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Are European Green Bond Yields Lower Than the
Yields of Their Conventional Counterparts?

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Abstract

In this paper, we contribute to the growing literature on sustainable finance by investigating whether European green bonds provide a lower issue yield compared to their conventional counterparts, i.e. if there exists a greenium in the primary market. We use the Nelson-Siegel model to investigate the term structures of green bonds compared to conventional bonds and a principal component analysis to determine if there is a yield difference between the two types of bonds. Our sample is constructed using a matching approach and contains bonds issued between 2013 and 2023. The results of the study are significant with an estimated greenium of 8 basis points and the term structure of green bonds indicates lower yields for green bonds at longer maturities. This means that issuers benefit from long term green bonds as the advantage of green bonds increases with the uncertainty associated with the passage of time. The existence of a greenium in the primary market implies that cheaper funding for green investments can be secured (Partridge & Medda, 2019).

Keywords: Green bonds, Greenium, Sustainable investment, Principal Component Analysis (PCA), Nelson-Siegel (NS).

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1 Introduction

Collaborations such as the Paris Agreement and the UN 2030 Agenda for Sustainable Development have emerged as a way to tackle the current environmental crisis (European Commission, 2018). In addition to these collaborations, the European Commission has also established an action plan for financing sustainable growth, which focuses on moving capital flows toward green investments (European Commission, 2018). This has culminated in an expansion of the green bond market, which has been increasing since 2019 (Caramichael & Rapp, 2022). Green bonds are in essence conventional bonds, meaning debt issued by an entity to the public, but with a particular feature where the proceeds need to be invested into green projects (Gianfrate & Peri, 2019). Green investments should be environmentally beneficial projects, for example, mitigating emissions and/or increasing resource efficiency (Gianfrate & Peri, 2019). The first green bond was issued in 2007 by the European Investment Bank (World Bank, 2016) and the market has grown to a value of USD 4 trillion in 2021 (Bloomberg, January 2022).

There are multiple studies examining how market forces affect the yields of green bonds and, in some cases, the increasing demand drives the yields of the green bonds below the yields of their traditional counterparts. Both the existence and driving factors of such a difference are frequently studied as the “greenium puzzle”. In this paper, we contribute to the growing literature on sustainable finance by investigating whether European green bonds provide a lower issue yield compared to their conventional counterparts, i.e. if there exists a greenium in the primary market. Some argue that investors do not seem to put any additional value on green bonds, meaning that no greenium should exist. Whereas, results indicating the presence of a greenium in the primary market imply that cheaper funding for green investments can be secured (Partridge & Medda, 2019). The research question is therefore: *Are European Green Bond Yields Lower Than the Yields of Their Conventional Counterparts?*

In standard economic theory (Markowitz, 1952), investors act rationally and solely focus on receiving the largest return. Evidence of a greenium can contradict this statement since it would indicate that investors are willing to receive a lower return if the proceeds are used for green investments. Investors do, however, tend to be risk averse as expectations change with the duration of bonds (Malkiel, 1989). Yields are expected to be higher when the bond is subject to more uncertainty (Partridge, 2019). It is consequently interesting to investigate the term structure of green bonds to identify if there is a difference in greenium at a given time for different maturities, which is especially crucial for bond portfolio management (Diebold & Li, 2006). The risk profile of green bonds, compared to their conventional counterparts, might differ due to regulations and restrictions regarding climate actions. The European market has higher sustainability reporting requirements, increasing the attractiveness of bonds issued in

this area (Charamichael & Rapp, 2022). This could explain why some researchers have found a greenium since green bonds aid investors and firms to comply with upcoming regulations (Gianfrate & Peri, 2019).

A greenium in the secondary market would pressure prices in the primary market while additionally indicating that investors prefer ESG investments (Partridge & Medda, 2019). This means that investors pay a premium for green bonds, which creates monetary incentives for firms' decision-makers to pursue sustainable development (Gianfrate & Peri, 2019). Though, a common critique is whether green bonds contribute to sustainable development or if the green investments would have occurred anyways using traditional bonds (Carmichael & Rapp, 2022). A similar issue is that issuers can take advantage of the intangible nature of the green label and deceive investors by falsely advertising something as "green" (Chiang, 2017). This practice is commonly known as greenwashing, where corporations misuse the term "green" and claim to be more sustainable to improve their reputation (Chiang, 2017). The addition of a greenium to this issue could create further incentives for such behavior, ultimately making the concept of sustainability more ambiguous within the context of finance (Carmichael & Rapp, 2022).

This study aims to identify if there exists a greenium in the primary market by including more recent data than earlier literature. Partridge and Medda (2019) argue that similar studies should be performed in the future, as the number of green issuers increases, which is expected to affect the results. The focus of our paper lies on European bonds since, to our knowledge, only one previous study investigate solely the European market, and the method in that study is different. Our research design consists of two models for examining bond yields, and the sample is constructed using a matching approach. We use a yield curve analysis to investigate the term structures of green bonds compared to conventional bonds and a principal component analysis (PCA) to determine if there is a yield difference. The results of our study are significant with an estimated greenium of 8.28 basis points (bps) for the PCA and the term structure of green bonds indicates lower yields for green bonds at longer maturities with a greenium estimate of 21.51 bps.

The most substantial limitation of this study is the availability of data on green bonds. The matching of green bonds with their conventional counterparts will therefore suffer from the limited availability of green bonds resulting in fewer observations of traditional bonds and further limiting the sample. Additionally, the sample is divided into subsamples, resulting in fewer data points. However, the sample size is large enough to test the hypothesis and draw reliable conclusions, though a larger sample would be preferable. Compared to earlier studies, our sample is not the smallest nor the largest in terms of number of green bonds.

The remaining sections of our paper are structured as follows: Section 2, *Literature Review*, provides a summary of previous studies on the greenium puzzle as well as a presentation of arguments made by the authors regarding which market to study and what approach to use for reducing bias. Section 3, *Method and Data*, presents the data collection, cleaning of the sample and the choice of methodology, including important theoretical information. Section 4, *Empirical Results and Analysis*, display the outcomes of section 3 and a thorough analysis of the results, together with the robustness tests. Lastly, this paper is finalized with section 5, *Conclusion*, which summarizes and provides general insights from our study.

2 Literature Review

There exists a vast amount of studies concerning the effect of social and environmental corporate governance on financial structure and the cost of debt. Klock et al. (2005) show that bond yield spreads decrease when the quality of corporate governance increases using U.S. data. Menz (2010) finds a contradicting effect where socially responsible European corporations face larger credit spreads on their bonds compared to firms who are not socially responsible. Oikonomou et al. (2011) look at U.S. corporate debt where they find that corporate social responsibility does reduce lending costs in the form of lower yields. They further strengthen these findings with the observed negative correlation between corporate social responsibility and financial risk.

With the appearance of the green label for bonds, more recent research looks into the differences in yields between green bonds and their conventional counterparts. This literature includes Baker et al. (2018), Gianfrate and Peri (2019), Zerbib (2019), Partridge and Medda (2019), Larcker and Watts (2020) and Carmichael and Rapp (2022). The samples in these papers include bond data from the years 2013/2014 up to 2021 and differ in geographic scope. Gianfrate and Peri (2019) study European bonds, Partridge and Medda (2019), Larcker and Watts (2020), and Baker et al. (2018) solely include U.S. bonds. Lastly, Zerbib (2019) and Carmichael and Rapp (2022) use international bonds.

The studies further differ in types of issuers. Partridge and Medda (2019), Larcker and Watts (2020) and Baker et al. (2018) study municipal bonds, where the latter include corporate bonds as well. Larcker and Watts (2020) argue that if a greenium exists in the market it is easiest to detect for municipal bonds due to the lower issue size which enables the marginal trader to play a larger role in the performance of the bonds. Gianfrate and Peri (2019) and Zerbib (2019) are more liberal in their data collection as they use various issuers such as corporates, financial institutions, municipalities, etc. The recent study by Carmichael and Rapp (2022) uses corporate bonds with a notional of at least USD 500,000, paying both zero and fixed coupons. To only study the market where the green bonds have the largest footprint they select bonds issued in EUR or USD and any green bonds where the company has not issued ordinary bonds are dropped from the sample.

The literature on the greenium is quite mixed, as stated by Larcker and Watts (2020). According to the authors, this is due to “methodological design misspecifications that produce biased estimates”. Baker et al. (2018) and Carmichael and Rapp (2022) use fixed effects regressions. Gianfrate and Peri (2019) use a combination of propensity scores and regressions whereas Zerbib (2019) uses two-stage OLS-regressions. Partridge and Medda (2019) diverge from the conventional regression approach by

instead utilizing two different methods, a yield curve analysis using the Nelson-Siegel model to construct monthly yield curves, and they introduce a green municipal bond index as their second approach. The authors emphasize the time aspect included in their study as most other studies do not take this into account. Larcker and Watts (2020) use two statistical tests, a t-test and a Wilcoxon rank-sum test, in order to test for differences in means of the yields between green bonds and their matched traditional bonds.

Although these studies differ in chosen methodology, they provide similar results indicating a small or non-existent greenium. Baker et al. (2018) find a significant greenium of 8 bps and Gianfrate and Peri (2019) discover this effect to be as large as 21 bps for corporate bonds. Gianfrate and Peri (2019) also study the secondary market where they find evidence of a greenium for corporate issuers around 8 bps after the 14th of December in 2017 and decreasing in size over time. Zerbib (2019) also finds a significant greenium, though smaller, of 2 bps. The author performs the same regression analysis on a monthly basis and provides evidence of the fact that the premium varies over time, but since May 2016 it remained negative.

In contrast, Partridge and Medda (2019) and Larcker and Watts (2020) find no significant greenium in the primary market. Larcker and Watts (2020) describe that green bonds have a low market penetration as they only make up a very small portion of the debt markets, which can explain why investors do not seem to put any additional value on green municipal bonds. The authors also argue that there is no greenium since sustainable investments produce adequate results that provide as much returns as non-green bonds. Another reason could be the higher issuance costs associated with green bonds, as studied by Chiang (2017), something Larcker and Watts (2020) also find evidence of. Instead, the only advantage of green bonds is that they help to diversify the investor base (Larcker & Watts 2020), a view shared by many practitioners, as mentioned by Braun (2019). Despite this, the authors remain optimistic about finding evidence of a greenium in the future as investors adopt different views and the market matures.

Lastly, Carmichael and Rapp (2022) find a yield spread differential between green bonds and their non-green counterparts of 8 bps, on average. By accounting for time-variation through the use of annual time-series data, they find that a greenium is first observed in 2019 at 14 bps and then it decreases to around 8 bps in 2020 and 2021. The authors consider this result to be in line with the emergence of the EU regulation along with the growth of sustainable asset management. They also observe that the greenium is highly concentrated to large investment grade issuers situated in developed economies and primarily active in the banking sector.

Liberati and Marinelli (2021) argue that studying the greenium after Covid-19 contributes to already existing literature since the health crisis has influenced the common perception of the urgency of climate change. The pandemic resulted in significant infrastructural changes regarding the use of resources, which served as a wake-up call concerning the enormous efforts required to mitigate climate change (Schumacher, 2020 & Schnabel, 2020). Studying the greenium puzzle could, therefore, be useful for firms, as investors may want to pay a premium for green bonds because they value green infrastructure development more highly than before the crisis (Liberati & Marinelli, 2021).

Market practitioners have raised concerns regarding the issue of greenwashing as a consequence of inadequate policies and the absence of requirements for third party certification of green bonds. This allows issuers to misuse the green label, which creates a lack of trust from investors as it is uncertain whether funds are invested in truly green and sustainable projects (Charamichael & Rapp, 2022), possibly affecting the yields of green bonds. Larcker and Watts (2020) test the impact of greenwashing and find no evidence of this issue affecting their results.

2.1 Approach for Reducing Bias

A major concern for studies attempting to disentangle the effect of a single factor, such as the green label, is whether or not the estimated effect is exhibiting any bias, and if so, how to reduce it. Previous literature goes in two ways to solve this issue, either by running fixed effects regressions or using a matching approach. This paragraph disentangles the arguments behind earlier literature's decisions on which approach to use and provides a foundation for the choice of using a matching approach in this study.

Gianfrate and Peri (2019) mention that the treatment in their study should ideally be assigned randomly to avoid bias arising from other bond characteristics. The matching procedure is a way to overcome this bias while also obtaining the counterfactuals and it is for this purpose the authors match the bonds in their sample. They do this by using a propensity score, which is a variable that encompasses all the relevant characteristics that should be used to match the bonds. Rosenbaum and Rubin (1983 & 1984) describe this as creating an index variable. Zerbib (2019), Partridge and Medda (2019), and Larcker and Watts (2020) also use a matching procedure to account for any factors which may bias their results. However, their approach is less mathematical as it does not involve the creation of a propensity score, rather they rely on manual matching based on important observable characteristics. Zerbib (2019) describes the matching procedure as a vital part of the study that works as a control for all the unobserved factors affecting bond yields. This model-free approach is used extensively in the

financial literature by researchers such as Kreander et. al. (2005), Bauer et.al. (2005) and Renneboog et. al (2008).

The authors who manually construct bond pairs base their matching on different characteristics. Crabbe and Turner (1995) are often referenced in other studies as the foundation for bond matching. They match a green bond with multiple traditional bonds when possible, but if exact matches exist, they omit any cases with different maturities. All studies base their matching on the issuer and then complement with other important variables, such as issue date, maturity, rating, seniority and more. This limits the sample of possible matches but minimizes the differences associated with the slope of the credit spreads (Larcker & Watts, 2020). Larcker and Watts (2020) argue that their matching approach completely controls for any factors affecting the yields since they are able to find exact matches among the municipal bonds. They further explain that a fixed-effects approach cannot ensure that all factors are controlled for in the same way as with a matching technique.

Baker et al. (2018) and Carmichael and Rapp (2022), on the other hand, do not match their bonds, rather they use fixed-effects regression to control for unobservable characteristics. Carmichael and Rapp (2022) choose this approach since bond matching limits the sample and creates a bias towards issuers with a strong position in the bond market. Consequently, SME:s and emerging markets will be underrepresented. They also argue that matching restricts the sample further to only contain issuers of green bonds, and thus creates a bias in the control group because these issuers are different from the “gray” issuers who have not issued any green bonds.

The previous literature presents mixed views on the choice of the approach used to control for bias in their data. Relevant arguments both in favor and against each method are mentioned, resulting in the absence of a common way of testing for a greenium. Therefore, it is important to thoroughly analyze the gathered data before deciding which method is more suitable.

2.2 Primary vs Secondary Market

The choice of whether to analyze the primary or the secondary market is a crucial part of our paper, and there are arguments both in favor and against studying either market. This is another fundamental way in which previous studies differ. Baker et. al. (2018), Gianfrate and Peri (2019), and Partridge and Medda (2019) all study both the primary and secondary market, while Zerbib (2019) only examine the secondary market, and Larcker and Watts (2020) and Carmichael and Rapp (2022) investigate

the primary market. Below, we present arguments regarding both reference markets to identify the problems arising from either market and to clarify the reasoning behind the choice of investigating the primary market in this study.

Partridge and Medda (2019) argue that it is easier to detect a greenium in the secondary market due to the larger set of observations available. They also state that investors' green interest has a greater impact on the secondary market than on the primary market. The secondary market appears to be more sensitive to investors' environmental preferences which aligns with the increase in ESG investing activities described by Fama and French (2007); Friedman and Heinle (2016); Riedl and Smeets (2017); Hartzmark and Sussman (2017), and Brodback et al. (2018). Another issue with the primary market, as pointed out by Hale (2018), is related to the green halo effect. The green halo effect causes the overall yields of an issuer to decrease following the issuance of a green bond. As a result, it could be difficult to measure and detect a greenium, since not only the yields of the green bonds will be lower, but also all yield curves of that issuer.

Conversely, some authors argue that the possibility of observing a greenium is higher in the primary market. Carmichael and Rapp (2022) mention the fact that it is the primary market that determines the actual borrowing cost of the issuer, as the interest rate (yield) is decided beforehand. Additionally, most investors prefer investing in bonds directly in the primary market, further strengthening the case for this market (Flanagan, Kedia, & Zhou, 2021). The choice of the primary market is also supported by illiquidity problems in the secondary market. Liquidity differences between green and non-green bonds are expected to exist since bond prices are affected by liquidity (Hachenberg & Shiereck, 2018) and green bonds have even lower liquidity compared to traditional bonds since they are often bought and held by large institutional investors in the primary market (Gianfrate & Peri, 2019). These liquidity issues impose larger transaction costs on investors in the secondary markets, and thus investors tend to express their opinions more in the primary market (Carmichael & Rapp, 2022).

3 Method and Data

3.1 Choice of Methodology

We choose to study the primary market based on the information in section 2.2, *Primary vs Secondary Market*. There are benefits and drawbacks with either choice of reference markets. Given that the liquidity issues are mainly related to the secondary market, this study focuses on the primary market as a way to address such issues.

To analyze the yield differences between green and ordinary bonds, we choose a two-fold approach including both a yield curve analysis, namely the Nelson-Siegel (NS) model, and a principal component analysis (PCA). We bootstrap the zero-coupon rates from the bonds in our sample and use them as input for the NS, while the issue yields of the bonds in our dataset serve as input for the PCA. The PCA is more flexible as this method is less data-dependent compared to the yield curve analysis, making it possible to implement the PCA on a larger scale.

The yield curve analysis provides a different approach because of its ability to capture the time component in the form of term structures, which is a vital aspect when studying the yields of bonds. According to Partridge and Medda (2019), most other studies disregard this aspect of time. The yield curve analysis produces two yield curves that illustrate the term structures of green and conventional bonds, enabling a graphical interpretation of the greenium. Graphical analysis of data is important because it can reveal properties of the data which sometimes statistics fail to capture, as illustrated by the statistician Francis Anscombe in 1973 with his now famous Anscombe's quartet (Anscombe, 1973).

The hypotheses tested in our study are related to the different methods with one null hypothesis for each method and, consequently, one alternative hypothesis for each method:

H₀a. There is no difference in yields between green and conventional bonds.

H₁a. There is a difference in yields between green and conventional bonds.

H₀b. There is no difference in the Nelson-Siegel factors between green bonds and their counterparts.

H₁b. There is a difference in the Nelson-Siegel factors between green bonds and their counterparts.

3.2 Data Collection

Data on European green bonds and their conventional counterparts are collected from the Thomson Reuter Refinitiv Eikon database (Refinitiv, 2023) between January 2013 and February 2023 since no bonds were formally issued as green before this period (Larcker & Watts, 2020). The dataset is collected as a cross-section given that we study the primary market. To run the subsequent analysis, as much data as possible is gathered on the bonds, and the sample is not restricted to any particular type of institution. It contains corporate, government, municipal and agency bonds including inactive bonds, i.e. bonds that have already reached their maturity as of the date for data collection. For comparison purposes, the data is restricted to investment grade bonds that pay plain vanilla fixed coupons. A complete summary of the variables included in the analysis can be found in Appendix B. The categorical variables (denoted by * in Table 13), such as country of issue, are transformed into dummy variables.

Since we rely upon two separate methods for detecting if there is a premium associated with green bonds, the data needs to be fitted with both these methods. The inclusion of solely investment-grade bonds with plain vanilla fixed coupons is consequently relevant to both methods. For the yield curve analysis, the bonds must be issued in the same year around the same date with a variety of maturities for the bootstrap to work smoothly. Thus, the sample used in the yield curve analysis is only a portion of the entire matched sample used in the PCA. For the yield curve analysis, we have selected a sub-sample based on the year of issue, specifically focusing on the year 2022. This year has the highest number of green bonds issued (161 green bonds), and it is also the most recent year, allowing us to more closely examine the recent development of the term structures in relation to the greenium. The selection of the year 2022 is also interesting from another point of view because it is after the Covid-19 pandemic, allowing us to study its effect on the common perception of sustainability, as described by Liberati and Marinelli (2021). We use all the data gathered from 2013 to February 2023 for the PCA and match it according to the procedure described in section 3.2.4 *Matching of the Bonds*, to arrive at the final sample.

3.2.1 Yield Data

In order to perform the PCA in this study, the key variable of interest is the bond yields, which can be defined in different ways. One option is to collect data on the yield to maturity (YTM) from the Refinitiv Eikon database. However, these YTM:s have approximately 18% missing values and are thus not used in this study. Refinitiv also provides issue yields, but the same problem with data availability exists for these yields. To avoid reducing the already limited sample size by removing missing values,

we instead calculate the issue yields for each observation using the following method:

$$Issue\ Yield = \frac{I + \frac{F-P}{n}}{\frac{F-P}{2}} \cdot 100 \quad (1)$$

where I is annual interest payments, F is the face value of the bond (principal), P is the issue price and n is the number of days until maturity (expressed in years).¹ There are multiple day-count conventions used in fixed income markets to express time in years. For this study, we adopt the common convention of actual/360 (James & Webber, 2000 & Tuckman & Serrat, 2011).

3.2.2 Cleaning the Data

We remove some data points from the sample because of missing values in crucial variables. The largest reduction concerns the variable “issue price” where 536 bonds are removed from the entire dataset, and 99 of these are also related to the subsample containing only observations from the year 2022. Other data points, although fewer, are removed due to missing values for the variables “amount issued”, “next coupon payment” and “time to maturity”. Lastly, one dual currency bond is erased to avoid any additional complications that may arise due to the exchange rates.

We identify extreme outliers using the method of 3 times interquartile range. This method is preferred over identifying outliers with standard deviation because the outliers impact the standard deviation and create a definition of outliers depending on themselves. A total of 70 bonds with extreme values for the coupon are removed from the dataset. The same method is used for issue price, where 3 extreme outliers are identified and removed at prices of 99 598, 0.6625 and 0.8580. We remove extreme outliers since all observations considered as outliers are valid and genuine, thus removing these outliers would affect the results of the analysis. However, there are some observations where the values are less valid (as in the examples of issue price), and those extreme outliers can negatively impact the results of the analysis and are therefore removed.

To improve the quality of the subsequent statistical analysis and address sparsity in the data, categorical variables with a negligible amount of observations (below 2%) are moved to a residual category. In this process, three residual categories are created: “other currencies”, “other countries”, and “other sectors”. Three variables “Has sinking fund”, “Inflation protected” and “Puttable” only contain observations with

¹The calculated issue yields are similar to the ones provided by the Refinitiv Eikon database, confirming the accuracy of this approach for estimating the issue yields. A plausible explanation for the small differences is that the Refinitiv Eikon database uses the day-count conventions associated with each bond whereas we only use one convention for all observations.

“no” and are thus considered redundant and excluded from the dataset. Lastly, the variables “modified duration to maturity”, “option adjusted spread” and “yield spread (OTR) to maturity” each has 28 missing values that are replaced by the respective sample median, enabling them to be included in the analysis. Lastly, observations appearing multiple times as a result of the matching, described in section 3.2.4 *Matching of the Bonds*, are deleted in order for each observation to only appear once in the final sample.

3.2.3 Descriptive Statistics

The following tables provide a summary of the most important characteristics of the two datasets used in this study. The total sample contains 562 bonds, equally divided between green and conventional bonds, with a subset of 20 bonds each for the yield curve approach. There are a total of 115 different issuers in the dataset, and the most frequent one is Norddeutsche Landesbank Girozentrale with 38 observations. Additional descriptive statistics can be found in Appendix C.

Table 1: Summary of Bonds Issued at Discount, Par and Premium

	Discount	Par	Premium
Green bonds	172 (56)	57 (23)	52 (7)
Conventional bonds	145 (25)	83 (50)	53 (10)

This table displays the distribution of bonds issued at a discount (<100), at par ($=100$), and at a premium (>100). Values in brackets represent recently issued bonds for the period 2022-2023

The trend with an increase of green bonds issued at par found by Partridge and Medda (2019), cannot be observed in our data, as shown in Table 1 above. The average issue price is lower for the green bonds, indicating that if a greenium exists, it is likely due to differences in coupons, as the average coupon is lower for green bonds, resulting in the average yields being lower for green bonds as well. Otherwise, it is possible to infer from the descriptive statistics that the two sets of bonds are similar to each other, as expected of the research design using matching.

Table 2: Summary Statistics

	Green Bonds	Conventional Bonds
Amount	281 (20)	281 (20)
Mean yield	1.2455% (1.7627%)	1.2459% (1.4379%)
Main yield	-0.4085% (-0.4085%)	-0.5602% (-0.1054%)
Max yield	5.0715% (3.6657%)	5.9800% (3.3694%)
Average mod. duration	6.0920 (7.7941)	5.1483 (7.6074)
Min mod. duration	0.0582 (1.3255)	0.0416 (1.2900)
Max mod. duration	30.5759 (19.7700)	27.1200 (1.2900)
Average issue amount (EUR)	886 463 960 (1 308 703 941)	2097 647 947 (2 361 805 541)
Min issue amount	507 793 (24 999 999)	2 577 632 (5 031 988)
Max issue amount	30 941 000 000 (4 999 999 999)	47 050 999 999 (27 000 000 000)
Average issue price	99.7664 (99.9238)	99.9540 (99.5566)
Min issue price	95.1944 (98.2110)	96.3450 (97.5800)
Max issue price	108.6200 (108.6200)	117.4940 (100.0670)
Average coupon	1.2194 (1.7307)	1.2386 (1.8158)
Min coupon	0.0100 (0.1000)	0.0100 (0.1250)
Max coupon	5.7500 (3.5000)	5.9800 (4.3750)

The table displays a selection of summary statistics for the entire dataset used in the study, values in brackets () display the corresponding statistic for the subsample of bonds used for the yield curve analysis. Mod duration is short for modified duration to maturity, min is short for minimum value and max is short for maximum value.

A large portion of our sample stems from the banking sector, as visible in Figure 8 in Appendix C. This could potentially lead to biased results since the banking sector has unique characteristics and SME:s and emerging markets become structurally underrepresented (Charamichael & Rapp, 2022). Charamichael and Rapp (2022) find a significant greenium when running their model solely on bonds issued by banks. However, they do not find any significant results for the other sectors. The authors provide some possible explanations, such as banks receiving higher payments for the additional work they need to put into granting green loans in terms of research and monitoring. Another possible explanation mentioned by the authors is the probable borrowing cost that arises from green loans that the lender may inherit. Additionally, banks are required to disclose sustainability reports to a greater degree, which may lead to an increased trust that investors could pay a premium for.

Most of the bonds in our dataset originate from France and Germany (and Switzerland for the PCA data), as shown in Figure 8 in Appendix C. This concentration of bonds is not expected to have any major effects on the results since we study the eurozone where countries share similar characteristics, currency and ease of trade. This distribution of countries is present since most of the green bonds are issued in those countries.

3.2.4 Matching of the Bonds

Matching is a widely used approach in financial studies to mitigate the impact of unobservable characteristics that may introduce bias into the estimation process. Therefore, we adopt this method in our study. Additionally, we include a robustness test of the matching to determine whether the approach produces different results when restricting the matching conditions, similar to Zerbib (2019).

To conduct PCA and the yield curve analysis, we first create matched pairs. We use manual matching with replacement, meaning that multiple conventional bonds can be matched to multiple green bonds to improve the matching quality (Smith & Todd, 2005). Green bonds are matched as closely as possible to traditional bonds with similar characteristics, such as issuer, maturity and issue size. The difference in issue date/maturity date is around 1-2 years, and the maximum difference in issue size is by a factor of 4, similar to Zerbib (2019). Zerbib (2019) also performs robustness tests on these thresholds and finds that differences arising from a more stringent policy are negligible. Therefore, imposing more conservative thresholds to restrict the sample further may negatively affect estimation quality. When there are no matching bonds from the same issuer, the matching criteria move to the same sector.

Table 3: Example of an Exact Match

Issuer	ISIN	Issue Date	Maturity Date	Issue Amount	Green	Covered
NLG	DE000DHY5256	2021-01-28	2026-01-28	10000000	Yes	No
NLG	DE000NLB3PN2	2021-01-27	2026-01-27	10000000	No	No

An example of a matched pair used in the sample. Matched as closely as possible based on issuer, issue date, maturity date, issue amount (expressed in EUR), and if covered or not. NLG is short for Norddeutsche Landesbank Girozentrale.

Similar to Crabbe and Turner (1995), we allow one green bond to be matched with multiple conventional bonds when there are no exact matches (see an example of an exact match in Table 3). This matching procedure is vital to the integrity of our results and is commonly used in earlier literature. Gianfrate and Peri (2019), Larcker and Watts (2020), and others use a similar matching procedure in their studies. This procedure allows us to better isolate the effect of a bond being green or not by helping to control for unobservable factors that affect bond performance. The bonds used in multiple matches are only included once in the final sample to avoid any issues related to the same observation appearing numerous times.

When using a matching approach, the sample inherits the characteristics of green bonds. This is described by Charamichael and Rapp (2022) to bias the sample to consist mostly of green issuers, which is not desirable because conventional bond issuers and green bond issuers differ greatly (Flammer, 2021), giving rise to the possibility of the green halo effect. However, this problem may only have a negligible effect on our results since Caramichael and Rapp (2022) find the results of a greenium to be significant regardless of the green halo effect. Larcker and Watts (2020) further emphasize the irrelevance of similar problems by providing insignificant results for greenwashing affecting their results.

3.3 Yield Curve Analysis

For the yield curve analysis, we use the parsimonious Nelson-Siegel model (Nelson & Siegel, 1987) to fit one curve for green bonds and another for conventional bonds. NS is often used to estimate yield curves, in order to describe the term structure of interest rates. As such, it is frequently used by market practitioners and central banks (Coroneo et al, 2011). This is also the model of choice for Partridge and Medda (2019) in their yield curve analysis.

NS assumes that yield curves can be described by three factors with different factor loadings. The factors include level (β_1), slope (β_2) and a curvature/hump factor (β_3), as interpreted by Diebold and Li (2006). This represents a flexible and interpretable yield curve formulation where these factors all

represent different aspects of the yield curve. In 1994, Svensson added a fourth term to the Nelson-Siegel in the form of a second hump/curvature factor, arguing that this formulation provides a better fit (Svensson, 1994). The addition of the extra term gives the Nelson-Siegel-Svensson (NSS) model. The in-sample fit using the NSS performs similar to NS and out-of-sample performance is not guaranteed to be improved. In fact, previous studies indicate that out-of-sample forecasting is better with NS (Diebold & Li, 2006).

The relationship between the factors and the yield curve is described by the factor loadings, and this relationship is used to estimate the factors and describe the dynamics of the yield curve. The parameters of the Nelson-Siegel model are estimated and then used to build the yield curve (Diebold & Li, 2006) through the NS functional form. Different estimation techniques, such as least squares and maximum likelihood, can be used to perform the estimation. Diebold & Li (2006) implement least squares in their estimation as it is commonly used due to its simplicity and reliability, consequently, least squares are used in this study as well. The parameters β_1 and λ need to be non-negative, and $\theta = [\beta_1, \beta_2, \beta_3, \lambda]$ (Svensson, 1994). Diebold and Li (2006) further describe λ as representing the decay rate, where a larger λ provides a better fit for shorter maturities and smaller values offer a better fit for longer maturities.

Yield curves provide interest rates for zero-coupon bonds, so it is necessary to obtain the implied zero-coupon discount factors (DFs) from our data, to then get the rates needed for the model. To achieve this, we use the bootstrapping approach, which establishes the term structure of DFs based on the available market data. Discount factors, in this context, refer to the zero-coupon DFs used for no-arbitrage pricing. This procedure is typically used to recover a yield curve inferred from market prices of zero-coupon government bonds or interest rate swaps. However, it can also be performed using coupon bonds by exploiting the no-arbitrage pricing relationship (Tuckman & Serrat, 2011). The bootstrap method allows us to obtain zero-coupon bond prices, i.e., discount factors, with the following recursive approach:

The first DF for period t to T_1 is obtained in the following way

$$P(t, T_1) = \frac{B_1}{F + c_1 \cdot \alpha_{0,1}} \quad (2)$$

And then for $i > 1$

$$P(t, T_n) = \frac{B_n - c_n \cdot A_{n-1}}{F + c_n \cdot \alpha_{n-1,n}} \quad (3)$$

$$A_1 = P(t, T_1)\alpha_{0,1} \quad (4)$$

Where the annuities (A) are obtained through the recursion:

$$A_n = \sum_{i=1}^n P(t, T_i)\alpha_{i-1,1} = A_{n-1} + P(t, T_n)\alpha_{n-1,n} \quad (5)$$

The equations are derived following Tuckman and Serrat's (2011) approach. The gross bond price is denoted by B , F represents the principal, and the coupon payment is denoted by c in the above equations, where time is expressed in years. The coupon day-count fraction is represented by α in the equations and, as previously described, we use the common day-count convention of actual/360 (James & Webber, 2000 & Tuckman & Serrat, 2011).

Our sample do not contain any green bonds with a one-year tenor, instead, a German one-year zero-coupon treasury bond is used to obtain the first discount factor, $P(t, T_n)$, in the above formulation. German bonds are known for their low credit risk and are considered close to default-free. As a result, they can be used to approximate the European risk-free rate, according to Damodaran (2008). Linear interpolation of bonds is used to create synthetic bonds for which there are no observations in our sample, similar to how Zerbib (2019) creates synthetic bonds. This is done in order for us to be able to sequentially bootstrap the DFs by following the approach specified by equations (2)-(5).

The bootstrapped DFs constitute the prices of the zero-coupon bonds that share the same maturities as the respective instruments they were bootstrapped from, representing the time value of money (Tuckman & Serrat, 2011). This relationship is used to reprice the bonds in our sample to ensure the accuracy of the bootstrapping procedure. Converting from zero-coupon bond prices to their interest rates (yields) depends on the compounding convention used, with common conventions including; simple, periodic, annual, continuous, and discount (Tuckman & Serrat, 2011). We convert from the DFs to interest rates using the simple compounding convention, which is the most frequently used convention in money markets (Tuckman & Serrat, 2011). This allows us to obtain spot rates from the DFs as follows:

$$Y(t, T) = \frac{1}{T-t} \frac{1 - P(t, T)}{P(t, T)} \quad (6)$$

Another common way of obtaining the implied zero-coupon rates is by directly applying the formulation for the continuously compounded (CC) rate and bootstrapping in a sequential way (Hull, 2022), similar to the sequence described by equations (3) - (6) above. The CC rates will be used to test the robustness

of our results in the yield curve analysis. The first CC rate is obtained through its definition, using the German zero-coupon treasury bond:

$$R(t, T) = -\frac{\ln P(t, T)}{T - t} \quad (7)$$

The following CC rates are obtained as the rate that equals the price of the bond with the principal, where all the coupons are annual. The coupons are then stripped off the bonds sequentially using the CC rate obtained from the respective previous periods to find the rates for the bonds with tenors longer than one year. The no-arbitrage relationship between the bond price and the coupons and principal is described by:

$$B = \sum_{k=1}^{T-1} c_k e^{-R(t, T-k)} + (F + c_T) e^{-R(t, T)}. \quad (8)$$

Which gives the following expression for the bootstrapped CC rate:

$$R(t, T) = \frac{\ln \left(B - \sum_{k=1}^{T-1} c_k e^{-R(t, T-k)} \right)}{F + c_T} \left(-\frac{1}{T - t} \right) \quad (9)$$

Here, all rates except $R(t, T)$ are known, given that this is a recursive process starting from the first rate. The same linear interpolation of bonds as in the previous approach is used when observations are missing. $T - t$ measures the maturity of the bond where $T \in [2, 27]$ for the set of green bonds and $T \in [2, 30]$ for the set of conventional bonds. This recursion allows us to find the term structure of treasury zero rates implied from the bonds in our sample using the continuous compounding convention (Hull, 2022).

The bootstrapped rates are used as inputs for the yield curve analysis. To fit the Nelson-Siegel model to these rates, we minimize the squared errors between the yields estimated by the NS and the bootstrapped rates. It is common to fix the value of λ in the estimation, as this dates back to Nelson and Siegel (1987). The advantage of this is that the model becomes more easily tractable, and Diebold and Li (2006) argue that the trustworthiness increases. λ is the term that determines the location of the hump, as it decides at which maturity the curvature factor (β_3) reaches its maximum (Diebold & Li, 2006). However, we do not fix the parameter λ in our estimation, as this results in a suboptimal fit of the model, which is not desirable given our research question. We describe the procedure for fitting the two NS curves using the following equations, where the first equation represents the NS functional form (Diebold & Li, 2006) and the second equation represents the minimization of the squared errors used to estimate the vector of parameters, θ :

$$\hat{y}(t) = \beta_1 + \beta_2 \frac{1 - e^{-\lambda t}}{\lambda t} + \beta_3 \left(\frac{1 - e^{-\lambda t}}{\lambda t} - e^{-\lambda t} \right) \quad (10)$$

$$\min_{\theta} \sum_{t=1}^T (\hat{y}(t) - y(t))^2 \quad (11)$$

We compare the two NS curves and their respective predicted yields to investigate differences in their dynamics, with a focus on whether a greenium is present in the yield curve data and how it behaves for different maturities. We perform the analysis by testing for differences in the factors (level, slope, and curvature) using both statistical tests and visual inspection of the term structures. To quantify the yield difference between the two curves, we use an OLS regression with robust standard errors on the predicted yields.² We use a t-test for testing the significance of the differences in the factors as well as a more rigorous approach utilizing a combined least squares between the two NS curves. For this approach a weighting matrix is imposed in order to weigh observations based on their errors, counteracting any heteroscedasticity which may be present. Three additional terms are added in the least squares estimation, in the form of interaction terms, to test for the differences in level, slope and curvature allowing us to incorporate these tests directly into the existing model framework.

3.4 Principal Component Analysis

The other method we employ to investigate the greenium is principal component analysis, and for this approach, we use the entire dataset of bonds. PCA is a multivariate statistical technique that is typically used to identify trends and patterns in data, similar to traditional regression analysis. PCA can handle large sets of control variables and regressors as it reduces the dimensionality of the multivariate data. This makes it suitable for our study as the data consists of a large set of variables, and in these cases, traditional OLS tends to overfit by estimating unreasonably large coefficients. Having multiple similar variables also calls for concern when it comes to multicollinearity. However, PCA is designed to deal with multicollinearity through the principal components and how they are constructed. PCA combines the variables into orthogonal principal components (PC) through linear combinations. The linear combinations are chosen to maximize the variance orthogonally for the first two PCs. Thereafter, the selection is performed so that they are uncorrelated with the previous components while maximizing the variance (Stock & Watson, 2020).

²See Appendix D for the mathematical formulation.

PCA is often used for prediction and, as such, a common way of determining the number of principal components is by minimizing the mean squared prediction error (MSPE) (Stock & Watson, 2020). However, the components and their relationships can be examined using other metrics than the MSPE. Plotting the variance explained by each component, or equivalently, the eigenvalues of each principal component, results in a scree plot that can be useful for determining how many PCs to retain. PCs with eigenvalues above one indicate that the principal component explains more variance than the average variance across all variables (Kaiser, 1960). This is a common rule of thumb for how many PCs to retain, formally known as the Kaiser-Guttman rule or the eigenvalue-one criterion. Therefore, we use the eigenvalue-one criterion as the threshold for how many PCs we retain in our analysis. The cumulative variance explained is also plotted to determine the performance of the PCA at the given threshold.

All variables need to be standardized to the same scale in order to combine them into principal components. Therefore, the first step in PCA is to standardize all variables by subtracting their mean and dividing by their standard deviation, resulting in z-scores:

$$z = \frac{x - \mu}{\sigma} \quad (12)$$

The linear combinations used to form the principal components are formulated using matrix algebra, where the weights for the linear transformations are calculated based on the covariance matrices for the variables included in the PCA. The squared sum of these loadings needs to sum to one in order for all the variance in the original data to be accounted for. Another restriction for the PCA is that the PCs cannot be correlated with the previous PCs. This gives a constrained maximization problem that solves for the weights, \mathbf{W}_j (Stock & Watson, 2020).

$$\max_{\mathbf{W}_j} \mathbf{PC}'_j \mathbf{PC}_j = \mathbf{W}'_j \mathbf{X}' \mathbf{X} \mathbf{W}_j \quad \text{subject to} \quad \mathbf{W}'_j \mathbf{W}_j = 1 \quad \text{and} \quad \mathbf{PC}'_j \mathbf{PC}_i = 0 \quad \text{for} \quad i < j \quad (13)$$

For PC_1 this constrained maximization is only subject to the first constraint and it is solved by taking the derivative with respect to \mathbf{W}_1 and setting it to zero, i.e. maximizing the Lagrangian:

$$\mathbf{W}'_1 \mathbf{X}' \mathbf{X} \mathbf{W}_1 - \lambda_1 (\mathbf{W}'_1 \mathbf{W}_1 - 1) \quad (14)$$

This gives:

$$\mathbf{X}'\mathbf{X}\mathbf{W}_1 = \lambda_1 \mathbf{W}_1 \quad (15)$$

where \mathbf{W}_1 is an eigenvector of $\mathbf{X}'\mathbf{X}$ representing the loadings for PC_1 and λ_1 is the eigenvalue. For the second PC, there are two constraints:

$$\mathbf{W}_2'\mathbf{W}_2 = 1 \quad \text{and} \quad \mathbf{PC}_2'\mathbf{PC}_1 = \mathbf{W}_2'\mathbf{X}'\mathbf{X}\mathbf{W}_1 = 0 \quad (16)$$

So the lagrangian becomes:

$$\mathbf{W}_2'\mathbf{X}'\mathbf{X}\mathbf{W}_2 - \lambda_2 (\mathbf{W}_2'\mathbf{W}_2 - 1) - \gamma_{21} \mathbf{W}_2'\mathbf{X}'\mathbf{X}\mathbf{W}_1 \quad (17)$$

Solving the above Lagrangian by taking the derivative with respect to \mathbf{W}_2 and equaling it to zero yields the eigenvector and eigenvalue for the second PC (Stock & Watson, 2020). The aforementioned steps constitute the recursion we use to obtain the remaining weights and eigenvalues for the dataset. This process is performed using functions in the machine learning library "scikit-learn" for Python (Pedregosa et al., 2011). The loadings from the eigenvectors are used to construct the PCs for the dataset, and the variance explained by each principal component is calculated from the eigenvalues:

$$\text{Variance explained by the } i\text{-th PC} = \frac{\lambda_i}{\sum \lambda_j} \quad (18)$$

The PCA is performed separately for green bonds and conventional bonds, essentially splitting the dataset into two smaller subsamples with 281 observations each. The above process is used to determine the PCs for each subsample. PC_1 and PC_2 are compared between the subsamples by plotting the PCs, as this is a way to assess the components and their relationships (Abdi & Williams, 2010). This is done because PC_1 and PC_2 explain the most variance by design (Stock & Watson, 2020). The comparison is based on the weights assigned to each variable for PC_1 and PC_2 in the two subsamples, as well as the corresponding values of these PCs, to analyze the underlying driving factors between the two groups of bonds.

The PCs obtained from the PCA can be incorporated into a model estimated by conventional methods, such as OLS, in the form of PC regressions (Stock & Watson, 2020). Consequently, the PCs for each subsample are used in two separate PC regressions, with the respective yields as the dependent variable

and robust standard errors.³ The PC regressions allow us to link the PCA with the yields and thus study the yield difference conditional on the PCA. The essential part of these PC regressions is the estimated coefficients and whether they demonstrate significant differences between the yields of the two groups. However, this approach lacks the tools to quantify the difference in yields. As such, one last econometric technique is needed to complete the analysis.

By treating our dataset as a time series, where each observation represents a specific time period, we can perform a Vector Autoregression (VAR) on the entire concatenated dataset of estimated PCs and observed yields (562 observations). This technique allows us to analyze the dynamic relationship between the PCs and the yields in our dataset. By examining this relationship, we can see how the PCs affect the observed yields and whether there is a difference between the green bond and conventional bond yields, as well as the magnitude of this difference, conditional on the PCs obtained from the PCA.

VAR assumes stationarity, and to check for this condition, we use the Augmented Dickey-Fuller (ADF) test with the null hypothesis of unit roots. In other words, we want to reject the ADF test to conclude stationarity (Stock & Watson, 2020). VAR is fitted to the dataset using the Akaike Information Criterion (AIC) to determine the order of the VAR, and the maximum number of lags is set at 10. AIC is commonly used in practice as it is the preferred method (Lütkepohl 2007, Tsay 2013). After confirming that the necessary assumptions are satisfied, such as stationarity, VAR can be estimated. We estimate the model parameters using maximum likelihood estimation, which is the usual method used in the VAR context (Stock & Watson, 2020), and this method is used by Ang and Piazzesi (2003), who also study bonds. The VAR fitted to the yields and the PCs is formulated as follows:

$$Z_t = \mu_t + \sum_{j=1}^k \sum_{i=1}^p A_{ij} \cdot Z_{j,t-i} + \epsilon_t \quad (19)$$

$Z_t = (z_{1t}, \dots, z_{kt})' \in \mathbb{R}^k$ is a k -dimensional vector, where k is the number of variables in the dataset (number of PCs + 1) and μ is the intercept. A represents the matrix of coefficients for variables and p is the number of lags. The residuals are checked for heteroscedasticity through visual inspection and using the ARCH-LM test, where a large p-value indicates that homoscedasticity cannot be rejected.

We calculate the impulse response functions (IRFs) of the VAR separately for the set of green bonds (G) and for the set of conventional bonds (C), where the yields are used as the only response variable. The IRFs describe the dynamic response of one variable (yields) to shocks in another (the PCs) (Kirchgässner

³See Appendix D for the mathematical formulation.

& Wolters, 2007), i.e., how the yields change when the PCs change. The IRFs are calculated over five periods, where a shock of one standard deviation is applied to each PC, observing the effect on the yields. The individual IRFs, which show how the yields react in each case, are added up to achieve the cumulative IRFs, following the equations below.

$$\text{Cumulative IRF}(G, t) = \sum_{j=1}^m \sum_{i=1}^h \gamma(G_{ji}) \cdot \varepsilon(G_{j,t-i}) \quad (20)$$

$$\text{Cumulative IRF}(C, t) = \sum_{j=1}^k \sum_{i=1}^h \gamma(C_{ji}) \cdot \varepsilon(C_{j,t-i}) \quad (21)$$

The value of m represents the number of PCs in the green bond set, k is the number of PCs in the conventional bonds set and h is the number of periods. $\gamma(G_{ji})$ denotes the IRF of the j -th green bond PC following a shock of one standard deviation to the PC, and ε represents the corresponding innovation term. The VAR estimation and IRF calculations are performed using the "statsmodels" library in Python (Seabold & Perktold, 2010). Finally, to quantify the difference in the respective yields' reactions to changes in the PCs, we calculate the average difference between the cumulative IRFs for the green bonds and the conventional bonds, resulting in a single number defined as the greenium conditional on the PCs. The statistical significance of this greenium is determined through a t-test.

4 Empirical Results and Analysis

In this section, we present and discuss the results of the two chosen methods. Paragraph 4.1 presents the results and implications of the yield curve analysis, while paragraph 4.2 discusses and displays the output from the principal component analysis. Each method is followed by robustness tests, which serve to strengthen the reliability of the findings and provide a more comprehensive assessment of them.

4.1 Yield Curve Analysis Results

To perform the yield curve analysis, we first need to establish the DFs through bootstrapping, in order to calculate the zero-coupon rates for the Nelson-Siegel model. Our bootstrapped DFs and associated rates can be found in Appendix D. The green bond rates range from -0.42% to 4.34%, with an average rate of 2.00%, and the conventional bond rates range from 0.18% to 4.84%, with an average rate of 2.05%. As shown in Tables 19 and 20 in the Appendix, repricing the bonds using the bootstrapped DFs and rates provides exactly the same prices as the original, indicating that the bootstrapping was successful.

The bootstrapped DFs show an overall decreasing trend for both green and conventional bonds, although not monotonic in any of the cases. The same, but increasing trend can be observed for the associated rates. A notable deviation is one green DF bootstrapped from a 16-year French government bond, which is above one (1.07), resulting in a negative rate for this tenor (-0.42%).

We use the bootstrapped rates to perform the yield curve analysis by fitting two NS curves. From these curves, we obtain the predicted yields, which we use as the dependent variable in an OLS regression to get a crude estimate of the greenium. This approach provides a significant result, where the OLS regression estimates a greenium of 21.51 bps, as shown in Table 4. The OLS regression is performed with the group label (equal to 1 for green bonds, 0 for conventional bonds) as the key independent variable, where tenor is included as a control variable to account for the structure of the yield curve.

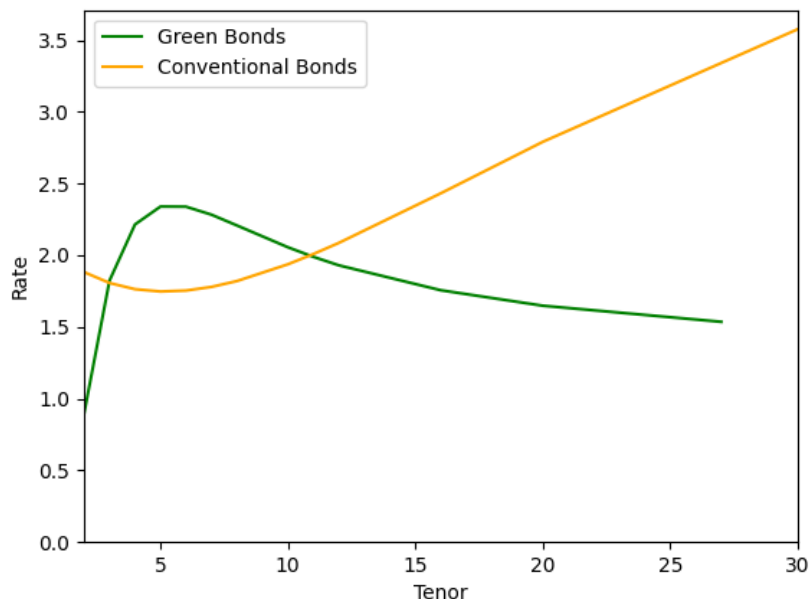
Table 4: OLS Regression Results for Measuring Greenium from NS Curves

	Coefficient	Std. Error	z	P> z	[0.025	0.975]
Const	1.8762***	0.126	14.856	0.000	1.629	2.124
Tenor	0.0221*	0.012	1.905	0.057	-0.001	0.045
group_label	-0.2151**	0.102	-2.117	0.034	-0.414	-0.016
R-squared	0.211					
Adjusted R-squared	0.166					
Covariance type	HC0					
Skew	-0.029					
Kurtosis	2.647					

Output from the OLS regression where predicted yields are the dependent variable. The regression is specified by equation (22). Const refers to the intercept and group_label is the key independent variable and it is a dummy variable that takes the value of 1 for green bonds and 0 for conventional bonds. The yields are limited to only contain the tenors 2-20 in order to avoid spurious results in the regression given the lack of original data observations for tenors outside of these bounds. Standard Errors are heteroscedasticity robust (*HC0*) and * = $P < 0.1$, ** = $P < 0.05$, *** = $P < 0.01$

The NS results match the findings of Gianfrate and Peri (2019), the only study to our knowledge that includes solely European bonds. However, the results conflict with the outputs from Carmichael and Rapp (2022), who observed a greenium of 14 bps for 2019, followed by a decreasing trend with 8 bps for 2020 and 2021. In contrast, our results of 21 bps for 2022 do not follow this trend and show a rather dramatic increase in relation to their results. Compared to other papers, our results provide a slightly higher greenium, which could be explained by the large number of bonds issued by banks in our dataset (as discussed in 3.2.3 *Descriptive Statistics*), our choice of methodology, and our inclusion of more recent data. Another argument supporting the larger greenium observed for our sample of bonds from 2022 is the fact that it directly follows the Covid-19 pandemic. The thoughts expressed by Liberati and Marinelli (2021) regarding the change in the common perception of sustainability may affect our results, resulting in a rather large greenium estimate for the year 2022.

Figure 1: Term Structures of Green and Conventional Bonds



The figure visualizes the term structures of green bonds and conventional bonds. The curves are fitted to the bootstrapped rates according to equations (10) and (11). The green bond curve ends at the 27-year tenor while the curve for the conventional bond ends at the 30-year tenor due to the last matched pair in the sample being roughly 3 years apart in maturities.

The graphical output of the NS curves (Figure 1) shows a difference between green and conventional bond yields. The conventional bonds exhibit what is usually referred to as a normal term structure, where long maturities have larger uncertainty reflected by an upward-sloping curve. The term structure for the green bonds is, on the contrary, downward-sloping, indicating lower rates for longer maturities, i.e. an inverted yield curve. The complete opposite term structures can occur due to multiple reasons. The risk profiles of the two sets of bonds may be completely different, causing the yields to behave differently. Green bonds could be seen as safer in the long term, whereas the risks of climate change (such as carbon taxes, etc.) increase with time for conventional bonds (Löffler, Petreski, & Stephan, 2021). Investors could regard non-green investments as required short-term for companies to manage current, important, and non-green necessary investments. Though, in the long run, they expect companies to fully plan on performing green investments only. Investors might believe that companies have enough time to perform green projects compared to today when time is more scarce. Additionally, it could be expected that non-green investments will be subject to additional costs imposed by regulators or other market practitioners leading to a higher risk of the issuing company not paying back its debt. With this, lenders require a higher return for those longer-term non-green bonds.

Table 5: Goodness-of-Fit Metrics

	RSS	TSS	R-squared	RMSE	MAE
Green	27.1157	29.7015	0.0871	1.1644	0.9821
Conventional	24.9883	29.4225	0.1507	1.1178	0.5593

Various measures of the accuracy for the two Nelson-Siegel curves are presented in this table. RSS = Residual sum of square and TSS = total sum of squares, R-squared is calculated using RSS and TSS for the respective curves using the following definition: $R\text{-squared} = 1 - \text{RSS}/\text{TSS}$. RMSE = Root mean square error, MAE = mean absolute error (Stock & Watson, 2020).

The goodness-of-fit measures for the two NS curves can be seen in Table 5, where the R-squared for the green subsample is 0.09 and the R-squared for the traditional bonds is 0.15. Partridge and Medda (2019), who also use NS, receive a range of R-squares from 0.15 to 0.99. One explanation for the relatively low R-squared could be the limited data availability and small sample size. Due to the model's structure, only data from one year and matched pairs could be included in constructing the yield curves, resulting in a sample of 20 unique matched pairs. The small sample also leads to one abnormal data point for green bonds, which is the rate of -0.42%. Since this rate is negative and the dataset is small, it lowers the green yield curve and affects how the model estimation performs. Another study by Cao, Jin, and Ma (2021) on the Chinese market, utilizing the same model on green bonds, achieve an R-squared of 0.21, which is rather close to our results. As there are few studies using the Nelson-Siegel model to investigate green bonds, with varying R-squares, it is unclear whether the R-squares in our study should be considered low or adequate.

The difference in yield curves is tested by a t-test on the differences in slope, level, and curvature, where level and curvature are statistically significant at 1%, as shown in Table 6 below. A more rigorous test utilizing a combined least squares and a weighting matrix is also performed considering the indications of heteroscedasticity, as seen in Appendix D, Figure 10. The statistical significance of these interaction terms provides a more conservative result, where the difference in the level is significant at 5%, whereas slope and curvature are not significant, as seen in Table 6. The two approaches thus partially reject the null hypothesis, $\mathbf{H}_0\mathbf{b}$: *There is no difference in the Nelson-Siegel factors between green bonds and their counterparts*, since some but not all differences in the NS factors are statistically significant.

Table 6: Statistical Tests on the Differences in the NS Factors

	T-test	Interaction Terms
Level	0.0078 ***	0.0117 **
Slope	0.2214	0.4810
Curvature	0.0000 ***	0.7517

The table provides the p-values for the factors from the Nelson-Siegel output. Level refers to β_1 , slope to β_2 and Curvature to β_3 in the NS specification described by equation (2) in section 3.3 *Yield Curve Analysis*. * = $P < 0.1$, ** = $P < 0.05$, *** = $P < 0.01$. Statistical significance of the interaction terms refers to the p-values associated with the added coefficients for the more robust test of the differences.

The difference in level, which can be interpreted as the average yield (Gilli, Grosse, & Schumann, 2010), implies the existence of a greenium, strengthened by the observation that the two curves start from different points. The slope could be understood as the difference in yields between bonds with longer and shorter maturities (Gilli, Grosse, & Schumann, 2010), but the result of the tests implies that there is no significant difference in slopes. The curvature in the less conservative test is significant and by examining the graph and the estimated coefficients, it is possible to see that the curvature of green bond yields is positive, while the curvature of conventional bond yields is negative. This can have implications for the entire term structure. Even though the slope difference is insignificant, it is possible to draw conclusions based on the shape of our estimated term structure, as seen in Figure 1. The opposite signs for the curvatures of the two term structures provide further evidence for the inverted yield curve of the green bonds. The negative curvature of the conventional bonds causes their term structure to exhibit the "normal" increasing convex shape. The positive curvature factor of the green bonds implies a concave decreasing shape where the yields for longer maturities are lower, i.e. an inverted yield curve, as seen in Figure 1.

4.1.1 Robustness Result for Yield Curve Analysis

We perform a robustness test on the Nelson-Siegel approach to identify whether our results are robust in the sense that the outcome does not change even if the input to the model changes. One important investigation is to bootstrap the rates using continuous compounding instead of simple compounding, as described in section 3.3 *Yield Curve Analysis*. The bootstrapped rates are similar to the ones obtained with simple compounding, with an average absolute difference of 0.21% for the green subsample and 0.19% for the conventional bonds. Given that bootstrapping is a recursive process, we naturally observe the differences to increase along with the maturities of the bonds. For the green bonds, the largest absolute difference is 0.68% for the 12-year tenor, and for the conventional bonds, the largest absolute difference is for the 20-year tenor at 1.27%, seen in Tables 26 and 27 in Appendix F.

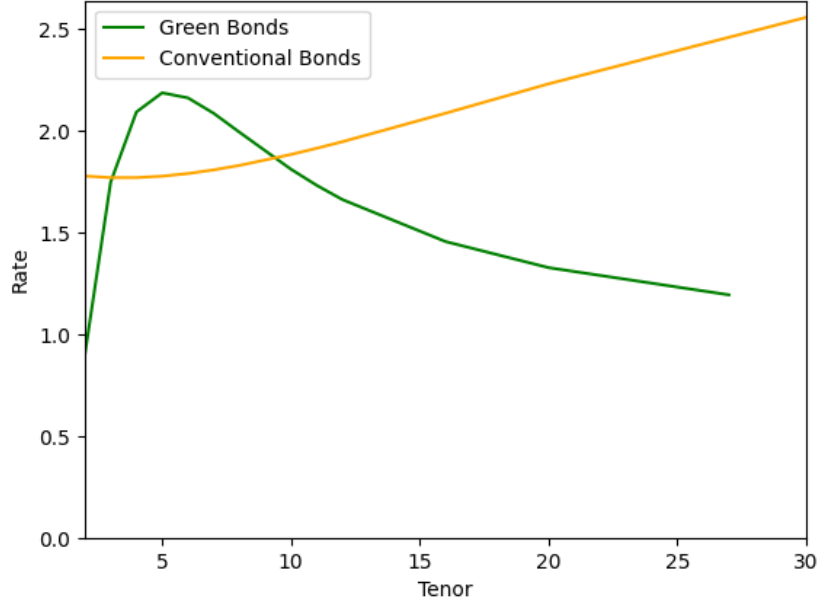
Table 7: OLS Robustness Regression Results for Measuring Greenium from CC NS Curves

	Coefficient	Std. Error	z	P> z	[0.025	0.975]
Const	1.9569***	0.110	17.770	0.000	1.741	2.173
Tenor	-0.0011	0.010	-0.111	0.911	-0.020	0.018
group_label	-0.2682***	0.084	-3.203	0.001	-0.432	-0.104
R-squared	0.213					
Adjusted R-squared	0.168					
Covariance type	HC0					
Skew	-0.250					
Kurtosis	3.680					

Output from the OLS regression where predicted yields from the continuously compounded Nelson-Siegel curves are the dependent variable. The regression is specified by equation (22). Const refers to the intercept and group_label is the key independent variable and it is a dummy variable that takes the value of 1 for green bonds and 0 for conventional bonds. The yields are limited to only contain the tenors 2-20 in order to avoid spurious results in the regression given the lack of original data observations for tenors outside of these bounds. Standard Errors are heteroscedasticity robust (*HC0*) and * = $P < 0.1$, ** = $P < 0.05$, *** = $P < 0.01$

The small differences in the rates lead to slight differences in the outcome when we apply the yield curve analysis with the NS for these new rates. The greenium is now strongly significant with a larger value of 26.82 bps, compared to the previous value of 21.51 bps. The yield curves maintain the expected dynamics with an inverted yield curve for the green bonds and an increasing "normal" yield curve for the conventional bonds, as shown in Figure 2 below.

Figure 2: Term Structures of Green and Conventional Bonds Using CC Rates



The figure visualizes the term structures of green bonds and conventional bonds from the bootstrapped continuously compounded rates. The curves are fitted according to equations (10) and (11). Tenor is on the x-axis and the associated rate in percent is on the y-axis. The green bond curve ends at the 27-year tenor while the curve for the conventional bond ends at the 30-year tenor due to the last matched pair in the sample being roughly 3 years apart in maturities.

Table 8: Statistical Tests on the Differences in the NS Factors for the CC Rates

	T-test	Interaction Terms
Level	0.0624 *	0.0000 ***
Slope	0.0383 **	0.3924
Curvature	0.0000 ***	0.8530

The table provides the p-values for the factors using the bootstrapped continuously compounded rates for the Nelson-Siegel. Level refers to β_1 , slope to β_2 and Curvature to β_3 in the NS specification described by equation (2) in section 3.3 “Yield Curve Analysis”. * = $P < 0.1$, ** = $P < 0.05$, *** = $P < 0.01$. Statistical significance of the interaction terms refers to the p-values associated with the added coefficients for the more robust test of the differences.

Curvature remains strongly significant, as before, while the slope is significant and the level is only weakly significant. When applying the more rigorous test, the results are similar, with no significance for slope and curvature using either compounding convention, but a strong significance is evidenced for the level compared to only a 5% significance level with simple compounding.

Table 9: Goodness of Fit Metrics for the CC Rates

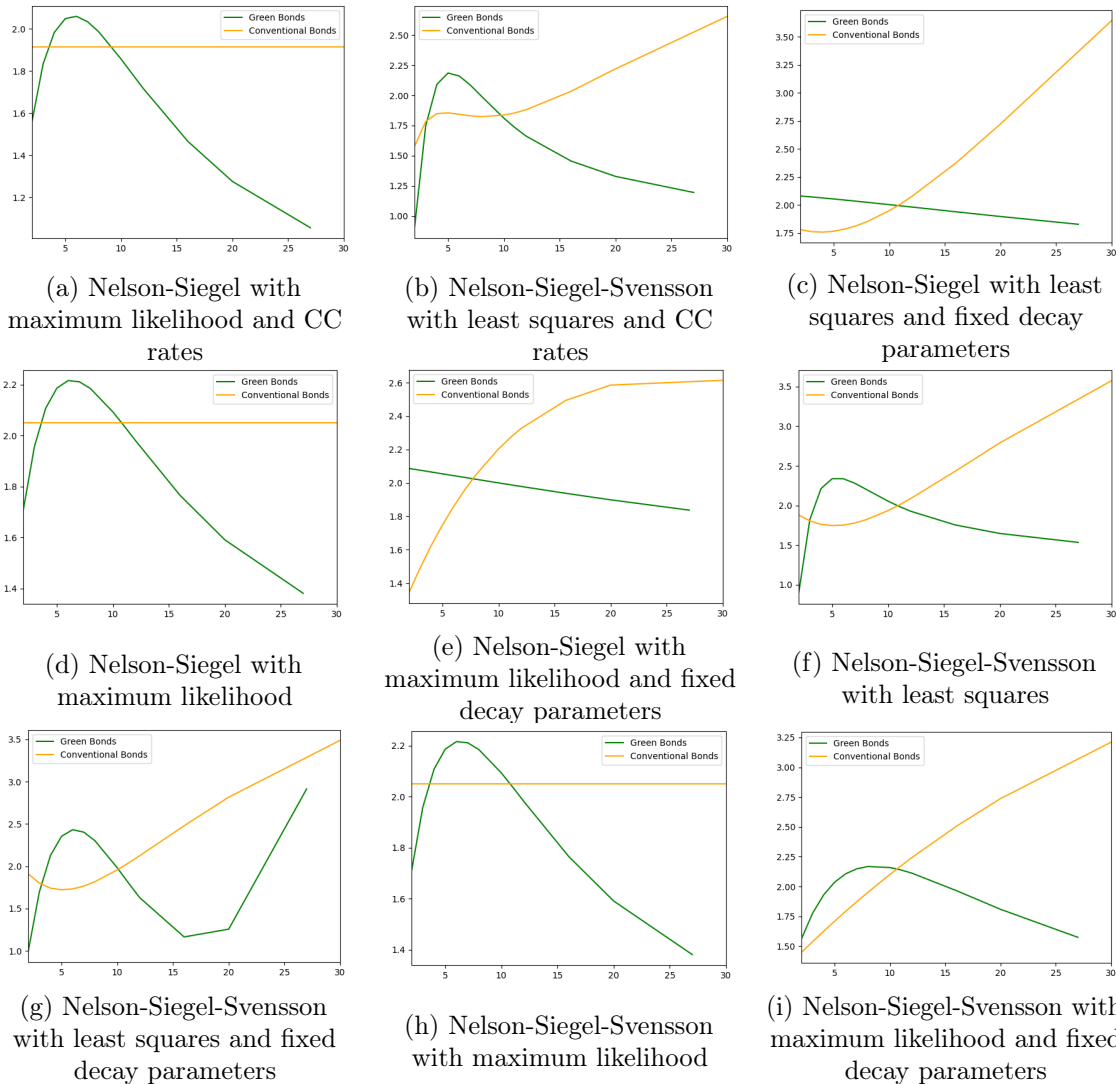
	RSS	TSS	R-squared	RMSE	MAE
Green	27.1157	29.7015	0.1218	0.9830	0.8269
Conventional	24.9883	29.4225	0.0381	1.0158	0.7434

Various measures of the accuracy of the two Nelson-Siegel curves from the bootstrapped continuously compounded rates. RSS = Residual sum of squares and TSS = total sum of squares, R-squared is calculated using RSS and TSS for the respective curves using the following definition: $R\text{-squared} = 1 - \text{RSS}/\text{TSS}$. RMSE = Root mean square error, MAE = mean absolute error (Stock & Watson, 2020).

The overall goodness of fit for the model remains quite low, with an R-squared of 0.12 for the green bonds and 0.04 for the conventional bonds. A notable change is that the model appears to fit the green bonds better when using the CC rates, while the fit for the conventional bonds is worse, as shown in Table 9.

Another important decision in the yield curve analysis is related to the NS model. In this study, we apply the NS model without the Svensson extension, estimating λ together with the other parameters, and a least-squares approach for estimation. These choices might be a topic for discussion, and hence, we test the robustness of our findings with respect to these choices in research design by performing multiple iterations of the yield curve analysis. The outcomes of this process are summarized in the figure below.

Figure 3: Summary of Yield Curves With Different Estimation Techniques



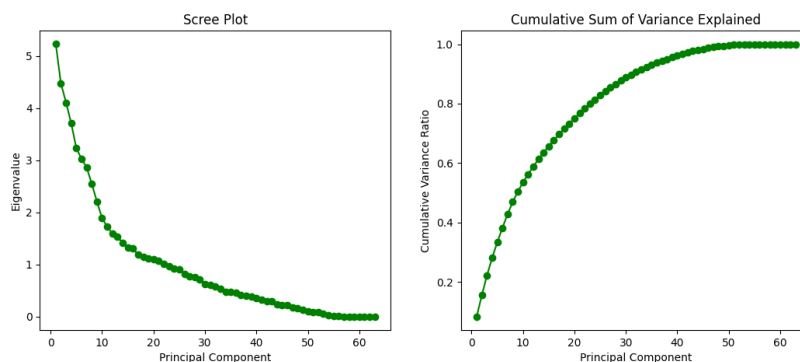
The figure describes nine alternate scenarios for the yield curve analysis by using different estimation techniques as well as different bootstrapped rates (continuously compounded, CC, vs simple compounded). The estimation technique used is described in the associated text for each figure. The simple rates are used for all graphs except for (a) and (b). The decay parameters are λ for NS and λ and μ for NSS. The fixings are performed according to the information provided by Diebold and Li (2006) and Svensson (1994).

For all iterations, the trend is similar with green bond yields being higher than traditional bond yields for shorter maturities and lower for longer maturities. The yield curve observed in our primary results implies an inverted yield curve for the green bonds and this observed dynamic is consequently robust to changes in estimation technique.

4.2 Principal Component Analysis Results

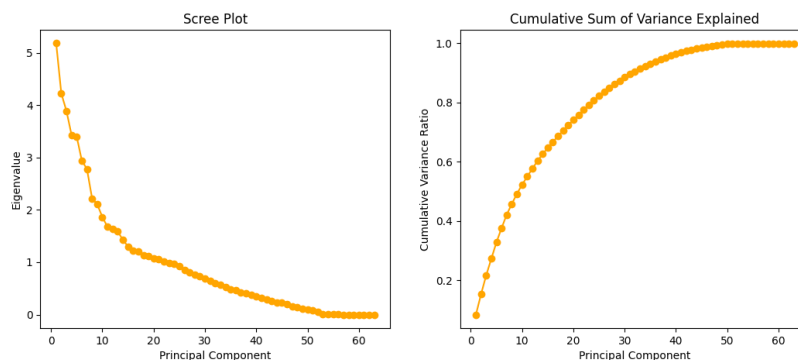
The PCA is performed separately on green bonds and conventional bonds and provides similar results between the two groups. A total of 63 principal components (PCs) are created for both green bonds and traditional bonds. The PCs obtained from the PCA show that all variables included in the dataset are needed to capture the variance for both green bonds and traditional bonds. Given the threshold of one for the eigenvalues we retain 22 PCs for each group and this results in a cumulative variance explained of 0.7838 for green bonds, as illustrated in Figure 4, and 0.7754 for traditional bonds, as illustrated in Figure 5. The fact that the PCA provides very similar results is expected considering the data structure with observations sharing similar traits as this is a consequence of the research design using a matching approach.

Figure 4: Screeplot and Cumulative Variance Explained for Green Bonds



The scree plot, the figure to the left, plots the eigenvalues and the corresponding principal component for green bonds. The values are extracted as the λ :s from the process described by equations (13)-(17) using the "scikit-learn" library in Python (Pedregosa et al., 2011). The threshold of one for the eigenvalues retains 22 PCs. The figure to the right plots the cumulative variance explained calculated using the concept provided by equation (18). At 22 PCs, the variance explained is 0.7838.

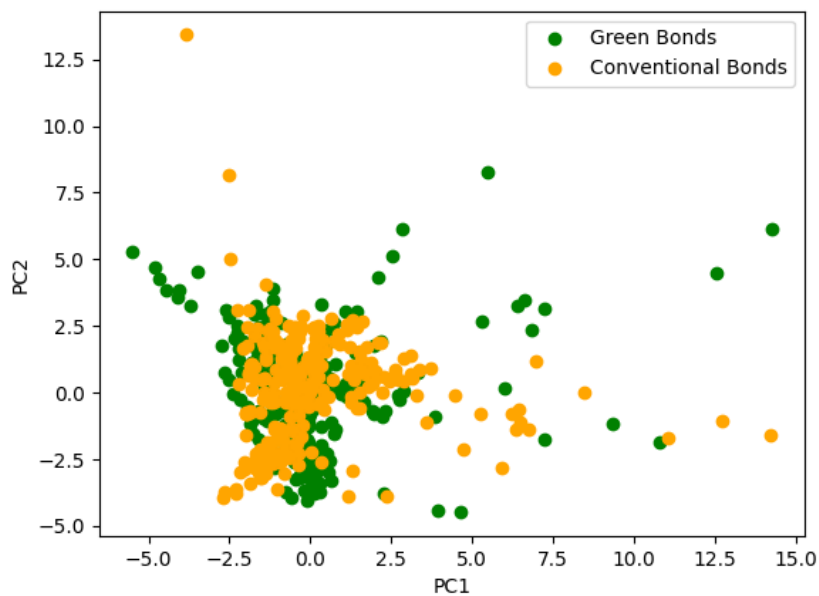
Figure 5: Screeplot and Cumulative Variance Explained for Conventional Bonds



The scree plot, the figure to the left, plots the eigenvalues and the corresponding principal component for conventional bonds. The values are extracted as the λ :s from the process described by equations (13)-(17) using the "scikit-learn" library in Python (Pedregosa et al., 2011). The threshold of one for the eigenvalues retains 22 PCs. The figure to the right plots the cumulative variance explained calculated using the concept provided by equation (18). At 22 PCs, the variance explained is 0.7754.

Examining PC_1 and PC_2 more closely, we can see that the PCs for both subsamples include all variables with different loadings. The loadings of the different variables included in PC_1 and PC_2 for both subsamples can be seen in Figures 11 and 12 in Appendix E. The factor loadings exhibit a tendency towards conformity, where the average absolute difference is low between the bond types, and in some cases, the difference is zero, indicating that some variables receive identical weights.

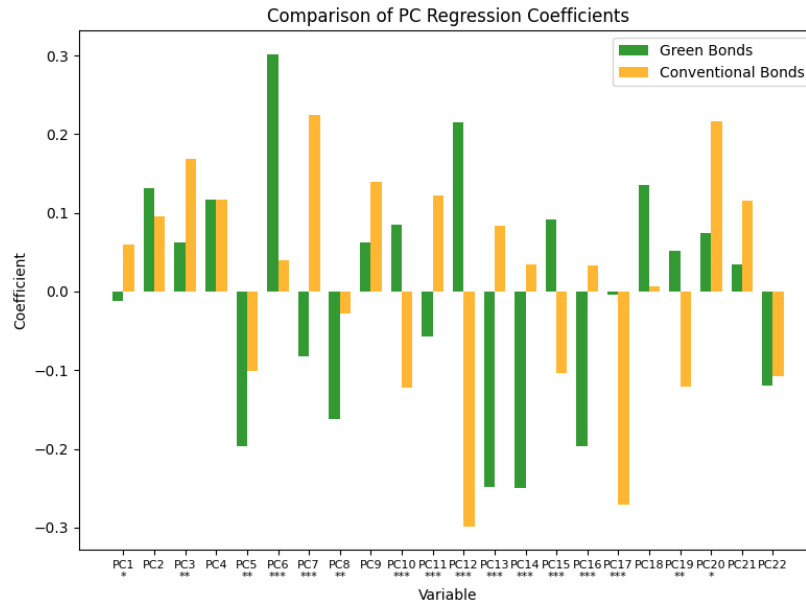
These loadings are used to create the values for PC_1 and PC_2 . Figure 6 shows the result of plotting the values of PC_1 against the values of PC_2 in the dataset, providing a further dimension to the analysis of the two first PCs. The construction of the PCs using the respective loadings results in homogeneous PCs between the two groups, as shown in Figure 6. However, some PCs deviate from this homogeneity, indicating that there are differences in the underlying factors between green bonds and conventional bonds. The maximum difference is above 10 in absolute terms, as seen in Table 25 in Appendix E.

Figure 6: Scatterplot of PC_1 and PC_2 for Both Green Bonds and Conventional Bonds

A plot of the PCs for both green and conventional bonds, with PC_1 on the x-axis and PC_2 on the y-axis. PC_1 and PC_2 are constructed according to equations (13) - (17) using the machine learning library "scikit-learn" for Python (Pedregosa et al., 2011). The weights/loadings used in the creation of PC_1 and PC_2 refer to those seen in Figures 11 and 12.

We perform PC regressions separately for each subsample, using their respective yields as the dependent variable and 22 PCs. The R-squared for the green bond subsample is 0.6576, and the R-squared for the conventional bond subsample is 0.6347. The two subsamples thus perform similarly in the two PC regressions, but some differences between the two sets of bonds emerge. Figure 7 displays a graphical illustration of the size of the estimated coefficients and the statistical significance of the difference in the estimated coefficients between the two groups.

Figure 7: PC Regression Coefficients Comparison



The figure shows a comparison of the coefficients for both green and conventional bonds obtained from the two PC regressions described by equations (23) and (24), where $m = 22$ and $k = 22$. The stars underneath the variable names on the x-axis symbolize whether the difference in coefficients associated with that variable is significantly different between green bonds and conventional bonds. * = $P < 0.1$, ** = $P < 0.05$, *** = $P < 0.01$.

It is evident that more than half of the differences between the estimated coefficients from the two sets of bonds are statistically significant, and these differences range from 0.0005 to 0.5129. In some cases, the estimated sign is also different, with the sign for the first PC being negative for green bonds but positive for conventional bonds with a statistically significant difference. These observed differences imply that the underlying factors driving the yields of green bonds and conventional bonds are different, even though the PCs are homogeneous between the two sets of bonds. The underlying driving factors of the yields, as explained by the PCs, indicate the existence of a difference in yields, i.e. a greenium. However, considering the complex interpretability of PCA, it is difficult to investigate the differences in yields between green bonds and conventional bonds further with this PC regression approach, other than stating that the underlying driving factors seem to differ between them. A complete table of all the estimated coefficients is available in Table 22 in Appendix E.

The principal components from each subsample are combined into a single dataset, allowing us to conduct a VAR to further examine the relationship between the PCs and their corresponding yields for each group. The stationarity of the data is checked using the Augmented Dickey-Fuller (ADF) test, which shows that all variables have p-values below 0.05. The highest p-value is 0.01 for variable 22 (Y), as shown in Table 23 in Appendix E. Thus, we can reject the null hypothesis of unit roots and conclude

that the data is stationary. We conclude that the data does not exhibit heteroscedasticity, as evidenced by a p-value of 0.2428 from the ARCH-LM test, and through a visual inspection of the residuals in Appendix E Figure 13. The VAR can consequently be fitted to the concatenated dataset, and the impulse response functions (IRFs) are calculated using the estimated coefficients from the VAR.

The point of interest in VAR is the impulse response functions, which we use to quantify the difference between green bond yields and conventional bond yields. By examining the average difference between the IRFs for green bonds and conventional bonds, we find evidence of a greenium at 8.28 bps. This is because the response of green bond yields to shocks in the PCs is on average 0.0828 less than the response seen from non-green bonds, and this difference is statistically significant with a p-value of 0.0027. This method of estimating the yield difference between green bonds and conventional bonds is not as direct as those taken by other researchers. However, it enables us to better deal with the dimensionality of the bond data through PCA while still providing an estimate that is consistent with those of other researchers and reasonable when examining the data used in our analysis.

The greenium estimate of 8.28 bps is statistically significant and consequently, leads us to reject the null hypothesis H_{0a} : *There is no difference in yields between green and conventional bonds*, and accept the alternative hypothesis, which implies the existence of a greenium. Our results are consistent with those of Baker et al. (2018) and Carmichael and Rapp (2022), who also find a greenium of 8 bps. However, since our sample comprises a significant number of bank bonds, our results might not be entirely applicable to other bond issuers who may experience a smaller, non-existent, or even a reversed greenium.

The indications of a greenium mean that investors are willing to accept lower yields in order to invest in green bonds instead of their conventional counterparts. As Fama and French (2007) explain, since some investors have a preference for certain assets, otherwise similar assets can provide different returns. Our results suggest that investors do have a preference for green bonds, however, Zerbib (2019) argues that their significant but small greenium is not observed because of investor preferences, but rather due to lower risk. Another possible reason for the greenium could be the tax savings on green bonds. This is mostly observed in US municipal bonds, described by Larcker and Watts (2020) and Baker et al. (2018), though after the announcement from the European Commission regarding green bond tax incentives, governments are developing tax benefits for green bonds in Europe as well (Jakubik, Uguz, 2021).

While green bonds are used for beneficial environmental investments, this study shows that these bonds produce lower returns. There is evidence that the issuance costs of green bonds are higher compared to

conventional bonds (Chiang, 2017), limiting the monetary advantage of issuing green bonds. Green projects may consequently involve larger upfront investments contributing to the larger issuance costs. These investments can potentially reduce the profitability of the issuer in the short term compared to conventional projects. If the issuer’s financial performance is impacted by these lower returns, it could make it challenging for them to offer higher yields on their green bonds, thus providing an additional explanation for the observed lower yields on green bonds. The greenium discovered in this paper provides incentives for issuers to issue green bonds, but it is unclear whether this premium outweighs the larger issuance costs. This makes the financial advantage of green bonds for issuers ambiguous in relation to the greenium. However, they could benefit from issuing green bonds as a means to expand their bondholder base (Zerbib, 2019) by reaching more potential investors (Larcker & Watts, 2020).

4.2.1 Robustness Result for PCA/Matching

The matching procedure is a research design that has been used in the majority of previous research. Although it is described as a vital part of sample construction, there are some inherent issues, mainly concerning the subjective judgments made by the researchers. Gianfrate and Peri (2019) use propensity score matching to combat this issue, while Zerbib (2019) employs stricter matching as a way to measure the robustness of the matching approach. In our study, matching is performed using the guidelines set by previous researchers, as described in section 3.2.4 *Matching of the Bonds*. To test the robustness of the sample, we use a similar methodology to Zerbib (2019) by only including observations with close to identical traits, such as the same issue date, maturity, issuer, and a maximum difference in issue size by a factor of two.

Table 10: Summary of Perfectly Matched Bonds Issued at Discount, Par and Premium

	Discount	Par	Premium
Green bonds	61	41	21
Conventional bonds	70	25	28

This table displays the distribution of bonds issued at discount (<100), at par ($=100$), and at a premium (>100) for the sample containing exact matches.

Table 11: Descriptive Statistics of Perfectly Matched Bonds

	Green Bonds	Conventional Bonds
Amount	123	123
Mean yield	1.0151%	1.0300%
Main yield	-0.4085%	-0.5602%
Max yield	5.0211%	5.9800%
Average mod. duration	5.2507	4.3873
Min mod. duration	0.3849	0.0416
Max mod. duration	30.5759	20.2877
Average issue amount (EUR)	546 508 544	537 450 385
Min issue amount	507 793	2 577 632
Max issue amount	4 999 999 999	8 030 999 999
Average issue price	99.9824	100.0108
Min issue price	95.1944	96.3450
Max issue price	108.6200	117.4940
Average coupon	1.0104	1.0236
Min coupon	0.0100	0.0100
Max coupon	5.7500	5.9800

The above table displays a selection of summary statistics for the dataset for the principal component analysis consisting of only exact matches. Mod duration is short for modified duration to maturity, min is short for minimum value and max is short for maximum value.

The new sample consists of 123 bond pairs, which is 158 fewer than the initial sample used for PCA. The descriptive statistics are similar to those of the original sample, though with some small differences. In general, the average amount for each variable is smaller compared to the descriptive statistics of our original sample. The most important difference to discuss is the average yield for both green and traditional bonds, as perfect matching reduces the mean yield by approximately 22 bps. This might have implications for our results, as the difference in yields between green and traditional bonds is not large enough to produce significant results given the smaller sample size. This decrease in mean yields can be explained by the green halo effect, which causes the overall yields of an issuer to decrease. This effect could be more pronounced in our perfectly matched sample since there are no deviations from each pair, all containing the same issuer. Moreover, this effect could also be more visible when the sample is smaller.

The outputs obtained from performing PCA on this new sample are visible in Appendix F. The PCA includes fewer principal components (19), using the same threshold with eigenvalues above one while explaining a larger variance (0.83) for both subsamples. This is expected considering the increased homogeneity in the sample resulting from stricter matching. The sign for the average difference in

the IRFs is negative, as in our original sample, indicating the presence of a greenium. However, the estimated greenium of 0.8 bps is not statistically significant. A possible reason for the insignificant results could be the smaller sample size coupled with the green halo effect. Tighter matching can result in lower quality results (Zerbib, 2019), which could explain the insignificant results of our robustness check.

5 Conclusion

The European Commission (2018) has developed an action plan to shift the trend of climate change towards a more sustainable future, which has resulted in the expansion of the green bond market from 2019 (Caramichael & Rapp, 2022) to a value of USD 4 trillion in 2021 (Bloomberg, January 2022). There are multiple studies examining how market forces affect the yields on green bonds and, in some cases, the increasing demand drives the yields of the green bonds below the yields of their traditional counterparts. In this paper, we contribute to the literature on sustainable finance by investigating whether there is a negative premium (i.e., greenium) on European green bonds in the primary market.

We utilize two different methods, a yield curve analysis and a principal component analysis, and both provide evidence for the existence of a greenium. The yield curve analysis applied to the year 2022 estimates a significant greenium of 21.51 bps, whereas the PCA, which includes bonds between 2013 to February 2023, provides results of a strongly significant greenium of 8.28 bps. Several factors can contribute to the observed greenium, such as the distinct risk profiles of green bonds, investor preferences, regulatory incentives, tax benefits, and the potential lower returns on green projects, which subsequently lead issuers to offer lower yields on green bonds. Partridge and Medda (2019) argue that the existence of a greenium in the primary market means that cheaper funding for green investments can be secured. This could, in turn, motivate more entities to issue green bonds and thus fund green projects, although this has not yet been proven. Due to the limitations of this study, the greenium is mainly present for European banks, hindering the generalization of our results to a broader global market. However, as other markets evolve in the same sustainable direction as Europe, results similar to this study could be expected for the global market as well.

The estimated yield curves display a greenium for longer-term bonds, while green bonds with shorter maturities have higher yields. This means that issuers benefit from long-term green bonds, as the advantage of green bonds increases with the uncertainty associated with the passage of time, indicating two separate risk profiles for green bonds and conventional bonds. The advantage this provides for issuers is not as clear since it does not take into account the additional costs associated with issuing green bonds (Chiang, 2017). These benefits are thus a bit more vague for issuers, who instead view green bonds as a signaling strategy (Caramichael & Rapp, 2022). There are also concerns for investors as a greenium could increase the risks of greenwashing, where issuers take advantage of the intangible nature of the green label and exploit the system to acquire cheaper funding by deceiving investors. This is an important area for policymakers to focus on in the future, as complete eradication of greenwashing would benefit the reliability of sustainable finance and provide further incentives to invest in green

bonds (Caramichael & Rapp, 2022).

The common problem of small sample bias remains in this study. Therefore, we suggest that future studies should aim to enlarge the dataset, preferably when more green bonds are issued, which the current bond issuance trend suggests will happen. Additionally, a larger sample would minimize the effect of the green halo on the results (Partridge & Medda, 2019), further emphasizing the need for this study to be conducted again in the future. Moreover, the relevance of this topic has grown over the years, increasing the importance to continue studying the differences in yields over time. Additionally, it would be interesting to test how large the greenium must be for investors to switch to conventional bonds. This could help identify the breaking point for investors, providing indications of how much they would be willing to sacrifice for green investments compared to non-green investments.

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Appendices

Appendix A

Table 12: Literature Overview

Literature	Type of Issuer	Geographical Scope	Method	Time Frame	Results (in Bps)
Baker et al. (2018)	Corporate and municipal	U.S.	Fixed-effects regressions	2014–2016	8 bps
Caramichael and Rapp (2022)	Corporate	International	Fixed-effects regression and PCA	2014–2021	8 bps on average
Gianfrate and Peri (2019)	Multiple issuers	European	Matching propensity scores and regressions	2013–2017	20 bps
Larcker and Watts (2020)	Municipal	U.S.	Statistical tests for differences	2013–2018	No significant greenium
Partridge and Medda (2019)	Municipal bonds	U.S. bonds	NS and green bond index	2014–2018	No significant greenium in primary market.
Zerbib (2019)	Multiple issuers	International bonds	OLS - regression	2013–2017	2 bps

A summary of the related literature presented in the literature review.

Appendix B

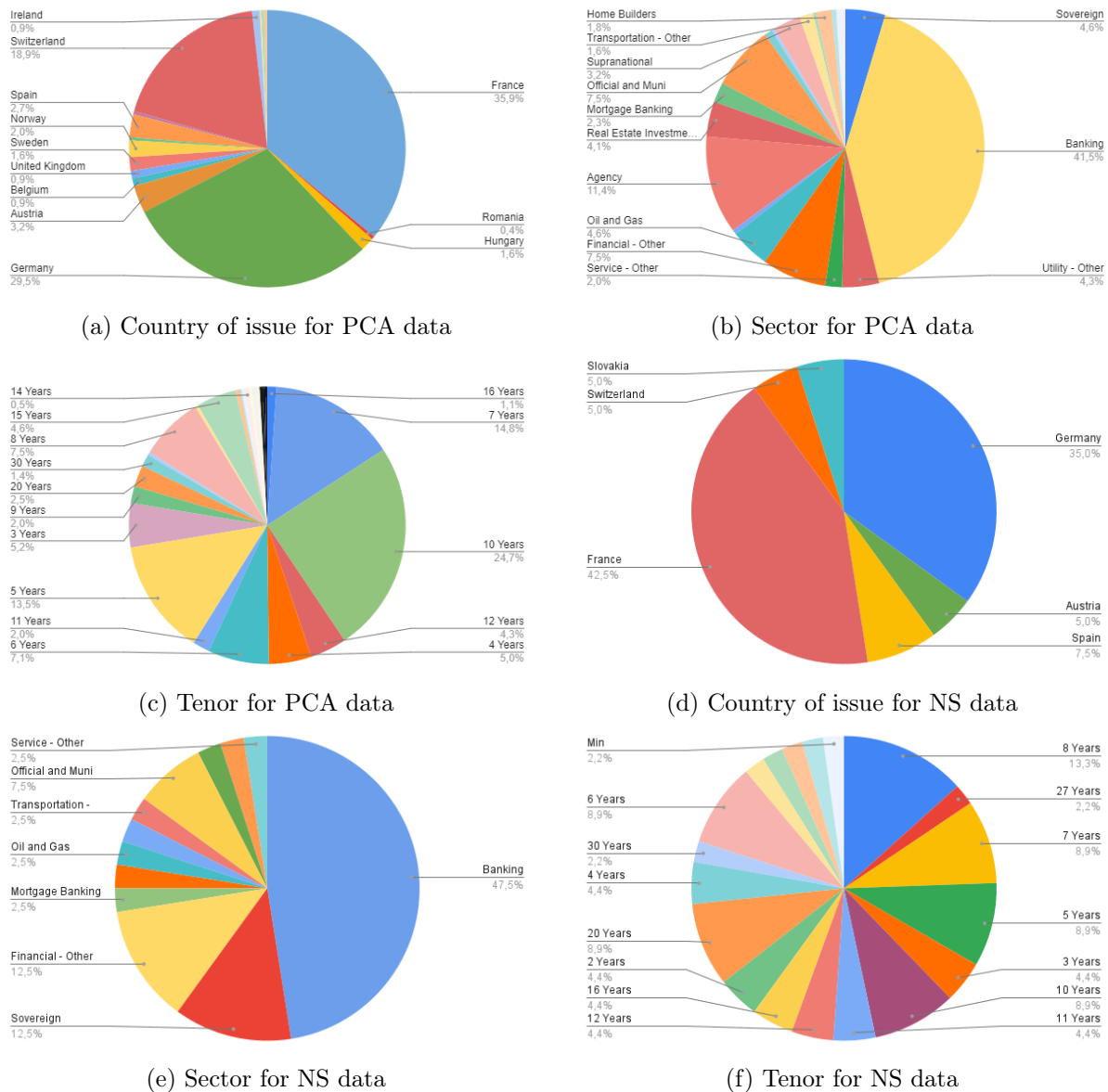
Table 13: List of Variables Included in the Analysis

Coupon	Issue date	Maturity	Tenor
Country*	Issuer type*	Amount issued (EUR)	Green bond*
Coupon frequency*	Sector*	Parent/Subsidiary*	Guaranteed*
ECB eligible*	Straight*	Prospectus available*	Is active*
Ownership (denomination)*	Private placement*	Executable*	Mod duration to maturity
Yield spread (OTR) to maturity	Callable*	Extendible*	Offering type*
Principal	Issue yield	Principal currency*	Issue price
Seniority*	Exchange listed*	Option adjusted spread	Covered bond*

All variables used in the analysis. A star (*) indicates a categorical variable which is deconstructed into one or multiple dummy-variables according to the process described under 3.2 *Data Collection*.

Appendix C

Figure 8: Circle Diagrams of Summary Statistics



Figures (a) - (c) provides descriptive statistics of the sample used for the principal component analysis. Figures (d) - (f) provides descriptive statistics of the sample used for the yield curve analysis.

Table 14: Summary Statistics of Principal Currency

	# of Green Bonds	# of Conventional Bonds
Euro	205 (19)	216 (19)
Swiss franc	57 (1)	47 (1)
Hungarian forint	5	4
British pound	2	5
Romanian leu	1	1
Norwegian krone	1	1
US dollar	1	1
Australian dollar	2	2
Swedish krona	7	2
Hong Kong dollar	0	2

Number of bonds in the sample for each currency divided between green and conventional bonds. The numbers without parentheses display the number of bonds in the dataset for PCA whereas, the numbers in parentheses show the numbers of bonds in the dataset for NS. No numbers in parentheses means that there are no bonds for that type in the dataset of NS.

Table 15: Summary Statistics of Country of Issue

	# of Green Bonds	# of Conventional Bonds
France	106 (9)	96 (8)
Hungary	5	4
Germany	72 (7)	94 (7)
Sweden	7	2
Switzerland	58 (1)	48 (1)
Slovakia	1 (1)	1 (1)
Romania	1	1
Malta	0	1
Finland	1	1
Belgium	2	3
UK	2	3
Ireland	2	3
Spain	9 (1)	6 (2)
Austria	8 (1)	10 (1)
Norway	5	6
Italy	1	1
Netherlands	1	1

Number of bonds in the sample for each country of issue divided between green and conventional bonds. The numbers without parentheses display the number of bonds in the dataset for PCA whereas, the numbers in parentheses show the numbers of bonds in the dataset for NS. No numbers in parentheses means that there are no bonds for that type in the dataset of NS.

Table 16: Summary Statistics of Sector

	# of Green Bonds	# of Conventional Bonds
Banking	113 (8)	121 (11)
Utility - other	12	12 (1)
Conglomerate/Diversified mfg	1	94 1
Agency	34 (1)	30
Official and muni	19	23 (3)
Life insurance	1	1
Service - other	6 (1)	5
Mortgage banking	6 (1)	7
Electronics	1	2
Transportation - other	6 (1)	3
Home builders	6	4
Telecommunications	2	2
Financial - other	21 (3)	21 (2)
Chemicals	2	3 (1)
Supranational	10	8
Real estate investment trust	11 (1)	12
Sovereign	12 (3)	14 (2)
Securities	2	1
Oil and gas	15 (1)	10
Property and casualty insurance	1	1

Number of bonds in the sample for each sector divided between green and conventional bonds. The numbers without parentheses display the number of bonds in the dataset for PCA whereas, the numbers in parentheses shows the numbers of bonds in the dataset for NS. No numbers in parentheses means that there are no bonds for that type in the dataset of NS.

Table 17: Summary Statistics of Issuer Type

	# of Green Bonds	# of Conventional Bonds
Corporate	206 (16)	206 (15)
Govt/Treasury/CB/Other Gov/Supra	38 (3)	41 (4)
Non-US munis	3	4 (1)
Agency	34 (1)	30

Number of bonds in the sample for each issuer type divided between green and conventional bonds. The numbers without parentheses display the number of bonds in the dataset for PCA whereas, the numbers in parentheses show the numbers of bonds in the dataset for NS. No numbers in parentheses means that there are no bonds for that type in the dataset of NS.

Table 18: Summary Statistics of Covered Bonds

	# of Green Bonds	# of Conventional Bonds
Yes	54 (9)	70 (9)
No	227 (11)	211 (11)

Number of bonds in the sample that are covered or not divided between green and conventional bonds. The numbers without parentheses display the number of bonds in the dataset for PCA whereas, the numbers in parentheses show the numbers of bonds in the dataset for NS. No numbers in parentheses means that there are no bonds for that type in the dataset of NS.

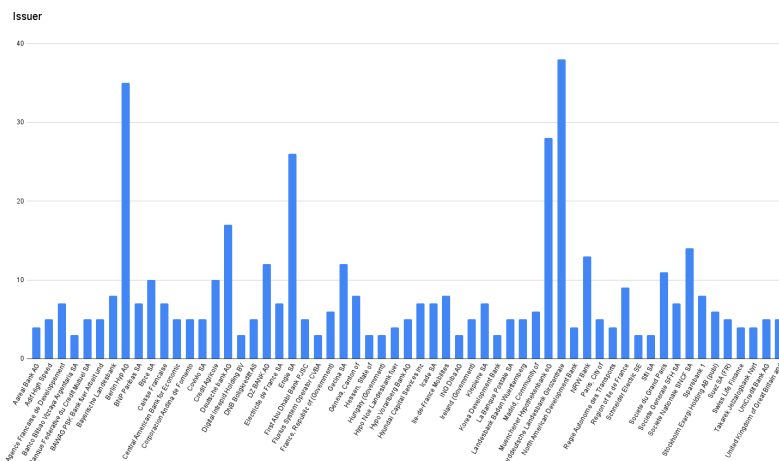


Figure 9: Summary of Number of Issuers

Number of bonds from each issuer used in the sample for the principal component analysis. The most frequent issuer is Norddeutsche Landesbank Girozentrale with 38 observations

Appendix D

The first part of Appendix D is dedicated to describing the additional regressions used. More specifically the OLS regressions mentioned in section 3.3 *Yield Curve Analysis* and 3.4 *Principal Component Analysis*. Heteroscedasticity robust standard errors are used in all OLS regressions.

The OLS regression in the yield curve analysis is used to estimate the magnitude of the greenium conditional on the two NS curves described in section 3.3 *Yield Curve Analysis*. The results of this regression can be found in Table 4 (and Table 7 for the robustness test), and equation (22) below describes the mathematical formulation used for the OLS estimation.

$$Y_i = \beta_0 + \beta_1 Tenor_i + \beta_2 group_label_i + \varepsilon_i \quad (22)$$

$i = 1, \dots, n$ where n is the amount of observations, Y is the dependent variable (the predicted yield), β_0 is the regression intercept and is referred to as the constant in Tables 4 and 7. $Tenor$ and $group_label$ are the independent variables where $group_label$ is a dummy variable taking the value of 1 for green bonds and 0 for conventional bonds, lastly ε represents the error/innovation term.

The second set of OLS regressions performed in this study are the two related to the PCA, namely the PC regressions. They are used to relate the PCA to the yields as described in section 3.4 *Principal Component Analysis* and the results obtained from the PC regressions are visible in Figure 7 (and Figure 19 for the robustness test). Equations 23 and 24 below describes the mathematical formulation used for the OLS estimation.

$$Y_i(G) = \gamma_0 + \gamma_1 PC_{1i} + \gamma_2 PC_{2i} + \dots + \gamma_m PC_{mi} + \nu_i \quad (23)$$

$$Y_i(C) = \gamma_0 + \gamma_1 PC_{1i} + \gamma_2 PC_{2i} + \dots + \gamma_k PC_{ki} + \nu_i \quad (24)$$

The first equation represents the regression performed for the set of green bonds (G) and the second equation represents the regression performed for the set of conventional bonds (C). We define the set $G = \{i : i \in G\}$ as the collection of observations denoted by i that belong to set G . Similarly, the set $C = \{i : i \in C\}$ represents the observations denoted by i that belong to set C . The set D is defined as the union of sets G and C , where $D = G \cup C$. This means that D contains all the observations from both G and C , and thus represents the entire dataset for PCA. m denotes the number of PCs in the green bond set and k denotes the number of PCs in the conventional bond set. The PC regressions are performed with an intercept/constant in the form of γ_0 , and the dependent variable, Y , is the issue yields calculated according to equation (1). The PCs are the independent variables and lastly the term ν represents the error/innovation term. The estimated coefficients (γ) for each set are compared against each other in Figure 7 (and Figure 19) and the coefficient estimates related to Figure 7 can also be found in Table 22.

The second part of Appendix D presents tables and figures containing results from the yield curve analysis.

Table 19: Bootstrap Results (Green)

Issuer	Coupon	Tenor	Issue Price	A	DF	Rate	Repricing
Hyundai Capital Services Inc	1.158	2 Years	100	1.98	0.98	1.17%	100
Berlin Hyp AG	1.25	3 Years	99.682	2.94	0.96	1.38%	99.682
Muenchener Hypothekenbank eG	1.305	4 Years	100	3.89	0.95	1.34%	100
Bayerische Landesbank Germany, Federal Republic of	3.125	5 Years	99.913	4.74	0.85	3.50 %	99.913
Slovenska Sporitelna as	1.3	5 Years	99.675	4.82	0.93	1.41%	99.675
UniCredit Bank AG	3.5	6 Years	99.98	5.58	0.80	4.05%	99.98
Engie SA	2.625	6 Years	99.936	5.63	0.85	2.90%	99.936
Caja Rural de Navarra S	3.5	7 Years	99.021	6.37	0.77	4.34%	99.021
DZ Hyp AG	0.75	7 Years	99.729	6.55	0.95	0.78%	99.729
ING Diba AG	0.75	8 Years	99.813	7.41	0.94	0.76%	99.813
La Banque Postale Home Loan	2.375	8 Years	99.597	7.29	0.82	2.69%	99.597
Bpce SA	1.625	8 Years	99.429	7.34	0.88	1.79%	99.429
BPCE SFH SA	1	10 Years	98.908	9.11	0.90	1.14%	98.908
Gecina SA	1.75	10 Years	98.996	9.05	0.83	2.02%	98.996
Suez SA (FR)	0.875	11 Years	98.211	9.97	0.89	1.07%	98.211
France, Republic of	2.875	12 Years	99.132	10.66	0.68	3.83%	99.132
Societe du Grand Paris	0.1	16 Years	108.62	14.31	1.07	-0.42%	108.62
Ile-de-France Mobilites	1.625	20 Years	98.537	17.74	0.70	2.18%	98.537
Austria, Republic of	1.275	20 Years	98.842	17.82	0.77	1.49%	99.842
	1.85	27 Years	99.454	22.32	0.58	2.67%	99.454

The output from the bootstrapping for green bonds using simple compounding. DF stands for discount factor and they are obtained following equations (2) - (5). A represents the annuity which is used in the bootstrapping process and it is calculated according to the recursion formulated by equation (5). The DFs are converted into rates according to equation (6). Repricing is used to test the quality of the bootstrap, and it indicates a successful bootstrapping considering that the repricing provides us with the exact issue price. There are no observations for a couple of tenors and for the first tenor a German government bond is used as explained in section 3.3 *Yield Curve Analysis*. Synthetic bonds are created using linear interpolation for the tenors without observations (e.g 9-year and 13-year), this is done in order for the bootstrapping process to work in the sequential manner explained in section 3.3 *Yield Curve Analysis*. The table only displays the rates which originates from observations in our sample and these are consequently the rates used as inputs in the yield curve analysis.

Table 20: Bootstrap Results (Conventional)

Issuer	Coupon	Tenor	Issue Price	A	DF	Rate	Repricing
Sika AG	1.6	2 Years	100.067	1.97	0.97	1.59%	100.067
Berlin Hyp AG	1.875	3 Years	99.784	2.91	0.94	2.01%	99.784
Muenchener Hypothekenbank eG	1.314	4 Years	100	3.86	0.95	1.34%	100
Bayerische Landesbank Germany, Federal Republic of	2.35	5 Years	99.868	4.75	0.89	2.55 %	99.868
	1.3	5 Years	99.43	4.79	0.93	1.46%	99.430
Slovenska Sporitelna as	2	6 Years	100	5,66	0.89	2.13%	99.98
UniCredit Bank AG	0.7	6 Years	100	5,73	0.96	0.70%	100
Electricite de France SA	4.375	7 Years	99.381	6,41	0.75	4.84%	99.381
Banco de Sabadell SA	1.75	7 Years	99.863	6,58	0.90	1.64%	99.863
Aareal Bank AG	0.125	8 Years	99.441	7,48	0.99	0.18%	99.441
Erste Group Bank AG	2.5	8 Years	99.607	7,31	0.83	2.52%	99.607
Societe Generale SFH SA	1.62	8 Years	100	7,37	0.89	1.50%	100
Bpce SA	2.375	10 Years	99.298	9,05	0.80	2.57%	99.298
BPCE SFH SA	0.375	10 Years	99.873	9,23	0.97	0.34%	99.873
Credit Agricole SA	1.125	11 Years	99.795	10,02	0.89	1.08%	99.7951
Credit Agricole SA France, Republic of	2.5	12 Years	99.54	10.66	0.74	2.86%	99.54
	1.25	16 Years	97.58	14.31	0.81	1.46%	97.58
Paris, City of	3	20 Years	98.336	17.74	0.51	4.86%	98.336
Madrid, Community of	1.931	20 Years	100	17.82	0.69	2.24%	100
North-Rhine Westphalia, State of	2.25	30 Years	99.268	22.32	0.51	3.19%	99.268

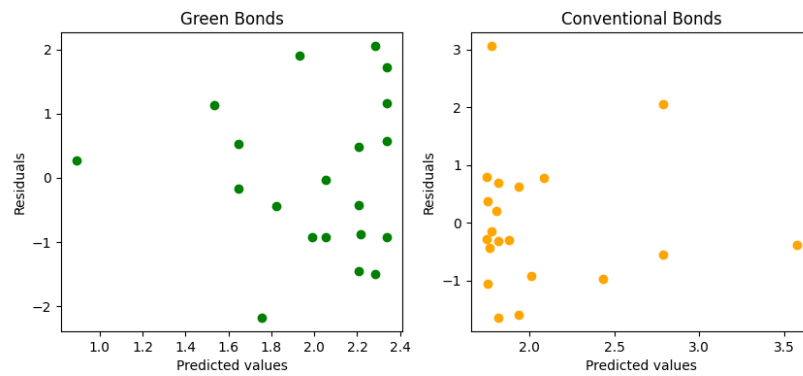
The output from the bootstrapping for conventional bonds using simple compounding. DF stands for discount factor and they are obtained following equations (2) - (5). A represents the annuity which is used in the bootstrapping process and it is calculated according to the recursion formulated by equation (5). The DFs are converted into rates according to equation (6). Repricing is used to test the quality of the bootstrap, and it indicates a successful bootstrapping considering that the repricing provides us with the exact issue price. There are no observations for a couple of tenors and for the first tenor a German government bond is used as explained in section 3.3 *Yield Curve Analysis*. Synthetic bonds are created using linear interpolation for the tenors without observations (e.g 9-year and 13-year), this is done in order for the bootstrapping process to work in the sequential manner explained in section 3.3 *Yield Curve Analysis*. The table only displays the rates which originates from observations in our sample and these are consequently the rates used as inputs in the yield curve analysis.

Table 21: Predicted Yields

Tenor	Green Bonds		Conventional Bonds	
	Predicted Yield	Residual	Predicted Yield	Residual
2	0.8923	0.2785	1.8822	-0.2924
3	1.8228	-0.4383	1.8063	0.199
4	2.2142	-0.8789	1.7629	-0.427
5	2.3391	1.1594	1.7468	0.7982
5	2.3391	-0.9281	1.7468	-0.2875
6	2.3381	1.714	1.7534	0.3728
6	2.3381	0.5565	1.7534	-1.057
7	2.2824	2.0535	1.7788	3.0639
7	2.2824	-1.0501	1.7788	-0.1421
8	2.2072	-1.4458	1.8197	-1.6425
8	2.2072	0.4827	1.8197	0.6998
8	2.2072	-0.4221	1.8731	-0.3198
9	2.129	–	1.8731	–
10	2.0551	-0.9187	1.9366	0.6327
10	2.0551	-0.0308	1.9366	-1.5945
11	1.9883	-0.9199	2.0082	-0.9238
12	1.9291	1.9055	2.0859	0.7735
13	1.8770	–	2.1684	–
14	1.8312	–	2.2542	–
15	1.7910	–	2.3423	–
16	1.7554	-2.1745	2.4319	-0.9722
17	1.7239	–	2.5220	–
18	1.6957	–	2.6122	–
19	1.6705	–	2.7018	–
20	1.6477	0.5286	2.7905	2.0537
20	1.6477	-0.1627	2.7905	-0.5488
21	1.6271	–	2.8778	–
22	1.6084	–	2.9636	–
23	1.5913	–	3.0476	–
24	1.5756	–	3.1296	–
25	1.5611	–	3.2096	–
26	1.5478	–	3.2873	–
27	1.5355	1.1321	3.3628	–
28	1.5241	–	3.4361	–
29	1.5134	–	3.5070	–
30	1.5034	–	3.5757	-0.3858

The predicted yields and residuals obtained from the two Nelson-Siegel curves. The predicted yields for each tenor are obtained using equation (10) and the estimated vector of coefficients, θ . The residuals are calculated as the difference between the predicted yield and the observed yield, i.e. $\hat{y}(t) - y(t)$. The residuals can consequently only be calculated for the tenors where we have observations of the yields, explaining the blank cells in the table (the yields for the synthetic bonds are not calculated).

Figure 10: Visualization of the Residuals for the NS Curves



To the left is a scatterplot of the residuals and predicted values for the NS fitted to green bonds and to the right is a similar plot, but for the conventional bonds. The scatterplots exhibits a "cone-like" shape where the variability does not seem to be equal, i.e. exhibiting heteroscedasticity. Small sample size may also be affecting this results, but evidently, these graphs suggest the use of a heteroscedasticity robust approach.

Appendix E

Table 22: Coefficients for the PCs and P-values of the Difference in Values

PC	Green Bonds	Conventional Bonds	P-value
	Coef	Coef	
1	-0.0116	0.0602	0.0532
2	0.1319	0.0958	0.3742
3	0.0621	0.1691	0.0117
4	0.1171	0.1166	0.9906
5	-0.1962	-0.1014	0.0417
6	0.3015	0.0398	0.0000
7	-0.0830	0.2241	0.0000
8	-0.1627	-0.0280	0.0145
9	0.0617	0.1399	0.1745
10	0.0845	-0.1218	0.0009
11	-0.0574	0.1220	0.0059
12	0.2147	-0.2983	0.0000
13	-0.2483	0.0834	0.0000
14	-0.2503	0.0347	0.0001
15	0.0915	-0.1043	0.0082
16	-0.1966	0.0331	0.0024
17	-0.0042	-0.2714	0.0006
18	0.1360	0.0065	0.1034
19	0.0522	-0.1204	0.0313
20	0.0738	0.2164	0.0792
21	0.0340	0.1150	0.3235
22	-0.1195	-0.1072	0.8832

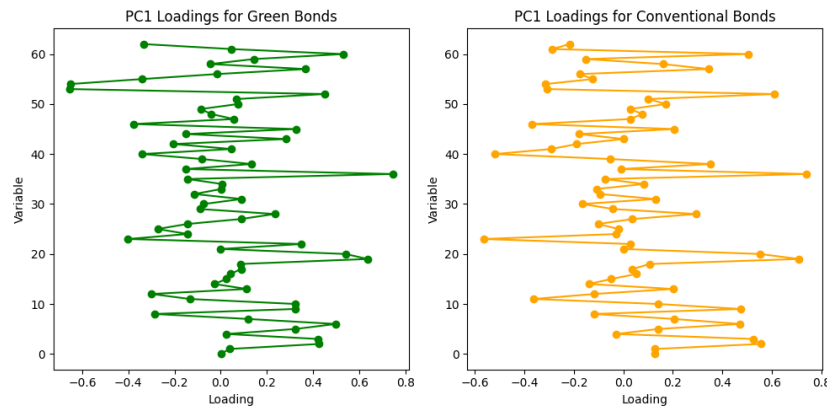
A summary from the PC regressions fitted to the two subsamples of green and conventional bonds. 22 PCs are included in the regression as a result from using the eigenvalue-one criterion as a threshold for how many PCs to retain in the analysis. The regressions are performed according to equations (23) and (24) where $m = 22$ and $k = 22$. The column for the p-values display the p-values for testing the significance of the difference between the estimated coefficients using a t-test. The first row in the p-value column shows the p-value for the difference between the estimated coefficients for PC_1 .

Table 23: Result of the Augmented Dickey-Fuller Test

Variable	ADF Statistic	P-value
0	-20.495	0.000
1	-5.817	0.000
2	-7.507	0.000
3	-7.832	0.000
4	-14.229	0.000
5	-22.227	0.000
6	-7.495	0.000
7	-15.258	0.000
8	-20.789	0.000
9	-13.869	0.000
10	-6.209	0.000
11	-9.908	0.000
12	-24.353	0.000
13	-14.211	0.000
14	-15.931	0.000
15	-23.721	0.000
16	-21.989	0.000
17	-25.251	0.000
18	-23.193	0.000
19	-22.703	0.000
20	-9.508	0.000
21	-10.105	0.000
22 (Y)	-3.408	0.011

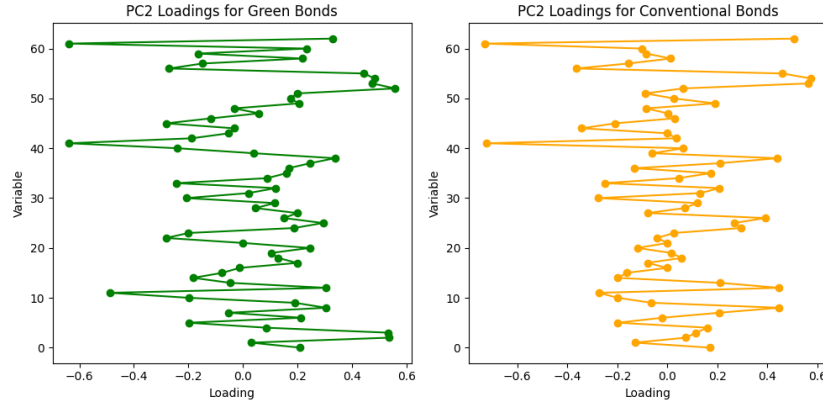
The results from the Augmented Dickey-Fuller test for all the variables included in the VAR (the 22 PCS + the yields (Y)). The ADF test is performed using the "adfuller" function from "stattools" module in the "statmodels" library for Python (Sebold & Perktold, 2010).

Figure 11: Variable Loadings for PC_1



The loadings associated with each variable used to construct PC_1 for green bonds is shown in the figure to the left and the one to the right depicts the same but for conventional bonds. The loadings represents the weights associated to each variable obtained as the eigenvector, \mathbf{W}_1 , explained by equations (13)-(15), further described in section 3.4 *Principal Component Analysis*. This is performed using functions in the machine learning library "scikit-learn" for Python (Pedregosa et al., 2011).

Figure 12: Variable Loadings for PC_2



The loadings associated with each variable used to construct PC_2 for green bonds is shown in the figure to the left and the one to the right depicts the same but for conventional bonds. The loadings represents the weights associated to each variable obtained as the eigenvector, \mathbf{W}_2 , explained by equations (16)-(17), further described in section 3.4 *Principal Component Analysis*. This is performed using functions in the machine learning library "scikit-learn" for Python (Pedregosa et al., 2011).

Table 24: Difference in Factor Loadings in PC_1 and PC_2 Between Green Bonds and Conventional Bonds

	PC_1	PC_2
Max difference	0.3384	0.4620
Min difference	0	0
Avg abs difference	0.1206	0.1433

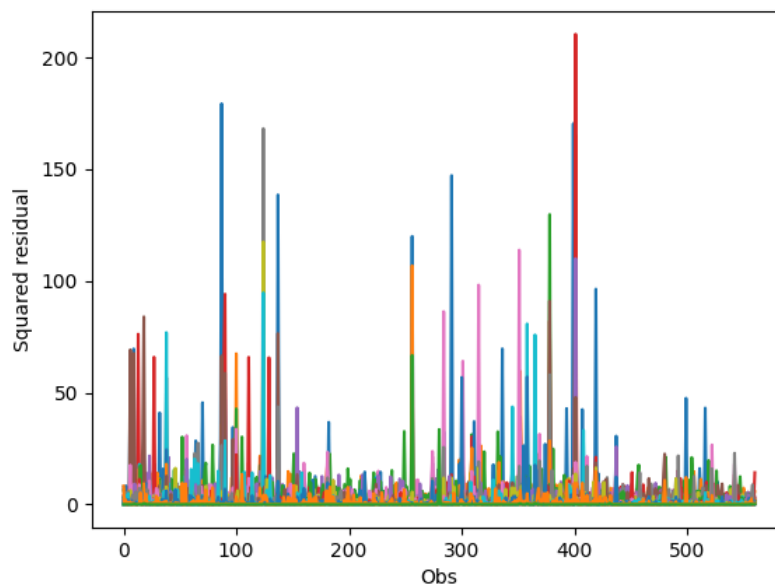
The maximum and minimum differences between the loadings for PC_1 and PC_2 for green and conventional bonds as well as the average absolute difference. These differences refer to the loadings seen in Figures 11 and 12.

Table 25: Difference in Values for PC_1 and PC_2 Between Green Bonds and Conventional Bonds

	PC_1	PC_2
Max difference	13.9281	-10.5813
Min difference	-0.0089	0.0045
Avg abs difference	2.0056	2.1895

The maximum and minimum differences between the values for PC_1 and PC_2 for green and conventional bonds as well as the average absolute difference. These differences refer to the values of PC_1 and PC_2 seen in Figure 6 and they are constructed using the weights/loadings seen in Figures 11 and 12.

Figure 13: Visual Inspection of the Residuals for the VAR



A visualization of all residuals for the VAR, the residuals are squared to better see any differences in the variability. Overall the variability looks equal and this observation is strengthened by the ARCH-LM test which show a p-value of 0.2428. This large p-value indicates that we cannot reject homoscedasticity and we can consequently perform the VAR. The ARCH-LM test is performed using the "het_arch" function from the "sm_diag" module in the "statsmodels" library for Python (Seabold & Perktold, 2010).

Appendix F

Table 26: CC Bootstrap Results (Green)

Issuer	Coupon	Tenor	Issue Price	CC rate	Simple rate	ABSdiff
Hyundai Capital Services Inc	1.158	2 Years	100	1.16%	1.17%	0.01
Berlin Hyp AG	1.25	3 Years	99.682	1.36%	1.38%	0.03
Muenchener Hypothekenbank eG	1.305	4 Years	100	1.3%	1.34%	0.03
Bayerische Landesbank	3.125	5 Years	99.913	3.22%	3.50%	0.27
Germany, Federal Republic of	1.3	5 Years	99.675	1.36%	1.41%	0.05
Slovenska Sporitelna as	3.5	6 Years	99.98	3.63%	4.05%	0.43
UniCredit Bank AG	2.625	6 Years	99.936	2.68%	2.90%	0.23
Engie SA	3.5	7 Years	99.021	3.79%	4.34%	0.55
Caja Rural de Navarra S	0.75	7 Years	99.729	0.76%	0.78%	0.02
DZ Hyp AG	0.75	8 Years	99.813	0.74%	0.76%	0.02
ING Diba AG	2.375	8 Years	99.597	2.43%	2.69%	0.26
La Banque Postale Home Loan	1.625	8 Years	99.429	1.67%	1.79%	0.12
Bpce SA	1	10 Years	98.908	1.08%	1.14%	0.06
BPCE SFH SA	1.75	10 Years	98.996	1.84%	2.02%	0.18
Gecina SA	0.875	11 Years	98.211	1.01%	1.07%	0.06
Suez SA (FR)	2.875	12 Years	99.132	3.15%	3.83%	0.68
France, Republic of	0.1	16 Years	108.62	-0.43%	-0.42%	0.01
Societe du Grand Paris	1.625	20 Years	98.537	1.83%	2.18%	0.35
Ile-de-France Mobilites	1.275	20 Years	98.842	1.31%	1.49%	0.17
Austria, Republic of	1.85	27 Years	99.454	2.03%	2.67%	0.64

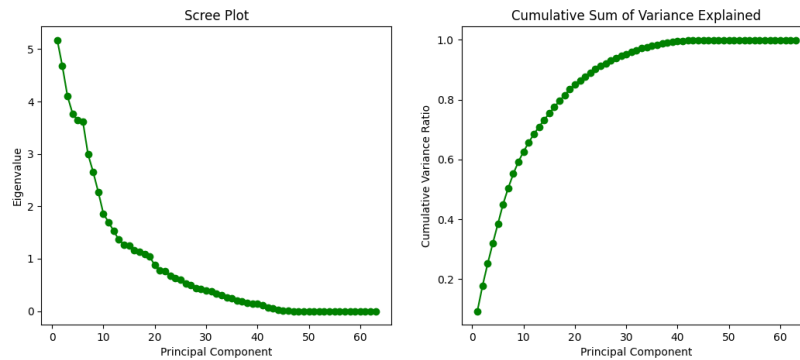
The output from the bootstrapping for green bonds using continuous compounding (CC). The CC rates are used as inputs for the NS in the robustness tests for the yield curve analysis. The simple rates are included for comparison purposes and they are the same rates as found in Table 19. ABSdiff provides the absolute difference between CC rates and simple rates. There are no observations for a couple of tenors and for the first tenor a German government bond is used as explained in section 3.3 *Yield Curve Analysis*. Synthetic bonds are created using linear interpolation for the tenors without observations (e.g 9-year and 13-year), this is done in order for the bootstrapping process to work in the sequential manner explained in section 3.3 *Yield Curve Analysis*.

Table 27: CC Bootstrap Results (Conventional)

Issuer	Coupon	Tenor	Issue Price	CC rate	Simple rate	ABSdiff
Sika AG	1.6	2 Years	100.067	1.57%	1.59%	0.02
Berlin Hyp AG	1.875	3 Years	99.784	1.95%	2.01%	0.06
Muenchener Hypothekenbank eG	1.314	4 Years	100	1.30%	1.34%	0.03
Bayerische Landesbank	2.35	5 Years	99.868	2.40%	2.55 %	0.15
Germany, Federal Republic of	1.3	5 Years	99.43	1.62%	1.46%	0.16
Slovenska Sporitelna as	2	6 Years	100	2.00%	2.13%	0.13
UniCredit Bank AG	0.7	6 Years	100	0.68%	0.70%	0.01
Electricite de France SA	4.375	7 Years	99.381	4.82%	4.84%	0.03
Banco de Sabadell SA	1.75	7 Years	99.863	1.77%	1.64%	0.13
Aareal Bank AG	0.125	8 Years	99.441	0.19%	0.18%	0.01
Erste Group Bank AG	2.5	8 Years	99.607	2.58%	2.52%	0.06
Societe Generale SFH SA	1.62	8 Years	100	1.60%	1.50%	0.10
Bpce SA	2.375	10 Years	99.298	2.50%	2.57%	0.07
BPCE SFH SA	0.375	10 Years	99.873	0.36%	0.34%	0.02
Credit Agricole SA	1.125	11 Years	99.795	1.11%	1.08%	0.02
Credit Agricole SA	2.5	12 Years	99.54	2.66%	2.86%	0.20
France, Republic of	1.25	16 Years	97.58	1.38%	1.46%	0.08
Paris, City of	3	20 Years	98.336	3.57%	4.86%	1.27
Madrid, Community of	1.931	20 Years	100	1.94%	2.24%	0.3
North-Rhine Westphalia, State of	2.25	30 Years	99.268	2.31%	3.19%	0.88

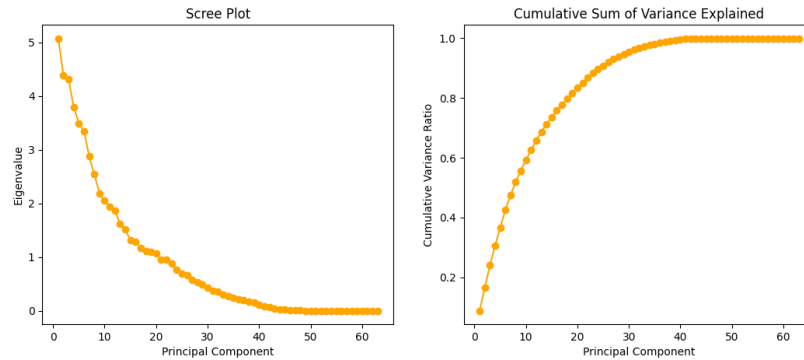
The output from the bootstrapping for conventional using continuous compounding (CC). The CC rates are used as inputs for the NS in the robustness tests for the yield curve analysis. The simple rates are included for comparison purposes and they are the same rates as found in Table 20. ABSdiff provides the absolute difference between CC rates and simple rates. There are no observations for a couple of tenors and for the first tenor a German government bond is used as explained in section 3.3 *Yield Curve Analysis*. Synthetic bonds are created using linear interpolation for the tenors without observations (e.g 9-year and 13-year), this is done in order for the bootstrapping process to work in the sequential manner explained in section 3.3 *Yield Curve Analysis*.

Figure 14: Robustness Scree Plot and Plot of Cumulative Variance Explained for Green Bonds



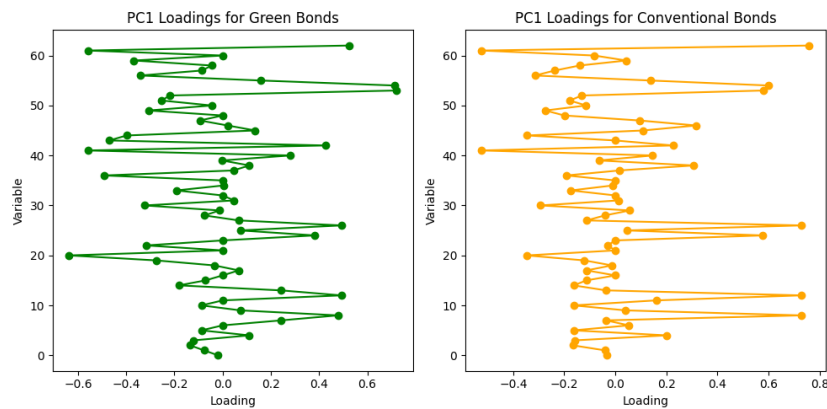
These two figures represents the results from performing the PCA on the restricted sample containing only exact matches as mentioned in section 4.2.1 *Robustness Result for PCA/Matching*. The scree plot, the figure to the left, plots the eigenvalues and the corresponding principal component for green bonds. The values are extracted as the λ s from the process described by equations (13)-(17) using the "scikit-learn" library in Python (Pedregosa et al., 2011). The threshold of one for the eigenvalues retain 19 PCs. The figure to the right plots the cumulative variance explained calculated using the concept provided by equation (18). At 19 PCs, variance explained is 0.8346.

Figure 15: Robustness Scree Plot and Plot of Cumulative Variance Explained for Conventional Bonds



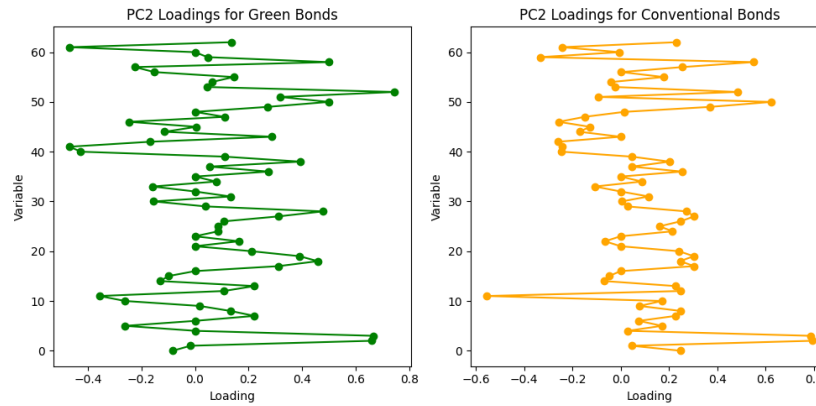
These two figures represents the results from performing the PCA on the restricted sample containing only exact matches as mentioned in section 4.2.1 *Robustness Result for PCA/Matching*. The scree plot, the figure to the left, plots the eigenvalues and the corresponding principal component for conventional bonds. The values are extracted as the λ :s from the process described by equations (13)-(17) using the "scikit-learn" library in Python (Pedregosa et al., 2011). The threshold of one for the eigenvalues retain 19 PCs. The figure to the right plots the cumulative variance explained calculated using the concept provided by equation (18). At 19 PCs, variance explained is 0.8350.

Figure 16: Robustness Plot of Loadings for PC_1



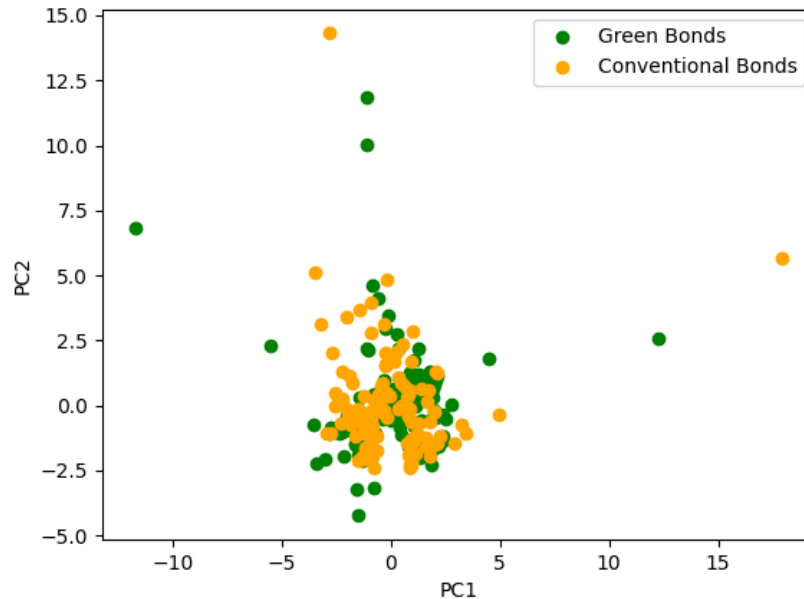
These two figures represents the results from performing the PCA on the restricted sample containing only exact matches as mentioned in section 4.2.1 *Robustness Result for PCA/Matching*. The loadings associated with each variable used to construct PC_1 for green bonds is shown to in the figure to the left and the one to the right depicts the same but for conventional bonds. The loadings represents the weights associated to each variable obtained as the eigenvector, W_1 , explained by equations (13)-(15), further described in section 3.4 *Principal Component Analysis*. This is performed using functions in the machine learning library "scikit-learn" for Python (Pedregosa et al., 2011).

Figure 17: Robustness Plot of Loadings for PC_2



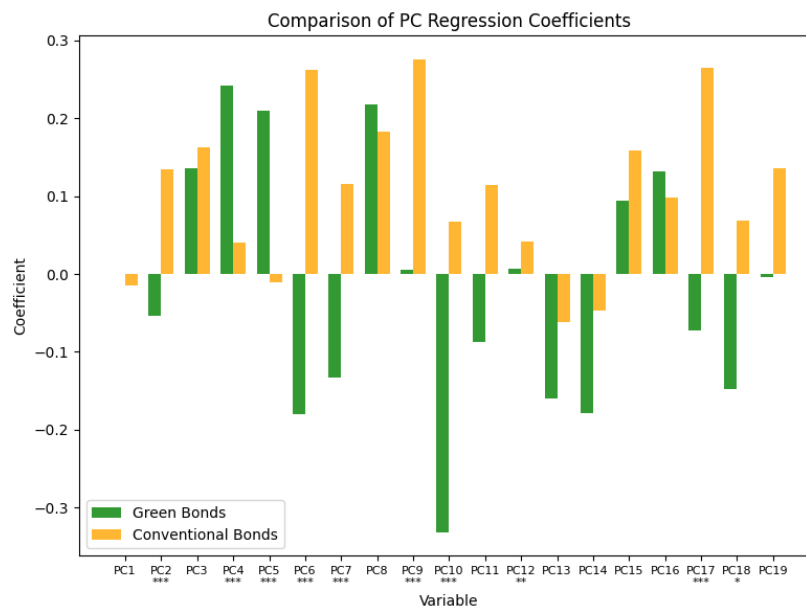
These two figures represents the results from performing the PCA on the restricted sample containing only exact matches as mentioned in section 4.2.1 *Robustness Result for PCA/Matching*. The loadings associated with each variable used to construct PC_2 for green bonds is shown to in the figure to the left and the one to the right depicts the same but for conventional bonds. The loadings represents the weights associated to each variable obtained as the eigenvector, \mathbf{W}_2 , explained by equations (16)-(17), further described in section 3.4 *Principal Component Analysis*. This is performed using functions in the machine learning library "scikit-learn" for Python (Pedregosa et al., 2011).

Figure 18: Robustness Scatter for Comparison of PC_1 and PC_2 Between Green and Conventional Bonds



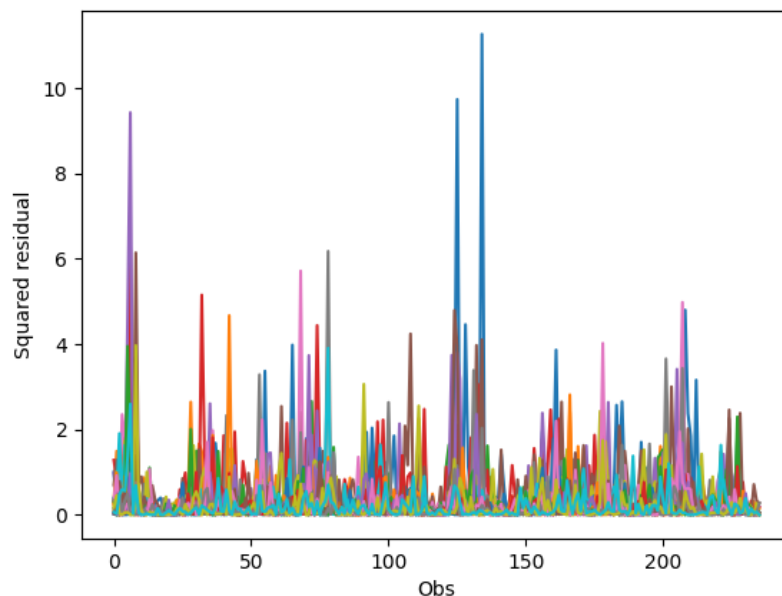
A plot of the PCs of both green and conventional bonds, with PC_1 on the x-axis and PC_2 on the y-axis. This scatterplot represents the results obtained from running the PCA on the more restricted sample according to section 4.2.1 *Robustness Result for PCA/Matching*. PC_1 and PC_2 are constructed according to equations (13) - (17) using the machine learning library "scikit-learn" for Python (Pedregosa et al., 2011). The weights/loadings used in the creation of PC_1 and PC_2 refers to those seen in Figures 16 and 17

Figure 19: Robustness Comparison of Coefficients from PC Regressions



This histogram represents the results obtained from running the PCA on the more restricted sample according to section 4.2.1 *Robustness Result for PCA/Matching*. It shows a comparison of the coefficients for both green and conventional bonds obtained from the PC regression described by equations (23) and (24), where $m = 19$ and $k = 19$. The stars underneath the variable names on the x-axis symbolizes whether the difference in coefficients associated with that variable is significantly different between green bonds and conventional bonds. $*$ = $P < 0.1$, $**$ = $P < 0.05$, $***$ = $P < 0.01$.

Figure 20: Robustness Visualization of Residuals for VAR



A visualization of all residuals for the VAR, the residuals are squared to better see any differences in the variability. The plot refers to the restricted sample where only "perfect matches" are included as explained in section 4.2.1 *Robustness Result for PCA/Matching*. Overall the variability looks equal and this observation is strengthened by the ARCH-LM test which show a p-value of 0.4405. This large p-value indicates that we cannot reject homoscedasticity and we can consequently perform the VAR for the smaller restricted sample as well. The ARCH-LM test is performed using the "het_arch" function from the "sm_diag" module in the "statsmodels" library for Python (Seabold & Perktold, 2010).

Table 28: Result of the Augmented Dickey-Fuller Test for Robustness Sample

Variable	ADF Statistic	P-value
0	-11.798	0.000
1	-6.315	0.000
2	-5.597	0.000
3	-3.665	0.005
4	-6.515	0.000
5	-11.344	0.000
6	-4.322	0.000
7	-14.008	0.000
8	-3.478	0.009
9	-12.776	0.000
10	-15.760	0.000
11	-15.800	0.000
12	-6.723	0.000
13	-15.293	0.000
14	-15.706	0.000
15	-16.773	0.000
16	-12.278	0.000
17	-6.193	0.000
18	-12.410	0.000
19 (Y)	-16.239	0.000

The results from the Augmented Dickey-Fuller test for all the variables included in the VAR for the smaller restricted sample constructed according to section 4.2.1 *Robustness Result for PCA/Matching*. It includes the 19 PCs and one extra variable for the yields (Y). The ADF test is performed using the "adfuller" function from "stattools" module in the "statmodels" library for Python (Seabold & Perktold, 2010).