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Why known unknowns may be better than knowns, and how that matters for the evolution of happiness

Johan Stennek

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UNIVERSITY OF GOTHENBURG
SCHOOL OF BUSINESS, ECONOMICS AND LAW

Why known unknowns may be better than knowns, and how that matters for the evolution of perception and happiness*

Johan Stennek
University of Gothenburg

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Abstract

Rayo and Becker (2007) model happiness as an imperfect measurement tool: It provides a partial ordering of alternative courses of actions. In this paper, decision-makers use their inability to rank two actions, to rationally infer rankings of other pairs of actions. I demonstrate that rational inference generates violations of the "rationality axiom" (independence of irrelevant alternatives). Moreover, coarser happiness information increases the power of inference. Therefore, a "Reasoning Man," endowed with an unlimited ability for rational inference, would be characterized by muted feelings and blurred perception. Behavior would nevertheless maximize happiness and evolutionary fitness, and not be merely satisficing.

JEL: D01; D80; D91

Keywords: Indirect evolutionary approach; utility function; perception; inference.

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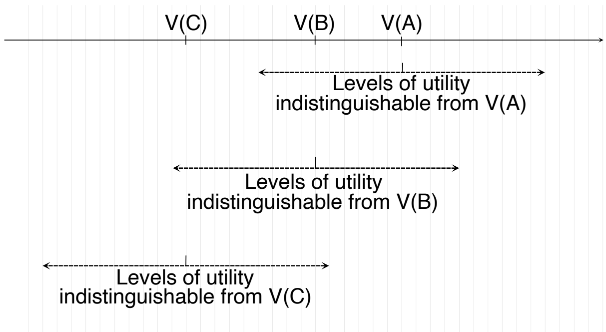
1 Introduction

Rayo and Becker (2007) view happiness as a decision-making device, allowing individuals to rank alternative courses of action. But our nervous system is subject to physical limitations, reducing the precision of sensory information, and therefore the quality of our decisions. There are (i) bounds on how happy or sad we can be, and (ii) we are unable to perceive small differences in the happiness to be expected from alternative actions. Therefore, human behavior is only satisficing, meaning that we must choose at random among the actions that we cannot distinguish from the best. Figure 1 provides an example. The decision-maker ranks action A over C, but she cannot perceive any difference in expected happiness (or utility) between A and B or between B and C. The decision-maker's perception sensitivity - indicated by the length of the arrows - is not sufficient, given the narrow range of utility levels. As only C is *perceived* inferior, the decision-maker must choose at random between A and B. Rayo and Becker go on to argue that

Figure 1: Utility-levels and perception-sensitivity.



the human happiness-function is innate, selected by nature to maximize fitness, rewarding the actions producing the most expected offspring. They demonstrate that to maximize fitness, subject to the two neurological constraints above, nature must assign maximum happiness to all fitness-levels above some performance benchmark and minimum utility to all fitness-levels below it. Extended versions of their model may explain several "behavioral regularities" documented in empirical work.[1]

This paper ads inference as an intermediate step between sensing happiness information and taking decisions. Knowing the limitations of her sensory system, the decision-maker above understands that the difference in utility between actions B and C must be smaller than that between A and C. It follows that A is also better than B and therefore the only maximizing choice.[2] The paper explores the implications of the decision-maker

[1]In the dynamic version the performance benchmark varies over time. Habits and peer comparisons arise as special cases. Utility is volatile and continuously reverts to its long-term mean.

[2]If the inference process is unconscious, the decision-maker may experience it as a part of perception. In that case, the decision-maker would so-to-say see the ranking between A and B in a similar way as that between A and C.

having an unlimited ability to draw such inferences, and produces three surprising results.

First, given the neurological limitations on the sensory system described above, adding the ability to draw rational inferences from known unknowns leads to violations of an important axiom of rational choice, namely the *Independence of Irrelevant Alternatives* (*IIA*). Consider the decision-maker above as an example. She can only infer that A is better than B when action C is in the choice set. While dominated, action C is not irrelevant to a decision-maker with limited perception. It provides a valuable "contrast," accentuating the difference between A and B. Violations of *IIA* by humans and other animals are well-documented in empirical studies (see e.g. Cohen et al., 2019). The present explanation differs from previous ones by suggesting that the presence of dominated alternatives may increase the decision maker's expected utility and fitness.

Second, a more precise sensory system – that is, a wider range of happiness-levels or a higher perception sensitivity – reduces the power of inference. As a result, a less precise sensory system can be better than a more precise. Consider the decision-maker above again. She would not have inferred that A is better than B, if her sensory system had been sufficient to perceive that B is better than C. As it turns out, there does not exist a tradeoff between the metabolic cost of the sensory capacity and the quality of decisions,

Third, if nature selects an unlimited ability for inference, then the decision-maker's behavior will be fully utility-maximizing, thereby fitness-maximizing, and not merely satisficing, while at the same time only using a minimum of sensory information. To minimize the metabolic cost of the sensory system, the decision-maker will be equipped with the lowest possible perception sensitivity generating useful information given her utility function, and a utility step-function where the fitness benchmark maximizes the contrast between the best and the worst actions. As a result, the expected utility of actions will reflect the probability with which the actions meet the fitness benchmark.

Finally, I argue that nature is more apt to endow the decision-maker with an ability for inference in an environment with a plethora of available actions where the difference in outcome is small even between the best and the worst action. Moreover, the decisions based on an unlimited ability for inference, when coupled with a minimum perception capacity, can be replicated by a surprisingly simple and possibly cheap decision-making process.

2 A model of decision-making with inference

A decision-maker selects an action $i = \{1, 2, ...N\}$ to maximize *expected* utility, $V_i \in [V_{\min}, V_{\max}]$. Label actions so that $V_1 \leq V_2 \leq ... \leq V_N$ and assume $V_1 < V_N$.[3] Following

[3] Rayo and Becker let the set of actions be a compact and convex subset of $\mathbb{R}^{\mathrm{N}}$ and expected utility a continuous function. However, as explained below, with these assumptions, inference would always lead to maximizing choice.

Rayo and Becker, the decision-maker compares pairs of actions and perceives which action is better if, and only if, the difference in expected utility is larger than her perception threshold, $|V_i - V_j| \geq \Theta$. A lower perception sensitivity means that the threshold is higher. A positive affine transformation of the utility function normalizes all expected utilities as $v_i = \frac{V_i - V_{\min}}{V_{\max} - V_{\min}} \in [0, 1]$. Let $\theta = \frac{\Theta}{V_{\max} - V_{\min}}$ be an (inverse) measure of the overall capacity of the sensory system.

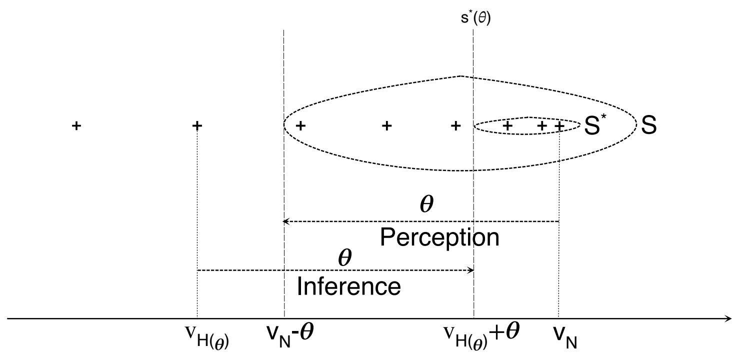
The key idea is that the decision-maker has a limited ability to compare the utilities to be expected from different actions. For a concrete interpretation, in terms of perception, it is helpful to think about some (unmodeled) attributes of the actions that the decision-maker can observe and which she has learned are predictors of utility. Consider a forager choosing an apple to bring back home. Ripe apples contain more nutrients, including sugar, which increase fitness. Ripe apples also provide more utility, by tasting sweeter. The decision-maker can assess the expected sweetness (expected utility) by the size, color, and fragrance of the apple. The problem is that the decision-maker's ability to perceive differences in these attributes is limited.

As any *sensorily dominated* action is dominated by the best action, the set of sensorily undominated actions is $S(\theta) = \{i : v_N - v_i < \theta\}$. This is the decision-maker's satisficing set, absent inference. With choice probabilities $\Pr\{i\,|\theta\} = \frac{w_i}{\sum_{j \in S(\theta)} w_j}$, $\forall i \in S(\theta)$, and fixed decision-weights $w_i > 0$ for all i, a higher capacity of the sensory system (lower θ) increases expected utility, $\sum_{i \in S(\theta)} \Pr\{i\,|\theta\} \cdot v_i$. Let $\underline{\theta}(v) = v_N - v_1$ represent the smallest usable sensory capacity. This is the minimum capacity required to perceive a difference between the best and the worst actions, given a certain utility function. A lower sensory capacity ($\theta > \underline{\theta}$) would not affect decisions, but still require energy to maintain. I will regularly suppress the dependency on the utility function and simply write $\underline{\theta}$.

The inability to rank two actions informs the decision-maker that the difference in expected utility is small. Additional rankings can then be inferred. This paper explores the consequences of the decision-maker correctly drawing all inferences that are possible. While I do not describe the details of this inference process, the end-result is easily characterized. For a decision-maker with $\theta \leq \underline{\theta}$, the best sensorily dominated action is $v_{H(\theta)}$ where $H(\theta) = \max\{i : v_N - v_i \geq \theta\}$. Any action $i \in S(\theta)$ which is sensorily indistinguishable from some sensorily dominated action is sensorily indistinguishable from $v_{H(\theta)}$. Thus, the set of *undominated* actions is $S^*(\theta) = \left\{i \in S(\theta) : v_i - v_{H(\theta)} \geq \theta\right\}$. Figure 2 provides an example. The two worst actions are sensorily dominated, as indicated by the left-pointing arrow of length θ from the best action. The best sensorily dominated action, $v_{H(\theta)}$, cannot be distinguished from the three subsequent actions, as indicated by the right-pointing arrow of length θ. Thus, $S^* = \{N-2, N-1, N\}$.[4] Given the decision-weights above, choice probabilities are given by $\Pr\{i\,|\theta\} = \frac{w_i}{\sum_{j \in S^*(\theta)} w_j}$, $\forall i \in S^*(\theta)$. In

[4]Note that I only use $v_{H(\theta)}$ to characterize $S^*(\theta)$. The decision-maker may not know which action is the best sensorily dominated action.

Figure 2: Perception and inference



conclusion:

Lemma 1. *A decision-maker with unlimited ability for inference, and sensory capacity θ, selects an action j with positive probability if, and only if, $v_j \geq s^*(\theta)$, where the threshold is defined by*

$$s^*(\theta) = v_{H(\theta)} + \theta. \tag{1}$$

and $H(\theta) = \max\{i : v_N - v_i \geq \theta\}$.

Clearly, inference increases expected utility whenever it leads to the elimination of some actions.

2.1 Why irrelevant alternatives matter

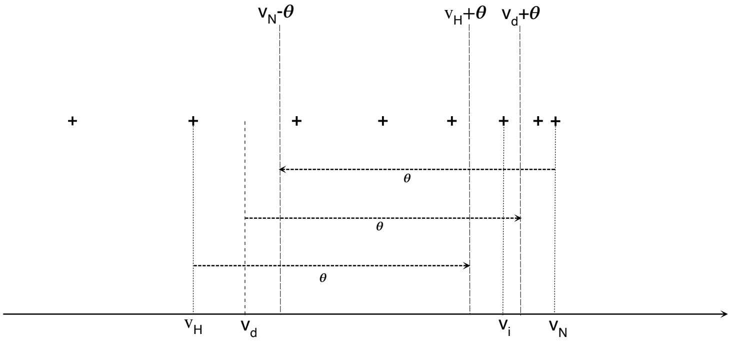
One of the axioms of rational choice is *Independence of Irrelevant Alternatives* (*IIA*).[5] Luce (1959) formulated *IIA* in probabilistic terms as follows. Let $P_X(i)$ denote the probability that the decision-maker selects an action i from a choice set X. Let X and Y be two choice sets, both containing actions i and j. Then, the relative probability of selecting i and j is unaffected by what other actions are included in the two choice sets, i.e.

$$\frac{P_Y(i)}{P_Y(j)} = \frac{P_X(i)}{P_X(j)}. \tag{2}$$

The present model predicts violations of *IIA*. These violations arise when rational inference is added to a limited sensory capacity. This is illustrated in Figure 3. The addition of a new best dominated action d leads to the elimination of action i, thereby reducing the relative choice probability of i relative to N.

[5]For a summary of various alternative definitions (including the principle of regularity, Sen's property a, the constant-ratio-rule and Luce's choice axiom) and their logical differences, see Ray (1973).

Figure 3: Adding dominated action d leads to the elimination of i.



Proposition 1. *Consider a decision-maker with unlimited ability for inference. Adding a new sensorily dominated action d to the choice set increases the decision-maker's expected utility, if* $v_d \in \big(v_{H(\theta)}, v_N - \theta\big)$ *and there exists some action i with* $v_i \in \big(v_{H(\theta)} + \theta, v_d + \theta\big)$. *The increase in utility is coincident with a violation of Independence of Irrelevant Alternatives.*

Proof. Note that the new action d is sensorily dominated as $v_N - v_d > \theta$ and that it replaces $v_{H(\theta)}$ as the best sensorily dominated action as $v_d > v_{H(\theta)}$. The new action allows the decision-maker to eliminate any action i with $v_i - v_d < \theta$, which was previously in S^* due to $v_i - v_{H(\theta)} > \theta$. The elimination of i from S^* increases the probability of choosing all the remaining actions in S^*, which all have higher expected utility than i. For any j remaining in S^*, $\frac{P_{\{1,\ldots,N\}}(i)}{P_{\{1,\ldots,N\}}(j)} = \frac{w_i}{w_j} > \frac{P_{\{1,\ldots,N\}\cup\{d\}}(i)}{P_{\{1,\ldots,N\}\cup\{d\}}(j)} = 0$, which constitutes a violation of IIA. □

Humans and other animals have been observed to violate *IIA* in empirical studies (see e.g. Cohen et al. 2019, and the references therein). Such effects (referred to as asymmetric dominance, attraction, decoy, or menu dependence) have been found to be robust, sizable and of practical significance e.g. in consumer choice (Doyle et al., 1999). Other examples include voting and hiring (see Bateman et al., 2005).

Previous theories suggest that adding actions increases complexity which may distract or confuse the decision-maker, creating "cognitive overload" or "biased attention". It appears likely that irrational choice behavior could be the cost of a more global optimization over both behavior and neural constraints (Louie et al., 2012). However, the neuronal mechanisms that lead to irrational behaviors are still unknown (Cohen et al. 2019). And proposition 1 even suggests that violations of *IIA* in some instances may be a feature rather than a bug. A dominated action is not irrelevant to a decision-maker with limited perception, as it may provides a valuable "contrast" accentuating the difference between other actions.[6]

2.2 Why a higher sensory capacity reduces the power of inference

Recall that the threshold for undominated actions is given by $s^*(\theta) = v_{H(\theta)} + \theta$. As inference leads to the elimination of all actions with expected utility $v_i \in \left(v_{H(\theta)}, s^*(\theta)\right)$, we may take the length of this interval, ie $s^*(\theta) - v_{H(\theta)}$, as a measure of the power of inference. And, as $s^*(\theta) - v_{H(\theta)} = \theta$, it follows immediately that a higher sensory capacity reduces the power of inference.

Moreover, the $s^*(\theta)-$equation reveals that the threshold is determined by two opposing forces. The first term of the threshold-function, ie $v_{H(\theta)}$, is the expected utility of the best sensorily dominated action. This term captures the direct benefit of the sensory system. A higher sensory capacity (lower θ), weakly increases $v_{H(\theta)}$, meaning that more actions are eliminated on the basis of sensorial domination. In Figure 2, this is represented by a short left-pointing arrow. However, a higher sensory capacity (lower θ) also makes fewer actions indistinguishable from $v_{H(\theta)}$; it reduces the power of inference. This is represented by the second term of equation 1 being lower. It is also illustrated by the right-pointing arrow of Figure 2 being shorter.

Lemma 2. *As a higher sensory capacity reduces the power of inference, the threshold-function $s^*(\theta)$ oscillates. In particular, for $j = 1...N-1$, and $v_{j+1} \neq v_j$, the threshold-function is given by $s^*(\theta) = v_j + \theta$ over the interval $\theta \in (v_N - v_{j+1}, v_N - v_j]$, thus increasing from $s^*(v_N - v_{j+1}) = v_N - (v_{j+1} - v_j) < v_N$ to $s^*(v_N - v_j) = v_N$. Moreover, $s^*(\theta) \leq v_1$, for $\theta > \underline{\theta} = v_N - v_1$.*

An example of a threshold-function is illustrated in Figure 4.

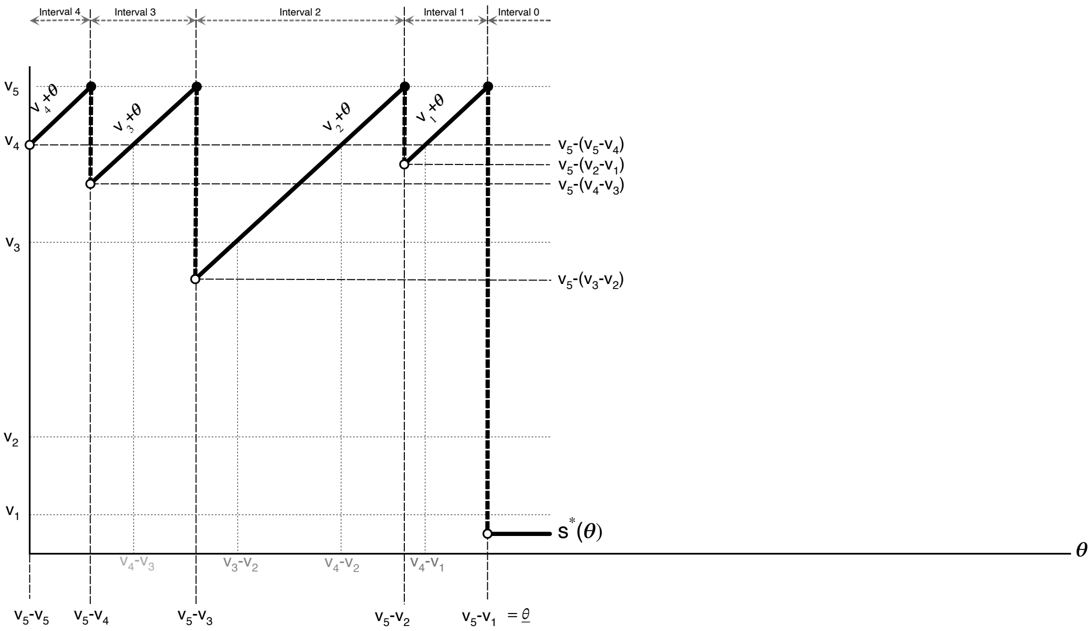
Proof. Recall that for a decision-maker with $\theta \leq \underline{\theta}$, the best sensorily dominated action is $v_{H(\theta)}$ where $H(\theta) = \max\{i : v_N - v_i \geq \theta\}$. It follows that $H(\theta) = j$, for $j = 1...N-1$, if $v_N - v_j \geq \theta$ and $v_N - v_{j+1} < \theta$ (when all N utility levels are distinct). Thus, over the interval $\theta \in (v_N - v_{j+1}, v_N - v_j]$, the threshold-function is given by $s^*(\theta) = v_j + \theta$. A final interval $j = 0$ is defined by $\theta > \underline{\theta} = v_N - v_1$. A decision-maker who cannot even distinguish between the best and the worst actions can also not infer any rankings. Thus, $s^*(\theta) \leq v_1$. □

The following proposition is a corollary.

Proposition 2. *Consider decision-makers with an unlimited ability for inference. A higher capacity of the sensory system reduces the power of inference. As a consequence,*

[6] The experiment in Proposition 1 changes the set of alternatives while keeping constant the utility-function and the capacity of the sensory system. One may think of this as short-term variations in the choice set, so that evolution does not change the human cognitive makeup.

Figure 4: The threshold function $s^* (\theta)$ oscillates



expected utility is a non-monotonic function of sensory capacity. The smallest sensory capacity that induces the decision-maker to select an action with the highest expected utility is the smallest usable capacity, $\underline{\theta} = v_N - v_1$.

Sensory capacity and inference are partly complements and partly substitutes in the production of good decisions. They are complements as some minimum of sensory information ($\theta \leq \underline{\theta}$) is a necessary seed for inference. More surprisingly, inference is also a substitute for sensory capacity. The best action would be chosen by either a "hyper sensitive man" with no ability for inference but a capacity for strong emotions (large $V_{\max} - V_{\min}$) and a high sensitivity (small Θ), so that $\theta \leq v_N - v_{N-1}$, or a "reasoning man" characterized by the ability to draw inferences based on muted feelings and blurred perception. Sensory capacity and inference are even antagonistic to some extent, as sensory information beyond the smallest usable level may harm a decision-maker with the capacity for inference. The key implication is that, given an unlimited ability for inference, there is no tradeoff between sensory capacity beyond the smallest usable level and the quality of decisions.

Remark A decision-maker will only suffer small utility losses as a result of her limited sensory capacity, if she has an unlimited ability for inference. The losses may even be trivial, e.g. if the choice set contains many actions with expected utilities that are spread relatively evenly over $[v_1, v_N]$.

To see this, let $\delta_i = v_{i+1} - v_i$ be the difference in expected utility between two adjacent actions i and $i+1$, given a certain utility function v. Let $\delta = \max_{1 \leq i \leq N-1} \delta_i \in [0, 1]$ be the largest such difference. Then, if a decision-maker with an unlimited ability for inference and sensory capacity $\theta \leq \underline{\theta}$ selects action i with positive probability, then the loss of utility is limited by $v_i > v_N - \delta$. If the difference is largest at the top, ie $v_N - v_{N-1} = \delta$, then the decision-maker selects the best action independent of the sensory capacity.[7]

As a consequence, if the decision-maker could include randomizations over "pure" actions, and if the decision-maker would be able to compare the expected utility of pairs of "mixed" actions according to the same standard as she compares "pure" actions, then δ could be made arbitrarily small. Moreover, in case there is a continuum of actions and the associated set of expected utilities is an interval $[\underline{v}, \overline{v}]$, and $\theta \in (0, \overline{v} - \underline{v})$, the best sensorily dominated action is $H(\theta) = \overline{v} - \theta$. As any action $v - H(\theta) < \theta$ cannot be distinguished from $H(\theta)$, inference eliminates all actions $v < \overline{v}$.

3 Natural Selection

3.1 The cognitive makeup and behavior of "Reasoning Man"

There are M outcomes cum fitness levels $y^1 < y^2 < ... < y^M$. Each action $i = \{1, 2, ...N\}$ is characterized by its cumulative distribution function over fitness levels, $F_i(y^j) = \Pr\{y \leq y^j \,|i\}$. The actions are ordered in terms of (strict) first-order stochastic dominance, ie $F_{i+1}(y^j) < F_i(y^j)$ for all $j < M$, so that $E\{y\,|1\} < E\{y\,|2\} < E\{y\,|3\}$.

Rayo and Becker let nature select a non-decreasing utility function $V(y^1) \leq V(y^2) \leq ... \leq V(y^M)$ maximizing the decision-maker's fitness, subject to a given range $V(y^j) \in [V_{\min}, V_{\max}]$ and a limited perceptions sensitivity $\Theta > 0$. I will not take the sensory capacity as given, but instead assume that the metabolic cost of the sensory system is increasing in both the range of utility levels (large $V_{\max} - V_{\min}$) as well as in the perception sensitivity (low Θ). I also assume that the ability to draw inferences is costly.

Proposition 3. *If nature selects an unlimited ability to draw inferences, then the decision-maker is induced to choose the action maximizing expected utility, thereby also maximizing fitness. Moreover, nature selects*

(i) *the utility step-function, with* $V(y^j) = V_{\max}$ *for all* $j \geq b+1$ *and* $V(y^j) = V_{\min}$ *for all* $j \leq b$*, and performance benchmark* $y^b = \arg\max[F_1(y^j) - F_N(y^j)]$,

(ii) *the smallest usable sensory capacity,* $\theta = \underline{\theta} = \frac{\Theta}{V_{\max} - V_{\min}} = F_1(y^b) - F_N(y^b)$.

The difference in (transformed) expected utility between two actions is equal to the difference in the probability that the benchmark fitness-level is reached, $v_i - v_j = F_j(y^b) - F_i(y^b)$.

Proof. First, note that if nature selects an unlimited ability for inference, then $E\{V\,|i+1\} > E\{V\,|i\}$, so that an action with strictly higher fitness also provides the decision-maker

[7] To prove these claims, recall that by the definition of $v_{H(\theta)}$, it follows that $v_N - v_{H(\theta)+1} < \theta$. By the definition of δ, $v_{H(\theta)+1} - v_{H(\theta)} \leq \delta$. Combining the two inequalities, $v_N - \theta < v_{H(\theta)+1} \leq v_{H(\theta)} + \delta$, i.e. $v_N - \delta < v_{H(\theta)} + \theta$. As $v_i \in S^*(\theta)$ implies $v_i \geq v_{H(\theta)} + \theta$, it follows that $v_i > v_N - \delta$ for all $v_i \in S^*(\theta)$. If $v_N - v_{N-1} = \delta$, then $v_i > v_N - (v_N - v_{N-1})$, so that only $v_i > v_{N-1}$, ie v_N, is selected.

with strictly higher expected utility. To see this, note that

$$E\{V|i\} = V\left(y^M\right) - \sum_{j=1}^{M-1}\left[V\left(y^{j+1}\right) - V\left(y^j\right)\right]\cdot F_i\left(y^j\right),$$

so that

$$E\{V|i+1\} - E\{V|i\} = \sum_{j=1}^{M-1}\left[V\left(y^{j+1}\right) - V\left(y^j\right)\right]\cdot\left[F_i\left(y^j\right) - F_{i+1}\left(y^j\right)\right] > 0.$$

The inequality follows from the actions being ordered according to first-order stochastic dominance and the utility function being non-decreasing. The strictness of the inequality follows from the fact that $V\left(y^{j+1}\right) > V\left(y^j\right)$ for some j, since otherwise the costly cognitive makeup would be redundant, as it would lead to random choice among all actions.

Second, it follows from Proposition 2 that if nature selects an unlimited ability for inference, it also selects Θ so that $\theta = \underline{\theta}(v)$, thereby inducing fully fitness-maximizing behavior. The reason is that $\theta = \underline{\theta}(v)$ is sufficient to induce fully maximizing behavior. A capacity beyond this level, $\theta < \underline{\theta}$, requires more energy and may even reduce the fitness of the behavior. A capacity below this level, $\theta > \underline{\theta}$, would make the costly cognitive makeup redundant, as it would lead to random choice among all actions.

Third, to minimize the metabolic cost of the sensory system, nature selects the utility function to minimize the sensory capacity, ie $\theta = \underline{\theta} = \frac{E\{V|N\}-E\{V|1\}}{V_{\max}-V_{\min}}$, where the difference in expected utility between the best and the worst actions is given by

$$E\{V|N\} - E\{V|1\} = \sum_{j=1}^{M-1}\left[V\left(y^{j+1}\right) - V\left(y^j\right)\right]\cdot\left[F_1\left(y^j\right) - F_N\left(y^j\right)\right].$$

To maximize this difference, given $V_{\max}$ and $V_{\min}$, nature selects $V\left(y^M\right) = V_{\max}$ and $V\left(y^1\right) = V_{\min}$. Moreover, as the utility range is bounded, any increase in the difference in utility between y^k and y^{k-1} must be balanced by a reduction in the difference in utility between some other fitness levels y^l and y^{l-1}. It follows that the only two adjacent fitness levels that should be assigned different utility levels is a y^{j+1} and y^j where $F_1\left(y^j\right) - F_N\left(y^j\right)$ is the largest. Thus,

$$E\{V|N\} - E\{V|1\} = \left[V_{\max} - V_{\min}\right]\cdot\left[F_1\left(y^b\right) - F_N\left(y^b\right)\right],$$

where

$$y^b = \arg\max_{y^j}\left[F_1\left(y^j\right) - F_N\left(y^j\right)\right],$$

so that $\theta = \underline{\theta} = F_1\left(y^b\right) - F_N\left(y^b\right)$.

Finally, as $E\{V|i\} = V_{\max} - F_i\left(y^b\right)\cdot\left[V_{\max} - V_{\min}\right]$, it follows that $E\{V|i\} - E\{V|j\} =$

$\left[F_j\left(y^b\right) - F_i\left(y^b\right)\right] \cdot \left[V_{\max} - V_{\min}\right]$, implying $v_i - v_j = F_j\left(y^b\right) - F_i\left(y^b\right)$. □

The main point is that, with an unlimited ability for inference, the decision-maker's behavior is fully utility-maximizing, thereby fitness-maximizing, and not merely satisficing, while at the same time using the smallest usable sensory capacity.

The utility function maximizing the power of inference takes on the same step-shape as Rayo's and Becker's utility function, for essentially the same reason. In both cases the utility function is constructed to assist the decision-maker's perception by maximizing the difference in expected utility between the best action and some alternative action. For Rayo and Becker the alternative action is the worst satisficing action. With inference the alternative action is the worst available action (which is also the best sensorily dominated action).

A potentially testable prediction of Proposition 3 is that a very high capacity for inference should be coupled with a low sensory capacity.

3.2 Will "Reasoning Man" be selected?

It is possible that the ability to draw inferences from inconclusive comparisons of expected utility is an add-on to the utility function, rather than an application of some general ability for inference. If so, this ability would emerge only after nature has already selected some utility function and some sensory capacity $\theta \leq \underline{\theta}$. And, unless inference emerges as a saltation, it would have to increase fitness, even when taking this existing utility function and sensory capacity as given. In what environment could inference overcome this hurdle?

Recall that the actions are ordered in terms of first-order stochastic dominance. Let the distance between two adjacent actions i and $i{+}1$ be denoted by $\phi_i = \max_j \left\{F_i\left(y^j\right) - F_{i+1}\left(y^j\right)\right\}$ for $1 \leq i \leq N-1$. The overall difference between adjacent actions in the environment can then be measured by $\phi = \max_{1\leq i\leq N-1} \phi_i \in [0,1]$, ie the largest distance between any pair of adjacent actions.

Inference gives rise to a gross gain in fitness (not counting the metabolic cost), if it leads to the elimination of some sensorily undominated action.

Proposition 4. *Consider a decision-maker with a given utility function and sensory capacity. If the difference between adjacent actions is sufficiently small compared to the sensory capacity, $\phi < \theta$, then unlimited inference gives rise to a gross gain in fitness.*

Proof. First, I demonstrate that for any utility function, $\delta \leq \phi$. That is, the distance between actions is smaller in terms of expected (transformed) utilities than in terms of probabilities. To see this, note that the expected utility of action i is

$$E\left\{V\,|i\right\} = V\left(y^M\right) - \sum_{j=1}^{M-1}\left[V\left(y^{j+1}\right) - V\left(y^j\right)\right] \cdot F_i\left(y^j\right).$$

The difference in expected utility between two adjacent actions is thus given by

$$E\{V|i+1\}-E\{V|i\}=\sum_{j=1}^{M-1}\left[V\left(y^{j+1}\right)-V\left(y^{j}\right)\right]\cdot\left[F_i\left(y^{j}\right)-F_{i+1}\left(y^{j}\right)\right].$$

The utility function that would maximize the difference in expected utility between actions $i+1$ and i is a step-function, so that

$$E\{V|i+1\}-E\{V|i\}\leq\left[V_{\max}-V_{\min}\right]\cdot\max_j\left[F_i\left(y^{j}\right)-F_{i+1}\left(y^{j}\right)\right].$$

The argument is the same as in the proof of Proposition 3. Thus, dividing by $V_{\max}-V_{\min}$, applying the affine transformation, and using the definition of ϕ_i,

$$v_{i+1}-v_i\leq\phi_i.$$

Using the definitions of δ and ϕ it follows that $\delta\leq\phi$.

Second I demonstrate that, if the decision-maker cannot perceive the difference between any pair of adjacent actions, i.e. $\delta<\theta$, then there exists an action i characterized by $v_i\in\left(v_N-\theta, v_{H(\theta)}+\theta\right]$, meaning that i is not sensorily dominated but that it will be eliminated by inference. To see this, note that by the definition of δ, there must exist some action i with $v_{H(\theta)}<v_i\leq v_{H(\theta)}+\delta_v$. If $\delta<\theta$, it follows that $v_i\leq v_{H(\theta)}+\theta$, meaning that i is sensorily indistinguishable from $v_{H(\theta)}$, so that it will be eliminated by inference. By definition of $v_{H(\theta)}$ this action must also be characterized by $v_i>v_N-\theta$, since otherwise v_i would be the best sensorily dominated action, meaning that v_i is not sensorily dominated.

Third, combining the two points above, namely that all utility functions are characterized by $\delta\leq\phi$, and that $\delta<\theta$ implies a gain from inference, it follows that $\phi<\theta$ implies a gain. □

Proposition 4 suggests that there may be an evolutionary pressure to provide inference, even if nature has already provided the decision-maker with a high sensory capacity, in case the decision-maker must choose from a plethora of available actions and the differences in outcome is small even between the best and the worst action.[8]

[8] Consider the case with only two fitness levels. Let p_i be the probability of the lower fitness level when action i is chosen. Then $F_i\left(y^{Low}\right)=p_i$ and $F_i\left(y^{High}\right)=1$. Thus, $\max_j\left[F_i\left(y^{j}\right)-F_{i+1}\left(y^{j}\right)\right]=p_i-p_{i+1}$. If the actions are all on equal distance from one another, $\phi=p_i-p_{i+1}=\frac{p_1-p_N}{N-1}$. Then, if

$$\frac{p_1-p_N}{N-1}<\theta$$

unlimited inference gives rise to a gain in fitness. That is, even if the sensory capacity is high (θ is low), there are gains from inference, if there are many actions or if the difference in expected fitness even between the best and the worst actions is small.

4 Concluding Remarks

This article studies the possible emergence and consequences of an unlimited ability to draw inferences. A natural next step would be to consider degrees of inferential ability. To do so, one would need to describe the process of inference in more detail. And, as the metabolic cost of inference is presumably increasing in the number of steps taken, one may expect there to be a tradeoff between this decision-making cost and the quality of decisions. An interior solution appears plausible.

Surprisingly, however, the decisions based on an unlimited ability for inference, when coupled with a minimum perception capacity, can actually be replicated by a simple and possibly cheap decision process. The decision-maker does not even need to compare all pairs of actions sensorily, before inference starts. With N alternative actions, there are $N \cdot (N-1)/2$ potential sensory comparisons to be made. When the decision-maker has the smallest usable sensory capacity, all comparisons will be inconclusive, except the one between the best and the worst actions. Thus, in order to select the best action, the decision-maker can simply continue to search for the one pair of actions that can be ranked, and once found, choose the better one. If the pairwise comparisons come in random order, then the probability that the conclusive comparison is the i'th comparison is $\frac{1}{N \cdot (N-1)/2}$ for all i. Thus the expected number of comparisons before a choice is $\frac{N \cdot (N-1)+2}{4}$.

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