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The Relationship Between Idiosyncratic Volatility and Portfolio Return within Swedish Stock Markets.

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Abstract

Main results suggest there is a statistically and economically significant positive relationship between idiosyncratic volatility and portfolio return within the Swedish stock markets. This relationship is detected despite the low idiosyncratic volatility climate of Sweden. This is surprisingly true in the case of applying the methodology of Ang, Hodrick, Xing, and Zhang (2006), where a negative relationship was expected and not found. This is also true in the case of applying the exponential GARCH methodology of Fu (2009), where a positive relationship was expected and found, consistent with traditional theory. The key difference between the methods—ignoring the time-varying property of idiosyncratic volatility—leads to an overestimation of portfolio return. We demonstrate that the main results are sensitive to weighting-scheme, market specification, and chosen asset pricing model.

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1. Introduction

This paper has the objective of analysing the relationship between idiosyncratic volatility (IV)[1] and portfolio returns in the Swedish stock market. The connection between risk and return has long been a field of research in academic finance literature, especially the risk return trade off. This established a positive relationship between the two — to obtain a larger return a higher risk must be tolerated. When we discuss risk we have to differentiate between systematic and unsystematic (idiosyncratic) risk, the latter being the type of risk that will be the focus of this paper. Consisting of a specific risk that an asset or a group of assets may face, to estimate idiosyncratic risk we use a natural proxy denominated IV. After the estimation we construct portfolios of returns using the IV as a sorting variable, meaning that stock returns are ordered according to their IV values thus providing the possibility to easily verify the nature of the relationship.

This relationship was only analysed for Sweden as part of studies that took a wide look country wise. A problem with these multinational studies is that by spreading the focus so wide they might overlook country specific details. Switzer & Picard (2015) looked at Sweden as part of analysis of the relationship between IV and returns in developed and emerging countries, but the stock sample used is very small which could hinder the depth of the analysis. Vo (2018) analysed the relationship between volatility and returns in Nordic countries, Sweden included, but didn't look at firm level instead used the OMX Stockholm index from the Stockholm Stock Exchange. To fill this gap in literature we shall add data from non-primary stock markets that would be available to the investor. Specifically, data from five Swedish stock exchanges, instead of one. This will provide us the opportunity to have a more in depth look at this relationship in Sweden. This also remedies the issue of too small of a sample, which could be a problem if we only use primary market stocks.

Our timeline of study is from 1983 to 2020, with some differences depending on the chosen method. The data is obtained from the Swedish House of Finance data centers. The reason behind the choice of this specific period is in part by limitation, as it comprises the available factor data. Still the sample size has enough length to provide us with the opportunity of analysing the relationship.

[1] Idiosyncratic volatility will be denominated IV throughout the remainder of the paper.

Our methods of study are a replication of the ones used by Ang, Hodrick, Xing and Zhang (2006), referred to in this paper as "AHXZ"[2], and Fu (2009). The reason for this choice is due to the fact the latter paper was conducted as a response to the former. AHXZ (2006) found a negative relationship between IV and portfolio returns in US stocks using one month lagged IV estimates as a sorting variable for monthly returns. Fu (2009) deemed AHXZ (2006) method as flawed saying that it failed to capture the time-varying property of IV. Using an Exponential Generalised Autoregressive Conditional Heteroskedastic (EGARCH) model to estimate IV, Fu found a positive relationship between IV and returns also in US stocks.

The chosen topic is of primarily academic interest, born out of the current academic discussion of the relationship between IV and returns. Still, this analysis is done from an investor perspective, specifically how an investor takes advantage of the relationship and which method could be chosen. The method of portfolio sorting is chosen with the investor perspective in mind, as it allows us to present easy to interpret results within the context of a feasible investment strategy.

This will lead us into our main research question: Will we obtain in Sweden the same results seen in AHXZ (2006) and Fu (2009) or will we observe a relationship of a different nature?
While the research question comprises the main objective of this study, it is of our interest to see if we have contradictory results from the two methods and if there's anything characteristic of Sweden that could help us explain the obtained results.

[2] The term AHXZ is used because their paper is referred to so often in this thesis and is easier to identify than writing "Ang et al. (2006)", which is an equivalent way of referring to their paper. Using the AHXZ term also continues the praxis set out by Fu (2009) amongst others.

2. Theory

This section is designed to provide theoretical context to our research. This overview has the desired effect to present the concept of risk/return tradeoff, which is crucial for understanding our intended object of study. Followed by the asset pricing models such as the Capital Asset Pricing Model (CAPM) and the Fama and French three factor model (FF3FM) that will enable us to analyse the relationship.

2.1 Total Risk/Return trade-off

Risk return trade-off is part of much of the foundation of the financial theory studied today. Some of the theories that are used in this paper are built upon this trade off. The formalisation of this relationship was stated by Markowitz (1952), where the investor sees the expected return as a desirable concept and risk, measured by the variance of the return, as an undesirable concept. This was further demonstrated theoretically in Merton (1973) and Merton (1987), which presented the relationship between the conditional variance and the conditional expected excess return on the market as a positive one.

Even though in theory the connection between risk and return is a positive one, empirically this has not been easy to demonstrate (Bali and Peng, 2006). Although there seems to be a consensus of a positive relationship in risk-return trade-off within a certain period, there's less consensus about that relationship across time specifically during periods where a security is more risky (Glosten, Jannathan and Runkle, 1993). Different asset pricing models with distinct definitions of volatility and distinct samples seem to provide inconclusive and sometimes even negative results, contrary to established theory.

We can briefly present some of the main contributions done in this field to demonstrate the ambiguity of the achieved results. French, Schwert & Stambaugh (1987) found a positive relationship between expected returns and volatility in the US market. On the other hand, Glosten, Jannathan and Runkle (1993) found a negative relationship between conditional expected returns and conditional variance. More recently, Bali and Peng (2006) used high-frequency data to determine the significance and existence of the connection between risk and return. Using different specifications for the volatility process and for market returns they found a significant and positive relationship between risk and return.

2.2 Systematic (Market) Risk

Total risk can be split into two categories: systematic (market) risk and unsystematic (idiosyncratic) risk. Market risk concerns the risk that an investor may suffer financial losses due to unfavourable price changes that will affect the market. The relationship between market risk and return can be observed in asset pricing models such as the CAPM. In it an investor can use a coefficient denominated beta to draw a comparison between the risk on a specific investment with the market risk (Hull, 2012). Even though we'll use the FF3FM and not the CAPM, the market risk factor will still be present. As we will explore further, the FF3FM is built upon the CAPM by adding two factors that take into account value and size risk.

Market risk includes changes in interest rates, stock prices, commodity prices and exchange rates. A change in those variables could affect a specific market segment, thus its impact is wide. Systematic risk is very difficult to avoid and is impossible to use diversification to mitigate it. But, hedging could still be used to protect against this type of risk.

2.2.1 CAPM

The CAPM is the most well known asset pricing model and the one most finance students are usually introduced to first. According to Fama & French (2004) the attractiveness of the CAPM is due to the fact that it offers significantly intuitive and simplified ways of measuring risk and the relationship between expected return and risk. Because it assumes that idiosyncratic risk is fully diversifiable there's only the need to focus on market risk.

Built upon the work done by Markowitz (1959) that established the modern portfolio theory, this theory assumed investors were risk averse and consequently choose their portfolios according to a "mean-variance strategy", picking the expected return conditional on the variance and the variance conditional on the expected return. One of the innovations that the CAPM brought was turning the previous algebraic statement of the asset weight in the mean variance portfolio into a possible testable condition about the connection between risk and expected return (Fama & French, 2004). Sharpe (1964) and Lintner (1965) expanded the Markowitz model with two assumptions, the first one complete agreement, whereas investors agree on the joint distribution of the asset return from t-1 to t, given the asset clearing prices in t-1. The second assumption is that borrowing and lending is done at a risk free rate, equal for all investors and independent of the amount borrowed or lent (Fama & French, 2004).

This expansion gives us the CAPM formula from Sharpe (1964) and Lintner (1965):

$$E(R_i) = R_f + \beta * (E(R_m) - R_f)$$

Where the $E(R_i)$ is the expected return of asset i and R_f is the risk-free rate. The β is the sensitivity of the return on investment to the market portfolio and $(E(R_m) - R_f)$ is the market risk premium - the extra return demanded for increased risk.

Even though the CAPM has been widely used as a useful theoretical tool that simplifies the relationship between expected returns and risk, empirically it does not have a good track record. The shortcomings of the CAPM have been exposed in Basu (1977) and Bhandari (1988), both found evidence that the variation in the excess returns are unrelated to the market betas. Additionally, Fama & French (1992) synthesised more explicitly how other variables such as size and book to market ratio helped to explain excess returns. The simplicity of the CAPM is at the same time its biggest advantage and disadvantage, allowing for a clear view of expected return and risk but failing to capture the complexities seen in real markets.

2.2.2 FF3FM

As we mentioned in the section before, even though the CAPM is a useful theoretical portrait of the relationship between market risk and return this has not been translated empirically. Fama & French (1992) expanded upon these failures and found that two simply measured characteristics such as size and value provide a strong explanation of the variation of stock returns. Consequently, this led to the addition of two factors to control for size and value: SMB (Small minus Big) and HML (High minus Low), respectively. With the size factor measured by the market equity and the value factor with the book to market ratio. Thus we would have a model with three factors: $(E(R_m) - R_f)$, $E(SMB)$ and $E(HML)$.

This leads to the following formula:

$$E(R_i) = R_f + \beta * (E(R_m) - R_f) + \beta_{SMB} * E(SMB) + \beta_{HML} * E(HML)$$

As stated prior, the FF3FM is built upon the CAPM. All the variables presented are maintained and $E(SMB)$ and $E(HML)$ are added, the respective size and value factors. In addition to β_{SMB} and β_{HML} being the coefficients for the size and value factor.

Since its introduction it has become a widely used model for research in the topic of expected returns (Fama & French, 2004). This has led to some debates about its performance in comparison with the CAPM. Fama & French (1996) found that the FF3FM captures much of the variation of the CAPM. Also, Fama & French (1998) looked at international data and found that the FF3FM was highly superior in explaining the value premium than the CAPM. On the other hand, Black (1993) found issue with the FF3FM saying that the "size effect" captured by Fama and French (1992) is the result of data mining and has no theoretical justification for its inclusion. Black (1993) expands its criticism regarding the value factor, arguing that this may be the product of data mining and that the "value effect" may vanish in the long run.

2.2.3 Other pricing models: Carhart four factor model, Fama and French five factor model and q-factor model

As Fama & French used the CAPM as a basis to expand and construct their model, so did others. Jegadeesh & Titman (1993) had previously demonstrated that portfolio strategies constructed on buying stocks that are performing well and selling underperforming stocks generate positive short-term results. Using these previously seen results, Carhart (1997) added a momentum factor to the FF3FM, to capture the persistence seen in stock returns. The Carhart four-factor model was found to significantly better capture the average pricing errors in comparison with the CAPM and FF3FM (Carhart, 1997).

In addition to the carhart model, other asset pricing models were developed in response to some limitations that the FF3FM was shown to have. The Fama and French five factor model added probability and investment factors. While the q-factor model consists in a practical application of the investment CAPM by shifting the focus of the perspective of the pricing of risky assets from the buyers to the sellers (Zhang, 2019). These models were shown to perform better in capturing systematic risk than the FF3FM and CAPM.

3. Literature Review

3.1 Unsystematic (Idiosyncratic) Risk

With the knowledge that total risk is systematic risk plus unsystematic risk, the focus is now shifted onto unsystematic risk, also known as idiosyncratic risk. This risk is described as idiosyncratic because it is firm-specific and associated with a particular individual investment, as opposed to systematic risk which is market-specific and associated with broad factors inherent to the market. Idiosyncratic risk is diversifiable, whilst systematic risk is not.

In theory idiosyncratic risk can be mitigated via diversification, but in practice this is often not the case, often leading to investors to be under-diversified. Goetzmann & Kumar (2008) support this claim with their finding that the vast majority of 62,000 US investors are typically under-diversified. Any idiosyncratic risk left after under-diversification should then be compensated with return, suggesting the existence of a positive relationship. It is this positive relationship that the majority of previous researchers find, most notably Levy (1978), Merton (1987), and Malkiel & Xu (2002).

Idiosyncratic risk is commonly referred to by researchers as its natural proxy—idiosyncratic volatility, or simply IV for short. Researchers may agree on terminology, however there is disagreement on how to define the estimation of expected IV and it is this lack of consensus on the estimation which makes this topic interesting. The choice of estimation by the researcher can greatly influence the outcome of the predicted relationship between IV and stock returns. The most influential recent authors are presented below, along with their IV definitions and results.

3.1.1 Ang, Hodrick, Xing, & Zhang (2006): The Cross-Section of Volatility and Expected Returns

Ang, Hodrick, Xing & Zhang (2006)—referred to in this thesis as "AHXZ"—is a paper with upgraded methodology that seriously challenged the positive relationship between IV and returns found by the majority of researchers. They claim that previous researchers had not examined IV on the firm level, or had not sorted stocks into portfolios ranked by IV, hence upon performing a method which took these into account a strong significantly negative relationship is found between IV and US stock returns.

The method uses stock data from the NYSE, AMEX, and Nasdaq between July 1963 to December 2000 to estimate each firms' monthly IV values. This is done by performing OLS regression of the firms' daily excess returns on the FF3FM, then monthly IV is defined as the standard deviation of the daily regression residuals within each month. Monthly IV values are then used to sort firms into value-weighted quintile portfolios which are rebalanced every month throughout the sample period. This is designed to represent a strategy where an investor holds a long position in the portfolio with the highest IV values and holds a short position in the portfolio with the lowest IV values. From this strategy a relationship between IV and returns could be determined.

Due to the surprising result, extra effort is made by the authors to support their discovery. They controlled for value, size, liquidity, volume, dispersion of analysts' forecasts, and momentum effects. They found their results were robust to different formation periods of IV, different holding periods, and a variety of market conditions. A complete explanation to this negative relationship could not be offered, and the authors settled with describing the finding as a "substantive puzzle", leaving large motivation for further investigation.

3.1.2 Fu (2009): Idiosyncratic Risk and the Cross-Section of Expected Stock Returns

Further investigation is taken up by Fu (2009), who offered a substantial critique of the method used by AHXZ. Fu described AHXZ's method of estimating IV as using a lagged value, which he reasons as inappropriate because IV is not highly persistent. With high IV persistence Fu means that any effect from the previous month will carry on to the following month, which he claims is a false presumption. It is the presumption of high persistence that is required to justify the use of a lagged value, therefore the lagged value is not suitable if the presumption is broken. He explains that IV cannot be highly persistent because it contains firm specific information which is time-varying due to circumstances such as for example: periodical and infrequent disclosure of earnings information, seasonal variations in supply and demand, or actions made by market competitors. Fu continues by stating that the method AHXZ used to estimate IV incorrectly assumes that a random walk process can approximate the time-varying property of IV, and therefore this mistake could introduce severe measurement errors. He statistically supports his reasoning in a Dickey-Fuller unit-root test by rejecting the null hypothesis of a random walk in 90% of the firms in his sample.

Fu presents an IV estimation method that takes the time-varying property of IV into account which involves an exponential generalised autoregressive conditional heteroskedasticity (EGARCH) model. This model was first introduced by Nelson (1991) and is an expansion of autoregressive conditional heteroskedasticity (ARCH) & generalised autoregressive conditional heteroskedasticity (GARCH) models. These models are best known for replacing the assumption of constant error term variance in OLS modelling (homoskedasticity) with conditional error term variance. Specifically these models perform well in the presence of heteroskedasticity, which commonly occurs in financial time-series data. Fu motivates the choice of EGARCH over other models with the work from Pagan & Schwert (1990) who compared several models for monthly US stock return volatility and found that the EGARCH model was superior. In addition, EGARCH modelling has the benefit of capturing the asymmetric property of volatility, which happens when negative shocks impact volatility more than positive shocks.

Fu uses Fama-Macbeth regression and portfolio sorting methods simultaneously. The portfolio-based approach he uses is similar to that of AHXZ and produces easy to interpret results from a feasible investment strategy. Fama-Macbeth regression has the advantage of being able to retain power when working with too few stocks—a scenario where the portfolio sorting method leads to noise, even so ensuring a minimum number of stocks can remedy this problem.

3.1.3 Weighting Scheme

Previous researchers face the dilemma of reporting equally-weighted (EW) or value-weighted (VW) returns. Fu (2009) reports both schemes and AHXZ (2006) reports exclusively VW, without much motivation for their decisions. Hou, Xue, & Zhang (2020) discuss their motivation for including both weighting schemes, yet they prefer VW and stress the extreme nature of microcaps and their influence on the results in an EW scheme. Microcaps are associated with a larger risk/return compared to larger caps, and using EW means a higher representation of microcaps within a portfolio compared to a VW scheme. Using VW shifts the portfolio representation more towards the large cap stocks, by doing so the risk/return of the portfolio is reduced. From a 1967-2016 US sample, Hou, Xue, & Zhang (2020) state microcaps which account for 60.7% of the total number of firms represent only 3.21% of the total market cap. Despite the preference for VW, when Hou, Xue, & Zhang (2020) replicate the results from AHXZ (2006) they report both weighting schemes even if the original authors only reported VW. Bali and Cakici (2008) also highlight why reporting both is good practice, because they find a significantly negative relation between IV and stock returns in the VW case but a insignificantly positive relation in the EW case, showing that returns can be sensitive to weighting scheme.

3.1.4 Sweden

No Swedish specific previous studies are found for the relationship between IV and portfolio returns, however a couple of studies exist which include Sweden as part of their multinational analysis. Vo (2018) is one of these studies and includes Sweden as part of his Nordic country analysis, but one major drawback of the study design is that it looks only at the returns from the OMX Stockholm index instead of using returns from stocks on the firm level. Because portfolio sorting cannot be applied in the case of an index return, Vo's approach is too different from the methods used by AHXZ (2006) and Fu (2009) who investigate on the firm level and use portfolio sorting. Another problem with using index returns is that if an investor suspects the existence of an IV and return relationship, in reality they would likely prefer to maximise their profits by investing in IV sorted stocks rather than investing in an index.

Switzer & Picard (2015) is another paper which includes Sweden in their large multinational analysis. They do look at returns on the firm level rather than an index, however their focus is spread over a wide range of developed and emerging countries. By spreading their focus too wide the sample sizes for certain countries are insufficient, Sweden in particular. Here we note that their sample size for Sweden is smaller than if they had included all available stocks on the Stockholm Stock Exchange, Sweden's primary stock market. Therefore their investable universe does not only overlook all of Sweden's nonprimary stock markets, but is also incomplete when using the specification of Sweden's primary stock market.

3.1.5 Summary

The methods proposed by AHXZ (2006) and Fu (2009) are so popular that most other papers researching this topic either replicate their methods, or allow for minor adjustments. In this paper these two key methods are replicated side by side, the purpose is by treating them equally and comparing them further discussion may be found. Much of the previous literature describe a preference for using value-weights but also include equal weights for completeness, therefore the results in this paper will reflect this approach. These methods will be applied for all available Swedish stocks, and not just those from the primary market (Stockholm Stock Exchange) as this was identified as a large gap in the literature.

4. Data

In this section we present the data that we will use in the paper. Initially we shall describe our stock universe, then we will move on to the data used in building the FF3FM.

4.1 Data Description

Table 1: Data description

Monthly data	*Timeline*
Stock Returns	*February 1983 - January 2020*
Market capitalization	

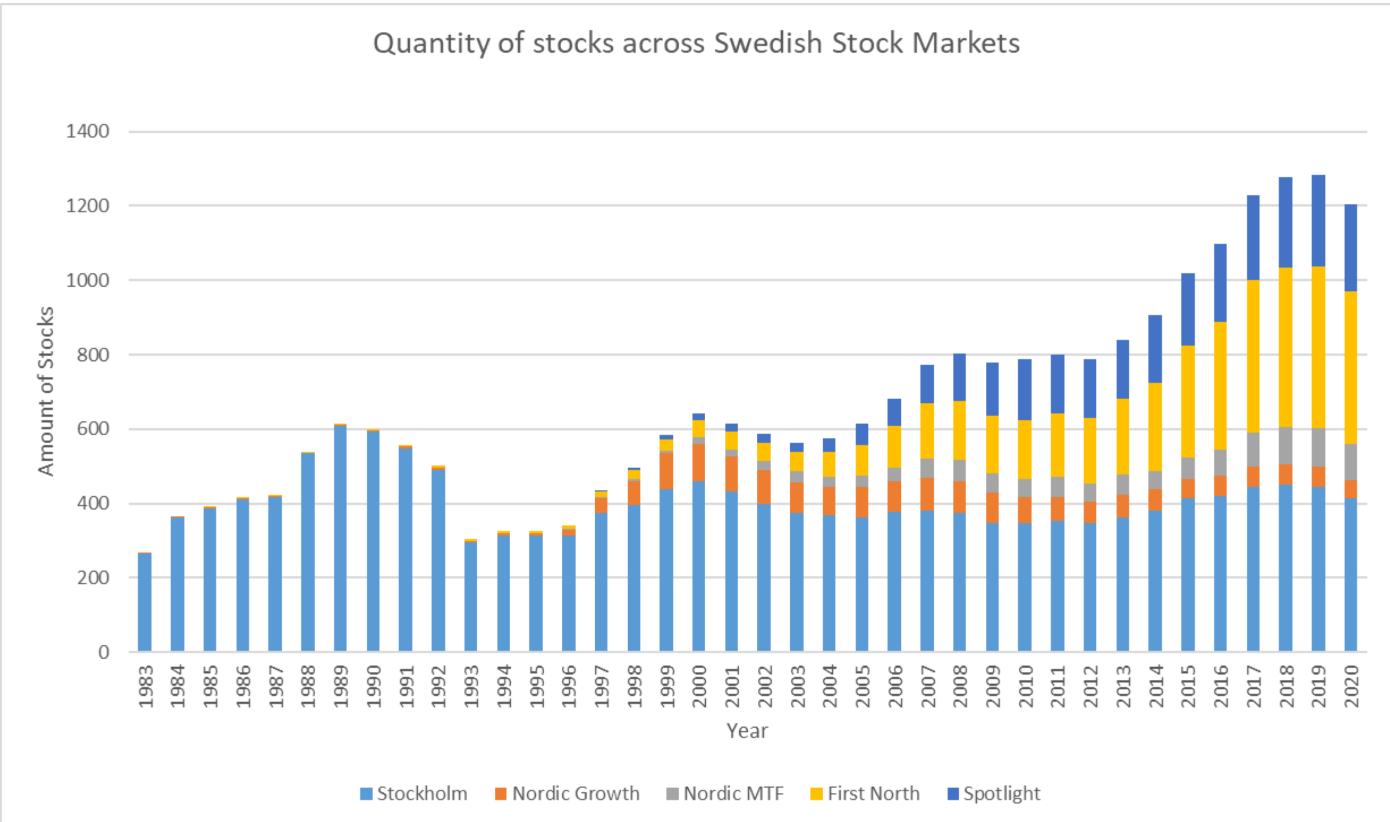
Daily data	*Timeline*
Stock Returns	*4rd of January 1983 - 30th December 2019*
Risk-free rate	
Small-minus-Big factor (SMB)	
High-minus-Low factor (HML)	
Momentum factor (MOM)	
Market return in excess of risk-free rate	

The tables above specifies the daily and monthly data needed to proceed with our investigation of the relationship between the IV and stock returns**.** The reason for this differentiation resides in the fact that daily and monthly data are used at different stages for our methods. All of the data is obtained from the Swedish House of Finance Datacenter datasets, specifically the FinBas dataset and the Fama French Factors (FFF) dataset. The daily and monthly returns and the market capitalization are obtained from the FinBas datacenter while the risk-free rate, market return in excess of the risk-free rate, and the SMB and HML factors are obtained from the FFF dataset. Our chosen period of studying is from 1983 to 2020. The reason is partly due to limitation, as it comprises all the available data for the FFF dataset. But upon reflecting on this limitation we decided it provided us with an ample time sample to study.

The data utilised is from all available stock exchanges in the FinBas database, which coincides with all the active stock exchanges in Sweden:

- **Stockholm Stock Exchange -** Also known as Nordic Stockholm AB due to being part of Nasdaq Nordic, it is the largest stock exchange in Scandinavia. It's with the Nordic Growth Market the only regulated markets in Sweden subject to supervision by the Swedish Financial Supervisory Authority (SFSA).
- **Nasdaq First North Growth Market -** It's an alternative marketplace operated by Nasdaq Stockholm AB with less stringent requirements to companies.
- **Spotlight Stock Market -** Another alternative marketplace for companies who lack the requirements to enter the main stock exchanges.
- **Nordic Growth Market** - One of the two regulated stock exchanges in Sweden supervisioned by the (SFSA). It is part of the Stuttgart Börse Group.
- **Nordic MTF Stockholm -** Part of Nordic Growth Market. It is a multilateral trading facility for companies that follows less stringent rules.

Graph 1: Annual quantity of stocks across Swedish stock markets



We can see that Stockholm Stock Exchange was practically the only financial stock exchange in Sweden from 1983 to 1995. As it saw a significant decrease in listed companies in 1993, its size remained constant throughout the remaining years of the sample. On the other hand there was an increase in size of the remaining exchange platforms. Specifically Spotlight and Firth North, as the latter in 2020 seems to have the same amount of stocks as the Stockholm Stock Exchange.

Class A and B stocks for the same company are included. On a more general level, the difference between these two classes usually is due to the different voting rights that each grants and their priority in case of bankruptcy. The level of these differences is defined by each company and will have different pricing.

As shown in graph 1 there was a big decrease in listed companies in 1993. This is due to the fact that up until the end of 1992, there was a specific type of stock that was limited to Swedish nationals - meaning that only investors who possessed Swedish nationality could invest in those stocks. With the reforms enacted to liberalise their financial markets, this restriction was lifted. These restricted stocks are included to increase the sample size, however, this implies that our results represent the returns for a Swedish-national investor rather than a foreign-international investor.

To construct the Fama and French three-factor model we need the daily data for the risk-free rate and the market return in excess of the risk-free rate. These are obtained directly from the Swedish House of Finance Fama and French datasets. The risk-free rate is the return obtained from an investment with zero risk. Because no such investments exist in practice, usually government treasury bills are used as a proxy, the more famous ones being US T-bills. In our case, we will use the risk-free rate of the one-month Sweden treasury bill. The market return in excess of the risk-free rate consists of the difference between the expected market return and the risk-free rate. The expected market return will be measured using the SIXRX index, which includes all the shares in the Stockholm Stock Exchange and the dividends. The market return in excess of the risk-free rate will represent the market risk premium, the relationship between extra return and the risk that will be tolerated for it.

The main limitation we found with the data was missing values, this was apparent for all of the data we downloaded and was to be expected since there are thousands of stocks spread over 38 years. Concerns for these missing values impacting the results are limited since a missing stock observation should not greatly influence the average of a portfolio of a hundred stocks, and a missing month observation should not greatly influence a 38 year monthly average. However we do notice that market capitalization values are more often missing for small cap stocks than for large cap stocks and would lead to more small cap stocks being omitted from the analysis compared to large cap stocks. This risk of bias to the results is worth keeping in mind, yet it is also worth remembering that small cap stocks outnumber large cap stocks.

4.2 Fama and French factors

The Fama and French factors are obtained from the Swedish House of Finance dataset. These factors have already been calculated and their timeline range is from January 1983 to December 2019. Aytug, Fu and Sodini (2020), who created the factors for the Swedish market, explain their methodology behind the construction of the factors, being almost similar as Fama and French (1993) with some exceptions specifically the size factor breakpoint. These factors are needed to construct the FF3FM and obtain the IV estimates.

Building upon the CAPM, the FF3FM adds two factors that take into account size and value risk, SMB and HML respectively. SMB measures the "size effect" whereas, in theory, smaller firms with a smaller market capitalization will outperform large firms. It represents the return on a portfolio of small stocks minus the return on a portfolio of larger stocks. The HML factor will represent the value premium and it will be the return on a portfolio of stocks with a high book to market ratio minus the return on a portfolio of stocks with a low book to market ratio.

5. Methodology

In this thesis, we investigate what type of relationship there is between IV and stock returns in the Swedish stock market. This particular chapter presents the models used to obtain the IV and subsequent methods utilised to scrutinise the relationship. First, AHXZ's (2006) method of estimating IV is used to obtain the IV from the Swedish stock market. Second, Fu's (2009) EGARCH method of estimating IV is an alternative way of obtaining the IV and takes into account its time-varying properties. Followed by the description of the method of portfolio sorting which we will use to sort the IV estimates obtained from the previous models. Finally, we'll go into the chosen weighting scheme for our portfolios.

5.1 AHXZ's (2006) Method of Estimating IV

To obtain the IV values we will use the Fama French three factor model as presented in AHXZ (2006). Building upon the CAPM, the Fama French three factor model adds two factors that take into account the size and value risk, SMB and HML respectively. For this, we are going to use all the stocks listed in the Swedish stock exchanges in all of the sample period.

5.1.1 Obtaining the factors

While the factors are obtained directly from the Swedish House of Finance database, it seems appropriate to get a better understanding of the methodology that goes into obtaining them. Aytug, Fu and Sodini (2020) present the methodology that goes into obtaining the SMB and HML factors. In that method, the factors are determined by first establishing six portfolios with the use of breakpoints: for size, the 80th percentile of market capitalization, meaning that stock returns above the breakpoint will be marked as Big, while companies below will be marked as Small. For value, the previous breakpoints of size are used and sorted by their book-to-market ratio, with 30th and 70th percentile being their breakpoints. They'll be named Growth if below the 30th percentile, Neutral if between 30th and 70th percentile, and Value if above 70th percentile. Finally, their denomination will be:

SG: Small-Growth	SN: Small-Neutral	SV: Small-Value
BG: Big-Growth	BN: Big-Neutral	BV: Big-Value

Using these portfolios the size factor is obtained, SMB. It is equal to the difference between the average returns on three small portfolios and the average return on the three big portfolios:

$$SMB = (SG + SN + SV)/3 - (BG + BN + BV)/3$$

The value factor, HML, is equal to the difference between the average return on the two value portfolios with the two growth portfolios:

$$HML = (SV + BV)/2 - (SG + BG)/2$$

5.1.2 Obtaining the IV

The IV consists of the part of the risk that is not explained by the market, also known as the firm-specific risk. The estimate for the IV is obtained from the residuals of an OLS regression using the Fama and French three-factor model. This model is an expansion of the CAPM by adding size and value factors.

The Fama and French three factor model will be equal to:

$$R_{i,t} - r_{f,t} = \alpha_{i,t} + \beta_{i,t} (R_{M,t} - r_{f,t}) + s_{i,t} SMB_t + h_{i,t} HML_t + \varepsilon_{i,t} \quad (1)$$

Where i is a firm's stock, t is the time period of one trading day. $R_{i,t} - r_{f,t}$ will represent the daily excess returns, with $R_{i,t}$ being the daily return of a stock and $r_{f,t}$ the risk free rate. $(R_{M,t} - r_{f,t})$ is the excess return on the market portfolio, with $R_{M,t}$ being the market portfolio return. $s_{i,t}$ is the factor loading for the SMB and $h_{i,t}$ is the factor loading for the HML factor. $\varepsilon_{i,t}$ is the error term of the regression.

To obtain the IV of stock i, a regression for each month is constructed with the FF3FM formula shown in (1). The IV of a stock i is obtained as the standard deviation of the regression residuals. To guarantee viability of the results a condition of a minimum of 15 days is used in the construction of the OLS regression. The timeline of the daily stock returns used for the regression are from 3rd of January 1983 to 29th of December 2019.

These IV estimates are then used as a sorting variable for the following month´s returns. Consequently, the monthly returns are from February 1983 to January 2020. Thus, one-month lagged monthly IV estimates are used to perform our portfolio construction strategy.

5.2 Fu's (2009) EGARCH Method of Estimating IV

We now introduce an alternative method for estimating IV. Fu (2009) improves upon AHXZ's (2006) method by using an exponential generalised autoregressive conditional heteroskedasticity (EGARCH) model, which was first introduced by Nelson (1991). The EGARCH model expands upon autoregressive conditional heteroskedasticity (ARCH) and generalised autoregressive conditional heteroskedasticity (GARCH) models, which are all designed to perform well in the presence of heteroskedasticity. Heteroskedasticity commonly occurs in financial time series data and is alleviated by replacing the OLS assumption of a constant error term variance with a conditional error term variance.

As outlined in the literature review, Fu (2009) argued that use of EGARCH is the superior method of estimation of IV, but for comparison and discussion reasons we perform AHXZ's method too which gives us two sets of results. The key distinction between AHXZ's method which uses OLS and Fu's method which uses EGARCH is that the former does not capture the time-varying property of IV, whilst the latter does. More practically, AHXZ's method uses one month *lagged* IV estimates whilst Fu's method uses *expected* IV estimates. When using a lagged IV estimate a high level of persistence for IV is also presumed, however according to Fu this presumption lacks reasoning and is neglecting the time-varying property of IV. In other words, instead of simply taking the standard deviation of the previous month's set of daily residuals as in the *lagged* estimate case, this method now applies the EGARCH model to a set of preceding monthly residuals to obtain an *expected* IV estimate.

The functional forms for EGARCH(p,q) to estimate IV as described by Fu (2009) are as follows:

$$R_{i,t} - r_{f,t} = \alpha_{i,t} + \beta_{i,t}\ (R_{M,t} - r_{f,t}) + s_{i,t} SMB_t + h_{i,t} HML_t + \varepsilon_{i,t} \quad (2)$$

$$\varepsilon_{i,t} \sim N(0, \sigma_{i,t}^2)$$

$$\ln \sigma_{i,t}^2 = a_i + \sum_{l=1}^{p} b_{i,l} \ln \sigma_{i,t-l}^2 + \sum_{k=1}^{q} c_{i,k} \left\{ \theta\left(\frac{\varepsilon_{i,t-k}}{\sigma_{i,t-k}}\right) + \gamma\left[\left|\frac{\varepsilon_{i,t-k}}{\sigma_{i,t-k}}\right| - (2/\pi)^{1/2}\right]\right\}$$

(3)

Where i is a firm's stock, t is the time period of one month. Excess returns are given by the specification (2) where the conditional distribution of residual $\varepsilon_{i,t}$ is assumed to be normal with mean of zero and variance of $\sigma_{i,t}^2$. The IV estimate is the conditional variance $\sigma_{i,t}^2$ given by specification (3). The conditional variance is a function of past *p*-period of residual variance and *q*-period of return shocks, where $1 \le p \le 3$ & $1 \le q \le 3$. This implies that nine different EGARCH models are available for our utility, as shown in table 2:

Table 2: Nine EGARCH models used

	p=1	p=2	p=3
q=1	EGARCH(1,1)	EGARCH(2,1)	EGARCH(3,1)
q=2	EGARCH(1,2)	EGARCH(2,2)	EGARCH(3,2)
q=3	EGARCH(1,3)	EGARCH(2,3)	EGARCH(3,3)

From the nine models, we select only one IV estimate for each individual stock and each time period given by the model with the lowest Akaike Information Criterion (AIC). AIC is one of many ways of determining which model from a set fits best. Whilst the selection can be made by maximum log likelihood or the Schwartz information criterion, Fu (2009) claims that his final results are not sensitive to these aforementioned selection criteria. AIC can be calculated with the following formula:

$$AIC = 2k - 2ln(L) \qquad (4)$$

where *k* represents the number of model parameters used and L represents the log-likelihood, which is a measure of model fit.

Following Fu (2009), we have a requirement that there must be at least 30 months of observations. By doing this our investable universe as described in the data section decreases from 3114 to 1774 stocks.

5.3 Portfolio Sorting

In order to test for a relationship between IV and stock returns we create quantile portfolios of stocks ranked by their IV estimates. The difference in holding-period returns between these extreme portfolios then informs the sign and magnitude of the relationship with IV. Portfolio sorting is performed for both AHXZ's (2006) method and Fu's (2009) method of estimating IV, giving two sets of results. Although an alternative to portfolio sorting―Fama-Macbeth regressions―has its advantages, portfolio sorting is chosen due to easier to interpret results which are more interesting since they are presented within the context of a feasible investment strategy.

Both quintile and decile portfolios have been used in existing literature, but here we follow AHXZ's (2006) method of quintile portfolios. Stocks are assigned evenly to portfolios 1,2,3,4, or 5 based on their IV estimate, implying 20 percent of stocks within each portfolio. The stocks with the highest IV estimates are grouped in portfolio 5 and the stocks with the lowest IV estimates are grouped in portfolio 1. The extreme portfolios are subtracted from one another to obtain the portfolio return difference, referred to as the "5-1" portfolio. This is designed to represent a strategy of a long buy position of stocks in portfolio 5 and a short sell position of stocks in portfolio 1, also known as a zero investment strategy.

The time duration of the holding period for the "5-1" portfolio strategy is from February 1983 to January 2020, in the case of AHXZ's method. In the case of Fu's method the holding period is from July 1985 to January 2020, this is due to the requirement of minimum 30 months of observations. The strategy will remain constant throughout this time span, but the stocks within the portfolios will routinely update due to rebalancing. Portfolio allocations will be held for one month at a time and rebalanced at the end of each month, this is because IV is estimated on a monthly basis.

5.4 Weighting Scheme

Finally once sorted portfolios are obtained, a weighting scheme must be chosen. We report value-weighted (VW) portfolio return in the main results and equal-weighted (EW) portfolio return as a robustness check, we prefer VW since it alleviates the extreme effect microcaps can have on the results. Further explanation of this phenomenon is provided in the discussion found in the literature review. VW returns are applied in following manner:

$$R_{jt} = \Sigma_{i \in D_{jt}} \frac{ME_{it}}{\Sigma_{i \in D_{jt}} ME_{it}} r_{it} \tag{5}$$

'ME_{it}' is the market equity of stock 'i' in month 't'. 'D_{jt}' denotes the set of stocks that belong to quintile 'j'. 'R_{jt}' is the return on the quintile portfolio, whilst 'r_{it}' is the return on stock 'i' in month 't'.

6. Results and Discussion

Now the main results for the two methods of IV estimation are presented. Apart from the difference in IV estimation method (as described in the method section), the results are presented almost identically. In addition to IV estimates and value-weighted returns, we present the FF3FM alpha value(excess return) and its corresponding robust t-statistic. FF3FM-alpha(intercept) values are obtained via time series regression of the value weighted (VW) portfolio returns on the Fama-French three factors. The benefit is that alpha values are risk-adjusted and take into account exposure to risk factors, whilst the simple returns are not risk-adjusted and carry a more naive implementation of the “5-1” trading strategy.

6.1 AHXZ’s (2006) Method of Estimating IV

Table 3: Results table. Portfolios are sorted based on one month lagged estimated IV of individual stocks, ranging from portfolio 1 with lowest lagged IV to 5 with highest lagged IV. “5-1” is the difference between the extreme portfolios and represents a zero investment strategy involving a long position in portfolio 5 and a short position in portfolio 1. The “VW-Return” row reports average monthly value-weighted portfolio returns in percent x.xx%. The “IV Estimate” row shows each portfolio’s average monthly lagged IV in percent x.xx%. The last row reports the alphas (intercepts) from the time-series regressions of the value-weighted portfolio excess returns on the Fama-French three factors in percent x.xx%. The reported t-statistics in square brackets are calculated using robust Newey-West HAC standard errors. The sample period is from February 1983 to January 2020. Asterix indicates level of significance: * = 90% , ** = 95%, *** = 99%.

Portfolio	1 (low)	2	3	4	5 (high)	5-1
VW-Return	1.51	2.02	1.99	2.45	3.28	1.77
IV Estimate	0.92	1.39	1.86	2.58	5.22	4.30
FF3FM-alpha [t-stat]	0.76*** [7.1538]	1.12*** [8.7054]	1.01*** [6.0960]	1.30*** [5.2475]	2.03*** [4.4755]	1.27*** [2.7132]

The IV estimate shows a pattern of constant growth until the last two portfolios, in those we see steeper increases notably for the “high“ portfolio where the return value more than doubles from the previous portfolio. The return portfolios follow in some way the same pattern as the IV estimate values, in exception of portfolio 3 which sees a small decrease. As in the IV values, the last two portfolios register the highest increases and consequently have the highest return values. Overall we see how the returns and IV have a positive parallel path. The 5-1 portfolio strategy values prove this, with 1.77% for the average monthly value weighted returns and 4.30% for the average lagged monthly IV estimate values. The FF3FM-alpha values for the "low" and "high" portfolios have highly statistically significant values of 0.76% and 2.03%, respectively, implying a positive relationship. This is further confirmed by our 5-1 strategy registering a statistically significant value of 1.27%. This contradicts the efficient market hypothesis that states that the share prices reflect all information at all times, thus positive long run alphas are impossible to obtain.

This is an unexpected result if we compare it to other papers who have also analysed this relationship. In terms of magnitude our obtained result is similar as AHXZ (2006) produced a negative result for the 5-1 portfolio of -1.06%. Looking into the results difference that causes the disparity in results we can see how in AHXZ (2006), portfolios 4 and 5 have the lowest returns. In the completely opposite direction of our produced results. It may be the case that stocks with higher IV in the US sample seem to not have the relationship with higher returns as in Sweden. Switzer & Picard (2015) used AHXZ‘s method and applied it to different developed and emerging markets, among them Sweden. They also found a negative relationship between IV and portfolio returns, with the 5-1 portfolio strategy recording a negative result of -0.46%. All of the developed markets that Switzer & Picard (2015) analysed registered a negative relationship with the sole exception of a few countries, since we obtain a positive relationship for Sweden we can describe Sweden as an outlier.

The sample used in Switzer & Picard (2015) is much smaller than the one used in this paper and because Sweden is part of the multi-national study we do not have a thorough discussion of their methodology process. A wide multi-national study is too broad and leads to small samples for each country, this leads to the omission of stocks which would otherwise be available to the investors. A study that overlooks the presence of these will not be able to capture the true essence of this relationship for the country of interest. From what we have obtained it seems that this indeed causes the results to differ and the inclusion of non-primary markets has a large impact on the nature of the obtained relationship.

6.2 Fu's (2009) EGARCH Method of Estimating IV

Table 4: Results table. Portfolios are sorted based on next month's expected conditional IV of individual stocks, ranging from portfolio 1 with lowest expected IV to 5 with highest expected IV. "5-1" is the difference between the extreme portfolios and represents a zero investment strategy involving a long position in portfolio 5 and a short position in portfolio 1. The "VW-Return" row reports average monthly value-weighted portfolio returns in percent x.xx%. The "IV Estimate" row shows each portfolio's average monthly expected IV in percent x.xx%.

The last row reports the alphas(intercepts) from the time-series regressions of the value-weighted portfolio excess returns on the Fama-French three factors in percent x.xx%. The reported t-statistics in square brackets are calculated using robust Newey-West HAC standard errors. The sample period is from July 1985 to January 2020. Asterix indicates level of significance: * = 90% , ** = 95%, *** = 99%.

Portfolio	1 (low)	2	3	4	5 (high)	5-1
VW-Return	1.64	1.72	1.79	2.09	2.60	0.96
IV Estimate	0.13	0.33	0.62	1.28	8.55	8.42
FF3FM-alpha [t-stat]	0.70*** [6.0975]	0.84*** [6.3304]	0.93*** [5.4466]	1.13*** [5.3485]	1.49*** [4.9483]	0.79** [2.4743]

From the table a positive relationship between IV and returns can be observed, the IV for portfolio 1 starts low at 0.13%, then approximately doubles each time as the portfolios increase up to portfolio 4. At portfolio 5, IV accelerates with a value of 8.55% which is over six times the value of portfolio 4. For portfolio 1 there is a 1.64% average monthly VW-portfolio return, the VW-returns also routinely increase as the portfolios and IV increase. Similar to the IV, the largest gap difference is from 2.09% in portfolio 4 to 2.6% in portfolio 5. This pattern clearly suggests there is a positive relationship between expected IV and portfolio returns. This relationship is epitomised when looking at the results from the “5-1” strategy, where the 0.96% average monthly VW-portfolio return is associated with the 8.42% average monthly difference of expected IV. The magnitude of this return can be large enough to be considered as an arbitrage opportunity and is therefore economically significant for an investor, however this return from Swedish stock markets is still 0.79% less than the return Fu (2009) found for the US stock markets.

When taking a closer look at the alpha values which can be interpreted as risk-adjusted returns, further confirmation of the positive relationship is found. The alpha is 0.70% for portfolio 1 and 1.49% for portfolio 5, both highly statistically significant. This means that for the “5-1” strategy there is a statistically significant 0.79% difference in monthly return between portfolio 5 and portfolio 1 alphas, where the robust t-statistic is 2.4743. The presence of this alpha is in conflict with the efficient market hypothesis, which states that over the long run alpha cannot be reasonably and consistently produced because market prices should incorporate all available information at all times.

The positive portfolio return makes sense because according to traditional theory an investor should be rewarded with return for holding firm-specific risk. Also, we note that both the IV estimates and returns within Swedish stock markets are smaller in magnitude than the US stock market results from Fu (2009). This means that less firm-specific risk is observed within Swedish stock markets compared to the US, a situation which is associated with less return. Sweden is not just associated with low IV in comparison to the US, but is also known to have relatively low IV compared to other European countries. This perspective is shared with Swanson & Lindberg (2018)[3], who also suggest Sweden's low IV could possibly be due to greater equity market development and less country risk. This implies that the statistically and economically significant positive relationship between IV and return found by this paper is found within a low IV climate, suggesting further analysis with Fu's (2009) method on countries with a larger IV climate may lead to even starker results.

6.3 Method Comparison

We argued that Fu's EGARCH method is a superior method of estimating IV, which is a method that expects to find a positive relationship between IV and portfolio. As we have just shown, our results match this hypothesised conclusion. AHXZ's theoretically inferior method of estimating IV also finds a positive relationship between IV and portfolio returns, surprisingly this result does not match with the hypothesised result where a negative relationship is expected. This means that: (i) the claim of a positive relationship between IV and portfolio returns within Swedish stock markets is supported by two prominent methods, and (ii) the results from the inferior AHXZ method does not match its hypothesised results, whilst the results from the superior Fu method does match its hypothesised results. Since the results from AHXZ (2006) could not be replicated, this raises doubt towards their method specification, whilst the results from Fu (2009) could be successfully replicated, thereby confirming their method specification and making it more favourable in this comparison.

[3] This paper is a master thesis and not introduced in the literature review, however the paper is very relevant and niche in reasoning for Sweden's low IV in particular.

7. Robustness Tests

Now three robustness checks are used to test the sensitivity of the main results for both IV estimation methods. Firstly, the portfolio weighting scheme is changed to equal weights. Secondly, the investable universe is limited only to stocks listed on the Stockholm Stock Exchange. Thirdly, a momentum factor is included so the asset pricing model applied represents the Carhart model instead of the FF3FM.

7.1 Controlling for: Weighting Scheme

Our main results report value-weighted portfolio returns, which shifts the portfolio representation more towards the large cap stocks, in theory by doing so the risk/return of the portfolio should be reduced. Now we test how the main results respond when using equal-weighted portfolio returns, which leads to a higher representation of microcaps within a portfolio compared to a value-weighted scheme. In theory microcaps are associated with a larger risk/return compared to large caps, and therefore we expect an increase in portfolio returns. We don't expect changes in IV estimates because the list of stocks within each portfolio remains identical when changing the portfolio weighting scheme.

7.1.1 AHXZ results

Table 5: Equally-weighted portfolio returns sorted by lagged IV. Otherwise, description from table 3 applies.

Portfolio	1 (low)	2	3	4	5 (high)	5-1
EW-Return	1.69	1.88	1.59	1.31	0.61	-1.08
IV Estimate	0.92	1.39	1.86	2.58	5.22	4.30
FF3FM-alpha [t-stat]	0.92*** [7.3211]	1.03*** [9.4595]	0.67*** [5.7859]	0.31* [1.6565]	-0.50* [-1.7050]	-1.45*** [-3.9303]

The IV estimate values remain the same as in the original results. With equal weighted returns, portfolios 1 and 2 increase slightly in comparison to the value weighted returns. Portfolio 3 has a small dropoff from 1.79% to 1.59%. This decreasing trend is seen in both Portfolios 4 and 5 with more pronounced drops: from 2.45% to 1.31% and 3.28% to 0.61%, respectively. These results are surprising as we expected an increase in Portfolios 3, 4 and 5. This large decrease in the returns of portfolio 5 is the main cause that leads to the return in the 5-1 strategy of -1.08%. The alpha value for 5-1 strategy also decreases to a highly significant value of -1.45%. From our results we can clearly see a negative relationship between the IV estimate and the value-weighted returns. The alpha values follow the same pattern seen in the return values, slight increases in portfolios 1 and 2, a small decrease in portfolio 3 and accentuated drop offs in portfolios 4 and 5.

These results are more in line with what was expected in our original results, a negative relationship between returns and estimated IV values. The larger change, going from positive to negative implies that the relationship is sensitive to the weighting scheme. While AHXZ (2006) did not report equal weighted returns, Bali & Cakici (2008) also found that their return results were sensitive to the weighting scheme. They found a significant negative relationship with value weighted returns and a insignificant positive relationship with equal weighted returns. Because Bali & Cakici (2008) also used US stock data it seems to imply that this sensitivity to the weighting scheme is independent of sample differences.

7.1.2 Fu results

Table 6: Equally-weighted portfolio returns sorted by expected IV. Otherwise, description from table 4 applies.

Portfolio	1 (low)	2	3	4	5 (high)	5-1
EW-Return	1.66	1.59	1.51	1.32	0.91	-0.75
IV Estimate	0.13	0.33	0.62	1.28	8.55	8.42
FF3FM-alpha [t-stat]	0.78*** [6.8197]	0.71*** [6.9695]	0.60*** [4.6578]	0.38* [1.8182]	-0.09 [-0.3603]	-0.87*** [-3.2419]

The IV estimate values in the equal-weighted case are the same as the value-weighted case. After replacing value-weighted returns in the main results with equal-weighted returns, only portfolio 1 increases in average monthly return. Portfolios 2 to 5 all decrease in average monthly return, which conflicts with what we expected. The largest decrease—from 2.6% to 0.91%—can be found in portfolio 5, which suggests it is the stocks with highest expected IV values which are the most sensitive to a change in weighting schemes. It is this large decrease in portfolio 5 which is responsible for creating a negative relationship between portfolio returns and IV, since the return from a 5-1 zero investment strategy now leads to a -0.75% average monthly return. There is a significant alpha of -0.87% from the 5-1 strategy with a robust t-statistic of -3.2419.

There remains a strong preference by researchers for value-weighting as it alleviates the overinfluence of microcaps, but our main results are over-sensitive to switching to equal-weighting and only portfolio 1 behaves consistently with theory. Our robustness test results are quite concerning, however they are not an isolated finding. Fu (2009) also obtains over-sensitive results and only a fifth of his portfolios behave consistently with theory. This equal-weight phenomenon is worth keeping in mind when interpreting the main results from this paper and others.

7.2 Controlling for: Using Only Stockholm Stock Exchange Stocks

We control for firm size by restricting the investable universe exclusively to stocks listed on the Stockholm Stock Exchange (Sweden's primary market), which mostly consists of large stocks. This means for this control we exclude stocks listed on First North Growth, Spotlight, Nordic Growth, and Nordic MTF markets. These markets typically consist of stocks smaller than those found on the Stockholm Stock Exchange and by removing them from the analysis we can investigate how sensitive our results are to small stocks. Small stocks are associated with larger risk/return and we can therefore expect a decrease in both IV and returns due to their removal from the analysis. This method is not perfect because it may be possible that there is some overlap between the smallest stocks in the Stockholm Stock Exchange and the largest stocks on the alternative markets.

7.2.1 AHXZ results

Table 7: Value-weighted portfolio returns sorted by lagged IV, using only Stockholm Stock Exchange stocks. Otherwise, description from table 3 applies.

Portfolio	1 (low)	2	3	4	5 (high)	5-1
VW-Return	1.51	1.98	2.10	2.18	2.80	1.29
IV Estimate	0.89	1.30	1.69	2.24	4.12	3.24
FF3FM-alpha [t-stat]	0.76*** [6.7004]	1.11*** [9.1270]	1.12*** [6.7032]	1.07*** [4.5440]	1.52*** [3.9221]	0.76* [1.8778]

While portfolio 1 monthly average value weighted returns are the same value as our original results, as the portfolios increase from “low” to “high” the differences in returns increase.

The 5-1 portfolio has a decrease in value from 1.77% to 1.29%, thus indicating that the exclusion of non-primary markets leads to an decrease in returns. Our IV estimates also follow this pattern of different values but in line with the previously obtained positive relationship seen in the original results. The exclusion of the non-primary markets also leads to the decrease of the alpha for the 5-1 strategy to 0.76%. The robust t-statistic value also drops to a less conclusive 1.8778 narrowly reaching significance at the 90% confidence level, and being insignificant at the 95% level.

If we look at AHXZ (2006) as the sample is restricted only to NYSE stocks there's a change in the return results from -1.31% to -0.66%. It seems that excluding non-primary markets leads to a decrease in magnitude in the results. Bali & Cakici (2008) also observed this pattern as they restricted their sample to NYSE stocks only, from -0.37% to -0.23%. This suggests that small stocks from non-primary markets contribute to a stronger relationship between IV and portfolio returns, regardless if a negative or positive relationship is found. While the nature of the relationship found in our analysis differs from the previously mentioned papers, there is a similarity of the responses to restricting the sample size to the primary market.

7.2.2 Fu results

Table 8: Value-weighted portfolio returns sorted by expected IV, using only Stockholm Stock Exchange stocks. Otherwise, description from table 4 applies.

Portfolio	1 (low)	2	3	4	5 (high)	5-1
VW-Return	1.66	1.67	1.69	2.15	2.09	0.43
IV Estimate	0.12	0.29	0.52	0.98	4.75	4.63
FF3FM-alpha [t-stat]	0.73*** [6.2058]	0.79*** [5.8171]	0.85*** [5.4532]	1.22*** [6.5702]	0.97*** [3.9613]	0.25 [0.9129]

We notice that portfolios 1 to 4 remain largely unchanged compared to the main results. Portfolio 5 however does change noticeably, the average expected IV falls from 8.55% to 4.75%, and value-weighted average monthly return falls from 2.6% to 2.09%. In response, the average monthly return from a 5-1 strategy also falls from 0.96% to 0.43%. This suggests that small stocks are driving the expected IV and returns upwards. Removal of the non-primary markets leads to a reduction in alpha for the 5-1 strategy from a significant 0.79% to an insignificant 0.25%, with a robust t-statistic of 0.9129. This means an investor may need to think twice about employing a 5-1 IV strategy only on socks listed on the Stockholm Stock Exchange. Overall this reaction to the removal of small stocks with this method mostly fits in line with our theoretical expectations.

7.2.3 Comparison with equal-weights control (section 7.1)

An interesting discussion topic is how is it possible for the 5-1 portfolio returns to *decrease* upon removal of the non-primary markets (representing small stocks), when in section 7.1 increasing the influence of the small stocks leads also to a *decrease* in 5-1 portfolio return. If we ignore the theoretical expectation for a moment, this is an inconsistency because extrapolating on our results from 7.1, when we remove the non-primary markets we should expect an increase in the 5-1 portfolio return but instead we observe a decrease. One explanation to this inconsistency is found in the details of how these methods interact with the portfolio sorting system. In 7.1 by using equal weights, the influence of each stock within each portfolio is changed, however the list of stocks within the portfolios are unaffected. This means that regardless of using equal or value weights for the portfolios, the list of stocks within each portfolio remain absolutely identical, this is why the IV estimates in section 6 and 7.1 are identical. Here in 7.2 the list of stocks within each portfolio is not static, and will change considerably depending on if only the primary market or all markets are specified. This is why we notice a difference in IV estimates between these two specifications. In summary, even though the results between 7.1 and 7.2 can be described as inconsistent, it is worth remembering that the methods interact very differently with the portfolio sorting system.

7.3 Controlling for: Momentum Factor

We can control for the momentum factor (MOM) by including this factor in the FF3FM regression. This causes our chosen pricing model to reflect the Carhart model. Using more advanced pricing models should be encouraged, since it should lead to increased accuracy of IV estimation due to a higher quality set of factors explaining the systematic risk. This additional factor tries to capture the performance inertia that exists in the market, that is stocks that are rising continue to rise and stocks that are decreasing continue to decrease. As the regression becomes a better fit we should have a reduction in the values of the residuals and consequently the reduction of the IV values. This is due to the fact that the IV is the standard deviation of the residual values. These changes should also be impactful in the portfolio returns as IV is the sorting variable. The nature of such changes will depend on how the returns respond to the reduction in the IV values.

Similarly to the size and value factor, the momentum factor is obtained from the Swedish House of finance database. According to Aytug, Fu and Sodini (2020) the momentum factor is constructed by sorting the two size portfolios in two momentum sorted portfolios formed on their past one year return. The breakpoints will be the 10th and 90th percentile. Companies above the 90th percentile are denominated Winners while companies below the 10th percentile are denominated Losers. Consequently 4 portfolios are created:

Small-Winners (SW)	Small-Losers (SL)
Big-Winners (BW)	Big-Losers (BL)

The momentum factor consists of the difference of the average return of the 2 high-return portfolios with the 2 low-return portfolios:

$$MOM = \frac{(SW + BW)}{2} - \frac{(SL + BL)}{2}$$

7.3.1 AHXZ results

Table 9: Value-weighted portfolio returns sorted by lagged IV including the momentum factor. Otherwise, description from table 3 applies.

Portfolio	1 (low)	2	3	4	5 (high)	5-1
VW-Return	1.85	1.87	1.96	2.17	2.49	0.63
IV Estimate	0.89	1.42	1.92	2.64	5.15	4.26
FF3FM-alpha [t-stat]	1.01*** [5.2474]	1.01*** [3.8463]	0.83*** [2.8639]	1.18*** [3.7307]	1.52*** [4.5370]	0.51 [1.3594]

It was our expectation that the estimated IV values had a reduction in their values in comparison to the original results, thus when we observe the table above we can see that it fails to fulfil that forecast. Only portfolios 5 and 1 have a small drop in their IV values while the remaining portfolios all register minor increases. On the contrary, with the exception of portfolio 1 all of the portfolios see a reduction in their returns. The addition of the momentum factor in our IV estimation led to a reduction of the average return value weighted returns of the 5-1 strategy from 1.77% to 0.63%. This reduction is caused by the increase of portfolio 1 and the decrease of portfolio 5. The alpha values for portfolios 5 and 1 have accentuated differences with our original results, while the remaining portfolios remain similar. This led to a large decrease in the alpha value for the 5-1 portfolio from 1.27% to a statistically insignificant value of 0.51%. In summary, while there's minor changes in the IV estimated values there is sharp reduction in the value-weighted returns.

The addition of the momentum factor provides us with lower return values for the majority of the portfolios and the 5-1 investment strategy while imposing only minor changes in the IV values in comparison with our original results. It seems that the new IV specification caused by the additional factor provokes a decrease in returns due to capturing lower performing firms. On the contrary, the value weighted returns portfolio 1‘s increase is due to the opposite reasoning: the capture of high-performing firms.

Switzer & Picard (2015) show that results can be sensitive to asset pricing models. We see that when they switch from FF3FM to Carhart, some but not all returns change considerably in economic and statistical significance, and even change sign of the relationship. When they switch to the Carhart model their total significant relationships found drops from 8 to 6, the magnitudes of these relationships change too but not in any systematic way. This situation mirrors our results, where a relationship loses its significance by switching to the Carhart model, and the magnitude changes non-systematically. The implications for the attractiveness of this strategy for an investor depends heavily on which asset pricing is chosen, at least in the case of using AHXZ’s method.

7.3.2 Fu results

Table 10: Value-weighted portfolio returns sorted by expected IV including the momentum factor. Otherwise, description from table 4 applies.

Portfolio	1 (low)	2	3	4	5 (high)	5-1
VW-Return	1.63	1.59	1.67	2.16	2.89	1.27
IV Estimate	0.13	0.33	0.62	1.26	7.38	7.25
FF3FM-alpha [t-stat]	0.71*** [5.8998]	0.67*** [5.3204]	0.80*** [5.5239]	1.21*** [6.5372]	1.85*** [6.0193]	1.14*** [3.3746]

The returns and IV estimates for portfolios 1 to 4 remain similar to the main results. Instead, for portfolio 5 we notice that the IV estimate reduces from 8.55% to 7.38%, and the return increases from 2.6% to 2.89%. The reduction in IV estimate for portfolio 5 behaves as expected, and could be due the new IV specification of the Carhart model explaining more systematic risk and explaining less unsystematic risk. The increased return for portfolio 5 is due to the new IV specification identifying higher performance stocks. Since portfolio 5 now contains higher performing stocks, the 5-1 strategy here of 1.27% naturally outperforms the main result of 0.96%. The same is true when looking at the 5-1 strategy alpha, which increases from a significant 0.79% to a significant 1.14%, with a higher robust t-statistic of 3.3746. The takeaway here is that the choice of pricing model in an analysis of this type should demand careful consideration as it can have a strong influence on the results. The preferred choice of pricing model is one which has a well established empirical consensus within academics regarding its performance.

8. Conclusion

The main results show a significantly positive relationship between idiosyncratic volatility and portfolio returns within Swedish stock markets, despite Sweden's low idiosyncratic volatility climate. This result is surprising in the case of applying Ang, Hodrick, Xing, and Zhang's (2006) method to the Swedish stock markets, since their method repeatedly finds a negative relationship within US markets and also internationally. This result is less surprising in the case of applying Fu's (2009) exponential GARCH method, since a positive relationship was to be expected. In both methods, the statistically significantly positive relationship can be used together with a zero-cost portfolio investment strategy to take advantage of economically significant arbitrage opportunities.

Regarding robustness checks, we found that the main results were extremely sensitive to a change in portfolio weighting scheme from value to equal weights, which justifies caution before using an investment strategy based on the main results. The main results were sensitive to limiting the investable universe to include only primary market stocks, leading to inconclusive results and raises questions for studies which may exclude non-primary markets. The main results were also sensitive to a change in asset pricing model specification, therefore the model specification should be chosen very carefully before analysis.

For further research it would be interesting to see the results for a high IV climate country other than the US. It's also worth revisiting countries and including all available stocks in the analysis, instead of just the stocks from the primary market. It would also be of value to utilise many different asset pricing models (q-factor model in particular) and to see how the results differ.

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