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Abstract
The time of retirement is analyzed in a theoretical framework taking capability and health into account. Capability if formalized as a stock characteristic which determines the attained amounts of a composite good which yields utility. The model is purposely simple and comprises one choice variable – the time of retirement. The core assumption is that inherited capability influences the rate of evolvement of health, and vice versa, and that the rates of change of the stocks differ between the pre- and post-retirement periods. The optimal retirement timing decision is characterized and the effects of the model's exogenous variables on this decision are examined. We derive refutable comparative statistics results with respect to the model's exogenous variables, and, for example, show – for a specified version of the model – how the timing of retirement depends on the inherited amounts of capability and health.

Introduction

The concept of capability has been used in several different contexts all pertaining more or less closely to attained, or the capability of attaining, fulfillment of human needs. It has been employed in empirical research in different fields. It has been criticized for lacking the quantitative precision needed for, for instance, theoretical analysis undertaken with the purpose of guiding public policy (Agee and Crocker, 2013). Quantitative definitions of individual capability involve, somewhat simplified, realized potential in relation to some exogenous full potential. For instance, Sen's definition of capability distinguishes between resources in themselves and the ultimate goals that are attained using these resources. Capability measures the ability of extracting useful characteristics of available resources (Sen, 1985). D'Agata (2007) argues, based on Sen's work, that capability is a dynamic notion that may be thought of and conceptualized in a way similar to that of human capital. To best of our knowledge, no theoretical analysis of individual economic decisions relate to capability has been published in the peer-reviewed literature.

People tend to differ both when it comes to what is considered valuable in life (preferences), and when it comes to the ability to convert resources into valuable achievements (Sen, 1992; Sen, 1995). The capability approach as described by Sen (Sen, 1993, 1985a, 1985b, 1990) theorises the notion of health in relation to the relative freedom people have to do and to be what they have a reason to value, i.e. their capabilities. By putting focus on people's freedom to achieve rather than actual achievements, the capability approach allows us to understand people's real opportunities to lead the kind of life they want to lead (Sen, 1992). Its main constituents are functionings and capabilities, representing the different outcomes people have a reason to value (functionings), and the opportunities people have to achieve a valued outcome in relation to their personal characteristics and environmental aspects (capabilities) (Sen, 1992; Sen, 1995). Thus, the capability approach distinguished between what people strive to achieve and their freedom to achieve it, while the normative focus of the healthy ageing discourse's, in which definitions of a good life with emphasise the instrumental value of achievements. In contrast, capability approach focuses both the instrumental value and the intrinsic value of freedom for a person's health and wellbeing (Sen, 1992; Sen, 1995). It is important to remember, however, that the capability approach is an evaluative framework and not a specified theory (Robeyns, 2006). The objective of this paper is to analyse the timing of retirement and how this decision is determined jointly by individual capability and health. To this end the notion of capability is formalised and integrated in a human-capital type theoretical model.

In order to quantify the ensuing analysis, we adopt the view that the core feature of the capability concept can be said to be that specific individual capabilities determine how much welfare an individual derives from a given amount of resources. In a generalized consumer production perspective, capability can be thought of as the degree of allocative efficiency, i.e., the individual's ability to choose the cost-minimizing input vector in order to produce a given amount of an output, or as the degree of productive efficiency, i.e., the efficiency of the particular technology used. Less than perfect allocative efficiency means that an input vector other than the cost-minimizing input vector is used to produce at the same isoquant, resulting in a higher cost for producing a given amount of the output. Similarly, less than perfect productive efficiency means that a technology which is less effective than the most effective available technology is used.

In the next section we develop a formalized framework for analyzing, primarily, the relationship between capability and retirement. However, as other factors are seemingly important in the retirement decision, the model includes, implicitly, pension rights and health. The theoretical model adopts Sen's (1997) views about the relationship between human capital and human capability. The theoretical model is purposely simple and models one decision: when to retire. Capability and health are inherited by circumstances during upbringing and genes, and we assume that both these characteristics of the individual evolves over time by rates that are exogenously given, but may differ between pre- and post-retirement periods.

The model

The formal model involves an individual who faces a deterministic length of life (T), and who has to make the decision about when to retire from the labour market. The decision is assumed to be made so as to maximize lifecycle utility, taking the evolvement over time of capability, health and post-retirement income (pension rights) into account. Health depreciates at exogenous rates, pre- and post-retirement, over the entire planning period, while capability growths as long as the individual does not retire, and starts depreciating thereafter. Thus, being active in the labour market affects both health and capability. Post-retirement income is given by a simple pension system which yields higher payouts the longer the individual remains in the labour market. Thus, the choice of time of retirement, R, is the only decision variable in the model. Time supplied to the labour market before retirement is a fixed share of total time. In the ensuing paragraphs we lay out the details of the model.

Capability is conceptualized as an inherent individual characteristic, which facilitates domestic production. We adopt a heuristic approach in modelling the actual doubtless complex relationship between capability and health, assuming that the amount of each stock accumulated during early life determines the rate at which the other stock evolves over time. Of course, some personal traits may be possible to influence by actions undertaken as a result of individual decision making. For instance, health has been viewed in this way in the health economic literature. That is not to say that exogenous factors are unimportant. In fact, there are reasons to believe that, to a considerable extent, health evolves over a person's life due to exogenous factors. Analogously, a person's capability is likely to be determined, to a significant extent, by circumstances outside the influence of that person. Here, we assume that labour-market activity improves a person's capability and that from the point in time when that person retires his or her capability starts to decline. CAPABILITY AND SOCIAL CAPITAL - REF

More formally, we assume that utility at each point in time is derived from a consumption good, $C(t)$. The commodity is produced by the individual using a production technology $C(t) = (\theta(t)H(t)(1-\ell)y)$, where $\theta(t) \in \mathbb{R}_+$ is capability, $H(t) \in \mathbb{R}_+$ is health at time t, ℓ, $0 \leq \ell \leq 1$, is time supplied to the labour market (time is normalized to 1), and y_ℓ is income from time supplied to the labour market. For simplicity, we assume that preferences are represented by a monotonically strictly increasing and strictly concave utility function, $U\big(C(t)\big)$. Preferences are identical before and after retirement and, hence, state-dependent preferences do not explain individual behaviour. Income from time supplied to the labour market, y_ℓ, and income after retirement (earned pension rights), y_R, are exogenous and constant.

A more formal definition of individual capability is as follows:

Definition capability and health: Capability, $\theta(t)$, and health, $H(t)$, are individual characteristics with the following productivity and stock properties:

(i) $\frac{\partial C(t)}{\partial \theta(t)} > 0, \frac{\partial C(t)}{\partial H(t)} > 0$

(ii) $\dot{\theta}(t) = \delta|_{H_0}\theta(t)$, where $\delta|_{H_0}$ is a constant and H_0 is inherited health, and

(iii) $\dot{H}(t) = \sigma|_{\theta_0}H(t)$, where $\sigma|_{\theta_0}$ is a constant, and θ_0 is inherited capability.

This follows the established view of health in the health-economic literature, with the addition that inherited health influences the rate of change of capability. Capability is defined analogously, but with a critical difference: while health declines, albeit with different rates, during the two stages of the lifecycle, capability grows as long the individual stays in the labour market.

Let's turn our attention to the details of what distinguishes the two stocks. So, we assume that the capability stock is added to by labour-market participation, and starts to depreciate after retirement (in both cases at exogenous rates), and, in contrast, that the health stock depreciates both before and after retirement (again, in both cases at exogenous rates). The stocks of capability and health at the start of the individual's planning period are formed by childhood and adolescent experiences and circumstances and genetic inheritance. Formally, the levels of capability and health at the outset are given by $\theta_0 \in (0,1]$, where $\theta_0 = 1$ is fully developed childhood potentials, and $H_0 \in (0,1]$, where $H_0 = 1$ is perfect health.

Capability over the lifecycle

The rate of capability growth before retirement is $\delta_\ell \in \mathbb{R}_+$, while the rate at which the stock of capability declines after retirement is $\delta_R \in \mathbb{R}_+$. We assume that inherited health improves the mechanism by which capability is accumulated. Thus, before retirement capability appreciates according to $\dot{\theta}(t) = (\delta_\ell + \alpha_H H_0)\theta(t)$, and depreciates after retirement according to $\dot{\theta}(t) = -(\delta_R - \beta_H H_0)\theta(t)$, where $\alpha_H \in \mathbb{R}_+$ and $\beta_H \in \mathbb{R}_+$ reflects the importance of health for the accumulation of capability before and after retirement, respectively. It follows that the capability time-path is completely determined by the initial level of capability and the time of retirement, R. For $t < R$, we have $\theta_\ell(t) = c_1 e^{(\delta_\ell + \alpha_H H_0)t} \Rightarrow \theta_\ell(t) = \theta_0 e^{(\delta_\ell + \alpha_H H_0)t}$. Similarly, for $t \geq R$, we have $\theta_R(t) = c_2 e^{(\delta_R - \beta_H H_0)(R-t)}$. Then, using the matching condition that $\theta_\ell(R) = \theta_R(R)$, we get $c_2 = \theta_0 e^{(\delta_\ell + \alpha_H H_0)R} \Rightarrow \theta_R(t) = \theta_0 e^{(\delta_\ell + \alpha_H H_0)R} e^{(\delta_R - \beta_H H_0)(R-t)}$.

Health over the lifecycle

The evolvement of the health stock over time is assumed to depend on inherited capability: the larger inherited capability the slower health depreciates. Thus, the health stock depreciates from its initial value, H_0, according to $\dot{H}(t) = -(\sigma_\ell - \alpha_\theta \theta_0)H(t)$, $t < R$, and $\dot{H}(t) = -(\sigma_R - \beta_\theta \theta_0)H(t)$, $t \geq R$, where $\alpha_\theta \in \mathbb{R}_+$ and $\beta_\theta \in \mathbb{R}_+$ reflects the importance of capability for the evolvement of health before and after retirement, respectively. Thus, $H_\ell(t) = H_0 e^{-(\sigma_\ell - \alpha_\theta \theta_0)t}$, and $H_R(t) = H_0 e^{-(\sigma_\ell - \alpha_\theta \theta_0)R} e^{(\sigma_R - \beta_\theta \theta_0)(R-t)}$. We assume that $\sigma_R > \sigma_\ell$. , *Income as retired*

Pension rights are earned as long as the individual remains in the labour market. The regulated time for retirement is R_L. Income after retirement is added to if the actual time of retirement, R, is $R > R_L$. Thus, income after retirement is equal to $y_R(R) = y_0 + y_\ell \mu (R - R_L)$, where $y_0 \in \mathbb{R}_+$ is the level of retirement income if $R = R_L$. We assume that $y_R(0) = y_0 - y_\ell \mu R_L > 0$.

Consumption before and after retirement

Consumption before and after retirement is given by:

$$C_\ell(t) = \theta_\ell(t) H_\ell(t)(1-\ell) y_\ell, \tag{1}$$

and

$$C_R(t;R) = \theta_R(t;R) H_R(t;R)(y_0 + y_\ell \mu (R - R_L)). \tag{2}$$

Pulling all this together, the decision problem becomes

$$V(\boldsymbol{\beta}) \stackrel{\text{def}}{=} \max_R \left[\int_0^R U(C_\ell(t)) e^{-\rho t} dt + \int_R^T U(C_R(t)) e^{-\rho t} dt \right], \tag{3}$$

where $\boldsymbol{\beta} \stackrel{\text{def}}{=} (H_0, \theta_0, \delta_\ell, \delta_R, \sigma_\ell, \sigma_R, \mu, T, \rho)$, and ρ is the rate of time preferences. The ensuing assumptions are placed on problem (1).

Assumption 1: There exists an interior solution, $R^0 \in (0, T)$, to the first-order necessary condition when $\boldsymbol{\beta} = \boldsymbol{\beta}^0 \in \mathbb{R}^9_{++}$.

Assumption 2: The second-order sufficient condition holds at $(R^0, \boldsymbol{\beta}^0) \in (0, T) \times \mathbb{R}^8_{++}$ (see the appendix).

Assumption 1 in conjunction with Assumption 2, means that the prerequisites of a well-known local sufficiency theorem are satisfied. Thus, it may be invoked to conclude that the value $R^0 \in (0,T)$ is the unique local solution to problem (3). Moreover, the conditions of the implicit function theorem are met under in this situation. Consequently, it follows from the implicit function theorem that the first-order necessary condition given in Eq. (4) may, in principle, be solved for R as a locally $C^{(1)}$ function of $\boldsymbol{\beta}$ in an open neighborhood of the point $(R^0,\boldsymbol{\beta}^0)$, yielding $R = R^*(\boldsymbol{\beta})$. Accordingly, a differential comparative statics analysis of problem (3) may be carried out.

The maximum present value of lifetime utility is defined as:

$$V(\boldsymbol{\beta}) \stackrel{\text{def}}{=} \int_0^{R^*} U\big(C_\ell(t)\big)e^{-\rho t}dt + \int_{R^*}^{T} U\big(C_{R^*}(t)\big)e^{-\rho t}dt. \tag{4}$$

Notice, that $V(\boldsymbol{\beta})$ is at least a $C^{(1)}$ function locally. The first term on the right-hand side is the discounted lifetime utility up until retirement, while the second term is the discounted lifetime utility from the time of retirement until the last moment in life.

Preliminary results and intuition

Recall that $R = R^*(\boldsymbol{\beta})$ satifies the first-order necessary condition of problem (3). The first-order condition is found by applying Liebniz's rule and the chain rule to (4):

$$U\big(C_\ell(R)\big)e^{-\rho R} - U\big(C_R(R;R)\big)e^{-\rho R} + \int_R^T \frac{\partial U\big(C_R(t;R)\big)e^{-\rho t}}{\partial R}dt = 0, \tag{5}$$

or more explicitly

$$U\big(\theta_R(R)H_R(R)(y_0 + y_\ell\mu(R-R_L))\big)e^{-\rho R} - U(\theta_\ell(R)H_\ell(R)(1-\ell)y_\ell)e^{-\rho R} =$$

$$\int_R^T \frac{\partial U\big(\theta_R(t)H_R(t)(y_0+y_\ell\mu(R-R_L))\big)e^{-\rho R}}{\partial R}dt = 0. \tag{6}$$

The left-hand side is the instantaneous marginal cost of delaying retirement, while the right-hand side is the marginal utility of delaying retirement. The sign of the integrand depends on the effect on $C_R(t;R)$ of a change in the time of retirement, R. Inspecting the expressions for $\theta_R(t)$ and $H_R(t)$ reveals how the sign of the derivative depends on the relative size of the parameters of the optimization problem (1).

To derive the comparative statics, substitute $R = R^*(\boldsymbol{\beta})$ in Eq. (5) to create an identity in the parameter vector, and then differentiate the identity with respect to the parameter of interest using Leibniz's rule and the chain rule. This is the approach followed to derive the ensuing comparative statics expressions. Note, that all such ensuing expressions are evaluated at R^*, and that $D < 0$ by assumption (A2), where D is defined in the Appendix.

Consider a change in parameter x. The resulting change in $R = R^*(\boldsymbol{\beta})$ is given by:

$$\left[\frac{\partial}{\partial x} U\big(C_\ell(R)\big)e^{-\rho R} - \frac{\partial}{\partial x} U\big(C_R(R)\big)e^{-\rho R} + \frac{\partial}{\partial x}\int_R^T \frac{\partial U\big(C_R(t)\big)e^{-\rho t}}{\partial R} dt\right] dx +$$

$$\underbrace{\frac{\partial}{\partial R}\left[U\big(C_\ell(R)\big)e^{-\rho R} - U\big(C_R(R)\big)e^{-\rho R} + \int_R^T \frac{\partial U\big(C_R(t)\big)e^{-\rho t}}{\partial R} dt\right]}_{D} dR = 0. \tag{7}$$

Thus, the effect of a change in parameter x on the timing of retirement is given by

$$\frac{\partial R}{\partial x} = -\frac{U'\big(C_\ell(R)\big)\partial_x C_\ell(R)e^{-\rho R} - U'\big(C_R(R)\big)\partial_x C_R(R)e^{-\rho R} + \int_R^T \frac{\partial^2 U\big(C_R(t)\big)e^{-\rho t}}{\partial R \partial x} dt}{D}. \tag{8}$$

From the first-order condition and the assumption of an interior solution it clear that if $\int_R^T \frac{\partial U(C_R(t))e^{-\rho t}}{\partial R} dt \neq 0$, then either $U\big(C_\ell(R)\big)e^{-\rho R} - U\big(C_R(R;R)\big)e^{-\rho R} > 0$ or $U\big(C_\ell(R)\big)e^{-\rho R} - U\big(C_R(R;R)\big)e^{-\rho R} < 0$. Then, concavity of the utility functions implies that the sign of $U'\big(C_\ell(R)\big)\partial_x C_\ell(R) - U'\big(C_R(R)\big)\partial_x C_R(R)$ is ambiguous, since if $U\big(C_\ell(R)\big)e^{-\rho R} - U\big(C_R(R)\big)e^{-\rho R} > 0$ it must be the case that $U'\big(C_\ell(R)\big)e^{-\rho R} -$

$U'\big(C_R(R)\big)e^{-\rho R} < 0$ while $\partial_x C_\ell(R) - \partial_x C_R(R) >< 0$. Notice, that one might proceed by (i) assuming either $C_\ell(R) > C_R(R)$ or $C_\ell(R) < C_R(R)$, (ii) from the assumption in (i) making the possible inference about the sign of the integrand, and (iii) making an assumptions about the marginal rate of substitution between the retired and not retired states at $t = R$, which in some cases will be related to the income related terms in the two states, i.e., $y_0 + y_\ell \mu(R - R_L)$ and $(1-\ell)y_\ell$. The required additional assumptions have no obvious practical or empirical meaning. Therefore, we instead proceed by specifying the utility function in more detail, retaining the monotonicity and concavity assumption.

An example – logarithmic utility

A relatively simple case arises when the utility function is logarithmic, i.e., $U\big(C(t)\big) = lnC(t)$. This preserves the complementarity between capability, health, income, and leisure time, in the production of the consumption commodity, but restricts the class of utility functions that represents the individual's preferences. In this case we have:

$$U\big(C_\ell(t)\big) = ln\theta_0 + (\delta_\ell + \alpha_H H_0)t + lnH_0 - (\sigma_\ell - \alpha_\theta \theta_0)t + ln\big((1-\ell)y_\ell\big), \text{ and}$$

$$U\big(C_R(t;R)\big) = ln\theta_0 + (\delta_\ell + \alpha_H H_0)R + (\delta_R - \beta_H H_0)(R-t) + lnH_0 - (\sigma_\ell - \alpha_\theta \theta_0)R + (\sigma_R - \beta_\theta \theta_0)(R-t) + ln(y_0 + y_\ell \mu(R - R_L)). \quad (9\text{-}10)$$

The first-order condition (5) becomes:

$$ln\big((1-\ell)y_\ell\big)e^{-\rho R} - ln(y_0 + y_\ell \mu(R-R_L))e^{-\rho R} + \int_R^T (\delta_\ell + \alpha_H H_0 + \delta_R - \beta_H H_0 - \sigma_\ell + \alpha_\theta \theta_0 + \sigma_R - \beta_\theta \theta_0 + (y_0 + y_\ell \mu(R-R_L))^{-1}\mu y_\ell)e^{-\rho t}dt = 0. \quad (11)$$

Generally, depending on the relative size of the parameters of Eq. (11), the individual will choose to retire at $0 < R < T$, at $R = 0$, or to never retire. Obviously, for our purposes the first choice is the most interesting.

However, before we proceed let's briefly consider two cases that characterize the importance of capability which will prove helpful when considering comparative statics of the optimal timing problem.

Observation 1: There exists a α_θ, $\bar{\alpha}_\theta$, such that the integrand in Eq. (11) is always positive. Thus, for $\alpha_\theta \geq \bar{\alpha}_\theta$, an interior solution necessitates that $ln[(1-\ell)y_\ell] - ln[y_0 + y_\ell \mu(R - R_L)] < 0$.

Observation 2: There exists a β_θ, $\bar{\beta}_\theta$, such that the integrand in Eq. (11) is always negative. Thus, for $\beta_\theta \geq \bar{\beta}_\theta$, an interior solution necessitates that $ln[(1-\ell)y_\ell] - ln[y_0 + y_\ell \mu(R - R_L)] > 0$.

Where appropriate in the ensuing analyses we will distinguish between type 1, $\alpha_\theta \geq \bar{\alpha}_\theta$, and type 2, $\beta_\theta \geq \bar{\beta}_\theta$, individuals referring to these two cases. A type 1 individual is characterized by the initial stock of capability being more important for health development when in the labour force than when retired.

Proposition 1: A type 1 individual will retire later than a type 2 individual, ceteris paribus.
Proof: The proposition follows from observations 1 and 2 and from inspecting the first two terms in the first-order condition Eq. (11).

Qualitative results

The ensuing comparative statics results are derived using the approach outlined above. Again, note, that all such ensuing expressions are evaluated at R^*, and that $D < 0$ by assumption (A2), where is D defined in the Appendix. For ease of denotation, in what follows we omit function arguments where suitable.

Inherited stocks of capability and health

The effect of an increase in inherited capability and in inherited health, respectively, on the optimal timing of retirement is given by

$$\frac{\partial R^*}{\partial \theta_0} = -\frac{\int_{R^*}^{T}(\alpha_\theta - \beta_\theta)e^{-\rho t}dt}{D}. \tag{12}$$

$$\frac{\partial R^*}{\partial H_0} = -\frac{\int_{R^*}^{T}(\alpha_H - \beta_H)e^{-\rho t}dt}{D}. \tag{13}$$

Assuming that inherited capability is more important for health in the first part of the lifecycle, while in the labour force, i.e., $\alpha_\theta > \beta_\theta$, it follows that $\frac{\partial R^*}{\partial \theta_0} > 0$. That is, an individual with a larger stock of inherited capability will retire later than an individual with a lower stock, ceteris paribus. Moreover, if initial capability is relatively more important for health development when retired, i.e., $\alpha_\theta < \beta_\theta$, $\frac{\partial R^*}{\partial \theta_0} < 0$. The intuition behind these results is straightforward. In the first case, an increase in the initial amount of capability will produce relatively more benefits while in the labour force and, hence, the individual will delay his or her retirement. A direct analogous interpretation can be done of expression (13).

Also, note that from the Envelop theorem we have

$$\frac{\partial V(\boldsymbol{\beta})}{\partial \theta_0} = \int_0^{R^*} U'\big(C_\ell(t)\big)\frac{\partial C_\ell(t)}{\partial \theta_0}e^{-\rho t}dt + \int_{R^*}^{T} U'\big(C_{R^*}(t)\big)\frac{\partial C_R(t)}{\partial \theta_0}e^{-\rho t}dt > 0. \tag{14}$$

$$\frac{\partial V(\boldsymbol{\beta})}{\partial H_0} = \int_0^{R^*} U'\big(C_\ell(t)\big)\frac{\partial C_\ell(t)}{\partial H_0}e^{-\rho t}dt + \int_{R^*}^{T} U'\big(C_{R^*}(t)\big)\frac{\partial C_R(t)}{\partial H_0}e^{-\rho t}dt > 0. \tag{15}$$

Thus, the lifecycle utility increases with inherited amounts of capability and health, respectively, which has a straightforward intuition.

The importance of inherited capability and health

Now, let's turn our attention to the importance of the two stocks – as reflected by the α and β parameters – for the timing of retirements. There are four different comparative statics results, which are as follows

$$\frac{\partial R^*}{\partial \alpha_H} = -\frac{\int_{R^*}^{T} H_0 e^{-\rho t} dt}{D} > 0. \tag{16}$$

$$\frac{\partial R^*}{\partial \beta_H} = \frac{\int_{R^*}^{T} H_0 e^{-\rho t} dt}{D} < 0. \tag{17}$$

$$\frac{\partial R^*}{\partial \alpha_\theta} = -\frac{\int_{R^*}^{T} \theta_0 e^{-\rho t} dt}{D} > 0. \tag{18}$$

$$\frac{\partial R^*}{\partial \beta_\theta} = \frac{\int_{R^*}^{T} \theta_0 e^{-\rho t} dt}{D} < 0. \tag{19}$$

The greater the pre-retirement importance of the stocks, the later the time of retirement, analogously, the greater the post-retirement importance of the stocks – for given amounts of capability and health, health and capability will depreciate slower when retired – the sooner the time of retirement. These results are perfectly in line with intuition.

The rate of change of capability and health

$$\frac{\partial R^*}{\partial \delta_\ell} = \frac{\partial R^*}{\partial \delta_R} = \frac{\partial R^*}{\partial \sigma_R} = -\frac{\int_{R^*}^{T} e^{-\rho t} dt}{D} > 0. \tag{20}$$

$$\frac{\partial R^*}{\partial \sigma_\ell} = \frac{\int_{R^*}^{T} e^{-\rho t} dt}{D} < 0. \tag{21}$$

The greater the rate of increase of the stock of capability prior to retirement, the later the optimal time of retirement and, analogously, the greater the post-retirement rate of depreciation of the stock, the sooner the optimal time of retirement. These predictions are also in line with intuition. In the first case, an increase in the rate of increase in capability prior to retirement decreases the cost of delaying retirement. In the second case, an increase in the post-retirement depreciation increases the cost of delaying retirement.

Time preferences

An increase in the rate of time preferences (heavier discounting of the future) has the following effect on the timing of retirement

$$\frac{\partial R^*}{\partial \rho} = \frac{R^*[ln((1-\ell)y_\ell) - ln(y_0 + y_\ell\mu(R-R_L))]e^{-\rho R^*}}{D} + \frac{\int_{R^*}^{T} kte^{-\rho t}dt}{D}, \quad (22)$$

where $k = (\delta_\ell + \alpha_H H_0 + \delta_R - \beta_H H_0 - \sigma_\ell + \alpha_\theta\theta_0 + \sigma_R - \beta_\theta\theta_0 + (y_0 + y_\ell\mu(R - R_L))^{-1}\mu y_\ell)$.

The first term on the right-hand side captures the marginal change in instantaneous utility from retirement, while the second term reflects the total discounted change in the marginal utility of postponing retirement by dR. From the first order condition (11) it is clear that the two terms in Eq. (22) have opposite signs. Integrating the second term by parts gives $kD^{-1}\left(\left[-\frac{t}{\rho}e^{-\rho t}\right]_R^T + \int_R^T \frac{1}{\rho}e^{-\rho t}dt\right)$. The second term in this expression together with the numerator of the first term in (22) is identical to the first-order necessary condition, if $R = \frac{1}{\rho}$. Thus, if $R < (>)\frac{1}{\rho}$ the sum of these two terms are positive (negative) for a type 1 individual and negative (positivie) for a type 2 individual. Moreover, notice that the first term is $\left[-\frac{t}{\rho}e^{-\rho t}\right]_R^T = -\frac{T}{\rho}e^{-\rho T} + \frac{R}{\rho}e^{-\rho R}$. Obviously, this expression is zero if $R = T$, and its derivative is positive for all R such that $R < T < \frac{1}{\rho}$, and negative for R such that if $\frac{1}{\rho} < R < T$, implying that $\left[-\frac{t}{\rho}e^{-\rho t}\right]_R^T < 0$ in the first case, and $\left[-\frac{t}{\rho}e^{-\rho t}\right]_R^T > 0$ in the second case. Pulling all this together leads to the following results:

$\frac{\partial R^*}{\partial \rho} < 0$ for a type 2 individual if $\frac{1}{\rho} < R < T$,

$\frac{\partial R^*}{\partial \rho} > 0$ for a type 2 individual if $R < T < \frac{1}{\rho}$. (23)

Thus, a type 2 individual – inherited capability is relatively important for alleviating health deterioration of health after retirement – will retire sooner if he or she is relatively shortsighted (large ρ) and becomes even more short sighted. Similarly, a type 2 individual will retire later if he or she is relatively farsighted (small ρ) and becomes less farsighted. The intuition in the first case is that the effect of the change in time in time preferences on the discounted marginal cost of delaying retirement outweighs the effect on the marginal utility of delaying retirement. An analogous intuitive understanding applies to the second result.

Discussion

A model describing the time at which an individual will choose to retire when considering the (exogenous) evolvement over time of capability and health was developed and analyzed for its (in principle) refutable predictions. Capability was operationalized as another stock – health is also assumed to have stock properties, which is a less unorthodox assumption. It was shown that an individual choosing the optimal time of retirement cannot in general be unambiguously predicted to respond to changes in exogenous parameters. However, applying specific preferences – represented by a logarithmic utility function – that individual could be unambiguously predicted to respond to changes in those parameters. A salient feature of the model is the assumption that inherited capability influences the rate of change of the health stock, and vice versa. We demonstrated that an increase in inherited capability will delay the optimal time of retirement if capability is relatively more important for pre-retirement evolvement of the health stock. Similarly, we showed that the optimal time of retirement is delayed when inherited health increase if the health is relatively more important for pre-retirement evolvement of capability.

In order to test the relationship $R = R^{*}(\boldsymbol{\beta})$, cross-sectional and/or longitudinal individual-level data could be employed. The information required includes (i) observed time of retirement, (ii) information about initial health state and capability, (iii) the rates of change over time of health and capability, (iv) (expected) length of life, (v) time preferences, and (vi) estimates of the amounts and effect of inherited amounts of capability and health on the rate of change of health and capability. Given this information, reduced form equations are possible to estimate the results from which would provide tests of the derived predcitions. An adequate question is: are there such data available? We argue that they (in principle) are.

We are going to make the argument for the existence of such data from Scandinavian point of view, since the Scandinavian context is that which with we are most familiar, and since Scandinavian individual level data and the possibilities to link information between different data sources using unique personal idenficiation numbers are arguably or probably unparalleled in the world. So, information about retirement and health and changes of health is available national registers and from national the longitudinal content of medical records, and this information can be complemented with information about timing of retirement by linking health records to other national registers. As regards empirical measures of capability, the literature gives hints as to how to proceed. For example, Muffels and Headey (2013) defines capability as comprised of human, social, psychological, and cultural capital. In this study we made a distinction between health capital and capability (capital) and, hence, for empirical purposes measures of the other components of human capital and/or social, psychological and cultural capital is one option. Eventhough potential empirical measures of capability are unlikely to be identified in nation wide registers in any of the Scandinavian countries, there exists representative survies containing potential such measures. For example, the Swedish Survey of Living Conditions. Given longitudinal data on health and capability, the varaibles H_0 and θ_0 may be estimated. Further, measures of time preferences may be inferred from available financial individual data. Thus, exploring the type of data available in the Nordic countries it is possible to estimate the relationship $R = R^*(\boldsymbol{\beta})$ and, based on the estimated parameters, test the derived comparative statics predictions.

The peer-reviewed literature that applies the economic toolbox to capability and the influences that capability might have on individual decisions that are traditionally analyzed as economic issues is relatively small and seems to be growing slowly, if at all. In this paper we have focused on the relationship between capability and health. The canonical theoretical economic framework for analyzing health and health-related individual decisions is the human-capital model of health (Grossman, 1972). The human-capital model of health has been extended in various directions (see e.g., to include the family as a decision maker, Bolin et.al., 2001, 2002a,b, to take different health production functions into account, Bolin and Lindgren, 2016, and to take uncertainty regarding the evolvement of the health into account, Bolin and Caputo, 2020). No attempts to integrate capability in the human capital model of health and health-related behavior have been published. It seems plausible to assume that capability affects the individual's health-related decisions, since, for example, the productivity of given efforts undertaken in order to influence health is, at least partly, dependent on that individual's capability. Thus, future research efforts that integrate capability in the theoretical human-cpaital model of health may be worthwhile.

Appendix

The second-order sufficient condition for problem (3) is given by

$$D \overset{\text{def}}{=} \frac{\partial}{\partial R} \underbrace{\left[U\big(C_\ell(R)\big)e^{-\rho R} - U\big(C_R(R)\big)e^{-\rho R} + \int_R^T \frac{\partial U\big(C_R(t;R)\big)e^{-\rho t}}{\partial R} dt \right]}_{D} < 0.$$

Thus, we have

$$D \overset{\text{def}}{=} U'\big(C_\ell(R)\big)\partial_R C_\ell(R)e^{-\rho R} - \rho U\big(C_\ell(R)\big)e^{-\rho R} - U'\big(C_R(R)\big)\partial_R C_R(R)e^{-\rho R} + \rho U\big(C_R(R)\big)e^{-\rho R}$$

$$- \frac{\partial U\big(C_R(R)\big)e^{-\rho t}}{\partial R} + \int_R^T \frac{\partial^2 U\big(C_R(t)\big)e^{-\rho t}}{\partial R^2} dt < 0.$$

The effect of time preferences

Integrating the second term of Eq. (22) by parts gives

$$\frac{\left[-\frac{t}{\rho}e^{-\rho t}\right]_R^T + \int_R^T \frac{1}{\rho}e^{-\rho t}dt}{D}.$$

The integral in Eq. (22) equals

$$\frac{(\delta_\ell+\alpha_H H_0+\delta_R-\beta_H H_0-\sigma_\ell+\alpha_\theta\theta_0+\sigma_R-\beta_\theta\theta_0+(y_0+y_\ell\mu(R-R_L))^{-1}\mu y_\ell)\left(\left[-\frac{t}{\rho}e^{-\rho t}\right]_{R^*}^T+\left[-\frac{1}{\rho^2}e^{-\rho t}\right]_{R^*}^T\right)}{D} =$$

$$\frac{(\delta_\ell+\alpha_H H_0+\delta_R-\beta_H H_0-\sigma_\ell+\alpha_\theta\theta_0+\sigma_R-\beta_\theta\theta_0+(y_0+y_\ell\mu(R-R_L))^{-1}\mu y_\ell)\left(-\frac{T}{\rho}e^{-\rho T}+\frac{R^*}{\rho}e^{-\rho R^*}-\frac{1}{\rho^2}e^{-\rho T}+\frac{1}{\rho^2}e^{-\rho R^*}\right)}{D}$$

Thus, the sign of the comparative static expression is given by the sign of

$$\frac{(\delta_\ell+\alpha_H H_0+\delta_R-\beta_H H_0-\sigma_\ell+\alpha_\theta\theta_0+\sigma_R-\beta_\theta\theta_0+(y_0+y_\ell\mu(R-R_L))^{-1}\mu y_\ell)\left(\frac{1}{\rho}e^{-\rho T}-\frac{1}{\rho}e^{-\rho R^*}\right)}{D}$$

Comparative statics of the general model

In the general formulation, the comparative statics of a parameter, x, is:

$$\left[\frac{\partial}{\partial x}U\big(C_\ell(R)\big)e^{-\rho R}-\frac{\partial}{\partial x}U\big(C_R(R;R)\big)e^{-\rho R}+\frac{\partial}{\partial x}\int_R^T\frac{\partial U\big(C_R(t;R)\big)e^{-\rho t}}{\partial R}dt\right]dx+$$

$$\underbrace{\frac{\partial}{\partial R}\left[U\big(C_\ell(R)\big)e^{-\rho R}-U\big(C_R(R;R)\big)e^{-\rho R}+\int_R^T\frac{\partial U\big(C_R(t;R)\big)e^{-\rho t}}{\partial R}dt\right]}_{D}dR=0$$

Thus,

$$\frac{dx}{dR}=\frac{\frac{\partial}{\partial x}U\big(C_\ell(R)\big)e^{-\rho R}-\frac{\partial}{\partial x}U\big(C_R(R;R)\big)e^{-\rho R}+\frac{\partial}{\partial x}\int_R^T\frac{\partial U\big(C_R(t;R)\big)e^{-\rho t}}{\partial R}dt}{D}.$$

The sign of the numerator is in general inconclusive, since $\frac{\partial}{\partial x}U\big(C_\ell(R)\big)e^{-\rho R}-\frac{\partial}{\partial x}U\big(C_R(R;R)\big)e^{-\rho R}=U_x\big(C_\ell(R)\big)\frac{\partial C_\ell(R)}{\partial x}-U_x\big(C_R(R;R)\big)\frac{\partial C_R(R;R)}{\partial x}$ cannot be signed without additional assumptions about the utility function.

The first-order is found by applying Liebniz's rule and the chain rule to (4):

$$U(\theta_\ell(R)H_\ell(R)(1-\ell)y_\ell)e^{-\rho R} - U\big(\theta_R(R)H_R(R)(y_0 + y_\ell\mu(R-R_L))\big)e^{-\rho R} +$$

$$\int_R^T \frac{\partial U\big(\theta_R(t)H_R(t)(y_0+y_\ell\mu(R-R_L))\big)e^{-\rho R}}{\partial R}dt = 0.$$

More explicitly, this is equal to

$$U\left(\left(\theta_0 e^{(\delta_\ell+\alpha_H H_0)R} H_0 e^{-(\sigma_\ell-\alpha\theta_0)R}\right)(1-\ell)\right)e^{-\rho R} - U\left(\left(\theta_0 e^{(\delta_\ell+\alpha_H H_0)R} H_0 e^{-(\sigma_\ell-\alpha\theta_0)R}\right)(y_0 + y_\ell\mu(R-R_L))\right)e^{-\rho R} + \int_R^T \frac{\partial U\big(\theta_R(t)H_R(t)(y_0+y_\ell\mu(R-R_L))\big)e^{-\rho t}}{\partial R}dt = 0,$$

where the second term is

$$\theta_R(t) = \theta_0 e^{(\delta_\ell+\alpha_H H_0)R} e^{(\delta_R-\beta_H H_0)(R-t)}$$

$$H_R(t) = H_0 e^{-(\sigma_\ell-\alpha\theta_0)R} e^{(\sigma_R-\beta\theta_0)(R-t)}$$

$$\int_R^T \{U'\big(C_R(t)\big)[(\delta_\ell + \alpha_H H_0 + \delta_R - \beta_H H_0 - \sigma_\ell + \alpha\theta_0 + \sigma_R - \beta\theta_0)\big(y_0 + y_\ell\mu(R-R_L)\big) + y_\ell\mu\}\theta_R(t)H_R(t)e^{-\rho t}dt.$$

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