

UNIVERSITY OF GOTHENBURG school of business, economics and law

Title: Does Bitcoin hedge inflation risk? A multivariate time series analysis

Author: Domenico Roberto Curciarello

Supervisor: Jian Hua Zhang

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Graduate School, School of Business, Economics and Law, University of Gothenburg, Sweden

Abstract

This thesis investigates whether Bitcoin can be considered a valuable hedge against inflation risk. The research examines the relationship between Bitcoin and inflation, the two variables' forecast ability, and how an exogenous shock simulated on one variable affects the other. The research firstly analyzes the relationship between the Bitcoin and 5 years - 5 years (5y5y) forward inflation expectation rate from July 2010 to February 2022. This period is split into two sub-period. The first goes from July 2010 to December 2019, the second sub-period goes from January 2020 to February 2022. Firstly, we explore the relationship between the two variables by applying the VAR model. We implement the VARX model, which allows for the inclusion of an exogenous variable in the multivariate regression. The forecast ability of each variable is investigated through the Granger-causality test, while the effect of the exogenous shock is measured by applying the Impulse response function and the Forecast error variance decomposition. Results show that 5y5y forward inflation expectation affects Bitcoin only in the first sub-period. In all the considered periods, Bitcoin affects 5y5y forward inflation expectation, and there is evidence of Granger causality from Bitcoin to 5y5y forward inflation expectation, both with VAR and VARX. The results show that 5y5y forward inflation expectation affects and Granger cause Bitcoin in the first sub-period only. The Impulse response function reports a strong response of 5y5y forward inflation expectation to a shock on Bitcoin during the second sub-period. Even if a correlation between the variables is founded within all the periods, it appears that Bitcoin has acted as an inflation hedging, particularly in the last two years.

Keywords: Bitcoin, 5 years– 5 years forward inflation expectation rate, VAR, VARX, Granger causality, Impulse response function, Forecast error variance decomposition.

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Chapter 1: INTRODUCTION

Bitcoin is a virtual currency that became popular after its introduction into the financial system in 2008 by Satoshi Nakamoto (2008). It was introduced to allow people to send money on the internet, keeping it independent from government and central banks' decisions. The Bitcoin supply is fixed at 21 million, a limit expected to be reached around 2140; the process through which Bitcoin is created is called mining, and it requires the solution of a complicated algorithm. As a reward for the algorithm solution, the miners receive a block of Bitcoin, the number of Bitcoin within each block is cut in half every four years through a process that is called halving. The fixed nature of the Bitcoin supply and the halving process attribute a deflationary connotation to it.

Inflation is defined as an increase in the price level or a decrease in the purchase power of the money. The two definitions are related, if the price level increase, a certain amount of money cannot buy as many goods as before. Given the fact that the economy experiences continued inflation, investors have to consider its risk and try to hedge it (Reilly et al, 1970).

After the Covid-19 pandemic many countries conducted expansionary monetary policies to support the economy, fueling the risk of rising inflation. In this regard, the question arose whether Bitcoin could be used as hedging for inflation risk. This thesis aims to test the following hypotheses: *Can Bitcoin be considered a valuable asset to hedge the inflation risk? Are the two variables related? How does one variable react to a shock on the other?*

As suggested by Blau et al. (2021), we examine the relationship between the two variables (i.e., Bitcoin and inflation) to test the above hypotheses. In Blau et al. (2021), the time frame considered goes from 2019 to 2020. It is enriched in the following research by adding the years from 2010 to 2019, 2021, and the first two months of 2022. The thesis compares the pre-pandemic (2010-2019) and the pandemic period (2020-2022). The result of the analysis derives from the implementation of the VAR and VARX model. The VAR is estimated considering Bitcoin (BTC) and the 5 years-5 years (5y5y) forward inflation expectation rate (T5YIFR) as endogenous variables, the VARX is similar to the VAR model, but it adds an exogenous control variable, the S&P500, that according to literature could be related to the

Bitcoin and inflation expectations. The thesis aims to contribute to the literature by implementing the two models mentioned above and comparing the association between the variables during the pre and pandemic periods. The research starts by testing the stationarity of the time series. Having verified the non-stationarity, we proceed with the analysis of the potential cointegration. As there is no evidence of cointegration, the models are estimated in first differences. They are implemented considering three different time horizons, the first is the whole period, it goes from July 2010 to February 2022, then two sub-periods, the first from July 2010 to December 2019, and the second from January 2020 to February 2022. This distinction allows us to separate two sub-periods during which the perception of Bitcoin appears to change. Since 2020 Bitcoin has experimented with increasing institutional demand. The Granger causality test, Impulse Response function (IRF), and Variance decomposition implementation follows the model estimation.

We find evidence of Granger causality from BTC to T5YIFR during the whole period and in the second sub-period, while in both directions during the first sub-period. The IRF suggests a positive relationship between the two variables during all the time frame, with some differences. It only appears strong within the Covid-19 period, the IRF of the VAR shows that a shock on BTC produces a 20 bp increase in T5YIFR log return, while 10 bp in the case of VARX. The Variance decomposition of the VAR model within the second subperiod highlights that an exogenous shock on BTC explains 22% of the T5YIFR forecast error variance. It appears, from both VAR and VARX, that a positive shock to the log return of Bitcoin price leads to an increase in the inflation expectation rate during the pandemic period, confirming the results of Blau et. al (2021). A positive relationship also characterizes the whole period and the first sub-period, but the related IRF in VAR and VARX shows a positive but briefly significant correlation.

The rest of the paper is organized as follows. Chapter 2 illustrates the development of literature on Bitcoin and its hedging properties, providing different definitions of hedging. Chapter 3 describes data and summarizes the behavior of the Bitcoin and T5YIFR time series. Chapter 4 profoundly illustrates the methodology, focusing on the VAR and VARX model, the ADF and PP test, and all the econometric tools used to interpret the estimation results. Chapter 5 will present the results, while chapter 6 concludes.

Chapter 2: LITERATURE REVIEW

The following section briefly summarizes the evolution of the literature around Bitcoin and its hedging properties. It illustrates the case in which, according to literature, a financial tool can be considered as a valuable inflation hedging.

2.1 Bitcoin hedging properties

The research, as already said, aims to investigate the hedging properties of Bitcoin with respect to the 5y5y forward inflation expectation rate. It is essential to estimate the relationship between the two variables. Following the idea of Blau et al. (2021) and Hoang et al. (2016), we report on the definitions of hedging that is given by Bodie (1976). According to Bodie (1976), there are three possible definitions of inflation hedging: firstly, an asset has hedging capabilities if it can eliminate or reduce the possibility of having a negative real rate of returns, this is present in the two papers of Really, Johnson and Smith (1970, 1971) and it is implicitly sustained by Cagan (1974); secondly the asset holds a hedging property if it is able to reduce the variance of the real returns when it is used in combination with other assets, or if its real returns are independent of the inflation rate. This definition is adopted, as written in Blau et al. (2021), by Branch (1974), Fama and MacBeth (1974) and Oudet (1973); lastly the asset is able to hedge from the inflation if the two variables are positively correlated. With respect to the last definition, Hoang et al. (2016) rely on the argumentation of Arnold and Auer (2015), they sustain that an asset can be considered as perfect hedging from inflation if the correlation coefficient is equal to 1, in that case, an increase in the level of inflation is perfectly absorbed by an increase in the value of the asset. However, they add that even if the correlation coefficient is lower than one and positive, the asset can be considered valuable in hedging from the inflation.

The argument of Bitcoin's ability to hedge inflation risk has only recently gained interest, mainly due to the pandemic period that has seen the application of expansionary monetary policies in the United States and Europe. As written by Choi et al. (2021), researchers studied the statistical properties of Bitcoin, trying to understand its behavior, and determining its

safe-haven status from a portfolio diversification perspective. Literature is about the efficiency of the Bitcoin market; according to Urquhart (2016), the inefficiency of the Bitcoin market was too strong between 2010 to 2016, the paper justifies the inefficiency considering that at the time the Bitcoin was an emerging market, highlighting that the market was starting a process of efficiency. Like Kristoufek (2018), others agreed to define inefficient the Bitcoin market, explaining the findings by sustaining that the market was shallow, and the institutional players were not attracted, while now different institutional investors have decided to invest in the crypto market. It is also sustained that the introduction of Bitcoin- derivatives could help solve the market inefficiency. In line with that, as shown by Köchling et al. (2019), there was a difference in the predictability of the Bitcoin price before, and after the introduction of the futures, indeed, the hypothesis of weak efficiency could not be rejected after the introduction of derivatives instruments. In accordance with the last research, Blau, Griffith, and Whitby (2019) prove the benefits gained in terms of information after introducing the derivatives.

Other papers tried to understand in which category it was possible to collocate Bitcoin, asking if it is a speculative asset or a medium of exchange. Mainly agreed not considering Bitcoin as a medium of exchange, Yermack (2013), Baur et al. (2018) based their analysis on the transaction data, Peetz and Mall (2018) showed that Bitcoin is more a speculative asset than a currency.

However, this research aims to enrich the literature with regards the hedging capabilities of Bitcoin. In particular, Dyhrberg's paper (2016) demonstrates the non-correlation between Bitcoin and the FTSE index. It was concluded that the cryptocurrency could be used to cover part of the market risk. On the other hand, different results have been found considering a VAR approach instead of the GARCH approach used by Dyhrberg (2016). Indeed according to Choi and Shin (2021), the Bitcoin price increased significantly in response to a shock on the US stock market, represented by the S&P 500, showing that the cryptocurrency cannot be used as a hedge for investment in the stock market. In accordance with Choi and Shin (2021), Bouri et al. (2017) showed a positive correlation between Bitcoin and the US stock market, concluding that the cryptocurrency is not a weak hedge against movements in the US stock market. Conversely, Wang et al. (2019) found that Bitcoin can be useful as a hedge

against stocks, bonds, and the SHIBOR, meaning the Chinese money market. In accordance with Wang et al. (2019), the paper of Bouri et al. (2017) reports that Bitcoin is a strong hedge against the movements in the Chinese stock market, results confirmed both in the case of daily and weekly data.

Still, the ability of crypto to hedge from global uncertainty has been investigated. On the one hand, Bitcoin's ability to hedge against the uncertainty of economic policies was highlighted (Demir et al., 2018). In other cases, such property was found only in the growing market and only in the short term (Bouri et al., 2017). It still turns out that the uncertainty of global economic policy has a significantly negative impact on the Bitcoin-bonds correlation while a positive impact on the Bitcoin-equities and Bitcoin-commodities correlations, highlighting the possibility that Bitcoin acts as a hedging tool under specific conditions of economic uncertainty (Fang et al., 2019). Wu et al. (2019) found that Bitcoin cannot be useful as a strong hedge against policy uncertainty but could be a weak hedge against the EPU index in extreme bearish or bullish markets. In the case of Choi and Shin (2021), instead, the results stated that the uncertainty about future government policy does not negatively affect the Bitcoin, hence it is highlighted that the absence of correlation between Bitcoin and world economic policy is due to the intrinsic nature of the cryptocurrency, of being independent from government decisions. It is known that the origin is to be placed after the financial crisis of 2008, and the reason was connected to the desire to offer a payment instrument and a currency that was disconnected from the decisions of governments (Choi, Shin 2021). Continuing to analyze the aspect related to uncertainty, moving to investigate the aspect of the uncertainty of financial nature, it turns out that the price of the cryptocurrency reacts negatively to shocks of a financial nature measured by the VIX index, highlighting how Bitcoin does not constitute a safe-haven asset.

As previously noted, however, the analysis concerning the correlation between Bitcoin and inflation has instead always passed into the background, in fact, the literature on the subject is not very wide, also, from this comes the interest and the desire to deepen the theme. Inflation is currently a real problem around the world. According to an article in the Wall Street Journal in February 2022, there is a surge in inflation that has reached an annual rate of 7.5% currently compared to last year, the price increase is equal to 7.9%, while the rate

on the 10-year Treasury note has for the first time since mid-2019 reached 2%, following expectations of a more restrictive future monetary policy. In a situation like the current one, there have been conflicting judgments about the possibility of using Bitcoin as a hedge against inflation risk. On the one hand, Paul Tudor at Forbes and JP Morgan have expressed favorable opinions to an investment in crypto, seen as potentially better than gold. Instead, the judgments of Morningstar and Bank of America were negative, which made it clear that the evidence clashed with previous statements. Even better, we can refer to the literature, which, as previously highlighted, is not very vast.

Returning to the analysis of the relationship between BTC and inflation expectations, according to Blau et al. (2021), the application of models for multivariate analysis showed that variations in the Bitcoin Granger cause changes in the 5 year- 5 year forward inflation expectation rate, while there is no evidence of Granger causality in the opposite direction. Even the Impulse Response Function application has returned a result that highlights how an exogenous and unexpected shock on Bitcoin increases inflation by 20/30 percentage points, depending on the number of lags included for multivariate analysis. It follows that not only can Bitcoin hedge against inflation, but it is also useful in predicting the behavior of expected inflation since it anticipates its movements. It seems, in fact, that it behaves like a commodity. Feyen et al. (2022) use also the 5 year- 5 year forward inflation expectation rate as measure of longer-term inflation outlook in the United States. In their paper sustained that if crypto are considered as macro hedge, then an increase in inflation expectation would generate an increase in crypto volume. Their results give evidence that cryptocurrency might be considered as an emerging longer-term macro hedge, by exhibiting that an increase in long-term inflation expectation produces a rise in crypto volumes. According to Vo et al. (2021) 5 year- 5 year forward inflation expectation rate is an elastic variable, a variable with an absolute coefficient value greater than one, which is significantly correlated to Bitcoin and cause more than a proportional change in Bitcoin price. Furthermore, other results seem to corroborate what has been said about the hedging property, although differently. The OPI index flanks the expected inflation (Cavallo A., Rigobon R. 2016), used to measure inflation at high frequencies, it is built by collecting prices from different websites. It results that the price of Bitcoin responds positively to both an exogenous shock on expected inflation and one that affects the OPI index, confirming the hedging property against inflation (Choi, Shin

2021). Further results seem to support what has been previously illustrated, although still in a partly different way. Martkovskyy and Jalan (2021) analyzes inflation quantiles and Bitcoin, using the CPI as a proxy for the general price index. It turns out that the relationship between Bitcoin's returns and inflation is asymmetrical and linked to the different markets that are considered and to the level of inflation. The relationship is related to the magnitude of the inflation shock and the type of market being considered, i.e., a bullish or bearish market. Specifically, the crypto in question acts as a hedge against inflation in the case of the Bullish market, specifically in the euro area, GBP and JPY. On the contrary, it has shown poorer performance in the case of the US market with rising inflation. The idea of a valuable tool to combat inflation is intrinsic to the very nature of Bitcoin and was one of the reasons for its introduction. The issuance of Bitcoin is not linked to human decisions but to an algorithm that controls its quantity. The mining process is highly complex, and the complexity increases with the amount of money in circulation. In addition, the extraction process is halved (halving) every four years (Meynkhard A., 2019).

2.2 The paper by Blau et al. (2021)

The thesis inherits the methodology followed by Blau et al. (2021) to study the relationship between Bitcoin and the expected inflation rate. The Blau et al. (2021) paper highlights the lack of literature that examines the association between Bitcoin and inflation. In the paper, the writers, following the idea of Bodie (1976), define an *asset* as inflation hedging if it is positively correlated with the rate of inflation. The analysis applies the Vector Autoregressive model (VAR) to examine the interactions between the two variables, furthermore, it allows estimating the IRF that helps understand how one variable reacts to an exogenous shock on the other variable. The estimation is preceded by steps required to analyze the characteristics of the Bitcoin price and the 5y5y forward inflation expectation rate. The Augmented Dickey-Fuller tests applied on the simple return of the Bitcoin price and 5y5y forward inflation expectation rate reject the null hypothesis of the presence of unit root. Hence it is possible to conclude that the time series of simple net return of both the time series are stationary. Once the stationarity of the time series is ensured, the VAR models are estimated. The time frame considered goes from January 1st, 2019, and December 31st, 2020, within which a portion of the Covid-19 pandemic is included. The number of lags is chosen by applying Akaike's information criterion, which indicates that the optimal number of lags is between one and five. The results of the Granger causality test highlight that changes in the 5y5y forward inflation expectation rate (*i.e.* $\%\Delta$ T5YIFR_t) do not Granger cause a change in Bitcoin price (*i.e.* $\%\Delta$ Bitcoin_t).

In contrast, a change in Bitcoin price can Granger cause the 5y5y forward inflation expectation rate. The estimation of the IRF follows the granger causality test. The impulse response function is estimated by applying an exogenous shock on both variables to analyze how the other variable would react. The IRF shows that a positive shock on the Bitcoin price produces a significant increase in the 5y5y forward inflation expectation rate, while the opposite is not recorded. A positive shock on the 5y5y forward inflation expectation does not significantly affect the Bitcoin price. The study results are important because they concluded that Bitcoin is helpful as a strategy to hedge against inflation since it is positively correlated with the forward inflation rate and is also able, in such a sense, to predict the latter's behavior.

Chapter 3: METHODOLOGY

The previous section briefly summarized the methodology and results obtained by Blau et al. (2021). The following chapter will describe the methodologies applied to conduce our analysis. It starts with a focus on the VAR model and VARX model, proceeding with the unit root tests and the Johansen cointegration test for potential long run relationship. Finally the Granger causality test, the IRF and Forecast error variance decomposition are described.

3.1 The Vector Autoregressive model (VAR)

The VAR model is one of the most useful, flexible and successful models in the time series analysis. It becomes particularly well known after Chris Sims' presentation in the paper "Macroeconomics and Reality" (1980). It helps conduct a dynamic analysis between two or more time series. The most common type of Var model is the one that includes two time series, although the analysis can be extended to a larger number. Theoretically, a p-th order vector autoregression is an extension and generalization of a univariate vector autoregression. It has the following equation:

$$Y_t = c + \Pi_1 Y_{t-1} + \Pi_2 Y_{t-2} + \dots + \Pi_p Y_{t-p} + \varepsilon_t$$
(1)

where *c* is a $(n \times 1)$ vector of constant and Π_i is a $(n \times n)$ matrix of autoregressive coefficient for j = 1, 2, ..., p, where *p* identifies the number of lags. Still ε_t is a vector generalization of white noise that respects the following condition:

- $E(\varepsilon_t) = 0$ The expected value of ε_t is equal to 0
- $E(\varepsilon_t \varepsilon'_{\tau}) = \begin{cases} \Omega & \text{for } t = \tau \\ 0 & otherwise \end{cases}$

Where $cov(\varepsilon_{1t}, \varepsilon_{2\tau}) = \sigma^{12} for t = \tau$. The matrix Ω is a $(n \times n)$ symmetric positive definite matrix.

If we assume that c_i is identified as the i-th element of the $(n \times 1)$ vector c and Π_{ij}^1 represent the row i, column j of the matrix Π_1 , then it is possible to write the previous model in the following way:

$$y_{1t} = c_1 + \pi_{11}^1 y_{1,t-1} + \pi_{12}^1 y_{2,t-1} + \dots + \pi_{1n}^1 y_{n,t-1} + \pi_{11}^2 y_{1,t-2} + \pi_{12}^2 y_{2,t-2} + \dots + \pi_{1n}^2 y_{n,t-2} + \dots + \pi_{1n}^p y_{n,t-2} + \dots + \pi_{1n}^p y_{n,t-p} + \varepsilon_{1t}$$
(2.1)

$$y_{2t} = c_2 + \pi_{21}^1 y_{1,t-1} + \pi_{22}^1 y_{2,t-1} + \dots + \pi_{2n}^1 y_{n,t-1} + \pi_{21}^2 y_{1,t-2} + \pi_{22}^2 y_{2,t-2} + \dots + \pi_{2n}^2 y_{n,t-2} + \dots + \pi_{2n}^p y_{n,t-2} + \dots + \pi_{2n}^p y_{1,t-p} + \varepsilon_{1t}$$
(2.2)

As it is possible to visualize in the above equation, the variable is regressed on a constant and on its own p lags and the p lags of the other variables in the system.

The VAR model van be written using the lag operators, it would have the following form:

$$\Pi(\mathbf{L})Y_t = c + \varepsilon_t \quad \longrightarrow \ \Pi(\mathbf{L}) = I_n - \Pi_1 L - \dots - \Pi_p L^p$$

A condition required of the VAR is its stability. The pattern is stable if the roots of det $(I_n - \Pi_1 L - \dots - \Pi_p L) = 0$ are located outside the unit circle, or modulus greater than one. A stable Var is stationary and ergodic and has time invariant means, variances and autocovariance. Now, assuming that it is covariance stationary, and there are no restrictions on the parameters of the model. If the SUR (seemingly unrelated regression) is adopted than each equation of the VAR can be written as:

$$y_i = Z\pi_i + e_i, i = 1, ..., n$$

 y_i is a vector $(T \times 1)$ of observations on the equation i^{th} , while Z is a matrix $(T \times k)$ of t^{th} rows given by $Z'_t = (1, Y'_{t-1}, ..., Y'_{t-p})$ with k = np + 1, while π_i it is a vector $(k \times 1)$ of parameters with e_i which is an error $(T \times 1)$ with covariance variance matrix $\sigma_i^2 I_T$. Each equation has the same explanatory variables, so each equation can be estimated individually via the OLS. Several advantages derive from using the Var model to study the relationship between time series:

- 1. it is not necessary to specify which variable is considered endogenous, moreover, the application of any specific condition is not required.
- More importantly, the model allows for extending the univariate model, we find both the lags of log Bitcoin return and the lags of the log return of 5y5y forward inflation expectation rate.
- The problem of endogeneity is avoided using only the lags of the variables on the RHS.

With regard to the point 1, it is possible to confirm that both variables of interest have been considered endogenous within the model. Although the positive aspects that characterize the Var model are remarkable, it also presents problems:

- 1. What approach to use when choosing the number of lags within the model
- 2. The number of parameters to be estimated is very high; indeed, if we consider a number of variables equal to g with lag k, the number of parameters to be estimated will be equal to $(g + kg^2)$. The problem should not arise in the case of the sample used in the following research since the models are mostly bivariate, and the available sample is large.
- Finally, the issue that fuels the greatest contrasts concerns the need for time series to be stationary.

The implementation of the VAR-X model will allow to control for an exogenous variable. It could be interpreted as the extension of a VAR model with exogenous variables.

The general form of the VAR-X model will be the following:

 $Y_t = c + \prod_1 Y_{t-1} + \prod_2 Y_{t-2} + \dots + \prod_p Y_{t-p} + B_1 X_{t-1} + \dots + B_p X_{t-q} + \varepsilon_t$ (3) Where X_t is a vector of exogenous variables, c is a vector of intercepts, B are (k × m) coefficient matrices.

3.2 Unit root tests

Proceeding by steps, the time series of the Bitcoin Price presents a high skewness, therefore it may be helpful to use the log transformation that is used to unskew highly skewed data (Roy et al. (2018)). The same transformation is also applied to the time series of 5y5y forward inflation expectations and S&P500 to use the same scale for all the variables. Once the Log transformation is adopted, we proceed by valuating the stationarity of the time series, which is an essential condition that time series in the VAR model must respect to avoid meaningless regression. The condition of stationarity that time series must hold could have two forms:

Strictly stationary if for any *t*, *k*, *h* ∈ *Z*: (*y*_t, *y*_{t+1}, ..., *y*_{t+h}) ^d₌ (*y*_{t+k}, *y*_{t+k+1}, *y*_{t+h+k}), meaning that the series owns the stationarity if the distribution of its value rest the same as time pass.

- Weakly stationary if:
 - $E(y_t) = \mu$
 - $E(y_t \mu)(y_t \mu) = \sigma^2$
 - $E(y_t \mu)(y_{t-k} \mu) = \gamma_{t_2 t_1}$

Meaning that a series must have a constant mean, variance and covariance, so if neither mean and autocovariance depends on t.

In order to investigate the stationarity of the time series, we proceed to perform two tests, the Augmented Dickey-Fuller and the Philippe-Perron. Notably, the ADF test is a unit root test, it is the augmented version of the basic Dickey-Fuller test, expanded in 1984 to be applied to more complex models. The ADF test works precisely like the DF test, but it is improved by adding the lag values of the dependent variable on the RHS. The test aims to analyze whether there is a unit root in the following equation:

 $y_t = a_0 + a_1y_{t-1} + a_2y_{t-2} + a_3y_{t-3} + \dots + a_{p-2}y_{t-p+2} + a_{p-1}y_{t-p+1} + a_py_{t-p} + \varepsilon_t$ To make the computation easier, Dickey and Fuller suggest providing some changes in the above equation by adding and subtracting a_py_{t-p+1} , again $(a_{p-1} + a_p)y_{t-p+2}$, continuing until the equation has the following form:

$$\Delta y_t = a_0 + \psi y_{t-1} + \sum_{j=2}^p \phi_j \Delta y_{t-j+1} + \varepsilon_t$$
(4)
where $\psi = -(1 - \sum_{i=1}^p a_i)$ and $\phi_j = \sum_{i=j}^p a_j$

The coefficient that the test analyzes is the ψ , the hypotheses are the following:

$$\begin{cases} H_0: \ \psi = 0 \ \Rightarrow y_t \text{ is non stationary} \\ H_1: \ \psi < 0 \ \Rightarrow y_t \text{ is stationary} \end{cases}$$

if the null hypothesis of $\psi = 0$ is not rejected, the time series presents a unit root, thus is non-stationary. The ADF test could include the case in which the presence of a drift or a trend is tested:

$$\Delta y_t = \psi y_{t-1} + \sum_{j=2}^p \phi_j \Delta y_{t-j+1} + \varepsilon_t \text{ No Drift & No trend}$$

$$\Delta y_t = \psi y_{t-1} + \mu + \delta t + \sum_{j=2}^p \phi_j \Delta y_{t-j+1} + \varepsilon_t \text{ Drift & Trend}$$

Focusing on the time series that characterized this study, following the indication of Blau et al. (2021), we apply the ADF test over the basic one, considering it more suitable, since drawing the ACF of both log of BTC price and log of 5y5y forward inflation expectation it is noticeable the strong lag autocorrelation. The same ADF test is applied to investigate the stationarity of the log of S&P500. Considering that the ADF test has been selected, it is

essential to choose the right lags order to include in the regression. In response to this requisite, it is possible to apply a rule of thumb that suggests using five lags in the case of weekly data. It is also possible to use the Information Criteria to select the proper lags order in implementing the ADF test.

Besides the ADF test, we decided to implement the Philippe-Perron test to deeply investigate the stationarity of the time series. The ADF may not perform well if the analyzed time series shows some structural breaks that can affect either the slope or the intercept. Conversely, the Philippe-Perron test could be useful when the time series under investigation shows some structural breaks. The general equation proposed by the test is the following:

$$\Delta y_{t} = \psi y_{t-1} + \mu + \alpha_{1} D_{t} + \alpha_{2} (t-\tau) D_{t} + \lambda t + \sum_{i=2}^{p} \alpha_{i} \Delta y_{t-i} + u_{t}$$
(5)

In the equation, if τ is defined as the break, then D_t is a dummy variable defined as following:

$$D_t = \begin{cases} 0 \ if \ t < \tau \\ 1 \ if \ t \ge \tau \end{cases}$$

The structural break could interest the intercept of the series, so we would have $\alpha_1 D_t \neq 0$, while if the structural break interest the slope, we would have $\alpha_2(t-\tau)D_t \neq 0$. Even in the case of the PP test we used the rule of thumb in order to test for the unit roots.

3.3 Cointegration analysis

The cointegration analysis follows the stationarity test. We decided to study cointegration because the time series of Log Bitcoin Price, Log 5y5y forward inflation expectations and S&P500 are I(1), hence they could also be cointegrated, meaning that there exists a linear combination among the time series that is I(0). In that case the VAR model is not the best suitable, it should be replaced by the VECM model that take into consideration the long-run relationship among the time series. We proceed with the Johansen test to examine the presence of cointegration. Before continuing with the Johansen test, it is important to recall that the test involves the matrix Π which, considering the VAR form described in the section 3.1, will be equal to: $\Pi = (I_n - \Pi_1 - \dots - \Pi_p)$. Having estimated the matrix, its rank is studied, and the possible conclusion are 3:

- $rank(\Pi) = 0 \implies$ This result implies that $\Pi = 0$, the time series are I(1) and there is

no cointegration, it is possible to proceed estimating a Var model in first difference.

- 0 < rank(Π) = r < n ⇒ This result implies that the time series within the Var are I(1) and there is cointegration, particularly there are r linearly independent cointegration vectors.
- rank(Π) = n ⇒ This result implies that the matrix has full rank, the variables must be stationary, hence it is possible to estimate a stationary Var(p).

Besides the estimation of the Johansen test, we apply the sequential procedure proposed by Johansen, which helps in consistently defining the number of cointegrating vectors. First, test the null hypothesis $H_0: r_0 = 0$ against the alternative hypothesis $H_1: r_0 > 0$, if the test fails to reject the null hypothesis, then it is concluded that there are no cointegrating vectors among the variables. Conversely, if the null hypothesis is rejected then it is possible to conclude that there is at least one cointegrating vector, hence the procedure continues testing the null hypothesis $H_0: r_0 = 1$ against $H_1: r_0 > 1$. The procedure continues until the test fails to reject the null hypothesis. Stata returns the values of the Trace statistic, given by the following formula:

$$LR_{trace}(r_0) = -T\sum_{i=r_0+1}^n \ln\left(1 - \hat{\lambda}_i\right) \tag{6}$$

If $rank(\Pi) = r_0$ then $\hat{\lambda}_{r_0+1}, ..., \hat{\lambda}_n$ should all be close to 0 and $LR_{trace}(r_0)$ should be small since $\ln(1 - \hat{\lambda}_i) \approx 0$ for $i > r_0$.

Having analyzed stationarity and cointegration between the time series of the log price of Bitcoin, 5y5y forward inflation expectation rate and S&P500, we can focus on the model specification problem. According to Walter Enders, in case the variables are I(1) and they are not cointegrated it is better to use the first difference in order to estimate the VAR model. On the contrary, if a VAR model in level is estimated the consequences can be summarized in three points:

- Tests lose their power because the model estimate one extra lag of each variable in each equation, hence n^2 parameters.
- Granger causality test applied on the I(1) variables do not have a F-distribution.
- If the VAR has *I*(1) variables that are not cointegrated, the impulse response at long forecast are not consistent estimates of the true response.

3.4 Model estimation

Once all these tests are performed, we proceed with the estimation of the VAR and VARX models. To estimate the VAR, it is important to choose the right lag order to include in the model. Hence, the Multivariate form of the Information Criterion will be applied to select the right VAR length. Three are the type of Information Criterion: the Akaike Information Criteria, the Bayes Information Criteria and the Hannan-Quinn Information Criteria. The M-AIC tends to choose the less parsimonious model than the SBIC model, while the HQIC is a middle way, indeed M-AIC has a non-null probability of choosing an overparameterized model, contrary to the other two IC that asymptotically choose the true order with probability one. In the analysis, the M-AIC model will be preferred. As theory suggests, the cost of underparameterization (lack of consistency) is larger than the overparameterization (lack of efficiency) in the empirical application. It is worth noting that sometimes, in estimating the models, the lags will be improved or decreased. Once the order is selected, it is possible to proceed with estimating the VAR and VARX models.

The first VAR model is estimated with the aim of investigate the relationship between the log return of Bitcoin Price and log return of 5y5y forward inflation expectation rate during the period July 2010 and February 2022, with a total of 600 weekly observations. The VAR model referred to the whole period has the following form:

$$\begin{split} \Delta(\log (\text{Bitcoin}_{t})) &= c_{1} + \pi_{11}^{1} \Delta(\log (\text{Bitcoin}_{t-1})) + \pi_{12}^{1} \Delta(\log (\text{T5YIFR}_{t-1})) \\ &+ \pi_{11}^{2} \Delta(\log (\text{Bitcoin}_{t-2})) + \pi_{12}^{2} \Delta(\log (\text{T5YIFR}_{t-2})) \\ &+ \pi_{11}^{3} \Delta(\log (\text{Bitcoin}_{t-3})) + \pi_{12}^{3} \Delta(\log (\text{T5YIFR}_{t-3})) \\ &+ \pi_{11}^{4} \Delta(\log (\text{Bitcoin}_{t-4})) + \pi_{12}^{4} \Delta(\log (\text{T5YIFR}_{t-4})) + \varepsilon_{1t} \end{split}$$
(7.1) $\Delta(\log (T5YIFR_{t})) \\ &= c_{2} + \pi_{21}^{1} \Delta(\log (\text{Bitcoin}_{t-1})) + \pi_{22}^{1} \Delta(\log (\text{T5YIFR}_{t-1})) \\ &+ \pi_{21}^{2} \Delta(\log (\text{Bitcoin}_{t-2})) + \pi_{22}^{2} \Delta(\log (\text{T5YIFR}_{t-2})) \\ &+ \pi_{21}^{3} \Delta(\log (\text{Bitcoin}_{t-3})) + \pi_{22}^{3} \Delta(\log (\text{T5YIFR}_{t-3})) \\ &+ \pi_{21}^{4} \Delta(\log (\text{Bitcoin}_{t-4})) + \pi_{22}^{4} \Delta(\log (\text{T5YIFR}_{t-4})) + \varepsilon_{2t} \end{aligned}$ (7.2) The VAR model for the whole period is estimated with 4 lags, since the M-AIC is minimized in correspondence of 4 lags. The form adopted is confirmed by the post-estimation procedure that will be introduced in the next paragraph. The model is stable, all the eigenvalues lie inside the unit circle as reported by tables in the results section.

The second VAR model referred to the first sub-period is implemented to analyze the relationship between the BTC and T5YIFR from July 2010 to December 2019 with a total of 487 observations, it excludes the pandemic period. Its equations will be similar to (7.1) - (7.2), but with one more lag for each endogenous variable since the M-AIC is minimized at lag order five. The post-estimation procedures confirm the estimation, the model is stable and there is no autocorrelation at lag order five.

The last VAR model that includes the second sub-period, from January 2020 to February 2022 with 112 observations, has equations equal to 7.1 and 7.2, the M-AIC is minimized in correspondence of lag order 4. In this case also the post-estimation procedures confirm the estimation, the model is stable and there is no autocorrelation at lag order four.

Focusing on the VARX model estimation, the VARX model for the whole period will have the following equations:

 $\begin{aligned} \Delta(\log (\text{Bitcoin}_{t})) &= c_{1} + \pi_{11}^{1} \Delta(\log (\text{Bitcoin}_{t-1})) + \pi_{12}^{1} \Delta(\log (\text{T5YIFR}_{t-1})) + \\ \pi_{11}^{2} \Delta(\log (\text{Bitcoin}_{t-2})) + \pi_{12}^{2} \Delta(\log (\text{T5YIFR}_{t-2})) + \pi_{11}^{3} \Delta(\log (\text{Bitcoin}_{t-3})) + \\ \pi_{12}^{3} \Delta(\log (\text{T5YIFR}_{t-3})) + \cdots + \pi_{11}^{5} \Delta(\log (\text{Bitcoin}_{t-5})) + \pi_{12}^{5} \Delta(\log (\text{T5YIFR}_{t-5})) + \Delta \log (\text{S\&P500}_{t-1}) + \cdots + \Delta \log (\text{S\&P500}_{t-3}) + \varepsilon_{1t} \end{aligned}$ (8.1)

$$\begin{split} \Delta(\log (T5YIFR_t)) &= c_2 + \pi_{21}^1 \Delta(\log (\text{Bitcoin}_{t-1})) + \pi_{22}^2 \Delta(\log (\text{T5YIFR}_{t-1})) + \\ \pi_{21}^2 \Delta(\log (\text{Bitcoin}_{t-2})) + \pi_{22}^2 \Delta(\log (\text{T5YIFR}_{t-2})) + \pi_{21}^3 \Delta(\log (\text{Bitcoin}_{t-3})) + \\ \pi_{22}^3 \Delta(\log (\text{T5YIFR}_{t-3})) + \cdots + \pi_{21}^5 \Delta(\log (\text{Bitcoin}_{t-5})) + \\ \pi_{22}^5 \Delta(\log (\text{T5YIFR}_{t-3})) + \cdots + \pi_{21}^5 \Delta(\log (\text{Bitcoin}_{t-5})) + \\ \pi_{22}^5 \Delta(\log (\text{T5YIFR}_{t-3})) + \cdots + \Delta \log (\text{S\&P500}_{t-3}) + \\ \epsilon_{2t} \end{split}$$
(8.2)

The model will be estimated on 599 observations, the S&P500 is significant at lag order three, we decide to introduce three lags, while the M-AIC suggests choosing five lags for the two endogenous variables.

The model that includes the first sub-period from July 2010 to December 2019 has exactly the same equations as 8.1-8.2. It will be estimated with a total of 487 observations.

The last VARX estimated to analyze the second sub-period has two lags for the exogenous variable and the M-AIC suggests including one lag for the endogenous variables. The VARX models estimated for each period are all stable and there is not residual autocorrelation as will be illustrated in the results section.

The estimation of the model is followed by a post-estimation check, which is important to validate the model. Firstly the stability of the model is investigated, the models are stable if the roots of det $(I_n - \prod_1 z - \dots - \prod_p z^p) = 0$ lie outside the complex unit circle, or in the same way if the eigenvalues of the companion matrix have modulus less than one.

The stability test is followed by the test on the autocorrelation of residuals, whose null hypothesis is H_0 : no autocorrelation at lag residual.

Once the models are estimated, it could seem difficult to interpret the coefficient and clearly define the relationship among the variables because of the many lags involved in the regression. In the thesis, we will use the Granger causality test to measure the forecast ability that belongs to a certain variable. It is important to recall that the term causality does not mean that the test allows us to comprehend the true causality among the variables, it just permits to investigate the forecast ability of a variable. If we consider a bivariate model, as the one in this study, and define two variables as y_1 and y_2 , if y_1 is useful in predicting the values of y_2 , then it is possible to say that y_1 Granger-cause y_2 , on the contrary, if it does not help predict, it is said that y_1 fail to Granger-cause y_2 . Formally speaking, y_1 fails to Granger-cause y_2 if for all s > 0 all the MSE of a forecast $y_{2,t+s}$ based on $(y_{2,t}, y_{2,t-1}, ...)$ and $(y_{1,t}, y_{1,t-1}, ...)$.

Talking in matrix term, if we still consider a bivariate model, we said that y_2 fails to Granger-

cause y_1 if all the coefficient matrices are lower triangular:

$$\begin{pmatrix} y_{1t} \\ y_{2t} \end{pmatrix} = \begin{pmatrix} c_1 \\ c_2 \end{pmatrix} + \begin{pmatrix} \pi_{11}^1 & 0 \\ \pi_{21}^1 & \pi_{22}^1 \end{pmatrix} \begin{pmatrix} y_{1t-1} \\ y_{2t-1} \end{pmatrix} + \dots + \begin{pmatrix} \pi_{11}^p & 0 \\ \pi_{21}^p & \pi_{22}^p \end{pmatrix} \begin{pmatrix} y_{1t-p} \\ y_{2t-p} \end{pmatrix} + \begin{pmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \end{pmatrix}$$
(9)

Conversely, y_1 fails to Granger-cause y_2 if all the coefficient matrices are upper triangular. The null hypothesis of a Granger-causality test is: $H_0: y_2$ does not Granger cause y_1 , which is the same as testing $H_0: \pi_{12}^1 = \cdots = \pi_{12}^p = 0$, with the test statistic which is an F-statistic. The Granger-causality test is useful to investigate if there is an effect between a variable and the lags of the other variable. However, it does not help understand the sign of the effect.

It is essential to estimate the Impulse Response Function (IRF), this is an important tool that helps in understanding the dynamic behavior of the dependent variable in response to shocks imposed on each of the variables. The IRF is estimated by writing the models in terms of a Vector Moving Average (VMA), considering that the model is stable, the shock should gradually go away. The VMA form is obtained starting from the VAR form presented in the chapter on theoretical models (1). This could be written in the following form:

$$\Pi(\mathbf{L})Y_t = c + \varepsilon_t \text{ with } \Pi(\mathbf{L}) = I_n - \Pi_1 L - \dots - \Pi_p L.$$

Since Y_t is stationary, $\Pi(L)^{-1}$ exists, hence it is possible to write $Y_t = \Pi(L)^{-1}c + \Pi(L)^{-1}\varepsilon_t$. Then define $\Pi(L)^{-1}c = \mu$ and $\Pi(L)^{-1} = \Psi(L) = \sum_{k=0}^{\infty} \Psi_k L^k$ with *L* that is the lag operator. In the case of the VAR model $\Psi_0 = I_n$, hence contemporaneous shock on variables different from the variable of interest does not affect the variable of interest. The VMA has the following form:

$$\begin{bmatrix} \Delta(\log(\operatorname{Bitcoin}_{t})) \\ \Delta(\log(\operatorname{T5YIFR}_{t})) \end{bmatrix} = \begin{bmatrix} \mu_{1} \\ \mu_{2} \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \end{bmatrix} + \begin{bmatrix} \theta_{11}^{1} & \theta_{12}^{1} \\ \theta_{21}^{1} & \theta_{22}^{1} \end{bmatrix} \begin{bmatrix} \varepsilon_{1t-1} \\ \varepsilon_{2t-1} \end{bmatrix} + \dots + \begin{bmatrix} \theta_{11}^{p} & \theta_{11}^{p} \\ \theta_{11}^{p} & \theta_{11}^{p} \end{bmatrix} \begin{bmatrix} \varepsilon_{1t-p} \\ \varepsilon_{2t-p} \end{bmatrix}$$
(10)

The matrix Ψ_k has the following interpretation:

$$\frac{\delta y_{t+s}}{\delta \varepsilon'_t} = \Psi_s \tag{11}$$

As reported in Hamilton (1994), the row *i*, column *j* element of Ψ_s represents the effect of a one unit increase in the j_{th} variable's innovation at date $t \varepsilon_{jt}$ for the value of the i_{th} variable at time $t + s (y_{i,t+s})$, with all the other variables that are kept constant. If any element changes at the same time by δ_1 , the second by δ_2 and so on, the combined effect is:

$$\Delta y_{t+s} = \frac{\delta y_{t+s}}{\delta \varepsilon_{1t}} \delta_1 + \frac{\delta y_{t+s}}{\delta \varepsilon_{2t}} \delta_2 + \dots + \frac{\delta y_{t+s}}{\delta \varepsilon_{nt}} \delta_n = \Psi_s \delta \tag{12}$$

In order to avoid the problem due to the fact that the errors are correlated, it is possible to use the orthogonalized impulse response function. This tool consists in decomposing the original VAR innovation ($\varepsilon_{1t}, ..., \varepsilon_{nt}$) into uncorrelated components ($u_{1t}, ..., u_{nt}$) and hence study the effects of a unit variation of this uncorrelated components on the variable y_{t+s} .

Besides the orthogonalized impulse response function, the interpretation of the results of the VAR model is helped also by the Variance decomposition. The Forecast error variance decomposition is able to decompose for a dependent variable, the proportion of the movements caused by its own shocks, from the proportion due to shocks on other variables. The variance decomposition is able to distinguish how much of the s - step ahead forecast error variance of a given variable is explained by each variable in the system, for s = 1, 2, ...

Chapter 4: DATA DESCRIPTION

The following section explains illustrates the data useful for the research, highlighting the sources and examining their behavior within the analyzed period.

4.1 Time-series description

The analysis follows the concept of Blau et al. (2021), it is conducted using the time series of Bitcoin Price and 5 year-5 year forward inflation expectation rate, the study is extended by adding the estimation of models that include S&P500 as an exogenous variable. The S&P500 time series helps understand the economy and stock market's behavior (Choi and Shin, 2021). According to Choi and Shin (2021), Bitcoin is correlated to S&P500, their results show that Bitcoin price increases significantly after a positive shock on the stock market.

The time series consist of weekly observations that involve the period from 25th July 2010 to 20th February 2022. It is split into two sub-periods, the first from July 2010 to December 2019, and the second from January 2020 to February 2022. The time series of Bitcoin and S&P500 is downloaded from the website Investing.com, while the observations for the 5y5y forward inflation expectation rate are downloaded from the website of the Federal Reserve Economic Data. The inflation expectation measure is chosen based on Blau et al. (2021)'s study, which uses a 5y5y forward inflation expectation rate. It measures expected inflation (on average) over the five-year period that begins five years from today. The formula is the following:

$$T5YIFR = \left\{ \left[\left(\frac{\left(1 + \left(\frac{BC_{10Year} - TC_{10Year}}{100} \right) \right)^{10}}{\left(1 + \left(\frac{BC_{5Year} - TC_{5Year}}{100} \right) \right)^{5}} \right)^{0.2} \right] - 1 \right\} \times 100$$
(13)

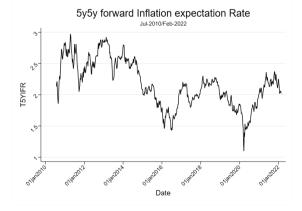
It is obtained from the 5-year and 10-year Treasury Inflation-Protected Securities (TIPS) yields and the 5-year and 10-year bond. The forward rate passes over the short-term noise that affects consumer prices. However, it remains tied to them because of the presence of TIPS that are linked to the consumer price (Bloomberg). The time series of the 5y5y forward Inflation Expectation rate is visible in figure (1), which shows the behavior of the 5y5y forward inflation expectation rate during the period between the 25th of July 2010 and the

20th of February 2022. Following the idea of Chapman et al. (2021) even if Bitcoin is an international currency, which trade all over the world, we decided to use US-based financial indicators to implement our analysis regarding the Bitcoin hedging property. As stated by Chapman et al. (2021), the US- based financial indicators (e.g. T5YIFR and S&P500) are good proxies for studying the movement of the Bitcoin, since the US are the world's leading economy. The U.S. economy often acts as a benchmark for measuring Bitcoin.

From the figure, it is possible to observe that the expected inflation increases between August 2010 and April 2011, passing from a rate of 1.87%, reaching almost 3% in April 2011. The rate returned to stabilize around the 2% threshold in September 2011 and then experienced a further increase until it almost broke through the 3% ceiling in February 2013. It then suffered a reduction until 2016 when it fell below 1.5%. The reduction was probably due to a weak economy, the expected growth of global GDP was equal to 2.8% in 2016. Therefore a weak economy may imply a reduction on the demand side, with consequences on the level of inflation (Forbes, 2015). In 2016 the 5y5y forward inflation expectation rate experienced a jump back to 2%, probably due to the expected expansionary fiscal policy under the incoming Trump administration (Forbes, 2016). In March 2020, expected inflation fell below 1.5% to 1.08%. Inflation expectations moved along with inflation rate developments, energy prices in 2020 experienced their sharpest slump in five years, along with airline tickets, hotel accommodation, and clothing. Since April 2020, however, expectations on the inflation rate have increased, returning to around 2% in December 2020 and then exceeding 2%, growing over 2021, reaching 2.36%. Nowadays, although the inflation rate is recording remarkable highs, reaching about 8%, the 5y5y forward inflation rate registers a value of around 2%, that is the one projected by the Fed.

Figure 1: Times series 5 year- 5 year forward inflation expectation rate

The graph describes the time series of the 5y5y forward inflation expectation rate from July 2010 to February 2022.



Once we have talked about the behavior of the inflation expectation measures, I move on to talking about the Bitcoin price Time Series. As previously said, it was introduced in 2009 and took parity with the dollar for the first time around February 2011. Between 2011 and 2013, there is no particular information to mention, indeed Bitcoin never crosses the hundred dollars until March 2013. The price experimented with an important peak in late 2013, rising from \$150 to over \$1000 (Gandal et al., 2018) in about two months. According to Gandal et al. (2018), the sudden increase in the price of Bitcoin was due to some suspicious activities on the Mt. Gox exchange platform. A crash around December 5th followed the increase, the price lost more than 20%, reaching the value of \$889 after China's first cryptocurrency ban, which prohibits financial institutions from carrying out transactions in Bitcoin. Still, in 2014 rumors spread about the possibility that China would penalize banks that took part in Bitcoin transactions, causing another price drop. The long-term investors bring the price up to 7.345%, throwing off the effects of the rumors.

After the events of 2014, Bitcoin needed two years to come back to its previous values, until January 2017 (Decrypt, 2022), when it went over the threshold of \$1000, giving way to a year of enormous growth with the market capitalization that increased from 7 Billion USD to 28 Billion USD, about 300 percent rise in one year (Gandal et al., 2018).

The 2017 was an important year for crypto. Indeed, it exceeded the 5000 dollars for the first time around September, that event was followed by a drop in price to \$3400. Once September was left behind, BTC started rising again until it reached the first all-time high on December 17th, when it recorded a value of \$19783.21 (Wall Street Journal, 2020). This historical moment was followed by one of the biggest market corrections, which brought the price down to \$11000, corresponding to an approximately 30% drop (Decrypt, 2022). The price continued to drop until 7000\$ in February 2018, stabilizing within a band between \$6000 and 10000 dollars within the whole year, until December 2018, when the price was worth less than \$3300. It is, therefore, recorded a crash approximately equal to 83% with respect to December 2017 (Decrypt, 2022).

Despite the slow start, 2019 was a positive year for Bitcoin, the price reacted to the news that Facebook wanted to launch its digital currency, passing the \$9000 (Independent, 2019).

Besides Facebook, AT&T has also shown its intention to enter the digital currency market, announcing that it would start accepting crypto markets (CNBC, 2019). The intentions of big companies help build the trust for a new sector that had to overcome many obstacles. In 2019, even the Chinese President XI Jinping, who had distanced from the crypto market, prohibiting its use in the financial institution, showed himself in favor of blockchain, stating that China should enter the sector. This opening helped the Bitcoin price, which reached \$10332 in October 2019 (CNBC, 2019).

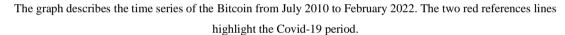
The 2020 did not start well for the cryptocurrency, during the first days of March it lost 50% of its value, reaching \$4000, its lowest value since March 2019. Despite these results, Bitcoin price starts to increase over the year, ending the year with a value equal to \$29374, the historical highest value at that moment, with a percentage increase that overcome the 300% (NextAdvisor, 2022). Besides the motivations related to policies implemented to counter the effects of the pandemic, the price increase can also be linked to an increase in the credibility of the currency itself and greater confidence in the blockchain project. To highlight is the entry of Microstrategy, which has invested hundreds of thousands of dollars in Bitcoin and has also borrowed money from its investors to increase the amount of bitcoin owned (Forbes, 2022).

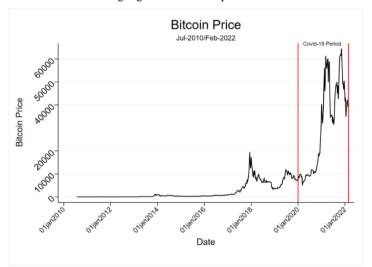
MicroStrategy's purchasing policy continued in 2021 despite the decline suffered by the cryptocurrency. Alongside Microstrategy, other companies have bought Bitcoin, Tesla has claimed to accept payments in digital currency and has supported investment in Bitcoin equal to 1.5 billion, while PayPal has declared to accept transactions in Bitcoin. Having described the Bitcoin time series from 2010 to 2021 is useful in distinguishing some common behaviors that occurred in different periods. For instance, it is important to highlight the differences between the Bitcoin bull market of 2017 and the ones of 2021. According to Andrew Urquhart (The Conversation, 2021), in 2021, the price increase is tied to large-scale institutions entering the bitcoin market, like pension schemes, university endowment funds, and investment trust. Besides that, also important investors entered the market, Paul Tudor Jones and Big insurance funds such as MassMutual invested in the cryptocurrency. Even JP Morgan changed its idea with respect to Bitcoin, sustaining the fact that the companies' market entry may generate a high increase in the demand for the cryptocurrency (CNBC,

2020). The entry of financial institutions and big companies helps change the perception of the crypto, they buy Bitcoin not with the aim of trading it, conversely to accumulate the digital money.

The Covid-19 period was seen as a proper environment for the crypto, indeed, to deal with the financial crisis, governments implemented expansionary monetary policies to lower the interest rate, making credit easier to obtain, and enhancing the functioning of the market. In a situation like this, BTC has been seen as a store of value, as previously said, it seems to not react to economic policy uncertainty, it is independent of government decisions, and its supply is fixed to 21 million. Besides those reasons, even introducing instruments like bitcoin futures and options allowed investors who were afraid of volatility to enter. Nevertheless, 2022 did not start well, Bitcoin reached \$36000 at the end of January.

Figure 2: Time series Bitcoin price





Chapter 5: EMPIRICAL RESULTS and ANALYSIS

This section of the thesis will be organized as follows: it starts with the summarized statistics results and continues illustrating the results of the unit root test and the cointegration test. Finally, the model estimations are presented, firstly the results of the VAR model, and secondly, the results of the VARX model.

5.1 Summarize statistics

The tables (1a)-(1b)-(1c) report some statistics that help in summarize the behavior of the time series that are used into the study in order to implement the models. As it is possible to visualize, within all the period the Bitcoin price record an average value of \$7395.421, with a median that is highly lower than the mean value, thus the Bitcoin price distribution presents a high and positive skewness. The value of the standard deviation is also high, the Bitcoin record during a period of just 12 years a minimum of \$0.1 and a maximum of roughly \$64398. The high value of standard deviation is able to describe the high volatility that characterized Bitcoin during its life. Dividing the whole period into two parts it is possible to notice that the average value is strongly different between the pre-pandemic period and the Covid-19 period. The average value between 2010 to 2019 for BTC price is equal to \$2186, while \$30322 in the second part during which the standard deviation is even higher than the ones during 2010 to 2022. The average value of the 5y5y forward inflation expectation rate oscillates around the 2%, the threshold that is fixed by the FED. The average value is lower during the Covid-19 period than the value that the rate has reported between 2010 to 2019, probably because of the minimum experimented in the March of 2020. The S&P500 reported a higher average value in levels during the pandemic period, with respect to the previous ten years, but a slightly higher standard deviation. Focusing on return, Bitcoin recorder a slightly lower average and standard deviation of log return during the second subperiod, with respect to the first. Moving to the other two variables, the average log returns for both the T5YIFR and S&P500 are very similar within the three different analyzed period. In contrast the standard deviations of log returns are slightly higher during the Covid-19 period.

Table 1a: Summarize statistics whole period (2010-2022)

	Bitcoin, 5v5v Forward inflation Expectations, S&P500										
	Mean Median SD Min Max Skewness										
Bitcoin Price	7395.421	627.900	14052.294	0.100	64398.602	2.490					
T5YIFR	2.175	2.150	0.367	1.100	2.970	0.031					
S&P500	2346.850	2108.100	905.474	1064.590	4766.180	0.821					
Log Return of Bitcoin, 5y5y Forward inflation Expectations, S&P500											
Bitcoin	0.021	0.010	0.158	-0.716	0.823	0.783					
T5YIFR	-0.000	0.000	0.031	-0.226	0.238	0.175					
S&P500	0.002	0.004	0.022	-0.162	0.114	-1.045					

The table presents the statistics of the variables between July 2010 and February 2022

Table 1b: Summarize statistics first sub-period (2010-2019)

The table presents the statistics of the variables between July 2010 and December 2019

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Bitcoin, 5y5y Forward inflation Expectations, S&P500										
	Mean	Median	SD	Min	Max	Skewness				
Bitcoin Price	2186.914	415.400	3478.499	0.100	19345.500	1.781				
T5YIFR	2.231	2.190	0.359	1.430	2.970	0.031				
S&P500	2015.7 50	2023.040	570.544	1064.590	3240.020	0.178				
Log Return of Bitcoin, 5y5y Forward inflation Expectations, S&P500										
Bitcoin	0.023	0.009	0.167	-0.716	0.823	0.873				
T5YIFR	-0.000	0.000	0.026	-0.098	0.099	0.045				
S&P500	0.002	0.003	0.019	-0.075	0.071	-0.575				

Table 1c: Summarize statistics second sub-period (2020-2022)The table presents the statistics of the variables between January 2020 and February 2022

Bitcoin, 5y5y Forward inflation Expectations, S&P500										
Mean Median SD Min Max Skewnes										
Bitcoin Price	30322.155	33667.701	19235.701	5182.700	64398.602	0.154				
T5YIFR	1.931	2.020	0.294	1.100	2.370	-0.589				
S&P500	3804.280	3833.075	623.182	2304.920	4766.180	-0.250				
Log Return of Bitcoin, 5y5y Forward inflation Expectations, S&P500										
Bitcoin	0.015	0.012	0.110	-0.539	0.237	-1.155				
T5YIFR	0.001	0.002	0.045	-0.226	0.239	0.216				
S&P500	0.003	0.007	0.033	-0.162	0.114	-1.256				

5.2: Stationarity results

The stationarity analysis helps in understanding how the models must be estimated. As presented in section 3.2, the stationarity check is conducted by applying two tests, the ADF and the Philippe Perron. The tables below present the ADF and PP test results of the time series, both in levels and in the first difference.

The table 2a summarizes the ADF test results whose null hypothesis is that the variable contains a unit root, while the alternative is that a stationary process generates the variable. For the variables in levels, the test fails to reject the null hypothesis in all the cases, a unit root process generates the variables. The same test is implemented with the variables in the first difference, the null hypothesis for the presence of a unit root is rejected in all the cases at 5% and 1% confidence levels.

Table 2a: ADF test H_0 : Random walk without drift

The table reports the results of the ADF test implemented for the BTC, T5YIFR and S&P in levels and in first difference. The ADF tests if the time series follow a unit-root process. The null is that the variables include a unit root. The alternative is that the variables are stationary.

			L(BTC)	L(BTC)		L(T5YIFR)	L(T5YIFR)		L(S&P)	L(S&P)
		L(BTC)	2010/2019	2020/2022	L(T5YIFR)	2010/2019	2020/2022	L(S&P)	(2010/2019)	2020/2022
Lonola	t-stat	-2.565	-2.558	-1.331	-2.205	-1.875	-1.153	-0.549	-0.901	-0.796
Levels	probability	0.1005	0.1020	0.6150	0.2044	0.3437	0.6936	0.8821	0.7876	0.8204
		>0.05	>0.05	>0.05	>0.05	>0.05	>0.05	>0.05	>0.05	>0.05
First	t-stat	-8.670	-7.788	-3.861	-10.170	-9.228	-4.308	-11.639	-10.408	-4.959
Diff.	probability	0.000	0.000	0.0023	0.0000	0.0000	0.0004	0.000	0.000	0.000
		< 0.05	< 0.05	< 0.05	< 0.05	< 0.05	< 0.05	< 0.05	< 0.05	< 0.05

The same conclusions are drawn from the ADF test that includes a trend in the regression. In the table 2b is clear that all the time series are non-stationary in levels, while the null hypothesis is strongly rejected at 5% and 1% levels in case of first differences.

Table 2b: ADF test H_0 : Random Walk with or without drift – Include a trend in the regression

The table reports the results of the ADF test implemented for the BTC, T5YIFR and S&P in levels and in first difference. The ADF tests if the time series follow a unit-root process. The null is that the variables include a unit root. The alternative is that the variables are stationary. A trend term is included in the regression.

		L(BTC)	L(BTC) 2010/2019	L(BTC) 2020/2022	L(T5YIFR)	L(T5YIFR) 2010/2019	L(T5YIFR) 2020/2022	L(S&P)	L(S&P) (2010/2019)	L(S&P) 2020/2022
Levels	t-stat	-3.322	-2.905	-1.508	-3.226	-3.226	-2.312	-3.133	-3.093	-2.751
	probability	0.0628	0.1605	0.8265	0.0794	0.0794	0.4274	0.0985	0.1078	0.2156
		>0.05	>0.05	>0.05	>0.05	>0.05	>0.05	>0.05	>0.05	>0.05
First	t-stat	-8.820	-7.967	-3.931	-10.170	-9.269	-4.287	-11.629	-10.403	-4.935
Diff.	probability	0.000	0.000	0.011	0.0000	0.000	0.0033	0.000	0.000	0.0003
		< 0.05	< 0.05	< 0.05	< 0.05	< 0.05	< 0.05	< 0.05	< 0.05	< 0.05

The PP test that allows for a shock within the time series seems to confirm the results of the ADF test. Table 3a reports the results of the PP that test the presence of unit root as the null hypothesis, while the alternative establishes that a stationary process generates the variables. The test fails to reject the null hypothesis when the variables are in levels, but in the S&P500. In case of log(S&P) the null is rejected when the whole period and the first sub-period are considered. Still, the null is strongly rejected when the first differences are taken.

Table 3a: PP test H_0 : Random Walk without drift

The table reports the results of the PP test implemented for the BTC, T5YIFR and S&P in levels and in first difference. The PP tests if the time series follow a unit-root process. The null is that the variables include a unit root. The alternative is that the variables are stationary.

		L(BTC)	L(BTC) 2010/2019	L(BTC) 2020/2022	L(T5YIFR)	L(T5YIFR) 2010/2019	L(T5YIFR) 2020/2022	L(S&P)	L(S&P) (2010/2019)	L(S&P) 2020/2022
Lovola	Z(rho)	-11.639	-9.416	-4.782	-17.189	-16.417	-13.044	-26.342	-24.000	-16.902
Levels	Z(t)	-2.922	-2.502	-1.305	-3.006	-3.060	-2.645	-3.660	-3.556	-3.000
	probability	0.1551	0.3271	0.8867	0.1304	0.1161	0.2596	0.0252	0.0338	0.1320
		>0.05	>0.05	>0.05	>0.05	>0.05	>0.05	< 0.05	< 0.05	>0.05
First	Z(rho)	-629.13	-513.133	-114.377	-519.323	-382.467	-106.772	-641.393	-513.390	-121.652
Diff.	Z(t)	-23.706	-21.405	-10.196	-22.376	-18.287	-11.142	-27.089	-24.738	-11.377
	probability	0.000	0.000	0.000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
		< 0.05	< 0.05	< 0.05	< 0.05	< 0.05	< 0.05	< 0.05	< 0.05	< 0.05

Finally, the PP test that allows for the presence of a trend within the regression is implemented. Table 3b reports results that agree with the ADF test, the test fails to reject the null hypothesis of non-stationarity for all the time series in level, while the stationarity of the time series in the first difference is confirmed at 5% and 1% level of confidence.

Table 3b: PP test H_0 : Random Walk with or without drift – Include a trend in the regression

The table reports the results of the PP test implemented for the BTC, T5YIFR and S&P in levels and in first difference. The PP tests if the time series follow a unit-root process. The null is that the variables include a unit root. The alternative is that the variables are stationary. A trend term is included in the regression.

		L(BTC)	L(BTC) 2010/2019	L(BTC) 2020/2022	L(T5YIFR)	L(T5YIFR) 2010/2019	L(T5YIFR) 2020/2022	L(S&P)	L(S&P) (2010/2019)	L(S&P) 2020/2022
Levels	Z(rho)	-3.315	-3.398	-2.124	-10.038	-7.343	-3.517	-0.779	-1.052	-1.872
	Z(t)	-2.568	-2.545	-1.320	-2.239	-1.844	-1.321	-0.603	-0.857	-0.949
	probability	0.0998	0.1050	0.6198	0.1923	0.3590	0.6197	0.8704	0.8018	0.7715
		>0.05	>0.05	>0.05	>0.05	>0.05	>0.05	>0.05	>0.05	>0.05
First	Z(rho)	-630.2	-514.340	-114.354	-519.343	-382.700	-106.775	-641.388	-513.487	-121.674
Diff.	Z(t)	-23.616	-21.297	-10.175	-22.396	-18.299	-11.198	-27.112	-24.762	-11.428
	probability	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
		< 0.05	< 0.05	< 0.05	< 0.05	< 0.05	< 0.05	< 0.05	< 0.05	< 0.05

5.3: Cointegration results

The unit root tests have shown that the time series are I (1), therefore, following the idea of Anders (2014) reported in section 3.2, testing the cointegration between the variable is essential to understand what model should be estimated. Table 4 reports the results of the Johansen cointegration test performed with an unrestricted constant in the model, a linear trend in the cointegrating equation and a restricted trend in the model. The tests analyze the cointegration between Log (BTC), Log(T5YIFR) and Log(S&P500) first, then the cointegration between the Log (BTC) and the Log(T5YIFR). The cointegration between the time series is tested within all the periods considered in the analysis. As it is possible to visualize from table 4, the null hypothesis of rank = 0 is mainly rejected both at 5%, in the other cases the null is not rejected at 1% level. We conclude that there is no evidence of a long-run relationship between the variables. Therefore, following the idea of Anders presented in the section 3.2, the models are estimated in the first difference. The variables will be computed in the following way:

$$\Delta \log(BTC_t) = \log(BTC_t) - \log(BTC_{t-1}) = \log\left(\frac{BTC_t}{BTC_{t-1}}\right)$$
(14)

The previous formula computes the log return of Bitcoin, the same is applied to the time series of log(T5YIFR) and log (S&P500).

Table 4: Johansen cointegration test with unrestricted constant, restricted trend, and linear trend

The table reports the results of the Johansen cointegration test estimated including a constant, a restricted trend and a trend in the regression for BTC, T5YIFR and S&P500. It reports the value of the trace statistics in correspondence of rank = 0. It is possible to notice that the null hypothesis of rank = 0 is not rejected, therefore there is no evidence of cointegration.

				Trace statistics	
	Lags	Max. rank	Constant	Rtrend	Trend
L(BTC) L(T5YIFR) L(S&P)	5	0	26.292**	37.644**	33.977**
L(BTC) L(T5YIFR) L(S&P) (2010-2019)	5	0	32.007***	40.9724**	36.3965***
L(BTC) L(T5YIFR) L(S&P) (2020-2022)	5	0	15.197**	31.2709**	30.2685**
L(BTC) L(T5YIFR)	5	0	17.273***	24.4770**	21.3238***
L(BTC) L(T5YIFR) (2010-2019)	5	0	19.983***	27.3640***	23.0517**
L(BTC) L(T5YIFR) (2020-2022)	5	0	10.808**	11.358**	12.320**

*** p<0.01, ** p<0.05

5.4: Multivariate regressions results

The following section will present the result of the model estimation. The section reports the results obtained applying the VAR model and VARX model for the whole period firstly, then the first and second sub-period. The test for the model checking follows the table of the coefficient estimation. Finally, the Granger causality and Impulse response function results are reported.

5.4.1: VAR model estimation for the whole period (2010-2022)

The following results derived from the application of the VAR model for the whole period between July 2010 to February 2022. The Akaike Information Criterion suggests estimating the model considering five lags for both the endogenous variables. The model estimated with five lags shows autocorrelation at the fifth lag. The estimate is then repeated with four lags for both Bitcoin and 5y5y forward inflation expectation log return. The results are summarized in Table 5. Focusing on the first column, the coefficients of the lags of BTC log-returns are significant at 1% and 10% levels. In contrast, the coefficients of the lags of T5YIFR log return seem to have a mostly not significant relationship with the $\Delta Log (BTC_t)$. Considering the second column, which has $\Delta Log (T5YIFR_t)$ as the dependent variable, the coefficients linked to the lag of ΔLog (BTC) are mostly significant at the 1% level, suggesting that there is probably a relationship between the two variables. Before proceeding with the estimation of the test that can better explain the relationship between the two variables, it is essential to analyze the stability of the VAR model. As it is possible to visualize from table 5, all the eigenvalues lie inside the unit circle, hence the VAR model satisfies the stability condition. The residuals autocorrelation is checked by applying the multiplier with hypothesis: Lagrange test the null H_0 : no autocorrelation at the lag order. Table 5 shows that the test fails to reject the null hypothesis at lag 4.

Table 5: VAR model estimation for the whole period (2010-2022) Equation 7.1-7.2 Table 5 displays the results of the VAR 7.1-7.2. The sample period goes from 25th of July 2010 to 20th of February 2022. The estimation is performed considering 4 lags for $\Delta \text{Log}(BTC_t)$ and $\Delta \text{Log}(T5YIFR_t)$. The first column has the $\Delta \text{Log}(BTC_t)$ as dependent variable, while the second column has $\Delta \text{Log}(T5YIFR_t)$ on LHS. Table reports the coefficients estimates, the eigenvalue stability condition, the Lagrange multiplier test for residuals autocorrelation, provides the Granger causality test.

Variables	$\Delta \text{Log}(BTC_t)$	$\Delta \text{Log} (T5YIFR_t)$	
$\Delta \text{Log}(BTC_{t-1})$	0.039	0.026***	
	(.041)	(.008)	
$\Delta \text{Log}(BTC_{t-2})$	0.068*	0.005	
	(.041)	(.008)	
$\Delta \text{Log}(BTC_{t-3})$	0.111***	-0.021***	
	(.041)	(.008)	
$\Delta \text{Log}(BTC_{t-4})$	-0.077*	-0.004	
	(.041)	(.008)	
$\Delta \text{Log}(T5YIFR_{t-1})$	0.055	0.080**	
	(.211)	(.041)	
$\Delta Log (T5YIFR_{t-2})$	0.396*	-0.024	
	(.210)	(.040)	
$\Delta \text{Log}(T5YIFR_{t-3})$	0.138	-0.031	
	(.210)	(.040)	
$\Delta \text{Log}(T5YIFR_{t-4})$	-0.293	-0.081**	
	(.208)	(.040)	
Constant	0.019***	-7.11E-05	
	(.007)	(.001)	
Observations	600	600	

Standard errors in parentheses						
*** p<0.01, ** p<0.05, * p<0.1						
	Eigenvalue stability condition					
Eigenva	lue	Modulus				
0.452±.3	399	0.603				
-0.43±.3	99	0.566				
-0.298±.4	462	0.55				
0.336±.1	.89	0.384				
	Lagrange-multiplier test					
lag	lag chi2					
1	10.873	0.028				
2	7.584	0.108				
3	11.118	0.025				
4	5.55	0.235				
	Granger causality test					
	No Granger causality from T5YIFR	No Granger causality from BTC to				
	to BTC	T5YIFR				
	[1]	[2]				
χ^2	6.511	17.699				
$P > \chi^2$	0.164	0.001				

The analysis proceeds by performing the Granger causality Wald test to analyze if ΔLog $(T5YIFR_t)$ does not Granger Cause $\Delta \text{Log} (BTC_t)$ $(i. e. \pi_{12}^1 = \pi_{12}^2 = \pi_{12}^3 = \pi_{12}^4 = 0)$ and if $\Delta \text{Log} (BTC_t)$ does not Granger cause $\Delta \text{Log} (T5YIFR_t)$ $(i. e. \pi_{21}^1 = \pi_{21}^2 = \pi_{21}^3 = \pi_{21}^4 = 0)$.

Table 5 shows that the test fails to reject the null hypothesis of no Granger causality from $\Delta \text{Log}(T5YIFR_t)$ to $\Delta \text{Log}(BTC_t)$, while rejects H_0 in the other direction, hence $\Delta \text{Log}(BTC_t)$ seems to help predict the $\Delta \text{Log}(T5YIFR_t)$.

The Granger causality test is helpful in understanding if there is a relationship between the lags of the variables used in the estimation of the model. The IRF could be helpful in understanding the sign of the relationship. The figure (3) shows the effect that an exogenous shock imposed on Bitcoin has on the T5YIFR. It exhibits the weekly steps on the x - axisand the basis point changes on the y - axis. The graph highlights that a one standard deviation exogenous positive shock on the ΔLog (BTC) generates an increase in ΔLog $(T5YIFR_t)$ of more than two basis points after one week. The effect starts reducing, having reached the 2-basis point change, becoming negative after about three weeks. After the three weeks the shock effect is not significant anymore at 95% level. The figure suggests a positive relationship between the variables, since the T5YIFR appears to positively respond to a shock on BTC. The orthogonalized impulse response function, which works by transforming the variance-covariance matrix into a lower triangular matrix, drawn in the figure (4), follows the same pattern as the Generalized IRF represented in the figure (3). The shock on the $\Delta Log (T5YIFR)$ is lower, as expected. A unit shock on the $\Delta Log (BTC)$ generates a 0.5 basis points increase on ΔLog (T5YIFR). As in figure 3 the effect of the shock decreases after two weeks becoming non-significant at 95% level after three weeks.

Figure 3: IRF impulse of Log Return (BTC) on Log Return (T5YIFR) (2010-2022)

The figure describes the response of T5YIFR to an exogenous shock on BTC during the period 2010-2022.

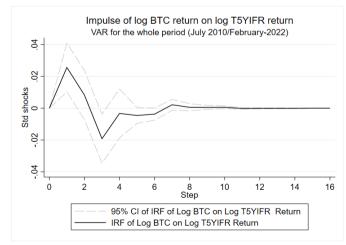


Figure 4: Orthogonal IRF impulse of Log Return (BTC) on Log Return (T5YIFR) (2010-2022)

The figure describes the response of T5YIFR to an orthogonal exogenous shock on BTC during the period 2010-2022

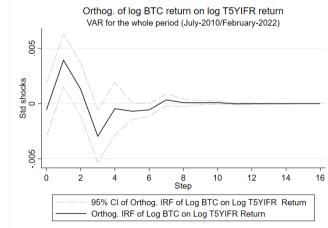
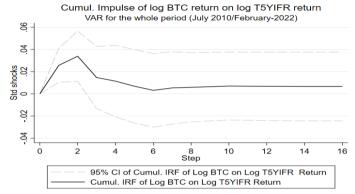


Figure 5 considers the cumulative IRF of the Δ Log (BTC) on the Δ Log (*T5YIFR*). As expected, considering the figure (4), the shock on the Bitcoin produces an increase in the T5YIFR of more than 2 bp. After the 2 bp increase, the effect remains positive but lower. After three weeks, the effect of the shock is not significantly different from 0. The relationship between the two variables seems to be positive. The IRF, in the other direction, does not show evidence of a significant response of BTC after a shock on T5YIFR.

Figure 5: Cumul. IRF impulse of Log Return (BTC) on Log Return (T5YIFR) (2010-2022) The figure describes the cumulative response of T5YIFR to an exogenous shock on BTC during the period 2010-2022.



5.4.2: VAR model estimation for the first sub-period (2010-2019)

The following section presents the result of model for the first sub-period, from July 2010 to December 2019, before the Covid-19 pandemic. It is estimated with five lags for each endogenous variables since the M-AIC is minimized in correspondence to the lag order five. The estimation results of VAR model for the first sub-period are presented in table 6. If we go through it, a difference is visible with respect to the model for the whole period. The coefficients of the Δ Log ($T5YIFR_t$) in the first equation becomes significative, the second lag and the fifth lag show a positive relationship with Δ Log (BTC_t) and are respectively significative at 1% and 5% levels. The fourth lag shows a negative relationship with Bitcoin and a significance at a 5% level. Examining the relationship from Bitcoin to T5YIFR, it is observable that, in contrast with the previous model, the coefficient significance decreased from 1% to 5%; in addition, the coefficient of the first lag of Bitcoin log return decreased in terms of size. The analysis proceeds by investigating the stability condition and residuals autocorrelation. The model is stable because the eigenvalues have a modulus lower than one, as shown in the table 6. The Lagrange multiplier test rejects the null hypothesis of autocorrelation in residuals at the lag order five, as displayed by table 6.

Variables	$\Delta Log (BTC_t)$	$\Delta \text{Log} (T5YIFR_t)$		
$\Delta \text{Log}(BTC_{t-1})$	0.060	0.016**		
	(.045)	(.007)		
$\Delta \text{Log}(BTC_{t-2})$	0.067	-0.007		
	(.045)	(.007)		
$\Delta \text{Log}(BTC_{t-3})$	0.108**	-0.016**		
	(.044)	(.007)		
$\Delta \text{Log}(BTC_{t-4})$	-0.101**	-0.004		
	(.044)	(.007)		
$\Delta \text{Log}(BTC_{t-5})$	0.122***	0.010		
	(.044)	(.007)		
$\Delta \text{Log}(T5YIFR_{t-1})$	0.113	0.179***		
	(.287)	(.045)		
$\Delta Log (T5YIFR_{t-2})$	0.853***	-0.026		
	(.290)	(.046)		
$\Delta \text{Log}(T5YIFR_{t-3})$	0.14	-0.067		
	(.289)	(.046)		
$\Delta \text{Log}(T5YIFR_{t-4})$	-0.908***	0.047		
	(.290)	(.046)		
$\Delta Log (T5YIFR_{t-5})$	0.702**	-0.011		
	(.286)	(.045)		
Constant	0.017**	-2.82E-05		
	(.008)	(.001)		
Observations	487	487		
	Standard errors in parentheses			
	*** p<0.01, ** p<0.05, * p<0.1			
	Eigenvalue stability condition			
Eigenva	alue	Modulus		
-0.566±	.474	0.739		
0.67	5	0.675		
0.326±.	583	0.668		
-0.111±		0.607		
-0.59	9	0.599		
0.432±.	226	0.488		
	Lagrange-multiplier test			
lag	chi2	$P > \chi^2$		
1	3.036	0.552		
2	5.895	0.207		
3	8.218 0.084			
4	5.158	0.271		
5	4.236	0.375		
	Granger causality test	No Company and 1'de from DTC de		
	No Granger causality from T5YIFR	No Granger causality from BTC to		
	to BTC	T5YIFR		
2	[1]	[2]		
χ^2	23.271	11.499		
$P > \chi^2$	0.00	0.042		

 Table 6: VAR estimation for the first sub-period (2010-2019) Equation 7.1-7.2

 Table 6 displays the results of the VAR model 7.1-7.2. The sample period goes from July 2010 to December

2019. The estimation is performed considering 5 lags for $\Delta \text{Log}(BTC_t)$ and $\Delta \text{Log}(T5YIFR_t)$. The first column has the $\Delta \text{Log}(BTC_t)$ as dependent variable, while the second column has $\Delta \text{Log}(T5YIFR_t)$ on LHS. Table reports the coefficients estimates, the eigenvalue stability condition, the Lagrange multiplier test for residuals

The results obtained from the model estimation suggest a relationship in both the direction, from Bitcoin to T5YIFR and vice-versa. I proceed by applying the Granger causality Wald test that has the following null hypotheses: $\Delta \text{Log} (T5YIFR_t)$ does not Granger Cause $\Delta \text{Log} (BTC_t)$ (*i.e.* $\pi_{12}^1 = \pi_{12}^2 = \pi_{12}^3 = \pi_{12}^4 = \pi_{12}^5 = 0$), and $\Delta \text{Log} (BTC_t)$ does not Granger cause $\Delta \text{Log} (T5YIFR_t)$ (*i.e.* $\pi_{12}^1 = \pi_{21}^2 = \pi_{21}^3 = \pi_{21}^4 = \pi_{21}^5 = 0$). Table 6 reports the results of the Granger causality Wald test. It is observable a difference with respect to the model for the whole period; in this case, the test for no Granger causality rejects the null hypothesis in both directions, from T5YIFR to BTC, it is rejected at a 1% level, while in the other direction it is rejected at 5% level. Therefore, it appears that there is evidence of Granger causality in both the directions.

We estimate the IRF to examine the sign of the relationship that links the two variables. As it is possible to visualize from figure (6), a one standard deviation shock on the log return of Bitcoin generates an increase in the log return of T5YIFR of about 2 bp in one week, immediately after the effect starts decreasing becoming non-significant at a 95% level. The orthogonal IRF (figure 7) returns a similar result in terms of significance, an exogenous shock on BTC is positive and significant in two weeks, while after, it is not significant at a 95% confidence level. The cumulative response function could help understand the effect of the shock of Bitcoin.

Figure 6: IRF impulse of Log Return (BTC) on Log Return (T5YIFR) (2010-2019)

The figure describes the response of T5YIFR to an exogenous shock on BTC during the period 2010-2019

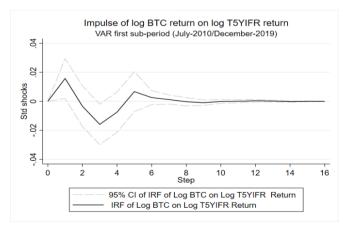
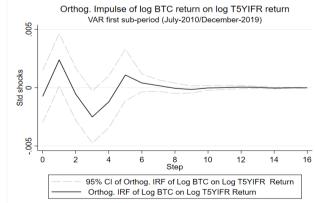


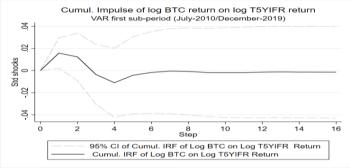
Figure 7: Ort. IRF impulse of Log Return (BTC) on Log Return (T5YIFR) (2010-2019)

The figure describes the response of T5YIFR to an orthogonal exogenous shock on BTC during the period 2010-2019



The Cumulative IRF (figure 8) seems to agree with the Granger causality test that rejects the null hypothesis at a 5% level instead of 1% level. The shock effect is positive and significant for less than two weeks, while there is no evidence of a relationship between the two variables after the two weeks. Excluding the Covid-19 period it seems that there is a weaker connection from Bitcoin to T5YIFR.

Figure 8: Cumulative IRF impulse of Log Return (BTC) on Log Return (T5YIFR) (2010-2019) The figure describes the cumulative response of T5YIFR to an exogenous shock on BTC during the period 2010-2019.



The IRF is also estimated in the other direction to analyze Bitcoin's response to a shock in inflation expectation. The generalized IRF in figure (9) shows that an exogenous positive shock on T5YIFR during the pre-pandemic sub-period increments the log Bitcoin returns by about 100 bp after two weeks. A decrease follows this maximum value in the extent of the effect that reaches -100 bp after four weeks. After five weeks, the effect is not significant at 95% level. Taking a look at the Orthogonal IRF (figure 10), the path is similar to figure (10), but as expected, the effect size on the Bitcoin is smaller than the Generalized IRF. The shock on the T5YIFR produces a positive increase of 2 bp on the Log BTC return that starts to decrease, becoming negative in four weeks. As in figure (9), the effect loses its significance after five weeks.

Figure 9: IRF impulse of Log Return (T5YIFR) on Log Return (BTC) (2010-2019) The figure describes the response of BTC to an exogenous shock on T5YIFR during the period 2010-2019

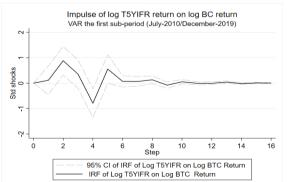
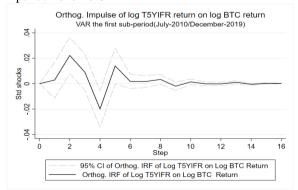
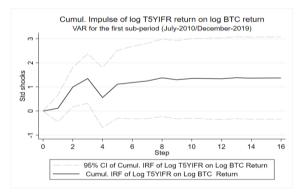


Figure 10: Orthogonal IRF impulse of Log Return (T5YIFR) on Log Return (BTC) (2010-2019) The figure describes the response of BTC to an orthogonal exogenous shock on T5YIFR during the period 2010-2019



The result derived from the Cumulative IRF, in figure 11, seems to suggest that Bitcoin could be used as a hedge against inflation expectations according to the definition previously sustained in the literature review. The Cumulative IRF shows that during the sub-period from 2010 to 2019, the relationship between the inflation expectation and the Bitcoin is positive. The one standard deviation shock on the ΔLog ($T5YIFR_t$) positively affects Bitcoin Log Return. It is worth noticing that the cumulative IRF becomes non-significant after three weeks, suggesting that the relationship between BTC and T5YIFR is not too strong between 2010 and 2019. Moreover, even if the relationship is positive as in Choi and Shin (2020), in this case the significance of the response of Bitcoin to a shock on T5YIFR is limited to a short period. The difference could be represented by the fact that Choi and Shin use the 5year breakeven inflation rate as a measure for inflation expectation, instead of the 5y5y forward inflation expectation rate.

Figure 11: Cumulative IRF impulse of Log Return (T5YIFR) on Log Return (BTC) (2010-2019) The figure describes the cumulative response of BTC to an exogenous shock on T5YIFR during the period 2010-2019



5.4.3: VAR model estimation for second sub-period (2020-2022)

The last estimated model is VAR model which considers the Covid-19 pandemic period and goes from January 2020 to February 2022. It is still a VAR (2) with $\Delta Log (BTC_t)$ and ΔLog $(T5YIFR_t)$ on the LHS and the lags of the two variables on the RHS. The M-AIC suggests estimating the model with four lags. As previously mentioned, we proceed by valuing the results of the estimations that are summarized in table 7. The first column has $\Delta Log (BTC_t)$ as the dependent variable, and it is clear that there are no significant coefficients. The lag of log BTC return affects the log return of T5YIFR. As results from the table 7, the sub-period of the Covid-19 pandemic shows a significant relationship from BTC to T5YIFR, the first, second, and third lags of Bitcoin in the second equation are significant, at least at a 5% level, in addition, the first two coefficients show a strong positive relationship between the two variables. Both variables, BTC and T5YIFR, registered a strong negative shock at the beginning of March in response to uncertainty due to the spread of Covid-19, but both strongly recovered after the strong negative shock of March 2020, exhibiting a high positive shock. In a sense, a positive relationship between the variables is expected if their pattern is analyzed. As observed in table 7, the model satisfies the stability condition, all the eigenvalues lie inside the unit circle. Furthermore, the Lagrange multipliers test fails to reject the null hypothesis of no autocorrelation in residuals at lag order four.

The results obtained from the model estimation suggest a relationship in only one direction, from Bitcoin to T5YIFR. To better test the jointly significance of the coefficient estimated, we proceed applying the Granger causality Wald test that has the following null hypotheses: $\Delta \text{Log} (T5YIFR_t)$ does not Granger Cause $\Delta \text{Log} (BTC_t)$ (*i.e.* $\pi_{12}^1 = \pi_{12}^2 = \pi_{12}^3 = \pi_{12}^4 = 0$) and $\Delta \text{Log} (BTC_t)$ does not Granger cause $\Delta \text{Log} (T5YIFR_t)$ (*i.e.* $\pi_{21}^1 = \pi_{21}^2 = \pi_{21}^3 = \pi_{21}^4 = \pi_{21}$

Table 7 shows the results of the Granger causality Wald test. It is identifiable a difference with respect to the model for the first sub-period, indeed the test fails to reject the null hypothesis of Granger causality from T5YIFR to BTC, while the no Granger causality hypothesis in the opposite direction is rejected at 1% level. The results of this test are in line with the ones obtained by Blau et al. (2020), even adding the 2021 and the first two months

of 2022 and considering weekly data against the daily observations implemented by Blau et

al. (2020).

Table 7: VAR estimation for the second sub-period (2020-2022) Equation 7.1-7.2Table 7 displays the results of the VAR model 7.1-7.2. The sample period goes from January 2020 to February2022. The estimation is performed considering 4 lags for $\Delta Log (BTC_t)$ and $\Delta Log (T5YIFR_t)$. The first columnhas the $\Delta Log (BTC_t)$ as dependent variable, while the second column has $\Delta Log (T5YIFR_t)$ on LHS. Panel Areports the coefficients estimates, Panel B provides the Granger causality test, Panel C the eigenvalue stabilitycondition, Panel D the Lagrange multiplier test for residuals autocorrelation.

Variables	$\Delta \text{Log}(BTC_t)$	$\Delta \text{Log} (T5YIFR_t)$
$\Delta \text{Log}(BTC_{t-1})$	0.0256	0.150***
	(.094)	(.033)
$\Delta \text{Log}(BTC_{t-2})$	0.0676	0.103***
	(.101)	(.036)
$\Delta \text{Log}(BTC_{t-3})$	0.00311	-0.0766**
	(.104)	(.037)
$\Delta \text{Log}(BTC_{t-4})$	0.0615	0.0274
	(.105)	(.037)
$\Delta Log (T5YIFR_{t-1})$	-0.0864	-0.0906
	(.255)	(.091)
$\Delta Log (T5YIFR_{t-2})$	-0.102	-0.0216
	(.252)	(.089)
$\Delta Log (T5YIFR_{t-3})$	0.0973	-0.0111
	(.243)	(.086)
$\Delta Log (T5YIFR_{t-4})$	0.315	-0.248***
	(.226)	(.080)
Constant	0.0121	-0.00174
	(0.0106)	(0.00374)
Observations	112	112

	Standard errors in parentheses					
	*** p<0.01, ** p<0.05, * p<0.1					
	Eigenvalue stability condition					
Eigenva	lue	Modulus				
-0.55±.:	534	0.767				
0.404±.	526	0.664				
0.604	l l	0.604				
-0.60	1	0.601				
0.112±.	491	0.503				
	Lagrange-multiplier test					
lag	chi2	$P > \chi^2$				
1	1.792	0.774				
2	5.315	0.257				
3	3.195	0.526				
4	1.173	0.883				
	Granger causality test					
	No Granger causality from T5YIFR	No Granger causality from BTC to				
	to BTC	T5YIFR				
	[1] [2]					
χ^2	2.383	35.202				
$P > \chi^2$	0.666	0.00				

The analysis proceeds to estimate the IRF by first imposing an exogenous shock on the Bitcoin log return and then applying an exogenous shock on the T5YIFR log return. The results that derive imposing an exogenous shock on the Bitcoin log return and treating both BTC and T5YIFR as endogenous variables are similar to the ones obtained by Blau et al. (2020). The Generalized IRF (figure 12) displays an increase of 15 bp in terms of T5YIFR log return in one week after a one standard deviation shock on BTC log return. The effect needs more than two weeks to go below 0 and then stabilize after about six weeks. The Orthogonal IRF (figure 13) follows the same course as the Generalized, showing an effect on the T5YIFR that is smaller in terms of size. The increase after one week is more than one bp.

Figure 12: IRF impulse of Log Return (BTC) on Log Return (T5YIFR) (2020-2022)

The figure describes the response of T5YIFR to an exogenous shock on BTC during the period 2020-2022

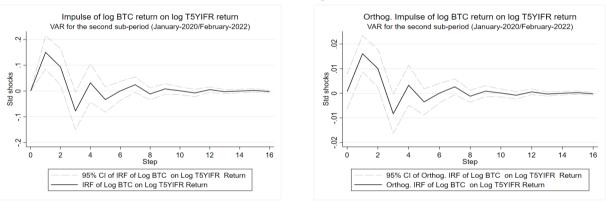


Figure 13: Orthogonal IRF impulse of Log Return (BTC) on Log Return (T5YIFR) (2020-2022)

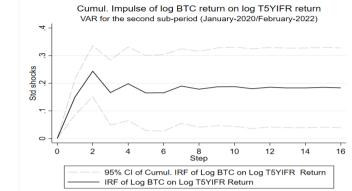
The figure describes the response of T5YIFR to an

orthogonal exogenous shock on BTC during the

period 2020-2022

The Cumulative IRF (figure 14) is very similar to the ones in Blau et al (2020). The periods included in Blau et al. (2020) and in the following study are different, the first one goes from 2019 to 2020, while this one includes 2020, 2021 and the first two months of 2022, both the studies include the pandemic period. It shows an increase of more than 20 bp after two weeks, then it stabilizes around the 20 bp during the next periods. It is essential to notice that for the first time the cumulative IRF remains significant during the whole weeks, suggesting that the relationship between the BTC and T5YIFR became stronger in the last two years. The IRFs of the previous time period analyzed report that there was a relationship between the variables, however it was limited to a short period. The results that follow from the model on the second sub-period, instead, sustaining that Bitcoin leads increasing in inflation expectation, indeed a positive shock on Bitcoin generate an increase in T5YIFR that is significant for 16 weeks in the case of the figure (14).

Figure 14: Cumulative IRF impulse of Log Return (BTC) on Log Return (T5YIFR) (2020-2022) The figure describes the cumulative response of T5YIFR to an exogenous shock on BTC during the period 2020-2022.



It is not found evidence of a significative response of BTC to a shock on T5YIFR. The study results are in line with the findings of Blau et al. (2020), as previously said. Following Blau et al.'s idea, it is worth reporting the results of Narayan et al. (2019), according to which the Bitcoin growth rate is related to Indonesia's monetary aggregate, generating increasing inflation. The positive correlation from Bitcoin to inflation expectation could also depend on the common behavior that the two variables have shown during the initial stages of the spread of Covid. It might be that Bitcoin anticipated the expected inflation both in the downward phase in March 2020 and in the upward phase that affected both variables during the entire 2020 and 2021. In both the sub-period, we found a positive correlation between the two variables, with a difference in the leading one. In the pre-pandemic period, T5YIFR seems to lead increase in BTC, in opposition to the pandemic where BTC leads increase in inflation expectation. Even if in model on first sub-period T5YIFR seems to lead Bitcoin, as aforementioned, it is essential to highlight that the response in IRF is limited to a short period, in contrast with the result derived from the model on the second sub-period. These differences are observable in table 8 where are reported the results of Forecast error variance decomposition. In table 8 VAR model first sub-period, a shock on the T5YIFR explains more of the BTC's behavior, if compared to the portion of T5YIFR explained by a shock on BTC. This result is strongly reversed in case of VAR model second sub-period, where results that a shock on Bitcoin can explain the 22% of the behavior of the T5YIFR log return after 16 weeks.

Table 8: Forecast error variance decomposition for VAR model (2010-2019) and VAR model (2020-2022)

unira	third the portion of BTC variation due to a shock on T5YIFR. The fo					1			Ι ΙΓΚ.
VAR model first sub-period				VAR n	nodel second sub	-period			
	(1) fevd	(2) fevd	(3) fevd	(4) fevd		(1) fevd	(2) fevd	(3) fevd	(4) fevd
0	0	0	0	0	0	0	0	0	0
1	1	0.0008	0	0.999	1	1	0.0003	0	0.999
2	0.999	0.009	0.0003	0.991	2	0.999	0.150	0.0009	0.849
3	0.981	0.009	0.019	0.990	3	0.998	0.197	0.002	0.803
4	0.978	0.019	0.022	0.981	4	0.996	0.226	0.003	0.774
5	0.964	0.021	0.036	0.979	5	0.985	0.221	0.015	0.779
6	0.958	0.023	0.042	0.977	6	0.985	0.224	0.015	0.776
7	0.958	0.023	0.042	0.977	7	0.985	0.224	0.015	0.776
8	0.958	0.023	0.042	0.977	8	0.985	0.227	0.015	0.773
9	0.957	0.023	0.043	0.977	9	0.985	0.227	0.015	0.773
10	0.957	0.023	0.043	0.977	10	0.985	0.227	0.015	0.773
11	0.957	0.023	0.043	0.977	11	0.985	0.227	0.015	0.773
12	0.957	0.023	0.043	0.977	12	0.985	0.227	0.015	0.773
13	0.957	0.023	0.043	0.977	13	0.985	0.227	0.015	0.773
14	0.957	0.023	0.043	0.977	14	0.985	0.227	0.015	0.773
15	0.957	0.023	0.043	0.977	15	0.985	0.227	0.015	0.773
16	0.957	0.023	0.043	0.977	16	0.985	0.227	0.015	0.773

Table 8 reports the results of Forecast error variance decomposition for VAR for first and second sub-period. For each model the first column displays the portion of BTC variation due to a shock on Bitcoin. The second the portion of T5YIFR variation due to a shock on BTC. The third the portion of BTC variation due to a shock on T5YIFR. The fourth the portion of T5YIFR variation due to a shock on T5YIFR.

5.4.4: VARX model estimation for the whole period (2010-2022)

The VARX model for the whole period is estimated with five lags for the endogenous variables and three lags for the exogenous variable. The three lags of S&P500 are chosen based on the significance, while the lags of the endogenous variables are selected applying the IC. The results of the model estimation are reported in table 9. The lags of log return of S&P500 have a highly significant positive relationship with the log return of the T5YIFR, meaning that an increasing economy might positively affect inflation expectation. In contrast Bitcoin does not record any significant relationship with the S&P500 log return in the period between 2010 to 2022. The second lag of the T5YIFR has a higher coefficient in the case of VARX model with respect to the VAR model which analyze the whole period, while the significance is still equal to 10% level. Focusing attention on the second column the significative coefficient of the lag of log return of Bitcoin are in absolute value lower with respect to the VAR model estimated for the same period. The model, as results from the table 9 is respectively stable and does not show autocorrelation at lag residuals.

Variables	$\Delta \text{Log}(BTC_t)$	$\Delta \text{Log} (T5YIFR_t)$	
$\Delta Log (BTC_{t-1})$	0.051	0.018**	
	(0.0408)	(0.00706)	
$\Delta Log (BTC_{t-2})$	0.054	-0.003	
	(0.0410)	(0.00709)	
$\Delta Log (BTC_{t-3})$	0.096**	-0.026***	
	(0.0408)	(0.00706)	
$\Delta Log (BTC_{t-4})$	-0.086**	-0.005	
	(0.0410)	(0.00710)	
$\Delta Log (BTC_{t-5})$	0.106***	0.002	
	(0.0410)	(0.00709)	
$\Delta Log (T5YIFR_{t-1})$	0.048	0.040	
	(0.233)	(0.0403)	
$\Delta Log (T5YIFR_{t-2})$	0.436*	-0.091**	
	(0.232)	(0.0401)	
$\Delta Log (T5YIFR_{t-3})$	0.131	-0.025	
5. LU /	(0.209)	(0.0362)	
$\Delta Log (T5YIFR_{t-4})$	-0.380*	-0.068*	
	(0.209)	(0.0362)	
$\Delta Log (T5YIFR_{t-5})$	0.317	0.082**	
	(0.209)	(0.0361)	
$\Delta Log (S\&P500_{t-1})$	-0.025	0.580***	
	(0.291)	(0.0504)	
$\Delta Log (S\&P500_{t-2})$	0.149	0.111**	
	(0.323)	(0.0559)	
ΔLog (S&P500 _{t-3})	0.098	0.204***	
ALog (See 500(=3)	(0.321)	(0.0555)	
Constant	0.016**	-0.002	
Constant	(0.00663)	(0.00115)	
Observations	599	599	
0.000000000	Standard errors in parentheses		
	*** p<0.01, ** p<0.05, * p<0.1		
	Eigenvalue stability condition		
Eigenva	ĕ •	Modulus	
563±		0.706	
305 ± .		0.700	
		0.629	
$.583 \pm .0$		0.583	
.152 ± .		0.524	
563±		0.706	
	Lagrange-multiplier test	··· · · ·	
lag	chi2	$P > \chi^2$	
1	3.540	0.472	
2	3.913	0.412	
2 3	1.252		
		0.869	
4	7.648	0.105	
5	4.347	0.361	
	Granger causality test		
	No Granger causality from T5YIFR	No Granger causality from BTC to	
	to BTC	T5YIFR	
	[1]	[2]	
v ²	9.1827	19.098	
$\frac{\lambda}{P > \chi^2}$	0.102	0.002	

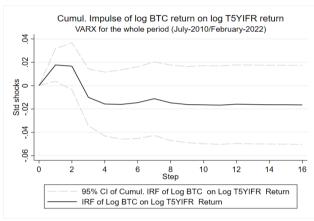
Table 9: VARX estimation for the whole period (2010-2022) Equation 8.1-8.2Table 9 displays the results of the VARX model 8.1-8.2. The sample period goes from 25^{th} of July 2010 to 20^{th} of
February 2022. The estimation is performed considering 4 lags for $\Delta Log (BTC_t)$ and $\Delta Log (T5YIFR_t)$. The first

column has the $\Delta Log (BTC_t)$ as dependent variable, while the second column has $\Delta Log (T5YIFR_t)$ on LHS. The

The results of Granger causality test in table 9 reports evidence of Granger causality from BTC to T5YIFR, the null is rejected at 1% level. In contrast, there is no evidence of Granger causality from T5YIFR to BTC.

The Cumulative IRF is estimated to understand the sign of the relationship after controlling for the S&P500. Figure 15 shows that a one standard deviation shock on log return of Bitcoin has a positive effect on the log return of T5YIFR, after one week the shock produce a 2 bp increase in the T5YIFR. Comparing the results with the ones of figure 5, in this case the effect is lower, and its significance is restricted to one week only. There is no evidence of a significative effect on BTC of a shock on T5YIFR.

Figure 15: Cumulative IRF impulse of Log Return (BTC) on Log Return (T5YIFR) (2010-2022) The figure describes the cumulative response of BTC to an exogenous shock on T5YIFR during the period 2010-2022 for the VARX model.



5.4.5: VARX model estimation for first sub-period (2010-2019)

The VARX model for the first sub-period is estimated including three lags for the exogenous variable, and five lags for each of the endogenous variables based on the results of the IC. The lags of log return of S&P500 have still a positive relationship with the inflation expectation, while does not show any significative relationship with the Bitcoin log return. During the first sub-period the lags of log return of T5YIFR report a significative relationship with the BTC confirming the result of VAR model estimated for the same time period. On the other side Bitcoin show a significative correlation with the T5YIFR. The first lag is significant at 10% level, less than the case of VAR model for first sub-period, while the third lag has a size higher in absolute value with respect to VAR model for 2010-2019. The model is stable and does not show autocorrelation at lag order as it is clear from table 10.

Variables	$\Delta \text{Log}(BTC_t)$	$\Delta \text{Log} (T5YIFR_t)$		
$\Delta Log (BTC_{t-1})$	0.059	0.011*		
	(0.0449)	(0.00662)		
$\Delta Log (BTC_{t-2})$	0.065	-0.0098		
	(0.0446)	(0.00658)		
$\Delta Log (BTC_{t-3})$	0.104**	-0.020***		
	(0.0445)	(0.00657)		
$\Delta Log (BTC_{t-4})$	-0.106**	-0.003		
	(0.0447)	(0.00660)		
$\Delta Log (BTC_{t-5})$	0.121***	0.007		
	(0.0447)	(0.00660)		
$\Delta Log (T5YIFR_{t-1})$	0.019	0.133***		
	(0.304)	(0.0449)		
$\Delta Log (T5YIFR_{t-2})$	0.802***	-0.052		
	(0.302)	(0.0446)		
$\Delta Log (T5YIFR_{t-3})$	0.162	-0.052		
	(0.291)	(0.0430)		
$\Delta Log (T5YIFR_{t-4})$	-0.919***	0.054		
	(0.290)	(0.0429)		
$\Delta Log (T5YIFR_{t-5})$	0.721**	0.017		
	(0.287)	(0.0424)		
$\Delta Log (S\&P500_{t-1})$	0.005	0.422***		
	(0.395)	(0.0583)		
$\Delta Log (S \& P500_{t-2})$	0.296	0.227***		
	(0.416)	(0.0614)		
ΔLog (S&P500 _{t-3})	0.380	0.178***		
$\Delta \log (500 \text{ f} - 3)$	(0.415)	(0.0612)		
Constant	0.016**	-0.001		
Constant	(0.00767)	(0.00113)		
	Standard errors in parentheses	(0100110)		
	*** p<0.01, ** p<0.05, * p<0.1			
	Eigenvalue stability condition			
Eigenva		Modulus		
562 ± .4		0.729		
.316 ± .6		0.679		
0.626		0.626		
092 ±		0.566		
-0.514		0.514		
0.512		0.512		
0.245	Lagrange-multiplier test	0.245		
lag	chi2	$P > \chi^2$		
1	4.006	0.405		
2	1.271	0.405		
2 3	1.271 1.147	0.880		
4	3.191 0.526			
5	4.167	0.320		
	Granger causality test			
	No Granger causality from T5YIFR	No Granger causality from BTC to		
	to BTC	T5YIFR		
	[1]	[2]		
χ^2	21.605	14.565		
$P > \chi^2$	0.001	0.012		

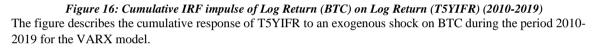
 Table 10: VARX estimation for the first sub-period (2010-2019) Equation 8.1-8.2

 Table 10 displays the results of the VARX model 8.1-8.2. The sample period goes from July 2010 to December 2019.

The estimation is performed considering 5 lags for $\Delta \text{Log}(BTC_t)$ and $\Delta \text{Log}(T5YIFR_t)$. The first column has the $\Delta \text{Log}(BTC_t)$ as dependent variable, while the second column has $\Delta \text{Log}(T5YIFR_t)$ on LHS. The variable $\Delta \text{Log}(S\&P500)$ is exogenous. Table reports the coefficients estimates, the eigenvalue stability condition, the Lagrange multiplier test for

Table 10 reports the results of Granger causality test. It is found evidence of Granger causality from T5YIFR to BTC at 1% level, while from BTC to T5YIFR at 5% level. The results are similar to the VAR for the first sub-period.

The results of the Cumulative IRF seem to not confirm the ones obtained from the VAR model for the first sub-period. In that case impulse on BTC had a slightly positive effect on the T5YIFR, this effect is completely cancelled in the case of VARX model. From figure 16 we observe that the effect is not significant at 95% level. The result is different if the focus moves on the effect on BTC of an impulse on T5YIFR. The effect is significant after three weeks and produce an increase in log return of BTC of about 100 bp (figure 17). The significance is lower than the ones recorder in VAR model for first sub-period, and it is very limited to a short period.



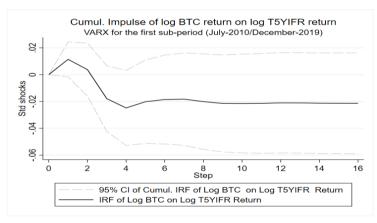
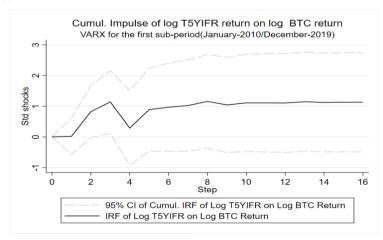


Figure 17: Cumulative IRF impulse of Log Return (T5YIFR) on Log Return (BTC) (2010-2019) The figure describes the cumulative response of BTC to an exogenous shock on T5YIFR during the period 2010-2019 for the VARX model.



5.4.6: VARX model estimation for second sub-period (2020-2022)

The last VARX model aims to analyze the second sub-period, the ones referred to the Covid-19 pandemic. As previously done, the lag of the exogenous variable included in the model are three, based on their significance. The M-AIC suggests using one lag for both the endogenous variable. The lag of the S&P500 shows for the first time a significant relationship with the log return of Bitcoin at 10% level, while the relationship with the log return of T5YIFR is at 1% level. The T5YIFR has not a significant effect on the BTC, while the Bitcoin seems to have a positive relationship with T5YIFR with a significance of 1% level. The model is stable and does not show autocorrelation at lag order one (Table 11).

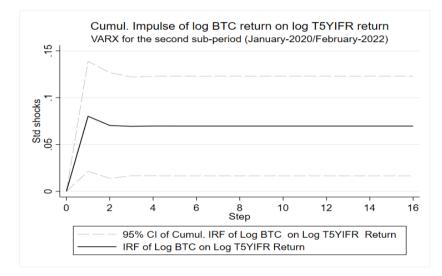
Table 11 displays the results of the VARX model 8.1-8.2. The sample period goes from January 2020 to February 2022. The estimation is performed considering 4 lags for $\Delta \text{Log}(BTC_t)$ and $\Delta \text{Log}(T5YIFR_t)$. The first column has the $\Delta \text{Log}(BTC_t)$ as dependent variable, while the second column has $\Delta \text{Log}(T5YIFR_t)$ on LHS. The variable $\Delta \text{Log}(S\&P500)$ is exogenous. Table reports the coefficients estimates, the eigenvalue stability condition, the Lagrange multiplier test for residuals autocorrelation, provides the Granger causality test.

Variables	$\Delta \text{Log}\left(BTC_{t}\right)$	$\Delta \text{Log} (T5YIFR_t)$				
$\Delta Log (BTC_{t-1})$	0.036	0.0797***				
	(0.0964)	(0.0298)				
$\Delta Log (T5YIFR_{t-1})$	-0.395	-0.160*				
	(0.302)	(0.0934)				
$\Delta Log (S\&P500_{t-1})$	0.058	0.830***				
	(0.334)	(0.103)				
$\Delta Log (S\&P500_{t-2})$	0.733*	0.005				
	(0.409)	(0.127)				
Constant	0.013	-0.002				
	(0.0103)	(0.00320)				
Observations	112	112				
	Standard errors in parentheses					
	*** p<0.01, ** p<0.05, * p<0.1					
	Eigenvalue stability condition					
Eigenval	Eigenvalue Modulus					
$-0.062 \pm 0.$	148	0.160				
	Lagrange-multiplier test					
lag	chi2	$P > \chi^2$				
1	4.108	0.392				
	Granger causality test					
	No Granger causality from T5YIFR	No Granger causality from BTC to				
	to BTC	T5YIFR				
	[1] [2]					
χ^2	1.707	7.134				
$P > \chi^2$	0.191	0.008				

The Granger causality test for the VARX for the second sub-period confirms the results of the VAR model for the same time period, indeed as the table 11 illustrates, it is found evidence of Granger causality from Bitcoin to T5YIFR at 1% level, while there is no evidence in the other direction, confirming the possibility that Bitcoin is able to lead inflation expectation increase during the pandemic period. Finally the Cumulative IRF result is reported in figure 18. As in the case of the VAR model for the second sub-period, the effect of a shock on Bitcoin produces an increase in T5YIFR of about 7-8 bp, lower than the 20 bp reported by the Cumulative IRF of the VAR model.

Figure 18: Cumulative IRF impulse of Log Return (BTC) on Log Return (T5YIFR) (2020-2022)

The figure describes the cumulative response of BTC to an exogenous shock on T5YIFR during the period 2020-2022 for the VARX model.



Chapter 6: CONCLUSION

The thesis investigates whether the Bitcoin can be a valuable asset to hedge inflation risk, if the Bitcoin and inflation expectations are correlated and how one variable reacts to an exogenous shock on the other.

The VAR and VARX models are implemented to study the correlation between the variables, while the Granger causality test is estimated to examine the forecast ability of the variables. Finally, the response of each variable to an exogenous shock on the other is examined by applying the IRF and FEVD. The research is conducted over a period ranging from 25th July 2010 to 20th February 2022, which is split into two sub-period, the first from July 2010 to December 2019, the second from January 2020 to February 2022.

The analysis of the whole period displays a significative influence of BTC on T5YIFR, with a significant Granger causality from BTC to T5YIFR. The Cumulative IRF reports a positive but shortly significant positive response from expected inflation to a positive shock on BTC.

The first sub-period displays a weaker influence of BTC on T5YIFR, while the T5YIFR seems to affect the BTC. Furthermore, in both directions there is evidence of Granger causality. Still, the Cumulative IRF reports a weak response of T5YIFR to a shock on BTC in the case of the VAR model, while after the VARX, the significance of the response of T5YIFR disappears. On the other side, the response of BTC to a shock on T5YIFR is positive and high, equal to 100 bp after three weeks, but still, its significance does not last long.

The second sub-period exhibits an influence of BTC on T5YIFR, furthermore, the BTC Granger causes T5YIFR. The cumulative IRF, for the first time, displays a strong response of T5YIFR to a shock on BTC. The response of T5YIFR lasts a long time both in case of VAR and VARX model. Further, from the FEVD results that a shock on BTC explains 22% of the variation of T5YIFR. These results confirm the idea of Blau et al (2021).

These finding suggests that, particularly during the pandemic period, Bitcoin behaves as inflation hedging. Regarding the first sub-period, although a positive correlation was found, this did not prove to be significant for a long time.

It seems that Bitcoin was able to lead by anticipating increases in the inflation expectation rate in the second sub-period. The development of hedging property might be due to the changes that have affected the Bitcoin market over the past two years. The entry of institutional investors may have influenced the perception of Bitcoin, stimulating the concept of Bitcoin as a hedging tool. Alongside this, also a greater adoption as a payment method.

It is essential to point out that Bitcoin market is still in constant evolution and poorly regulated. In addition, the available sample is not very vast because of the recent introduction of Bitcoin. It is, therefore, important to point out that the results obtained, and their interpretation must be taken with caution.

It is worth notice the drop in value that Bitcoin has suffered in 2022. The positive correlation between Bitcoin and inflation expectation might be confirmed by the last results. Fed, indeed, expected to hike rate in 50 basis-point steps in coming months to beat inflation, generating expectations of decreasing inflation. The positive correlation found in the research seems to be confirmed by the common behavior that Bitcoin and inflation expectations experimented in 2022.

For future research could be interesting to compare Bitcoin and Gold behavior with respect to inflation, having a larger sample size. Gold is considered a valuable hedging tool against inflation in the US and UK (Hoang, Lahiani, and Heller, 2016). Despite this, Baur and Glover (2012) showed that investors behavior could erode gold's hedging potential due to the rise of speculative investments. The interest in Bitcoin has been boosted in the last few years. The two assets share some characteristics: both are independent of political influence; no central authority controls their mining, Bitcoin's supply is tied to the solution of an algorithm; the supply of both assets is fixed, and their value is also related to their scarcity. The thesis followed, in part, the methodology of Blau et al. (2021); the analysis was conducted by applying log returns for the variables, while in Blau et al. (2021), simple net returns are implemented. Having available a larger sample, it would be useful to repeat the analysis using risk-adjusted returns, given the volatility that has characterized the cryptocurrency. In this regard, it is essential to underscore that Bitcoin has also been impressively valuable in terms of risk-adjusted returns. It has outperformed, in terms of risk-adjusted returns, assets such as gold, real estate, and stocks for every four-year holding period since 2013 (Woolbool, 2022).

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