UNIVERSITY OF GOTHENBURG
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Financial Economics

Portfolio Optimization

The search for an optimal portfolio with cryptocurrencies and S&P 500

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Abstract

This thesis' aim is to create an optimal portfolio consisting of Bitcoin, Ethereum and S&P 500. We also examine the minimum variance portfolio with the framework of Markowitz's mean variance optimization model. We evaluate the performance of the optimal portfolio and compare it to the minimum variance portfolio based on the risk measures Sharpe ratio and Conditional Value at Risk. We find that the optimal portfolio consists of almost a third position towards cryptocurrency. Compared to how previous studies of portfolios have diversified towards cryptocurrencies our study finds that the optimal portfolio consists of a higher percentile amount of cryptocurrencies. Our second major finding is that a smaller diversification towards cryptocurrencies can give less volatility compared to investing fully in a traditional index, such as, S&P 500. The latter result contradicts some previous investment advice that cryptocurrency has too high volatility for diversification benefits.

Keywords: Portfolio optimization, Minimum variance portfolio, Capital allocation line, Cryptocurrency, Diversification

1. Introduction

This chapter introduces a background description of the three assets Bitcoin, Ethereum and S&P 500 as well as the risk free asset. Furthermore, this chapter contains the purpose of the thesis and a problem discussion. Lastly the structure of the thesis is presented.

1.1 Background Description

Investing in the stock market has for long been seen as a long-term secure investment to achieve a positive return on the invested capital. The S&P 500 is an index that follows the 500 largest corporations listed on stock exchanges in the United States. It has a historical average return of 10 % with inflation included. Therefore, the real average return has decreased by about 2-3 % yearly, but overall there is still a historical positive return on the S&P 500 index. (Maverick, 2022)

An investment that generates an almost risk free return is the investment in a US treasury bill. Which is a security that is issued by a government. In this thesis we have looked further into the 3 Months Treasury bill and the yield that is generated from this investment. The US 3 Months Treasury is a bill that guarantees a set interest rate as well as repayment of the invested funds 3 months after the Treasury was bought. In order to track the daily prices of the Treasury bill, one could track the Treasury yield, which is the current rate Treasury bills would pay the investor if it was bought at that date. (Baldridge & Curry, 2021)

Satoshi Nakamoto designed Bitcoin in 2008 and since Bitcoin launched in January 2009 it has been the market leader of cryptocurrency. Cryptocurrency is a peer to peer technology that makes it possible to send digital currency online without the need for intermediaries. Bitcoin uses a technology that is called blockchain technology and is complex to explain. But a simplification of blockchain technology is that it is a shared database that stores data over multiple users. In the case of Bitcoin, the mentioned data is transaction information. Transactions on Bitcoins blockchain are placed in a queue and have to be validated by miners, creators of new blocks in the blockchain, within the network at the same time. New blocks in the blockchain are created when a miner completes a cryptographic calculation that is hard to generate but easy for the other miners to verify. When other miners have verified

that the calculation is correct, the block will be added to the blockchain and the miner that managed to calculate the hash is rewarded by 6.25 Bitcoins. The amount that is rewarded for solving a block is halved for every 210 000 blocks that are mined. It was previously halved for the third time on the 11th of May 2020. A big risk that is associated with investing in cryptocurrency is the volatility of the cryptocurrency market; the previous price fluctuations can be an indication of the volatility of the overall market. (Bitcoin, 2022)

The crypto market consists of other cryptocurrencies as well as Bitcoin, for example did another cryptocurrency, Ethereum, have 11.6 % of the market capitalization on the crypto market. It was placed as the second largest cryptocurrency on the crypto market in 2020 with only Bitcoin being larger with 60.23 % of the market capitalization on the crypto market in 2020 (Levulyté et al, 2021). However, the cryptocurrency market is volatile and there have been several crashes and rises of different coins, tokens and the market as a whole (Lapin, 2021). With that being said, several investors have seen their investment rise and fall quickly, the same could be said about the investors that have invested in the stock market. The stock market has incorporated some certificates that track and follow the prices of cryptocurrencies (Hedqvist, 2022). The market capitalization of cryptocurrencies have increased greatly during the last decade and therefore the possibility to diversificate between different digital assets such as cryptocurrencies or digital tokens has also increased. (Antonakaskis et.al, 2019)

Diversification between assets in a portfolio is an investment strategy that has existed for a long time. In order to diversify a portfolio, the investor has to spread its investment across different sectors of the market in order to eliminate the idiosyncratic risk of a portfolio. However, with the introduction of cryptocurrencies, a debate arose whether or not cryptocurrencies were supposed to be a part of that diversified portfolio (Guzun, 2021). Previous research has shown that historically, some portfolios that have diversified with cryptocurrencies have had a lower return then the portfolios that haven’t had parts of their investment diversified with cryptocurrencies (Tavares et al, 2020).

1.2 Purpose of the thesis

The purpose of this thesis is to analyze how the optimal portfolio is constructed with the constraint of 3 variables; S&P 500 Index, Bloomberg Galaxy Bitcoin Index and Bloomberg Galaxy Ethereum Index, during the period August 2017 (08/04/17) to the 4th of February 2022 (02/04/22). The 3 Month Treasury Bill will also be analyzed during this period as a risk free investment in contrast to the portfolio. The optimal portfolio is defined by the portfolio that has the best risk to return relationship. This thesis aims to give a better insight into the volatility and returns that exist when combining the stock market with the crypto market. The target group of the thesis is investors that have limited knowledge about these possibilities.

1.3 Research Question

The research question that will be examined in this thesis is the following:

> *How is the allocation of assets in an optimal and minimum variance portfolio with regards to three indexes; S&P 500 Index, Bloomberg Galaxy Bitcoin Index and Bloomberg Galaxy Ethereum Index?*

1.4 Problem Definition and Problem Analysis

In both the crypto market and the stock market, the general focus of every investor is to generate a greater return on the market than what the invested capital would do anywhere else. Since the stock market has existed for a longer time than the crypto market, there is no surprise that multiple portfolio- and investment strategies have been created and used to maximize the return that can be achieved on the stock market without taking the crypto market into account. Therefore, one could assume that some portfolio theories and investment strategies would look different in the modern economy then how they did when they were created. (Rowland, 2010) One investment strategy that we will look further into is the strategy of a diversified portfolio over two different markets.

Diversification is the concept of pooling different financial assets together in order to generate a portfolio that isn't overly weighted into a specific sector of the economy. The causality that exists within a separate market makes diversification an appealing option for the reason that a recession in a specific sector of the economy will not make the entire portfolio collapse. Some sections of the stock market are correlated with the crypto market and both are impacted by certain changes in the outside world. A well diversified portfolio has in theory eliminated the idiosyncratic risk and only holds the market risk. (Segal, 2021)

When comparing the crypto market and stock market, the fact that the crypto market is fairly new has to be taken into consideration. There is often a higher risk of investing in a new market then there is in an older, more well established market. One of the reasons is that the regulations are often well established in the older market, while they might not be as definite in the newer markets. Another reason is that there might be limited information regarding the new market and the actors on it.

1.5 Structure of the thesis

The sequent part of the thesis is structured in the following order: The upcoming section explains the fundamental theories that are used within this thesis and what previous research has achieved. The third and fourth part of the thesis looks over the different methods and measurements that are crucial in order to get a well-founded analysis, as well as the data that is analyzed. The fifth part is the results that were achieved with the collected data and after the established methods could develop further on. The next part after the results is the discussion, where the results are analyzed. The final part of this thesis is the conclusion where the authors conclude the findings of the thesis.

2. Theory and previous research

In this chapter the theories about portfolio optimization are presented. It also sets to present previous research about the stock market, crypto market and the relationship between these. The chapter starts with a presentation of the modern portfolio theory and the mean variance optimization followed by the presentation of the previous research.

2.1 Theory

There has been a substantial rise of overall market capitalization in the crypto market as a whole but also in Bitcoin as well as in Ethereum in the previous years. Simultaneously more and more investors have gained an increased interest in the crypto market and the returns that can be generated on it. Some theories were created before the creation of cryptocurrencies and were therefore created for another market but might work on the crypto market as well. Therefore this thesis will look to more well established theories to come to a conclusion regarding this area.

2.1.1 Modern Portfolio Theory

In this thesis the well established Modern Portfolio Theory, written by Markowitz, will be used, in order to estimate parameters and draw conclusions from verifiable data on a variation of portfolios. The basics of modern portfolio theory is that it is used in order to get a higher overall portfolio return without increasing the risk. This is possible by diversifying the portfolio towards different sectors. Modern portfolio theory is a theory that argues that all investments should be viewed not only by itself, but how it affects the overall portfolio in terms of risk and return. (Markowitz, 1952)

For this comparison to be possible, an efficient frontier between portfolios that include Bitcoin, Ethereum and S&P 500 must be created. An efficient frontier is the optimal weight of assets in a portfolio that are expected to generate the highest possible return for a set level of risk (Markowitz, 1952). A portfolio will follow the efficient frontier if no other portfolio that offers higher return has the same or less risk as the original portfolio. The efficient

portfolio is defined as the ones that achieves maximum expected return for a desired level of risk (Corporate Finance Institute, 2022).

2.1.2 Mean Variance Optimization

Mean variance optimization is a part of the modern portfolio theory. It takes ground in the idea that investors will make rational decisions when having full information about an investment. The goal of mean variance optimization is to create a portfolio that generates a maximum return compared to a set risk of the portfolio. (Markowitz, 1952) Therefore the optimized portfolio will generate the highest possible return based on the investors chosen set risk. The risk is measured based on the volatility and the return is measured in terms of expected return of the portfolio. (Chen, 2021)

$$max \sum_{i=1}^{i=n} \mu_i \omega_i$$

$$min \sum_{i=1}^{i=n} \sum_{j=1}^{j=n} \sigma_{ij} \mu_i \omega_i$$

$$\sum_{i=1}^{i=n} \omega_i = 1 \qquad (2.1)$$

n = number of assets

ωi = the weight of each asset in the portfolio

μi= expected return on asset i

σij = covariance between the returns of j, ωi and i

2.2 Previous research

The previous research that this thesis focuses on is either articles studying cryptocurrency, the stock market or a comparison of them both. The articles presented in this thesis have in various ways examined the relationship between cryptocurrencies, the stock market and the surrounding environment.

R. Tavares, J Caldeira and G. Raimundo (2020) writes in their report '*It's all in the Timing Again: Simple Active Portfolio Strategies That Outperform Naive Diversification in the Cryptocurrency Market'* that according to the modern portfolio selection theory, it is possible to obtain greater results in terms of risk and return on a portfolio that is diversified with different assets. The report took ground in the comparison between the S&P 100 index and 20 leading cryptocurrencies to see whether it would bring greater return to the timing portfolio other than the mean and minimum variance portfolios. A conclusion this thesis came to was that there was no performance gain in adding cryptocurrencies to stock portfolios and that instead several of them performed worse with the addition of digital currencies. However, they found that the implementation of portfolio selection on a base composed of only digital currencies showed a greater average return then a portfolio that didn't diversify between digital assets.

The paper '*Investigating the Relationships between Volatility of Cryptocurrencies and Other Financial Assets*' written by Ghorbel and Jeribi (2021) discusses the relationship between volatilities for five cryptocurrencies, oil, gold and American Indices (S&P 500, Nasdaq and VIX). One major finding is that there is a higher volatility spillover between cryptocurrencies then the volatility spillover between cryptocurrencies and other financial assets. Bitcoin and gold have some similarities, such as, neither has a specific nationality or is controlled by a government and both are mined by several different independent operators. Cryptocurrencies can be used as diversification and can therefore give the diversification benefits for investors during stable periods. The conditional volatility of Bitcoin and other cryptocurrencies such as Dash or Monero have a positive and significant effect on the conditional volatility of the S&P 500 and NASDAQ. The coefficient is small, which indicates that it has a low effect on the indexes. Another finding in the article was that both Bitcoin and gold are considered a hedge for US investors when there is a stable period, in other words previous to the corona crises. The last finding was that digital assets wasn't, contrary to gold, a safe haven for US investors during the coronavirus crisis.

Uzonanne (2021) studies in his article the presence of return and volatility spillovers over five major markets and the bitcoin market. Uzonwanne study observed significant return

spillovers and volatility spillovers across these five markets. The author of the article presents that the study shows that volatility spillovers were bi-directional in some markets and unidirectional in others. It also concludes that investors switch between these market pairs at the high and low of the stock market in order to maximize return and reduce risk, resulting in return and volatility spillover effects between market pairs.

In the text '*The role of bitcoin in well diversified portfolios: A comparative global study'* the authors A. Kajatazi and A. Moro has pursued research that explores the effect of adding bitcoin to an optimal portfolio. The article investigates how Bitcoin fits into the portfolios of U.S, European and Chinese assets. The authors use the Conditional Value at Risk (CVaR) and portfolio optimization in the search for the answer. For each scenario, the autor do a back-test to compare the performance of portfolios with and without bitcoin. The data demonstrate that adding bitcoin to a portfolio improves its performance, but that it is more because of an increase in in return rather than the reduction of volatility. The conclusion of the research is that bitcoin may have a role in portfolio diversification even though the analysis in this text confirms bitcoins speculative characteristics.

The three authors; Anarkulova, A., Cederburg, S., and Doherty, M., wrote 2022 an article named '*Stocks for the long run? Evidence from a broad sample of developed markets'* that questions whether or not stocks are a safe investment over long holding periods. The writers use a bootstrap simulation to characterize returns for stock market investors that invest with a long horizon. The historical record of stock market performance dates back to 1841 and the observations are conducted in 2019. The analysis finds three primary findings. Firstly, the long-term outcomes from a diversified portfolio are highly uncertain based on historical records over developed stock markets. Secondly, they find that "catastrophic investments" are common even when investors use a longer investment horizon and that it is a 12.1 % possibility that the investor will lose relative to inflation. Thirdly, it shows that historical records of market performance across different developed markets can vary much and does vary much within the data for the historical US experience.

3. Methodology

During the methodology chapter, some key measuring methods as well as how these key figures are calculated are presented. The chapter ends with a presentation of how the theories and methods are combined in order to present the data.

3.1 Sharpe ratio

This thesis aims to analyze the Sharpe ratio by William F. Sharpe as a performance metric on the different portfolios to find the optimal portfolio. The Sharpe ratio is a measure that is used to help investors understand the risk/return relationship of an investment and helps portfolio managers combine the risk and return variables into one single value. (Sharpe, 1966) The ratio describes how much excess return the investor receives for the increased volatility that it means to hold a riskier asset. As an investor needs to be compensated for the additional risk they take when choosing to not hold a risk-free asset. Therefore, the risk free rate is included in the formula to prove that the investor is sufficiently compensated for the additional risk. (Sharpe, 1994; Benninga, 2008)

The formula is as follows:

$$S_a = \frac{E\left[R_a - R_b\right]}{\sigma_a} \tag{3.1}$$

S_a = Sharpe ratio

$E[R_a]$ = Expected return of portfolio a

$[R_b]$ = Risk free return

σ_a = Standard deviation of asset a

A Sharpe ratio with a high number indicates that the investment has a higher risk -adjusted return, which is preferable for investors in terms of a risk to return perspective. (Lioudis, 2021

3.2 Capital Allocation Line

The capital allocation line is similar to the capital market link (CML) and is a line on a graph that represents the possible combinations of a risk free asset and riskier assets. The difference between the CAL and CML is that the CML is where the risk portfolio, is the market portfolio and the CAL can be any portfolio that holds risk. The capital allocation line holds all combinations of risk free assets and riskier assets and can therefore be seen as the reward to volatility ratio. The slope of the capital allocation line measures the risk to reward trade-off. The CAL helps investors choose how much to invest in the risk free asset such as Treasuries as well as the riskier assets such as stocks or bonds. (Chen, 2020)

The equation for the capital allocation line is as follows:

$$CAL = R_f + (S_a * E[R]) \quad (3.2)$$

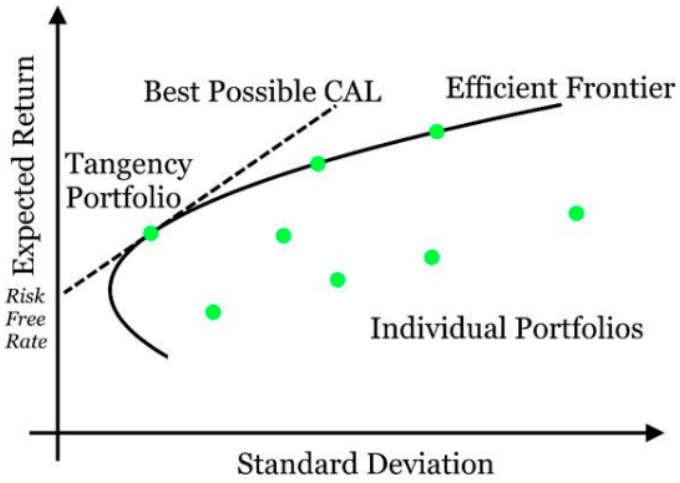
CAL = Capital Allocation Line

R_f = Risk Free Rate

S_a = Sharpe Ratio

$E[R]$ = Expected Return

On graph 1 below, the optimal portfolio is found at the point where a portfolio in the efficient frontier and the capital allocation line is tangent. The portfolio that is found is the one that gives the best risk-to-reward ratio, since the slope is the highest at this point. The rational investor seeks to invest at the point where the “Tangency Portfolio” is presented, since this is the portfolio that generates the highest return for a set level of risk. (Corporate Finance Institute, 2022). The other points on graph 1 shows the expected return and standard deviation if an investor would invest in the individual assets.



(Stockton, S., 2018)

Graph 1. Portfolio optimization illustration. Illustration of how the capital allocation line and the efficient frontier will be combined.

3.3 Variance and Conditional Value at Risk (CVaR)

The portfolio variance is a commonly used model to calculate asset return. But the portfolio variance or mean-variance has the disadvantage that it oversimplifies the risk-preferences of certain investors. Variance is a symmetric measure that incorporates both the upside and the downside volatility, whereas in the real-world, assuming investors are rational, only the downside component is undesirable. (Kajtazi & Moro, 2019)

For the empirical methodology an alternative risk measure called Value-at-risk (VaR) can be used. It is an asymmetric measure that is expressed as a minimum loss (value or percentage) for a given probability and time horizon. The value at risk is a suffisent measurement if the investment has shown stability over time. However, the crypto market can be seen as a fairly volatile market and therefore there is a risk that the VaR will not give a full picture of the risk, as the VaR is indifferent to anything beyond its own threshold (Alexander & Baptista, 2004). For that reason the Conditional Value at Risk (CVaR) will also be used as a risk measure on the optimal and minimum variance portfolio. The CVaR attempts to address the shortcomings of the VaR model. The Conditional Value At Risk (CVaR) will be used because it is shown to have a number of advantages compared to other similar measurements. Mainly the CVaR

satisfies a set of four desirable properties, namely: Monotonicity, Translation invariance, Homogeneity and Sub-additivity wheresas variance and VaR does not. (Artzner et al, 1999)

$$CVaR_p^2 = \sum_i \sum_j w_i w_j \, CVaR_i \, CVaR_j \, p_{ik} \tag{3.3}$$

From this calculation the portfolio risk is derived:

$$CVaR_p = \sqrt{CVaR_p^2} \tag{3.4}$$

Which give:

$$CVaR = \frac{1}{1-c} \int_{-1}^{VaR} xp(x)dx \tag{3.5}$$

where:

p(x)dx = The probability density of getting a return with value "x"

c = The cut-off point on the distribution where the analyst sets the VaR breakpoint

VaR = The agreed-upon VaR level. (Chen, 2020)

3.4 Excel solver

To find the optimal portfolio and minimum variance portfolio for the three assets we use the solver function in Excel. The Excel solver can be used to optimize a portfolio in the framework of Markowitz's mean variance optimization model. We use the excel solver to construct different portfolios to find the optimal weights for the assets in the portfolio by our chosen constraints. One constraint that will be consistent during all calculations is that shorting of assets is not allowed. These portfolio calculations derive from data based on the covariance matrix in Appendix 4, as well as expected return and the standard deviation of

each asset. The expected return and the standard deviation of each asset can be found in table 2. The Excel solver will alter the portfolio weights with consideration of the set constraints. The excel solver will thereafter give the weights, expected returns, standard deviation and Sharpe ratio for the different portfolios. These results will make it possible to find the optimal portfolio and minimum variance portfolio. The results will also provide the information needed to create the efficient frontier.

4. Data

In the following chapter the approach to collect reliable data and how it is presented throughout the thesis is explained.

4.1 Data Collection

The data that was collected for this thesis was extracted from the Bloomberg terminal database and Yahoo finance database. From Bloomberg's database, the data for three indexes were extracted; S&P 500 index (SPX), Bloomberg Galaxy Bitcoin index and Bloomberg Galaxy Ethereum index. The data that was analyzed in this thesis spans from the 4th of August 2017 (08/04/17) to the 4th of February 2022 (02/04/22). In total this is a timespan of 18 quarters or 1,135 trading days. The observations of the S&P 500 index are on a daily basis with an end of the day price observation. Since the crypto market does not follow the same opening hours of the stock market, the crypto index calculates the daily simple average of prices between 16:00 and 16:15 in order to get the closing price. During the observed period, both the stock market and crypto market have had price fluctuations that will have an effect on the overall results. The 3 Month Treasury Bill was extracted from Yahoo finance and follows the same timespan as the indexes that was extracted from Bloomberg.

Historical data for Bitcoin starts in January 2009, but the early period of Bitcoin is characterized by very low trading volumes and liquidity. Ethereum was created in July 2015 but the Bloomberg Ethereum Galaxy Index that tracks the price of Ethereum starts to follow the cryptocurrency on the 4th of August in 2017 (Bloomberg, 2020b). For that reason the data collection will start from the 4th of August 2017 and continue for the following 18 quarters and end on the 4th of February 2022 for both Bitcoin and Ethereum. In total this is 1,135 daily observations of each asset.

Table 1. Variable explanation. Table 1 describes the variables that were used in this thesis. Some variables are explained further within the report.

Variables	Descriptive
Mean Price	Average price of the security
Minimum/maximum price	Minimum and maximum price of the security
Mean Return	Average daily return*
Variance	Variance of the sample
Std dev	Standard deviation of the sample
Minimum/maximum return	Minimum/maximum daily return of the security
Average monthly return	Average return on a monthly basis**
Alpha	Alpha value
Count	Observations
Location	Amount of times the portfolio experiences Var(p)
VaR(p)	Return of portfolio at alpha times (See 3.)
CVaR(p)	Return of portfolio at probability alpha (See 3.)

*Closing price of asset divided by opening price of the asset, quota is then subtracted by 1 to get the percentage return.

**Average monthly return is the average daily return multiplied with 22, the average days the stock market is open per month.

4.2 Bloomberg Galaxy Bitcoin Index

The Bloomberg Galaxy Bitcoin Index is designed to track the price of a Bitcoin, traded in US dollars. The Bloomberg CFIX rate fixings are used as a reference point for Bitcoin and other cryptocurrencies. CFIX is the average of Bloomberg Generic Price, a real time pricing data that accounts from multiple pricing sources, for the Bloomberg Galaxy Bitcoin Index that data is the simple average of the prices of Bitcoin between 16:00 and 16:15. The index follows the Bitcoin price and is divided by an index divisor. The index was set at 1.000 at the first of May 2018, this implies that it is not the price of a Bitcoin that is shown but the return is the same since it has the same percentile growth. (Bloomberg, 2020a)

4.3 Bloomberg Galaxy Ethereum Index

The Bloomberg Galaxy Ethereum Index follows the same methodology as the Bitcoin Index but tracks the Ethereum cryptocurrency instead of Bitcoin (Bloomberg, 2020b).

4.4 Risk Free Rate

The 3 Month Treasury Bill (^IRX) was gathered from Yahoo Finance and used as the risk free rate. The Treasury bill is the interest that a government pays an investor for borrowing a set amount. It is virtually a risk free investment, since the federal reserve controls the printing process, the only minor risk is that it defaults and can't repay the debt at maturity day. In this thesis the 3 Month Treasury Bill will be considered a risk free investment. The treasury bill is dependent on 3 values; the face value of the security, what amount the security was purchased for and how long it is til the bill reaches maturity. The treasury bill can simply be described as the rate that the state would buy back the security for, at that specific date (Hayes, 2021).

5. Results

Chapter 5 is the result when the theories, methods and data that previously been presented are combined. The time span stretches from the 4th of august 2017 to the 4th of february 2022.

5.1 Descriptive statistics

Table 2 presents key figures for the underlying securities in the indexes, as well as, the 3 Month Treasury bill. The data in chapter 5.1 present the amount of observations made on the daily price of the different assets. Table 2 also shows the mean price, minimum/maximum price, standard deviation, the minimum/maximum return and the average monthly return in each observation category. The values that are presented are extracted from the daily return of the assets, it is calculated as the closing price of an asset subtracted with the opening price of the same asset. The price of the 3 Month Treasury is excluded since it does not follow the same pricing possibilities as the indexes.

Table 2. Descriptive statistics. The observations, mean price, minimum price and maximum price are presented as standard units traded in US dollars. Mean return, variance, standard deviation, minimum/maximum return and average monthly return is presented in percentage.

	Bitcoin	Ethereum	S&P 500	3 Month Treasury
Observations	1135	1135	1135	1135
Mean price	2002,91	1442,74	3248,46	
Minimum Price	317,79	127,58	2237,40	
Maximum Price	7561,44	7395,45	4796,56	
Mean Return	0,0035	0,0042	0,0007	0,0000
Variance	0,0023	0,0038	0,0002	0,0000
Std deviation	0,0480	0,0616	0,0128	0,0000
Minimum Return	-0,2352	-0,2811	-0,1198	0,0000
Maximum Return	0,2335	0,3835	0,0939	0,0001
Average Monthly Return	0,0769	0,0914	0,0150	0,0009

Bitcoin and Ethereum have a similar maximum price but have a large percent difference in the minimum price.The steadiness within the cryptocurrencies can differ and is not represented in the prices but if one only analyzes the prices, Bitcoin does seem like an investment with lesser downside then Ethereum. It also seems like Bitcoin is the safer security to invest in since the major difference is in the minimum return and holds a lower volatility then Ethereum.

The largest mean return, variance and standard deviation is found in the column for Ethereum. These measurements show that the Ethereum currency has a broader data spread then the other assets, the standard deviation also indicates that the Ethereum currency has broader tails then the other assets. Even though the average mean return is higher for Ethereum the mean price of a Bitcoin has been higher. This gives part of an explanation to why there is a large difference in the variance of the two cryptocurrencies.

The smallest mean return, for the indexes, can be found in the S&P 500 column, which clarifies that the mean return is the lowest when investing in the S&P 500. Since the mean return is the lowest for this index, the average monthly return will also be the lowest for the S&P 500. The average monthly return of the S&P 500 is 1.5 %. The average return of Ethereum however is ~9.14 % which is more than 4 times higher than the return of the S&P 500. Of all the analyzed indexes the S&P 500 also has the lowest variations in standard deviation and variance. The variance at 0.00016 is considerably lower than the variance for the second lowest index, that of Bitcoin, at 0.00230.

Ethereum has the biggest span from minimum to maximum return, the span is from -28.11 % to 38.35 % and is a span of 66.46 percentage points. The span is 45.09 percentage points broader than the span for the S&P 500. The S&P 500 has the smallest average return span, which is to be expected since the S&P 500 has removed the separate risk and carries only the market risk.

The minimum return in the 3 Month Treasury gives a clear indication that it does not generate negative return over the analyzed time. The variance of the 3 Month Treasury is also close to 0, which makes it reasonable to consider the security as a risk free option.

Graph 3 illustrates the price of three different indexes over time, the three indexes are the S&P 500, Bitcoin and Ethereum.

Graph 3. Price development of individual assets

The illustration reinforces the previous facts shown, that Bitcoin and Ethereum have had a clear positive price development since the start of our data set. Graph 3 also shows that there seems to be a correlation between Bitcoin and Ethereum. There is also a notable difference in volatility between the cryptocurrencies and the S&P 500. Where the S&P 500 has had a stable upward trend with only a few dips compared to Bitcoin and Ethereum which has been much more volatile.

5.2 Correlation

Table 3 gives a numerical representation of how the separate assets correlate with the other assets. The 1 in the correlation table is how the asset correlates with itself.

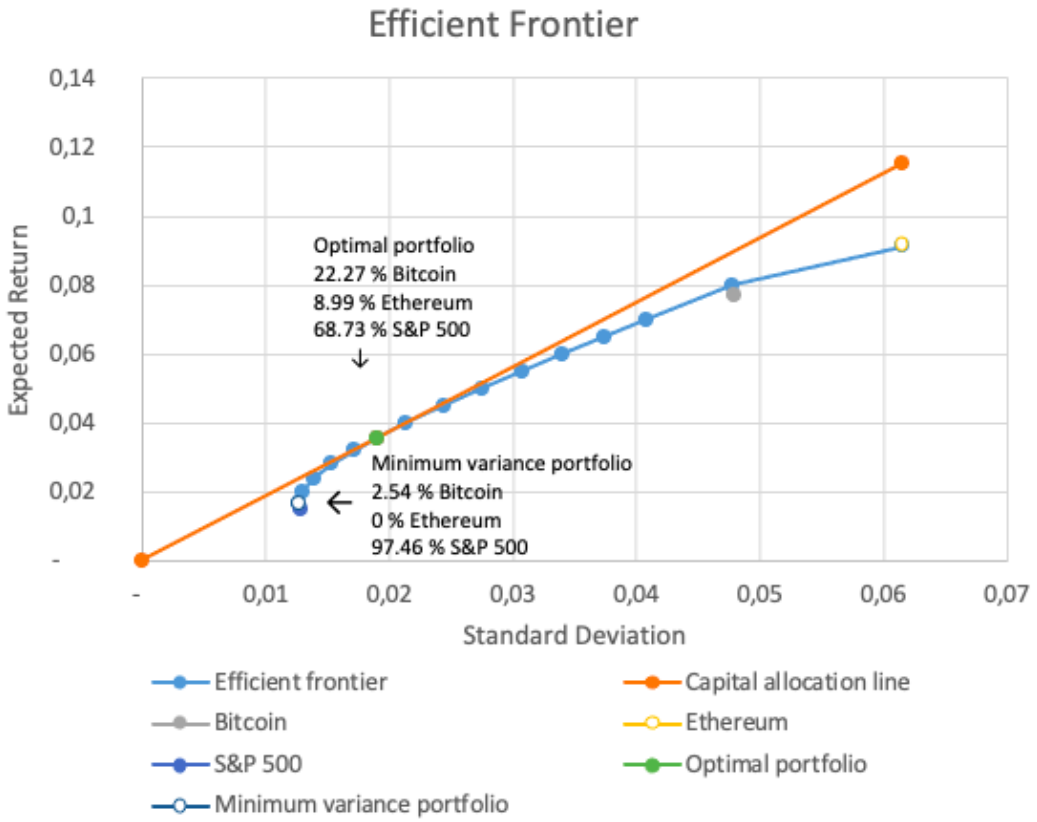
Table 3. Correlation Table. Presents the correlation between Bitcoin, Ethereum, S&P 500 and the 3 Month Treasury.

	Bitcoin	*Ethereum*	*S&P 500*	*3 Month Treasury*
Bitcoin	1			
Ethereum	0,7240	1		
S&P500	0,1735	0,1970	1	
3 Month Treasury	-0,0300	-0,0218	-0,0022	1

The correlation presented above indicates that there is a moderate positive correlation between the two cryptocurrencies; Bitcoin and Ethereum. The correlation is 0.7240 for these cryptocurrencies and shows that the relative movement is similar for the two cryptocurrencies. The cryptocurrencies have a similar correlation with the S&P 500, with Ethereum being a minuscule amount larger than Bitcoin. The low correlation indicates that the cryptocurrencies can be a good complement to diversifying a portfolio with S&P 500. As a low correlation between two assets has a greater effect on diversification. The correlation between the 3 Month Treasury and all assets is negative, indicating that they move opposite with the 3 Month Treasury. The correlation coefficient of the negative values are negligible since they are close to zero. The 3 Month Treasury can therefore also act as a diversification but towards all indexes since it barely moves with any correlation in either direction.

5.3 Efficient frontier

Graph 2 shows the efficient frontier for a portfolio that consists of Bitcoin, Ethereum and S&P 500. It also shows the separate assets that together create the efficient frontier. The capital allocation line starts at the risk free rate and increases with the optimal Sharpe ratio for each level of expected return. The final point on the capital allocation line is an extrapolation, which assumes that the risk-to-return relationship will follow the same trend.



Graph 2. Efficient frontier of the possible portfolios combined with the capital allocation line. The graph also shows the assets separately.

The point where the capital allocation line is tangential to the efficient frontier is where the Sharpe ratio is maximized and where the optimal portfolio is presented. The values behind the capital allocation line can be found in Appendix 2. In graph 2 one could extract that the capital allocation line crosses the optimal portfolio where the expected return is 0.0356 and the standard deviation 0.0190. This expected return and standard deviation of all portfolios on the efficient frontier can be found in Appendix 1. In total there are 15 different portfolios that

have different asset weights in order to achieve a set expected returns. The optimal portfolio with regard to Sharpe ratio consists of 22.27 % Bitcoin, 8.99 % Ethereum and 68.73 % S&P 500. This portfolio would generate on average a monthly expected return of 3.56 % and has, for this dataset, the highest possible Sharpe ratio at 1.8708.

The risk to return relationship can either be lucrative with a higher return or a lower risk. In the case of the optimal portfolio, it seems like the higher expected return compensates for the increased risk of the portfolio. In the minimum variance portfolio on the other hand, it is the consistent low risk that compensate for the loss of expected return. The portfolio that has the lowest variance is the portfolio that is the second portfolio in Appendix 3. It shows a portfolio that consists mostly of S&P 500 (97.46 %) and a smaller post in Bitcoin (2.54 %). The minimum variance implies that this portfolio is the portfolio that has the lowest price fluctuations and is considered the safest investment of the portfolios. Even though the Bitcoin index has a 4 time higher standard deviation compared to S&P 500 index, the portfolio that holds a small post of the Bitcoin index (<2.54 %) will have a lower overall variance then one that holds only the S&P 500 index.

Ethereum has had a higher maximum value compared to the mean value still it will not generate the same diversification advantages that Bitcoin does in a portfolio. If an investor has a high risk tolerance and for some reason would like to invest in a portfolio that has a maximum variance, the investor should invest solely in Ethereum as the portfolio that only holds Ethereum has the highest standard deviation. It is the first portfolio in Appendix 1.

5.4 Value at Risk and Conditional Value at Risk

The value at risk and conditional value at risk calculation is derived from historical data and is used in our optimal portfolio. The results from our value at risk (VaR) and conditional value at risk (CVaR) calculations are made at three different confidence intervals 95%, 99% and 99.85%.

5.4.1 Optimal portfolio

Table 4 below shows the risk measurements VaR and CVaR for the optimal portfolio at different confidence intervals.

Table 4. Value at risk and Conditional value at risk for the optimal portfolio.

	Confidence interval 99,85 %	Confidence interval 99 %	Confidence interval 95 %
Alpha	0,15%	1%	5%
Count	1135	1135	1135
Location	2	11	57
VaR(p)	-11,58%	-4,84%	-2,72%
CVaR(p)	-14,29%	-7,93%	-4,40%

At a confidence interval of 95% there is a 5% risk that the optimal portfolio will lose 2.72% or more of the portfolio's value in one day. During the 1,135 days that the portfolio has been observed, the portfolios should've dropped 2.72 % or more around 57 times, which is close to once every 20th day. To see how much the portfolio will lose on average in a worst case scenario, we evaluate the conditional value to risk value. On the worst 5 % of the days that the portfolio generates loss it will on average lose 4.40 % of its value with a confidence interval of 95%.

At a confidence interval of 99% there is a 1% risk that the portfolio will lose 4.84% or more in one day. Table 4 also show that in the worst 1% of the portfolio performance the average loss will be 7.93%

The highest confidence interval that we could apply this method on was 99.85%. At this confidence interval the VaR is 11.58% and the CVaR is 14.29%.

5.4.2 Minimum variance portfolio

In the minimum variance portfolio we evaluate the same risk measurements and at the same confidence intervals as for the optimal portfolio. We can see the results in table 5 below.

Table 5. Value at risk and Conditional value at risk for the minimum variance portfolio.

	Confidence interval 99,85 %	Confidence interval 99 %	Confidence interval 95 %
Alpha	0,15%	1%	5%
Count	1135	1135	1135
Location	2	11	57
VaR(p)	-9,85%	-3,62%	-1,86%
CVaR(p)	-11,91%	-6,26%	-3,21%

We can see in table 5 that the VaR is 1.86% and the CVaR is 3.21% with an alpha at 5%. Which indicates that there is a 5% risk at a confidence level of 95 % that the minimum variance portfolio will lose 1.86 % or more of the portfolio's value in one day. On the worst 5 % of the portfolio returns the portfolio will on average lose 3.21%

At the 99% confidence interval the VaR is 3.62% and the CVaR 6.26%.

Lastly for the 99.85% confidence interval. The risk of receiving a negative return of -9.85 or more is at 0.15 % and on the worst 0.15 % of the 1,135 days observed the portfolio would generate an average 11.91 % in negative return.

6. Discussion

The agenda for chapter 6 is to discuss the results that have emerged in the thesis. It starts with a critical discussion of the thesis and follows with a discussion of the results. Further the discussion of the results will be compared with the previous research.

6.1 Critical discussion

As mentioned in chapter 4, this thesis is investigating the period between the 4th of August 2017 and the 4th of February 2022, therefore, the result will also be limited to this period. The time span that this thesis is analyzing is during 18 quarters where the overall stock market and crypto market has increased greatly. During the period (spring of 2022) that this thesis was written, all indexes in this thesis had an overall negative return. This negative market trend may be an effect of external events but it can also be a compensation for abnormal returns from previous years. For that reason the return that is shown in the collected data may not be an accurate description of the return that can be found on the market today.

The S&P 500 Index (SPX) is used as an index that tracks the S&P 500, however, this index does not include dividends. This thesis does not aim to see how S&P 500 with dividends would change the results.

The crypto market is still fairly new and the rise and fall of cryptocurrencies can still be seen on a regular basis. The limitation to two cryptocurrencies that have survived the market does not make it a valid representation of the whole crypto market. Therefore this thesis is limited to analysis of Bitcoin and Ethereum as separate entities and not the crypto market as a whole.

6.2 Discussion of results

This thesis finds that the volatility increases when the portfolio holdings of Bitcoin increases above 2.54 % as well as the addition of Ethereum to a portfolio at any stage. Therefore, the optimal amount of diversification that a S&P 500 portfolio could do towards cryptocurrencies, with regard to the lowest possible volatility, is a diversification with precisely 2.54 % Bitcoin. From graph 1, one could extract that the portfolio that consists of solely S&P 500 has the lowest expected return. The low standard deviation that this portfolio holds does not compensate enough for the lack of expected return since the minimum variance portfolio is giving a greater return with a lower volatility. Ghorbel and Jeribi (2019) also found similar diversification advantages with cryptocurrency but they only found it during stable periods, or before the corona crisis. Since our data spans from before the corona crisis and after the crisis it is valid that we have similar findings.

Another finding is that, if the amount of Bitcoin is decreased in the minimum variance portfolio, from a weight of 2.54%, the standard deviation will increase. In this scenario it does not matter that the variance for S&P 500 is considerably lower then Bitcoin when analyzed separately. This is in direct contrast to the finding that Kajatazi & Moro (2019) presented in their study. They found that it is rather an increase in return that compensates for the added volatility of Bitcoin, which is not solely the case in this thesis, with regards to the minimum variance portfolio.

When one analyzes the results from the portfolio optimization, it can be concluded that the optimal portfolio has almost a third of its investment invested towards cryptocurrencies. A rational investor looking for the highest return in relation to the risk taken should therefore have a majority holding in S&P 500 and a third of its position towards Bitcoin and Ethereum combined. The optimal portfolio generates on average 3.56 % in expected monthly return. When that portfolio is compared to the 3 Month Treasury average monthly return of 0.09 %, it can easily be seen that the optimal portfolio generates a comparably sizable average monthly return. However, the fact that the 3 Month Treasury is almost risk free and the optimal portfolio carries risk also has to be taken into account.

The portfolio's expected return increases with the addition of cryptocurrency in a portfolio, which can best be seen in the portfolios that have a larger variance, but the overall volatility

of the portfolio is not increased massively in all cases. In the case of the minimum variance portfolio, it has a holding of 2.54 % Bitcoin and still holds the lowest standard deviation of all portfolios even though it has a holding in cryptocurrency. This shows that the addition of some cryptocurrency does not only increase average return but in some portoflios it can also lower the volatility. Kajatazi & Moro (2019) found that a portfolio's performance is improved when adding Bitcoin to a portfolio but the overall volatility is increased. Which is not found true in this thesis in cases where the portfolio consists of S&P 500 and Bitcoin, but Bitcoin holdings are 2.54 % or lower.

The portfolio with the optimal Sharpe ratio consists of all three indexes and the standard deviation is also higher than the minimum variance portfolio. Even though the standard deviation would drop if the portfolio consisted of more S&P 500, the expected return that cryptocurrencies generate does compensate for the increased volatility of the portfolio. For that reason the return that comes from cryptocurrencies is preferable compared to lowering the risk of the portfolio. It could be an indication of the abnormal returns that could be found on the crypto market for the 18 quarters that the data spans over.

Tavares, et.al (2020) found that there was no performance gain while adding cryptocurrency to stock portfolios and that several portfolios performed worse when cryptocurrencies were added. It differs quite a lot from our results, since the portfolios did not ever perform worse with the addition of cryptocurrency, quite the contrary, it performed better with the addition of cryptocurrencies. Some difference is expected since the time era of the observation differs between the two studies. There is also a difference in the amount of assets that are analyzed, in the case of Tavares study, it only investigates S&P 100 while this thesis investigates S&P 500.

The conditional value at risk shows the possible major downsides that can become the negative return when holding the optimal portfolio. Even though 0.15 % is a relatively small percentage risk, the negative return that happens at the 0.15 percentile is quite massive. The loss of the portfolio would be on average 14.29 % and the return that would be needed in order to get the same original value would then be 16,67 %. And since 14.29% only is an average of the worst losses, the investor needs to consider that there is a possibility that the losses will be even greater. But overall the portfolio has a positive return over time and a long term investment in this portfolio should for that reason, still be profitable.

The loss that happens for 5 % of the occasions for the optimal portfolio is 2.72 % which can be compared to the average monthly return of 3.56 %. The values are quite similar which indicates that almost an entire month's profit can be diminished in a single day. That information should make an investor fairly cautious before investing in the optimal portfolio, at least, short term. A comparison of the value at risk and the conditional value at risk between the optimal portfolio and the minimum variance portfolio shows that both the CVaR and the VaR is higher in the optimal portfolio. This specific finding is not that surprising since the descriptive statistics in Table 2 have previously shown that cryptocurrencies tend to be much more volatile than the S&P 500. It is also a similar finding to what Kajtazi and Moro (2019) found in their study, they found that Cvar increased when crypto was added to an already diversified portfolio.

7. Conclusion

The chapter below ties the thesis together and explains the conclusion that has been found. The thesis ends with examples for further studies within the field.

The thesis aims to analyze how an optimal portfolio is constructed when it had the possibility to use three different indexes; Bitcoin index, Ethereum index and S&P 500 index. The optimal portfolio is the one that held the highest sharpe ratio and consisted of 22.27 % Bitcoin, 8.99 % Ethereum and 68.73 % S&P 500. The optimal portfolio holds almost one third of its holdings towards the crypto market, even though the crypto market is described as a fairly volatile market and that is represented in the volatility of its price. When the cryptocurrencies are compared towards each other, the portfolio opts to hold a higher share of Bitcoin since it has a lower volatility than Ethereum. That should also be the case for any investor that wants the highest possible return for a set level of risk.

Besides, we also study the minimum variance portfolio consisting of 97.46 % S&P 500 and a smaller but significant post of 2.54 % in Bitcoin. The allocation of assets is constructed in this way because of the overall lower volatility that the S&P 500 holds. However, despite the low holding of Bitcoin it played a crucial role in the findings of this thesis. This thesis found that if you decrease the holdings of bitcoin below 2.54% in the minimum variance portfolio the standard deviation will increase. The conclusion from this discovery is that diversification with Bitcoin is desirable to minimize the idiosyncratic risk. This specific finding is new to this thesis, since the previous research found that the diversification with Bitcoin increased the overall volatility of the portfolio. Based on this, we can also interpret that Bitcoin is preferable to Ethereum, despite the fact that Ethereum has had a higher return during this period. The reason for this is that bitcoin has a lower standard deviation over the studied time period and hence has been more stable.

When it comes to the optimal portfolio, it holds a higher risk to lose a bigger percentage of the portfolio then the minimum variance portfolio or an investment in the 3 Month Treasury bill. It is set to lose at least 4.84 % of its total value once every 100th day on average, while the 3 Month Treasury bill has zero days with a negative return. The optimal portfolio is also set to lose at least 11.58 % of its value once over the observed data. However, the optimal portfolio's average monthly return does compensate for these losses. Since the average return

is 3.56 % monthly it should recover fairly soon from its losses. For that reason, and that the optimal portfolio has the highest return per set amount of risk, the rational investor should invest in the optimal portfolio.

For further studies within the field, it could be interesting to follow more cryptocurrencies or analyze the crypto variables in this thesis with the Bloomberg Galaxy Crypto Index. The addition of a similar index as the S&P 500 but for the crypto market would correlate with the portfolios and one could see if the portfolios would have a different weighting if it had an option to diversify further within the crypto market.

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9. Appendix

Appendix 1. Efficient frontier data

St dev	Expected return
0,0616	0,0914
0,0477	0,0800
0,0408	0,0700
0,0374	0,0650
0,0341	0,0600
0,0307	0,0550
0,0275	0,0500
0,0244	0,0450
0,0214	0,0400
0,0190	0,0356
0,0171	0,0320
0,0153	0,0280
0,0139	0,0240
0,0130	0,0200
0,0127	0,0166

Appendix 2. Capital allocation line (CAL)

	Standard deviation	Expected return
Risk free rate	0,0000	0,0002
Optimal portfolio	0,0190	0,0356
Extrapolation	0,0616	0,1154

Appendix 3. Weight and Sharpe ratio of the optimal portfolio and the minimum variance portfolio

Portfolio Weight	Bitcoin weight	Ethereum weight	S&P500 weight	St dev	Expected return	Sharpe ratio
100%	22,27%	8,99%	68,73%	0,0190	0,0356	1,8708
100%	2,54%	0,00%	97,46%	0,0127	0,0166	1,2869

Appendix 4. Covariance matrix

	Bitcoin	Ethereum	S&P 500
Bitcoin	0,0023	0,0021	0,0001
Ethereum	0,0021	0,0038	0,0002
S&P 500	0,0001	0,0002	0,0002