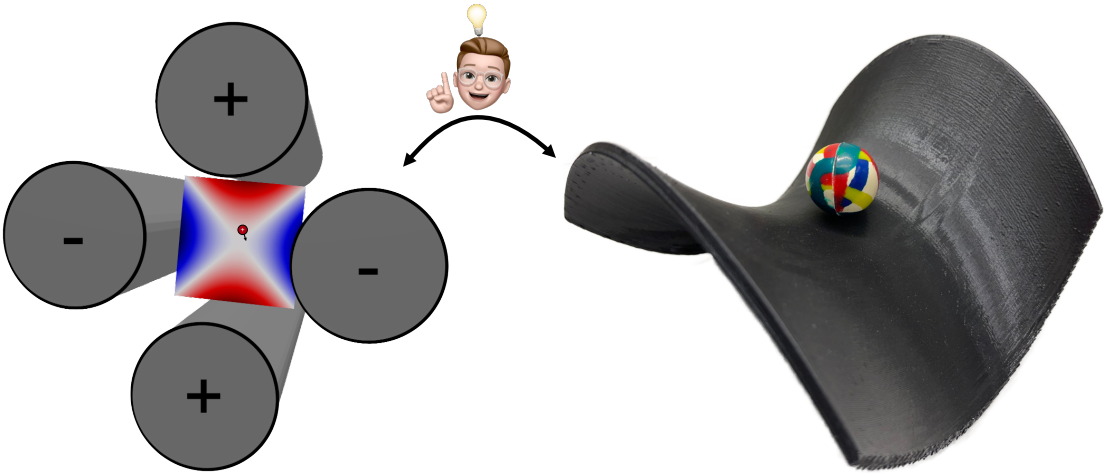
The mechanical Paul trap in the upper secondary physics laboratory



THESIS OF LICENTIATE DEGREE

The mechanical Paul trap in the upper secondary physics laboratory

SEBASTIAN KILDE LÖFGREN

Department of Physics
UNIVERSITY OF GOTHENBURG
Göteborg, Sweden 2023

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Sebastian Kilde Löfgren

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Department of Physics
University of Gothenburg
se-412 96 Göteborg, Sweden

Cover: To the left is a model of a linear Paul trap and a snapshot of the field inside with a particle. To the right is a picture of a mechanical Paul trap standing still with a ball balancing on the saddle point. The double-pointing arrow highlights that there exists some connection between the two and shows a cartoon resembling some learner who has discerned this connection

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'All models are wrong, but some are useful'

- George E. P. Box (1979)

Abstract

In physics, toy models are often used in popular science contexts. Further, toy models have also been used to communicate some important principles of abstract phenomena qualitatively. One such example can be found in the 1989 Nobel laureate Wolfgang Paul's Nobel lecture. Previous research on models in science education has identified that using models and engaging students in model development could improve learning outcomes. However, an important issue is making model development meaningful for students in similar ways that scientists can use models and the modelling process to learn. This thesis investigates the usefulness of a toy model of the Paul trap, the mechanical Paul trap, for the upper secondary physics classroom. To do this, two studies were conducted, considered part of one design experiment. The first study investigated the mechanical Paul trap and its similarities and differences to a linear Paul trap to develop a physical model and a simulation for the design experiment. The second study used a design-based research approach, guided by variation theory, to conduct a mixed-method study developing a laboratory exercise and investigating how students interacted with and what learning was made possible using the mechanical Paul trap. The laboratory exercise was developed over three phases, consisting of three to five laboratory sessions each. Results from the first study provided suggestions for constructing an accessible and affordable physical model of the mechanical Paul trap. In addition, it resulted in the development of two similar simulations but in different online environments. The second study identified patterns of variation, highlighting critical steps where struggling and successful lab groups differed. One example is that it was critical to discern the threshold frequency as a step to discern the rotational frequency-trapping time relation. Further, findings also support the argument that the level of openness in laboratory exercises should be chosen deliberately by the teacher, with both the current groups of students and the complexity of the laboratory exercise at hand. Finally, it contributes to the discussion regarding the use of models in physics education by exemplifying how working with models can better help students understand the limitations and use cases of different models in physics.

Keywords: Physics education research, discipline-based education research, physics laboratory, design-based research, variation theory, model-based science, modelling, mechanical Paul trap, Paul trap.

Sammanfattning

Inom fysik används ofta olika idealiserade modeller i populärvetenskapliga sammanhang. Liknande modeller har även använts för att beskriva viktiga principer tillhörande abstrakta fenomen kvalitativt. Ett exempel på detta återfinns i Wolfgang Pauls, Nobelpristagare i fysik 1989, Nobelföreläsning. Tidigare forskning om modeller inom undervisning i naturvetenskap har visat att användning av modeller och modellutveckling kan förbättra studieresultat. Dock finns en viktig fråga gällande hur elevers involvering i utveckling av modeller kan göras meningsfull på liknande sätt som forskare använder modeller och modellering för att lära sig. Denna avhandling undersöker hur en mekanisk modell av en Paulfälla kan användas i fysiklaborationer på gymnasienivå. För att göra detta genomfördes två studier inom ramen av ett designexperiment. Den första studien undersökte likheter och skillnader mellan den mekaniska Paulfällan och en linjär Paulfälla, med syftet att utveckla en fysisk modell och en simulering att använda i designexperimentet. Den andra studien följde en designbaserad forskningsansats tillsammans med variationsteori för att genomföra en mixed-method studie med syfte att utveckla en laboration och undersöka hur eleverna arbetade och vilket lärande som möjliggjordes med den mekaniska Paulfällan. Laborationen utvecklades i tre faser bestående av tre till fem laborationspass vardera. Resultaten från den första studien ledde fram till en design av en fysisk mekanisk Paulfälla som är billig och lätt för skolor att själva konstruera, då den utnyttjar 3D-skrivare och Arduino. Två simuleringar av den mekaniska Paulfällan i olika onlinemiljöer utvecklades också. Den andra studien identifierade variationsmönster som visar kritiska steg där laborationsgrupper som var framgångsrika särskilt skiljde sig från grupper som hade problem med att förstå laborationens olika delar. Till exempel var det kritiskt att identifiera tröskelfrekvens som ett steg att förstå kopplingen mellan rotationsfrekvens och fångtid. Resultaten från studien stärker även argumentationen gällande hur graden av öppenhet i laborationer bör väljas av läraren medvetet, där hänsyn tas till både den aktuella elevgruppen och laborationens komplexitet. Slutligen bidrar även studien till diskussionen gällande anvädning av modeller i fysikundervisning. Detta genom att visa hur arbete med modeller kan hjälpa elever att bättre förstå modellers användbarhet och begränsningar inom fysik.

Preface

In your hand, or, more probably, on the screen before you, you have a licentiate thesis in Physics Education Research (PER). A licentiate degree is formally recognised primarily in Sweden and Finland as a pre-doctoral degree equivalent to half the work of a doctoral thesis. For me, this thesis marks the halfway point on my way towards pursuing a PhD and should be read as me taking a step back and reflecting upon and synthesising the work I have done thus far.

I have always had a profound interest in education. During my years in primary education, I also noticed how interesting both mathematics and science, especially physics, were. This back-and-forth interest in science and education led me down a path where I started studying to become a teacher in physics and mathematics in 2012 at the University of Gothenburg. In 2017, I started working as a physics, mathematics, and later also programming, teacher at Alströmergymnasiet in Alingsås. However, during my teacher education years, an interest in educational research started to grow. So in the fall of 2020, in the midst of the Covid-19 pandemic, I started my journey to pursue a PhD in the cross-disciplinary field of PER.

The research presented in this thesis builds mainly on two papers, one accepted for publication and one under review. Due to the fortune of having great collaborators, I am not the sole author of these papers, and my contributions are presented in the list of publications. Preceding these papers, chapters providing necessary background information are provided, as well as framing the research done so far in relation to PER and the broader field of Discipline-Based Education Research.

The content of this thesis is to be seen as work done within PER. As such, the intended audience is one with an interest in teaching and learning physics in some capacity, either for educational or professional purposes as a researcher or teacher. This includes practising teachers, teachers in training, teacher educators, physics education researchers and science education researchers to name some.

Finally, before expressing my gratitude to all those who contributed to the completion of this thesis, there is one person that I must acknowledge first – you, the reader. Thank you for showing an interest in this work and for taking the time to read it. It is my hope that you enjoy reading this thesis as much as I did writing it!

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Without the support of many, this thesis would not have been.

I am grateful to my team of supervisors, all of whom I had the fortune to also meet in various courses during my teacher education. Jonas Enger, your steadfast support, never-ending positive attitude and eagerness to learn has been crucial to the work leading up to this thesis. Thank you for being, and continuing to be, an outstanding mentor. Jonathan Weidow, your interest in teaching and education is inspiring, and I am thankful for getting to collaborate with you and for all the insightful comments and discussions. Dag Hanstorp, your ability to lead and organise groups seemingly tirelessly never seizes to impress. Thank you for providing valuable discussions and scientific opportunities. Ann-Marie Pendrill, your passion for PER and outreach is marvellous. Thank you for providing valuable input both on the thesis text and the next steps towards my PhD.

The Department of Physics at the University of Gothenburg has been a great and joyful place to spend my days up until this point. This is in part thanks to all the fun and interesting fellow PhD students I get to share this journey with, but also thanks to the wonderful group Dag has developed over the years both within the department and in outreach with UNAM, Mexico City. For all the lunches, coffee discussions and everyday exchanges thank you to Miranda, Annie, Jakob, Ludi and Remi. Andreas, I could not have wished for a better fellow PhD student in PER and as a positive and supportive colleague in Sweden and Mexico. Meeting you before I applied to CUL made a big impact on my decision to start this journey. Rich, I feel fortunate that our paths crossed. Thank you for wanting to collaborate with me. Javier, you are a beacon of joy and an inspiration for me to also pursue research in physics when time allows for it. Thank you for all the conversations, lunches, beer etc.

I would also like to extend my gratitude to the graduate school CUL organisation for providing me with the opportunity to go from being a teacher to a PhD student. CUL is an inspiring environment and offers a good network due to the yearly conferences. Within CUL, I also find valuable support in the theme group MaNa with

its meetings and writing camps. Through MaNa, I also found my way into the research environment for Phenomenography, Variation theory and Learning Study, which provides much support and cross-disciplinary discussions with a common theoretical grounding. Some PhD students I've met through these environments that merit special recognition here are Fredrik, Anna, and Hanna. Thank you for always being up to discuss educational research and variation theory related to science and mathematics.

Lastly, I want to thank my parents for your support throughout my life, and recently for helping take care of Alle during tough times. And to my own little family, Amanda and Alle. Alle, thank you for forcing me to find balance and perspective. You are the rainbow in my life. Amanda, thank you for putting up with this crazy idea of starting to pursue a PhD and standing by my side with your unwavering support. Without you, I would never have been here.

Sebastian Kilde Löfgren
Gothenburg
May, 2023

Abbreviations

CTD	Concurrent Triangulation Design.
DBER	Discipline-Based Education Research.
DBR	Design-Based Research.
PER	Physics Education Research.
TA	Thematic Analysis.

List of publications and my contributions

This thesis consists of an introductory text and the following papers:

I **"The Mechanical Paul Trap: Introducing the Concept of Ion Trapping"**

Sebastian Kilde Lofgren[1], Ricardo Méndez-Fragoso[2], Jonathan Weidow[3], and Jonas Enger[1].

[1] Department of Physics, University of Gothenburg, Gothenburg, Sweden
[2] Facultad de Ciencias, Universidad Nacional Autónoma de México, México City, Mexico
[3] Department of Physics, Chalmers University of Technology, Sweden

Accepted for publication in: *The Physics Teacher*.

Contribution: Most development regarding designing the mechanical setup and classroom activities. Developed the first version of the simulation and the final version with Ricardo. The main writer of the manuscript.

II **"Rolling Balls or Trapping Ions? How Students Relate Models to Real-world Phenomena in the Physics Laboratory"**

Sebastian Kilde Lofgren[1], Jonathan Weidow[2], and Jonas Enger[1].

[1] Department of Physics, University of Gothenburg, Gothenburg, Sweden
[2] Department of Physics, Chalmers University of Technology, Sweden

Submitted.

Contribution: Most development regarding choosing the appropriate design and data collection tools. Responsible for data collection, analysis and interpretations. The main writer of the manuscript.

Supporting work

This thesis also draws from the following work of conference presentations and popular science publications.

Kilde Löfgren, S., Weidow, J., & Enger, J. (2022, January 6-8). *The Mechanical Rotating Saddle Trap in High School Physics Education* [Presentation]. AAPT Virtual Winter Meeting, online. https://www.aapt.org/Conferences/WM2022/upload/aapt_wm22_Program-1.pdf

Kilde Löfgren, S., Weidow, J., & Enger, J. (2022, June 15-17). *The Mechanical Paul Trap: A Playful Introduction to Ion Trapping for Upper Secondary Students* [Poster]. Fysikdagarna, Lund, Sweden. https://indico.cern.ch/event/1126478/timetable/#20220615

Kilde Löfgren, S., Weidow, J., & Enger, J. (2022, August 24-26). *The Mechanical Paul Trap in the Upper Secondary Physics Classroom: A Design-based Approach to Developing Laboratory Exercises* [Presentation]. EARLI SIG 9 Conference, Stockholm, Sweden. https://www.su.se/polopoly_fs/1.619064.1656861410!/menu/standard/file/Book%20of%20Abstracts%20SIG9%202022.pdf

Kilde Löfgren, S., Weidow, J., & Enger, J. (2023, January 14-17). *Using Mechanical Models to Teach Secondary School Students Particle Trapping*. AAPT Winter Meeting, Portland, OR, USA. https://www.aapt.org/Conferences/WM2023/upload/aapt_wm23_Program_final.pdf

Fäldt, Å., Fredlund, T., & **Kilde Löfgren, S.** (2023, April, 3) *Mathematical concepts in physics - the gyroscopic effect* [Presentation]. KUL2023, Gothenburg, Sweden. https://ui.ungpd.com/Events/44ecf437-a5bd-48d6-8bbd-0188c8488f11

Kilde Löfgren, S., Tello Marmolejo, J., Ringvall Moberg, A., Enger, J., & Hanstorp, D. (2023, May). Fysik på Ullevi: Levande experiment. *Fysikaktuellt,* (2023-2). 13.

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1

Introduction

With Physics Education Research (PER) being a relatively young field within physics (Beichner, 2009; Heron, 2018; McDermott, 2021), there is a need for me to start this thesis by providing a brief introduction to the field, popular research areas, and where my work fits into the bigger picture of this vibrant community. It is argued that with limited resources that come with being a relatively small field, it could hinder the field if the research is too broad in terms of topical focus (Heron & Meltzer, 2005).

Reviews aimed at identifying what topical areas of research are of interest within PER show an overall tendency towards studying problems that could be sorted into six categories: conceptual understanding, problem-solving, curriculum and instruction, assessment, cognitive psychology, and attitudes and beliefs about teaching and learning (Docktor & Mestre, 2014; Odden et al., 2020). According to the recent work in developing *The International Handbook of Physics Education Research: Learning Physics* (Tasar & Heron, 2023a), *Teaching Physics* (Tasar & Heron, 2023c), and *Special Topics* (Tasar & Heron, 2023b), there seems to be a general agreement to keep focusing on earlier research areas. Still, there are also hints of new interests. Two examples are the more socio-cultural-inspired issues regarding inclusivity and gender (e.g. Andersson & Johansson, 2016; Phillips & O'donnell, 2021; Potvin et al., 2023), and the use of modern, easily accessible technology to expand and transform laboratory work (e.g. Bodegom et al., 2019; Galán et al., 2019; Vieyra et al., 2020).

As a graduate student in physics, specialising in educational sciences connected to the graduate school in educational science at the Center for Educational Science and Teacher Research, one part of the journey up to this point has been tackling the question of where PER belong. Is it more related to the realm of physics or educational research? I am far from the first to ponder over this, and as initially

stated in this chapter, one might consider that I have already provided some notion of an answer.

However, where PER belong is not a simple this or that question. One view that seemingly agrees with this sentiment can be found in Redish (2003). There he argues for the need to have the nature of the knowledge being taught (or learnt) in mind to understand how people learn. Further, he identifies that educational research is valuable to understand how students learn physics in that it provides evidence for what does not work. By looking at Discipline-Based Education Research (DBER), which PER can be considered a sub-category, more credence is given to educational research (Docktor & Mestre, 2014; National Research Council, 2012). There, a more collaboratory view is suggested between physics and educational research to gain further knowledge about teaching and learning in physics. From the view of DBER, it is more clear that whether PER belongs is more a matter of what the focus is, either for a research group or a specific project. Thus it might be a sign of healthy collaboration and possible exchange of methodologies between physics institutions and educational institutions that the question of where PER belongs is left unanswered.

This thesis revolves around developing new and easily accessible physics laboratory exercises and equipment. In this process, it seeks to gain further insight into what is made possible for students to learn in the physics laboratory. I explore this by developing a lab exercise using a physical and simulated version of a mechanical Paul trap to help students learn about the principle behind the linear Paul trap. Paul traps and linear Paul traps refer to the technique of trapping ions using varying electric fields in two and three dimensions, respectively (Paul, 1990) (see Chapter 3). One motivation to focus on Paul traps can be related to the historical value of letting students encounter Nobel-prize awarded physics in the upper secondary school classroom. It is also a technique used in many modern physics apparatuses; from particle accelerators to quantum computers. As such, its applications in modern physics might spark the interest of students in a way that they find the physics subject more enjoyable or even consider a career related to physics.

Relating to the previously introduced areas within PER, research focusing on students understanding of models could fit several, depending on the specific focus. Therefore, I consider a broader categorisation of areas of interest, suggested by Heron (2018). Following this, research on models and students' reasoning and understanding thereof would fall under the *intellectual domain*. Thus it is an area of interest and something studied by the community already, and I am not going down a path astray from the general direction of PER.

1.1 Aim and guiding questions

The overall aim of this thesis can be broken down into two objectives covered in papers I and II, respectively: 1, to develop a hands-on model and a simulation of a mechanical Paul trap that can be used in everyday physics classrooms; 2, to study what students learn when interacting with the models. These objectives can be reformulated into the following research questions this thesis aims to answer:

- How can a mechanical model of a Paul trap be made accessible for use in the physics classroom?
- What is made possible for students to learn when interacting with a physical and simulated model of a mechanical Paul trap?

1.2 Structure of the thesis

The general structure of this thesis focuses on expanding on the work leading up to papers I and II. In chapter 2, I provide an overview of research on models in physics (and science) education. This work lays an important foundation for the background in Paper II. It draws from research and discussions on models educational contexts and how experts use and understands them, as well as how models are used in physics. Chapter 3 has a similar importance, but for Paper I. There, I introduce what Paul traps are and the mechanical Paul trap by explaining and deriving various relevant properties.

Chapters 5 and 6 are dedicated to describing the methods and results in papers I and II. In chapter 5, I first account for the development process of the mechanical Paul trap and the simulation that laid the foundation for Paper I. Then I detail the method and analysis process used in Paper II. Finally, chapter 6 summarises the results and conclusions from papers I and II.

To end it all, chapter 7 summarises answers to the research questions in relation to papers I and II. Further, I provide suggestions and reflections on the next steps as I continue on my path towards a PhD and how the work done so far, together with my supporting work not explicitly included in this thesis, guide me on the next leg of this journey.

2

Previous research on models in physics education

In this chapter, I will provide an overview of relevant research related to models in PER and DBER. However, the work in this thesis is set in naturalistic environments, more specifically, the context of Swedish upper secondary physics classrooms. As such, it is relevant first to introduce the role of models in the Swedish physics curriculum and PISA's (a program for international student assessment from OECD) influential view of scientific literacy.

Models in science come in various forms and names, such as models, mental models and representations. According to Neumann and Nordine (2021), large-scale assessments before the PISA program identified that students were generally successful in reproducing factual knowledge and solving routine tasks but failed when tasked with more complex problem-solving. Further, students had trouble with tasks related to modelling.

With the introduction of PISA, the following definition of scientific literacy was introduced:

> ... *the capacity to use scientific knowledge, to identify questions and to draw evidence-based conclusions in order to understand and help make decisions about the natural world and the changes made to it through human activity.* (OECD, 1999, p. 60)

The influence of PISA due to its ability to compare countries sparked reformulations of various national goals with science education. In the USA, for example, a core competency that includes modelling is science and engineering practices was introduced in a broader reformulation of goals in science education (Neumann & Nordine, 2021). In the current Swedish upper secondary physics curriculum,

teaching physics should help students '*develop knowledge of the concepts, theories, **models** and working methods of physics*' (Swedish National Agency for Education, 2011). Further, in the central content to be covered in the courses Physics 1, 2 and (the not so commonly taught) 3, the nature of models as simplifications of reality and how they are assessed through experimental work is highlighted.

The following discussion regarding models and modelling will draw from both physics in a more general sense and from research concerning science (although focusing on physics) education. The first section discusses how models and modelling are relevant to physicists. Then, models and modelling will be addressed in relation to science education and how it is and can be used within physics education more specifically.

2.1 Models in physics

Physics can broadly be divided into two categories, *experimental* and *theoretical physics*. The two are similar and dependent on one another to advance our understanding of the world. An experimental physicist deals with empirically probing and answering questions derived from theoretical models. Thus dealing with describing and predicting events and processes in space and time (Fischer et al., 2021, p. 7). Theoretical physicists instead strive to design and develop the corpus of physics concepts, and in doing so provide experimental physicists with theoretical models. They can be said to focus on describing, predicting and explaining space and time changes of physical objects (Fischer et al., 2021, p. 8). As such, models are important tools for developing a greater understanding and representing the world around us (Giere, 2004; Hesse, 1966; Nelson et al., 2002; Palmgren & Rasa, 2022; Redhead, 1980; Saltelli & Funtowicz, 2014).

2.1.1 Scientific models

'*What is a scientific model?*' refers to an essential philosophical question and a debate too broad to cover fully within this chapter. However, presenting some views and previous discussions related to this is relevant. Following, a definition of models related to the strategy used in model-based science is provided and related to how science is progressed, as well as the role of mathematics in physics.

Models can be defined as abstract or physical structures that can potentially represent real-world phenomena (Weisberg, 2007). This view draws upon work done by Giere (1988) and highlights the abstract, indirect relationship between a model and the phenomenon it is related to. Further, Giere (2004) argues for the pragmatic use of models as representations, meaning a focus on relationships between models and aspects of the world. Thus, models are used to form hypotheses regard-

ing aspects of the world in this way. Formalising this, we arrive at the following relationship between some scientist(s) (S), aspect of the world (W), model-system (X) and some purpose (P) (Giere, 2004, p. 743):

S uses X to represent W for purposes P

The purpose of model creation is usually thought of as to be able to make predictions. However, it is not the sole purpose (Bokulich, 2011). Epstein (2008) suggests no less than 16 example reasons to create models other than for making predictions. Three of which are common in educational and outreach contexts and on a surface level similar to predict: *explain*, *illuminate core dynamics* and *educate the general public*. This can be exemplified by the mechanical model of a Paul trap, introduced by Wolfgang Paul during his Nobel Prize acceptance speech (Paul, 1990). It relates the concept of Paul trapping (explained in depth in chapter 3) to something more tangible; trapping a ball on a horse-saddle-like-looking shape. It is (almost) impossible when the saddle shape is still. However, by letting it rotate with a certain speed, the ball will suddenly seem trapped.

2.1.2 Model-based science

The act of modelling is a challenging and complex endeavour. It requires one to move between constructing, using, evaluating and revising models, all whilst having epistemic considerations in mind (Tiberghien & Venturini, 2023). Moreover, as with most complex phenomena or systems, models are often constructed to understand and discuss, as is true even for modelling. One influential model of how scientists specify model systems in model-based science is presented by Giere (1988), and an adaption drawn from Godfrey-Smith (2006) can be found in Figure 2.1).

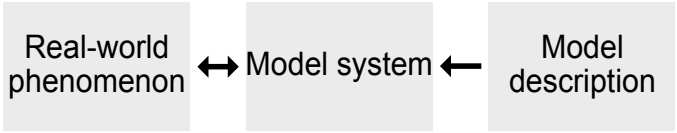

Figure 2.1: Visualising how a scientist specifies a model system. The double arrow highlights a back-and-forth process between the real-world phenomenon and the model system. The model description specifies the model system and is usually presented as mathematical relationships. Adapted from Giere (1988) and Godfrey-Smith (2006).

According to Godfrey-Smith (2006), model-based science is understood as a particular strategy for theorists to produce indirect representations of phenomena. For example, relating to Figure 2.1, a modeller (an agent applying the model-based

science strategy) seeking to understand a real-world system first seeks to specify and understand some hypothetical scenario. Following this, the agent continues onwards to consider the resemblance between the model system and the real-world phenomenon. However, in practice, the second step of resemblance is often vague and introduces a possibility for subjective bias and validity problems. This is because scientists, although using the same model system for the same real-world phenomenon, may have different resemblance relations in mind (Godfrey-Smith, 2006).

An example of the resemblance relations problem is that a model might be used purely as a predictive system for some and considered more of a system providing a causal map for others. To illustrate this using an exaggerated example, consider the mechanical model of the Paul trap, as suggested by Paul (1990). If one sees it and tries to use it as a predictive system, where properties can be mapped between the Paul trap and the mechanical model, one might try to use the mechanical model to predict features of the Paul trap. This will lead to identifying the model as useless and that it should be discarded. However, by thinking of the model as merely highlighting some qualitatively similar features, one might use the mechanical model to gain further insight into the Paul trap by thinking of the model as merely highlighting some qualitatively similar features. Not by some mapping and predictive structure, but rather recognising some important feature like the notion of threshold frequency (see for example Paper I for an explanation of this). By identifying this feature in the mechanical model and similar behaviour in the Paul trap, one might gain further knowledge both about the real-world phenomenon (the Paul trap) and regarding similarities and differences to the mechanical model. The moral of the story is that if the intended resemblance is not considered when reviewing scientific work involving modelling, results from such modelling could lead to different implications and conclusions than the intended ones.

2.1.2.1 The case of Volterra, the modeller

An example of a modeller and how one might work within model-based science is provided by Weisberg (2007). He considers Volterra, a mathematician exploring the cause of fish shortage in the Adriatic sea post World War I, a modeller. Volterra created two model populations, one to signify prey and one predators, in the form of a predator-prey model with some simple adjustable parameters. The model populations were constructed by assuming specific properties, the result of which could be represented by coupled differential equations, one for each population (Volterra, 1928). A surprising implication of the model was that significantly decreasing fishing in the area lowers the total population. This is because less fishing allows more predators in the sea, reducing the prey population and finally allowing fewer predators due to lack of food (available prey). However, the initial

model was deemed too simplistic and further study of the predator-prey model increased complexity to better model the complexity of the real-world phenomenon (Berryman, 1992; Weisberg, 2007).

Coming back to Figure 2.1, the example by Weisberg (2007) relatively clearly highlights the interaction between the *model system* and the *model description*, in terms of that the model description given by Volterra are the coupled differential equations of the predator-prey system. They specify the model system by making interactions between the populations explicit and the influence of assumed properties (Volterra, 1928). However, a step of resemblance is required to make use of the model to understand the problem of the dwindling fish populations. According to Weisberg (2007, p. 211), this was done in a somewhat back-and-forth manner. First, Volterra identified the prediction that a decrease in fishing ultimately would lead to a smaller total population. Second, by D'Ancona using facts from the real-world phenomenon (records of fish sales) to evaluate Volterra's model, identifying at least a qualitatively agreeing relationship between them.

2.2 Models and modelling in science education

In physics education, models come in various forms. Etkina et al. (2006, p. 35) suggests a classification of models that produce the following categories: Models of ...

- objects - after identifying a phenomenon to study, simplifications of identified objects involved are made (e.g. '*consider a spherical cow*').
- interactions - if multiple objects are involved, interactions between them are simplified. An example is Newton's law of gravity, a mathematical model of the gravitational interaction between two objects.
- systems - The billiard ball model of gas is an example of when the two models above are combined and form a model of the system, here being gas.
- processes (qualitative/quantitative) - Interactions between objects in a system might lead to some change. If the model includes descriptions (predictions) of what happens when some interaction occurs, it can be called a process model.

However, the types of models presented all need some level of predictive power, which from the previous discussion on models, does not incorporate all types of valuable models used in science. Thus, it is relevant to mention two additional categories brought up in Paper II, that are common in physics education, to consider as well:

- Ideal models - One example being considering the sun-moon-earth system as an isolated system (e.g. Chabay & Sherwood, 2015).

- Toy models - Such as the mechanical model of a Paul trap (Paul, 1990).

By acknowledging the various types of models that could take form, it is all but impossible to disagree with the first part of the realisation that '*Students often engage in modelling during our classes but are unaware of it*' (Etkina et al., 2006, p. 36). But how do we know if and how students become aware of these modelling practices? One way could be to introduce explicit tasks in lessons focused on highlighting various faces of modelling (e.g. Brewe, 2008; Dounas-Frazer & Lewandowski, 2018; Etkina et al., 2006; I. Halloun, 1996; I. A. Halloun & Hestenes, 1987; Hestenes, 1987; Lawrence, 2004; Wells et al., 1995)

2.2.1 Modelling in school: From novice to expert

In PER, a common approach to understanding what skills students need to develop and what the ideal end goal looks like is to study both learners and professionals. This approach allows researchers to form a hypothesis regarding intermediate steps and informs what suggested exercises look like. This idea of moving from novice to pro is also very much at the heart of the influential ISLE approach (Etkina et al., 2019).

In two consecutive studies, Tarciso Borges and Gilbert (1998, 1999) explored what models people construct when making sense of electricity and magnetism, respectively. The sample varied in expertise, with the various groups in the study ranging from first-year upper-secondary school students who had not studied electromagnetism to secondary school physics teachers and electrical engineers.

The study on models in magnetism identified five different models being used (Tarciso Borges & Gilbert, 1998). The models went from simple in the form of magnetism as pulling, where a magnet simply pulls some objects, which is close to how we experience them in everyday life. A more complex model was modelling magnetism as electricity. Then why objects are attracted is related to some internal distribution of charges within the magnet. It is hypothesised that this model relates to some over-generalisation of the notion '*opposites attract*'. Finally, the most advanced model sees magnetism as a field, a common model for physics teachers and electrical engineers. Tarciso Borges and Gilbert (1998) identifies this model as some combination of previous models in considering the movement of charges within the magnet and the relationship between electric polarity and magnetism. It seems to provide a greater value in terms of explanatory power. One important result of the study is to highlight that it seems common to make sense of magnetism in terms of electricity. This, together with identifying crucial differences between

the models identified, could help inform teachers on how they can help students in their journey of internalising more complex models of magnetism successively.

When studying models in electricity, a larger body of previous research is available. Amongst others, Kärrqvist (1985) identified six models students used, going from simpler ones similar to water flowing in some pipes to a more holistic view of the entire circuit as a system and current flows and transmits energy. Tarciso Borges and Gilbert (1999) made similar identifications but also identified a model of seeing electricity as a field, again, more common amongst the experts. However, even when identifying the field nature of electricity, the interviewees kept introducing the idea of electric current as the motion of charges. Similar to the study on models in magnetism, Tarciso Borges and Gilbert (1999) identified how models of a more complex and complete nature of how electricity behaves were built up by previous, simpler models. Thus, again this aggregate nature of more complex models could help inform teachers on how to guide their students to identify the need for more complex models.

When evaluating the quality of scientific work, epistemic criteria are used to evaluate essential aspects such as validity and accuracy. When judging the quality of scientific models, among the criteria used are that good models: have high levels of conceptual coherence and clarity; are compatible with theories in other fields; are appropriately parsimonious; are consistent with empirical evidence; have a history of making novel empirical predictions (Pluta et al., 2011, p. 486). When letting middle school students evaluate models, Pluta et al. (2011) found an overlap between some epistemic criteria scientists use and the epistemic criteria the students' identified. This would suggest that using models in physics education could already be included in early education because even young learners can judge and discuss the abstract notion regarding model quality. This result partially contrasts with findings from Harrison and Treagust (1998). They studied modelling in a secondary school setting and found that the involved students' notion of the quality of models was less developed than what Pluta et al. (2011) suggests. However, both agree that it is a good idea to use and start from students' notion of models and their development in the classroom.

On a parting note, reviews on using models in the science classroom have identified some pointers towards the path of future research. For example, Coll et al. (2005) employs a sociocultural theoretical viewpoint (Vygotskij, 1986) and identifies that a potentially fruitful avenue to enhance model-based learning could be to include discussion-focused activities. Furthermore, an essential insight from the synthesis of previous research is that effective teaching focusing on models seems to be related to students being actively involved in model construction and criticising (Coll et al., 2005, p. 194).

Oh and Oh (2011) is in agreement with Coll et al. (2005) and previously presented research in that it is important to let students participate in various mod-

elling activities. Further, they identify five types of modelling activities to make use of a more model-production-focused pedagogy (Oh & Oh, 2011). *Exploratory*, *expressive* and *inquiry* modelling is presented in earlier research, and the new additions are *evaluative* and *cyclic* modelling. Evaluative modelling aims to have students comparing multiple models related to the same phenomenon and train on evaluating the quality and identify limitations of the models. Cyclic modelling refers to the process of iteratively developing, evaluating, and improving models. Finally, it is suggested by Oh and Oh (2011) that the concept of didactic transposition (introduced by Chevallard, 2006; Chevallard & Bosch, 2014) could be a good idea to use to identify how scientific models are simplified and made available to students, while still having a relation to the expert view. Didactic transposition identifies that what is learnt is influenced by what is taught (i.e. lesson planning), which in turn is influenced by how some scholarly knowledge is transformed and made to fit into the curriculum.

3

Paul traps and the mechanical Paul trap

Paper I presents a 3D printed mechanical model of a linear Paul trap, called the mechanical Paul trap. This chapter presents an overview of the work leading up to this paper. First, the principle of Paul traps will be introduced, focusing on linear Paul traps. Second, the mechanical Paul trap as a model of linear Paul traps will be presented and discussed. However, before digging into Paul traps, let us first look at the underlying idea of particle trapping.

What do we mean by trapping? Consulting the online dictionary of Merriam-Webster, the word ”trap” is found to be either a noun or a verb. Trapping in particle trapping relates to the transitive verb definition ”to place in a restricted position” (Merriam-Webster, n.d.). Translating to physics, possibly the simplest particle trap that can be constructed works in one dimension and draws inspiration from Hooke's law

$$\mathbf{F} = -k\mathbf{r}\,, \tag{3.1}$$

where $\mathbf{F}$ is a force vector, k a spring constant and $\mathbf{r}$ a position vector denoting the displacement from equilibrium. The formulation of Hooke's law as in 3.1 shows that there exists a restoring force that increases linearly with distance acting on an object moving away from equilibrium. Going from the notion of trapping with springs to confining a particle in space relative to some point, a centre of the trap, can be seen in Figure 3.1.

Moving to three dimensions and using formula 3.1, it is possible to place a test particle on various locations to create a *'map'* of how the strength of the force varies in the space around the origin of the force (centre of the trap in Figure 3.1b).

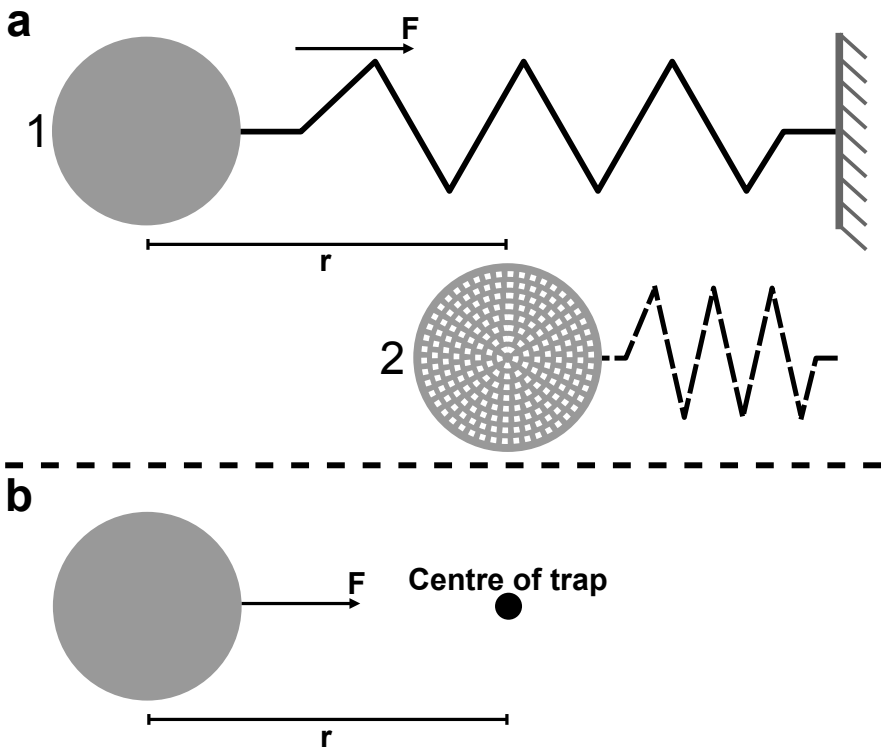

Figure 3.1: Visualising a one-dimensional particle trap obeying Hooke's law as formulated in equation 3.1. **a** shows how a particle-spring system is displaced from equilibrium (1) and its corresponding equilibrium position (2). **b** generalises the idea and replaces the spring with the notion of a centre of trap.

When considering the map, we are considering a force field. The relationship between force and field strength can be described as

$$\mathbf{F} \propto \textbf{field strength}\,. \tag{3.2}$$

Combining this proportionality with the fact that the field and some corresponding potential Φ have the relationship

$$\textbf{field strength} = -\nabla\Phi = -\left(\frac{\partial\Phi}{\partial x}, \frac{\partial\Phi}{\partial y}, \frac{\partial\Phi}{\partial z}\right)\,, \tag{3.3}$$

leads to the realisation that integrating formula 3.1 gives the corresponding potential for the particle trap in $\mathbf{r} \in \mathbb{R}^3$ as

$$\Phi \propto \alpha x^2 + \beta y^2 + \gamma z^2\,, \tag{3.4}$$

with α, β and γ being constants.

3.1 Paul traps

Constructing a particle trap following the proportionality 3.4 to trap charged particles can be done using electric or magnetic multipole fields (Paul, 1990). Using a quadrupole electric field, the potential becomes

$$\Phi = \frac{\Phi_0}{2r_0^2}\left(\alpha x^2 + \beta y^2 + \gamma z^2\right) , \tag{3.5}$$

where Φ_0 is the magnitude of the potential and r_0 denotes the separation of the poles as the radius of the trap.

Creating a potential 3.5 that gives rise to a force towards the centre in a static configuration requires that $\alpha > 0$, $\beta > 0$ and $\gamma > 0$. However, such a configuration would violate the Laplace equation (Lindqvist, 2017)

$$\nabla \cdot \nabla\Phi = \frac{\partial^2\Phi}{\partial x^2} + \frac{\partial^2\Phi}{\partial y^2} + \frac{\partial^2\Phi}{\partial z^2} = 0 . \tag{3.6}$$

Following the condition set by 3.6 generates the restriction

$$\alpha + \beta + \gamma = 0 , \tag{3.7}$$

which lets us arrive at the following solution for a two-dimensional configuration in Cartesian coordinates

$$\Phi = \frac{\Phi_0}{2r_0^2}\left(x^2 - z^2\right) , \tag{3.8}$$

with $\alpha = 1 = -\gamma$ (Paul, 1990).

A problem has arisen: there is a focusing effect along one axis and a defocusing effect on the other (see Figure 3.2). Using static fields, this is, due to the Laplace equation 3.6, unavoidable. Earnshaw noted it already in the 19th century (Jones, 1980; Maxwell, 1873; Tonks, 1940). One seemingly simple idea to solve this problem is not to use static fields. For the two-dimensional quadrupole configuration 3.8 the applied voltage, dictating the value of Φ_0, can be a combination of a DC- (U) and AC-voltage (V), thus arriving at

$$\Phi_0 = U + V\cos\omega t , \tag{3.9}$$

where ω is some driving frequency and t is time.

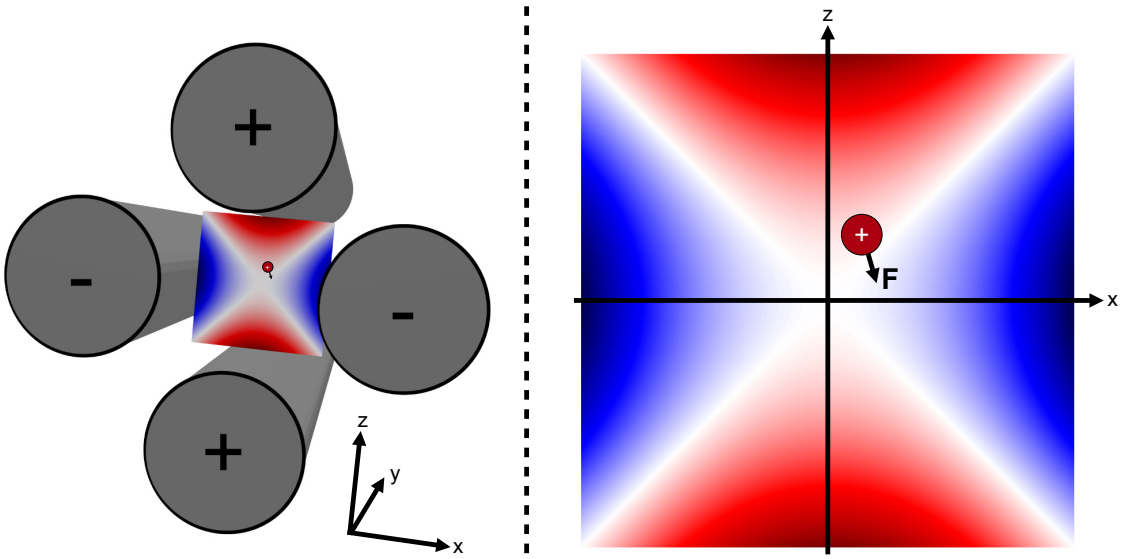

Figure 3.2: Visualising the electric potential inside a linear Paul trap following the configuration 3.8. The left image shows an electrode configuration to create the potential within the trap that is enlarged to the right. The colour scale goes from dark red to dark blue, with white corresponding to the potential being 0. Considering a positively charged particle in the field, blue regions give rise to a defocusing force and red regions to a focusing force. Adapted from Paul (1990, p. 532).

By combining the configuration 3.8 with the time-dependent magnitude of the applied voltage 3.9, the equations of motion for a particle in the linear Paul trap following from Newton's second law can be written in dimensionless parameters as

$$\frac{d^2x}{d\tau^2} = -x\left(a + 2p\cos 2\tau\right) , \tag{3.10a}$$

$$\frac{d^2z}{d\tau^2} = z\left(a + 2p\cos 2\tau\right) , \tag{3.10b}$$

where

$$a = \frac{4qU}{mr_0^2\omega^2} , \; p = \frac{2qV}{mr_0^2\omega^2} , \; \text{and} \; \tau = \frac{\omega t}{2} ,$$

with q being the charge and m the mass of the particle. The equations of motion 3.10 are Mathieu functions and exhibit both stable and unstable solutions (Brimacombe et al., 2021; Coisson et al., 2009; Frenkel & Portugal, 2001; Gutiérrez-Vega et al., 2003; Paul, 1990; Thompson et al., 2002).

3.2 The mechanical Paul trap

In his Nobel prize lecture, Wolfgang Paul (1990) introduces a mechanical analogue to the linear Paul trap. It draws inspiration from the fact that equipotential lines in the Paul trap form a saddle surface. Further, particles in the linear Paul trap exhibit a slow oscillatory motion around the centre of the trap, along with a much faster micromotion. This can also be noticed in the mechanical Paul trap. This behaviour and the fact that the rotation frequency of the trap has to match the potential parameters and the mass of the ball are noted as similarities between the linear Paul trap and the mechanical model (the mechanical Paul trap) (Paul, 1990).

The time-varying amplitude of the linear Paul trap described by 3.9 means that the field changes in a flapping manner (see Figure 3.3). This cannot be the case in the mechanical Paul trap as the potential is both a physical '*thing*' and thus has to change by rotation. To better understand the mechanical Paul trap, we need to reformulate the linear Paul trap potential 3.8 as a gravitational potential.

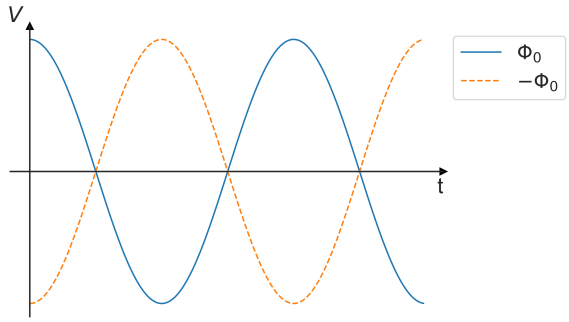

Figure 3.3: How the potential varies in a linear Paul trap as described by 3.9. Opposite electrodes are connected to the same polarity, with two following Φ_0 (initially with a positive charge) and two following $-\Phi_0$ (initially with a negative charge). In this configuration, there are times when the potential momentarily goes to zero, and the field varies in a flapping manner.

In the mechanical Paul trap, we need to consider both a rotating and a stationary (laboratory) frame of reference. The magnitude of the potential is related to the height of the trap (z), and the particle is considered to move in the x-y plane (in contrast to x-z in the linear Paul trap). The geometry of the mechanical trap is thus described by

$$z = \frac{h}{R^2}\left(x^2 - y^2\right) , \tag{3.11}$$

where z, x^2 and y^2 are spatial coordinates, and h the maximum height of the surface at the radial distance R from the centre. In this frame, the gravitational potential

becomes

$$U_g = \frac{mgh}{R^2}\left(x_r^2 - y_r^2\right) , \tag{3.12}$$

where m is the mass of the particle and g the gravitational acceleration.

Converting to a stationary frame, the frame we as observers experience, can be done by noting that each point in the x-y plane is rotating with some angular frequency Ω. Thus, the conversion can be done by introducing a rotation matrix that gives

$$\begin{pmatrix} x_r \\ y_r \end{pmatrix} = \begin{pmatrix} \cos\Omega t & -\sin\Omega t \\ \sin\Omega t & \cos\Omega t \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x\cos\Omega t - y\sin\Omega t \\ x\sin\Omega t + y\cos\Omega t \end{pmatrix} . \tag{3.13}$$

Substituting $x_r^2 - y_r^2$ in the potential 3.12 with the results from 3.13 the potential in the stationary frame is

$$U_g = \frac{mgh}{R^2}\left[(x^2 - y^2)\cos 2\Omega t + 2xy\sin 2\Omega t\right] . \tag{3.14}$$

From this, equations of motion in the mechanical Paul trap can be derived by solving the first-order Euler-Lagrange differential equation

$$\frac{\partial \mathcal{L}}{\partial \mathbf{r}} - \frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{\mathbf{r}}} = 0 , \tag{3.15}$$

where $\mathcal{L}$ is the Lagrangian. Introducing the substitution $\tau = \Omega t$, equation 3.15 becomes

$$\frac{\partial \mathcal{L}}{\partial \mathbf{r}} - \Omega\frac{d}{d\tau}\frac{\partial \mathcal{L}}{\partial \dot{\mathbf{r}}} = 0 , \tag{3.16}$$

and the Lagrangian can be constructed as

$$\mathcal{L} = \frac{m}{2}\left(\dot{x}^2 + \dot{y}^2\right) - \frac{mgh}{R^2}\left[(x^2 - y^2)\cos 2\tau + 2xy\sin 2\tau\right] . \tag{3.17}$$

Finally, the motion of a particle in the mechanical Paul trap can be described by solving equation 3.16, which produces the following equations of motion in the x-y-plane

$$\frac{d^2x}{d\tau^2} = -\frac{2q}{\Omega^2}\left(x\cos 2\tau + y\sin 2\tau\right) , \tag{3.18a}$$

$$\frac{d^2y}{d\tau^2} = \frac{2\bar{q}}{\Omega^2}\left(y\cos 2\tau - x\sin 2\tau\right) , \tag{3.18b}$$

with $\bar{q} = \frac{gh}{R^2}$.

The equations of motion for a particle in the mechanical Paul trap 3.18 are similar to the equations in the linear Paul trap 3.10, apart from a sinusoidal cross-dependent term for the mechanical Paul trap. This stems from the potential 3.14 having a sinusoidal term. This shows that the potential is rotating and not flapping (Thompson et al., 2002).

The derivation leading to the equations of motion for a particle in the mechanical Paul trap is simplified in several ways. First, the particle trapped is considered a point mass. Second, friction between the particle and the physical saddle surface is neglected. A limited body of work has been done investigating these due to in part the substantial increase in complexity coupled with the problematic nature of simulating chaotic systems (e.g. Borisov et al., 2018, 2019; Fan et al., 2017; Thompson et al., 2002). Further, it quickly starts to go outside the scope of this thesis, as it is less related to investigating how students can notice some qualitative similarities between the mechanical Paul trap and the linear Paul trap.

In this thesis, the focus is on noticing qualitative similarities between the mechanical Paul trap and the linear Paul trap. Alternative models exist that can be used for demonstration purposes, that argue for being closer to the linear Paul trap in some aspects (e.g. A. K. Johnson & Rabchuk, 2009; Kirillov & Levi, 2016). However, the mechanical Paul trap does exhibit some similarities to the linear Paul trap, such as the oscillatory motion and having the important feature of *threshold frequency*. Finally, it also visually shows what the potential looks like (see Figure 3.4), which was identified as something important for the aim of Paper II.

Figure 3.4: The mechanical Paul trap with a rubber ball balancing on the saddle point. Adapted from Paper I.

4

Variation theory and design-based research

In this chapter, I give an overview of the theoretical framework of variation theory. This was used as a lens to analyse and understand the learning and struggles of students presented in Paper II. Further, the design-based research approach is also introduced as the methodological approach for this thesis.

4.1 Variation theory

Improving learning outcomes and exploring new ways to visualise and convey the meaning of various concepts is a common focus in PER. Thus questions asked can often be considered quite hands-on in terms of having a close connection to the practice of teaching and learning physics. In this section, I am presenting the theoretical framework of variation theory and how it can serve as a powerful way to begin to answer questions relevant to PER.

Variation theory can be considered a learning theory (Runesson, 2006). It originated from the phenomenographic approach beginning to develop in Gothenburg, Sweden, in the 1970s (Marton & Säljö, 1976; Marton, 1981, 1986; Säljö, 1979). Variation theory began to see the light of day following Marton and Booth (1997) as somewhat separate but still connected to phenomenography. In the context of physics education, phenomenography has sporadically been used to investigate learning (e.g. Booth & Ingerman, 2002; Ling Lo et al., 2006; McBride, 2009; Prosser et al., 1996). Regarding variation theory, examples of fruitful ways of applying it have started appearing within the context of PER (e.g. Euler et al., 2020; Fredlund, 2015; Ingerman et al., 2009; Svensson, 2022).

When considering purposeful learning using the variation theory perspective, *what is to be learned* should be considered a central question (Marton, 2015). The answer to this question is within the theory called the *object of learning*. Marton (2015, p. 164) identifies the object of learning as something that can be defined and appear in three distinct yet similar ways: 1. The content, i.e. something quite concrete and the same for every student participating in the same lesson. 2. Educational objectives, which can be exemplified in terms of some skill or something else written in the course syllabus that the student should be able to recite or act in accordance with at the end of the course. 3. As something related to the content, but rather the focus should be on what a learner needs to learn on the way towards understanding the content.

The first two puts the focus of the teaching and lesson planning towards an 'end of the course'-style of planning, whereas the third way of seeing the object of learning focuses on both the lesson at hand and the students in front of the teacher (Lo, 2012; Marton, 2015). Variation theory acknowledges the third focus as an efficient way to plan and organise teaching. Having this focus recognises that reaching some educational objective consists of some necessary aspects students need to discern in order to reach the end goal. Further, necessary aspects become critical when a learner has yet to make the aspect their own.

4.1.1 Learning, the object of learning and its critical aspects

Learning in variation theory can be considered as a change in a person-world relationship, i.e. learning is learning to see the world in a new and more powerful way (Marton & Booth, 1997; Marton & Tsui, 2004). Thus, learning is closely tied to the actual learner, and the learner's current viewing of the world, and understanding what needs to be learned (discerned) can not be fully understood if only considering the phenomena (concept or something else handled during a lesson) when planning a learning segment. Following this comes the realisation that there is always something to be learned from a lesson. Even if a particular learner has understood the concept in a traditional sense, there may be skills, capabilities or modelling related to the concept that could be learned if appropriately guided.

To understand how students learn, it is essential first to understand what learning is made possible (Häggström, 2008; Kullberg et al., 2017). Lo et al. (2004) highlights that for teaching to be thought of as a rational activity, a teacher must consider not only the content to be taught but also particulars related to the actual students attending the lesson and the physical conditions of the location. These conditions and particulars need to be then considered in relation to the content and transformed into a set of actions that help the current group of students in the current lesson to achieve specific goals. Analysing what learning is relevant and related to the particular content for a particular group of learners in this way is the

path to identifying what the object of learning for the particular lesson or learning segment should be considered as. This leads to the realisation that the question of what is to be learned is closely related to what the object of learning is (Lo, 2012).

The object of learning appears in three distinct ways that can be ascribed to different parts of the teaching-learning situation (Marton, 2015; Pang & Marton, 2003): *intended* (the aim), *enacted* (what is taught), and *lived* (what is learned). From these, what learning is made possible is made separate from what learning is offered, i.e. what the teacher beforehand thinks is possible to learn. This provides another important insight into the object of learning, namely that the object of learning is not to be treated as synonymous with what is commonly known as learning objectives (or learning outcomes) in curriculums (Lo, 2012).

4.1.1.1 Critical aspects and features

Around the same time that variation theory started to see a distinction from phenomenography, the notion of what is now referred to in the literature as *critical aspects* and its constituent parts *critical features* (can also be called *values*) came about. Sometimes features and aspects are treated rather synonymously (e.g. Pang & Marton, 2003). In the context of this thesis, critical aspects are to be considered and understood as being part of some object of learning. Critical features are in turn to be seen as specific values within the domain of the critical aspect. Lo (2012) notes that a learner must discern all critical features in all critical aspects simultaneously in order to fully discern the object of learning using, which is exemplified in Figure 4.1.

Critical aspects are to be understood as aspects related to a phenomenon (the object of learning) that a learner has to take notice of (discern) in order to become aware of the said phenomenon but is currently unable to (Marton, 2015). When a critical aspect is experienced, it is done so via presenting the learner with some values, or features, within that aspect. This is an act of varying the aspect in focus and is synonymous with what is known as the act of opening up a dimension of

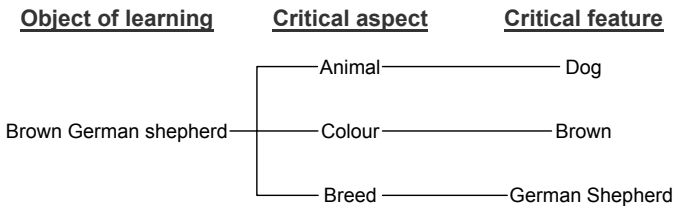

Figure 4.1: The relationship between the object of learning, critical aspects and critical features from an example introduced by Lo (2012, pp. 59–60). To fully discern a brown German shepherd, a learner must discern all three critical features simultaneously.

variation (Pang & Ki, 2016). If features within the critical aspect are necessary to experience to discern the aspect, it is called a critical feature (Lo, 2012).

4.1.2 Understanding the process of learning using variation theory

When a set of critical features and values have been identified, and thus defining an object of learning that needs to be discerned. How would a teacher go about using this knowledge to implement effective learning segments for their students? Marton (2015) proposes to focus on a three-step process (patterns of variation and invariance) if beforehand the object of learning has been presented. The first step is the act of *contrast*, where the focus is on identifying and focusing on a critical aspect. One way to achieve this is by placing two objects side by side, with every aspect identical except for one. By this simple juxtaposition, a learner is most likely to take notice of the one aspect that differs.

When an aspect has been noticed (separated) the next step in the learning process is to engage in some form of *generalisation* (Marton, 2015). That step involves again an act of variation to make learning possible, but this time the critical aspect in focus is kept constant (invariant). This could be thought of as beginning to better frame what or isolate that the aspect the teacher thinks is in focus really is in focus and related to the correct object of learning. By letting the discerned aspect be invariant and vary some other irrelevant aspect, the learner can get a better grasp of the object of learning via induction.

It is important to take note of the step-wise nature of the acts proposed. The act of generalisation needs to occur after a pattern of contrast. Further, the last step, *fusion*, needs to be preceded by the generalisation step. However, fusion is referred to as the act of simultaneously opening up multiple dimensions of variation. Thus all critical aspects need to be discerned and focused on before this step in a contrast $\rightarrow$ generalisation-like manner. The relationship between the three patterns can thus be visualised as in Figure 4.2.

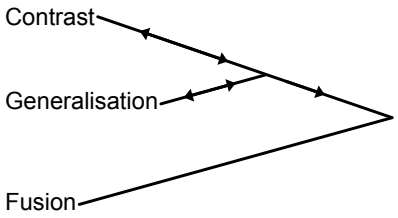

Figure 4.2: The relationship between the patterns of variation and invariance. It highlights the possible back-and-forth between contrast and generalisation before fusion can finally occur. Adapted from Marton (2015).

4.2 Design-based research

PER is no stranger to the design-based research approach (also known as design-experiments or design research, forthcoming DBR) (Hake, 2008). Further, it is also no stranger to variation theory, as it is one of the building blocks of the learning study method (Holmqvist et al., 2007). The work included in this thesis is the result of employing DBR to develop a new laboratory exercise for the upper secondary physics classroom.

An important goal with DBR is to provide ways to improve educational situations in a systematic way (Bannan-Ritland & Baek, John, 2014; Brown, 1992; Cobb et al., 2003; Guisasola et al., 2023; Joseph, 2004). It was originally developed to gain deeper knowledge about the nature of teaching and learning in naturalistic contexts (Brown, 1992). This is close to the intentions of both variation theory (Marton, 2015) and current trends in PER (Hammer, 2023). As such, using DBR has the potential to contribute to the development of both theory and practice (Brown, 1992; Cobb & Gravemeijer, 2008).

When applying DBR, it is important to have a reflexive position regarding being able to refine research questions during the development process (Joseph, 2004). This is because the nature of naturalistic experiments includes a level of uncertainty that is hard to fully prepare for beforehand. Taking note of this issue implies that successful reporting of research using DBR includes handling the trustworthiness criteria. This includes having a pragmatic outlook regarding the intentions and implications of a design process, which in turn better visualises the role of the researcher(s) reflections (Juuti & Lavonen, 2006).

Due to DBR having the focus on theory development in niche and specific learning processes (Cobb et al., 2003), it is well suited for use in PER together with variation theory. Guisasola et al. (2023) argues that DBR, with its emphasis on the role of teachers and learners, can help bring PER closer to its needs to develop teacher-learning sequences. It may also contribute to the available toolset for educators in providing *design tools* (Guisasola et al., 2023, p. 13). Design tools can be thought of as constructs helping to systematise the design process. Design studies that provide guidance towards the development of design tools that could be considered hypothesis-generating and working towards model formulation (Kelly, 2004). The work in this thesis falls under the category of using DBR to explore variation theory as a design tool for structuring lab exercises.

5 Method

The work in this thesis can be considered to consist of two parts within the same design experiment. The experimental setup presented in Paper I was developed with the study presented in Paper II in mind. In this chapter, I will first provide an overview of the experimental setup presented in Paper I. Then the method in Paper II is presented both in terms of data collection and a discussion regarding the analysis process.

5.1 Developing the experimental setup (Paper I)

The experimental setup students interacted with in the design study consisted of two representations of the mechanical Paul trap, a physical and a computer-simulated model.

Several points need to be considered to increase the potential of using experimental equipment in classrooms. The ones focused on when developing the mechanical Paul trap were for the physical trap: 1. It should be constructed with easily accessible and affordable components. 2. It should be hard to break anything accidentally. 3. There should be no need to install specialised software on the computer to read the output from the Arduino. In designing the simulation, the following points were considered: 1. No experience with coding should be required to run the simulation. 2. There should be no need to install specialised software on the computer to run the simulation.

5.1.1 Model 1: The physical trap

An important aim guiding the development of the physical trap was for it to be easily controlled and built using affordable and readily available components. With

this in mind, most parts were designed to be 3D-printed, and everything should be controllable with a simple Arduino. Figure 5.1 shows the final design for the physical model.

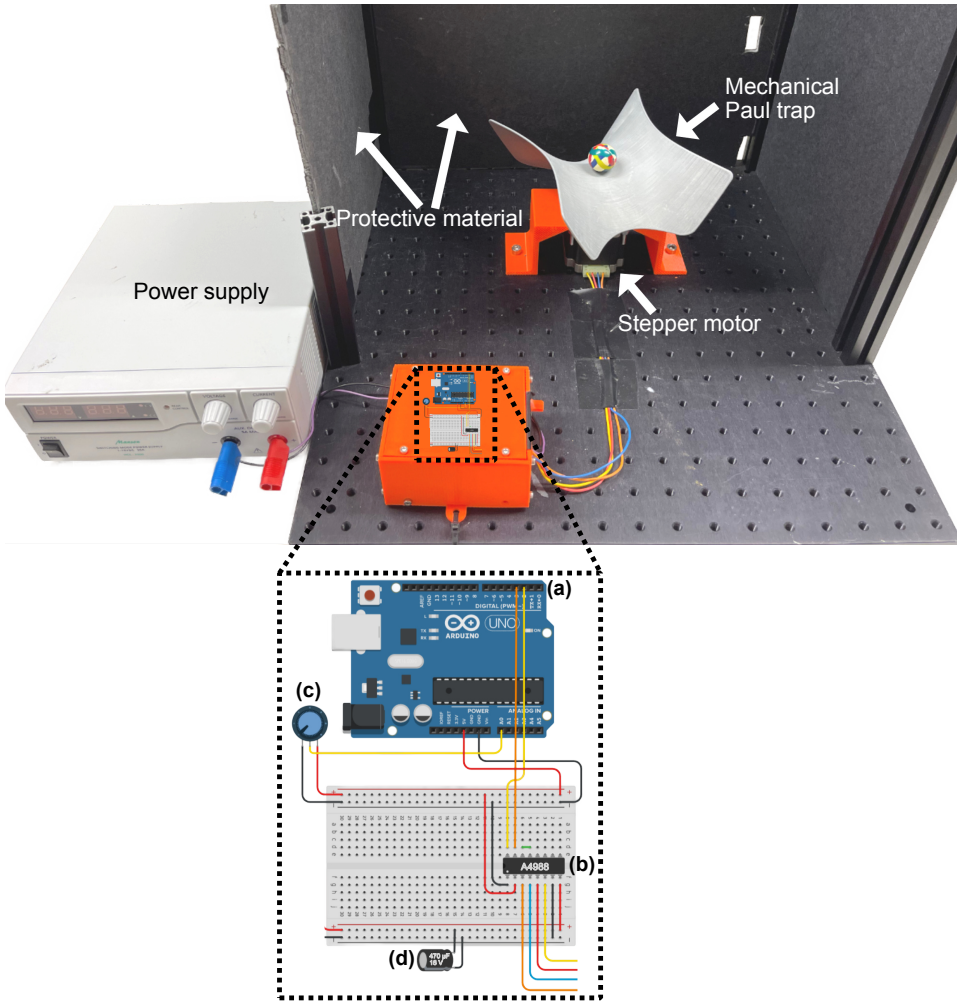

Figure 5.1: The experimental setup. A computer was connected to the Arduino (a) to read the rotational frequency via USB. The Arduino also drives the stepper motor with a motor driver (A4988) (b). The frequency is controlled by a potentiometer (c), and a capacitor (d) protects the motor from potential current spikes.

To drive the stepper motor (a NEMA 23 with 1 Nm torque and maximum load of 24V/1A), a power supply set to 16V/1A was connected to the motor driver. The V/A was chosen to allow the motor driver to operate for extended periods without worrying about overheating. It also was sufficiently high to provide enough torque for the stepper motor to run without noticeable stalling issues. In addition to the

trap, balls were provided varying in material and size. However, for most of the planned student investigations, glass beads with diameters of 22 mm or ping pong balls with diameters of 40 mm were used.

5.1.2 Model 2: The simulated trap

Two versions of the computer simulation were developed, both with the considerations mentioned earlier. The first iteration (which is the version used in II) used the online environment Glowscript[1]. The one presented in Paper I used the online environment Google Colaboratory[2]. Both had similar limitations in that friction was not included in the system, which marks a significant difference between the physical and simulated model of the mechanical Paul trap. Figure 5.2 shows how the simulations look when in use.

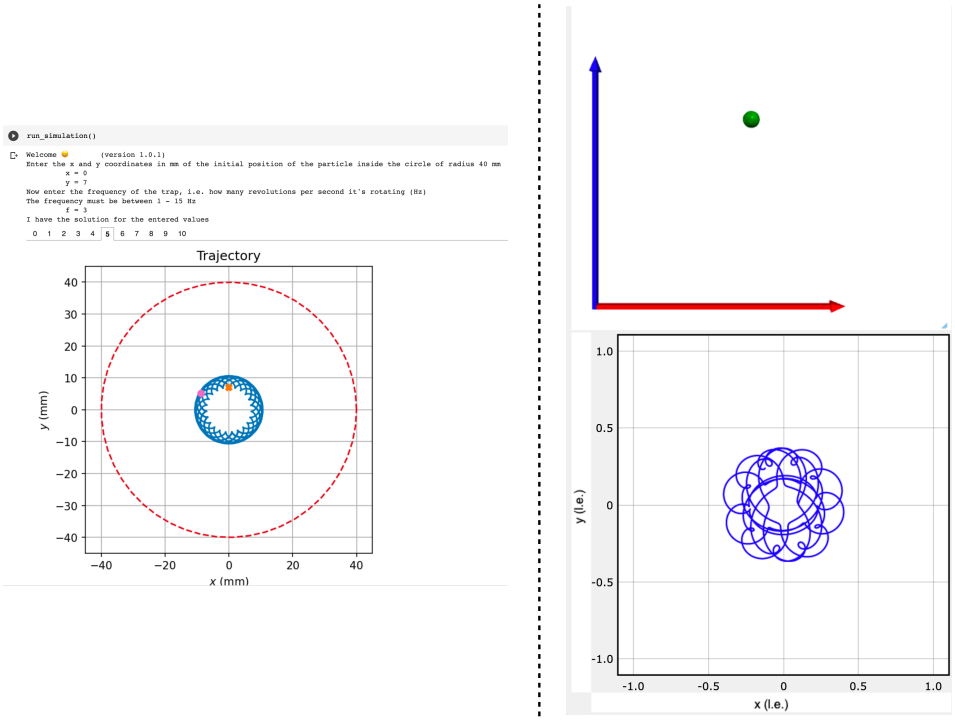

Figure 5.2: Screenshots of the simulations, using the frequency 3 Hz to visualise stable regions. The left image is the view in Google Colaboratory. There the particle starts at position $(0, 7)$ mm. The right image shows the output from Glowscript. In that version, the user is prompted with a popup window at the start of the simulation to provide a frequency. The particle starts at position $(0.1, 0)$.

[1] https://glowscript.org
[2] https:colab.research.google.com

Both simulations have pros and cons. One significant difference between the two simulations is that in the Glowscript version, the simulation runs continuously, whereas using Google Colaboratory only allows one to produce a set of static frames. Not seeing the simulation run continuously could make it harder for students to notice the similarities in how the ball behaves when moving in the simulated versus physical trap. However, in testing on various computers (running Windows, MacOS and ChromeOS) and in different levels of locked-down IT environments, the Google Colaboratory implementation could always run, whereas the Glowscript implementation got blocked in one incredibly limiting IT environment.

5.2 Conducting the design experiment (Paper II)

The study presented in Paper II uses DBR and the development of a laboratory exercise using the mechanical models of Paul traps developed in Paper I, with the difference being that the simulation used was the Glowscript version. This section presents methodological issues and choices regarding data collection and analysis.

5.2.1 Selection

The study was conducted in the Swedish upper-secondary physics course Physics 2 (Swedish National Agency for Education, 2011). It is generally offered to students studying a natural science or technological program during their third and last year of upper secondary school. To increase the willingness of teachers wanting to participate in the study, clear connections between the mechanical Paul trap to the following central content in the curriculum were highlighted:

- '*Models and theories as simplifications of reality. Models and their areas of applicability and how they can be developed, generalised or replaced by other models and theories over time.*
- *Planning and implementation of experimental investigations and observations, and formulating and testing hypotheses in connection with this.*
- *Physics principles underlying technical applications for communication and* ***detection****.*'

Six upper secondary schools in or close to a large city in Sweden participated in the study. Four of which were public schools and two private ones. It should be noted that differences between public and private schools in Sweden are limited due to both operating under the same regulations, for example, having to follow

the same curriculum and not being able to individualise how students apply to them.

Since the purpose of the study, having variation theory in mind, was to use students' varying interpretations and ways of understanding a real Paul trap using a mechanical Paul trap, purposeful sampling with an emphasis on variation was relevant for selecting participating schools (Palinkas et al., 2015; Suri, 2011). As such, schools were initially considered who have geographical variation, as that would naturally increase the variation regarding socioeconomic status. Further, discussions with the participating teachers regarding how they perceived their students and the general trend of accepted students' grades were also considered.

Within the six participating schools, eight teachers conducted 1-2 laboratory sessions each (depending on whether the class size required one or two laboratory sessions and still keeping small groups of 3-4 students). The number of students in each laboratory session varied from six to 25; there were 12 laboratory sessions in total. 124 students were enrolled in the classes who participated in the study, and 119 were present during the sessions.

5.2.2 Data collection

There were three types of data collected and used to varying degrees during the study: *semi-structured student interviews*, *field-notes from each laboratory session*, and *pre and post-tests*. The interviews and field notes are qualitative data, and the pre and post-tests are quantitative. It should be noted that the main researcher also took notes continuously during the design experiment. This is important when conducting DBR as the process, including meetings and analysis apart from the actual interventions, could include valuable data to support the final analysis (Cobb et al., 2003).

All data was gathered during the design experiment, which took place between October to December 2021. It was split into three phases; three lab sessions in the first, five in the second, and four in the final. The first phase was initially considered a practical pre-study phase, focused on testing the equipment and data collection, but was subsequently included in the final study. The major difference between phase 1 and the rest was in the end the amount of time available to conduct initial analyses and redesign. Further, the study presented in Paper II is in itself considered a pre-study situated in a larger project developing a remote laboratory version of the mechanical Paul trap.

The quantitative data used in the study was in the form of pre and post-tests. The pre-tests were conducted approximately one week before each laboratory session. The post-tests were done at the end of each laboratory session in all but three lessons, where time ran out during the lab, and the tests had to be done the next lesson. The questionnaire consisted of ten questions, seven multiple choice and three

free-text format questions. The free-text format questions asked the students to explain why it is impossible to trap particles using static fields, and what properties of the ball or the trap affect the possibility to trap and the trapping time. Three of the multiple-choice questions focused on forces. One was a baseline question about the direction of the electric force between equally charged particles. Two focused on what force acts as a trapping force in a real and mechanical Paul trap, respectively. The rest of the multiple-choice questions presented different scenarios and asked the students to predict the outcome related to some baseline (an example of such a question can be found in Figure 5.3).

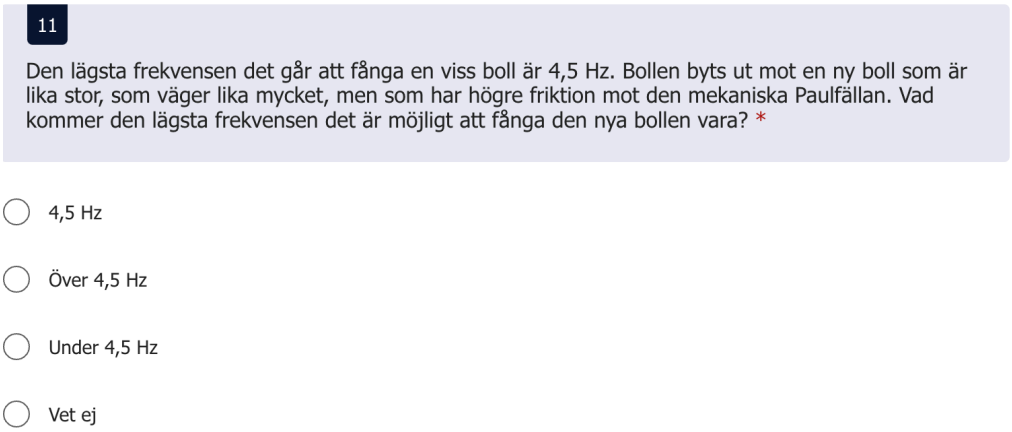

Figure 5.3: One scenario question used in the test. In English, the problem is the following: 'The lowest frequency a certain ball can be trapped is 4.5 Hz. The ball is replaced with a new ball equal in size and mass, but that has higher friction against the mechanical Paul trap. What will the lowest frequency it is possible to trap the new ball be?'. The alternatives: '4.5 Hz. Above 4.5 Hz. Below 4.5 Hz. Don't know.'

The qualitative data consisted of two parts. To collect field notes, one researcher took part in every lesson and focused on taking notes on the structure of the session in general. However, more specifically, the interest was to gather what the students were doing during the session during each of the tasks in the exercise. To do this, not all groups could be in focus simultaneously. As a result, two groups were randomly selected for each laboratory session. The researcher focused on being close to the role of a *non-participant observer* but sometimes moved into the more active role of a *participant-observer* (Otero et al., 2023). Having a *non-participant observer* role means only observing and not interacting or disturbing at all. In this role, it is possible to collect more detailed notes. However, moving into the role of a *participant-observer*, which includes being considered an assistant or student, allows the researcher to ask questions to gather more information about

what students are doing and why. During laboratory sessions, there are generally moments of slower activity. During those, the researcher could move towards being more participating and thus gather more information regarding students' choices and impressions during the lab.

After each laboratory session, one student from each observed lab group was asked to participate in an interview. The interview aimed to explore the students' experiences of the laboratory session and probe how they understand the mechanical trap and their ability to use it as a model for the real Paul trap (Otero et al., 2023). Each interview was audio recorded and followed a semi-structured approach (Kvale & Brinkmann, 2018; Otero et al., 2023; Runswick-Cole, 2011). An initial analysis was done shortly after the interviews as they were transcribed. Conducting an initial analysis soon after qualitative data is collected is important to keep the essence of the data (Bogdan & Knopp Biklen, 2007; Phillippi & Lauderdale, 2018).

5.2.3 Analysis

Table 5.1: An overview of when different parts of the analysis were conducted.

↓ Time	Where in the study	Quantitative	Qualitative
	Between phases 1 and 2, and 2 and 3	Initial descriptive analysis to guide redesign	Transcribing interviews and structuring field notes. Potential points of particular interest were identified to guide the redesign
	After phase 3	Descriptive and inferential analysis	Thematic analysis
		Comparing and combining results. Qualitative focus	

Using qualitative and quantitative data, the study is considered a mixed-method study (R. B. Johnson et al., 2007). With that comes a need to identify and choose how data will be handled and in what way. The study presented in Paper II is a concurrent triangulation design (forthcoming CTD), meaning that qualitative and quantitative parts are collected and analysed in part and then compared for deeper understanding (Creswell et al., 2003; Creswell & Plano Clark, 2017; La Braca & Kalman, 2021). Due to the novel nature of exploring how the mechanical

Paul trap can be used in the upper-secondary physics laboratory, CTD allowed for investigating the problem from slightly various points of view. As such, it opens up the possibility of discovering nuances in students' experiences that might otherwise be missed. An overview of the analysis process is provided in Table 5.1, which highlights when the different parts of the analysis were conducted in relation to each other.

5.2.3.1 Quantitative analysis

After each phase of the design experiment, initial analysis and exploration of the pre and post-test results were done using descriptive analysis (Stewart et al., 2023). This was done to get an overview to be used as a basis for redesigning the laboratory exercise between each phase. A combination of pre and post-test results and the researcher's experiences from observing the laboratory sessions guided the redesign changes.

After all phases of the design experiment, all available pre and post-test data were analysed. However, due to a combination of an emphasis on scenario questions and the chaotic nature of the mechanical Paul trap, no significant trend was observed, nor was a clear null result identified as the noise floor was high when looking at the complete test. Thus, those questions were put aside, and the remaining test was analysed. A descriptive analysis focusing on normalised gain (e.g. Coletta & Phillips, 2005), and inferential analysis using t-tests and the effect size using Cohen's d (Cohen, 1988). The analysis was conducted for each phase, and in studying differences in pre and post-test scores regarding the questions related to forces, relevant results regarding student understanding were identified.

5.2.3.2 Qualitative analysis

Thematic analysis (TA) following Braun and Clarke (2006, 2019) was chosen to analyse the qualitative parts of the data. Generally, analysing data can be viewed as a pattern-finding endeavour, whether quantitative or qualitative. Following the six-step process presented by Braun and Clarke (2006), pattern-finding can become more systematic. The aforementioned six-step process is presented in Table 5.2. It provides clear steps for the analysis process, starting from having collected the data and going all the way to reporting on the results.

When conducting qualitative analysis, it is important to consider the role of the researcher doing the analysis. In the process of coding and generating themes, the data is always filtered through the lenses of the particular researcher or group of researchers doing the analysis. This is why it is important not to mistake presented themes as something that emerges from the data (Braun & Clarke, 2019).

Table 5.2: An overview of the six-step process suggested by Braun and Clarke (2006) was implemented.

Step	What was done
1. Become familiar with the data	Transcribing interviews and structuring field notes, and taking note of potential points of interest
2. Generate initial codes	Focusing on the interviews initially, carefully reading the transcripts and producing all potentially relevant codes and sorting them together with the corresponding chunk of data in Excel.
3. Search for initial themes 4. Review themes	Step 3 and 4 was considered a back-and-forth process. Guided by the research question and variation theory, identified codes were put into categories corresponding to themes. These themes were then reviewed and revised based on discussions both in the research group and by presenting initial findings at seminars and conferences.
5. Define and name themes	After relevant themes and corresponding codes were identified and grouped, they were renamed and defined to better identify the link between them and to the research question.
6. Report on the study	A report from the study can be found in the method and findings section of Paper II

Further, Braun and Clarke (2019) also highlights the importance of having a reflexive approach when conducting TA. The initial coding was heavily influenced by more of a theoretical TA during the analysis. However, in the process of generating, reviewing and revising themes, an initial more inductive-inspired TA was used, which later in the process once again became more theoretically driven by assumptions from variation theory.

5.2.3.3 Merging and interpreting the results

The final results presented in Paper II are the result of merging insights from both the quantitative and qualitative data. With an emphasis on the qualitative analysis, as it was more relevant towards answering the research question, comparisons to both the quantitative analysis and the resulting redesigns during the design experiment. Using CTD provided valuable insights on how to continue developing the quantitative pre and post-test for further studies using the mechanical Paul trap. Further, it also provided better insights into some of the generated themes.

One particularly relevant outcome of having both quantitative and qualitative data was that it allowed us to better grasp the importance and struggle of identifying what force acted as a trapping force in the mechanical Paul trap. It was identified in the field notes that students tended to struggle with this. However, not until looking at all the available data was it clearly identified both the issue

students had with this task and how important it seemed to be for their ability to modelise the mechanical Paul trap.

5.2.4 Ethical considerations and limitations

During all steps in planning, designing, conducting and analysing, ethical considerations were made in accordance with the guidelines set by the Swedish Research Council (2017). All participants in the study were older than 15 years, and no sensitive information was gathered. Further, participating students and teachers received information regarding how data would be handled, information regarding the purpose of the study and that participation was completely voluntary before giving their consent to participate in the study. Confirmation that this was provided clearly and understandably was provided, as during two of the laboratory sessions, one student denied the request to be interviewed.

A struggle when conducting a mixed-method study is the need to consider what data to collect and how to analyse it in relation to the available time. Initially, this was a leading cause of why field notes were chosen in favour of video recording each laboratory session. Further, by not having video recordings of the sessions, there could be an increased threat to validity, as important details about what students did during the sessions could during the lesson be considered unimportant by the observing researcher. On the other hand, having video cameras in the classroom could make the sessions feel less naturalistic, making students act accordingly.

This issue can be related to a common critique towards DBR, namely its handling of trustworthiness. Juuti and Lavonen (2006) argues for the use of '*unity trustworthiness*' and '*partial trustworthiness*'. It is related to evaluating whether the design experiment generates something new and useful, as well as if appropriate data collection and analysis methods are used. The first part can be handled by properly specifying research questions before starting the design experiment. This forces the researcher to engage in a proper literature review to make sure the issue up for investigation is something worth investigating. The second part is handled in part by, when reporting results from a design experiment, properly disclosing choices made, and motivating them based on previous research. By providing clear and extensive background related to the current study, and providing discussions on how and why data was collected in a certain way, the trustworthiness of the current study should be considered acceptable.

Built into DBR is the focus on a niche context and solving a particular problem within that context. This could put the generalisability of such studies into question. In this study, I am indeed involved in the practice of working in a niche context. However, by using theoretical frameworks, in this case, variation theory, and identifying connections between findings from the current study to previous

work, some level of analytical generalisation is possible even in DBR (Kilpatrick, 1993).

6 Results

6.1 Paper I - The Mechanical Paul Trap: Introducing the Concept of Ion Trapping

Initial proposals for student investigations were developed along with developing the physical and simulated model of the mechanical Paul trap described in 5.1. These were inspired by a combination of synthesising previous research on the quality of the model as a representation of Paul traps, exploring simplified theoretical models of the effective electric and gravitational potential acting as the main trapping components and the pedagogical framework of variation theory.

This paper aimed to provide a design that is useful and easy to build. A physical model of the mechanical Paul trap was developed using only 3D-printed parts, a standard laboratory power supply and a relatively cheap but stable stepper motor controlled by an easily programmable Arduino. With the focus on the design being accessible and affordable, it is a setup that any school with a 3D printer could easily construct. Regarding the simulated model, using an online environment like Google Colaboratory meant that anyone with an internet connection and a web browser could open and run the simulation, no coding required. However, for those interested, the code is also plainly visible in the program and written with a partial focus to make it easily understood, even for amateur coders.

Finally, the usefulness of the developed material was further highlighted by providing suggested student activities. All three investigations aim to identify qualitative similarities between the mechanical Paul trap and an electric Paul trap. The first demonstrates the meaning of a saddle point. It shows that although it is possible to theoretically trap a particle in a stable field configuration, it is not to be considered stable because even the tiniest perturbation makes it unstable. The

second investigation highlights that there seems to be a relationship between what frequency the trap is rotating with and the trapping time. The last investigation connects the mechanical to the electric potential by showcasing another similarity, namely that both (given a fixed effective potential) have some region of frequencies where a particle can be trapped by having students identify the threshold frequency (see Figure 6.1).

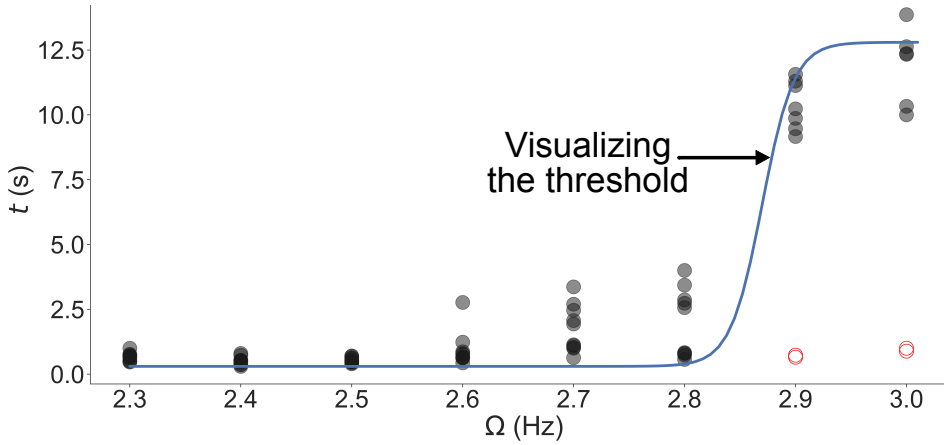

Figure 6.1: The threshold frequency is where the trap changes from being unable to trap a particle to suddenly being able to. This is highlighted in the physical mechanical Paul trap as an increase in trapping time of at least one order of magnitude. Adapted from Paper I.

6.2 Paper II - Rolling Balls of Trapping Ions? How Students Relate Models to Real-world Phenomena in the Physics Laboratory

In this paper, we investigate how students are able to use a toy model (the mechanical Paul trap) to understand the real-world phenomenon of how electric Paul traps function. The research question explored is how interacting with a physical and simulated model of a mechanical Paul trap affects students' ability to understand how a real Paul trap qualitatively works. This was explored in a mixed-method DBR study, using variation theory to analyse and understand the learning process.

Results indicate that the mechanical model is far from trivial, and many students struggled to make sense of it. However, at the end of the laboratory sessions, there

were also groups of students who could use their experiences with the mechanical Paul trap to make sense of and explain qualitatively how a real Paul trap is similar and different to the mechanical model. We identify this as that they were able to create an internal idealised version of the mechanical Paul trap. This process is referred to as students being able to *modelise* the mechanical Paul trap. From the results, the process of *modelising* was identified as the learning process crucial to explore further.

Following the analysis process presented in chapter 5.2.3, the findings indicate that the relevant object of learning students participating in the study needed to focus on was the ability to modelise the mechanical model. Following threads of struggling and successful students, several critical aspects were identified, both to discern and disregard. Aspects critical to discern were frequency, trapping time, and effective potential. Further, within the domain intersecting frequency and trapping time, a critical feature was the threshold frequency.

At first glance, the identified critical aspects can seem to rely solely on attributes related to the phenomenon under study, and not as existing in the learner-phenomenon interaction. Varying frequency affects the trapping time, and the critical feature of threshold frequency can be thought of as existing in the intersection. However, when taking the students into account, an intricate relationship between the three is identified, which makes it unfeasible to consider trapping time and frequency as part of some joint critical aspect. This is highlighted by the identified pattern of variation related to a contrast and generalisation step where the frequency-trapping time relation was discerned and solidified (see Table 6.1 for an overview of the identified patterns of variation and invariance). This was done most successfully by varying the friction after having introduced the threshold frequency. During this step, whether students focused on the frequency or trapping time was crucial for their success in this generalisation. Focusing on trapping time helped students identify that although increased friction decreases trapping time, it does not shift the threshold. Students focusing on frequency during this step generally struggled with seeing its relation to trapping time properly, thus having trouble discerning the effect of the physical properties of the ball.

Table 6.1: Identified patterns of variation and invariance from analysing successful and struggling students. Frequency, trapping time, model and effective potential were identified as dimensions of variation critical to be discerned. Adapted from Paper II.

Frequency	**Trapping time**	Ball initial	Friction	**Model**	Force	**Effective potential**	Pattern of variation and invariance
-	-	-	-	**V**	**V**	-	Attend to previous knowledge model-force relation
V	**V**	i	i	i	-	i	**Contrast** Frequency-trapping time relation
i	**I**	**V**	i	i	-	i	**Generalisation** Frequency-trapping time relation
v	**V**	-	**V**	i	-	i	**Generalisation** Threshold frequency
v	**V**	-	-	i	-	**V**	**Contrast** Effective potential and relation to threshold frequency
V	**V**	-	-	**V**	-	**V**	**Fusion**

Note. Varied aspects are marked 'v', invariant aspects 'i', and aspects not included '-'. Aspects in focus during each pattern of variation and invariance are bold and capitalized.

7

Conclusion and Outlook

In this concluding chapter, I will briefly summarise the licentiate thesis by answering the research questions. Implications for teaching and learning physics and contributions this thesis makes to PER are also suggested. Finally, thoughts on the way forward regarding future work are discussed as an outlook.

The overarching aim of this thesis revolves in part around how the mechanical Paul trap can be used in the upper secondary physics laboratory and to study what learning is made possible. Papers I and II address the aim by providing answers to the research questions posed at the beginning of this thesis in the following ways.

> How can a mechanical model of a Paul trap be made accessible for use in the physics classroom?

This question is explored in both Paper I and II. By developing an easy-to-build mechanical Paul trap, an accessible computer simulation and designing a laboratory exercise using the developed tools, its usefulness is explored. Paper I demonstrates the ease of building and using the setup. This is an important step for educators to increase the willingness to build and incorporate the setup in their teaching. Further, using DBR to develop and evaluate a laboratory exercise using the mechanical Paul trap, as in Paper II, pros and cons are explored more in-depth.

> What is made possible for students to learn when interacting with a physical and simulated model of a mechanical Paul trap?

Results from Paper II identify crucial struggles students experienced, mainly related to the chaotic nature of the ball's movement in the mechanical Paul trap. However, how these can be handled and made useful in the physics laboratory to learn about models and model development in science is also identified.

The relevant object of learning to focus on was the ability to modelise the mechanical model. This was successfully afforded to students following the patterns of variation and invariance presented in Table 6.1 (which also generally agrees with the process illustrated by Marton, 2015, presented in chapter 4.1.2).

7.1 Implications for teaching and learning physics

This thesis provides multiple contributions to researchers and practitioners when it comes to teaching and learning physics, such as

- suggesting a laboratory exercise that can be implemented to develop students' ability to modelise and showcase an important apparatus in modern physics.
- providing further arguments for using models and model construction in the physics classroom.
- highlighting the crucial role of teacher guidance when there is a high level of complexity regarding the interpretation of a phenomenon. This is related to the fact that the level of openness (level of inquiry as stated by Gericke et al., 2022) needs to be chosen deliberately with the intended learning goals in mind.
- identifying that it is not always necessary for students to make use of previous knowledge in subject matter 'correct' ways. This further agrees with the notion that the teacher needs to focus on the current object of learning in focus rather than overarching course-level learning objectives.

7.2 Contributions to PER

By using DBR informed by variation theory, this thesis makes contributions to PER by

- expanding the knowledge on how DBR and variation theory can be used for structuring educational science laboratories to reach the intended object of learning more effectively. Thus increasing the analytical tool set available to researchers in PER.

- contributing to the discussion on the use of models, and particularly toy models, in science education. Working with partly problematic models, and seeing how physicists use them to communicate science, can be a meaningful process to include in the physics classroom.

- relating the identified patterns of variation to the process of model-based science, the usefulness of variation theory in PER is strengthened.

7.3 Outlook

This licentiate thesis roughly marks the halfway point on my journey towards a PhD in PER. Starting from the work presented in Papers I and II, many possible strands exist that would be interesting to follow. In this section, I will reflect upon one such strand I am especially interested in: exploring the development of remote laboratories.

The initial project idea for my PhD was to develop remote laboratories targeted primarily towards the upper secondary school physics curriculum. However, developing and investigating remote laboratories is not something new (e.g. Agbo et al., 2021; Brinson, 2015, 2017; De La Torre et al., 2013; Heradio et al., 2016; Lowe et al., 2013; Ma & Nickerson, 2006; Tho et al., 2017). With this in mind, a potential relevant contribution was identified in employing DBR to develop both a hands-on and remote version of the same lab. Then, the differences in final designs can be explored using variation theory to get insight into how the afforded learning opportunities change depending on the location of the laboratory setup.

Papers I and II can within this project be related to a first pilot phase in developing a remote controllable version of the mechanical Paul trap. Little previous research has focused on how the mechanical Paul trap can be used in teaching about particle trapping, especially in an upper-secondary school setting. Further, remote laboratories can not be used as freely as a hands-on version of the same setup, simply due to that most relevant interactions with the remote setup need to be built in deliberately. Thus an initial design had to be developed to guide what interactions would be built into the remote version of the setup.

Moving forward with the next phase of the project, two designs will be developed in parallel. The hands-on design will be further developed as needed during the next DBR round. However, more focus will be towards developing the remote laboratory version. At the end of this round, what learning is afforded will be analysed using a mixed-method design similar to the one used in this thesis. Preliminary research questions guiding the next stage of my project are the following:

- What role does the physical presence of a toy model have on students' ability to internalise and link the mechanical Paul trap to the real Paul trap?
- How do the afforded learning opportunities differ when working with a mechanical model and a simulation of a Paul trap in a hands-on setting versus a remote laboratory setting?

Another relevant path to follow from this thesis revolves around the discussion of models and taking inspiration from model-based science in physics education. How models can be used to help students grasp abstract and otherwise hard-to-visualise or experience phenomena is something I am continuing to explore further through various side projects. Two such active projects are using drones to visualise how sound waves interact and propagate and making sense of the gyroscopic effect using abstract vector concepts. These projects could help to gain further knowledge in how and when various types of models are best suited to physics education. In addition, it could help to further highlight possibilities and challenges related to teaching and learning about abstract phenomena.

Bibliography

Agbo, F. J., Oyelere, S. S., Suhonen, J., & Tukiainen, M. (2021). Scientific production and thematic breakthroughs in smart learning environments: a bibliometric analysis. *Smart Learning Environments*, *8*(1), 1–25. https://doi.org/10.1186/s40561-020-00145-4

Andersson, S., & Johansson, A. (2016). Gender gap or program gap? Students' negotiations of study practice in a course in electromagnetism. *Physical Review Physics Education Research*, *12*(2), 020112. https://doi.org/10.1103/PhysRevPhysEducRes.12.020112

Bannan-Ritland, B., & Baek, John, Y. (2014). Investigation the Act of Design in Design Research: The Road Taken. In E. Kelly, Anthony, A. Lesh, Richard, & Y. Baek, John (Eds.), *Handbook of design research methods in education: Innovations in science, technology, engineering, and mathematics learning and teaching* (pp. 299–319). Taylor & Francis Group.

Beichner, R. J. (2009). An Introduction to Physics Education Research. In C. Henderson & K. Harper (Eds.), *Reviews in per volume 2: Getting started in physics education research*. American Association of Physics Teachers.

Berryman, A. A. (1992). The Orgins and Evolution of Predator-Prey Theory. *Ecology*, *73*(5), 1530–1535. https://doi.org/10.2307/1940005

Bodegom, E., Jensen, E., & Sokoloff, D. (2019). Adapting RealTime Physics for Distance Learning with the IOLab. *The Physics Teacher*, *57*(6), 382–386. https://doi.org/10.1119/1.5124277

Bogdan, R., & Knopp Biklen, S. (2007). *Qualitative research for education : an introduction to theory and methods* (5th ed.). Pearson Education.

Bokulich, A. (2011). How scientific models can explain. *Synthese*, *180*, 33–45. https://doi.org/10.1007/s11229-009-9565-1

Booth, S., & Ingerman, Å. (2002). Making sense of physics in the first year of study. *Learning and Instruction*, *12*(5), 493–507. https://doi.org/10.1016/S0959-4752(01)00028-7

Borisov, A. V., Kilin, A. A., & Mamaev, I. S. (2018). A Nonholonomic Model of the Paul Trap. *Regular and Chaotic Dynamics*, *23*(3), 339–354. https://doi.org/10.1134/S1560354718030085

Borisov, A. V., Kilin, A. A., & Mamaev, I. S. (2019). A Parabolic Chaplygin Pendulum and a Paul Trap: Nonintegrability, Stability, and Boundedness. *Regular and Chaotic Dynamics*, *24*(3), 329–352. https://doi.org/10.1134/S1560354719030067

Braun, V., & Clarke, V. (2006). Using thematic analysis in psychology. *Qualitative Research in Psychology*, *3*(2), 77–101. https://doi.org/10.1191/1478088706qp063oa

Braun, V., & Clarke, V. (2019). Reflecting on reflexive thematic analysis. *Qualitative Research in Sport, Exercise and Health*, *11*(4), 589–597. https://doi.org/10.1080/2159676X.2019.1628806

Brewe, E. (2008). Modeling theory applied: Modeling Instruction in introductory physics. *American Journal of Physics*, *76*(12), 1155–1160. https://doi.org/10.1119/1.2983148

Brimacombe, C., Corless, R. M., & Zamir, M. (2021). Computation and applications of Mathieu functions: a historical perspective. *SIAM Review*, *63*(4), 653–720.

Brinson, J. R. (2015). Learning outcome achievement in non-traditional (virtual and remote) versus traditional (hands-on) laboratories: A review of the empirical research. *Computers and Education*, *87*, 218–237. https://doi.org/10.1016/j.compedu.2015.07.003

Brinson, J. R. (2017). A Further Characterization of Empirical Research Related to Learning Outcome Achievement in Remote and Virtual Science Labs. *Journal of Science Education and Technology*, *26*(5), 546–560. https://doi.org/10.1007/s10956-017-9699-8

Brown, A. L. (1992). Design Experiments: Theoretical and Methodological Challenges in Creating Complex Interventions in Classroom Settings. *Source: The Journal of the Learning Sciences*, *2*(2), 141–178.

Chabay, R. W., & Sherwood, B. A. (2015). *Matter & interactions* (4th ed.). John Wiley & Sons, Inc.

Chevallard, Y. (2006). Steps towards a new epistemology in mathematics education. *Proceedings of the IV congress of the European society for research in mathematics education*, 21–30.

Chevallard, Y., & Bosch, M. (2014). Didactic Transposition in Mathematics Education. *Encyclopedia of Mathematics Education*, 170–174. https://doi.org/10.1007/978-94-007-4978-8_48

Cobb, P., Confrey, J., DiSessa, A., Lehrer, R., & Schauble, L. (2003). Design Experiments in Educational Research. *Educational Researcher*, *32*(1), 9–13. https://doi.org/10.3102/0013189X032001009

Cobb, P., & Gravemeijer, K. (2008). Experimenting to Support and Understand Learning Processes. In A. Kelly, R. Lesh, & J. Baek (Eds.), *Handbook of design research methods in education: Innovations in science, technology,*

engineering and mathematics learning and teaching (1st ed., pp. 86–113). Taylor & Francis Group.

Cohen, J. (1988). *Statistical Power Analysis for the Behavioral Sciences* (2nd ed.). Routledge. https://doi.org/10.4324/9780203771587

Coisson, R., Vernizzi, G., & Yang, X. (2009). Mathieu functions and numerical solutions of the Mathieu equation. *2009 IEEE International Workshop on Open-source Software for Scientific Computation (OSSC)*, 3–10.

Coletta, V. P., & Phillips, J. A. (2005). Interpreting FCI scores: Normalized gain, preinstruction scores, and scientific reasoning ability. *American Journal of Physics*, *73*(12), 1172–1182. https://doi.org/10.1119/1.2117109

Coll, R. K., France, B., & Taylor, I. (2005). The role of models/and analogies in science education: implications from research. *International Journal of Science Education*, *27*(2), 183–198. https://doi.org/10.1080/0950069042000276712

Creswell, J. W., Plano Clark, V. L., Gutmann, M. L., & Hanson, W. E. (2003). Advanced mixed methods research designs. In A. Tashakkori & C. Teddlie (Eds.), *Handbook of mixed methods in social and behavioral research* (pp. 209–240). Sage.

Creswell, J. W., & Plano Clark, V. L. (2017). *Designing and conducting mixed methods research*. Sage publications.

De La Torre, L., Heradio, R., Jara, C. A., Sanchez, J., Dormido, S., Torres, F., & Candelas, F. A. (2013). Providing collaborative support to virtual and remote laboratories. *IEEE Transactions on Learning Technologies*, *6*(4), 312–323. https://doi.org/10.1109/TLT.2013.20

Docktor, J. L., & Mestre, J. P. (2014). Synthesis of discipline-based education research in physics. *Physical Review Special Topics - Physics Education Research*, *10*(2), 020119. https://doi.org/10.1103/PhysRevSTPER.10.020119

Dounas-Frazer, D. R., & Lewandowski, H. J. (2018). The Modeling Framework for Experimental Physics: Description, development, and applications. *European Journal of Physics*, *39*(6), 064005. https://doi.org/10.1088/1361-6404/aae3ce

Epstein, J. M. (2008). Why model? *Journal of Artificial Societies and Social Simulation*, *11*(4), 12. https://www.jasss.org/11/4/12.html

Etkina, E., Brookes, D. T., & Planinsic, G. (2019). *Investigative Science Learning Environment: When learning physics mirrors doing physics*. Morgan & Claypool Publishers. https://doi.org/10.1088/2053-2571/ab3ebd

Etkina, E., Warren, A., & Gentile, M. (2006). The Role of Models in Physics Instruction. *The Physics Teacher*, *44*(1), 34–39. https://doi.org/10.1119/1.2150757

Euler, E., Gregorcic, B., & Linder, C. (2020). Variation theory as a lens for interpreting and guiding physics students' use of digital learning environments.

European Journal of Physics, *41*(4), 045705. https://doi.org/10.1088/1361-6404/ab895c

Fan, W., Du, L., Wang, S., & Zhou, H. (2017). Confining rigid balls by mimicking quadrupole ion trapping. *American Journal of Physics*, *85*(11), 821–829. https://doi.org/10.1119/1.5005927

Fischer, H. E., Girwidz, R., & Treagust, D. F. (2021). Topics of Physics Education and Connections to Other Sciences. In H. E. Fischer & R. Girwidz (Eds.), *Physics education* (pp. 1–23). Springer International Publishing. https://doi.org/10.1007/978-3-030-87391-2_1

Fredlund, T. (2015). *Using a Social Semiotic Perspective to Inform the Teaching and Learning of Physics* (Doctoral dissertation). Uppsala University. http://urn.kb.se/resolve?urn=urn:nbn:se:uu:diva-247771

Frenkel, D., & Portugal, R. (2001). Algebraic methods to compute Mathieu functions. *Journal of Physics A: Mathematical and General*, *34*(17), 3541.

Galán, D., Isaksson, O., Enger, J., Rostedt, M., Johansson, A., Hanstorp, D., & de la Torre, L. (2019). Safe Experimentation in Optical Levitation of Charged Droplets Using Remote Labs. *Journal of Visualized Experiments*, (143). https://doi.org/10.3791/58699

Gericke, N., Högström, P., & Wallin, J. (2022). A systematic review of research on laboratory work in secondary school. https://doi.org/10.1080/03057267.2022.2090125

Giere, R. N. (1988). *Explaining science : a cognitive approach*. Univ. of Chicago Press.

Giere, R. N. (2004). How Models Are Used to Represent Reality. *Philosophy of Science*, *71*(5), 742–752. https://doi.org/10.1086/425063

Godfrey-Smith, P. (2006). The strategy of model-based science. *Biology and Philosophy*, *21*(5), 725–740. https://doi.org/10.1007/S10539-006-9054-6/FIGURES/1

Guisasola, J., Zuza, K., Sarriugarte, P., & Ametller, J. (2023). Research-Based Teaching-Learning Sequences in Physics Education: A Rising Line of Research. In *The international handbook of physics education research: Special topics* (pp. 26–1 – 26–26). AIP Publishing. https://doi.org/10.1063/9780735425514_026

Gutiérrez-Vega, J. C., Rodrıguez-Dagnino, R. M., Meneses-Nava, M. A., & Chávez-Cerda, S. (2003). Mathieu functions, a visual approach. *American Journal of Physics*, *71*(3), 233–242.

Häggström, J. (2008). *Teaching systems of linear equations in Sweden and China: What is made possible to learn?* (Doctoral dissertation). University of Gothenburg. http://hdl.handle.net/2077/17286

Hake, R. R. (2008). Design-Based Research in Physics Education: A Review. In A. E. Kelly, R. A. Lesh, & J. Y. Baek (Eds.), *Handbook of design research methods in education* (pp. 493–508). Taylor & Francis Group.

Halloun, I. (1996). Schematic modeling for meaningful learning of physics. *Journal of Research in Science Teaching*, *33*(9), 1019–1041. https://doi.org/10.1002/(SICI)1098-2736(199611)33:9<1019::AID-TEA4>3.0.CO;2-I

Halloun, I. A., & Hestenes, D. (1987). Modeling instruction in mechanics. *American Journal of Physics*, *55*(5), 455–462. https://doi.org/10.1119/1.15130

Hammer, D. (2023). The Necessarily, Wonderfully Unsettled State of Methodology in PER: A Reflection. In M. F. Tasar & P. R. L. Heron (Eds.), *The international handbook of physics education research: Special topics* (pp. 22–1 – 22–12). AIP Publishing.

Harrison, A. G., & Treagust, D. F. (1998). Modelling in Science Lessons: Are There Better Ways to Learn With Models? *School Science and Mathematics*, *98*(8), 420–429. https://doi.org/10.1111/j.1949-8594.1998.tb17434.x

Heradio, R., de la Torre, L., Galan, D., Cabrerizo, F. J., Herrera-Viedma, E., & Dormido, S. (2016). Virtual and remote labs in education: A bibliometric analysis. *Computers & Education*, *98*, 14–38. https://doi.org/10.1016/j.compedu.2016.03.010

Heron, P. R. L. (2018). Identifying and addressing difficulties: Reflections on the empirical and theoretical basis of an influential approach to improving physics education. In C. Henderson & K. Harper (Eds.), *Reviews in per volume 2: Getting started in physics education research*. American Association of Physics Teachers.

Heron, P. R. L., & Meltzer, D. E. (2005). The future of physics education research: Intellectual challenges and practical concerns. *American Journal of Physics*, *73*(5), 390–394. https://doi.org/10.1119/1.1858480

Hesse, M. (1966). *Models and Analogies*. Blackwell Publishers Ltd. https://doi.org/10.1002/9781405164481.ch44

Hestenes, D. (1987). Toward a modeling theory of physics instruction. *American Journal of Physics*, *55*(5), 440–454. https://doi.org/10.1119/1.15129

Holmqvist, M., Gustavsson, L., & Wernberg, A. (2007). Generative learning: learning beyond the learning situation. *Educational Action Research*, *15*(2), 181–208. https://doi.org/10.1080/09650790701314684

Ingerman, Å., Linder, C., & Marshall, D. (2009). The learners' experience of variation: following students' threads of learning physics in computer simulation sessions. *Instructional Science*, *37*(3), 273–292. https://doi.org/10.1007/s11251-007-9044-3

Johnson, A. K., & Rabchuk, J. A. (2009). A bead on a hoop rotating about a horizontal axis: A one-dimensional ponderomotive trap. *American Journal of Physics*, *77*(11), 1039–1048. https://doi.org/10.1119/1.3167216

Johnson, R. B., Onwuegbuzie, A. J., & Turner, L. A. (2007). Toward a Definition of Mixed Methods Research. *Journal of Mixed Methods Research*, *1*(2), 112–133. https://doi.org/10.1177/1558689806298224

Jones, W. (1980). Earnshaw's theorem and the stability of matter. *European Journal of Physics*, *1*(2), 85.

Joseph, D. (2004). The Practice of Design-Based Research: Uncovering the Interplay Between Design, Research, and the Real-World Context. *Educational psychologist*, *39*(4), 235–242. https://doi.org/10.1207/s15326985ep3904_5

Juuti, K., & Lavonen, J. (2006). Design-Based Research in Science Education: One Step Towards Methodology. *Nordic Studies in Science Education*, *2*(2), 54–68. https://doi.org/10.5617/nordina.424

Kärrqvist, C. (1985). *Kunskapsutveckling genom experimentcentrerade dialoger i ellära : [Development of reasoning in electricity by means of experiments and discussions]* (Doctoral dissertation). Diss. Göteborg : Univ.
Diss. Göteborg : Univ.

Kelly, A. ((2004). Design Research in Education: Yes, but is it Methodological? *Journal of the Learning Sciences*, *13*(1), 115–128. https://doi.org/10.1207/s15327809jls1301_6

Kilpatrick, J. (1993). Beyond face value: Assessing research in mathematics education. In G. Nissen & M. Blomhøj (Eds.), *Criteria for scientific quality and relevance in the didactics of mathematics* (pp. 15–34).

Kirillov, O. N., & Levi, M. (2016). Rotating saddle trap as Foucault's pendulum. *American Journal of Physics*, *84*(1), 26–31. https://doi.org/10.1119/1.4933206

Kullberg, A., Runesson Kempe, U., & Marton, F. (2017). What is made possible to learn when using the variation theory of learning in teaching mathematics? *ZDM - Mathematics Education*, *49*(4), 559–569. https://doi.org/10.1007/s11858-017-0858-4

Kvale, S., & Brinkmann, S. (2018). Doing Interviews. http://digital.casalini.it/9781526426093%20http://digital.casalini.it/5017966
This eBook is not available for institutional purchase. Special access authorizations may apply; please contact us for further information. 208 p.

La Braca, F., & Kalman, C. S. (2021). Comparison of labatorials and traditional labs: The impacts of instructional scaffolding on the student experience and conceptual understanding. https://doi.org/10.1103/PhysRevPhysEducRes.17.010131

Lawrence, I. (2004). Modelling simply, without algebra: beyond the spreadsheet. *Physics Education*, *39*(3), 281–288. https://doi.org/10.1088/0031-9120/39/3/005

Lindqvist, P. (2017). *Notes on the p-Laplace equation* (2nd ed.). University of Jyväskylä.

Ling Lo, M., Chik, P., & Pang, M. F. (2006). Patterns of variation in teaching the colour of light to primary 3 students. *Instructional Science*, *34*(1), 1–19. https://doi.org/10.1007/s11251-005-3348-y

Lo, M. L. (2012). *Variation theory and the improvement of teaching and learning*. Acta Universitatis Gothoburgensis.

Lo, M. L., Marton, F., Pang, M. F., & Pong, W. Y. (2004). Toward a pedagogy of learning. In F. Marton & A. B. Tsui (Eds.), *Classroom discourse and the space of learning* (pp. 189–225). Routledge Taylor & Francis Group. https://doi.org/10.4324/9781410609762

Lowe, D., Newcombe, P., & Stumpers, B. (2013). Evaluation of the Use of Remote Laboratories for Secondary School Science Education. *Research in Science Education*, *43*(3), 1197–1219. https://doi.org/10.1007/s11165-012-9304-3

Ma, J., & Nickerson, J. V. (2006). Hands-On, Simulated, and Remote Laboratories: A Comparative Literature Review. https://doi.org/10.1145/1132960.1132961

Marton, F., & Säljö, R. (1976). On Qualitative Differences in Learning: I-Outcome and Process. *British Journal of Educational Psychology*, *46*(1), 4–11. https://doi.org/10.1111/j.2044-8279.1976.tb02980.x

Marton, F. (1981). Phenomenography - Describing conceptions of the world around us. *Instructional Science*, *10*(2), 177–200. https://doi.org/10.1007/BF00132516

Marton, F. (1986). Phenomenography-A Research Approach to Investigating Different Understandings of Reality. *Source: Journal of Thought*, *21*(3), 28–49.

Marton, F. (2015). *Necessary Conditions of Learning*. Taylor & Francis. https://doi.org/10.4324/9781315816876

Marton, F., & Booth, S. (1997). *Learning and Awareness*. Lawrence Erlbaum Associates, Inc.

Marton, F., & Tsui, A. B. (Eds.). (2004). *Classroom Discourse and the Space of Learning*. Routledge. https://doi.org/10.4324/9781410609762

Maxwell, J. C. (1873). *A treatise on electricity and magnetism* (Vol. 1). Oxford: Clarendon Press.

McBride, D. (2009). *Transfer of learning from traditional optics to wavefront aberrometry* (Ph.D. Dissertation). Kansas State University.

McDermott, L. C. (2021). *A View From Physics*. AIP Publishing LLCMelville, New York. https://doi.org/10.1063/9780735423831

Merriam-Webster. (n.d.). Trap. https://www.merriam-webster.com/dictionary/trap

National Research Council. (2012). *Discipline-Based Education Research*. National Academies Press. https://doi.org/10.17226/13362

Nelson, C. E., Nickels, M. K., & Beard, J. (2002). The Nature of Science as a Foundation for Teaching Science: Evolution as a Case Study BT - The Nature of Science in Science Education: Rationales and Strategies. In W. F. McCo-

mas (Ed.), *The nature of science in science education. science & technology education library, vol 5* (pp. 315–328). Springer Netherlands. https://doi.org/10.1007/0-306-47215-5_20

Neumann, K., & Nordine, J. (2021). Instructional Coherence and the Development of Student Competence in Physics. In H. E. Fischer & R. Girwidz (Eds.), *Physics education* (pp. 151–173). Springer International Publishing. https://doi.org/10.1007/978-3-030-87391-2_6

Odden, T. O. B., Marin, A., & Caballero, M. D. (2020). Thematic analysis of 18 years of physics education research conference proceedings using natural language processing. *Physical Review Physics Education Research*, *16*(1), 010142. https://doi.org/10.1103/PhysRevPhysEducRes.16.010142

OECD. (1999). *Measuring student knowledge and skills: A new framework for assessment*.

Oh, P. S., & Oh, S. J. (2011). What Teachers of Science Need to Know about Models: An overview. *International Journal of Science Education*, *33*(8), 1109–1130. https://doi.org/10.1080/09500693.2010.502191

Otero, V. K., Harlow, D. B., & Meltzer, D. E. (2023). Qualitative Methods in Physics Education Research. In M. F. Tasar & P. R. L. Heron (Eds.), *The international handbook of physics education research: Special topics* (pp. 25–1 – 25–32). AIP Publishing. https://doi.org/10.1063/9780735425514_025

Palinkas, L. A., Horwitz, S. M., Green, C. A., Wisdom, J. P., Duan, N., & Hoagwood, K. (2015). Purposeful Sampling for Qualitative Data Collection and Analysis in Mixed Method Implementation Research. *Administration and Policy in Mental Health and Mental Health Services Research*, *42*(5), 533–544. https://doi.org/10.1007/S10488-013-0528-Y/FIGURES/2

Palmgren, E., & Rasa, T. (2022). Modelling Roles of Mathematics in Physics: Perspectives for Physics Education. *Science and Education*, 1–18. https://doi.org/10.1007/S11191-022-00393-5/TABLES/1

Pang, M. F., & Ki, W. W. (2016). Revisiting the Idea of "Critical Aspects". *Scandinavian Journal of Educational Research*, *60*(3), 323–336. https://doi.org/10.1080/00313831.2015.1119724

Pang, M. F., & Marton, F. (2003). Beyond "lesson study": Comparing two ways of facilitating the grasp of some economic concepts. *Instructional Science*, *31*(3), 175–194. https://doi.org/10.1023/A:1023280619632

Paul, W. (1990). Electromagnetic traps for charged and neutral particles. *Reviews of modern physics*, *62*(3), 531–542.

Phillippi, J., & Lauderdale, J. (2018). A Guide to Field Notes for Qualitative Research: Context and Conversation. *Qualitative Health Research*, *28*(3), 381–388. https://doi.org/10.1177/1049732317697102

Phillips, A. M., & O'donnell, C. (2021). A critical examination of "expert-like" in physics education research. *Physics Education Research Conference Proceedings*, 339–346.

Pluta, W. J., Chinn, C. A., & Duncan, R. G. (2011). Learners' epistemic criteria for good scientific models. *Journal of Research in Science Teaching*, *48*(5), 486–511. https://doi.org/10.1002/tea.20415

Potvin, G., Hazari, Z., Khatri, R., Cheng, H., Head, T. B., Lock, R. M., Kornahrens, A. F., Woodle, K. S., Vieyra, R. E., Cunningham, B. A., Kramer, L., & Hodapp, T. (2023). Examining the effect of counternarratives about physics on women's physics career intentions. *Physical Review Physics Education Research*, *19*(1), 010126. https://doi.org/10.1103/PhysRevPhysEducRes.19.010126

Prosser, M., Walker, P., & Millar, R. (1996). Differences in students' perceptions of learning physics. *Physics Education*, *31*(1), 43–48. https://doi.org/10.1088/0031-9120/31/1/022

Redhead, M. (1980). Models in Physics. *The British Journal for the Philosophy of Science*, *31*(2), 145–163. http://www.jstor.org/stable/687183

Redish, E. F. (2003). *Teaching Physics with the Physics Suite*. Wiley.

Runesson, U. (2006). What is it possible to learn? On variation as a necessary condition for learning. *Scandinavian Journal of Educational Research*, *50*(4), 397–410. https://doi.org/10.1080/00313830600823753

Runswick-Cole, K. (2011). Interviewing. In P. Banister, J. Sixsmith, G. Bunn, E. Burman, J. Daniels, P. Duckett, D. Goodley, R. Lawthom, I. Parker, & K. Runswick-Cole (Eds.), *Qualitative methods in psychology : A research guide* (2nd ed., pp. 88–99). McGraw-Hill Education. http://ebookcentral.proquest.com/lib/gu/detail.action?docID=893040

Säljö, R. (1979). Learning about learning. *Higher Education*, *8*(4), 443–451. https://doi.org/10.1007/BF01680533

Saltelli, A., & Funtowicz, S. (2014). When All Models Are Wrong. *Issues in Science and Technology*, *30*(2), 79–85. http://www.jstor.org/stable/43315849

Stewart, J., Hansen, J., & Ding, L. (2023). Quantitative Methods in PER. In M. F. Tasar & P. R. L. Heron (Eds.), *The international handbook of physics education research: Special topics* (pp. 24–1 – 24–32). AIP Publishing. https://doi.org/10.1063/9780735425514_024

Suri, H. (2011). Purposeful sampling in qualitative research synthesis. *Qualitative Research Journal*, *11*(2), 63–75. https://doi.org/10.3316/QRJ1102063

Svensson, K. (2022). *Linking Programming to Representations: Understanding meaningnmaking in physics education through semiotic resources* (Doctoral dissertation). Lund University.

Swedish National Agency for Education. (2011). *Physics [Subject syllabus]*.

Swedish Research Council. (2017). *Good research practice*.

Tarciso Borges, A., & Gilbert, J. K. (1998). Models of magnetism. *International Journal of Science Education*, *20*(3), 361–378. https://doi.org/10.1080/0950069980200308

Tarciso Borges, A., & Gilbert, J. K. (1999). Mental models of electricity. *International Journal of Science Education*, *21*(1), 95–117. https://doi.org/10.1080/095006999290859

Tasar, M. F., & Heron, P. R. L. (Eds.). (2023a). *The International Handbook of Physics Education Research: Learning Physics*. AIP Publishing.

Tasar, M. F., & Heron, P. R. L. (Eds.). (2023b). *The International Handbook of Physics Education Research: Special Topics*. AIP Publishing.

Tasar, M. F., & Heron, P. R. L. (Eds.). (2023c). *The International Handbook of Physics Education Research: Teaching Physics*. AIP Publishing.

Tho, S. W., Yeung, Y. Y., Wei, R., Chan, K. W., & So, W. W.-m. (2017). A Systematic Review of Remote Laboratory Work in Science Education with the Support of Visualizing its Structure through the HistCite and CiteSpace Software. *International Journal of Science and Mathematics Education*, *15*(7), 1217–1236. https://doi.org/10.1007/s10763-016-9740-z

Thompson, R. I., Harmon, T. J., & Ball, M. G. (2002). The rotating-saddle trap: A mechanical analogy to RF-electric-quadrupole ion trapping? *Canadian Journal of Physics*, *80*(12), 1433–1448. https://doi.org/10.1139/p02-110

Tiberghien, A., & Venturini, P. (2023). Teaching Practices. In M. Tasar & P. R. L. Heron (Eds.), *The international handbook of physics education research: Learning physics* (pp. 24–1 – 24–36). AIP Publishing.

Tonks, L. (1940). Note on earnshaw's theorem. *Electrical Engineering*, *59*(3), 118–119. https://doi.org/10.1109/EE.1940.6434810

Vieyra, R., Vieyra, C., Pendrill, A.-M., & Xu, B. (2020). Gamified physics challenges for teachers and the public. *Physics Education*, *55*(4), 045014. https://doi.org/10.1088/1361-6552/ab8779

Volterra, V. (1928). Variations and Fluctuations of the Number of Individuals in Animal Species living together. *ICES Journal of Marine Science*, *3*(1), 3–51. https://doi.org/10.1093/icesjms/3.1.3

Vygotskij, L. S. (1986). *Thought and language* (Translation newly rev. and edited / by Alex Kozulin. ed.). MIT Press.

Weisberg, M. (2007). Who is a modeler? https://doi.org/10.1093/bjps/axm011

Wells, M., Hestenes, D., & Swackhamer, G. (1995). A modeling method for high school physics instruction. *American Journal of Physics*, *63*(7), 606–619. https://doi.org/10.1119/1.17849

Appended papers

”The Mechanical Paul Trap: Introducing the Concept of Ion Trapping”

The Mechanical Paul Trap: Introducing the Concept of Ion Trapping

Sebastian Kilde Lofgren*, Ricardo Méndez-Fragoso**, Jonathan Weidow***, Jonas Enger*

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Nobel laureate Wolfgang Paul showed, back in the 1950s, that charged particles can be trapped using alternating electric fields[1]. This technique is commonly referred to as Paul traps or radiofrequency traps (RF-traps) and is used in various areas of modern physics. This paper presents a 3D-printed mechanical Paul trap, a naïve simulation of the system in Python, and student investigations. The files for the 3D-printed trap are available for download and print[i], and the code for the simulation is available to run and tinker with[ii].

The mechanical Paul trap (***Fig. 1A***) can be used as a demonstration tool or an experimental setup to simulate how a real Paul trap works or to investigate interesting physical phenomena[2–5]. Previous work has mainly focused on higher education, and here we present investigations that can be incorporated into lower parts of the educational system. Proposed investigations can either be done in isolation or together to have the student(s) experience the physical phenomena in the mechanical Paul trap and visualize how a real Paul trap works.

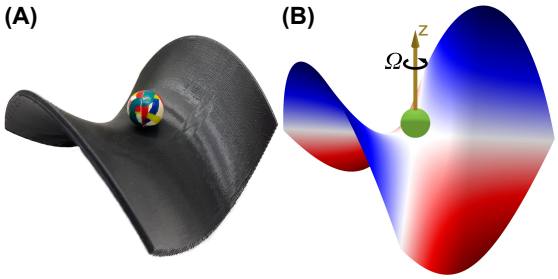

*Fig. 1: **(A)** 3D-printed mechanical Paul trap. **(B)** A particle in the quadrupole field. The potential of the field is illustrated as both the height of the saddle and the color scale that goes from dark red (large attractive force) to dark blue (large repulsive force).*

The Mechanical Paul Trap

A linear Paul trap confines particles by having four metal rods connected to an AC source, where opposite rods have the same polarity and continuously change their polarity with some frequency Ω, as seen in ***Fig. 2***.

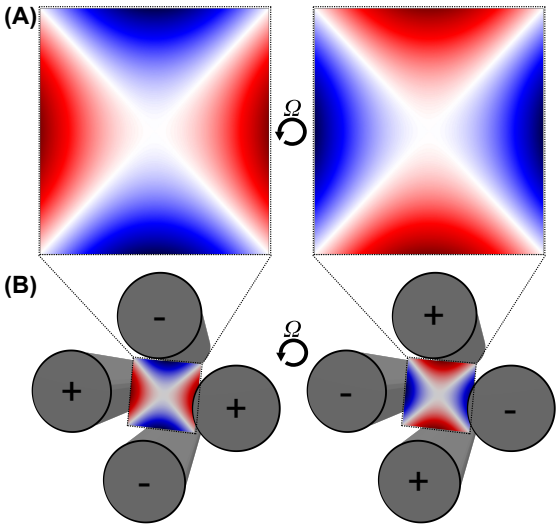

*Fig. 2: **(A)** Two configurations of the potential field in a linear Paul trap. The normalized potential goes from -1 (dark blue) to 1 (dark red). The field flips from left to right configuration during its rotation driven by the angular frequency Ω. The potential is zero along the diagonals, creating a saddle point in the middle. **(B)** A linear Paul traps using 4 charged rods.*

Moving from a two-dimensional representation to a three-dimensional one makes visualizing the field strength less abstract. ***Fig. 1B*** shows a saddle-looking shape with a particle in the middle. In this picture, the field strength is represented as the height of the z-axis. However, the field is not visible and thus becomes a rather abstract concept to communicate to students. This raises critical pedagogical questions about visualizing and making the idea more tangible for students. Also, why do we even need a varying electric field to trap a particle? One way is to take a pure mathematical route and let that be the representational tool to understand the physical situation. Another method can be finding some analog that students can see and relate to. This is where the mechanical Paul trap takes the stage, as it did during Wolfgang Paul's Nobel lecture[1]. Since then, others have constructed and studied the mechanical Paul trap for demonstration purposes or laboratory exercises [4–6].

* University of Gothenburg, Sweden
** Universidad Nacional Autónoma de México, Mexico
*** Chalmers University of Technology, Sweden
Corresponding author: Sebastian Kilde Lofgren, Sebastian.lofgren@physics.gu.se

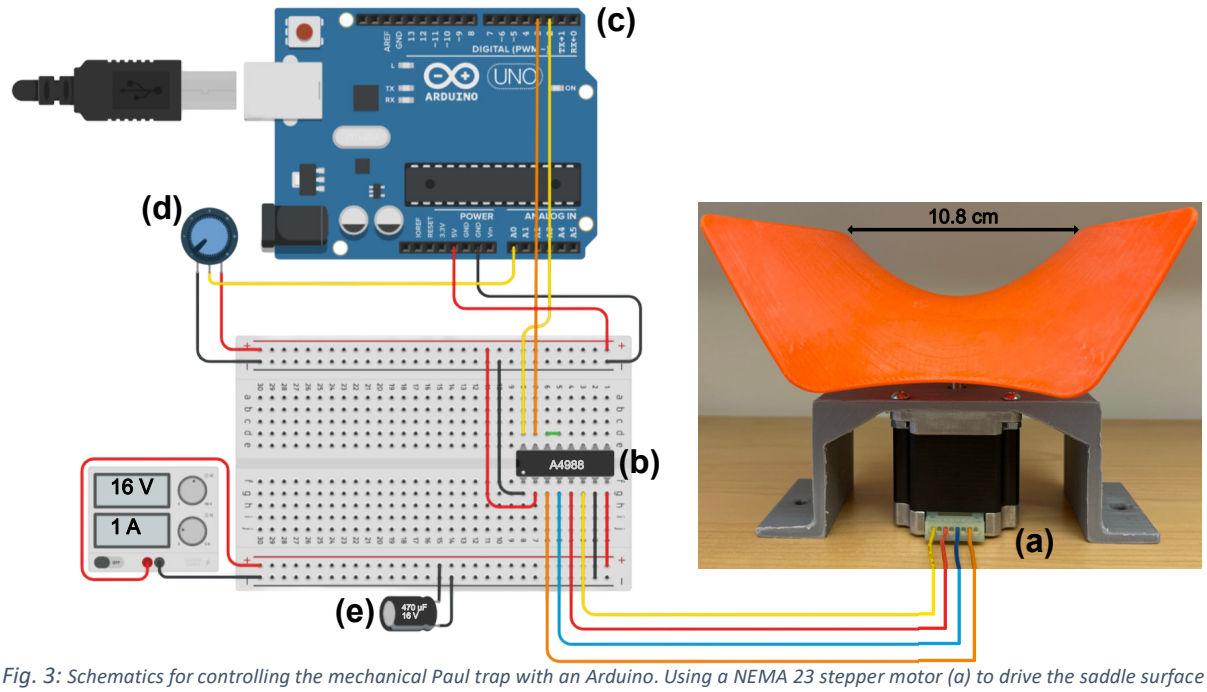

Fig. 3: Schematics for controlling the mechanical Paul trap with an Arduino. Using a NEMA 23 stepper motor (a) to drive the saddle surface requires a motor driver (b). The Arduino (c) is connected to a computer for displaying the frequency. The frequency is controlled by a potentiometer (d). A capacitor (e) protects the motor from current spikes.

However, the mechanical Paul trap is not a trivial analog because it has some crucial differences from a real Paul trap. Two such significant differences are the effect of rolling and friction, which can lead to trapping balls on various rotating surfaces[7–10]. The effects in a mechanical Paul trap have been discussed as simulating adding a magnetic field to the Paul trap, giving rise to a Lorentz force[3,6,11]. There have been several proposals for alternative models that can be used for demonstration purposes, such as a spring-loaded inverted pendulum[2], the rotating hoop trap[12], or a large-scale electrical quadrupole trap[13]. The first two examples solve some apparent problems with the mechanical Paul trap model but also remove some pedagogical strengths, such as the potential shape's physical appearance. The last example lets students see particles trapped in a varying electric field. However, since it uses high voltage, the equipment is dangerous, and once again, the visual component of the particle-field interaction is obscured. The focus on the visual aspect of particle-field interaction is why the mechanical Paul trap is further investigated as a pedagogical tool in this paper, despite its mentioned shortcomings.

A mechanical Paul trap can be constructed, as described by Rueckner et al.[5] or 3D-printed. There are several advantages of using an automated process like 3D printing, such as low cost, high precision, and easy modification. The proposed trap design only requires a few components that cannot be 3D-printed: a motor, an Arduino and some basic components, and a power source. See ***Fig. 3*** for schematics[iii].

Theory

For a real Paul trap, we can express the varying electric potential as

$$V(t) = \frac{V_0}{2R^2}(x^2 - y^2)\cos(\Omega t) \qquad (1)$$

where V_0 is the voltage of an AC source, R is half the distance between two opposing rods (***Fig. 1***), Ω is the angular frequency of the AC source, t the time, and x and y are cartesian coordinates.

For the mechanical Paul trap, in the frictionless case, we can construct the gravitational potential as

$$U_g(t) = \frac{mgh}{R^2}[(x^2 - y^2)\cos(2\Omega t) + 2xy\sin(2\Omega t)] \qquad (2)$$

where m is the mass of the trapped particle, g the gravitational acceleration, Ω the angular rotational frequency of the saddle surface, R the radius of the saddle, h the maximum height of the surface at a distance r from the center, t the time, and x and y are cartesian coordinates.

Equations (1) and (2) look alike, apart from a sinusoidal cross-dependent term for the mechanical trap. This term shows that the potential in the mechanical Paul trap is bounded[4], which must be true for a solid surface. The gravitational potential and the initial conditions dictate the motion of a "particle" in the trap. In equation (2) the shape of the trap, given by $\frac{h}{R^2}$ and the angular

frequency Ω are controllable terms that affect the "particle's" path. In the case of the real Paul trap, the shape of the trap and the frequency at which the electric potential is changed are terms affecting particle paths as well. Thus, to use the mechanical Paul trap to teach how real Paul traps work, the shape and frequency can be relevant factors to investigate.

Pedagogical framework

The theoretical framework that served as a guiding principle when developing the exercises is variation theory proposed by Marton[14,15]. A fundamental question to ask is what students need to have learned to succeed in a particular task[14]? I.e., what are the critical aspects when learning something? According to variation theory, learning something can be thought of as *learning to see*. Learning is about being able to discern features of the world, and the proposed process of going from *not seeing* to *seeing* can be separated into three stages that Marton[14] refers to as

1. **Contrast**, where different aspects of a concept of phenomena take turns being varied or invariant. This can be done via induction, where the studied aspect is invariant, or via contrast, where the studied aspect is varied.
2. **Generalization** lets the discerned feature in the previous stage be invariant and allows the student to vary other aspects.
3. **Fusion** requires that the student experiences variations of multiple focused aspects and puts different aspects together to understand the object of learning more deeply.

Student investigation 1: Physical properties of a saddle point

In Swedish upper secondary mathematics, students experience the concept of saddle points in the abstract world of sketching function graphs using derivatives. A saddle point exhibits the phenomena of having a derivative of zero, but when moving just a small distance away from that point, the derivatives point toward the saddle point in some directions and away in others. For example, the function $z = \frac{1}{a}(x^2 - y^2)$ constructs the surface of the mechanical Paul trap (***Fig. 1***). Using the mechanical Paul trap, one can intuitively understand the concept of saddle points by trying to place a ball on the trap carefully and have it not fall off when the trap is stationary. It may require some trial and error, but it is doable. To guide students, the teacher can ask follow-up questions: What does it take for the ball not to roll off? Why does the ball tend to roll off the surface?

This simple experiment highlights one crucial property of saddle points. Namely that it is possible to have something placed on such a point, but it is not a stable point.

Student investigation 2: Varying frequency

The mechanical Paul trap has a frequency region where, if a ball is placed close to the middle with some care, it will be trapped for some time. The purpose of varying the rotation frequency of the trap is to guide students to discern rotation frequency as a critical aspect through contrast.

We propose the following procedure to discover the importance of rotation frequency in the mechanical Paul trap:

a) Place a ball, for example, a glass bead with a diameter of 22 mm, on the saddle surface using the following rotational frequencies: 2 Hz and 3.5 Hz. Repeat the experiment at least ten times for each frequency and note the trapping time for each trial. Is it possible to catch the ball at any of the frequencies?

The trapping time can be defined as the time from letting go of the ball until the ball leaves the trap. The ball can be counted as trapped if the trapping time exceeds, for example, 10 seconds.

b) Using a computer simulation, investigate if a "particle" can be trapped using the following rotational frequencies: 2 Hz and 3.5 Hz, and the starting position $x = 0$ mm, and $y = 8$ mm.

The frequencies are chosen so students will experience one region where a ball cannot be trapped and one where trapping is possible. For example, when trying to trap the glass bead at 3.5 Hz, it is possible to trap it for at least 10 seconds. However, due to the chaotic nature of the mechanical Paul trap, the trapping time can fluctuate between 1 – 13 seconds (***Fig. 4***).

Student investigation 3: Threshold frequencies

There is a range of possible frequencies for which to trap a ball, and finding the lower bound could help highlight this feature. The lower bound is the threshold frequency, which we define as where the trapping time increases significantly.

For this investigation, let the students choose their approach and have them vary the size of the ball (for example between 10 – 45 mm in diameter). Some groups may rely heavily on computer

simulation, and others do not use it. Moreover, when using the mechanical Paul trap, one can find the threshold frequency in a couple of different ways: (1) starting with a frequency where it is easy to trap and with the ball stable in the trap, successively lowering the frequency until dropping the ball; (2) starting with a low frequency and systematically increasing the frequency in small increments. The threshold frequency should become somewhat apparent if plotting the trapping time at various frequencies, as in ***Fig. 4***.

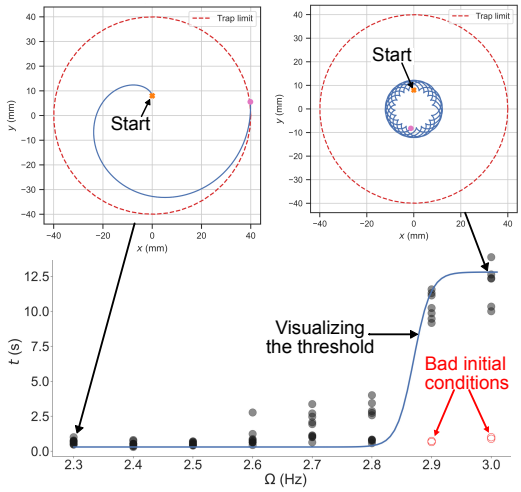

Fig. 4: Finding the threshold frequency for a glass bead with diameter of 22 mm. From 2.8 to 2.9 Hz the trapping time increases significantly, usually by at least one order of magnitude. The inclusion of bad initial conditions here highlights the sensitivity of initial conditions. With the simulation this shift is observable as the particle being either trapped indefinately or not by observing a few rotations.

The act of changing balls affects the system in various ways. If the mass or surface of the ball is changed, then friction is changed. However, small changes in friction do only appear to affect trapping time and not the threshold frequency. Generally, it is suggested to have low friction in the system to increase trapping time. When changing the size of the ball, the threshold frequency changes since the effective potential changes. The effective potential can be considered the allowed path of a ball's center of mass in the trap. The simulation can help highlight this by having the ball as a frictionless point particle.

Conclusions

Our proposed investigations allow students to explore the concept of trapping particles using varying fields. Furthermore, by having a mechanical analog to the Paul trap and a simulation, the abstract notion of field strength can be explained using multiple representations, thus opening more possibilities for students to grasp the concept through variation.

Acknowledgment

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References

1. Paul, W. Electromagnetic Traps for Charged and Neutral Particles(Nobel Lecture). *Angewandte Chemie International Edition in English* **29**, 739–748 (1990).

2. Kirillov, O. N. & Levi, M. Rotating saddle trap as Foucault's pendulum. *Am J Phys* **84**, 26–31 (2016).

3. Kirillov, O. & Levi, M. A Coriolis force in an inertial frame. *Nonlinearity* **30**, 1109–1119 (2017).

4. Thompson, R. I., Harmon, T. J. & Ball, M. G. The rotating-saddle trap: a mechanical analogy to RF-electric-quadrupole ion trapping? *Can J Phys* **80**, 1433–1448 (2002).

5. Rueckner, W., Georgi, J., Goodale, D., Rosenberg, D. & Tavilla, D. Rotating saddle Paul trap. *Am J Phys* **63**, 186–187 (1995).

6. Fan, W., Du, L., Wang, S. & Zhou, H. Confining rigid balls by mimicking quadrupole ion trapping. *Am J Phys* **85**, 821–829 (2016).

7. Weckesser, W. A ball rolling on a freely spinning turntable. *Am J Phys* **65**, 736–738 (1997).

8. Zengel, K. The electromagnetic analogy of a ball on a rotating conical turntable. *Am J Phys* **85**, 901–907 (2017).

9. Ehrlich, R. & Tuszynski, J. Ball on a rotating turntable: Comparison of theory and experiment. *Am J Phys* **63**, 351–359 (1995).

10. Romer, R. H. Motion of a sphere on a tilted turntable. *Am J Phys* **49**, 985–986 (1981).

11. Cox, G. & Levi, M. The ponderomotive Lorentz force. *Nonlinearity* **33**, 4030–4045 (2020).

12. Johnson, A. K. & Rabchuk, J. A. A bead on a hoop rotating about a horizontal axis: A one-dimensional ponderomotive trap. *Am J Phys* **77**, 1039–1048 (2009).

13. McGinness, L. *et al.* 3D-Printable Model of a Particle Trap: Development and Use in the Physics Classroom. *Journal of Open Hardware* **3**, (2019).

14. Marton, F. *Necessary conditions of learning*. (Routledge: London, 2015).

15. Marton, F. & Booth, S. *Learning and awareness*. (Abingdon, Oxon : Routledge, 1997).

[i] Link to stl-files ready to be 3D-rpinted: https://www.thingiverse.com/thing:5249350

[ii] Link to a Google Colaboratory notebook with the simulation, ready to run directly from the browser: https://colab.research.google.com/drive/1j8PF0aVwbNHfwCPKdJB4rMXY4U9XapK1?usp=sharing

[iii] Link to the Arduino code: https://github.com/SebKilL91/Mechanical-Paul-trap

”Rolling Balls or Trapping Ions? How Students Relate Models to Real-world Phenomena in the Physics Laboratory”

RESEARCH ARTICLE

Rolling Balls or Trapping Ions? How Students Relate Models to Real-world Phenomena in the Physics Laboratory

The creation and use of models in science is of great importance for knowledge production and communication. For example, toy models are often used as idealised explanatory models in physics education. Models can be a powerful tool for exploring phenomena in ways that facilitate learning. However, careful consideration of instruction and explanations needs to be considered to guide how students relate models to real-world phenomena in subject-correct ways. A design experiment was conducted to investigate how upper secondary school students can use models for learning in the physics laboratory. The intervention used in the study was a laboratory exercise developed over three phases where students worked with a mechanical Paul trap and a simulation to understand the principle behind a real Paul trap. Each phase of the study consisted of three to five laboratory sessions. The data were analysed using thematic analysis, and the learning process was understood using the theoretical framework of variation theory. From the results, it was possible to identify patterns of variation for successful lab groups and critical aspects and features students need to discern to effectively modelise the mechanical Paul trap. The findings also indicate that having students work with models can be a meaningful clarificatory process to develop a deeper understanding of the use and limitations of models in science.

KEYWORDS

upper secondary physics education; physics laboratory; variation theory; design-based research; toy models; paul trap

1 | INTRODUCTION

In science, models are everywhere. Consequently, there is a need to categorise and make sense of the role of different models (Hesse, 2017; Toon, 2012; Bokulich, 2011; Godfrey-Smith, 2006, 2009; Rosenblueth & Wiener, 1945). Some usually present in physics education are *analogue models*, such as the billiard ball model of a gas (e.g. PhET, 2022) and electron orbits in an atom (e.g. Beiser, 2003); *ideal models*, such as perfectly isolated systems like the sun-moon-earth interactions (e.g. Chabay & Sherwood, 2015); and *toy models*, often seen as mechanical models such as the inflating balloon being analogous to the expanding universe and the lack of a "centre" (e.g. Pålsgård, Kvist, & Nilson, 2012), or the mechanical model for the Paul trap (Paul, 1990).

Using the theoretical framework of variation theory and a design-based approach, how students can use toy models to understand a real-world phenomenon is investigated by developing and iteratively revising a laboratory exercise. To this end, the current study focuses on exploring how a mechanical analogue to the Paul trap, the mechanical Paul trap, as both a physical and computer-simulated model, can help upper secondary physics students learn the concept of trapping ions using varying electric fields, as in a real Paul trap. To relate the particular study to the discussion regarding the use of models in science education, the research question in focus is how interacting with a physical and simulated model of a mechanical Paul trap affects students' ability to understand how a real Paul trap qualitatively works.

This paper refers to the technique for trapping ions using a varying quadrupole electric field, developed by Nobel laureate Wolfgang Paul (1990) and collaborators, as either "Paul trap" or "real Paul trap". Furthermore, the two toy models for the Paul trap, a physical (mechanical) model and a computer simulation of the mechanical model, will be referred to as either "mechanical Paul trap" or "mechanical trap" and "simulation" respectively, with further clarifications as needed.

By reporting on the results from the design experiment study, contributions are made in three ways to both the physics education research community as well as the broader field of science education by identifying strategies and presenting an approach inspired by variation theory for how teachers can evaluate and improve laboratory exercises to fit their student groups better:

1. By constructing and evaluating a laboratory exercise in upper secondary physics classrooms, from the learners' point of view, using the toy models of the Paul trap and identifying critical aspects, suggestions are provided on how to use the artefacts to help students approach how a real Paul trap works.
2. Our use of variation theory contributes to the analytical toolset for structuring educational science laboratories to effectively reach the intended object of learning.
3. Our analysis provides further insight into the discussions on using models in science education.

1.1 | Models in science education

When investigating the scope of which young learners (middle schoolers) can judge the quality of scientific models, Pluta, Chinn, and Duncan (2011) found evidence of overlap between scientists' epistemic criteria, as proposed by Kuhn, and the investigated classes' ideas. The overlapping ideas were accuracy, explanatory scope, and the principle of parsimony (i.e. model complexity should be appropriate, see for example Sober,

1981). The findings suggest using models and student-led inquiry in laboratory exercises as a possible effective teaching strategy. Earlier work by, amongst others, Harrison and Treagust (1998) suggests a similarly positive outlook on using students' notion of models and their development in the classroom to improve learning outcomes, even though their stance is that secondary school students' ideas on the quality of models are less developed than Pluta et al. (2011) suggests. Moreover, in a study by Schwarz et al. (2009), also set in middle school, issues regarding creating fruitful learning progressions for scientific practice using models are discussed. A vital issue identified but left unanswered was trying to motivate students to develop models to learn, as scientists do, rather than "doing school". Therefore not fully grasping the meaning or usefulness of models in how they are used to develop knowledge influences students' abilities to evaluate a model (Schwarz et al., 2009). Thus, finding ways to make model development meaningful could be important for improving learning outcomes in science.

Further, in reviewing studies on using models in the science classroom, Coll, France, and Taylor (2005) suggest that discussion-focused activities can enhance model-based learning from a sociocultural theoretical standpoint (Vygotskij, 1986). Lastly, Oh and Oh (2011) identifies the use of *didactic transposition* (Chevallard, 2006) as a way to identify how and why models can be used to bridge the gap between what should be taught and what students are ready to face. For example, when relating to teaching in secondary school, *what is to be taught* is informed by, but ultimately a subset of, some *scholarly knowledge*. Moving to a particular lesson, *what is to be taught* and *what is learnt* are further reduced to some essential sub-concepts. The back-and-forth process from *scholarly knowledge* to *what is learnt* can be related to the style of science referred to as model-based science (Godfrey-Smith, 2006; Giere, 1988) in the sense that when using a model in the physics classroom, it is a reduction, but supposed to resemble, some *real-world phenomenon* and as such existing in both *scholarly knowledge* and *what is to be taught* (see Figure 1). The *model system* is informed by the *real-world phenomenon* and specified by the *model description*, which is how a model is understood and described in the classroom, living both in *what is taught* and *what is learnt*.

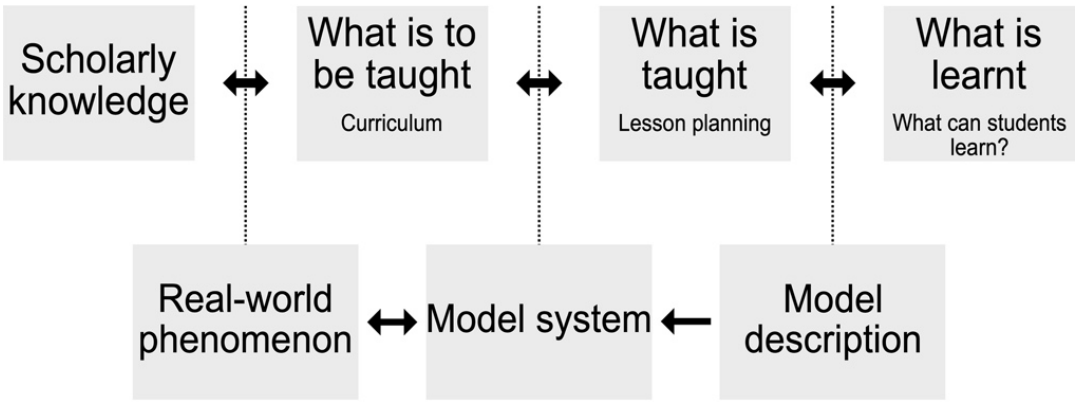

FIGURE 1 Visualising the relation between the use of models in science (education) and transformations of an object of knowledge in different parts of the educational system. Adapted the process of didactic transposition from Chevallard and Bosch (2014) [top process] and how a scientist specifies a model system from Giere (1988) [bottom process].

1.2 | Variation theory

Originating from the phenomenographic approach, which started its development in the 1970s (e.g. Marton & Säljö, 1976) but first appeared by name in the literature with Marton (1981), variation theory began to see the light of day in the early 2000s following Marton and Booth (1997). Variation theory can be viewed as a learning theory and described in the words of Runesson (2006, p. 397) as *"Variation theory is a theory accounting for differences in learning, and it provides a way to describe the conditions necessary for learning"*. According to variation theory, one of the most fundamental questions to ask is what students need to be able to discern for the intended learning to be possible (Marton, 2015). As such, variation theory recognises that the most effective way to teach should be linked to the specific learning objective (the object of learning, see below), and it is in studying how learners develop in connection to the learning objective that proposals for how to teach the specific topic effectively can be made (Holmqvist, Gustavsson, & Wernberg, 2007).

Following is an expansion of two core concepts in variation theory, the object of learning and critical aspects, and a prototype to promote learning that serves as a basis for our analysis.

1.2.1 | Object of learning

In order to understand how students learn, it is essential first to understand what learning is made possible. Considering teaching as a rational activity, Lo, Marton, Pang, and Pong (2004) state that teachers must have their particular students' characteristics and physical conditions in mind when planning a lesson. Further, these conditions need to be translated into particular actions that help the current class in the current lesson to achieve specific goals. This relates to the object of learning, which can be thought of as a particular way of experiencing something which develops a particular capability. Marton (2015) suggests three ways the object of learning appears: intended (the aim), enacted (what is taught) and lived (what is learnt) object of learning. These distinctions are necessary since what learning is made possible is not necessarily synonymous with what the teacher or instructional designer thinks is possible to learn. Furthermore, the object of learning can seem synonymous with what is known in curriculums as a learning objective due to their lexical similarities. However, as Lo (2012) emphasises, expected learning outcomes (learning objectives) easily lead to examination-oriented learning, whereas the object of learning focuses on the starting point and what students need to learn to achieve the desired learning objectives.

The current study focuses on the enacted (what the teacher and students are doing during the lesson) and lived object of learning (what the students learnt, as evident mainly from interviews and, to some extent, pre and post-tests) to inform how to change the lab design (constructed from the intended object of learning). It is done by defining the object of learning in terms of its critical aspects, and much like Ingerman, Linder, and Marshall (2009) focus on how the attended variations constitute student learning. By having this focus, we take on what is known in phenomenography as a second-order perspective (Marton & Booth, 1997), meaning that, as indicated previously, the students' experiences of the phenomenon are the focus, rather than the phenomenon itself. This has been shown to be a powerful way to identify qualitative different ways of experiencing a phenomenon (e.g. Marton & Booth, 1997; Booth & Ingerman, 2002; Ingerman et al., 2009; Patron, Linder, & Wikman, 2021).

1.2.2 | Critical aspects

Building on the definition of the object of learning used in this study, critical aspects are understood as what aspects are necessary to discern of a phenomenon in order to become aware of said phenomenon. More specifically, it is the aspects that the learner has to notice but is currently unable to (Marton, 2015). Further, a critical aspect can be seen as synonymous with a dimension of variation in the phenomenon (Pang & Ki, 2016), in the sense that critical aspects refer to a dimension of variation that needs to be experienced by the learner in order for them to be able to discern a particular phenomenon in a particular way (Lo, 2012; Kullberg, Runesson Kempe, & Marton, 2017).

When an aspect is varied, a dimension of variation is opened up by presenting a set of values, or features, in the aspect. If the aspect being varied is a critical aspect, then the dimension of variation is a critical aspect. Further, if certain features are required in order for the learner to discern a critical aspect, we call it a critical feature. According to Lo (2012, p. 30), when a child is learning to identify the colour "red", it's important to focus on its critical aspects and features. For instance, if the critical aspect is colour, one can show the child a red and a green ball to open up the dimension of variation. However, since the goal is to teach the child to recognise "red" specifically, it's important to discern the critical feature of "redness" by displaying different objects that are all red. This example highlights the inseparable nature of critical aspects and features. Meaning that it is impossible to discern a critical feature without identifying what critical aspects the feature belongs to.

According to variation theory, the proper way to understand critical aspects can be problematic when put into practice, as revealed by Pang and Ki (2016). They highlight that teachers can interpret critical aspects as either the essential aspects defined by the tradition of the field or the curriculum or related to what students usually struggle with or tend to forget. By linking back to the relationship with phenomenography and the qualitatively different way of experiencing something, Marton and Booth (1997) note that this can be understood as a difference in what aspects are discerned and focused on simultaneously. Thus pointing toward the relational nature of critical aspects, they visualise the critical differences between qualitatively different ways of experiencing something as a learner (Pang & Ki, 2016).

A synonymous view between critical aspects and dimensions of variation makes the core of the variation theory visible: *variation*. Holmqvist et al. (2007, p. 186) make this clear by pointing out that *"in the learning situation there must be a structure of relevance and variation to make it possible to discern critical aspects"*. Here they are implying that it is not enough to open up dimensions of variation related to the critical aspects; it must be done purposely.

1.2.3 | A prototype for designing learning activities

Marton (2015) specifies that if the process of learning is seen as learning to see, one can understand the learning process through the following three-step process (patterns of variation), prefaced by some instantiation where the object of learning is presented:

1. **Contrast**. It focuses on studying different aspects of the studied concept separately by letting the studied (critical) aspect vary with certain values. The aim is to discern a critical aspect for understanding the object of learning.
2. **Generalisation**. By letting the previously discerned aspect be invariant and varying other aspects, the goal here is to discard varied aspects as not relating to the object of learning.
3. **Fusion**. When going back and forth between **contrast** and **generalisation**, multiple critical aspects can be identified. However, to fully understand the object of learning, students must experience variations in multiple critical aspects simultaneously, and fusion can occur when this is done.

As an example of this process in the physics laboratory, consider a student learning about Ohm's law $U = RI$ during a laboratory session using a simple electric DC circuit. Focusing on how a resistor affects the current flowing in the circuit, a possible beginning (**contrast**) could be to let the student test multiple (including having no) resistors with different resistance to open up the dimension of variation of resistance. Next up, in the **generalisation** phase, since resistors can look quite different, the student might ascribe some physical properties of the resistor as a relevant aspect to how a resistor affects the current. Thus, letting the student try resistors that keep the resistance invariant but have different physical properties, such as size and seeing that those aspects do not affect the current in the circuit allows for a generalisation to be made through experiencing sameness in resistance. Later, **fusion** can occur when the student has made a similar exploratory journey regarding the dimension of variation of voltage and both critical aspects are simultaneously opened up as dimensions of variations to more deeply understand the object of learning, here being about understanding what affects the current in a simple DC circuit.

Another example linked to physics education can be found in Ling, Chik, and Pang (2006), which investigates patterns of variation used by a group of teachers participating in a learning study when teaching primary school students about the colour of light. The intended object of learning in the study was defined as understanding the relationship between sunlight and the component colours. After conducting a pre-test, the researchers and participating teachers identified two critical aspects: The prism will not create a rainbow, it merely splits up sunlight; and there exists a part-whole relationship between sunlight and the colours of a rainbow. To improve learning outcomes, teachers participating in the study used the concept of patterns of variation to help students better discern the critical aspects. However, the results indicate that only the first critical aspect identified in this study was successfully discerned by the majority of the students. The analysis by Ling et al. (2006) indicates that this could be related to the fact that the second critical aspect was not properly contrasted. In fact, it seemed that the second critical aspect was taught using a pattern of variation on the generalisation level, skipping the seemingly essential contrast step.

2 | METHOD

2.1 | Design-based research

Design-based research is used within the physics education research community (Hake, 2008). Further, it is also one of the building blocks for the popular method used within variation theory called learning study (Holmqvist et al., 2007). The usefulness of design-based research within the field of education, where interventions are made in naturalistic classroom settings, can be considered in terms of its goals.

Conducting design experiments is about contributing to theory and informing practice (Brown, 1992; Joseph, 2004; Bannan-Ritland & Baek, John, 2014). Brown (1992) argues that conducting intervention studies in naturalistic classroom settings in cycles, systematically changing certain parameters, and exploring the actual outcome versus the expected can generate theoretical advancements that are not possible in a laboratory setting. However, the aforementioned positive sides of design-based research also have to be viewed as problematic regarding generalisability and meaningfulness (Kelly, 2004). With a focus on generating powerful hypotheses and contributing to the extension of conceptual frameworks within variation theory to the physics laboratory setting, this study provides a focus that helps mitigate these problems. Moreover, it should be noted that the researchers role in the development is crucial and that connections to existing theory must be ever-present (Barab & Squire, 2004). Lastly, the main type of questions that can be asked using a design-based approach revolves around what is happening and why it is happening (Shavelson, Phillips, Towne, & Feuer, 2003). What is happening originates from the descriptions from observations and relates to the enacted object of learning, whereas why it is happening is understood from using the theoretical framework of variation theory to understand what students learnt from interviews and pre and post-tests.

The current study uses variation theory as a theoretical framework but sticks to a design-based research approach instead of the more commonly used learning study approach. One main difference between these approaches is the clear focus on practising teacher involvement in all stages of the learning study cycle (Lo, 2012; Marton, 2015). The high involvement of participating teachers in a learning study presents some problems if wanting to conduct a learning study cycle in a limited course given in Swedish upper secondary schools such as Physics 2. First, for example, it introduces a lot more work for participating teachers having to be able to collaborate and co-plan between several different schools. Thus possibly decreasing the number of willing participants. Second, it may change the focus of the study away from the intended one, here being the use of toy models in a laboratory setting, to something the participating teachers feel more at home with to teach the intended object of learning. Furthermore, the distance between research and practice, a clear advantage of using learning study, is kept small in the present study because the researchers involved in the project all have a teacher or teacher-educator background. The study was conducted in three phases (see Figure 2). The first phase was considered a pre-study in which the laboratory equipment was thoroughly tested, and instruments for data collection were reviewed. Figure 2 also shows a condensed version of the different investigations students were tasked with conducting during the lab. The various parts will be described and explained further throughout the text.

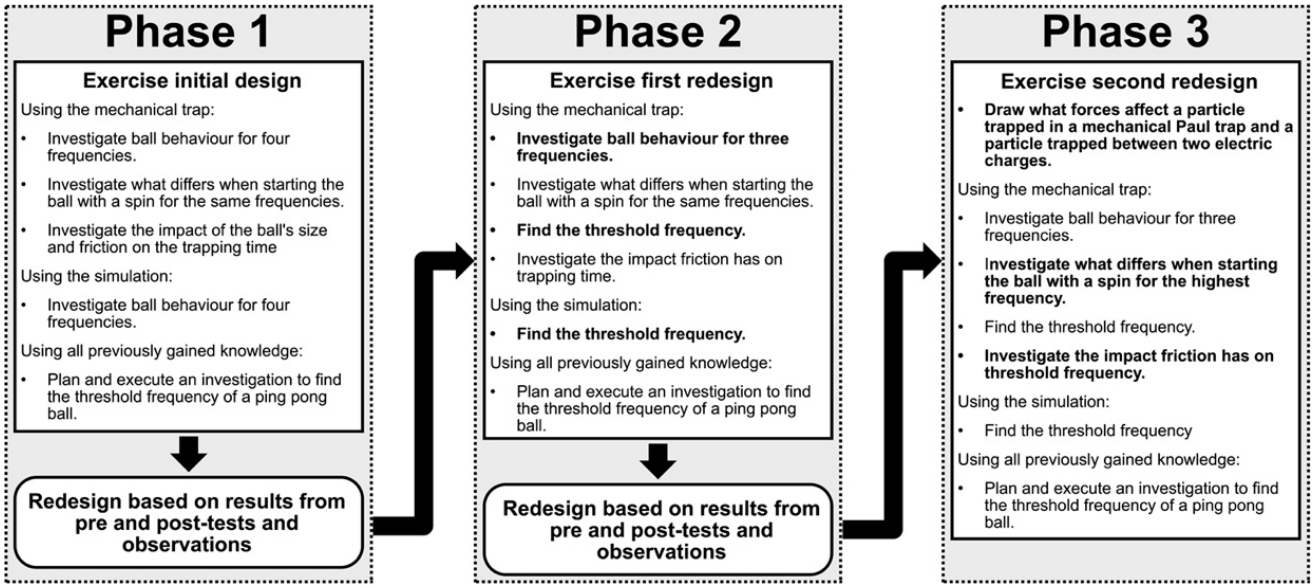

FIGURE 2 The design experiment cycle with an overview of the practical exercises included. After phases 1 and 2, an initial analysis was based on the pre and post-tests and the observations. After phase 3, all available data from all phases were analysed to provide the study's final results.

2.2 | Participants

The setting for the current study is the course Physics 2, generally taken during the last year in Swedish upper secondary school (Swedish National Agency for Education, 2011). Six upper secondary schools in Sweden participated in the study, four public schools and two private ones. In the Swedish upper-secondary school system, private and public schools operate under the same regulations in terms of not being allowed to charge tuition fees, how students apply to them, and following the same curriculum. Within those, eight teachers had twelve laboratory sessions, three during phase 1, five in phase 2 and four in phase 3. Participating schools were chosen such that there would be a variation of big city schools versus suburban and smaller city and a spread in terms of socioeconomic status. The number of students in each laboratory session varied from six to 25, but most sessions spanned from eight to 15 students. 119 students participated in the laboratory sessions. In total, there were 124 students enrolled in the classes who participated in the study. Two students from each session were interviewed in all but two sessions, 10 and 11, in which only one participated, meaning a total of 22 students were interviewed.

In designing the study, ethical considerations were made under the guidelines established by the Swedish Research Council (2017). Participating teachers and students received information regarding the purpose of the study, how data would be handled and used, and that participating was voluntary before giving their consent to participate in the study.

2.3 | Lab design

For the lab to have the potential to be utilised in the upper secondary physics classroom, several important factors had to be considered: First, it had to be of relevance to the current curriculum in the course. Second, the equipment has to be cheap, accessible and reliable. Third, the preparatory work before and after the laboratory session should be minimal for the teacher.

On the first point of consideration, the concept the students were supposed to learn about is how charged particles can be confined using charged rods (i.e. how a Paul trap works). In the Swedish upper secondary course Physics 2, students study movement in gravitational, electric and magnetic fields in two dimensions (Swedish National Agency for Education, 2011). Learning about Paul traps is relevant for understanding complex movement in electric fields. Further, the mechanical Paul trap also illustrates complex movement in a field, gravitational, thus allowing students to encounter similar complex movements in a gravitational field as they would have been able to identify in a real Paul trap.

Concerning the second and third points, five setups of the mechanical Paul trap were built using 3D printed parts and an Arduino connected to a stepper motor to allow students to control the rotational frequency of the trap (Figure 3). The design of the mechanical Paul trap and the simulation is similar to the one proposed by Kilde Lofgren, Méndez Fragoso, Weidow, and Enger (in press). The mechanical Paul trap allows one to experience how different objects behave when put on the surface of the 3D printed saddle surface, either stationary or rotating at some frequency. For this lab, students had access to balls of varying sizes, densities, mass distribution and material and could change the rotational frequency of the trap between 0–4 Hz using steps of 0.1 Hz. Furthermore, the simulation allowed students to study a simplified version of the mechanical Paul trap in which there was no friction, the "ball" was considered a point particle, and the initial conditions were kept constant.

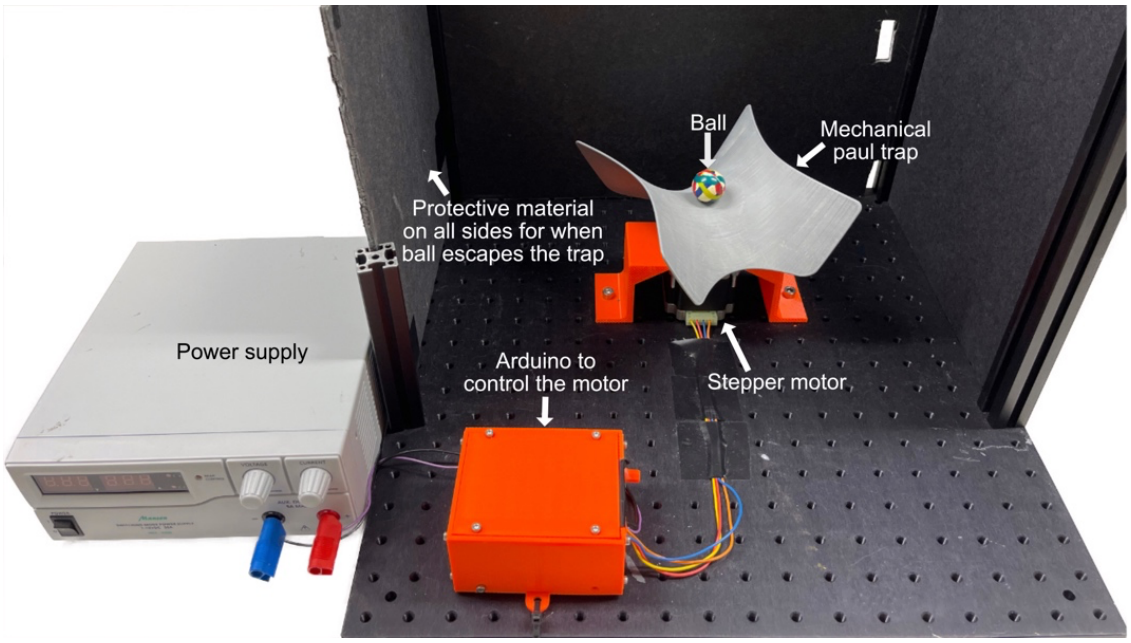

FIGURE 3 The experimental setup that students interacted with. To read the rotational frequency, they had a computer connected to the Arduino.

Before each laboratory session, the teacher in charge of the lesson was provided with a teacher guide consisting of: theoretical explanations regarding how a Paul trap works, what the similarities and differences are between it and the mechanical Paul trap, suggestions on their introduction of the lab and time management, some notes regarding the laboratory equipment, and suggested answers to the students' worksheet. In addition, the teachers had the opportunity to consult with one researcher and try the equipment before the session. The researchers provided all of the equipment needed for the laboratory session. Lastly, the student worksheet consisted of exercises that went from more closed to more open investigations as the session progressed, ending with an open-ended discussion question regarding the validity of the mechanical Paul trap as a model for a real Paul trap.

2.4 | Data collection and analysis

A pre-test was made approximately one week before each lesson to establish that the concept of Paul traps and the mechanical model was indeed novel to the participating students. It was similar to the post-test conducted after the laboratory session to analyse how students' understanding of the mechanical Paul trap developed. The pre and post-tests, analysed using SPSS, and the field notes were the primary informers of how the student worksheets changed between the three phases. However, the same material had a more passive role during the final analysis as the interview transcripts provided the richest insight into the students' lived object of learning. Why the different types of data had varying importance over the study can be motivated by the fact that since no studies have previously been made concerning upper secondary student perception when learning about Paul traps, it was warranted to gather data in such a way that multiple possible threads could be followed during the final analysis. Lastly, it should be noted that the response rate differed between the pre and post-tests due to factors such as students not being present on one occasion or not choosing to answer the post-test (see Table 1).

TABLE 1 Distribution of answers over the pre and post-tests over the three phases. The given percentages are in relation to the total number of students enrolled in the classes participating in each phase.

Test	Phase 1 (n = 37)	Phase 2 (n = 45)	Phase 3 (n = 42)
Pre	34 (92%)	45 (100%)	35 (83%)
Post	32 (86%)	33 (73%)	38 (90%)

In order to collect the qualitative data, the field notes and interview transcripts, one of the researchers took part in every lesson as an observer and interviewer. During each lesson, field notes were taken by focusing on two beforehand randomly selected lab groups, as well as taking notes on how the teacher interacted with the groups and started and ended the lesson. By having randomly selected groups to focus on, the potential for having a biased or unintentionally skewed set of observation data was avoided. Further, we argue that focusing only on two lab groups allowed the observations to examine how the groups worked in greater detail. Specifically if and how their discussions around the tasks and interactions with the material changed during the lesson. The process of taking field notes was partially done during the lesson, where

keywords and phrases regarding what happened were written down. In addition, shortly after each lesson, a more substantial written account of what happened was made, as well as some initial coding and highlighting of potential points of extra interest. Doing an initial analysis regarding qualitative data shortly after collection is an important step to keep the essence of the data (Bogdan & Knopp Biklen, 2007; Phillippi & Lauderdale, 2018). The choice of using field notes instead of video recording each lesson was made mainly for time-management reasons regarding available resources. This was important for the current study since there is a delicate process of how to divide the available time between each type of data collected and the analysis. In terms of validity, taking field notes could have the effect of being coloured by the observing researcher's beliefs on what would be important to recognise (i.e. the intended object of learning). The aforementioned process of focusing on a limited number of randomly chosen groups during each session was used to combat this issue. Another point of consideration was regarding keeping the sessions close to normal classroom conditions. Then filming the lesson can make the sessions feel less naturalistic for the students.

One student was randomly selected to be interviewed after the lesson within each observed group. The interviews followed a semi-structured approach and were audio-recorded so that the interviewer could focus on the conversation taking place. Like the field notes, an initial analysis was done soon after the interviews were conducted as the data was transcribed. Transcripts presented in the findings section of this paper have been translated from Swedish to English. In order to keep the transcripts authentic, the translations were done as directly as possible to keep the grammatical structure students use and still make the sentences make sense in English. Transcripts are provided from different phases of the study, and what student was in what phase can be seen in Table 2.

TABLE 2 Distribution of the presented student transcripts over the three phases. In total, nine interviewed students' transcripts are included as examples, labelled S1 through S9 in order of appearance.

Phase 1	Phase 2	Phase 3
S4, S6, S7, S8	S2, S5	S1, S3, S9

The final analysis of the data was done using thematic analysis (TA), following Braun and Clarke (2006, 2019). With a focus on the qualitative parts of the data (interview transcripts and field notes), emerging patterns and themes were also informed by previous initial data analysis that informed the design changes between the phases. By reanalysing all available data and then relating it to previous initial coding, it is possible to mitigate the fact that the initial analysis and coding of data may lead to prematurely partially closing or narrowing down the analysis (Vaismoradi, Jones, Turunen, & Snelgrove, 2016). With a grounding in variation theory, the TA was theory-driven and themes were identified on a latent level, meaning that the analysis is already theorised (Braun & Clarke, 2006). A theory-driven approach allows the later stages of the analysis, in which the themes are abstracted, to become less dependent on the researchers' subjective judgement (Vaismoradi et al., 2016). Finally, by combining the initial analysis (resulting in the redesign between each phase) with the themes found using TA, it was possible to discern patterns of variation for groups that succeeded with the lab during phase 3.

As with a phenomenographic approach (Marton, 1986), results from this study should be taken as something discovered in a specific context. The identified themes are coloured by the researchers different experiences regarding the phenomenon, teaching and research. This introduces problems with the replicability of this study. However, the identified themes initially proposed by the main author were discussed in relation to the available data and accepted only after the whole research group agreed with the analytical process. After agreement within the group, identified themes with the presented data were also discussed with other colleagues. Thus, there is analytical replicability due to the high intersubjective agreement regarding the identified themes and increased credibility (Nowell, Norris, White, & Moules, 2017).

3 | FINDINGS

The analysis identified what students needed to focus on to understand how a real Paul trap works using the mechanical Paul trap and the simulation and how the model can be problematic (see Figure 4). Further, the analysis also shows that students' views of what a simulation differs and that it is something teachers should be aware of. Finally, considering both the identified themes and how the laboratory exercise changed, patterns of variation for groups who succeeded with the laboratory exercise were identified.

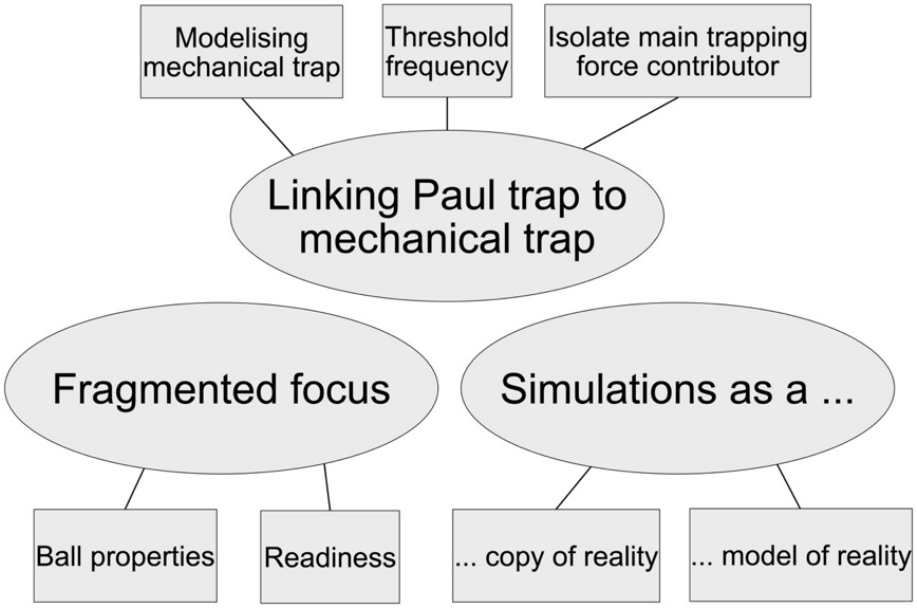

FIGURE 4 Identified themes from the thematic analysis.

3.1 | Linking Paul trap to the mechanical trap

In this section, we report on findings related to necessary discernments for students to make to better see the link between a real Paul trap and the mechanical Paul trap.

3.1.1 | Students' inability to recount the intended aim

To gauge what the students' experiences of what they were supposed to do during the lab were, they were asked to describe the lab to a hypothetical fellow student who did not attend. In doing so, students who, during the interviews, saw connections between the models and a real Paul trap generally provided short and non-metaphorical answers. However, they did not typically mention any purpose but mentioned frequency as something important to think about during the lab:

S1: We used a Paul trap... Or a model of a Paul trap, as I understood it. And then we placed a ball on it and tried to put it as close to the middle as possible and test different frequencies to see when it stays.

S2: I would say that we... I don't know how to explain it, but it looked a little bit like a saddle. It was a square where two edges were pulled up and two down, and the goal was to find the frequency when it rotates so different balls could stay and not roll down the edges. And see how long you can get it to stay, at what frequencies is it possible.

One student in phase 3 pointed more explicitly toward the intended object of learning:

S3: I would say that we tried to, understand the principle of a Paul trap. A mechanical one and how that phenomenon, I don't know how I should explain... Well when it stays in the middle from the beginning, when it's trapped, the ball. It was kind of like how electrical charges affect, like a real one.

As the lab session was led by each class' ordinary teacher, there were differences in how each lesson was started and framed. Student **S3** belonged to a group where the teacher stood out from the rest by being particularly thoughtful during the introduction of the lab to highlight the intended aim. Although some of the other teachers talked about links between the Paul trap and the mechanical model during the lesson, they did so without directly mentioning that this is something related to the focus or aim of the lab.

3.1.2 | Threshold frequency - The importance of critical features

When describing what they thought was the most important to investigate, varying the frequency to identify the threshold frequency was identified. The threshold frequency is a particular frequency where the trapping time suddenly increases significantly (see, for example, Kilde Lofgren et al., in press). One student more clearly verbalised their thought process leading up to this realisation:

S3: I thought you noticed how, for example weight affected. Like that you could have different frequencies for the light and the heavier. Meaning it was how you could have different high or low frequencies for different weights that was interesting... No, then it is probably the frequency that was the most important.

Others also indicated the importance of the frequency and mentioned the importance of seeing particular features in the dimension of variation of the frequency.

S1: I mean, the frequency, partly. Because had it only been a low frequency all the balls would have went out and then you wouldn't get a good understanding for it. And then a little bit with the size and such but... There I didn't really understand as much, what did what.

This particular kind of reasoning became apparent only in phase 3, during both interviews and group discussions during the lab. During phase 1, no task relating to the threshold was made explicit until the last task. That meant that no real chance was given to the students to experience the critical feature within the dimension of frequency in isolation. In phase 2, this problem was partially handled by introducing the feature threshold frequency earlier and in isolation. However, students could notice this feature when the threshold frequency was separated from friction in phase 3. When doing so, the link became easier to make between the Paul trap and the mechanical Paul trap.

3.1.3 | Trying to modelise the mechanical trap

In order to see past the differences between the models and a real Paul trap, students needed to view the mechanical model as a toy model of a real Paul trap. As a final guidance towards modelising the mechanical Paul trap, students also interacted with a simplified computer simulation of the mechanical Paul trap. During phases 1 and 2, groups generally struggled greatly with this. When asked whether one model was better than the other or if they complemented each other, students typically provided the following type of answers:

S4: Then I would say the simulation. Because when you trap an electron there is no friction. And then it is only a very small weight and a small size. Because in the simulation you couldn't set what type of ball we had in this case that were used in the simulation. But that was possible in the mechanical. But to compare between our experiment and a real Paul trap I think only the simulation would have been enough, but then you miss friction and weight differences. They weren't possible to investigate in the simulation.

S5: I think you could have understood it with only the computer or only, hmm, the mechanical trap. But the combination made the similarities to a real clearer.

However, some students during phase 3 were able to use the previously discerned critical feature of threshold frequency to help see similarities between the real Paul trap and the mechanical models:

S3: It felt like you understood more with both. That it became a combination that made you really, understand that it can be trapped at some frequencies and at some others not.

If they felt they had the time, groups often chose to conduct extra investigations, usually after studying variations in friction. The groups who did this and had managed to disregard friction as affecting the threshold frequency (three groups in phase 3) focused on studying the threshold frequency for different ball diameters. After discussions with their teacher, those groups identified this as that they varied the effective potential of the mechanical trap (See Figure 5) and identified a relation of that to the threshold frequency. Here the

effective potential is defined as the relative path the ball's centre of mass is allowed to travel, thus effectively changing the shape of the potential field affecting the ball.

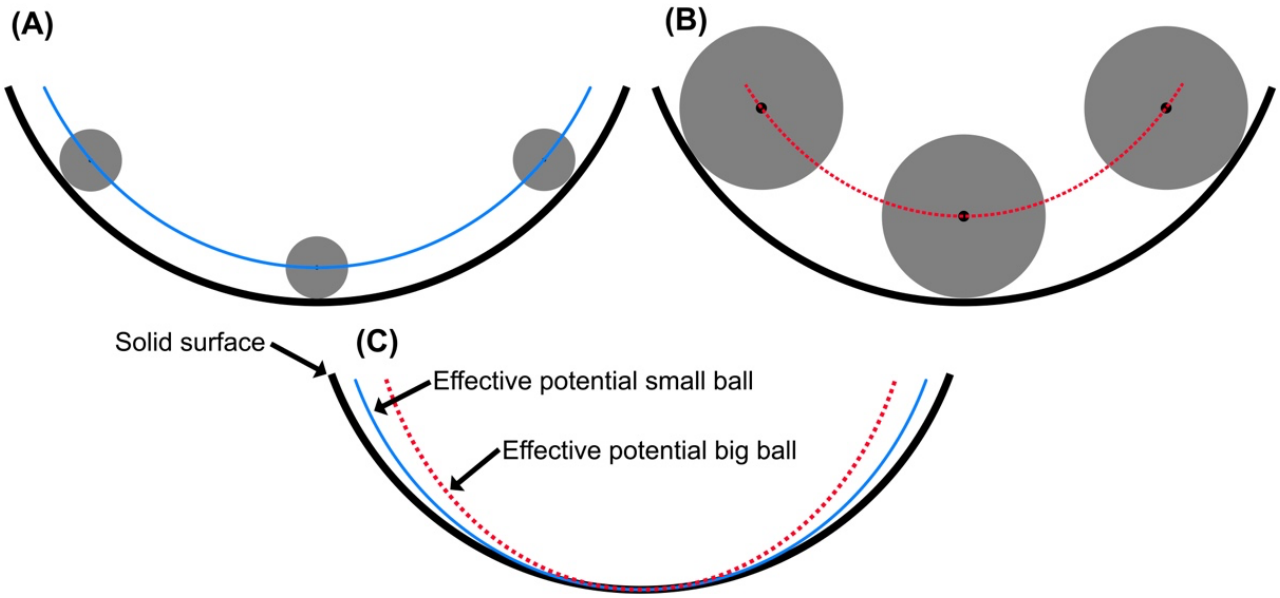

FIGURE 5 The effective potential (path of the centre of mass) is exemplified for balls with different radii, **(A)** and **(B)**, rolling on the same solid curved surface. In **(C)**, the curves are layered with a common lowest point, illustrating that each ball's centre of mass travels along different curved paths.

3.1.4 | Isolating main trapping force contributor - non-trivial previous discernments

During all the lab sessions in phase 3, a task was added where the students were asked to identify the relevant forces acting to keep the "particle" trapped in a real Paul trap and a mechanical Paul trap. Although no student interviewed in phase 3 made explicit mention of them, the ones who did manage to relate the models to a real Paul trap to understand the concept better were in groups and classes where both the group and the teacher during the introduction or at the end of the lesson discussed this. Further, during the laboratory work in all three phases of the study, lab groups who explicitly identified the likeness of the Coulomb force and Newtons force of gravity also managed to, to a greater extent, use the models as toy models to infer how a real Paul trap functions during group discussions.

In Figure 6 distribution of answers on the pre and post-test are compared for the questions regarding the main force contributors for the mechanical and the real Paul trap. When inspecting students understanding of how a mechanical Paul trap works in terms of what is to be regarded as the main trapping force contributor, there is a clear shift between the three phases. In the post-test in all three phases, students' answers all tend to be more towards the force of gravity, friction or normal force, i.e. towards forces student relate to mechanical systems. However, only during phase 3 did a significant change ($t[37]=3.285$, $p<0.05$) with a medium effect size ($d=0.507$) according to Cohen's d (Cohen, 1988) occur, that points toward the correct answer (gravitational force). Turning the focus to understanding what force is used to trap ions in a real Paul trap, there is no as prominent move toward the correct answer (electric force). Therefore, compared to the

observation data mentioned previously, this points toward that students in phase 3 were guided by the intervention to discern the main force contributor for the models of the mechanical Paul trap only.

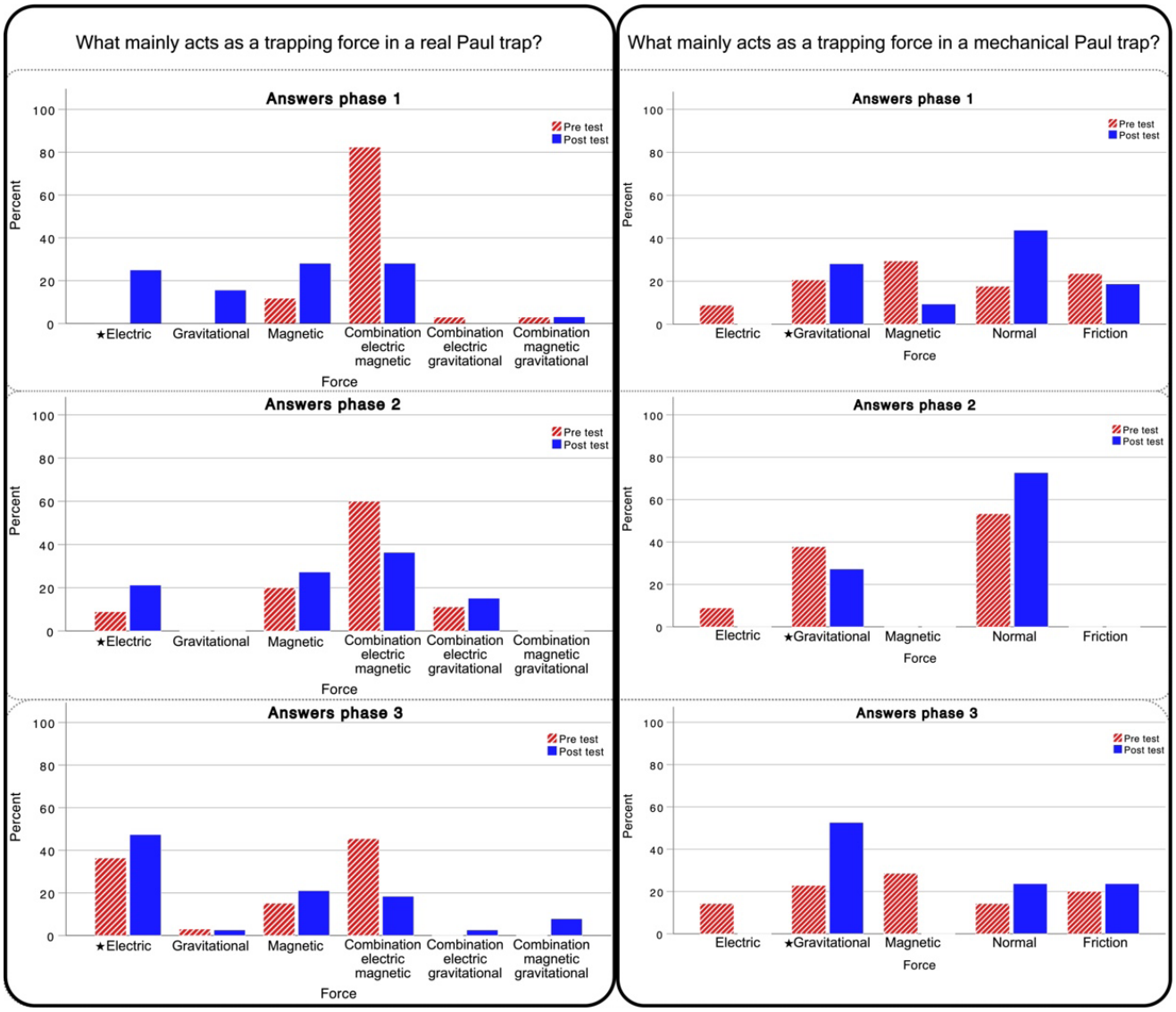

FIGURE 6 Student answers regarding what the main force contributor acting as a trapping force are in a real Paul trap and a mechanical Paul trap. The correct answer, marked with a star, was an electric force for the real Paul trap and a gravitational force for the mechanical Paul trap. As part of the evaluation regarding the intervention's effectiveness, a one-sample *t* test was conducted to evaluate if the relative increase of correct answers were significant, and if so, the effect according to Cohen's *d*. Regarding the real Paul trap, there is a small to medium significant change during phase 1 ($t[31]=3.215$, $p<0.05$. $d=0.44$) but no significant change during phase 2 or 3 ($t[32]=1.690$, $p>0.05$ and $t[37]=1.629$, $p>0.05$). For the mechanical trap there is no significant change during phase 1 or 2 ($t[31]=0.882$, $p>0.05$ and $t[32]=-1.363$, $p>0.05$), but a medium significant change during phase 3 ($t[37]=3.285$, $p<0.05$. $d=0.507$).

3.2 | Fragmented focus - Aspects critical to disregard

During four of the lessons, two in phase 2 and two in phase 3, the information provided by the teacher differed significantly from the rest. During these lessons, the teacher skipped some or almost all proposed background information about real Paul traps. The following interviews resulted in that no matter what was discerned by the students during the lab, they could not make the intended connections between the models and real Paul traps because it was a foreign concept for them to try to approach.

Some students ability to discern critical aspects was also hindered by other aspects, namely those relating to the physical properties of the different balls that could be used during the lab, usually relating to how the trapping time in the mechanical trap was affected. When asked about what was the most important to investigate, these students provided answers along the following lines:

S6: I think it was... It could have been to test different balls. So that you not only see how it works, but also how different factors affect trapping time, which is what it's for. So I think that was probably the most important.

S7: It was that you tested different sizes or weights, so that you deepened, or understood the factors that mattered.

Continuing, when asked if there were anything they did not study that they would have if given another attempt or other material, more focus was drawn to irrelevant properties.

S6: ...I don't think we studied balls with different mass properly. Or we did it but then it was size. And I guess that there were balls with same size but different masses, that was probably something we would have wanted to test actually.

S7: Hmm... The bouncy ball you maybe could have tried to bounce a little but, that we didn't do. ... We only tested to spin it (the ball) in the same direction as the saddle rotated. We didn't try the other way.

Several groups also expressed these views during the laboratory sessions, mostly during phases 1 and 2. As a result, the laboratory exercise was redesigned after phase 1 and phase 2 to allow students to be able to disregard these irrelevant aspects, namely ball properties. From the implemented changes and the fact that ball properties seemed to be disregarded by almost all groups during phase 3, hints of a meaningful change can be seen in the design changes. Namely, it was important to introduce contrast in one particular property, friction, against a background of sameness in threshold frequency to disregard ball properties when modelising the mechanical Paul trap. When changing the friction in the mechanical trap, trapping time will decrease, but the threshold frequency will remain practically unchanged.

3.3 | Simulations as a copy or a model of reality?

A couple of questions were dedicated to how students would explain what a simulation is, both in general and in physics specifically. The students generally had trouble answering these questions, more so if they did not think they had previously seen or worked with simulations in school. Two different ways students viewed simulations as were identified, giving rise to different levels of credence or insight into how simulations can be made: *copy of reality* and *model of reality*. A noticeable distinction between these two views is that within the copy of reality camp, students struggled to a higher degree with seeing the mechanical Paul trap as a model of a real Paul trap.

The most commonly expressed view boiled down to simulations as a real-world copy in which it is possible to conduct experiments. Students ascribing to this view commonly referred to it in terms similar to the following examples:

S6: It is when you... replicate something you can do, in the real world. But digitally, and seeing what the results would be.

S8: It is a copy... Or humanity's attempt to copy how it would happen in reality, but having control over everything such as, air resistance, gravity, friction.. eh.. All types of forces is possible to, put in by yourself. But if you make a simulation so that it has to be as real as possible then its, just reality in a computer that makes it easier to calculate the dimensions of the actual investigation you should conduct.

Students who expressed that simulations are merely models of reality usually pointed toward the fact that simulations are visualisations of equations or some simplified version of reality, as exemplified by **S7** and **S9**:

S9: Maybe like a laboratory. Or backwards, like contrary to a laboratory that you go from having the answers. ... As I understand it, a simulation is just a rewritten formula. Everything is in the little book we have (collection of formulas). But by having it, visualised, or animated you really see how the different factors contribute to the formula perhaps.

S7: First you have to start from reality and see what is happening, and you have to make some simplification of the system and see what.. eh.. variables that come about and such, and try to make as representative a model as possible in for example a computer program. And then you also have to... State its limitations because you can't.. A model is just a model of reality, it's not the reality you represent.

3.4 | An overview of the redesigns

During the first two phases of the intervention, the majority of the results can be seen as pointing towards that something critical seems to be missing for the intended learning to take place. In Table 3, the changes (bold parts in Figure 2) between phases 1 and 2, and 2 and 3 are highlighted together with brief motivations. Each change was motivated by a combination of initial analysis of collected data, practical (lab time) limitations, and informed by variation theory.

TABLE 3 Highlighting the changes to the laboratory exercise that was made after phase 1 and phase 2. For more detail on included tasks in each phase, see Figure 2.

	Action	Motivation
Phase 1 → 2	Removed 1 frequency to test in the trapped region	Save time
	Added finding threshold frequency with mechanical trap and simulation	Introducing threshold frequency earlier during lab
	Removed investigating impact of ball size	Remove focus on too complex aspect
Phase 2 → 3	Added drawing what forces acts on particle trapped in bowl and between two charges	Draw focus towards previous knowledge
	Changed investigation on spin to highest frequency	Save time
	Changed friction investigation to its effect on threshold frequency	Help students generalise threshold frequency

For example, after phase 1, it became clear that some values (features) in the dimension of variation of frequency were necessary (critical) for students to experience in order to discern the critical aspect properly. It was evidently not enough to include this in the last part of the exercise, where they were supposed to find the threshold frequency (but it was not stated explicitly) for another ball than what they used previously. However, it was not until the additional change after phase 2, where frequency was generalised by separating friction from threshold frequency, that students, to a greater extent, could use the notion of threshold frequency to better identify similarities between the mechanical Paul trap and a real Paul trap. Another change, that was not thought of before seeing results from the pre and posttests, was regarding the students' struggle to identify the main force contributor acting as a trapping force in the mechanical and real Paul traps (see Figure 6). This was initially overlooked as similar discussions about relevant forces are handled extensively in a previous physics course. However, being situated in a new and unfamiliar specific context, it seemed that students needed some nudge in the direction of actually considering what are the relevant forces in the systems they are working with. This led to the introduction of the initial task implemented in phase 3, where they had to bring previous knowledge into focus by explicitly drawing relevant forces acting on a particle in similar situations as in the mechanical and real Paul trap.

3.5 | Towards identifying patterns of variation

One of the core interests in this study, as with all action research related work, is to improve and inform practice. As such, and in line with having variation theory as a guiding principle, it is of interest to make sense of the varying experiences the students had reasoning about the lab and trying to find links between the mechanical Paul trap and a real Paul trap. We did this by identifying similarities in what students were successful in modelising the mechanical Paul trap and juxtaposing that with the reasoning and workflow of struggling students. According to Marton (2015), it is necessary to find both students who are able to discern aspects of an object of learning and those who are not, in order to identify potential critical aspects. This idea, together with the fact that the mechanical Paul trap is a model not commonly utilised within secondary physics education, meant that finding students struggling with the intervention is both expected and something that helps better pinpoint what aspects of the object of learning are in fact critical.

From the analysis, patterns of variation were identified that can be seen as building blocks towards helping students successfully modelise the mechanical model. The resulting patterns of variation can be found in Table 4, and following is an expanded explanation of them and their relation to previously presented findings. Starting from the top, including the task of drawing forces acting on particles seemed to be important, not to fully grasp what force is used as a trapping force, but rather to identify the two types of systems (models) being considered. Thus students will, in this step, be guided towards discerning the model-force relationship.

TABLE 4 Identified patterns of variation for successful groups during phase 3. Not all successful groups performed the investigation corresponding to the second to last row, but the groups who did not skip it were all able to modelise the mechanical model. Frequency, trapping time, model and effective potential were all identified as being domains critical to experience variation within to successfully being able to discern them simultaneously in the last pattern of variation. The aspects in focus during each part are bold and capitalised, a varied aspect is marked by v, and an invariant aspect is marked by i. Aspects not included are marked by -.

Frequency	**Trapping time**	Ball initial	Friction	**Model**	Force	**Effective potential**	Pattern of variation and invariance
-	-	-	-	**V**	**V**	-	Bring previously discerned model and force relations into focus
V	**V**	i	i	i	-	i	Contrast frequency and see relation with trapping time
i	**I**	**V**	i	i	-	i	Generalisation of frequency and trapping time relation by varying initial conditions of ball
v	**V**	-	i	i	-	i	Introducing the critical feature threshold frequency
v	**V**	-	**V**	i	-	i	Generalisation of threshold frequency by varying friction
v	**V**	-	-	i	-	**V**	Contrast effective potential and see relation with threshold frequency by varying ball size
V	**V**	-	-	**V**	-	**V**	Modelising mechanical model by contrasting model (Fusion)

Meaning that what separates the models under consideration during the lab session is what the trapping force is (i.e. that the mechanical model and the simulation can be thought of as both being different representations of the mechanical model since they are both using gravity as the main trapping force contributor). After coming to this realisation, it seemed as though having a focus on trapping force was not needed during the rest of the lab, and that is why it is marked as *not included* in the following patterns of variation in Table 4.

Next is a contrast pattern followed by a generalisation, both focusing on identifying the connection between frequency and trapping time. It was critical for students to identify this connection in part due to the chaotic nature of the mechanical Paul trap, where trapping time can fluctuate although keeping the rotational frequency unchanged (see Kilde Lofgren et al., in press for an example data collection run to identify the threshold frequency). Our suspicion, strengthened by data collected during phase 3, is that it was by a combination of guiding students to discern the critical feature of threshold frequency together with identifying that friction has no observable effect on this feature that the connection between frequency and trapping time became more clear.

As a final note, we turn our attention to the point of being able to modelise the mechanical model, and thus having it available to use as a tool to understand how a real Paul trap works. It is indicated that this seemed to happen to a high degree if: patterns of variation presented in Table 4 are offered so that the aspects critical to a particular student can be discerned both separately and together; and that the teacher is being mindful to both introduce the lab exercise in a way that the aim, or intended object of learning, is being presented in such a way that it becomes visible for the students, and shows flexibility during the lab. Then, if needed, a particular group of students may get guidance towards some discernment that seems to be critical for them not to miss out on, even though it may not be included among the particular tasks during the lab exercise.

4 | DISCUSSION

Toy models, despite their joyful tone, are far from trivial to use in teaching situations. Results from this study support this notion in that there were apparent struggles, especially when it came to phases 1 and 2, where few managed to successfully create an internal idealised version of the mechanical Paul trap to qualitatively understand how a real Paul trap works. This process, which we call to be able to *modelise* the mechanical Paul trap, was identified as a crucial step to properly be able to link the real Paul trap and the mechanical Paul trap. This became apparent due to a combination of realisations regarding both the struggle and successes of students, viewed through the lens of variation theory.

The following discussion is divided into two parts. The first part starts from the results and the identified patterns of variation to discuss implications for teaching and learning in a laboratory environment. The second part aims to provide further to the theoretical discussion around models and modelling in science education, both by relating to the findings from this particular study and to previous research.

4.1 | Implications for teaching and learning in laboratory environments

From our results, we propose patterns of variation highlighting important steps and relations between the different tasks in the lab exercise during phase 3 (see Figure 2) in Table 4, drawn from identifying differences between students who succeeded with the lab and those who did not. It should be noted that as with all models of learning processes, including the one presented in the introduction that follows Marton (2015) steps of instantiation, contrast, generalisation, and fusion, the reality is more complex. However, it poses a good starting ground, and the general form can be extrapolated from the identified learning process.

Keeping the focus on the patterns of variation, there are several points to emphasise from the students' learning process. First, it was identified that during phase 3, the groups who identified the relationship between the models and their particular trapping force via an initial fusion step also had an easier time later modelising the mechanical trap, meaning being able to use it as a means to understand the principle of a real Paul trap. However, this does not mean that the task helps students understand what *is* the main trapping force in the real and mechanical Paul trap, which is supported by Figure 6. Here hides an important highlight when it comes to critical aspects, object of learning and learning objects. Namely that, by identifying the need for students to attend to previously discerned aspects and use that for discerning something new (here being using the model-force relationship to see what representations that are presented during the lab should fall under the same model), it is not necessarily crucial to focus on whether it is being handled correctly in a general sense. It only needs to be understood in such a way that the current critical aspects can be discerned by the student. Thus we have a situation where the teacher would be encouraged to act differently depending on whether he or she has learning objectives (end of course goals) or a specific object of learning (specific aim) in mind. The former focus would promote an immediate intervention to try to combat the potential distribution of answers as seen in Figure 6. Whereas the latter would promote keeping the focus on providing a potential intervention in a way that, first and foremost, helps students towards seeing what different models are being utilised in the lab exercise and how the different representations can be linked to those models.

Second, much like Marton (2015) suggests, to understand the learning situation as a whole, it is relevant to experience both critical aspects to discern and aspects that are critical to disregard to generalise an aspect. When students were more clearly guided only to vary friction against a background of sameness of the feature threshold frequency was identified as an important generalisation step, something only presented to students in the exercise during phase 3. It seemed that it was critical to vary the friction by specifically varying the coefficient of static friction (i.e., the material of the ball) rather than varying the friction by changing the mass of the ball, as some tended to do. Changing other ball properties, such as size, also changes the effective potential of the trap, thus opening up more dimensions of variations than the students are ready to take on at that point during the lab.

Third, as the physics courses build on one another, students should continuously gain a deeper understanding of science and thus, amongst other things, develop knowledge about how models are used and developed in physics (Swedish National Agency for Education, 2011). Moving to the context of Physics 2, this will have the implication of students potentially having a large spread of depth regarding their development of this knowledge. This leads to students having a varying set of previously discerned dimensions of variations

in physics modelling, which they might draw upon to make sense of the lab covered in this paper. That includes students' ability to judge the quality of modelling a real Paul trap with a mechanical trap. In our results, this appears in the way that students expressed varying meanings to the purpose of simulations and that some groups had little trouble using the notion of the effective gravitational potential that the ball and the mechanical trap surface jointly created for the ball's centre of mass.

Moving away from patterns of variation, the evidence of student struggles presented in this study provides an opportunity to discussing model complexity and how to design laboratory exercises that appropriately meet and challenge students in ways appropriate to their current both practical and theoretical knowledge. In a recent review of research on laboratory work in schools, Gericke, Högström, and Wallin (2022) point towards, amongst other things, the importance of the role of the teacher, the need for proper introduction of the phenomenon being studied (or appropriate guidance during the lab), and that the level of openness (level of inquiry) should be chosen deliberately on the intended learning goals.

Regarding the role of the teacher and introduction of the studied phenomenon, the current study (thanks to the naturalistic setting of the lab sessions) finds agreement with the points identified by Gericke et al. (2022). Because Paul traps are, to the best of our knowledge, novel as something studied even qualitatively at the secondary physics education level, how the teacher introduced the phenomenon at the beginning of the lesson had a vital impact on students' ability to relate their findings and realisations during the lab and use that to understand how a real Paul trap works. This was particularly evident by the fact that many students were unable to recount what the intended aim, or actually important to had focused on during the lab was. We believe that this is related to the fact that few teachers chose to focus on thoroughly introduce the Paul trap at the beginning of the lab. A belief that is in agreement with findings from Gericke et al. (2022) and included in the general form of effective use of patterns of variation presented by Marton (2015).

Regarding the complexity of the model used in this study and the seemingly hard task of linking the mechanical Paul trap to how a real Paul trap works, it is relevant to draw upon the level of openness in laboratory work. Going from phase 1 through to phase 3 in this intervention, it is noticed how the level of openness seemed to decrease. At the same time, the number of lab groups that successfully managed to modelise the mechanical trap went up. The natural question to ask is whether there is any connection between these two trends? Drawing upon the previous discussion regarding identified patterns of variation, the answer is a cautious yes. This also falls in line with the discussion by Gericke et al. (2022), where it seems that even though guided inquiry is an evidently favorable way of structuring laboratory work, the level of openness and student-centered vs more teacher-centered is subject to change based on, amongst other things, the complexity and unfamiliarity of the phenomenon being studied. Based on the current study, we stand in favor of the argumentation that the level of openness and the use of student- vs teacher-centered approaches when planning laboratory work should be a mindful task planned by the class' teacher and taking the students' current knowledge and skills related to the intended object of learning for the lab exercise under careful consideration.

4.2 | On the use of models in science education

An essential goal of the laboratory exercise developed in this study was, when focusing on the lived object of learning, for the students to be able to modelise the mechanical trap. This is crucial as there are differences between the mechanical Paul trap and a real Paul trap, such as the inclusion of friction and how the potential changes over time to name a couple (see Kilde Lofgren et al., in press; Fan, Du, Wang, & Zhou, 2016; Kirillov & Levi, 2016; Thompson, Harmon, & Ball, 2002 for further discussions on the quality of the model). From the patterns of variation presented in Table 4, it can be seen that students were guided by the tasks to update their *model description* successively. Thus step by step specifying the *model system* and creating an *ideal model* they can eventually use to make sense of the *real-world phenomenon*, a Paul trap (see Figure 1 for the relationship between *real-world phenomenon*, *model system*, and *model description*).

As pointed out by Oh and Oh (2011), models are considered subsets of scientific theories in model-based science, there to enrich with systems of explanations. Thus, it is reasonable for Godfrey-Smith (2006) to assert that modelbased science should not be associated with idealisation. However, working to construct ideal models is important to include in the physics classroom since they are part of scientific communication. Levy (2018) specifies idealisations as intentional misrepresentations and closely connected to the constructor's knowledge and intentions. Levy also notes that how Newtonian mechanics is being used today is to be seen as an idealisation since it, by intention, falsely describe gravitational interactions.

Finally, returning to Oh and Oh (2011), who suggest that students should engage in the construction and revision of models, results from the current study contribute to this view and suggest that, much like has been found previously (e.g. Schwarz et al., 2009; Coll et al., 2005; Harrison & Treagust, 1998) that involving students in model construction could be a good practice to promote learning. Both in terms of drawing upon previously discerned dimensions of variations, which have also been pointed out by Euler, Gregorcic, and Linder (2020) in the context of working with a digital learning environment in physics, and providing the students with an opportunity to see the explanatory power as well as the limitations of idealised models such as the mechanical Paul trap being a toy model for a real Paul trap.

5 | CONCLUSIONS AND OUTLOOK

This study aimed to investigate if and how the mechanical Paul trap and a simulation could be used in a laboratory setting as artefacts to learn how a real Paul trap qualitatively works, within the context of the Swedish upper secondary course Physics 2. To do this, variation theory was used as a theoretical framework, thus deeming it necessary to identify the critical aspects to define the object of learning. The design experiment over three phases identifies an instruction process in terms of patterns of variation that made it possible for students to learn qualitatively how a real Paul trap works by exploring and developing an idealised model using a mechanical Paul trap.

The patterns of variation students exhibited who successfully used the mechanical Paul trap to understand a real Paul trap follow the general pattern suggested by Marton (2015). However, it can also be understood in terms of Figure 1 and how the students successively updated their *model description*, further specifying the *model system* and eventually were able to use it as an idealisation of a real Paul trap (the *real-*

world phenomenon). The results indicate that it was crucial that the students first got some introduction to a real Paul trap since the *model system* is not only created by the *model description* but also informed by the *real-world phenomenon*.

Science education is riddled with models, often various forms of idealised ones thereof. Our study shows agreement with Oh and Oh (2011) and Coll et al. (2005) in that the use of models can develop students understanding of them and their limitations when they are engaged in constructing and revising. We want to add to this discussion that, as indicated by our findings, working with idealised models could be a meaningful clarificatory process to help students better understand the use and limitations of models. However, further research in this area is needed, and our study can serve as an example of how variation theory is a fruitful approach to exploring the use of models as a tool for teaching conceptual understanding in the science laboratory.

References

Bannan-Ritland, B., & Baek, John, Y. (2014). Investigation the Act of Design in Design Research: The Road Taken. In E. Kelly, Anthony, A. Lesh, Richard, & Y. Baek, John (Eds.), *Handbook of design research methods in education: Innovations in science, technology, engineering, and mathematics learning and teaching* (pp. 299–319).

Barab, S., & Squire, K. (2004). Design-Based Research: Putting a Stake in the Ground. *The Journal of the Learning Sciences*, *13*(1), 1–14. https://doi.org/10.1207/s15327809jls1301_1

Beiser, A. (2003). *Concepts of modern physics* (6th ed.). McGraw-Hill.

Bogdan, R., & Knopp Biklen, S. (2007). *Qualitative research for education : an introduction to theory and methods* (5th ed.). Pearson Education.

Bokulich, A. (2011). How scientific models can explain. *Synthese*, *180*, 33–45. https://doi.org/10.1007/s11229-009-9565-1

Booth, S., & Ingerman, Å. (2002). Making sense of physics in the first year of study. *Learning and Instruction*, *12*(5), 493–507. https://doi.org/10.1016/S0959-4752(01)00028-7

Braun, V., & Clarke, V. (2006). Using thematic analysis in psychology. *Qualitative Research in Psychology*, *3*(2), 77–101. https://doi.org/10.1191/1478088706qp063oa

Braun, V., & Clarke, V. (2019). Reflecting on reflexive thematic analysis. *Qualitative Research in Sport, Exercise and Health*, *11*(4), 589–597. https://doi.org/10.1080/2159676X.2019.1628806

Brown, A. L. (1992). Design Experiments: Theoretical and Methodological Challenges in Creating Complex Interventions in Classroom Settings. *The Journal of the Learning Sciences*, *2*(2), 141–178. https://doi.org/10.1207/s15327809jls0202_2

Chabay, R. W., & Sherwood, B. A. (2015). *Matter interactions* (4th ed.). John Wiley Sons, Inc.

Chevallard, Y. (2006). Steps towards a new epistemology in mathematics education. In *Proceedings of the iv congress of the european society for research in mathematics education* (pp. 21–30).

Chevallard, Y., & Bosch, M. (2014). Didactic Transposition in Mathematics Education. *Encyclopedia of Mathematics Education*, 170–174. https://doi.org/10.1007/978-94-007-4978-8_48

Cohen, J. (1988). *Statistical Power Analysis for the Behavioral Sciences* (2nd ed.). Routledge. https://doi.org/10.4324/9780203771587

Coll, R. K., France, B., & Taylor, I. (2005). The role of models/and analogies in science education: implications from research. *International Journal of Science Education*, *27*(2), 183–198. https://doi.org/10.1080/0950069042000276712

Euler, E., Gregorcic, B., & Linder, C. (2020). Variation theory as a lens for interpreting and guiding physics students' use of digital learning environments. *European Journal of Physics*, *41*(4), 045705. https://doi.org/10.1088/1361-6404/ab895c

Fan, W., Du, L., Wang, S., & Zhou, H. (2016). Confining rigid balls by mimicking quadrupole ion trapping. *American Journal of Physics*, *85*(11), 821–829. https://doi.org/10.1119/1.5005927

Gericke, N., Högström, P., & Wallin, J. (2022). A systematic review of research on laboratory work in secondary school. *Studies in Science Education*. https://doi.org/10.1080/03057267.2022.2090125

Giere, R. N. (1988). *Explaining science : a cognitive approach*. Univ. of Chicago Press.

Godfrey-Smith, P. (2006). The strategy of model-based science. *Biology and Philosophy*, *21*(5), 725–740. https://doi.org/10.1007/s10539-006-9054-6

Godfrey-Smith, P. (2009). Models and fictions in science. *Philosophical Studies*, *143*(1), 101–116. https://doi.org/10.1007/S11098-008-9313-2

Hake, R. R. (2008). Design-Based Research in Physics Education: A Review. In A. E. Kelly, R. A. Lesh, & J. Y. Baek (Eds.), *Handbook of design research methods in education* (pp. 493–508). Taylor Francis Group.

Harrison, A. G., & Treagust, D. F. (1998). Modelling in Science Lessons: Are There Better Ways to Learn With Models? *School Science and Mathematics*, *98*(8), 420–429. https://doi.org/10.1111/j.1949-8594.1998.tb17434.x

Hesse, M. (2017). Models and Analogies. In W. H. Newton-Smith (Ed.). https://doi.org/10.1002/9781405164481.ch44

Holmqvist, M., Gustavsson, L., & Wernberg, A. (2007). Generative learning: learning beyond the learning situation. *Educational Action Research*, *15*(2), 181–208. https://doi.org/10.1080/09650790701314684

Ingerman, Å., Linder, C., & Marshall, D. (2009). The learners' experience of variation: following students' threads of learning physics in computer simulation sessions. *Instructional Science*, *37*(3), 273–292. https://doi.org/10.1007/s11251-007-9044-3

Joseph, D. (2004). The Practice of Design-Based Research: Uncovering the Interplay Between Design, Research, and the Real-World Context. *Educational psychologist*, *39*(4), 235–242. https://doi.org/10.1207/s15326985ep3904_5

Kelly, A. E. (2004). Design Research in Education: Yes, but is it Methodological? *Journal of the Learning Sciences*, *13*(1), 115–128. https://doi.org/10.1207/s15327809jls1301_6

Kilde Lofgren, S., Méndez Fragoso, R., Weidow, J., & Enger, J. (in press). The mechanical paul trap: Introducing the concept of ion trapping. *The Physics Teacher*.

Kirillov, O. N., & Levi, M. (2016). Rotating saddle trap as Foucault's pendulum. *American Journal of Physics*, *84*(1), 26–31. https://doi.org/10.1119/1.4933206

Kullberg, A., Runesson Kempe, U., & Marton, F. (2017). What is made possible to learn when using the variation theory of learning in teaching mathematics? *ZDM - Mathematics Education*, *49*(4), 559–569. https://doi.org/10.1007/s11858-017-0858-4

Levy, A. (2018). Idealization and abstraction: refining the distinction. *Synthese*, *198*(24), 5855–5872. https://doi.org/10.1007/S11229-018-1721-Z

Ling, L. M., Chik, P., & Pang, M. F. (2006). Patterns of variation in teaching the colour of light to primary 3 students. *Instructional Science*, *34*(1), 1–19. https://doi.org/10.1007/s11251-005-3348-y

Lo, M. L. (2012). *Variation theory and the improvement of teaching and learning*. Acta Universitatis Gothoburgensis.

Lo, M. L., Marton, F., Pang, M. F., & Pong, W. Y. (2004). Toward a pedagogy of learning. In F. Marton & A. B. Tsui (Eds.), *Classroom discourse and the space of learning* (pp. 189–225). Routledge Taylor Francis Group. https://doi.org/10.4324/9781410609762

Marton, F. (1981). Phenomenography - Describing conceptions of the world around us. *Instructional Science*, *10*(2), 177–200. https://doi.org/10.1007/BF00132516

Marton, F. (1986). Phenomenography-A Research Approach to Investigating Different Understandings of Reality. *Journal of Thought*, 21(3), 28–49.

Marton, F. (2015). *Necessary Conditions of Learning*. Taylor Francis. https://doi.org/10.4324/9781315816876

Marton, F., & Booth, S. (1997). *Learning and Awareness*. Lawrence Erlbaum Associates, Inc.

Marton, F., & Säljö, R. (1976). On Qualitative Differences in Learning: I-Outcome and Process. *British Journal of Educational Psychology*, *46*(1), 4–11. https://doi.org/10.1111/j.2044-8279.1976.tb02980.x

Nowell, L. S., Norris, J. M., White, D. E., & Moules, N. J. (2017). Thematic Analysis: Striving to Meet the Trustworthiness Criteria. *International Journal of Qualitative Methods*, *16*(1). https://doi.org/10.1177/1609406917733847

Oh, P. S., & Oh, S. J. (2011). What Teachers of Science Need to Know about Models: An overview. *International Journal of Science Education*, *33*(8), 1109–1130. https://doi.org/10.1080/09500693.2010.502191

Pålsgård, J., Kvist, G., & Nilson, K. (2012). *Ergo Fysik 2* (3rd ed.). Liber.

Pang, M. F., & Ki, W. W. (2016). Revisiting the Idea of "Critical Aspects". *Scandinavian Journal of Educational Research*, *60*(3), 323–336. https://doi.org/10.1080/00313831.2015.1119724

Patron, E., Linder, C., & Wikman, S. (2021). Qualitatively different ways of unpacking visual representations when teaching intermolecular forces in upper secondary school. *Science Education*, *105*, 1173–1201. https://doi.org/10.1002/sce.21662

Paul, W. (1990). Electromagnetic Traps for Charged and Neutral Particles (Nobel Lecture). *Angew. Chem. Int. Ed. Engl.*, *29*, 739-748. https://doi.org/10.1002/anie.199007391

PhET. (2022). *Gas Properties*. Retrieved 2022-09-19, from https://phet.colorado.edu/en/simulations/gas-properties/

Phillippi, J., & Lauderdale, J. (2018). A Guide to Field Notes for Qualitative Research: Context and Conversation. *Qualitative Health Research*, *28*(3), 381–388. https://doi.org/10.1177/1049732317697102

Pluta, W. J., Chinn, C. A., & Duncan, R. G. (2011). Learners' epistemic criteria for good scientific models. *Journal of Research in Science Teaching*, *48*(5), 486–511. https://doi.org/10.1002/tea.20415

Rosenblueth, A., & Wiener, N. (1945). The Role of Models in Science. *Philosophy of Science*, *12*(4), 316–321. https://doi.org/10.1086/286874

Runesson, U. (2006). What is it possible to learn? On variation as a necessary condition for learning. *Scandinavian Journal of Educational Research*, *50*(4), 397–410. https://doi.org/10.1080/00313830600823753

Schwarz, C. V., Reiser, B. J., Davis, E. A., Kenyon, L., Achér, A., Fortus, D., … Krajcik, J. (2009). Developing a Learning Progression for Scientific Modeling: Making Scientific Modeling Accessible and Meaningful for Learners. *Journal of Research in Science Teaching*, *46*(6), 632–654. https://doi.org/10.1002/tea.20311

Shavelson, R. J., Phillips, D. C., Towne, L., & Feuer, M. J. (2003). On the Science of Education Design Studies. *Educational Researcher*, *32*(1), 25–28. https://doi.org/10.3102/0013189X032001025

Sober, E. (1981). The principle of parsimony. *The British Journal for the Philosophy of Science*, *32*(2), 145–156.

Swedish National Agency for Education. (2011). *Physics [Subject syllabus]*.

Swedish Research Council. (2017). *Good research practice.*

Thompson, R. I., Harmon, T. J., & Ball, M. G. (2002). The rotating-saddle trap: A mechanical analogy to RF-electric-quadrupole ion trapping? *Canadian Journal of Physics*, *80*(12), 1433–1448. https://doi.org/10.1139/p02-110

Toon, A. (2012). Similarity and scientific representation. *International Studies in the Philosophy of Science*, *26*(3), 241– 257. https://doi.org/10.1080/02698595.2012.731730

Vaismoradi, M., Jones, J., Turunen, H., & Snelgrove, S. (2016). Theme development in qualitative content analysis and thematic analysis. *Journal of Nursing Education and Practice*, *6*(5). https://doi.org/10.5430/jnep.v6n5p100

Vygotskij, L. S. (1986). *Thought and language* (Translation newly rev. and edited / by Alex Kozulin. ed.). MIT Press.