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Exploring the Idiosyncratic Volatility Anomaly in the Swedish Stock Market: An Empirical Analysis of its Impact on Returns

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Abstract

We examine the cross-sectional relationship between idiosyncratic volatility relative to the Fama-French three factor model and expected stock returns. We find that portfolios containing the firms with the lowest idiosyncratic risk offers excess returns in relation to the prediction of the Fama-French three factor model, while those with the highest idiosyncratic risk do not. We test our findings to an E-GARCH estimated idiosyncratic volatility and find that the relation between low idiosyncratic risk and excess returns persist. The results are not explained by firm size or book-to-market ratio but when controlling for different weighting schemes and smaller time samples, weaknesses in the anomaly are apparent.

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1. INTRODUCTION

Factor anomalies have been a been a topic of interest in the realm of finance, as they present deviations from traditional asset pricing models and can offer possible opportunities for investors to generate returns. Mclean and Pontiff (2016) find that portfolios based on anomalies experienced on average a 32% decrease in returns post publication of the anomaly. Furthermore, the correlation between the post-public portfolio and other previous anomaly-based portfolios also increased after publication, indicating that investors learn about mispricing from academic publications. When traders exploit the proposed anomaly, the pattern starts to disappear until it is no longer mispriced. Disappearing or diminishing anomalies could also happen as a result of changing economic environments or from the fact that they originally appeared by chance or through a particular methodological choice. Consequently, anomalies cannot be assumed to persist without a constant re-evaluation, using both new data and new methodologies.

One factor anomaly that has received great attention and is well-documented is the idiosyncratic volatility anomaly. The idiosyncratic factor anomaly refers to the observation that stocks with low firm-specific risk generate on average higher returns than those with high idiosyncratic volatility (Ang et al, 2006). If factor models such as Fama-French's factors are correct, idiosyncratic volatility should not be priced in, thus Ang et al (2006) points to an anomaly in factor models.

Our first goal is to examine the relationship between idiosyncratic risk and expected returns using a sample of Swedish firms. We measure idiosyncratic risk in relation to the Fama-French-three-factor Model (1992) (FF-3) and then sort stocks into portfolios based upon the estimated idiosyncratic risk. If the FF-3 model is correct, or if firm-specific risk is not priced in, the portfolios containing different amounts of idiosyncratic risk should not yield different average returns. On the other hand, if factor models are incorrect, and idiosyncratic volatility is priced in, then there will be differences in average returns in the sorted portfolios. In that case, trading strategies that go long in low idiosyncratic firms and short in high idiosyncratic firms would yield positive excess returns.

By regressing the portfolio returns onto the FF-3 model, we find that the strategies investing in low idiosyncratic firms yield significant positive excess returns in relation to the factor model, while high idiosyncratic strategies do not. We test the results for different weighting schemes, use double sorting to further control for size and book-to-market-ratio, different holding and forming strategies as well as for different subsamples and find evidence for the anomaly, albeit weak.

Modern portfolio theory, such as the CAPM, relies on the assumption that investors are well-diversified such that any firm-specific risk is diversified away. Thus, in equilibrium investors do not get compensated for holding idiosyncratic risk, ultimately predicting a non-existent relationship between idiosyncratic risk and returns in contrast to our results. However, if in reality, investors are not fully diversified, they may demand compensation for holding idiosyncratic risk. For instance, both Levy (1978) & Merton (1987) shows that in a market with incomplete or segmented information, idiosyncratic risk is priced in and exerts a positive relationship with expected return.

Examining the relationship in the cross-section, most research finds either non-significant coefficients, or positive significant alphas in line with the theories of under-diversified investors. For instance, Lintner (1965) finds a significant positive relationship between idiosyncratic volatility and average returns. Furthermore, using a larger sample period, Lehmann (1990) strengthens the empirical evidence by again finding a positive relationship between firm-specific risk and returns.

However, Ang et al (2006) argue that these earlier studies do not explore idiosyncratic risk at the firm-level or do not sort portfolios directly based on the estimated idiosyncratic risk. Ang et al (2006) instead find that stocks with low idiosyncratic volatility have on average higher future returns than those with high idiosyncratic volatility, much in line with our results.

In criticism of utilizing the one-month lagged idiosyncratic volatility, Fu (2009) states that idiosyncratic volatilities are time-varying and therefore, the one-month lagged value is not a good proxy for the expected result. Fu (2009) explains that the findings of Ang et al (2006) can be explained by a small subset of firms with high idiosyncratic volatility. Instead of using the one-month lagged idiosyncratic volatility, Fu (2009) uses an E-GARCH model to estimate future volatility and finds a significant positive relationship between high expected idiosyncratic volatility and high returns, fully adverse to our findings and those of Ang et al (2006).

Thus, our second goal is to examine the robustness of the anomaly by applying the same methodology as before, but with a predicted value of idiosyncratic risk instead of the previous lagged value. We estimate idiosyncratic volatility by using several E-GARCH models and choose the value of the model with the lowest AIC-score. We then sort portfolios based on the predicted idiosyncratic volatility and again observe higher average returns for the portfolios investing in low idiosyncratic volatility firms. Regressing the portfolio returns on the FF-3, we once again observe a significant positive alpha for the lowest idiosyncratic volatility portfolios. The results are similar to those of the portfolio return analysis using the original estimation of idiosyncratic volatility. Thus, even if Fu (2009) is correct regarding the observation that using lagged idiosyncratic volatility is inappropriate, it does not alter the results in the way that Fu (2009) finds when conducting similar research on the US market.

The robustness of the results is evaluated by performing double sorting on idiosyncratic volatility and size and book-to-market respectively, offering additional control for these characteristics. Moreover, we divide the sample into subsamples and use different formation and holding strategies to evaluate the robustness. None of these regressions disprove the previous results. Rather the findings are more significant when controlling for characteristics that are generally known to affect returns.

The structure of the paper is split up in five sections barring the introduction. The first section covers the literature review in which fundamental theory from relevant previous academic research is mentioned. The second section presents the data used in the study, which includes a description of the source and sample size. The subsequent section presents the methodology used to analyze the data, in which the statistical models and estimation techniques are elaborated. Section five presents the findings and analysis of the main regressions. Section six presents the findings of the robustness checks in which various factors will be taken into consideration. Section seven concludes.

2. LITERATURE REVIEW

2.1 Systematic and unsystematic risk

The concepts of systematic and unsystematic risk are essential to the understanding of risk management in finance. It is widely known that risks affecting financial assets can be divided into two main classifications. *Systematic risk* refers to the risk that is associated with the broader markets in large and *unsystematic risk* refers to the individual risk that is specific for a particular company or sector.

2.2 Factor Anomalies

The Capital Asset Pricing Model (CAPM) is one of the most well-known theories in finance and explains the relationship between risk and expected return in financial markets. The theory states that investors should be compensated for higher risk in terms of higher returns (Sharpe, 1964).

The CAPM-relationship between risk and returns is used in portfolio allocation applications and it is thought to often be the case that investors who want to beat the market bases their stock picks on the idea that volatile stocks may yield larger returns (Dutt, Humphery-Jenner, 2013). However, Haugen & Heins (1975) point out shortcomings in the empirical efforts to explain the relationship between volatility and returns, as their findings show adverse results. Factor anomalies are used to describe such deviations from predictions in a financial model which can be attributed to a certain factor. Fama & French (1992) is one the most influential studies on factor anomalies where companies are divided into portfolios based on factors such as size and value in order to investigate whether these factors can explain stock returns.

2.3 Fama-French 3-Factor Model (FF-3)

While empirically investigating whether the CAPM can explain the cross-sectional variation in average stock returns, Fama & French (1992) finds that the CAPM alone is insufficient in explaining the variation. Expanding on the fundamentals of the CAPM, they add two additional factors to help explain the return of an asset. These additional factors are the size factor based on market capitalization and the value (book-to-market) factor. The authors divide monthly returns into portfolios based on size and value and found that smaller companies and higher book-to-market ratio companies generate on average higher returns (Fama & French, 1992).

However, there are discrepancies in the findings depending on the weighting schemes used to calculate portfolio returns. For equal-weighted portfolios, they find that both the size and value factors are significant with a positive alpha. On the other hand, for value-weighted portfolios, the size value is less significant, showcasing sensitivity to methodological choices (Fama & French, 1992).

2.4 Idiosyncratic volatility

Idiosyncratic volatility refers to the volatility of an asset that is not explained by general market factors and depending on which factors that are taken into consideration in the definition, the estimates can be vastly different. To achieve proper and reliable results, it is crucial that both the definition and the estimate of idiosyncratic volatility is done accurately. Our methodology is thus based on the frameworks found in Ang et al. (2006) and Fu (2009), two widely known papers with many replications in subsequent studies.

Given the shortcomings of the CAPM to explain cross-sectional returns and widespread adoption of the FF-3 model in the world of empirical finance, Ang et al (2006) decides to measure idiosyncratic volatility relative to the FF-3 model. That is, the idiosyncratic volatility measures the variation in a stock's return that is unique to that particular stock, controlling for the factors that appear in the FF-3 model.

Dividing stocks into five quintiles based on the one-month lagged idiosyncratic volatility and looking at the average return, Ang et al (2006) finds that stocks in the bottom quintiles outperforms the stocks in the top quintiles by more than 1% per month. And by regressing the portfolio returns onto the FF-3 model, they find that the stocks in the highest IVOL quintile have statistically significant and abysmally low returns in relation to the factor model. The results are robust for controlling for size, value, momentum, dispersion of analysts' forecasts and liquidity.

Building on the previous study, Ang et al (2009) expands the scope of the research into financial markets outside of the U.S. The authors find that across 23 developed financial markets in the G7 countries, stocks with high idiosyncratic volatility tend to generate low expected returns. The results are significant across all countries while controlling for private information, analysts' coverage, transaction costs, institutional ownership and even delay measures.

In criticism of utilizing the one-month lagged idiosyncratic volatility, Fu (2009) states that idiosyncratic volatilities are time-varying and therefore, the one-month lagged value is not a good proxy for the expected result. He explains that the findings of Ang et al (2006) can be explained by a small subset of firms with high idiosyncratic volatility. According to Fu (2009), firms with high idiosyncratic volatility generate high returns in the month of high idiosyncratic volatility but for the subsequent month, the returns reverse resulting in low returns for that period. Instead of using the one-month lagged idiosyncratic volatility, Fu (2009) uses EGARCH to estimate future volatility. Fu (2009) continues the analysis by conducting a Fama-Macbeth regression and finds a significant positive relationship between high expected idiosyncratic volatility and returns. Moreover, he extends the analysis by also sorting portfolios based upon the estimated idiosyncratic volatility, just like Ang (2006) but with 10 deciles instead of 5 quintiles. In the portfolio return analysis, Fu (2009) also finds that high idiosyncratic volatility stocks have positive significant alphas, strengthening his previous findings.

More recently, Chen et al. (2020) published a study regarding idiosyncratic volatility in a microstructural noise environment and find that the anomaly is only strong after excluding stocks that are prone to high microstructure noise. After excluding stocks with high microstructure noise such as microcaps and penny stocks, the anomaly is robust for both equal-weighted and value-weighted portfolios. This strengthens the findings of Ang et al (2006).

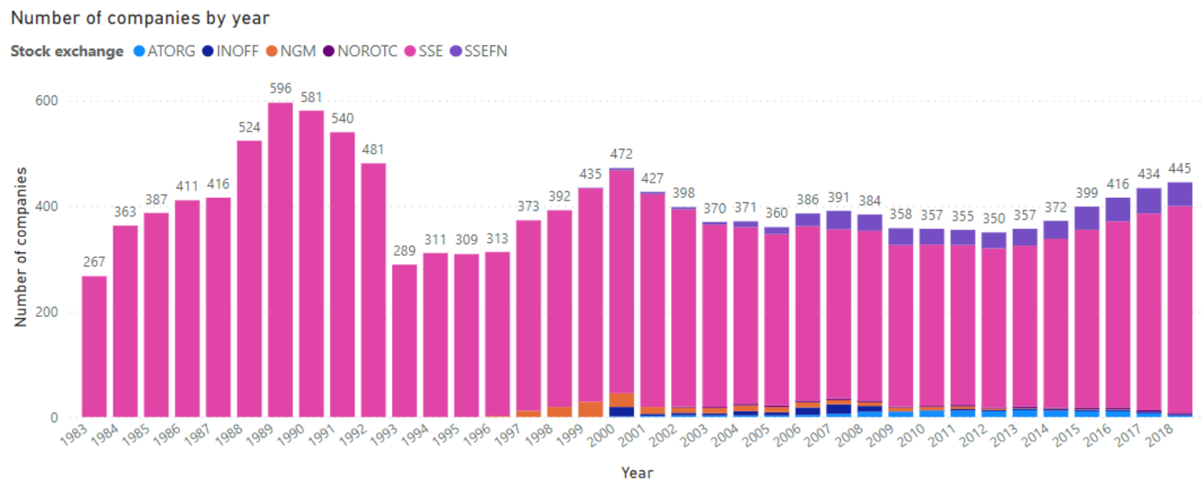
3. DATA

In order to conduct our analysis, we retrieve data from the Swedish House of Finance's (SHoF) database. The data include daily stock returns on firms listed on various Swedish markets, coupled with the FF-3 model factors. Moreover, we gather daily data on the one-month Swedish treasury bill as a proxy for the risk-free rate. All data cover the period from 1983 to 2019. Additionally, we gather monthly data on the same variables from the same database and over the same time period.

Table 1: Description of market names in full sample

Market name	Description
ATORG	Aktietorget
INOFF	Unofficial quotation list
NGM	Nordic Growth Market
NOROTC	NGM Nordic MTF
SSE	Stockholm Stock Exchange
SSEFN	SSE First North

Graph 1: Distribution of firms by market name throughout the whole sample period



As seen in the graph above, firms in SSE are overrepresented in the sample using the data retrieved from SHoF. Looking at the distribution of companies based on stock exchange, we can see that the sample taken from SHoF does not contain particularly many of the smallest firms which may impact the result.

4. METHODOLOGY

4.1 Estimation of idiosyncratic volatility

Idiosyncratic volatility (or risk) is firm-specific risk that is independent of market movements. Idiosyncratic volatility is not observed ex ante; hence it needs to be estimated. Inspired by earlier studies, we estimate the idiosyncratic volatility relative to the FF-3 model. We perform time-series regression for every firm and every month using daily excess returns from the previous month according to (Eq. 1).

$$r_t^i = a^i + \beta_{MKT}^i MKT_t + \beta_{SMB}^i SMB_t + \beta_{HML}^i HML_t + \epsilon_t^i \qquad \text{(Eq. 1)}$$

Where r_t^i is the daily excess return of the asset, a^i is the intercept, $\beta_{MKT}^i MKT_t$ represent the effect of the market return, $\beta_{SML}^i SML_t$ controls for the size effect, $\beta_{HML}^i HML_t$ controls for the value premium and ϵ_t^i is the error term representing idiosyncratic returns.

The three factors are proxied by using SHoF's daily data on each factor for the Swedish market. SHoF estimate the market return (MKT) by using the return of the SIX Return Index, which is a value-weighted index of all the stocks listed on the SSE-market, and that also includes reinvested dividends (Aytug, Fu, Sodini, 2020). The excess market return is computed by taking the market return minus the risk-free rate proxied by the one-month Swedish T-bill rate. The Small-minus-big factor is (SMB) is computed as follows:

$$SMB = \frac{(SG + SN + SV)}{3} - \frac{(BG + BN + BV)}{3} \qquad \text{(Eq. 2)}$$

Where SG, SN, SV is the returns of the Small-Growth, Small-Neutral and Small-Value portfolios respectively. Analogously, BG, BNN and BV represents the returns of the Big-Growth, Big-Neutral and the Big-Value portfolios. Thus, SMB is a factor controlling for the size of the companies.

The HML-factor is estimated by Eq. (3),

$$HML = \frac{(SV + BV)}{2} - \frac{(SG + BG)}{2} \qquad \text{(Eq. 3)}$$

Where SV and BV are the returns of the Small-Value and Big-Value portfolios, and SG and BG are the returns of the Small-Growth and Big-Growth portfolios. Ultimately, HML is a factor that controls for the value-effect on stock returns.

The idiosyncratic risk (IVOL) is then defined as $\sqrt{var(\epsilon_t^i)}$ from the FF-3 model found in (Eq. 1) and is transformed into monthly idiosyncratic volatility by multiplying with the square root of the number of trading days for ease of comparison with earlier literature. We repeat the estimation of the idiosyncratic volatility for every month for every firm. The mean idiosyncratic volatility of all the 155 738 firm-month observations is estimated to 11.81%, with a standard deviation of 7.98%, which is considerably lower than the 14.17% and

13.91%, respectively, that Fu (2009) observed in the US market. We sort portfolios based on the level of estimated idiosyncratic volatility in the next section. Lagged idiosyncratic volatility and IVOL are used interchangeably through the essay, meaning the idiosyncratic volatility estimated in line with the methodology of Ang et al. (2006).

4.2 Estimation of expected idiosyncratic volatility

To obtain reliable results, the estimated idiosyncratic volatility must be of high quality. The above estimation of idiosyncratic volatility uses IVOL from the previous month which inherently assumes that the idiosyncratic volatility follows a random walk process and hence is best predicted by simply using the lagged value. Fu (2009) performs a standard unit-root test and compares the t-stat with the Dickey-Fueller critical value to test the random walk hypothesis for US-stocks between 1963 and 2006. Fu (2009) rejects the hypothesis at the 1% for 90% of the companies. Hence, according to Fu's (2009) results, it is not appropriate to assume a random walk when estimating idiosyncratic volatility for US stocks. Although our sample does not contain US stocks and we have a different time horizon, we still take the concern of Fu (2009) into consideration by completing the analysis with another estimation of idiosyncratic volatility. Fu (2009) suggests estimating expected idiosyncratic volatility using an Exponential Generalized Autoregressive Conditional Heteroscedasticity (E-GARCH) model. Therefore, in addition to the lagged idiosyncratic volatility, expected idiosyncratic volatility is estimated by emulating the method of Fu (2009). The predicted or expected idiosyncratic volatility and EIVOL are used interchangeably through the essay, both referring to idiosyncratic volatility estimated in line with Fu's (2009) method.

4.2.1 E-GARCH

GARCH models are commonly used to estimate the volatility of financial data. It is an extension of the ARCH model that incorporates the tendency of persistency of volatility in financial data. That is, periods of high volatility are likely to be followed by periods of high volatility, and vice versa. Moreover, extending the GARCH model to E-GARCH Nelson (1991) explains that the model now captures the propensity of return volatility to increase after a stock price drop. E-GARCH models the conditional volatility of the data as a function of past squared error terms and past variances. Doing so, the model captures tendency of volatility clustering. One advantage of the E-GARCH model is that it models the leverage effect of the time series data. Meaning the tendency of volatility to react more to large decreases than large increases. Furthermore, EGARCH models have been proven to provide accurate forecasts of volatility in financial data in comparison to other GARCH models, making it an attractive choice (Pagan & Schwert, 1990).

We estimate the residuals for each firm and each month using the FF-3 model seen in (Eq.4) below.

$$r_t^i = a^i + \beta_{MKT}^i MKT_t + \beta_{SMB}^i SMB_t + \beta_{HML}^i HML_t + \epsilon_t^i , \quad \epsilon_t^i \sim N\left(0, \sigma_t^i\right) \qquad \text{(Eq. 4)}$$

In (Eq. 4), r_t^i is the monthly excess return of an asset i, which is explained by the intercept a^i, in addition to the estimated betas times the factor returns of MKT, SMB and HML as well as the monthly residual ϵ_t^i. The residuals are assumed to be normally distributed with mean zero and a conditional variance that is described with the explicit functional form below in (Eq.5).

$$ln\ \sigma_{it}^2 = \kappa_i + \sum_{l=1}^{P} \gamma_{it} ln\sigma_{i,t-l}^2 + \sum_{j=1}^{Q} \alpha_{i,j} \left[\frac{\left|\varepsilon_{i,t-j}\right|}{\sigma_{i,t-j}} - E\left\{ \frac{\left|\varepsilon_{i,t-j}\right|}{\sigma_{i,t-j}} \right\} \right] + \sum_{j=1}^{Q} \xi_j \left(\frac{\varepsilon_{i,t-j}}{\sigma_{i,t-j}} \right) \qquad \text{(Eq. 5)}$$

The conditional variance is a function of the past variances of the residuals (σ_{t-j}^2), as well as the past return shocks (ε_{t-j}). κ is the constant, γ is the GARCH-parameter, α is the ARCH parameter, and ξ captures the asymmetric response of volatility to positive and negative shocks. P and Q are the number of past variances and number of past return shocks that are used to estimate the conditional variance. We follow Fu's (2009) methodology and estimate nine different E-GARCH using all permutations of where the number of residual variance lag (P) is between 1-3 and the number of lags of returns shocks (Q) is between 1-3 illustrated in Table 2 below.

Table 2: Configurations of E-GARCH processes

		P		
		p=1	p=2	p=3
Q	q=1	E-GARCH (1,1)	E-GARCH (2,1)	E-GARCH (3,1)
	q=2	E-GARCH (1,2)	E-GARCH (2,2)	E-GARCH (3,2)
	q=3	E-GARCH (1,3)	E-GARCH (2,3)	E-GARCH (3,3)

We use an expanding window of data to estimate the conditional idiosyncratic variance using all available information up to t-1. In compliance with the method of Fu (2009) we require at least 30 monthly observations prior in order to qualify for estimation. The nine models are then compared for each firm and time period using Akaike Information Criterion (AIC). The AIC-score is computed with the following equation (Eq. 6):

$$AIC = 2k - 2\log(L) \qquad \text{(Eq. 6)}$$

Where k is the number of parameters (P+Q) in the model, and L denotes the maximum likelihood for the estimated model (Akaike, 1974). Hence, the AIC score takes both the accuracy, and the number of parameters in the model into account. Higher likelihood is encouraged in the score, while increasing the number of parameters in the model is penalized. The model with the lowest AIC score is then chosen in order to provide the best estimate of the expected idiosyncratic volatility.

We repeat the estimation of expected idiosyncratic volatility and AIC score for each month and each firm. The mean expected idiosyncratic volatility of all the 111 811 firm-month observations is 9.71% with a standard deviation of 7.05%, which is lower than for the lagged idiosyncratic volatility. The correlation between the expected idiosyncratic volatility and the lagged idiosyncratic volatility is 35.53%, showcasing that they are different but that there exists a relation between them. The expected idiosyncratic volatility is used in the next section to sort stocks into portfolios.

4.3 Portfolios

4.3.1 Portfolio weighting schemes

In order to compute the portfolio returns, the portfolio weighting need to be taken into consideration. We use the following two weighting schemes that both are commonly used in financial research (for instance, see Ang (2006 & 2009), Fu (2009) and Fama & French (1992)), as well as weighting schemes for investing funds.

Equal-weighted portfolio:

The asset weights for an individual stock (i) at time t in a portfolio is calculated as follows in (Eq. 7):

$$w_{i,t} = \frac{1}{N_t} \qquad \text{(Eq. 7)}$$

Equal-weighting means that all the stocks in the considered investment universe (in our case, the stocks of the SSE-index for which data is available) receive an equal weight. Thus, a small company is as heavily invested in as a large company, leading to a tilt towards smaller stocks in comparison to the value-weighted strategy. Equal-weighted strategies are known to perform well in good times as small companies generally are more volatile but with higher expected return than their larger counterparts.

Value-weighted portfolios:

For the value-weighted portfolio, the asset weights are proportional to the size of the companies' market capitalization. For an individual stock (i) at time t, the weight is calculated as follows in (Eq. 8):

$$w_{i,t} = \frac{Market\ cap_{i,t}}{\Sigma_{i=1}^{N_t} Market\ cap_{i,t}} \qquad \text{(Eq. 8)}$$

Thus, value-weighted strategies invest heavier in larger firms in relation to smaller firms. Moreover, when the portfolio is rebalanced, stocks that have done well will receive further investment while stocks whose price has declined will be sold off to an extent. Therefore, the strategy implicitly captures parts of the momentum effect as it sells losers and buys winners.

By calculating portfolio returns with both equal-weighting and value-weighting, we control for differences in results that arise because of how the weighting is calculated rather than the effect of idiosyncratic volatility.

4.3.2 Portfolio sorting

We sort the stocks in the sample into five different quintile portfolios based on the estimated lagged (or forecasted) monthly idiosyncratic volatility. Firms with missing information on any of the factors, returns, or market cap are excluded. We call the portfolio consisting of the lowest 20% idiosyncratic volatility-stocks the rank 1 portfolio, and the highest 20% the rank 5 portfolio.

Table 3: Portfolio ranking based on idiosyncratic volatility

Idiosyncratic volatility				
Portfolio 1	Portfolio 2	Portfolio 3	Portfolio 4	Portfolio 5
1st quintile (Bottom 20%)	2nd quintile (20-40%)	3rd quintile (40-60%)	4th quintile (60-80%)	5th quintile (Top 20%)

These portfolios are rebalanced each month in accordance with the new monthly estimates of the idiosyncratic volatility, just like in the method of Ang et al (2006) as well as Fu (2009). The excess return of each portfolio is calculated by multiplying the weights of each stock in the quintile portfolio with the respective monthly excess return for that stock. We calculate the returns using both an equal weighting scheme and value weighting for robustness.

4.3.3 Cross-sectional regression

We follow the methodology of Ang et al (2006) by comparing the average returns of the different quintile portfolios, as well as regressing the portfolio return against the FF-3 model. We focus mostly on the rank 1 and rank 5 portfolios in the comparison to see whether the portfolios investing in stocks with the lowest or highest amounts of firm-specific risk yield any excess returns over the FF-3 model. Moreover, the regression on the zero-cost-portfolio that invest in high idiosyncratic volatility stocks and short-sell low idiosyncratic volatility stocks shows whether there are statistically significant differences between the returns of those portfolios. The logic is that if the FF-3 model is correct, there would be no statistical differences between the portfolios sorted on different levels of idiosyncratic volatility. However, if idiosyncratic risk is in fact priced in and is not captured by the FF-3 model, there should be differences in average returns between the portfolios with disparities in exposure to firm-specific risk.

4.4 Robustness scenarios

4.4.1 Robustness over double sorting on Size and Book-to-market

In addition, we perform a set of robustness tests, starting with double sorts on idiosyncratic volatility together with size and book-to-market respectively. Size and book-to-market are commonly thought to affect the expected returns of a company and the double-sorting offers an additional way for the FF3-model to control for these characteristics. Specifically, we control for these firm characteristics by first sorting the firms into five quintiles based on the characteristic (size, BM-ratio), and then each characteristic quintile is sorted into five quintiles based on IVOL and EIVOL. Then the results are compared across the characteristic quintiles to see if only the portfolios with the smallest or largest companies yield significant alphas (or analogously, firms with the highest or lowest BM ratios). The logic is that if the result is robust, we would see the same patterns in portfolio alphas for the portfolios consisting of small firms as well as the large firms. Finally, the characteristic quintiles are averaged across to create five new portfolios with different levels of IVOL (low to high) and that contains firms with all different sizes. In this way, we control for size since every portfolio has a balance of small, medium, and large firms. The same procedure is used to control for book-to-market values.

Table 4: Double sorting on size and idiosyncratic volatility

Double sorting on size and idiosyncratic volatility						
	Idiosyncratic volatility					
Size		1 – Low	2	3	4	5 – High
	1 – Low					
	2					
	3					
	4					
	5 – High					

Table 5: Double sorting on book-to-market and idiosyncratic volatility

Double sorting on book-to-market and idiosyncratic volatility						
	Idiosyncratic volatility					
Book-to-market		1 – Low	2	3	4	5 – High
	1 – Low					
	2					
	3					
	4					
	5 – High					

4.4.2 Robustness over different subsamples

Ang et al (2006) raises a concern that the anomaly may only be apparent in specific parts of the business cycle, so that the results may be sensitive to how many recessions (or other relevant events) that happen during the sample period. Thus, in line with Ang et al (2006) we explore this by dividing the total sample into sub-samples of approximately ten years, regressing the portfolios sorted on idiosyncratic volatility onto the FF-3 model and then observe the results.

4.4.3 Robustness over different implementation strategies (L/M/N)

Lastly, we perform the same FF-3 model regression on idiosyncratic volatility as earlier but using different implementation strategies. These implementation strategies are in line with Ang et al's (2006) methodology and are described as L/M/N strategies. L denotes the number of months of daily data that is used to estimate the idiosyncratic volatility, M dictates which lag of the estimated IVOL months idiosyncratic volatility that is used, while N determines the number of months the portfolios are held. Our main results are estimated using the 1/0/1 strategy, which means that we use one month worth of daily data to estimate IVOL, which we then use for the upcoming month's portfolio sorting. This portfolio is then held one month before rebalancing and re-estimating the IVOL again. However, to test whether our results are robust for different strategies, we also estimate our regression with the returns from the 1/0/12, 12/0/12, 12/0/1 -strategies. The methodology is an emulation of Ang et al (2006); however, one difference is that we use M=0 instead of M=1 which Ang et al (2006) used, with the reasoning that M=0 is more fit to compare with our main strategy. To construct the 12/0/12 quintile portfolios, we follow a monthly process. First, we calculate the idiosyncratic volatility using daily data over a period of 12 months, ending at the formation date and forming portfolios based on that information. Next, we repeat this process for 12 consecutive months, each time using return data from an earlier time period. Specifically, we construct portfolios based on returns ending 1 months prior, 2 months prior, and so on, up to 12 months prior. The quintile portfolios are then calculated by taking the simple average of the 12 portfolios mentioned above. Additionally, a twelfth of the portfolio is rebalanced each month.

Robustness test to different L/M/N strategies is only performed in the section covering the results and analysis of IVOL, and not EIVOL. The estimation of EIVOL uses an expanding window of past data to estimate optimal weights for the forecast. Using fewer observations to estimate a worse set of optimal weight thus makes little sense.

5. RESULTS & ANALYSIS

5.1 Alphas on portfolios sorted on IVOL

Table 6: Descriptive statistics and alphas of portfolios based on lagged idiosyncratic volatility

The rank shows what quintile the portfolio invests in. For instance, the rank 1 portfolio contains the firms that are in the bottom 20:th percentile in idiosyncratic volatility, while the rank 5 portfolio contains the firms that are in the 80-100:th percentile of idiosyncratic volatility. 5-1 reports the values of a zero-cost portfolio. The table shows the mean excess return (in percentage), standard deviation of returns, as well as the size of the companies in each quintile portfolio. Size is calculated as the average log market cap of the firms in each portfolio. The column "FF-3 Alpha" shows the coefficient (in percentage) of the intercept when the corresponding portfolio's return is regressed on the Fama-French-three factor model. Robust Newey-West p-value is reported in the brackets []. Bold numbers indicate significance on the five percent level.

Rank	Mean	Std.dev	Size	FF-3 Alpha
Panel A: Portfolios sorted on Idiosyncratic Volatility - EW				
1	1.06	4.88	18.524	**0.5** **[0.001]**
2	1.05	5.54	18.84	**0.33** **[0.007]**
3	1.04	6.4	18.017	**0.275** **[0.04]**
4	0.88	7.38	18.198	-0.03 [0.83]
5	1.34	9.92	17.262	0.06 [0.79]
5-1	0.28	7.773		-0.43 [0.09]

Panel B: Portfolios sorted on Idiosyncratic Volatility using - VW				
1	0.62	5.69	18.524	0.06 [0.63]
2	0.89	5.92	18.84	0.23 [0.17]
3	0.72	7	18.017	-0.12 [0.51]
4	0.57	9.06	18.198	-0.43 [0.13]
5	1.17	10.19	17.262	-0.2 [0.537]
5-1	0.56	8.08		-0.25 [0.444]

Amongst the returns of the portfolios containing the different lagged idiosyncratic volatility quintiles, we observe average monthly excess returns ranging from 0.88 to 1.34% using an equal weighting scheme. The value-weighted strategy has lower average returns ranging from 0.57 to 1.17%. This is in line with the findings of Ang et al (2006) where they also find that equal-weighted strategy seems to outperform the value-weighted strategy. The equal-weighted portfolios show a pattern that lower idiosyncratic volatility seems to indicate higher average excess returns, with the exception of the quintile with the highest IVOLs. Examining Panel B with the value-weighted quintile portfolios, there is no apparent pattern in average excess returns.

It is clear that returns of the portfolios with the highest idiosyncratic volatility also have the largest standard deviation, and vice versa, indicating that idiosyncratic risk seems to correlate with the overall riskiness of a portfolio allocation strategy.

We find positive and significant alpha on the Rank 1 IVOL portfolio (0.5%), Rank 2 (0.33%) as well as the Rank 3 portfolio (0.27%) for the equal-weighted portfolios, showcasing that the low idiosyncratic volatility-strategies yield returns in excess to what the factor model predicts. The Rank 4 and Rank 5 portfolios are insignificant; hence there is no clear relation between high idiosyncratic volatility and expected returns in our sample. Furthermore, the zero-cost portfolio exhibits a negative alpha but is only significant at the 1% level, indicating a weak disparity between both ends of the idiosyncratic volatility-based portfolios. Thus, based on the regressions on the returns of the equal-weighted portfolios, we see evidence of the low volatility anomaly.

The anomaly takes the form of positive alpha for the quintiles with the lowest IVOL in comparison to Ang (2006), where instead the quintile with the highest IVOL had a significant negative alpha. Thus, instead of high IVOL being open to exploitation for excess returns by shorting it, we find that a better strategy for the Swedish market would have been avoid firms with high firm-specific risk and instead invest in low IVOL stocks. We don't find any evidence for high IVOL quintiles to yield positive alphas as found by Fu's (2009).

When incorporating a value-weighted strategy for the portfolios, the low rank portfolios have positive alphas while the high rank portfolios have negative alphas, both in line with the idiosyncratic volatility anomaly. However, none of the alphas are significant, thus indicating that the evidence for the anomaly in the Swedish market is weak given its sensitivity to chosen weighting schemes. Consequently, the results using value-weighting points to an absence of the anomaly rather than its existence, which is in line with the theory that holding idiosyncratic risk should not yield any extra compensation.

5.2 Alphas on portfolios sorted on EIVOL

Table 7: Descriptive statistics and alpha of portfolios sorted on EIVOL

The rank shows what quintile the portfolio invests in. For instance, the rank 1 portfolio contains the firms that are in the bottom 20:th percentile in expected idiosyncratic volatility, while the rank 5 portfolio contains the firms that are in the 80-100:th percentile of idiosyncratic volatility. The table shows the mean excess return, standard deviation of returns, as well as the size of the companies in each quintile portfolio. Size is calculated as the average log market cap of the firms in each portfolio. The column "FF-3Alpha" shows the coefficient of the intercept when the corresponding portfolio's return is regressed on the Fama-French-three factor model. In the brackets [], the robust Newey-West p-value is reported. Bold numbers indicate significance on at least the five percent level.

Rank	**Mean**	**Std. Dev**	**Size**	**FF-3 Alpha**
Panel A: Portfolios sorted on Idiosyncratic Volatility - EW				
1	1.08	5.01	18.78	**0.414** **[0.001]**
2	0.94	5.14	18.30	**0.24** **[0.04]**
3	0.85	5.8	18.7	0.06 [0.67]
4	0.79	6.46	19.4	-0.06 [0.7]
5	1.31	9.18	18.42	0.128 [0.6]
5-1	0.23	6.45		-0.43 [0.087]
Panel B: Portfolios sorted on Idiosyncratic Volatility - VW				
1	0.75	5.51	18.78	0.187

				[0.23]
2	0.82	5.68	18.30	0.169
				[0.27]
3	0.59	6.63	18.7	-0.112
				[0.528]
4	0.69	7.86	19.4	-0.14
				[0.531]
5	0.78	9.56	18.42	-0.16
				[0.643]
5-1	0.03	8.01		-0.351
				[0.34]

The average returns of the equal weighted portfolios sorted on expected idiosyncratic volatility demonstrate similar patterns as the lagged idiosyncratic portfolios. The mean returns increase as the expected idiosyncratic volatility increases. However, the rank 5 portfolio is an exemption from this rule as it yields the highest average return even though it also has the highest expected idiosyncratic volatility, just as it did in the case of IVOL. The standard deviation of the monthly returns is increasing in rank of the portfolios, thus the higher EIVOL a portfolio has, the higher risk it is associated with. Like the result using lagged IVOL, the pattern disappears when using value weighted returns for the portfolios.

The regressions also show similar results as in the situation with lagged IVOL. In the EW case, the two portfolios with the lowest EIVOL have a positive, significant alpha of 0.4% & 0.2%, showcasing that those two portfolios yield a higher return than the FF-3 model would predict. The rank 4 & 5 portfolios have negative and positive alpha respectively, but highly insignificant like in the last table. Furthermore, the zero-cost portfolio (5-1) has a significant (at the 0.1 level) negative alpha, which indicates that there is a slight difference between the returns of the highest EIVOL portfolio and the lowest EIVOL portfolio returns.

When calculating the portfolio returns using value-weighting we once again find very similar results as in the case of lagged IVOL. The low rank (low EIVOL) portfolios are observed to have positive alphas while the high rank (high EIVOL) portfolios yield negative alphas. However, none of these results are significant, thus highlighting weakness in the anomaly again.

There seem to be no real difference in results when sorting on EIVOL instead of IVOL. Even though the methodology of Fu (2009) is followed carefully, we find different results. Low EIVOL yield positive results while Fu (2009) finds that high EIVOL yields positive alpha. Thus, if the large differences in results in previous research on the US market is a consequence of what methodology is used for estimating the idiosyncratic volatility, we don't see the same effect on the Swedish market.

The results align more with Ang et al´s (2006) research, although instead of high IVOL having negative effect on returns, we see that low EIVOL affects returns positively, but only with equal weighting in the returns. Hence, a strategy focusing on low EIVOL, or low IVOL would historically have yielded excess returns in relation to the FF3-model assuming an equal weighted strategy. There is evidence for the anomaly but given that the results are conditional on the chosen weighting scheme; the evidence is weak.

6. ROBUST REGRESSIONS

6.1 Robustness IVOL

The disparity between both the average returns as well as the significance of the alphas between the two weighing schemes may be a result of heavier investing into small companies (which in general are thought to yield higher returns) when using equal weighting. Furthermore, as we observe in the data, companies with higher idiosyncratic volatility seem to be smaller companies and vice versa. Thus, to further control for the effect of size we create new portfolios sorted on size and then idiosyncratic volatility as described in the methodology. We end up with a total of 25 portfolios and run a regression of each of their returns on the FF3-model. Additionally, book-to-market is generally known to be a characteristic that explains stock returns; hence it is also controlled for further by averaging across the 25 different characteristics portfolios.

6.1.1 Double sorting on Size & IVOL

Table 8: Alphas of Portfolios sorted on Idiosyncratic Volatility

The table shows double-sorted portfolios based on first size and then idiosyncratic volatility. For instance, column “1 Low” reports the alpha and p-value of the five portfolios with the lowest idiosyncratic volatility but with all sizes (1-5). The row “Small” shows the alphas and p-values of the five portfolios containing the smallest firms, and with all (1-5) different values of idiosyncratic volatility. The alphas and p-values are estimated by regressing the portfolio returns onto the Fama-French three factor model. The rows denoted by “Control for size” and “Control for BM” are the portfolios that are the result of averaging over all of the quintile portfolios sorted on the characteristic variable. Bold numbers indicate significance at the five-percent level.

		Ranking on idiosyncratic volatility					
		1 Low	2	3	4	5 High	5-1
Panel A: Portfolios sorted on idiosyncratic volatility - EW							
Size quantile	Small 1	**0.89**	**0.47**	0.398	0.19	**0.206**	0.012
		[0.001]	**[0.048]**	[0.14]	[0.56]	[0.001]	[0.08]
	2	**0.54**	**0.57**	0.198	0.03	-0.27	-0.82
		[0.02]	**[0.009]**	[0.365]	[0.89]	[0.47]	[0.059]
	3	**0.55**	0.33	0.26	-0.31	**-0.73**	**-0.013**
		[0.033]	[0.08]	[0.211]	[0.168]	**[0.015]**	**[0.001]**
	4	**0.95**	**0.414**	0.17	-0.34	**-0.54**	**-1.49**
		[0.000]	**[0.015]**	[0.376]	[0.105]	**[0.042]**	**[0.001]**
	Large 5	-0.02	0.25	0.22	0.13	**-0.42**	-0.42
		[0.98]	[0.068]	[0.1252]	[0.394]	**[0.046]**	[0.058]
Control for Size		**0.59**	**0.41**	0.25	-0.06	0.019	**-0.57**
		[0.001]	**[0.003]**	[0.055]	[0.68]	[0.93]	**[0.021]**
Control for BM		**0.39**	**0.43**	0.17	-0.08	0.25	-0.13
		[0.002]	**[0.002]**	[0.22]	[0.57]	[0.25]	[0.525]
Panel B: Portfolios sorted on Idiosyncratic Volatility – VW							
Size quantile	Small 1	**0.603**	0.556	0.294	-0.11	0.63	0.028
		[0.046]	[0.055]	[0.30]	[0.726]	[0.238]	[0.96]
	2	**0.634**	**0.62**	0.123	-0.04	-0.28	**-0.92**
		[0.015]	**[0.007]**	[0.578]	[0.87]	[0.472]	**[0.047]**
	3	0.571	0.377	0.225	-0.35	**-0.68**	**-1.25**

		[0.06]	[0.051]	[0.278]	[0.138]	**[0.027]**	**[0.001]**
	4	**0.896**	**0.442**	0.075	-0.37	**-0.59**	**-1.49**
		[0.001]	**[0.013]**	[0.693]	[0.08]	**[0.038]**	**[0.001]**
	Large 5	-0.19	0.287	0.025	0.21	-0.33	-0.14
		[0.227]	[0.083]	[0.888]	[0.286]	[0.201]	[0.62]
Control for Size		**0.5**	**0.46**	0.14	-0.13	-0.25	**-0.75**
		[0.003]	**[0.001]**	[0.26]	[0.37]	[0.267]	**[0.002]**
Control for BM		**0.16**	**0.31**	-0.07	-0.3	-0.15	-0.32
		[**0.002**]	**[0.027]**	[0.65]	[0.08]	[0.51]	[0.2]

From the regressions on the double sorted portfolios, it is evident that the positive alpha of low idiosyncratic volatility portfolios is not unique to small companies. The patterns of the alphas and their significance levels are reminiscing of the patterns of the regressions on the portfolios sorted only on idiosyncratic volatility where we saw positive significant alphas on portfolio 1 and 2. Analogously that is the case for the portfolios first sorted on size and then thereafter on IVOL, except for the largest quintile firms with the highest IVOL. Moreover, a large majority of the zero-cost portfolios (5-1) where the portfolio is long in high IVOL-companies and short in low IVOL-companies showcase significant alphas, signifying statistically determined differences in the two trading strategies.

The two rows with “Control for Size” show the result of the regression on portfolios that are averaged across each characteristic portfolio and shows positive significant alphas of 0.5% and 0.46% for the lowest 1 & 2 IVOL portfolios in the value-weighted scheme, and 0.59% & 0.41% with equal weighting. The value-weighted alpha of the 5-1-portfolio is -0.75%, while the equal-weighted alpha is –0.57%, both significant, showcasing differences between high and low IVOL-strategies. Controlling for size by averaging each portfolio sorted on market cap yields significant results that are in line with earlier findings for both weighting schemes.

Two disparities from the results of the simple sorted regressions are that there are several portfolios in the highest quintile of IVOL with negative significant alphas and that the results are robust for when using a value-weighting scheme. Thus, the low volatility anomaly is even more apparent in the Swedish market when controlling for size through double sorting. When regressing the zero-cost portfolios which are long the largest companies and short the smallest companies in each IVOL quintile, three of the ten portfolios have a significant alpha. Hence, those three portfolios show significant differences in returns as a result of the sizes of the companies which are invested in and not as a result of different levels of IVOL. Consequently, the pattern of small companies having higher returns may drive some of the idiosyncratic volatility anomaly in our sample, pointing out a weak point in the anomaly.

When controlling for the book-to-market-ratio of the companies the regression on the returns of the equal weighted and value weighted portfolios yields returns of similar patterns as earlier. The high rank portfolios have negative insignificant coefficients, while the two portfolios with the lowest IVOL have significant positive alphas of 0.39% & 0.43%, using equal weighting, and 0.16% & 0.31% using value weighting, ultimately disproving the concern of the BM-ratio driving the previously found results.

6.1.2 The idiosyncratic volatility effect over different subsamples

In line with the methodology of Ang et al (2006) we explore whether the anomaly is sensitive to different time periods by dividing the total sample into sub-samples of approximately ten years and regressing portfolios sorted on idiosyncratic volatility onto the FF-3 model.

Table 9: The idiosyncratic Volatility Effect over Different Subsamples

The table report the Fama and French (1992) alphas and the square brackets below show the Newey-West calculated p-values. Each column displays the results corresponding to the five portfolios containing stocks with different levels of IVOL. Column 1 reports values for the portfolio with the lowest IVOL firms, while column 5 reports values for the portfolio with the highest IVOL firms. Similar to Ang et al (2006) column “5-1" refers to the difference in alphas between Portfolio 5 and Portfolio 1.

	Ranking on idiosyncratic volatility					
Subperiod	**1**	**2**	**3**	**4**	**5**	**5-1**
	Low				**High**	
Panel A: Portfolio sorted on idiosyncratic volatility - EW						
Feb 1983 – Dec 1990	0.11	0.15	0.38	0.17	0.03	-0.08
	[0.594]	[0.545]	[0.002]	[0.533]	[0.932]	[0.655]
Jan 1991 – Dec 2000	0.18	0.03	-0.02	-0.15	0.55	0.37
	[0.492]	[0.887]	[0.94]	[0.584]	[0.249]	[0.489]
Jan 2001 – Dec 2010	**0.81**	0.54	0.29	-0.04	0.12	-0.69
	[0.002]	[0.051]	[0.318]	[0.926]	[0.814]	[0.178]
Jan 2011 – Dec 2018	**0.66**	**0.43**	0.42	0.04	0.23	-0.43
	[0.026]	**[0.034]**	[0.114]	[0.882]	[0.492]	[0.243]

Panel B: Portfolio sorted on idiosyncratic volatility - VW						
Subperiod	**1**	**2**	**3**	**4**	**5**	**5-1**
	Low				**High**	
Feb 1983 – Dec 1990	0.27	0.03	0.14	-0.33	-0.59	-0.86
	[0.372]	[0.936]	[0.727]	[0.414]	[0.192]	[0.506]
Jan 1991 – Dec 2000	0.04	0.01	-0.06	-0.39	0.77	0.73
	[0.863]	[0.966]	[0.861]	[0.438]	[0.178]	[0.226]

Jan 2001 – Dec 2010	0.35	0.20	-0.28	0.25	-0.88	-1.23
	[0.209]	[0.638]	[0.541]	[0.733]	[0.249]	[0.131]
Jan 2011 – Dec 2018	0.10	**0.50**	-0.23	-0.22	0.26	0.16
	[0.552]	**[0.045]**	[0.402]	[0.542]	[0.616]	[0.467]

As shown in the tables above, when studying the factor anomaly throughout various sample periods the results are in large insignificant. However, in Panel A, the low idiosyncratic volatility anomaly seems to yield significant positive alpha for Portfolio 1 during Period 3 and for both Portfolio 1 & 2 during Period 4. When studying the results using value-weighted returns, the insignificance is further exacerbated. Weaker significance when using value-weighting is not surprising given the results of our main regression. On the other hand, the alphas are in general of the same sign and close in magnitude to previous findings; combining that with the fact that dividing the sample into subsamples with fewer observations increases the noise in the estimation, the results are not unexpected. The subsample-robustness test shows that the anomaly is not strong in the Swedish market given smaller samples but does not disprove the existence of the anomaly as a whole.

If Mclean and Pontiff's (2016) theory that anomalies become smaller after academic publications would be true in this case, we would see diminishing excess returns for the low idiosyncratic volatility portfolios. Instead, we find adverse results. Combining that with the insignificance of the coefficients, we don't find evidence for a diminishing anomaly as described by Mclean and Pontiff (2016), but rather a weak anomaly throughout time.

6.1.3 Regression on different implementation strategies (L/M/N)

There is a possibility that stocks with higher idiosyncratic volatility earn higher returns in the long run, but that it is not prevalent in the first month after a period of high IVOL. To further investigate whether this is the case, as well as to examine the sensitivity to different holding and formation strategies, we perform regressions on portfolio returns using different L/M/N-strategies that among other things, allow longer holding periods, as is explained in the methodology section.

Table 10: Regression on different implementation strategies

The table report the alpha as well as the Newey-West calculated p-values of a regression of the portfolio returns onto the FF-3 model. Each column represents the results of the 1-5 portfolios that has different strategies relating to the level of idiosyncratic volatility in their investments. Each row represents a strategy that is further explained in the methodology section.

	Ranking on idiosyncratic volatility					
Strategy	1 Low	2	3	4	5 High	5-1
Panel A: Portfolio sorted on idiosyncratic volatility - EW						
1/0/12	**0.4**	**0.3**	0.03	-0.19	-0.06	**-0.46**
	[0.003]	**[0.018]**	[0.83]	[0.23]	[0.73]	**[0.019]**
12/0/1	**0.45**	**0.03**	0.18	-0.15	0.4	-0.05
	[0.001]	**[0.02]**	[0.185]	[0.372]	[0.11]	[0.85]
12/0/12	**0.44**	**0.27**	0.09	-0.08	0.38	-0.06
	[0.001]	**[0.033]**	[0.51]	[0.59]	[0.11]	[0.79]

Panel B: Portfolio sorted on idiosyncratic volatility - VW						
Strategy	1 Low	2	3	4	5 High	5-1
1/0/12	0.06	**0.34**	-0.2	-0.5	**-0.7**	**-0.76**
	[0.72]	**[0.026]**	[0.27]	[0.068]	**[0.01]**	**[0.01]**
12/0/1	0.08	-0.09	-0.16	-0.3	-0.18	-0.26
	[0.51]	[0.58]	[0.42]	[0.4]	[0.56]	[0.42]

12/0/12	0.04	0.04	-0.12	-0.25	-0.29	-0.33
	[0.7]	[0.7]	[0.33]	[0.16]	[0.204]	[0.16]

For all the strategies, the two quintiles of the lowest IVOL have positive, significant alphas that are higher for the lowest quintile, offering counterevidence to the possibility of differences between long run and short run returns. However, these results are only robust for the equal-weighted portfolios. In the case of value-weighted portfolio returns, while we for the most part see similar coefficients, only the 1/0/12 strategy has significant results. The second to lowest IVOL quintile have a positive alpha of 0.34% and the highest IVOL portfolio has a negative significant alpha of –0.7%, while the zero-cost portfolio displays differences between the high and low IVOL portfolios. The regressions on the VW portfolios utilizing 12 months of data (L=12) to estimate IVOL has the same patterns in average returns, but the alphas are all statistically insignificant. Thus, looking at the results of the VW regressions, it does seem like recent (past month) data on idiosyncratic volatility is what allows a strategy to yield excess return, offering some credibility to the claim that strategies based on IVOL may only catch short term effects on returns. These results showcase that the anomaly have some persistence while accounting for different strategies, but only when using equal weighting.

6.2 Robustness EIVOL

In this section, the same analysis will be done for portfolios sorted on the expected idiosyncratic volatility that was estimated using E-GARCH models.

The disparity between the results of the two weighting schemes introduces concerns that the sizes of the companies may be what is driving whether a strategy exhibits significant alpha or not. On the other hand, in the case of IVOL portfolios, we saw a pattern that low IVOL companies usually were smaller companies and vice versa, but in the case of EIVOL the pattern is not evident. However, to combat the concern of size driving the results, we perform additional control for size with the same methodology as was used in the IVOL section. Additionally, in line with earlier methodology book-to-market ratio is also controlled for in the same manner, as it is a characteristic that are known to generally impact returns.

6.2.1 Double sorting on Size & EIVOL

Table 11: Alphas of portfolios sorted on expected idiosyncratic volatility

The table shows double-sorted portfolios based on first size and then expected idiosyncratic volatility. For instance, column "1 Low" reports the alpha and p-value of the five portfolios with the lowest EIVOL but with all sizes (1-5). The row "Small" shows the alphas and p-values of the five portfolios containing the smallest firms, and with all (1-5) different values of idiosyncratic volatility. The alphas and p-values are estimated by regressing the portfolio returns onto the Fama-French three factor model. The rows denoted by "Control for Size" and "Control for BM" are the portfolios that are the result of averaging over all of the quintile portfolios sorted on the characteristic variable. Bold numbers indicate significance at the five-percent level.

		Ranking on idiosyncratic volatility					
		1 Low	2	3	4	5 High	5-1
Panel A: Portfolios sorted on idiosyncratic volatility - EW							
Size quantile	Small 1	**0.934**	0.059	-0.19	-0.24	**0.099**	0.011
		[0.002]	[0.824]	[0.457]	[0.475]	**[0.002]**	[0.13]
	2	**0.432**	0.235	-0.04	0.125	-0.03	-0.46
		[0.032]	[0.247]	[0.875]	[0.728]	[0.935]	[0.284]
	3	0.2	0.284	-0.19	-0.01	-0.61	**-0.8**
		[0.178]	[0.159]	[0.345]	[0.995]	[0.058]	**[0.008]**
	4	**0.445**	**0.33**	0.28	0.18	-0.64	**-1.1**
		[0.009]	**[0.04]**	[0.109]	[0.371]	[0.07]	**[0.002]**
	Large 5	0.225	0.091	0.155	0.3	-0.09	-0.32
		[0.086]	[0.506]	[0.258]	[0.084]	[0.643]	[0.095]
Control for Size		**0.45**	0.2	0.001	0.067	0.125	-0.32
		[0.001]	[0.12]	[0.98]	[0.7]	[0.583]	[0.131]
Control for BM		**0.38**	0.08	0.06	0.06	0.11	-0.27
		[0.002]	[0.49]	[0.67]	[0.72]	[0.63]	[0.18]
Panel B: Portfolios sorted on Idiosyncratic Volatility – VW							
Size quantile	Small 1	**0.83**	-0.02	-0.34	-0.24	0.97	0.143
		[0.024]	[0.936]	[0.263]	[0.554]	[0.059]	[0.816]
	2	0.4	0.24	0.01	0.16	-0.01	-0.39
		[0.074]	[0.253]	[0.70]	[0.695]	[0.98]	[0.43]
	3	0.21	0.35	-0.22	0.14	**-0.66**	**-0.87**
		[0.199]	[0.087]	[0.275]	[0.712]	**[0.039]**	**[0.004]**
	4	**0.46**	0.31	0.3	0.174	**-0.81**	**-1.27**
		[0.008]	[0.054]	[0.087]	[0.397]	**[0.016]**	**[0.001]**
	Large 5	0.11	0.236	0.089	0.028	-0.16	-0.27
		[0.488]	[0.199]	[0.588]	[0.879]	[0.544]	[0.353]
Control for Size		**0.4**	0.22	-0.03	0.052	-0.13	**-0.53**
		[0.003]	[0.098]	[0.81]	[0.78]	[0.571]	**[0.002]**

Control for BM	**0.51**	0.27	0.52	0.21	-0.12	**-0.6**
	0.83	-0.02	-0.34	-0.24	0.97	0.143

When controlling for size through five size quintiles before sorting on EIVOL, the patterns are like the earlier results. With the equal-weighting scheme, we find that three out of the five quintiles with the lowest EIVOL have significant positive alphas, and not only the smallest companies. Moreover, when the characteristics portfolios are averaged across, creating five new quintile portfolios based on EIVOL, but that contains firms with all different sizes, we find that the lowest EIVOL portfolio has a significant alpha of 0.45%. Thus, the results of the main regression in regard to EIVOL using equal weighting is robust to controlling for the size of the firms.

Two out of the five size quintiles in the lowest EIVOL category have significant positive alphas on the 0.05 level when using value weighting, both the smallest companies, but also the top 60-80% largest firms. While the results would have been more convincing if all the size quintiles with the lowest EIVOL had positive significant alphas, the results are at least not unique for only the small firms. All of the size quintiles with the lowest EIVOL still have positive alpha, albeit three of them insignificant.

When averaging across the size quintiles, we find positive significant alpha of 0.4% for the lowest EIVOL quintile, as well as negative significant alpha of –0.53% for the highest EIVOL quintile. Moreover, the five minus one portfolio in the VW scheme, also has a significant alpha, indicating differences between a strategy focusing on low EIVOL versus one focusing on high EIVOL. Thus, company size is not what drives the persistency of the low idiosyncratic volatility anomaly when estimating EIVOL with our GARCH models. In contrast to the first EIVOL regression when size was not controlled for by double-sorting, the results are robust for both equal-weighted portfolios and value-weighted portfolios.

Averaging across the characteristic portfolios based on the book-to-market-ratio the regression produces positive significant alphas for the lowest EIVOL quintile portfolios in line with earlier finding. The equal-weighted returns have an alpha of 0.38% and the value-weighted returns have an alpha of 0.51%. Moreover, while the coefficients of the other rank portfolios are insignificant, the signs are very much alike earlier findings, all together suggesting that the book-to-market-ratios are not what is causing the positive alphas of the low EIVOL portfolios.

6.2.2 The expected idiosyncratic volatility effect over different subsamples

In line with the methodology applied in the case of lagged IVOL we explore whether the anomaly is sensitive to different time periods by dividing the total sample into sub-samples of approximately ten years and regressing portfolios sorted on expected idiosyncratic volatility (predicted using E-GARCH) onto the FF-3 model.

Table 12: The expected idiosyncratic Volatility Effect over Different Subsamples

The table report the Fama and French (1987) alphas and the square brackets below show the Newey-West calculated p-values. Each column displays the results corresponding to the five portfolios containing stocks with different levels of EIVOL. Column 1 reports values for the portfolio with the lowest EIVOL firms, while column 5 reports values for the portfolio with the highest IVOL firms. Similar to Ang (2006) column “5-1" refers to the difference in alphas between Portfolio 5 and Portfolio 1.

	Ranking on idiosyncratic volatility					
Subperiod	**1**	**2**	**3**	**4**	**5**	**5-1**
	Low				**High**	
Panel A: Portfolio sorted on idiosyncratic volatility - EW						
Feb 1983 – Dec 1990	0.35	0.41	0.15	0.66	**1.02**	**0.66**
	[0.246]	[0.187]	[0.675]	[0.072]	**[0.024]**	**[0.015]**
Jan 1991 – Dec 2000	0.07	-0.30	-0.28	-0.10	0.24	0.17
	[0.739]	[0.151]	[0.242]	[0.777]	[0.566]	[0.700]
Jan 2001 – Dec 2010	**0.76**	**0.73**	0.37	-0.08	-0.12	-0.88
	[0.002]	**[0.004]**	[0.237]	[0.800]	[0.817]	[0.061]
Jan 2011 – Dec 2018	**0.51**	0.26	0.14	-0.07	0.44	-0.07
	[0.011]	[0.127]	[0.496]	[0.762]	[0.320]	[0.867]

Panel B: Portfolio sorted on idiosyncratic volatility - VW						
Subperiod	**1**	**2**	**3**	**4**	**5**	**5-1**
	Low				**High**	
Feb 1983 – Dec 1990	0.20	0.35	0.41	**1.06**	-0.83	0.63
	[0.586]	[0.376]	[0.315]	**[0.022]**	[0.123]	[0.116]
Jan 1991 – Dec 2000	0.18	0.18	0.13	0.23	0.13	-0.05
	[0.571]	[0.564]	[0.664]	[0.503]	[0.834]	[0.656]
Jan 2001 – Dec 2010	0.50	0.60	-0.42	-0.69	-0.85	-1.35
	[0.163]	[0.072]	[0.337]	[0.216]	[0.271]	[0.107]
Jan 2011 – Dec 2018	0.31	-0.16	-0.31	-0.49	0.35	0.05
	[0.066]	[0.411]	[0.27]	[0.220]	[0.641]	[0.949]

Dividing the sample up into subperiods yield results similar to the findings when using IVOL. Using equal-weighted returns, the alphas are of the same pattern as most of the regressions thus far; low rank portfolios have positive alphas, while it is split between positive and negative alphas for the high rank portfolios. Only a few of the alphas are significant, such as the rank 1 portfolios for 2001-2010 and 2011-2018 offering evidence for the anomaly for later parts of the full sample periods. On the other hand, when value-weighting the returns, only one portfolio has a significant alpha. However, the alphas in general are close to what we have earlier found (with a few exceptions). Together with the fact that the subperiods are short, allowing for more noise into the regression results, the absence of significant results is not surprising. Thus, we don't deny the existence of the anomaly based on this robustness test, but it showcases that the anomaly is not very strong.

7. CONCLUSION

Theories such as the CAPM predict that investors should not be compensated for holding idiosyncratic risk given its possibility of being diversified away. On the other hand, theories assuming under-diversification, such as Merton (1987) predict that idiosyncratic volatility should be compensated with excess returns. However, examining US stocks Ang et al (2006) finds that portfolios sorted on idiosyncratic volatility in relation to the FF-3 model show a negative relationship between idiosyncratic volatility and stock returns. Emulating the methodology of Ang et al (2006) we find that the portfolios containing the stocks with the lowest idiosyncratic volatility exhibit excess returns in relation to the FF-3 model. These results are robust to the size of the companies, book-to-market ratios and certain formation and holding strategies. However, the results are weak when using value-weighted portfolio returns and when dividing the sample into subsamples in the size of decades.

In 2009, Fu raise concerns about the time-series properties of idiosyncratic volatility and instead suggest estimating the firm-specific risk using E-GARCH models. Thus, we also follow the methodology of Fu (2009) to estimate expected idiosyncratic volatility and perform the same analysis of portfolio returns in relation to the FF3-model as we did for lagged IVOL. The results once again show a positive relationship between low idiosyncratic volatility and returns and are strikingly similar in robustness to the analysis using Ang et al's (2006) methodology. Thus, we find no evidence that the two different estimates produce different results.

We find evidence of the low volatility anomaly being apparent in the Swedish market, but the lack of significance using value-weighting indicates that it is weak. There is a possibility that some other characteristic which is more apparent in an equal-weighted strategy is driving the results. Hence, future research could focus on testing for more characteristic that are generally known to impact returns of companies as well as completing the analysis with data from earlier years than 1983 in order to further strengthen the empirical evidence of the anomaly.

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