (Heath et al., 2005). This finding is related to Meisner et al. (2007) who discovered that, exhibit designs, although intended to facilitate simple and intuitive use, may also constrain interaction and collaboration. One explanation for this is that visitors immersed in hands-on exhibits tend to prioritize their focus on the task at hand, potentially resisting or being less receptive to support or participation from others (Hsi, 2003). This issue is also reflected in Tscholl and Lindgren's (2016) study which conveys that, interactive and visually rich technologies may attract children more than family members, with the latter becoming rather passive observers. The lack of social interaction for an interactive exhibit may impose challenges on children's learning since social interaction represents an important medium for them to receive help, but also to allow them to express their ideas and thoughts in the problem-solving process (Hurst et al., 2019).

The aforementioned considerations not only support the idea of collaboration as an important factor for learning with interactive technologies, but also provide valuable insights into how their design could be improved to support visitors' development. Heath et al. (2005) emphasize these aspects in their work stating that, a reconceptualization of visitor interaction in museums is necessary for creating effective interactive exhibits:

> In designing and developing exhibits for science centers and museums we have to reshape the ways in which we think of and conceptualize the visitor, breaking free from individualistic models which continue to pervade "interactives" and the very idea of "interactivity". (p.98)

Additionally, another perceived limitation of interactive exhibits is that, even if they may promote social interaction, this may not necessarily be learning-oriented. Cooper (2011) undertook an explorative study on the potential for mathematical experiences in informal learning environments, investigating 3 local museum settings. Her findings reveal that, while children and family members interacted with the mathematical exhibit in various ways, the conversations related to mathematical ideas were very limited. In addition to this, the study revealed that even though mathematics exhibits present potential for mathematics learning, this potential is not sufficiently emphasized. While mathematics learning opportunities may be apparent for educators and developers, this potential is not always evident for the visitors. Connected to this view is Cabello and Ferk Savec's (2018) study which underlines that learning opportunities in science centers and museums may be evident for teachers, but not necessarily by an unknowledgeable visitor.

Taken together, these studies reveal the significant potential of interactive exhibits for learning through social engagement, free exploration, and hands-on manipulation, but also their limitations as they can isolate visitors and may not effectively promote learning-oriented activities.

3.3 Parent Scaffolding for Mathematics Learning

Research in the area of learning and development indicates that parents who are involved in their children's mathematics learning can positively influence their understanding of the subject (Eason & Ramani, 2020; Bjorklund et al., 2004; Gyllenhaal, 2006; Cooper, 2011). Parent support, or otherwise known as parental scaffolding, relates to "any action or statement intended to provide guidance, actively help, or motivate the child" (Schnieders & Schuh, 2022, p.47). A considerable amount of research suggests various forms of scaffolding to support and enhance collaboration between parents and children, demonstrating how learning may be significantly improved if additional guidance is provided (Jant et al., 2014; Tscholl & Lindgren, 2016). Conversations have been argued to represent important scaffolds for learning, where parents provide structure and guidance to their children's actions, helping them understand what they would have not on their own (Tscholl & Lindgren, 2016). Another perceived advantage of conversations is that while children have the opportunity to verbalise their thoughts, parents are able to identify problems in their understanding and work towards improving it.

Prior studies also indicate that conversations together with object manipulation may significantly improve children's learning. For example, Jant et al. (2014) conducted a study on the effects of object manipulation and conversation on learning in a natural history museum. Their findings reveal that, the parent-child dyads who were assigned conversation cards prior to their visit engaged in more elaborative talk and engagement with the exhibit than those who did not. Related to this, Benjamin et al. (2010) demonstrated how previsit conversational guidelines can enhance parent-child interactions by emphasizing the use of elaborative questions, joint talk, and making associations, resulting in improved interactions, compared to the control group without guidelines. The findings of these studies support the sociocultural belief that children's takeaways from informal learning settings are influenced by a combination of what they talk and do (Jant et al., 2014).

Previous research conveys that, when parents are asked about what they do to support their children's learning of mathematics, they give as examples home activities such as play, conversations, commercial games (Anderson, 1997; Schuh & Schnieders, 2022; Bjorklund, 2004) and talking about money (LeFevre et al., 2009). Relating to mathematics games, several studies investigate the differences in parent-child interaction when playing the same mathematics-related game using two different formats, technology-based in the form of electronic games and non-technology such as traditional board games (Schnieders and Schuh, 2022; Krcmar & Cingel, 2014, Wooldridge & Shapka, 2012). These studies revealed that the two game formats afford diverse parental scaffolding behaviours, with technology-based games eliciting more numeracy-related scaffoldings than in the game board format (Schnieders and Schuh, 2022).

Parent scaffolding has been asserted to be quite complex, involving multiple steps and skills to ensure children's effective learning outcomes. Moreover, effective parent scaffolding has been argued to be even more difficult to provide when technology is involved. As emphasized by Allen (2004), adults need to not only interpret the information for themselves and effectively transmit it to the child, but also need to decipher how the device functions:

> Adults wanting to support their children must make sense of each novel device, decipher the instructions, guide their children toward the key experience, interpret this experience for themselves, translate the significance of it for their children, assess the

> result, and make on-the-fly adjustments as needed to optimize their children's learning. (Allen, 2004, p.20)

Building upon previous studies that have explored parent-child interactions and conversations, it becomes evident that parents play a multifaceted role in facilitating their children's learning. These studies have identified diverse approaches through which parents actively contribute to learning, one of them being conceptual talk. Conceptual talk has been previously discussed to serve as a crucial component within the framework of parent scaffolding, contributing to a better understanding of the problems at hand. This learning talk category has been defined as referring to "cognitive interpretations of whatever was being attended to in the exhibit" (Leinhardt & Knutson, 2002, p.275). Furthermore, conceptual talk comprises simple and complex inferences that extend beyond mere observations. Moreover, it is regarded as a sign of higher-level engagement where "children voice their understandings and parents engage with these understandings" (Tscholl & Lindgren, 2016, p.880). This category of talk has been often used in family conversations for analysing whether their verbal and physical interactions are conducive to deep learning in interactive informal environments (Kisiel et al., 2012; Tscholl & Lindgren, 2016; Tare et al., 2011). While the current research on parent-child conceptual talk focuses on science interactive exhibits, there has been very little focus on mathematics exhibits due to their very limited number. Conceptual talk in mathematics may relate to discussing the underlying concepts and principles, fostering problem-solving strategies, and connecting prior knowledge to the task at hand.

Within the category of conceptual talk, explanations are regarded as critical contributions to children's learning with several studies supporting this idea. According to Crowley et al. (2001), explanations "present excellent opportunities for children to articulate and revise their theories of scientific phenomena, with guidance from parents and other adults" (p.714). Also, they have been previously proven to help children significantly with gaining a deeper understanding of the underlying concepts and ideas (Chi et al., 1989). Having this in mind, parents' role in supporting children through explanations extends beyond simply providing information. They may also elicit explanations from their children encouraging them to develop their own cognitive abilities. This aspect is reflected in Fender & Crowley's (2008) work which indicates how parents' explanations helped considerably their children in their exploration and understanding of the mechanisms of the interactive exhibit. Moreover, it was discovered that, by engaging in meaningful conversations with their parents, children's thinking and language gradually align with those of their more experienced conversational partners. Additionally, explanations have been demonstrated to serve a vital role in supporting problem-solving strategies. For example, Crowley and Siegler (1999) discovered that, when children receive an explanation accompanied by a demonstration, it helps them better grasp the underlying concepts and develop their own strategies. Furthermore, Csibra and Gergely' (2009) research on "natural pedagogy" suggests that children tend to learn quickly and make broad generalizations when learning from an adult who is purposefully guiding their learning process.

Besides providing explanations, parents may scaffold their children through strategic support, prompting them to plan and reflect on their actions and understandings. Strategic support has been previously defined as "explicit discussion of how to use exhibits […] how to move, where to look, or how to listen to something" (Leinhardt & Knutson, 2002, p.276). Research in developmental psychology indicates that children's problem-solving skills are less systematic than adults' (Schauble, 1996). Taking this into consideration, parental support to improve children's mathematical thinking is essential since mathematics is heavily reliant on problem-solving abilities. Prior research on parent-child engagement with mathematics indicates that different parent scaffolding behaviours support different mathematical strategies and learning outcomes. For example, Bjorklund et al. (2004) show that, parents' prompts and cognitive directives supported considerably children's arithmetic strategies in an interactive mathematics game. Furthermore, parents' strategic support fostered a greater variety of strategies, confirming their important role in expanding children's problem-solving skills.

One crucial aspect to be mentioned regarding parents’ scaffolding of their children’s mathematics learning is that it is very much dependent on their background. For instance, in the study conducted by Siegel et al. (2007), parents with higher schooling were more likely to direct children’s actions compared to parents with lower schooling. Also, parents who have a passion for the subject were found to have more collaborative and open-ended conversations (Luce et al., 2013). In this view, unskilled parents in either mathematics or didactics can significantly affect the way their children perceive and understand the subject. For instance, parents who lack knowledge of specific mathematical topics may struggle to provide accurate explanations or guidance to their children. This can lead to misconceptions or incomplete understanding of the concepts being taught (Bjorklund et al., 2004). As Hurst et al. (2019) and LeFevre et al. (2009) state, parents’ mathematical talk has the potential to predict children’s mathematical thinking. Moreover, it can also be the case that parents are knowledgeable in mathematics, but they may have difficulty providing appropriate guidance to the young learners and getting their message through (LeFevre et al., 2009). Without a proper understanding of scaffolding techniques, parents might inadvertently hinder their children's learning or reinforce misconceptions.

4. Method

In the following section, a description of the procedures and methods used in the data production and analysis process will be presented. The study used a mixed-methods approach to explore participants' experiences with the interactive mathematics exhibit, examining both parent-child dynamics and parental scaffolding of children's learning using this technology. First, a detailed introduction of the research setting will be provided, including the exhibit used in the study. Next, the study design and participants will be introduced, along with the recruitment process and eligibility criteria. Then ethical considerations taken during data production and analysis will be discussed. Lastly, the coding scheme and further details on data analysis will be provided.

4.1 Research Setting

Located in the heart of Gothenburg, Universeum serves as Sweden's national science center for education in science, technology, and sustainable development (Universeum, 2023). Through its many interactive exhibits and engaging displays, the science center offers visitors unique opportunities to learn about various subjects, including chemistry, space, the human body, and most recently, mathematics. In recent years, Universeum has invested in designing and implementing a mathematics exhibition, which opened to the public in February, 2023. The exhibition is called "Mathrix" and is designed as an informal learning environment that allows visitors to see the way mathematics makes up the world they live in. The purpose of the exhibit is to provide visitors with opportunities to explore and apply mathematics in a way that resonates with their personal interests, abilities, and curiosities. Drawing from this, "Mathrix" has the objective of meeting multiple goals concerning what the educational setting should offer to its visitors. To begin with, the exhibition aims to contribute to increased interest, engagement, and motivation to learn mathematics among the primary target group, namely teenagers (13-18 years old), but also adults. Moreover, the exhibition strives to address the personal relationship people have with mathematics and encourage them to reflect on their own attitude toward the subject and what might have influenced it. Lastly, the learning environment helps provide more entry points into the world of mathematics for visitors, through areas that engage the audience.

4.2 Studied Exhibit

The exhibit used in this study is called "*Vågade Ekvationer*" ("Bold Equations" in English) and it is designed as a game that helps people understand the concept of equation of equivalence or equation of equality by working with coefficients and unknown values. The exhibit consists of a see-saw part where units are placed, and a screen where the current status of participants' actions is displayed (see Figure 1).

The goal of the exhibit is to reach equilibrium by manipulating abstract units, namely three different units that are each assigned a different letter, besides the white circle, which is the only known variable, assigned the value of 1. Additionally, these units are designed with different colours and shapes to aid visitors in distinguishing them from one another. The abstract units are the following: a yellow pentagon representing X, a blue triangle as Y, and a red square as Z (see Figure 1 and Figure 2).

Figure 1

"Bold Equations" Exhibit Design

Figure 2

"Bold Equations" Screen Interface

The see-saw section symbolizes the concept of force, namely the length from the center multiplied by weight, which is represented through coefficients that appear along the axis of the exhibit. As soon as units are placed on the axis, an equivalent equation is displayed on the screen. If one side of the axis is heavier than the other, the axis tilts on the heavier side (see Figure 3).

Figure 3

"Bold Equations" Imbalance Example

The "Bold Equations" exhibit can be regarded as an augmented reality exhibit, where the physical see-saw scale and the colored geometric bricks with unknown "weights" are augmented with sensors and motors. Also, once units are placed on the scale, the physical configuration is mapped to the corresponding mathematical equation, displaying it on the screen. This collaborative game aims to promote both learning and teamwork by requiring players to work together to achieve equilibrium and solve the unknowns that appear on the screen. In addition to fostering a sense of cooperation, the game also aims to encourage effective communication and problem-solving skills between partners.

4.3 Procedure

The following study was designed as a field experiment that aspired to capture an ordinary visit to the new exhibition. In doing so, several aspects were taken into account for minimizing participants' feelings of being examined which will be presented later in this section. The study used two kinds of data: video and audio recordings of participants' interaction with the exhibit and with each other, and observational field notes of aspects that could not be captured on video and/or audio. While initially serving to offer an overview to the researcher of how participants engaged with the exhibit, the field notes also yielded additional insights. For example, they allowed assessing whether participants were reading the instructions displayed on the right side of the exhibit, taking advantage of this form of scaffolding offered by it. Furthermore, the field notes provided invaluable for documenting parents' impressions and experiences with the exhibit, including the communication of any technical issues or thoughts to the researcher. A noteworthy aspect documented through the field notes, which will be discussed more in the discussion section, is parents' estimation of their children's ability to understand the mathematical concepts displayed by the exhibit. Some parents initially expressed doubts towards including their children in the study as they assumed that they would not be able to work with the exhibit. However, they were later surprised to see that their children were actually capable of understanding the underlying mathematical concepts.

Regarding audio-visual research, it has been previously shown to be highly efficient for capturing the way knowledge is revealed in informal settings where skills and practice are shared through social activities (Heath et al., 2010). Moreover, a significant body of research argues for the effectiveness of video research in museums and science centers (Heath et al., 2010; Ash, 2007; Callanan et al., 2007; Meisner et al., 2007). For example, Meisner et al. (2007) claim that, video recordings provide an important analytic resource in analysing how visitors interact both with the exhibits and with each other. They state that, video recordings allow:

> repeated and detailed access to the conduct and interaction of participants, and, more specifically the interplay of talk, bodily, and material conduct and the ways in which the visitors' engagement with exhibits contingently arises both in and through their emerging interaction with others. (Meisner et al., 2007, p. 1536)

In addition to this, Callanan et al. (2007) argue that, video research brings 3 main opportunities for analysing conversations in museum settings. These are the following: it captures spontaneous conversations while keeping the recording of data unobtrusive, allows analysis of unanticipated aspects, and captures both verbal and non-verbal communication (Callanan et al., 2007).

The data was collected on-site, during regular opening hours, over the course of the first two weeks from the inauguration of the exhibit. Moreover, these two weeks overlapped with the national school holiday "sportlovet", which represented a very opportune time to start the data production process. This holiday is considered to be one of the busiest times of the year at the science center, thus an increased number of participants was expected.

The equipment was positioned taking into account the broader ambition of the study and several general methodological considerations informed by relevant literature (Heath et al., 2010; Nordenström, 2019; Silverman, 2021). Moreover, several pilot studies were conducted prior to the data production for planning and testing the recording arrangements and equipment. First of all, the equipment was placed as to avoid interfering with participants' interaction with the exhibit. This aspect has been proven to be crucial since an improper placement can distract the participants and thus influence the results of the study (Heath et al., 2010). Following these considerations, the physical interaction with the exhibit was recorded with a GoPro camera which was placed above the screen (see Figure 4 below).

Figure 4

Recording equipment arrangement

The camera was positioned at an angle that captured the screen and participants' hand movements on the axis, framing the shot to exclude any other distractions from the scene (see Figure 5).

Figure 5

Video Framing of Participants' Interaction with the Exhibit

Since the main focus of the study is analysing how participants collaborate to reach equilibrium, the most important actions to be recorded were the hand movements. Also, the screen of the exhibit was recorded to see the equations that result from participants' placements on the see-saw scale. Furthermore, by recording only participants' hands, anonymity and confidentiality are further maintained, together with preserving the integrity of the data. This aspect was essential considering the vulnerable nature of the selected studied participants, namely underage visitors.

In addition to avoiding interference with participants' interaction with the exhibit, the equipment was positioned to optimize high-quality recordings with adequate lighting and reduced ambient noise. Since the camera alone proved to be insufficient for recording the conversations of the participants during the pilot tests, a Zoom H6 audio recorder was attached to the axis pillar, behind the see-saw section where units are being placed (see Figure 4). The recorder used omnidirectional microphones which have been confirmed to be optimal for recording sound coming from multiple directions (Nordenström, 2019). The entire process followed an iterative workflow where procedures were constantly evaluated and improved when needed after each recording (Nordenström, 2019).

4.4 Eligibility Criteria

In this study, a set of specific eligibility criteria was pre-established to minimize potential biases and enhance generalizability for the studied population. To begin with, only groups were accepted, either dyads or triads, for the collaborative learning aspect to be fulfilled. Also, since conversation is the main focus of the study, groups were the most suitable as participants tend to communicate out loud more often in groups than individually (Dierking & Falk, 1994; Eason & Ramani, 2020).

Another eligibility criterion was for participants to speak either Swedish or English with ease. Furthermore, children younger than 15 who wanted to participate in the study needed to be assisted by their legal guardians to give their legal consent. There was no set age limit for the young participants, however the children recruited in the research were carefully selected to ensure that they were old enough to work with mathematical equations and unknown variables. Initially, the age limit was decided to exclude participants younger than 13, as the target audience of the exhibition is 13 years and older. However, during the pilot tests, it was noticed that the "Bold Equations" exhibit sparked significant interest among children younger than 13, contrary to initial expectations. Also, during the trial sessions, it was noticed that the young visitors showed signs of understanding the arithmetic features of the exhibit while they were exploring with their parents, such as identifying the different values of the units depending on their position on the scale and multiplying numbers. In light of this, children younger than 13 were also included in the study.

Lastly, for the participants to be eligible for the study, they needed to communicate out loud with each other while interacting with the mathematical game. Given that communication is the focus of the study, visitors who verbally expressed their ideas, thoughts, and strategies were prioritized. Considering this, prior to inviting participants to take part in the study, a very brief trial session was conducted. During this session, visitors would come to the exhibit and try it freely for a short period of time, unaware that the exhibit is under study. In the meantime, the researcher would stay close and observe whether the participants communicate with each other and meet the language requirements. Once these aspects were met, the researcher would interrupt visitors' exploration and ask them if they would be interested in participating in the research.

While several previous studies on visitor conversation in science centers and museums have adopted the think-aloud method for encouraging groups to verbalise their thoughts (Tunnicliffe & Reiss, 2000; Allen, 2002; Lee & Kim, 2007; DeWitt & Hohenstein, 2010; Ma et al., 2020), the current study chose to not use it. The motivation behind this was to simulate a natural visit experience for participants, allowing them to explore the exhibit as they normally would and reducing added pressure to articulate their thoughts in a certain manner.

4.5 Recruitment of Participants

After ensuring that a group meets the established criteria, the participants would receive information about the study. In addition to the presentation given by the researcher, the participants also had a participation information form which they could read at their own pace and also take home. Once the participants accepted, they were given a consent form and a short demographics questionnaire to complete, prior to their exploration of the exhibit. After their completion, the groups were welcomed to explore the "Bold Equations" exhibit as they wished, not offering direct instructions on what should be done. The rationale behind it was to allow free, unguided exploration similar to what visitors would typically experience on a regular visit to Universeum. Furthermore, the lack of directives also served the purpose of preventing any potential influence on the study results. The participants were also reminded that they were not under any sort of examination. This is important in learning situations where the absence of regulations and assessments is part of the motivational force of the situation (Rogoff et al., 2016).

4.6 Participants

The study started as an exploratory investigation with no predetermined focus on either the target population or interaction to be studied, but which later emerged as the most interesting aspect to further investigate. During the pilot studies when the equipment's positioning was being decided and later during the data production process, it became apparent that parent-child groups were the visitors who expressed the most interest in the exhibit. Furthermore, their varied interactions with the exhibit sparked interest in seeing how families use interactive exhibits for learning mathematics. Based on these considerations, parent-child groups were chosen for further examination and for understanding how families develop a conceptual understanding of mathematical concepts with the help of interactive exhibits.

Having this in mind, out of the 24 groups participating in the study, only 10 were selected. The participants involved in the study were 9 parent-child dyads and one triad (one parent only). The underlying reason behind choosing one-parent-only groups is that they represent the focus of many studies on informal learning which allows comparisons to existing research on the topic (Tscholl & Lindgren, 2016). Among the 10 groups that participated in the study, 9 consisted of children aged below 13, while one group included an adolescent aged between 16 and 19. Also, out of the 11 young participants, 7 were male and 4 were female. In regard to the parents, 9 groups were comprised of 35+ adults, while one group contained a parent aged between 26 and 35. The gender distribution of the adults was 6 female and 4 male. In terms of nationality, 8 groups were Swedish, one was British, and one was Chinese.

4.7 Ethics

Audio-visual research in science centers raises several ethical issues which were also tackled in this study. The guidelines provided by the Swedish Research Council (2017) served as a starting point for designing the data production process and as a checklist for ensuring the ethical integrity of the study.

To begin with, it is essential that participants be informed about the study and the conditions of their participation. This means that the potential participants should be given information regarding the objectives of the research together with how the data will be collected, stored, protected, and whether it will be shared with other people. In addition to this, the participants need to be reminded that their participation is voluntary and that they have the right to withdraw from the study at any stage with no consequences involved. Also, participants need to be aware of the duration for which their data will be stored. All the aforementioned aspects were presented in the participation and consent form (see Appendix 1 and 2). Time was allocated for questions after the participants were given information about the study to address any potential concerns the participants may have related to the research (Nordenström, 2019). The contact details of the researcher were also included in the participation form in case participants had further questions or wished to retrieve from the study.

The forms were written using simple language to ensure that participants could easily understand them and to minimize possible misunderstandings. Furthermore, participants younger than 15 years old were required to be assisted by a responsible third party, such as a parent or legal guardian, who could allow participation and sign the legal consent form.

One problematic ethical aspect of this study was ensuring anonymity and confidentiality from the video recordings since video data is inherently non-anonymous (Derry et al., 2010). In connection to this, the Swedish Research Council (2017) states that, although individual integrity may be difficult to guarantee in video research, it is nevertheless imperative. Having this in mind, this study strove to protect participants' identities as much as possible by recording only their hands and blurring their faces when they appeared in the shot. Also, to keep participants' identities anonymous, each group was assigned a unique identification number on their participation form and in the recordings. Lastly, any personal identification information from the audio data such as the names of the participants was omitted from the transcripts.

4.8 Analysis

Both the audio and video recordings were used for transcribing participants' interactions with the exhibit. In the process of transcribing the audio data, each utterance was marked with the corresponding timestamp and identification label (Parent or Child). The transcription was done with the help of the online transcription software "Transcribe" from Microsoft 365 Word. The transcription was thoroughly revised and edited due to the many instances when the participants were talking simultaneously, which the tool could not properly catch. Any statement that was unintelligible from both the audio and video recordings was marked as "inaudible" in the transcripts.

The video recordings were transcribed in parallel with the audio and marked between square brackets in the transcript. Due to the vast amount of information that is naturally present in videos, only actions relevant to the study were chosen. These included, for example, pointing at the screen or the see-saw scale, placing units on the scale, and any other actions that the participants were performing for helping each other.

The conversations of the participants were segmented into utterances, accompanied by annotations of the other actions captured by the video. According to Carter and McCarthy (2006), an utterance is a complete communicative unit expressing an idea. Having this in mind, an utterance can comprise several expressions and propositions (Tscholl & Lindgren, 2016). To provide an example from the study, parents' demonstrations tended to come in sequences:

> "**(1) Parent**: It is like this. Now it is equilibrium of course, but now we will test which one corresponds to which one. We think that 2 red are a white [puts a red unit on top of the other red, right side-the bar lowers on the right] and we test if it's true [puts a white on the 5th position, left side, the bar does not move]. If it's true, it reaches equilibrium, but it was not." (author's translation from Swedish, original in Appendix 4)

4.8.1 Coding Scheme

The coding scheme used for the data of this study is an adaptation of two main studies: Tscholl and Lindgren's (2016) work on parents' learning talk around interactive exhibits and Schnieder and Schuh's (2022) research on parent scaffolding behaviours for mathematics learning using technology-based games. By merging the parental scaffoldings from both articles, a total of 11 codes were employed for analysing parents' support of their children's learning (see Table 1). Moreover, Tscholl and Lindgren's (2016) work was also used for defining and identifying learning talk categories to which parent scaffolding behaviours can be attributed (see Appendix 3: Data Overview). The main learning talk categories associated with these scaffolding behaviours are: conceptual, instructive, affective, and physical support. Within the conceptual talk category, the parent scaffolding behaviours included are: hints/prompts, modeling/demonstrating, re-representations, corrections/disaffirmations, explanations, and inquiry. Instructive talk solely comprises instructions. Affective support contains affirmations/encouragements, expressing enjoyment, expressing confusion, and promotion of independence. Lastly, physical support forms a distinct category, encompassing all physical actions that the parents undertook to support children's engagement with the exhibit, such as handing units.

Coding followed an iterative process where each utterance was identified and categorised according to the established codes from the literature (Tscholl & Lindgren, 2016). The utterances were coded and further analysed using the MAXQDA online software for qualitative research and mixed methods.

Table 1

Coding Scheme based on Schnieders and Schuh (2022) and Tscholl and Lindgren (2016)

Parent Scaffolding Behaviour	Description
Instructions	Parent instructs the child on what to do.
Hints/Prompts	Parent gives a hint or suggestion without mentioning a specific strategy.
Modeling/Demonstrating	Parent verbally or non-verbally demonstrate an action for the child to observe and imitate.
Re-representations	Parent reconceptualizes the problem, approaching it from a different perspective.
Corrections/Disaffirmations	Parent corrects child's actions or incorrect response.
Explanations	Parent gives comments, descriptions, predictions, or narrations of an action.
Inquiry	Parent asks the child to explain, reflect or produce some kind of information.
Affirmations/Encouragements	Parent agrees to a child's verbal or non-verbal action.
Express Enjoyment	Parent expresses positive emotions while exploring the exhibit.
Express Confusion	Parent expresses confusion or uncertainty about the exhibit or the steps to be taken.
Promotion of Independence	Parent encourages the child to work individually without giving direct instructions.

4.8.2 Groups' Performance with the Exhibit

In addition to the analysis of parents' learning talk, an overview of groups' performance will also be conveyed for getting an overall picture of how participants used the equation-focused exhibit. More specifically, the overview will focus on three main aspects: how many groups reached equilibrium, what was their main strategy for reaching equilibrium, and whether the parents and children participated in the activity.

In terms of strategies, the overview will examine whether the participants reached equilibrium by following either a mechanical or a semantic approach. As mentioned earlier in the description of the "Bold Equations" exhibit, the display consists of two primary components: the see-saw scale where units are distributed, and the screen showing the mathematical equation corresponding to the units placed on the scale. Therefore, visitors have the option to reach equilibrium by following a mechanical iterative approach of placing units on the scale, following its tilting motion, and adjusting their placements accordingly. Or they can use a semantic approach by engaging in deeper mathematical reasoning. This approach consists of working with the mechanisms of the exhibit and with the mathematical formulas displayed on the screen for understanding coefficients, calculating the unknown variables, and reaching equilibrium.

Investigating children and parents' participation in the activity serves a dual purpose: it unveils the dynamics of parental involvement in children's exploration of the mathematics exhibit and sheds light on children's self-driven engagement for reaching equilibrium. In this study, participation is defined as contributing through conversations, hands-on manipulation, or both.

5. Findings

In this section, the findings from the video, audio, and observational data will be presented. First, an overview of all groups' performance with the exhibit will be given. This has the purpose of getting an overall picture of how participants used the mathematics exhibit for reaching equilibrium. More specifically, understanding whether participants reached equilibrium by relying only on the mechanics of the exhibit, or they engaged with the mathematical formulas on the screen as well. After the general overview, parents' support of children's learning of mathematics will be conveyed both qualitatively and quantitatively. The quantitative analysis presents the number of coded utterances organized by learning talk categories. The qualitative analysis focuses on providing excerpts from the data to illustrate the different scaffolding behaviours, accompanied by images from the video data for better visualization of the actions undertaken by the participants.

5.1 Participants' Performance with the Exhibit

Before delving into participants' performance with the exhibit, an example of how it can be used will be provided for a better understanding of its features. To begin with, as outlined in the Method section, the "Bold Equations" exhibit is designed to help the public understand the concept of equality and inequality of equations by working with coefficients and unknown variables. The coefficients are physically represented by the different positions along the axis, with greater coefficients placed further away from its center. The design of the 3 coefficients on each side of the axis coincides with the notion of force, with units displaying more weight the further away they are placed from the center. The unknown variables are represented by the 3 sets of pieces assigned the letters X, Y, and Z. In addition to the pieces with unknown values, there is also a set of white units which are assigned the value of 1. With the help of the white units, visitors can identify the values of the unknown variables.

An example of how coefficients work and how participants can find the values of the unknown variables is shown in Figure 5 below. In the given picture, the red Z unit, whose value is unknown, is placed on the position with the coefficient 3, being presented on the screen as 3Z, i.e., three times Z. On the opposite side of the axis, three white units are placed in the corresponding position with coefficient 3. Since one white unit is known from the very beginning to be 1, the 9 value results from the 3 units multiplied by their coefficient which is 3. By knowing the value of one side, the user can easily calculate the other unknown variable from the opposite side. In the given example, visitors can calculate that Z has the value of 3 by dividing 9 by 3.

Figure 5

Identifying Unknown Variables Example

Given this example, understanding these mechanisms plays an essential role in finding the value of the unknown variables and reaching equilibrium using mathematical reasoning.

The primary goal of the exhibit was for visitors to reach equilibrium, which was achieved by all groups, looking at the results presented in Table 2. However, not all groups engaged in understanding the underlying mathematical principles promoted by the exhibit. Out of the 10 studied groups, 6 engaged in a more semantic approach by looking into the role of the coefficients, identifying the unknown variables, and using this knowledge for reaching equality of equations, and therefore equilibrium. The remaining 4 groups relied on a mechanical approach, by iteratively placing units along the scale and observing its motion until it reached equilibrium. These groups were found to not engage in discussions leading to mathematical strategies or mathematical insights. Instead, the conversations were primarily focused on the exhibit's mechanisms. Lastly, even though they identified relations between the units, they did not engage in calculating the actual value of the unknown variables.

Table 2

Parent-child Performance with the "Bold Equations" Exhibit

Group ID	Reached Equilibrium	Semantic Approach	Mechanical Approach	Parent Participation	Child Participation
ID2	X	X		X	X
ID5	X		X	X	X
ID7	X		X	X	X
ID10	X	X		X	X
ID11	X	X		X	X
ID13	X	X		X	X
ID14	X		X	X	X
ID15	X		X	X	X
ID16	X	X		X	X
ID18	X	X		X	X

Another noteworthy finding from Table 2 is that parents' participation does not necessarily result in the adoption of more advanced approaches. As observed in Table 2, parents engaged their children in both semantic and mechanical approaches to reaching equilibrium.

5.2 Quantitative Analysis of Parents' Learning Talk

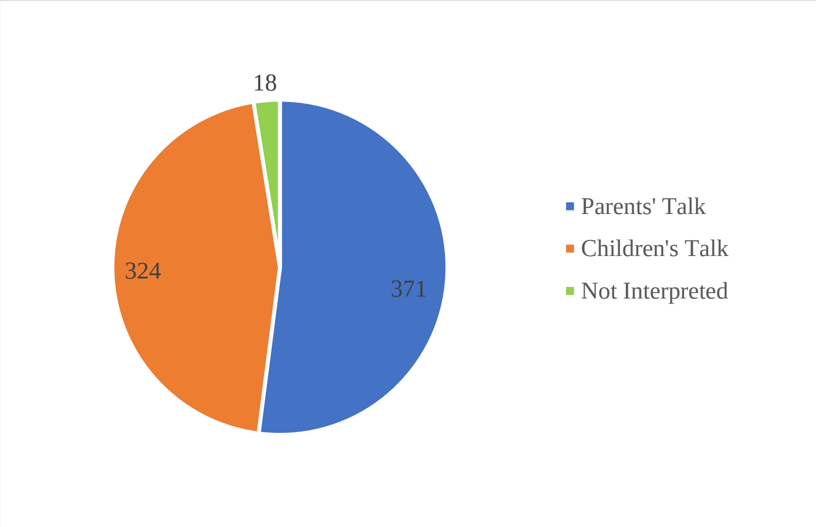
In the following section, a quantitative analysis of the participants' learning talk will be presented, by looking at both the overall utterance distribution and the conversation dynamic within each studied group. Subsequently, the focus will narrow to parents' identified learning talk, conveying both its frequency and distribution.

5.2.1 Parent-Child Utterance Distribution

Upon transcribing and coding all participant utterances, a total of 713 were identified. Out of these utterances, 371 belonged to parents, 324 belonged to children and 18 were categorised as uninterpreted (see Figure 6 below). The uninterpreted utterances relate to talk that was not related to the exhibit, as well as unintelligible utterances which could not be understood from either the audio or video recordings. The mean time spent by the participants at the exhibit was calculated to be 7.035 minutes.

Figure 6

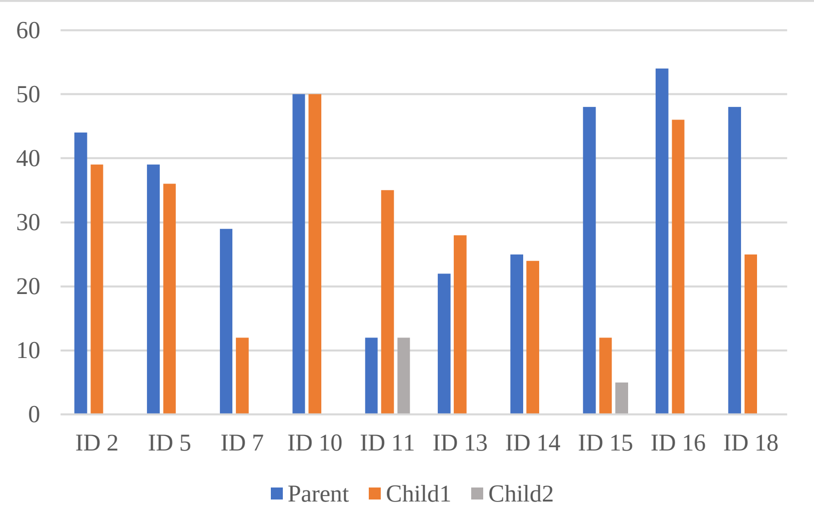
Parents' and Children's Overall Utterance Distribution



Given that the parents and children presented a nearly even number of utterances, a closer look at the dynamic of these conversations was needed. Looking at Figure 7 below, in 7 out of the 10 observed cases, parents talked more compared to their children during their interaction with the exhibit. There is also one occurrence where parents had the same number of utterances as their children, leaving 2 cases where children talked more than their parents.

Figure 7

Talk Distribution between Parents and Children



In two cases of children communicating more compared to their parents, it was observed that the young learners played a very active role in their learning, understanding the mathematical principles promoted by the exhibit and developing strategies for calculating the value of the unknown variables on their own. Moreover, it was noticed that they also helped their parents understand the mechanisms of the exhibit and how they relate to mathematical concepts. For instance, the following example illustrates how the child takes charge of her own learning and explains to her parent how to operate the exhibit using mathematical reasoning:

"[participants reach equilibrium- refer to Figure A1]

Figure A1

Child: So 5Z+1 is equal to 5X+3Y+5. Oh, what's that then?

Parent: So, what are we saying it is? So …

Child: Yeah, we need to figure it out, no, but we can't do it because there's two variables there [pointing at the right side of the formula- refer to Figure A2], one there [pointing at the left side of the formula- refer to Figure A3], you can't do it.

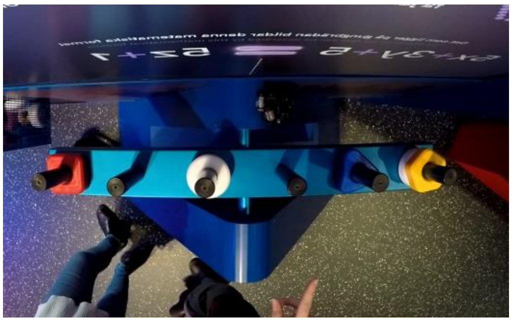


Figure A2

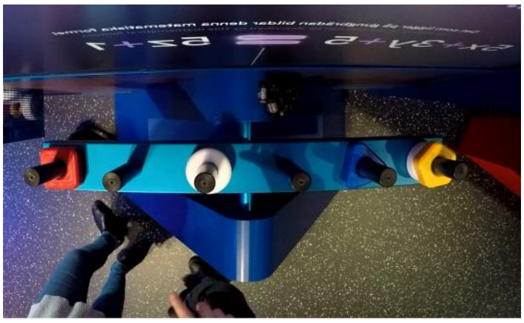


Figure A3

Parent: Right.

Child: I think we should take only one variable first [starts removing all the pieces off the see-saw].

Parent: It what?

Child: One variable.

Parent: Okay. So, do I just put it in the bag or what?

Child: OK, so the variables are the X, Y and Z [pointing at the four figures on the screen which indicate the colour, shape and letter assigned to each piece- refer to Figure A4].

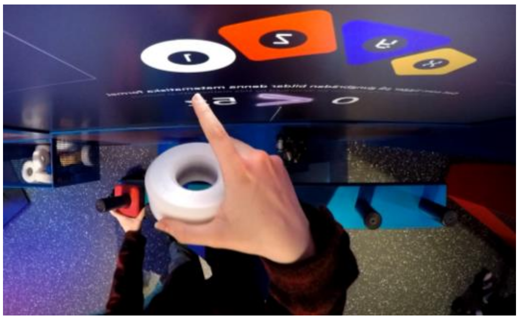


Figure A4

Figure A5

Parent: Yeah, so we just need to …

Child: So, we can try and figure out what like just one X or one Y or one Z is.

Parent: Ok, what do you want to start with?

Child: Z [places a red piece on coefficient 1- refer to Figure A5]

Parent: Z. So, Z is...

[child places a white piece on coefficient 5- refer to Figure A6]

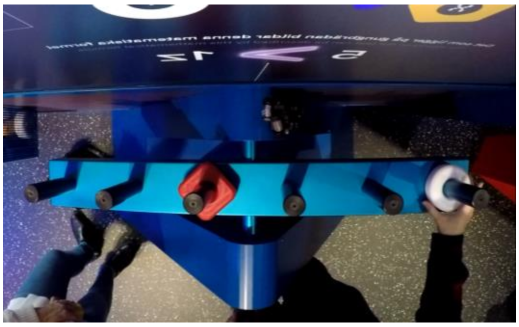


Figure A6

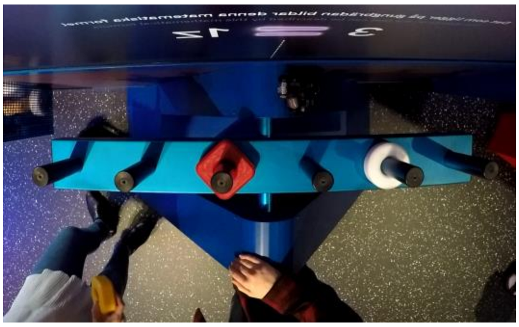


Figure A7

Child: 5 is greater than Z [child moves the white piece on coefficient 3, reaching equilibrium- refer to Figure A7]. So Z is 3!”

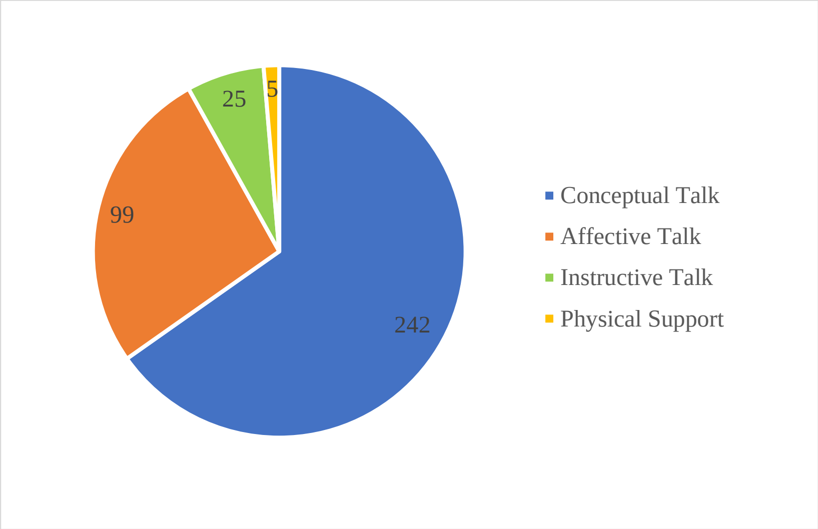
Although this study focuses solely on parents, presenting an example from these two cases is important in showing the diverse dynamics between parents and children as they explore the exhibit.

5.2.2 Parents’ Learning Talk Categories

Upon coding and analysing parents’ utterances and actions, four types of learning talk were identified: conceptual, affective, instructive, and physical support. Parents used a significant amount of conceptual talk (242 utterances), surpassing considerably affective talk (99 utterances), instructive talk (25 utterances), and physical support (5 utterances) (see Figure 8 below).

Figure 8

Parents' Learning Talk by Coded References



In terms of distribution, conceptual and affective talk have been found to be present in all 10 studied groups, with instructive talk occurring in 8 groups and physical support in 3 (see Table 3).

Table 3

Group Distribution of Parents' Learning Talk

Group ID	Conceptual Talk	Affective Talk	Instructive Talk	Physical Support
ID2	X	X	X	X
ID5	X	X		
ID7	X	X	X	
ID10	X	X	X	
ID11	X	X	X	
ID13	X	X		X
ID14	X	X	X	
ID15	X	X	X	
ID16	X	X	X	X
ID18	X	X	X	

5.3 Qualitative Analysis of Parents' Learning Talk

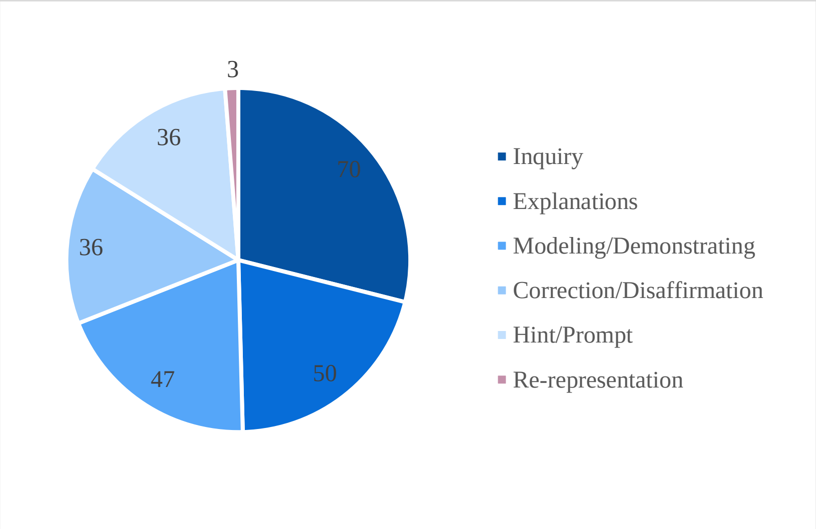
In the following section, a qualitative analysis of the parents' learning talk will be presented, giving examples from the transcripts, and incorporating images from the video data where necessary for better visualization of participants' actions. The purpose of the qualitative analysis is to examine how parents scaffolded their children's learning of mathematics using the interactive exhibit.

5.3.1 Conceptual Talk

The conceptual talk was defined by 6 subcategories or parent scaffolding behaviours: inquiry, explanations, modeling/demonstrating, correction/disaffirmation, hint/prompt, and re-representation (see Figure 9). A more in-depth analysis of these subcategories will be presented in the following section, looking at the way parents supported children's learning of mathematics using the exhibit. Images from the video data will be showcased for the three most frequently observed scaffolding behaviours: inquiry, explanations, and modeling/demonstrating. The remaining scaffolding behaviours will be briefly illustrated through excerpts from the transcripts. The decision to exclude images for these scaffolds is based on the nature of these interactions. In these cases, the textual description and context within the transcripts are sufficiently explanatory, without needing additional visual aid.

Figure 9

Conceptual Talk Subcategories by Number of Coding References



5.3.1.1 Inquiry

Parents were found to support their children's learning by asking questions about: mathematical operations and concepts, the mechanisms of the exhibit, and strategies that they have in mind for solving the tasks at hand. The following example shows how one parent uses inquiry to involve the child in the activity, by asking him to recall their previous findings and test his knowledge about arithmetics:

> **(2)** "**Parent**: Do you remember how much this was? What was this one? [pointing at the red unit- refer to Figure B]. This was 3 x 5, so we multiply it. What is 3 x 5?

Figure B

Child: 3 x 5 …15!

Parent: So, you need to add so that it gets up to … 15 only for the red one.” (author’s translation from Swedish, original in Appendix 4)

Parents’ inquiry about the mechanisms of the exhibit and mathematics concepts aimed to assess children’s understanding of the exhibit and provide explanations if needed. A relevant example is the following where the parent asks his daughter if she understands the meaning behind the different coefficients, providing an explanation after she admits that she doesn’t:

(3) “**Parent**: Do you know why it shows a 1 there [pointing at the coefficient 1- refer to Figure C1] and a 5 there? [pointing at the coefficient 5- refer to Figure C2]

Figure C1

Figure C2

Child: No.

Parent: The thing is that the closer you get here [touching the center if the see-saw scale- refer to Figure C3] the less, so this axis holds up a bit.” (author’s translation from Swedish, original in Appendix 4)

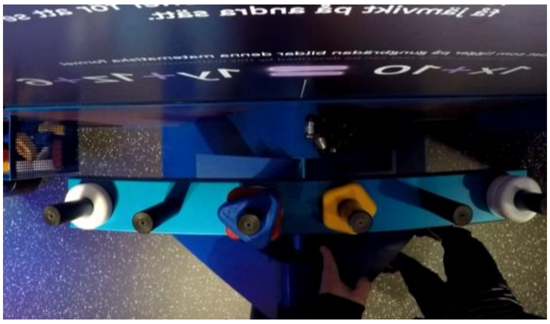


Figure C3

Another related example is when parents would ask their children to see if they understand the relation between the pieces placed on the scale and the formula displayed on the screen:

> **(4)** "**Parent**: So, 3Z [pointing at the red piece on the coefficient 3- refer to Figure D1], that's it. 5 white. What is this one then? [pointing at the yellow piece on the same coefficient– see Figure D2]. What letter is that?

Figure D1

Figure D2

> **Child**: I have no idea.
>
> **Parent**: 5... [pointing at the number on the bar which indicates the coefficient 5 and at the 5X from the equation on the screen- refer to Figure D3]

Figure D3

Figure D4

Child: Oh, 5X.

Parent: 5X. What letter is this? [pointing at the blue piece- refer to Figure D4]

Child: A Y.

Parent: A Y, exactly! And these? [pointing at the white pieces on the opposite side- refer to Figure D5]

Figure D5

Child: These are 5.

Parent: No. 5+5 is 10 [pointing at the 2 white pieces on coefficient 5]. 3+3 is 6 [pointing at the 2 white pieces on coefficient 3]. 10+6 is [pointing at the number 16 from the equation]

Child: Ahaa!" (author's translation from Swedish, original in Appendix 4)

There were also instances where parents' inquiry did not necessarily lead to improved knowledge in either mathematics or exhibit's mechanisms. For example, one parent asked her child how he can tell that units "weigh" differently, but provided no additional explanation after the child's response that he just "knows":

"**Parent**: So, does everything weigh differently then? [parent asks child while they are removing the pieces]

Child: Yes!

Parent: Ok, but do you know that by the colours?

Child: No, I just know.

Parent: Okay."

5.3.1.2 Explanations

During children's interaction with the exhibit, parents also actively supported their learning through explanations. However, the majority of these explanations were related to the mechanisms and features of the exhibit, rather than the underlying mathematical concepts promoted by it. Surprisingly, there were only 4 instances of parents giving explanations related to mathematical concepts, while exhibit-specific explanations were found in 46 utterances. It is also important to mention that explanations related to mathematical concepts occurred in only 3 groups out of 10, therefore the overall usage of math concepts was very limited.

The exhibit-specific explanations centered on parents clarifying how the coefficients from the see-saw section work, how the physical units are related to the formula displayed on the screen, as well as reinforcing the values of the units. In the following example, the father explains to his daughter the mechanism behind the coefficients, pointing out that units placed closer to the center of the axis are supported more than the ones at the extremities and therefore their values are lower:

> **(5) “Parent**: But it depends on … it’s because here [refer to Figure E1] the axis helps a bit to support this thing. Here [refer to Figure E2] it helps less, there is pushes down by 3 (refer to Figure E3).

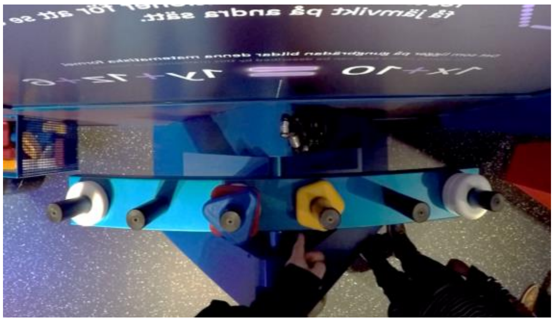


Figure E1

Figure E2

> **Parent**: And there (see Figure E4) at the back it doesn’t help at all.” (author’s translation from Swedish, original in Appendix 4)

Figure E3

Figure E4

Parents also explained the relationship between the physical units and the formula presented on the screen. One relevant example is when one parent placed all the unknown variables on one side of the scale to explain to the child how the coefficients assign different values:

> **(6) “Parent**: So, now you know that 3X is equal with 3. So, let’s take [placing one blue and one red- see Figure F1].

Figure F1

Figure F2

Parent: Here we have 3 unknowns. We go to them later [the child wants to leave to another exhibit]. 3X [pointing at the yellow piece- refer to Figure F2] +1Y [pointing at the blue piece- refer to Figure F3] +5Z [pointing at the red piece- refer to Figure F4].” (author’s translation from Swedish, original in Appendix 4)

Figure F3

Figure F4

The explanations related to mathematical concepts addressed mathematical operations such as multiplication, addition, and subtraction. For instance, one parent explained how equations can be simplified by subtracting the same value from each side:

> **(7)** “**Parent**: You know, these formulas can be simplified mathematically because if you have Y on both sides, then you can remove Y from both sides.” (author’s translation from Swedish, original in Appendix 3)

In addition to this, parents also helped the young learners make strategies for calculating the unknown variables, explaining how the white unit, which is the only given known variable with the value of 1, can be used to solve the unknown variable Z:

> **(8)** “**Parent**: Z+1 is equal to X+Y+1 [pointing at the formula on the screen- see Figure G1]. These white ones [touching one white unit underneath the yellow- see Figure G2], they are 1 so they can be removed.

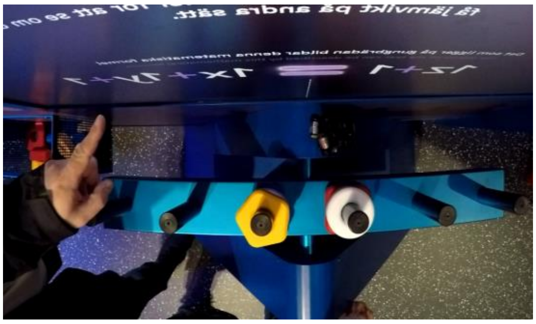


Figure G1

Figure G2

Parent: We remove both the white ones and see [refer to Figure G3]. Oh, we see that one Z, one Y…one X plus one Y and if we put only white ones here, put one white more, put one white more, yes [refer to Figure G4].

Figure G3

Figure G4

Parent: Then we see that Z is 3 because these ones are a 3. One red is 3 [refer to Figure G5].” (author’s translation from Swedish, original in Appendix 3)

Figure G5

5.3.1.3 Modeling/Demonstrating

This subcategory is represented by any verbal or non-verbal demonstration that the parent performed to help the child’s ongoing action or to give them an example that they could observe, imitate, or reproduce. Parents’ demonstrations focused mostly on steps or strategies that children can take to reach equilibrium:

(9) “**Parent**: Now we have 10 here, and there we don't know how much we have, but if we do it like this, then we have 11 [places one white piece on coefficient 1- refer to

Figure H1]. It's worth more than 11 [pointing at the red piece on the 5th position, left side]. And if we take that one... 12 [places another white piece- refer to Figure H2].

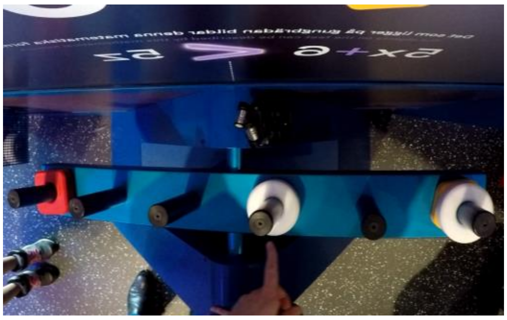


Figure H1

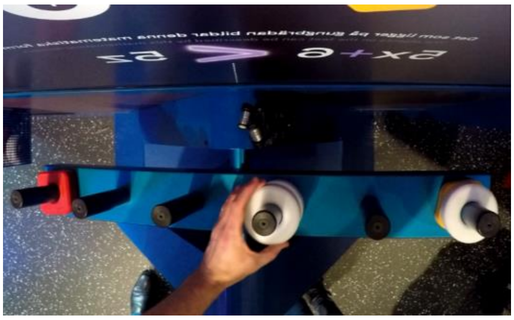


Figure H2

Child: If we add 3 [child places one white on the same spot-refer to Figure H3]. It's worth more than 13 [points at the red piece on the 5th position, left side]. If we add 14 [child places one more white on top of the other white pieces- refer to Figure H4].

Figure H3

Figure H4

Parent: But we can also move one like this [takes the white piece the child just placed and puts it on the next position- refer to Figure H5]. If we move it there, it's worth 3, right? So, then you'll soon know how much they are worth [pointing at the white piece just moved- refer to Figure H6].

Figure H5

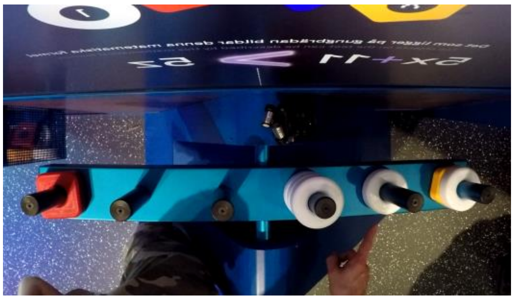


Figure H6

Child: What?

Parent: Because if we remove this one, then it will be balanced, right? [removes one white from the coefficient 1- refer to Figure H7] 1 or 2, look! [reaches equilibrium].

Now! So then we know how much [pointing at the red piece on the 5th position, right side] is worth. 5, 10, 13, 15 [counting the values of each brick from the left side].” (author’s translation from Swedish, original in Appendix 3)

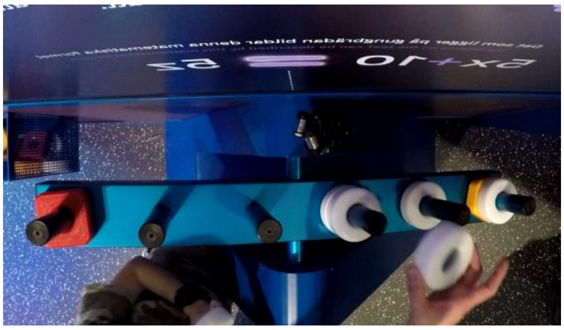


Figure H7

Parents also provided suggestions for different problem-solving strategies that the children could use for calculating the unknown variables. For instance, one parent guided his son in determining the value of the unknown variable Y once they had already reached equilibrium:

(10) “Child: I put this here. One more [places 2 white on coefficient 1, reaches equilibrium- refer to Figure J1). Look!

Parent: Yes, but we still haven’t figured out what Y is [pointing at the Y from the equation on the screen].

Child: Y?

Parent: What you can do is remove the same amount from each side now that we have balance. So, take one red.

[Child removes one red piece from the 5th position on each side simultaneously- refer to Figure J2]

Parent: Still balanced. There’s an equal amount on each side.

Child: Now I’m removing two 3s. Was this a 3?

Parent: We’ve calculated before that it’s a 1 [points at the yellow piece- refer to Figure J2].

Figure J1

Figure J2

[Child removes one yellow and one white simultaneously- refer to Figure J3]

Parent: So, now you still have balance.

Child: Yes, one Y is equal to 2 [refer to Figure J4].

Parent: Yes, exactly.” (author’s translation from Swedish, original in Appendix 3)

Figure J3

Figure J4

An additional problem-solving strategy that the parents demonstrated besides identifying the unknown variables, is helping them make estimations of how they can reach equilibrium without necessarily knowing the values of the units. An illustrative example is when one parent and his son were collaboratively working towards reaching equilibrium, with the parent noticing that the missing value for reaching equilibrium is between 15 and 13, judging the changing sign between the equations:

(11) “[Child places a white piece on the position with coefficient 3- refer to Figure K1)

[Child takes the same piece and moves it to the position with coefficient 1- refer to Figure K2)

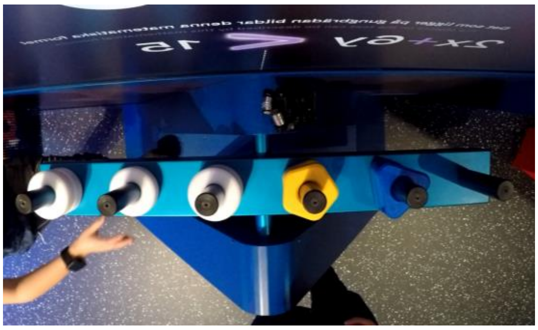


Figure K1

Figure K2

Parent: So, it is less than 3 and more than 1 [parent notices that the sign from the equation changes when the child moves the units to different positions]

[Child puts one white piece on coefficient 1, reaching equilibrium- refer to Figure K3)”

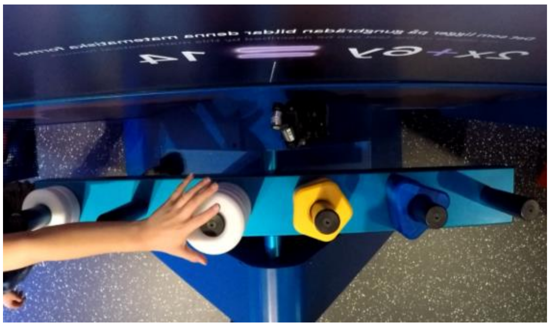


Figure K3

5.3.1.4 Correction/Disaffirmation

It was observed that parents provided corrections to their children's learning in 2 main ways. The first one is correcting the way they use the exhibit, such as stopping them from pushing down on the axis: "Don't push on that [parent removes child's hand from the bar]". Also, parents encouraged their children to place pieces gradually to see how each piece changes the equation on the screen and the position of the see-saw section: "**(12)** No, wait, wait a bit. Can you put one at a time? If you put many, you won't see what they do" (author's translation from Swedish, original in Appendix 3). The second type of correction is focused on guiding children's learning process. One example is parents asking the children to count before they put any more pieces: "Wait, wait, you need to count!".

5.3.1.5 Hint/Prompt

Hints or prompts refer to clues and suggestions that the parents provided when the child was at an impasse. One example of this would be when parents provide hints for their children when they are unsure of what to do, one illustrative example is the following: "Think that they are the same, try and put them on each side, we see".

Prompts were also given when children encountered drawbacks, for example when their strategies wouldn't work, but the parent would prompt them for different alternatives: "It was wrong, it was wrong, OK. But now we can test if a yellow is equal to a certain number of red, for example, or blue."; "It has become too much. We remove the blue and yellow ones, and we test."

5.3.1.6 Re-representation

Re-representation occurred in only one group where the parent re-represented the different values of the units by their weights:

> **(13)** "**Parent**: We can weigh them a bit. So [gives the child on the left a yellow unit and a red unit, the child holds them both in his hands and weighs them]. We can estimate a bit. What do you feel, what do you feel?
>
> **Child 2**: This is way heavier [holding up the red piece]"
>
> **Child 1**: I want to try.
>
> [the boy on the right gives his brother the red piece]
>
> **Parent**: It's heavier, exactly.
>
> **Child**: [the boy weighs the red piece in his hand] Okay, these have the same weight, I think."

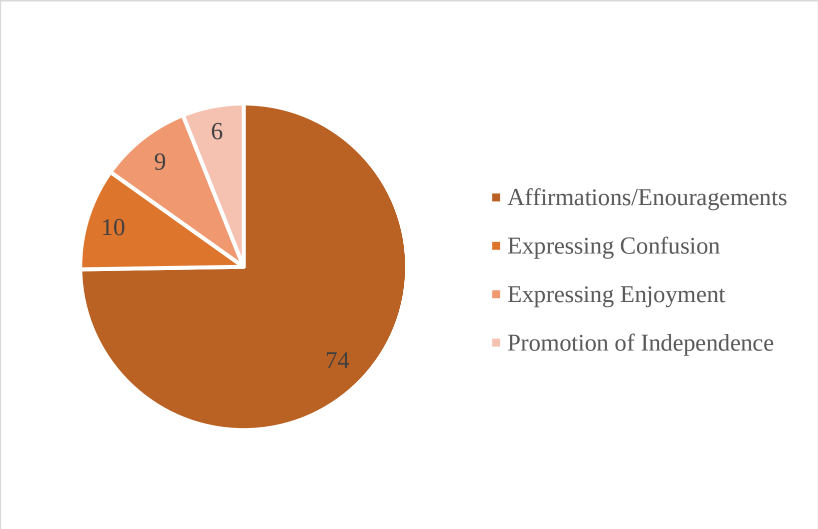
The interesting aspect about this re-representation is that the units do not have different weights, even though both the parent and the 2 children felt that some units are heavier than others. While this re-representation is not unreasonable since the see-saw could also have worked by weight, the notion is inconsistent with the concept of unknown variables. In such a scenario, the unknowns would consistently have the same weights, making it impossible to attribute different values to them, and therefore contradicting the fundamental concept of the unknown variable promoted by the exhibit.

5.3.2 Affective Talk

Parents' affective participation was represented by affirmations/encouragements, expressing confusion, expressing enjoyment, and promotion of independence (see Figure 10).

Figure 10

Affective Talk Subcategories by Number of Coded References



Parents provided a considerable number of affirmations/encouragements, being the most occurred parent scaffolding behaviour overall. Examples of affirmations/encouragements are the following: "Ok, so think through."; "Very good!"; "Right. Aahh, well done"; "Go on, try it [parent encourages the child to put a piece on the machine when she saw that he was hesitant to do it]"; "That's a very good observation!".

Expressing confusion mostly related to how the exhibit works: "Now we barely know what we're talking about". Expressing enjoyment relates to parents' positive experience with using the exhibit: "It was actually fun!"; "You love it so much!".

Promotion of independence is represented by parents encouraging their children to be in charge of the learning activity. For instance, in one group the parent put his daughter in charge, asking her to instruct him on what he needs to do: "Yes, but you can say what I need to do if I need to do anything".

5.3.3 Instructive Talk

Instructions were present in 25 utterances and were focused on guiding the child on what to do for achieving a specific outcome or giving instruction on how to perform an action. The instructions aimed at achieving a specific outcome referred to suggestions or directives on what the child could try for reaching equilibrium or for overcoming an obstacle: "It has become too much so now we remove the blue and the yellow ones". The instructions aimed at calling on the child's appropriate action or response related to, for example, asking the young participant to pay attention to the machine or to count the value of the units, rather than just place them aimlessly: "Look, look!"; "Wait, wait, you need to count!". These instructions were accompanied most of the time by gestures such as pointing at the screen, the axis, or at the different units.

6. Discussion

The findings of this study shed light on the multifaceted role parents undertake in their children's mathematics learning. Moreover, they unveil how the exhibit's design fostered an environment where participants could actively explore intricate mathematical concepts and ideas, promoting engagement and learning among children and parents. Guided by the sociocultural and constructionist theories of learning, this study aimed to answer the following research questions:

> RQ1*: How do parents support their children's learning of mathematics using an interactive exhibit in a science center?*
>
> RQ2: *Can the design of the interactive exhibit promote learning through conversations, hands-on manipulation, and problem-solving among parents and children, and if so, how?*

Having this in mind, the discussion will be divided into two sections. The first one will discuss how parents scaffolded children's interaction with the exhibit and subsequently their learning of mathematics. The second section will focus on the interactive exhibit's design implications in regard to learning defined by communication, hands-on manipulation, and problem-solving.

6.1 Parents' Scaffolding of Children's Learning using the Exhibit

Looking at parents' support of children's learning of mathematics, the data reveals that the adults fulfilled various roles. As presented in Table 2, parents engaged their children in systematic strategies for reaching equilibrium, by following either a semantic or a mechanical approach. The semantic approach is characterised by reaching equilibrium mathematically, more specifically, understanding the different coefficients to calculate the unknown values of the units and using this knowledge to place equal values on each side of the see-saw scale. The mechanical approach involves a step-by-step iterative process of placing units, guided by the behaviour of the scale. This means placing a unit, assessing the outcome, and then refining the attempts based on the insights from the previous placements.

These different approaches unveil the diverse strategies used by the parents to guide their children's exploration and learning with the interactive exhibit. Parents' adoption of different approaches may rely on several aspects, one of the main ones being their own interest and understanding of the subject, which can consequently influence their scaffolding approaches (Tare et al., 2011). Prior studies show that, parents who have a strong mathematical background may naturally use more systematic methods, teaching their children mathematical concepts in a more structured manner (Hart et al., 2016, Andersson, 1997). With this in mind, parents with less proficiency may opt for a less systematic and more exploratory approach (Hart et al., 2016; Bjorklund et al., 2004). These considerations highlight the importance of designing exhibits that are flexible and can adapt to varying levels of knowledge. Looking at Table 2, all parents reached equilibrium which represents the goal of the exhibit, indicating the exhibit's ability to accommodate visitors with varying levels of mathematical knowledge and interests. Lastly, this study reveals another factor that contributes to the varying degree of parent scaffolding, which will be later discussed in the paper, this being parents' estimations of their children's ability to grasp complex concepts.

Moving on to parents' learning talk, it was discovered that parents provided support to their children in four ways: conceptual, affective, instructive, and physical support. Conceptual support emerged as the prevailing form of assistance, covering scaffoldings for facilitating a deeper understanding of the mathematical principles promoted by the exhibit. These principles relate to mathematical operations such as adding, subtracting, multiplying, and dividing to calculate the unknown variables and ultimately reaching equality of equations. In addition to this, exploration strategies were also promoted through parents' scaffoldings, fostering children's problem-solving skills.

Parents' scaffoldings were mainly related to offering explanations and hints, asking questions, giving demonstrations, and correcting children's actions. These scaffolds closely align with the concept of zone of proximal development, representing actions that help bridge the gap between what children can accomplish on their own and what they can achieve with parental support (Bjorklund et al., 2004). Moreover, they emphasize the essential role of social interaction and of parents' guidance in children's learning process. The predominant use of conceptual talk in this study closely mirrors the findings of Tscholl and Lindgren's (2016) research on a science-related interactive exhibit. In their study, it was conveyed that parents offered significant conceptual support which helped children better understand the concept of gravitational force.

One notable finding regarding conceptual talk is parents' use of inquiry which represented the most occurred scaffolding category under this learning talk category. Parents asked children questions regarding the mechanisms of the exhibit, mathematical concepts, and equations, as well as their strategies for solving the tasks at hand. These questions aimed at both evaluating children's understanding of mathematics and the exhibit, as well as fostering improved reasoning. The high number of inquiries in this study contrasts Gutwill's (2008) research on several science exhibits, conveying that exhibit designs normally lack opportunities for visitor inquiry. Conversely, while the current study recorded a considerable number of inquiries, it is essential to note that not all questions led to enhanced reasoning and understanding. As shown in the example where the parent inquired about the child's finding that the pieces have different values, but didn't provide additional clarification when the child replied that he "just knows". In this case, parents' lack of scaffolding might have been caused by several reasons, such as lack of understanding of the exhibit's content, as well as misinterpretation or assumption that the child already has a solid grasp of the concept.

Another noteworthy finding of this research is the very limited number of explanations about the underlying mathematical concepts promoted by the exhibit. The explanations provided by the parents were discovered to focus predominantly on exhibit specific aspects, such as explaining its mechanisms, reinforcing the values of the units, and helping them understand how the units relate to the formula presented on the screen. These findings suggest a particular emphasis on the operational aspects of interactive exhibits, rather than the underlying mathematical principles. While this type of scaffolding behaviour can be valuable in terms of building familiarity with the exhibit and its functionalities, it also indicates a potential gap in parents' understanding or knowledge of how to connect the interactive experience with the underlying mathematical content. This aspect is also reflected in parents' very limited use of re-representations, which can potentially indicate parents' difficulty in bridging the gap between mathematical content and operational understanding. This aspect is closely related to Cooper's (2011) research on the potential for mathematical experiences in informal settings, which conveys that conversations related to mathematical ideas tend to be very limited, with learners focusing mostly on the tangible operational aspects then the underlying ideas. While this limitation may initially come as a surprise, it is also expected since parents are not typically trained educators or proficient in the subject promoted by an exhibit. Instead, they are also eager to learn together with their children, taking on a dual role as both facilitators of learning and learners themselves. Research on family learning in informal environments highlights this aspect, discussing that parents' lack of conceptual support can be significantly improved through conversational scaffolds, such as cards or objects, which can guide and structure their learning (Jant et al., 2014; Benjamin et al., 2010). These studies reveal that parents engage in more elaborative conversations and joint activities when they receive some sort of assistance.

In addition to parents' varying knowledge levels, a cause for the low number of explanations related to mathematical principles is that parents tend to underestimate their children's ability to understand mathematical concepts. This may influence them to provide basic to no explanations, assuming that the complex mathematical ideas may be too advanced for their children to understand. This aspect was noticed during field observations where 3 parents out of the 10 observed groups were uncertain whether they should participate, thinking that their children would not manage to work with the equations. They were later surprised to find out that their children not only identified the values of the unknown variables, but also managed to reach equilibrium on their own based on their mathematical reasoning and calculations. This further strengthens the idea that the exhibit can be a powerful tool for stimulating mathematical thinking and reasoning through exploration and experimentation.

Parents also played a significant role in offering affective support through affirmations and encouragements, being the most common form of scaffolding behaviour observed (see Appendix 3: Data Overview). This finding holds several implications for parent-child mathematics learning. First of all, it can suggest parents' strong interest in fostering a supportive and motivating environment, enhancing children's willingness to explore and engage with mathematical concepts, as well as increasing their interest in the subject. Second of all, the prominence of encouragements could reflect parents' understanding of developing their children's cognitive skills by encouraging them to explore and build their own conceptual understandings. These findings are consistent with previous work on parents' important role in providing affective comments for supporting children's interactions with technologies (Neumann, 2018; Wood et al., 2016; Danby, 2013; Kucirkova et al., 2015). In addition to this, the increased number of encouragements and affirmations has been previously proven to be essential for keeping children focused on the activity at hand and completing challenging tasks (Neumann, 2018; Dodici et al., 2003; Plowman & Stephen, 2007).

All in all, the findings of this study suggest that provided that exhibits offer opportunities for hands-on manipulation that encourages exploration, mathematical conversations, and strategy making, they can present great potential in fostering learning in the subject. Since informal environments have been previously argued to bring numerous contributions to mathematics learning, science centers, and museums may represent important settings where learning in this subject could be adopted. In addition to this finding, this study also presents the various roles parents play in their children's learning with mathematics interactive exhibits. Lastly, this study points out the impact the design of interactive exhibits may have on the interaction between users and the exhibit, as well as on the interactions among users themselves.

6.2 Learning Design Implications of the Exhibit

Gutwill and Allen (2010) discuss in their study that, current interactive exhibits do not offer enough options for visitor manipulation to sustain prolonged engagement among parents and children, limiting opportunities for hands-on manipulation and conversations on the displayed phenomena. Their study aligns with several previous works showing that, interactive exhibits fall short in providing social affordances which support interaction and collaboration between users (Narayanan & Hegarty, 2000; Heath et al., 2005; Meisner et al., 2007; Hsi, 2003; Hurst et al., 2019). In contrast to these studies, the "Bold Equations" exhibit was discovered to foster collaboration and conversations among participants. As revealed in Table 2, all parents and children participated in the interactive experience. Furthermore, looking at Figure 6 and 7, both parents and children engaged in conversations, with parents overall contributing a slightly higher number of utterances. This discovery is consistent with Tscholl and Lindren's (2016) study on an interactive exhibit about outer space, indicating that parents communicate more compared to their children during the informal interactive experience. These observations may underscore the significant role parents play in guiding their children's learning with interactive technologies. In these instances, parents are not only responsible for helping the young learners with the exhibit's content, but also with the underlying mechanisms. Thus, interactive exhibits may present a more demanding role for parents, requiring them to navigate both the content and operational aspects to effectively foster their children's learning.

Participants' engagement with the exhibit can be argued to have occurred due to several factors. Firstly, one factor can be represented by the interactive setup that invites visitors to engage in joint problem-solving activities, encouraging them to collaborate as they work together towards reaching equilibrium. Another significant factor contributing to the active engagement of participants can be attributed to the exhibits' personalised learning features. More specifically, the exhibit allows visitors to engage with it at their own pace, based on their own interests and curiosities. Also, the exhibit allows visitors to explore mathematical ideas that they themselves had produced by placing units along the see-saw scale, adjusting their positionings based on the tilting motion of the scale, and/or their mathematical calculations. Physical interactivity is considered an essential feature in science museums, which has been previously argued to promote engagement, motivation, and recall of actions and key concepts (Allen, 2003, Maxwell & Evans, 2002). Lastly, the presence of video recording equipment may have further encouraged participant engagement, possibly influencing them to be more involved than they would have been without being recorded.

From a constructionist point of view, the "Bold Equations" exhibit may serve as a useful "object-to-think-with" as theorized by Papert, promoting knowledge construction, with users actively constructing their understanding of mathematical concepts and ideas. Furthermore, the exhibit can encourage active, explorative, and extended engagement as evidenced by the considerable amount of time the participants dedicated to it, the average being 7.035 minutes. Morado et al. (2021) discuss this aspect in their study on constructionist learning, pointing out that learners are stimulated by the externalization of their actions with the tools, most of the time enacted through various media representations. This idea is closely related to this study, where participants' actions are visually represented through the feedback displayed on the screen and the tilting motion of the scale, allowing them to directly perceive the outcomes of their interactions. These features have been previously discussed to contribute to a state of being immersed in the activity, constantly connected to the changes in the environment (Morado et al., 2021).

The average time spent in this study considerably surpasses previous works on visitor behaviors at interactive science museums where it was discovered that families typically spend on average less than a minute per exhibit (Sandifer, 1997; Gutwill & Allen, 2010). Although it may seem intriguing at first glance, the notable average time difference between the current and previous studies may be attributed to the video recording method used in this study. This approach may have influenced participants to invest more time and effort, unlike the observational approaches adopted in previous studies where participants were not always aware that they were being observed.

Another significant discovery of this study is that children were able to engage in calculating the unknown variables and reaching equilibrium without their parents' assistance as well. The 2 instances where children engaged with the exhibit mostly independently, were also the instances where children communicated more than their parents (refer to ID 11 and ID 13 in Figure 7). In these cases, the exhibit served as the main scaffolder through the feedback on the screen and the motion of the see-saw scale, with parents providing help only when necessary. In this view, the exhibit allowed self-exploration and discovery, supported through immediate feedback. The exhibit's feedback allowed the participants to observe the consequences of their actions and make adjustments accordingly, allowing them to self-correct and refine their problem-solving strategies. From a constructionist point of view, the exhibit allowed exploration and construction of knowledge, with visitors actively constructing their understanding of mathematical concepts by exploring the exhibit, making connections, and manipulating equations. In this case, the participants were not merely passive recipients of information but active constructors of their knowledge. Moreover, in the 2 instances of children working on their own, the young learners actually helped their parents understand the mathematical principles of the exhibit. This aspect underlines the sociocultural emphasis on collaboration and co-construction of knowledge where learning is mediated through dialogic exchanges and tools. Also, it highlights the dynamic nature of learning within the context of the exhibit, where the roles of "learner" and "teacher" are fluid and interchangeable. This fluidity aligns with the sociocultural perspective on learning, reinforcing the idea that learning is a reciprocal process, where both parents and children contribute and benefit from the conversations and interactive experience that occur during their engagement with tools.

The findings of this study are consistent with previous research indicating that interactive technologies sustain the development of problem-solving skills by engaging in exploring various mathematical strategies (Pareto et al., 2012; Pareto et al., 2023). Moreover, the findings reinforce Heath's (2005) idea of interactive exhibits encouraging visitors to engage with the content for a longer time. Also, participants' use of exhibit's scaffolds in this study aligns with Sommerauer and Müller's (2014) and Yoon et al.'s (2012) reasoning on the importance of providing user scaffolds that offer real-time feedback and a concrete representation of their actions.

6.3 Limitations and Future Recommendations

It is hoped that the insights gained from this study provide a better understanding of how parents support children's learning of mathematics using an interactive exhibit in a science center. This may be of special interest to museum educators and designers as the present study aimed to also contribute to a deeper understanding of the social and learning affordances of interactive exhibits and technologies overall for facilitating mathematics learning.

However, it is important to interpret the results of this research considering its limitations as well. In this section, the perceived limitations will be conveyed, together with their potential implications for the study. To start with, the extent to which the results of the study can be generalized will be discussed. Then, further constraints relating to the design of the study will be presented.

Generalisability of Results

Several factors impact the generalizability of the results of the study. To begin with, it is important to acknowledge that, due to time constraints, the present research involved a small number of participants. In addition to this, as can be seen in the demographic information, the participant sample shows quite limited diversity in terms of nationality and age of both parents and children. Having this in mind, the 10 studied groups may not be representative of the broader public of science centers and museums in Sweden or worldwide, but rather provide an insight into how parent-child groups interact with mathematics-oriented exhibits. Having this in mind, further research needs to be conducted to either support or oppose these results, but also to offer a more comprehensive view of parents' role in supporting children's mathematical learning in interactive settings.

One more perceived issue that may affect the generalizability of the results is participants' awareness of being recorded which may have influenced them into performing differently than they normally would. This phenomenon is otherwise known as the Hawthorn effect which has been previously argued to impose challenges on observational studies (McCambridge et al., 2014). What indicated this effect in this study is the fact that participants, especially children, seemed to be highly aware of the researcher's presence although positioned away from the exhibit, but close enough to offer support if needed. While some young participants seemed to be unbothered, others tended to look at the researcher quite often, especially after placing units on the scale. For instance, one child seemed to be considerably influenced by the recording equipment in the sense that it prompted her to verbalize her thoughts extensively, although she frequently admits that she did not understand what she was talking about.

Encountered Technical Issues with the "Bold Equations" Exhibit

Another notable limitation of the study is represented by the technical problems encountered by the participants with the exhibit. Since the data production started on the first week of the opening of the exhibition, some unexpected technical problems with the exhibit arose during participants' exploration, issues which were eventually fixed. The problems that the participants faced were related to units not being recognized by the machine, and the axis getting stuck or starting to move without participants placing units on it. These technical difficulties hindered parents' and children's participation and therefore may have affected the results of the study as well.

Design Study Limitations

To ensure the anonymity and confidentiality of the participants, their faces were not recorded during their interaction with the exhibit. However, recording faces could have yielded valuable insights, especially when participants were engaged with the formula displayed on the screen. Examining this visual data could have revealed whether children and parents reached equilibrium by checking the formula on the screen.

Another limitation of the study is represented by the explicit focus on one-parent groups. Since science centers and museums often attract families comprising more than one parent or two children, future research could include families with more members as well. This inclusion is necessary to ensure a comprehensive overview of parent scaffolding behaviours. In what concerns this study, one-parent groups were chosen as the results can be more easily compared to previous studies that have focused on similar family structures.

Lastly, the specific focus on only parents represents an additional limitation of the study. While the primary objective of the research was to investigate the way parents support their children's learning of mathematics, an exploration of children's interaction with the exhibit would have also added valuable insights. For example, an examination of children's talk would have contributed to a further understanding of how parents adjust their talk and reasoning in regard to children's talk and actions. Since learning in this context is considered a collaborative process, an exploration of the interaction of both children and parents would have offered a more comprehensive view. Due to the limited time of this research, this aspect was not possible, but represents a potential start for future research in the area.

Another recommendation for future research would be to also analyse whether such exhibits may improve attitudes and performance in mathematics. Since museums and science centers are generally designed to facilitate fun and engaging learning experiences, it would be interesting to explore whether these settings can positively influence visitors' attitudes toward mathematics and potentially enhance their performance in the subject.

7. Conclusion

One of the aims of the study was to explore the way parents support children's learning of mathematics using an interactive mathematics exhibit in a science center. The study discovered that parents engaged their children in reaching equality of equations by following two main approaches, either relying only on the mechanical features or combining it with the mathematical representation. Moreover, the study revealed that all observed parents provided significant conceptual support, guiding their children's mathematical understanding, as well as their use of the exhibit. This support was predominantly provided through explanations, asking questions, prompting different strategies, demonstrating, and correcting children's actions. Additionally, parents provided considerable affective support, fostering a supportive and motivating environment, where children can explore and try different strategies for reaching equilibrium. Lastly, parents also fulfilled the role of instructors, giving directives and encouraging children to engage with the exhibit in a more systematic way. On the other hand, the research also uncovered a significant challenge faced by parents when guiding children's learning with the exhibit. Despite their significant support in facilitating exploration and problem-solving, parents encountered difficulties in connecting the interactive experience with the underlying mathematical concepts. While this study examines only 10 groups and the findings may be suggestive, it uncovers notable implications for future parent-child studies on mathematics learning in informal settings.

The study also aimed to examine whether the design of the exhibit facilitates learning of mathematical concepts through conversations, hands-on manipulation, exploration, and problem-solving. The study revealed that the design of the exhibit scaffolded participants' learning through two main features, the interactive see-saw scale and the screen which shows the mathematical representation of visitors' actions. While the see-saw scale promoted hands-on collaborative exploration, engaging the participants in joint problem-solving activities, the screen feedback prompted conversation on the mathematical implications of their actions. Another notable finding related to the design of the exhibit is that although the exhibit is designed as a collaborative game, children were also able to work on their own for calculating the unknown variables for reaching equilibrium. This aspect was reflected in 2 out of the 10 observed groups, where children took ownership over their learning, as well as helped their parents understand the mechanisms of the exhibit. This aspect underscores the sociocultural theory, which posits that learning is a transactional process that mutually benefits all participants in the activity.

Considering the scarce literature on interactive mathematics exhibits, this study addresses the current research gap by shedding light on both mathematics exhibit design and the parent-child dynamic when engaging with them. In addition to this, this research offers valuable insights for designing studies on future works on parent-child interactions around interactive mathematics exhibits. One notable suggestion for future research is that the analysis of mathematics exhibits should not focus solely on the outcome. While analyzing outcomes has been previously argued to play an important role in analysing mathematics development (LeFevre et al., 2009), researchers need to also consider examining the steps taken toward these outcomes. This approach can reveal various mathematical understandings and strategies. As demonstrated in this study, all participant groups reached equilibrium using the interactive exhibit. However, not all of them relied on mathematical reasoning to reach this outcome. Instead, some groups mostly relied on a trial-and-error approach, by iteratively placing units along the see-saw scale and adjusting their placings based on the movement of the scale. In such cases, reaching equilibrium may not necessarily indicate a deep understanding of the underlying mathematical concepts, but it is the steps taken toward achieving it that contribute to the development of greater mathematical skills. This insight holds essential implications for future research on mathematics-oriented exhibits, as it allows us to better understand whether such displays enhance mathematical reasoning and problem-solving skills. Besides this, the findings of this study hold particular significance for designers of interactive exhibits, as well as for museum and science center educators. The design of the “Bold Equations” exhibit highlights the importance of facilitating exploration, as well as collaborative learning defined by meaningful conversations and hands-on manipulation. Since conversations and hands-on manipulation have been previously demonstrated to bring numerous benefits to mathematics learning, future designs may consider these elements as integral components. Additionally, the findings of this study suggest that connecting visitors’ actions with the mathematical representation can serve as a bridge between abstract mathematical concepts and real-world experiences. However, this exhibit reinforces previous research in the field which argues that current mathematics exhibits present limited opportunities for conversations about mathematics concepts and theories.

Overall, this study strengthens the idea that design decisions affect visitors’ interaction not only with the exhibit, but also with each other, influencing considerably the way learning occurs. The researched exhibit proved to serve as a practical tool that allows visitors to explore mathematical ideas and develop their problem-solving skills. Additionally, the exhibit fostered active engagement from both children and parents, encouraging them to actively participate in constructing their own learning experiences.

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Appendix 1: Participation Form

Study of Mathematics Talk in “Vågade Ekvationer” exhibit

You are being invited to take part in a master thesis research project from the department of IT and Learning, University of Gothenburg. Before you decide, it is important for you to understand why the research is being done and what it will involve. Please take time to read the following information carefully and discuss it with others if you wish. Ask me if there is anything that is not clear or if you would like more information. Take time to decide whether or not you wish to take part. If you decide to take part, you will be given a copy of this information sheet.

Thank you for considering participating in this study. Your contribution to our understanding of mathematical learning is greatly appreciated!

What is the study about?

The study wishes to explore what kind of discussions and thoughts the “Vågade Ekvationer” exhibit at Universeum gives rise to. The study thereby examines the exhibit and not you. Nothing you do is either wrong or right so you are more than welcome to interact with the exhibit however you please.

Why study this?

The aim for the “Vågade Ekvationer” exhibit is to produce learning opportunities for visitors regarding mathematical equations. I would like to examine how visitors interact with the mathematical content of this exhibit and how this may impact their understanding and learning.

How is the study conducted?

If you wish to participate, you will have to fill in a very short questionnaire stating your age, gender and nationality. After answering these questions, you are free to explore the “Vågade Ekvationer” exhibit for as long as you like and you are also more than welcome to communicate and cooperate with your partner during your exploration. Your interaction will be recorded audio and video (only the screen and your hands will be visible). These recordings will then be transcribed, analysed and compared to other visitors’ and will be shared only with my supervisor and examiner.

Data Protection & Confidentiality:

Any personal identifying information will be removed from the data. The data production process will comply with the General Data Protection Regulation (GDPR). Once the data is gathered, it will be stored in a password-protected computer. The data will be kept until the thesis is submitted and presented. Afterwards, it will be deleted entirely.

Withdrawal of consent:

You are free to withdraw your consent at any moment by contacting me and stating your unique ID-number.

Your ID-number is: ___________

Contact details:

Student’s name: Iuliana Badica
E-mail: iuliana.mihaela.badica@gmail.com

Supervisor’s name: Lena Pareto
E-mail: lena.pareto@gu.se

Appendix 2: Consent Form & Demographic Information

Consent Form

I, ________________________________, agree to participate or agree the participation of my child,______________________________________, in the research project titled "Study of Mathematics Talk in "Vågade Ekvationer" exhibit", conducted by Iuliana Badica, who has discussed the research project with me.

I have received, read and kept a copy of the information letter. I have had the opportunity to ask questions about this research and I have received satisfactory answers. I understand the general purposes, risks, and methods of this research.

I consent to participate in the research project and the following has been explained to me:

- the research may not be of direct benefit to me;
- my participation is completely voluntary;
- my right to withdraw from the study at any time without any implications to me;
- the risks including any possible inconvenience, discomfort or harm as a consequence of my participation in the research project;
- the steps that have been taken to minimise any possible risks;
- what I am expected and required to do;
- whom I should contact for any complaints with the research or the conduct of the research;
- I am able to request a copy of the research findings and reports;
- security and confidentiality of my personal information.

In addition, I consent to:

- audio-visual recording of any part of or all research activities;
- publication of results from this study on the condition that my identity will not be revealed.

Name: ____

Signature: ____

Date: ____

Demographic Information Questionnaire

1. **How old are you?**

☐ <13

☐ 13-15

☐ 16-19

☐ 20-25

☐ 26-35

☐ 35 or older

2. **What gender do you identify as?**

☐ Male

☐ Female

☐ Other (please specify) ___________

☐ Prefer not to say

3. **What is your nationality?**

____

Appendix 3: Data Overview

	Parent Scaffolding Behaviours	Total Utterance Number	Frequency of Sessions where the Scaffolding Type was Present
Instructive Talk	1. Instructions	25	8
Conceptual Talk	2. Correction/Disaffirmation	36	9
	3. Re-representation	3	1
	4. Hint/Prompt	36	9
	5. Modeling/Demonstrating	47	7
	6. a. Local/Environment specific Explanations	45	9
	b. Math concepts-related Explanations	5	3
	7. Inquiry	70	10
Affective Talk	8. Affirmation/ Encouragement	74	10
	9. Promotion of Independence	6	4
	10. Express Confusion	10	5
	11. Express Enjoyment	9	6

Appendix 4: Original Quotes in Swedish

Example 1:

Parent: Det är så här. Nu är det jämvikt, såklart, men nu ska vi testa vilken som motsvarar vilken. Vi tänker att två röda var en vit [puts a red unit on top of the other red, right side-the bar lowers on the right] och vi testar om det stämmer [puts a white on the 5th position, left side, the bar does not move]. Om det stämmer så blir det jämnvikt, men det blev det inte.

Example 2:

Parent: Kommer du ihåg den här? Vad var den? [pointing at the red unit] Den var 3 * 5 så vi ska få ihop det. Vad är 3*5?

Child: 3 * 5...15! Parent: Så då måste du lägga på så att du får upp ihop...15 bara for den röda.

Example 3:

Parent: Vet du varför det står en etta där [pointing at the coefficient 1] och en femma där? [pointing at the coefficient 5]

Child: Nej.

Parent: Grejen är att desto närmare du kommer här [touching to the center of the axis] desto mindre, alltså den där axeln håller upp lite.

Example 4:

Parent: Så, 3Z, det är det [pointing at the red piece on the coefficient 3]

Parent: Vad är den då för nåt? [pointing at the yellow piece from the same position] Vad är det för bokstav?

Child: Jag har ingen aning.

Parent: 5 ...[pointing at the number on the bar which indicates coefficient 5 and then at the 5X from the equation]

Child: Ahaa, 5X!

Parent: 5X. Vad är den här för bokstav? [pointing at the blue piece on the right side]

Child: Ett Y.

Parent: Ett Y, precis! Och de här är ju? [pointing at the white pieces on the left side]

Child: Här är ju 5.

Parent: Nej.

Parent: 5+5 är 10 [pointing at the 2 white pieces on the 5th position, left side]. 3+3 är 6 [parent pointing at the 2 white pieces on the 3rd position, left side]. 10+6 är [pointing at the 16 from the equation]

Child: Jahaa!

Example 5:

Parent: Men det beror på, det beror alltså att här [touches the part in the middle that holds the bar] hjälper axeln till att hålla upp den här grejen lite [points at the 1st position, left side]. Här [points at the 3rd position] hjälper till mindre, där trycker den med 3. Och där borta hjälper den inte alls.

Example 6:

Parent: Så nu vet du att 3X är lika med 3. Alltså ska vi ta [parent places one blue on the 1st position and one red on the 5th position] här har vi 3 okända här. Du, vi går till dem sen. 3X+1Y+5Z [points at each piece individually as he is reading the formula on the screen]

Example 7:

Parent: Vet du att de här formerna [pointing at the screen], de kan man förenkla matematisk för om du har Y i bägge sidorna, då kan du ta bort ett Y på bägge sidorna.

Example 8:

Parent: Z+1 är lika med X+Y+1. De här vita [pointimg at the white piece from the 1st position, right side], de ...är ettor så de kan du i princip bara ta bort. Vi tar bort både de vita [removing the white pieces from the axis]. Så ser vi här [parent reaches equilibrium]. Oj, vi ser att ett Z, ett Y...ett X + ett Y [pointing at the formula on the screen which shows 1Z= 1X+1Y] och om vi lägger bara vita här [takes away the yellow and the blue pieces from the 1st position, right side, and puts 2 white] lägg på en vitt till, lägg på en vitt till [child puts another white] så [they reach equilibrium]. Då ser vi att Z, den är 3 för de här ettor är en trea. En röd är en 3.

Example 9:

Parent: Nu har vi 10 där och där vet vi inte hur mycket vi har men om vi gör så här då, då har vi 11 [places one white piece on the 1st position, left side] den är värd mer än 11 [pointing at the red piece on the 5th position, right side] och om vi tar den då... 12 [places one white piece on top of the other white one, 1st place, left- the machine doesn't move]

Child: Om vi lägger 3 [places one white as well on the same spot]. Den är värd mer än 13 [points at the red piece on the 5th position, right side]. Om vi lägger 14 [places one more white on top of the other white pieces, nothing happens]

Parent: Men vi kan också flytta en sådan [takes the white piece the child just placed and puts it on the next position- 3rd, left side]. Om vi flyttar den dit är den värd 3, eller hur? Så [the bar gets heavier on the left side] då vet du hur mycket de är värda snart [pointing at the white piece just moved].

Child: Vad?

Parent: För om vi tar bort den här så blir det jämnvikt, va? [removes one white from the 1st position, left side] 1 eller 2, titta! [reaches equilibrium]. Nu! Så då vet vi en sådan [pointing at the red piece on the 5th position, right side] är värd vad då? 5, 10, 13, 15 [counting the values of each brick from the left side].

Example 10:

Child: Jag lägger den här. En till [reaches quilibrium]. Titta!

Parent: Ja, men vi har fortfarande inte räknat ut vad Y är [points at the Y from the equation on the screen].

Child: Y?

Parent: Den är den blåa och den är vad? Kom, kolla [the child leaves the scene, but comes back].

Parent: Vad du kan göra är att plocka bort lika mycket från varje sida nu när vi har jämnvikt.

Parent: Så ta en röda.

[child removes one red piece from the 5th position, each side, at the same time]

Parent: Fortfarande jämnvikt. Man har lika mycket på varje sida.

Child: Nu tar jag bort två 3.

Parent: [agrees]

Child: Eller en 3. Är det här?

Parent: Den har vi räknat ut tidigare att den var en etta [points at the yellow piece].

[Child removes one yellow from LS and one white RS, both 3rd position]

Parent: Så då har du fortfarande jämvikt.

Child: Ja, ett Y är lika med 2.

Parent: Ja, precis.

Example 11:

Parent: Så det är mindre än 3 och mer än 1.

Example 12:

Parent: Nej, vänta, vänta lite [the child tries to take the red piece as well, but then puts a blue piece on the 3rd position and another on the 5th]. Kan du bara sätta en i taget? Om du sätter flera, vet du inte vad de gör.

Example 13:

Parent: Vi kan bedöma lite. Nej, nej, vad du känner, vad du känner?

Child 2: Den är mycket tyngre.

Child 1: Jag vill känna.

[the boy on the right gives his brother the red piece]

Parent: Den är mycket tyngre, precis.

Child 1: [the boy weighs the red piece in his hand] Okej, de här är lika tunga, tror jag.