Charity, Status, and Optimal Taxation: Welfarist and Non-Welfarist Approaches

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Department of Economics, June 2021 & rev November 2021
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December 2021

Abstract
This paper analyzes optimal taxation of charitable giving to a public good in a Mirrleesian framework with social comparisons. Leisure separability and zero transaction costs of giving together imply that charitable giving should be subsidized to such an extent that government contributions are completely crowded out, regardless of whether the government acknowledges the warm glow of giving. Stronger concerns for relative charitable giving and higher transaction costs support lower marginal subsidies, whereas relative consumption concerns work in the other direction. We also characterize how the marginal subsidy on charitable giving varies with income under optimal taxation. Moreover, a dual screening approach to optimal taxation, where charitable giving constitutes an indicator of wealth, is also presented. Extensive numerical simulations supplement the theoretical results.

Keywords: Conspicuous consumption, conspicuous charitable giving, optimal taxation, public good provision, warm glow, multiple screening.

JEL Classification: D03, D62, H21, H23

1 The authors would like to thank Tommy Andersson, Nathalie Bolh, Dawei Fang, David Granlund, Randi Hjalmarssson, Etienne Lehmann, Mikael Lindahl, Volker Meier, Katarina Nordblom, Andreas Peichl, and Tomas Sjögren for very helpful comments and suggestions. Earlier versions of this paper have been presented at seminars at the University of Arizona, the Free University, Berlin, the University of Gothenburg, Linnaeus University, University of Osnabrück, Umeå University, and the University of Vermont, as well as at the 111th Annual Conference of the National Tax Association in New Orleans 2018, the 2019 Meeting of the European Public Choice Society in Jerusalem, the 75th Annual Congress of the International Institute of Public Finance 2019 in Glasgow, and the 77th annual meeting of the International Institute of Public Finance (held virtually) 2021. The authors would like to thank the participants for valuable discussions. Research grants from the Marianne and Marcus Wallenberg Foundation (MMW 2015.0037) are gratefully acknowledged.
I. Introduction
What is the optimal tax treatment of charitable giving? Individuals and organizations donate substantial amounts to charities, not least in the United States where charitable giving amounted to almost $500 billion, or more than 2% of GDP, in 2020 (Giving USA, 2021). The bulk of these donations can be described as voluntary contributions to public goods or community services, e.g., religious and environmental organizations, research, education, and public-society benefit charities (Giving USA, 2021). Thus, the importance of the above question is a given. The purpose of the present paper is to answer the question by integrating the tax treatment of charitable giving in a Mirrleesian model of optimal redistributive taxation. That the answer is non-trivial is indicated by the fact that tax policies related to charitable giving vary largely both between countries and within countries over time (Fack and Landais, 2012).

To our knowledge, the present study is the first to integrate voluntary contributions to a public good, to which there is also potential public provision, into a continuous-type Mirrleesian framework. In such a framework, any deviation from first-best taxation is driven solely by information limitations and not by arbitrary linearity restrictions. This approach allows us to derive richer, sharper, and more easily interpretable policy rules than in the existing literature.

In doing so, we also distinguish between a conventional welfarist government that respects all aspects of consumer preferences and forms the social objective thereupon, and a non-welfarist government that does not attach any social value to the warm glow of giving. Although welfarism is a standard assumption in normative economics, it is by no means obvious when warm glow preferences are involved. Based on arguments presented in Diamond (2006), which we refer to in Section II below, one can discuss whether consumer preferences for warm glow ought to be reflected in a social cost-benefit analysis of charitable giving. Therefore, we consider one case where the government respects all aspects of consumer preferences and forms the social objective thereupon and one case where it does not respect the consumer preference for warm glow.

Each such government collects revenue and redistributes through a nonlinear tax based on both gross income and charitable giving. We present our results using a modified version of Diamond’s (1998) and Saez’s (2001) ABC formulation, extended with private and public contributions to a public good. An important benchmark result appears in the special case where the utility functions are weakly leisure separable (a common assumption in the optimal taxation literature), and where there are no transaction costs of charitable giving (as in
previous literature; see below). In this special case, we show that a welfarist government should subsidize charitable contributions to such an extent that they completely crowd out governmental provision of the public good. The intuition is based on logic similar to the Atkinson and Stiglitz (1976) theorem: the marginal subsidies on charitable giving are in this case based on a first-best policy rule guided solely by concerns for economic efficiency. Such a policy favors charitable giving over public provision, since the former comes with the additional benefit of warm glow to the givers. Even more strikingly, we show that this complete crowding-out result continues to hold also when the government is non-welfarist and does not attach any social value to the warm glow of giving. The logic here is that subsidizing charitable giving for high-income earners constitutes a cost-efficient means of redistribution. Moreover, we show that a welfarist government implements a flat rate subsidy on charitable giving, whereas a non-welfarist government implements income-varying marginal subsidies, under leisure separability.

A second novelty is that we allow for transaction costs of charitable giving, such that a fraction of the contribution is lost in the process. Experimental research by Null (2011) shows that donors tend to divide their gifts among charities with similar aims despite that these charities differ in terms of marginal public goods productivity, thus forfeiting a significant social surplus.\(^2\) Such transaction costs turn out to play a critical role also in our study by reducing the marginal subsidies on charitable giving, and (if the costs are not too small) imply that the government should also contribute directly to the public good in accordance with a modified Samuelson rule. Thus, transaction costs are not only vital for the optimal tax treatment of voluntary contributions, they are also instrumental for whether the government should contribute directly to the public good.

A third novelty is that the model also, simultaneously, encompasses conspicuous charitable giving and conspicuous consumption, such that people derive well-being from giving more to charity and consuming more than referent others, and vice versa. Several studies suggest that charitable giving is a means of signaling status or prestige (e.g., Glazer and Konrad, 1996; Harbaugh, 1998a; Cartwright and Patel, 2013; Karlan and McConnell, 2014). Empirical evidence supports this idea by showing that the size and distribution of donations depend on whether they are made publicly observable and, if observable, on the way in which they are reported (e.g., Harbaugh, 1998b; Alpizar et al., 2008), as well as on the contributions made by other people (Alpizar et al., 2008). Harbaugh (1998b) also argues that

\(^2\) Empirical evidence presented by Caviola et al. (2020, 2021) indicates huge differences in the efficiency of charities.
this prestige motive is likely to be empirically important in the sense that “a substantial portion of donations can be attributed to it” (p. 281), and Dannenberg et al. (2021) found, based on a charity experiment in the field, that the status motive is not important only for the wealthy.

Questionnaire-experimental and happiness research points to a similar status-driven motive for private consumption (influencing the incentives underlying income formation), such that people derive well-being from their relative consumption or income compared with that of referent others. In fact, as much as 20–60% of the increase in well-being from an additional dollar spent on consumption may be attributable to increased relative consumption (see Section IV). There are also revealed-preference studies finding that people are willing to pay for ways of signaling wealth. In a way similar to status motives for giving to charity, relative consumption concerns are directly relevant for policies aimed at influencing charitable giving, since giving reflects a tradeoff between donations and private consumption.

The model then contains three simultaneous externalities: i) A contribution to the public good by an individual increases the size of the public good and thus induces a corresponding benefit for all individuals. ii) The same contribution also decreases others’ relative contribution, and correspondingly decreases their utility. iii) Increased consumption by an individual reduces the relative consumption of all others, and therefore decreases their utility. The strength of the concerns for relative charitable giving typically works in the direction of supporting lower marginal subsidies on charitable giving, whereas the strength of the concerns for relative consumption works in the other direction. An interesting exception arises if zero bunching in charitable giving is sufficiently prevalent, in which case an increase in the positional gifts externality may actually motivate a higher marginal subsidy (or a lower marginal tax) on charitable giving. For government contributions to the public good and the marginal tax treatment of charitable giving, respectively, leisure separability simplifies the optimal policy rules dramatically, again based on logic similar to the Atkinson and Stiglitz

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4 For example, Bursztyn et al. (2017) find that the demand for platinum credit cards greatly exceeds that for a regular credit card with the exact same functional properties.
(1976) theorem. Moreover, the crowding-out results described above continue to hold under relative comparisons.

In the general case where leisure separability does not apply, we demonstrate how the corresponding additional components of the policy rules can be written directly in terms of the optimal marginal income tax. This way of writing the policy rules constitutes a fourth novelty and further emphasizes the roles of governmental provision to the public good and the marginal subsidy/tax on charity as supplemental instruments for income redistribution.

As a fifth novelty, we present, as an extension, a generalization in the form of a dual screening model, where individuals now differ in two dimensions: ability (or gross wage), as before, and wealth, where wealth is also unobservable to the government. To our knowledge, the present paper is the first to integrate externalities and externality correction in a dual screening model of optimal taxation. Assuming that charitable giving is a normal good, we use it as a second screening device in order to redistribute from individuals with higher ability and higher wealth. We express the optimal policy rules using a two-dimensional ABC formulation, where the positional externalities caused by concerns for relative consumption and relative charitable giving enter in a way similar to the simpler single-screening model. However, since the logic behind the Atkinson-Stiglitz theorem is not applicable to the screening mechanism in the wealth dimension, leisure separability will no longer imply the same drastic simplifications of the policy rules.

Finally, the theoretical results are supplemented with extensive numerical simulations based on specific functional forms for the utility functions and the ability distribution, which illustrate the theoretical results and examine how sensitive the optimal tax policy is to underlying key parameters.

The remainder of the paper is organized as follows: After a brief review of earlier research on the optimal tax treatment of voluntary contributions to public goods in Section II, Section III presents our basic Mirrleesian model, whereas Section IV generalizes the model to allow for concerns for relative charitable giving and relative consumption. Section V presents the dual screening model and Section VI the numerical simulations, and finally, Section VII concludes the paper; proofs are presented in the Appendix.

II. A Brief Literature Review on Optimal Taxation and Charitable Giving
A number of studies have examined the policy implications of charitable giving in models of optimal (albeit not Mirrleesian) taxation, where the donations to charity are described in terms of voluntary contributions to a public good (e.g., Feldstein, 1980; Warr, 1982; Saez, 2004;
In a model with optimal linear taxation, Saez (2004) shows that the optimal subsidy on voluntary contributions can be expressed as a sum of three elements: the positive externality that each contributor imposes on other people (as in Warr, 1982), the price sensitivity of the contribution good, and the extent to which direct public contributions crowd out private contributions. We discuss, in some detail, a key result by Saez (2004) in relation to our Proposition 2, and show that his result can be seen as a special case of our model, given that he makes a number of rather strong assumptions.

Diamond (2006) uses a two-type model of optimal nonlinear taxation, developed along the lines of Diamond (1980) where the jobs available differ among skill types while the hours of work at a given job are fixed, to derive a second-best argument for marginal subsidies on charitable giving at the top of the income distribution. He shows that voluntary contributions by high earners relax the incentive compatibility constraint and hence motivate a marginal subsidy. In our Mirrleesian framework, the corresponding mechanisms can go in either direction (depending on whether the marginal valuation of charitable giving increases or decreases with the time spent on leisure) and would vanish under leisure separability.

Blumkin and Sadka (2007) examine a status motive for charitable giving (albeit with no concerns for warm glow or relative consumption) as well as tax policy implications thereof. They consider a model where charitable donations signal status and find that the welfare effect of introducing a small tax on charitable giving, when the income tax is optimal, is non-negative; it is positive if status concerns lead to overprovision of the public good compared with the Samuelson condition and zero otherwise. In the present paper, their finding follows as a special case.

If we were to disregard warm-glow welfare effects in the optimal policy analysis, the case for supporting private charity would naturally be weakened, as will be shown. Diamond (2006) argues against using warm-glow preferences as a basis for social cost benefit analysis. A key reason for this stance is that warm glow is likely to be context dependent, suggesting that it is not straightforward to measure this source of welfare benefit in practice. In addition, by recognizing warm glow as a source of benefit, there may also be reasons to devote resources to produce the contexts in which warm glow arises, which is not necessarily the best use of resources. Further arguments against including such benefits are related to the social

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Footnote: There is also a much smaller literature modeling charitable giving as direct redistribution from richer to poorer groups; see Atkinson (1976), Kaplow (1995), and Aronsson, Johansson-Stenman, and Wendner (2019). Whereas the former two studies focus on the tax treatment of such gifts in a first-best environment, the latter examines optimal redistributive income taxation in a two-type model where the high-ability type can transfer income to the low-ability type.
pressure to contribute (e.g., DellaVigna et al., 2012). However, other authors such as Kaplow (1998) and Kaplow and Shavell (2001) present arguments suggesting that benefits arising from the warm glow of giving should nevertheless be accounted for. We therefore follow Diamond (2006) and present results reflecting both when the government values and when it does not value changes in the warm glow of giving. In our case, the latter means that the government would like individuals to behave as if the concerns for (absolute and relative) warm glow were absent.\(^6\)

### III. A Mirrleesian Model with Charitable Giving and Public Goods

Consider an economy with linear production and competitive markets, implying that marginal productivity, \(w\), reflects an ability-specific, fixed wage rate per unit of labor, which we refer to as *ability* in what follows. Ability is distributed continuously, and the population is fixed and normalized to one, i.e., \(\int_0^{\infty} f(w)dw = 1\). The size of the public good, \(G\), depends on the sum of individual contributions, \(g\), and the amount provided directly by the government, \(G^{\text{Gov}}\).

Contrary to earlier studies on the optimal tax treatment of charitable contributions to public goods (see Section II), we introduce a transaction cost attached to charitable giving. We formalize this cost in the simplest possible way by assuming a potential discrepancy between the donation and its contribution to the public good, such that the overall size of the public good is given as follows:

\[
G = G^{\text{Gov}} + (1 - \mu) \int_0^{\infty} g(w)f(w)dw,
\]

where \(\mu < 1\) thus reflects the transaction cost. Since there may be transaction costs also for governmental provision of public goods, it is natural to interpret \(\mu\) as a measure of additional transaction costs associated with charitable giving. In this perspective, one cannot rule out that \(\mu < 0\). Yet, for simplicity, in the subsequent analysis we focus on the case where \(\mu \geq 0\) (where interpretations thus change accordingly when \(\mu < 0\)). One may also interpret \(\mu\) more

\(^6\) Another approach to non-welfarism would be to assume that the government respects neither absolute and relative warm glow nor relative consumption. This is because relative consumption and relative charitable giving are interpretable as expressions of envy. Harsanyi (1982) uses envy to exemplify an anti-social preference that should not be part of the social welfare function. Aronsson and Johansson-Stenman (2018) examine a two-type model where the policy maker does not respect the individual preferences for relative consumption and derive the optimal tax policy implications thereof. Their model does not include any charitable giving. If we extend the notion of non-welfarism in the present paper to the case where the policy maker respects neither absolute and relative warm glow nor relative consumption, the major qualitative results derived below continue to hold, and are available from the authors upon request.
broadly as reflecting that charitable giving may result in a less optimal distribution of public goods from society’s point of view.

Individuals are endowed with one unit of time and supply \( l \) units of labor. Their utility depends on consumption, \( c \), labor, \( l \) (and hence leisure, \( 1-l \)), and the overall level of the public good, \( G \). In addition, it depends on their own charitable contribution to the public good, \( g \), implying a corresponding warm glow (Andreoni, 1989):

\[
u_w = u(c_w, l, g, G).
\]

We assume that \( u(\cdot) \) is twice continuously differentiable, increasing in \( c, g \), and \( G \), decreasing in \( l \), and strictly concave.\(^7\) To ensure implementability of the optimal tax schedule described below, we also assume single crossing properties in both the consumption/gross-income and the charitable-giving/gross-income dimensions. Let \( V(c, y, g, G, w) = u(c, y / w, g, G) = u(c, l, g, G) \) and define the corresponding slopes of the indifference curves as

\[
\Phi_{y,c}(c, y, g, G, w) \equiv \frac{V_y(c, y, g, G, w)}{V_c(c, y, g, G, w)} \quad \text{and} \quad \Phi_{y,g}(c, y, g, G, w) \equiv \frac{V_y(c, y, g, G, w)}{V_g(c, y, g, G, w)}.
\]

The condition \( \partial \Phi_{y,c}(c, y, g, G, w) / \partial w < 0 \) means that the indifference curves in the gross-income/net-income space become flatter when the before-tax wage rate, \( w \), increases (such that the single-crossing property applies). It is then easy to verify that, for individuals supplying labor, gross income as well as consumption increase monotonically with ability. Correspondingly, \( \partial \Phi_{y,g}(c, y, g, G, w) / \partial w < 0 \) implies that, for individuals contributing to charity, charitable giving increases monotonically in ability. Together, these conditions also imply that charitable giving is a normal good.\(^9\)

Following, e.g., Saez (2004) and Blumkin and Sadka (2007), individuals are assumed to be atomistic agents by treating the level of the public good, \( G \), as exogenous;\(^10\) in Section IV, we make the analogous assumption that individuals treat externalities as exogenous. We

\(^7\) Alternatively, one may assume that people derive utility from their net contribution to the public good, \((1-\mu)g\). Since \( \mu \) is exogenous to the individual, we can still think of (2) as a reduced form. All qualitative insights would hold also with this alternative formulation.

\(^8\) Note that these expressions apply generally and not just in the optimal equilibrium solution. For this reason, we do not write \( V(c, y, g, G) = u(c, l, g, G) \) here.

\(^9\) We will verify these properties in the numerical application in Section VI as well as that the second-order conditions for social welfare maximization are satisfied.

\(^10\) This is in contrast to, e.g., Bergstrom, Blume, and Varian (1986), where individuals treat \( G \) as endogenous. In their study, an individual’s contribution is not negligible to \( G \).
want to emphasize that this should not be interpreted to mean that people solely care about how much they contribute and not about the public good to which they contribute. If the model would be extended to include several public goods, which is quite straightforward, the warm glow per dollar contributed may differ among charities.\textsuperscript{11}While we will present all propositions in terms of general utility functions, we will also, following the seminal contribution of Atkinson and Stiglitz (1976), present results for the important special case of weak leisure separability:

**Definition.** The utility function is denoted *leisure separable* if it can be written $u_w = V(k(c_w, g_w, G), I_w)$.

Thus, leisure separability implies that the marginal rates of substitution between $c$, $G$, and $g$ are independent of labor (and hence leisure). Note that this assumption is weaker than additive leisure separability, which is often assumed (see, e.g., Tuomala, 2016).

The individual budget constraint implies that the sum of private consumption and charitable giving equals gross income, $y = w l$, minus the taxes paid:

$$y_w - T(y_w, g_w) = c_w + g_w,$$

where $T(y, g)$ is a general, nonlinear tax function through which the tax payment depends on both gross income and charitable giving.\textsuperscript{12} Each individual chooses consumption, work hours, and charitable giving to maximize utility given by (2) subject to their respective budget constraint (3). In addition to (3), an interior solution satisfies the individual first-order conditions for labor supply and charitable giving:

$$MRS_{l,c}^{(w)} \equiv \frac{u_c^{(w)}}{u_l^{(w)}} = -w \left(1 - T_y^{(w)}\right); \quad MRS_{g,c}^{(w)} \equiv \frac{u_g^{(w)}}{u_c^{(w)}} = 1 + T_g^{(w)}.$$  

Single subscripts attached to the utility function or tax function reflect partial derivatives (unless being $w$), where $T_y$ denotes the marginal income tax and $T_g$ the marginal tax or subsidy (if negative) on charitable giving; $(w)$ refers to ability type.

The social welfare function is a generalized utilitarian welfare function as follows:

\textsuperscript{11} Such an extension then implies that the optimal marginal subsidy may vary between charities. Most qualitative results would prevail under such a generalization.

\textsuperscript{12} This formulation means, for instance, that the marginal and average income tax rates are allowed to depend on the level of charitable giving; thus, the model encompasses the case where gifts can be used for tax avoidance purposes.
\[ W = \int_0^\infty \psi(u_w) f(w)dw, \]  

where \( \psi \) is (weakly) concave. We will consider two versions of social welfare: a conventional welfarist objective where the government takes individual utility at face value (as in [5]) and a non-welfarist version where the government does not value utility changes caused by the warm glow of giving. Whether the government is welfarist or not, it faces the same resource constraint and incentive compatibility constraints. The resource constraint means that the aggregate production equals the sum of aggregate private consumption and charitable giving plus the government contributions to the public good,

\[ \int_0^\infty w l_w f(w)dw = \int_0^\infty (c_w + g_w) f(w)dw + G^{Gov}. \]  

We assume that the government observes income and charitable giving at the individual level, whereas ability is private information. The incentive compatibility constraints serve to prevent individuals of any type \( w \) from mimicking the ability type just below them in terms of observable income and charitable giving,

\[ du_w / dw = -l_{ith^{(w)}} / w. \]  

Our tax instruments and informational assumptions imply that the government can implement any desired combination of labor supply, consumption, and charitable giving for each type subject to the resource and incentive compatibility constraints. Therefore, we follow much earlier work in formulating the social decision problem as a direct decision problem throughout the paper. In doing so, we treat utility, \( u_w \), as a state variable, while \( l_w, g_w \), and \( G^{Gov} \) are control variables. Consumption, \( c_w \), is defined by the inverse of the function \( u(\cdot) \) in (2) such that \( c_w = h(l_w, g_w, G, u_w) \), the properties of which are \( h^{(w)}_l = -MRS_{l,c}^{(w)} \), \( h^{(w)}_g = -MRS_{g,c}^{(w)} \), \( h^{(w)}_G = -MRS_{G,c}^{(w)} \equiv -u^{(w)} / u^{(w)}_c \), and \( h^{(w)}_u = 1 / u^{(w)}_c \), where a subscript attached to the function \( h(\cdot) \) denotes a partial derivative. The policy rules for marginal income taxation and the marginal subsidization/taxation of charitable giving can then be derived by combining the private and social first-order conditions.

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13 That charitable giving is observable is, of course, a stronger assumption than observability of income. The long history of tax deductions attached to charitable giving, accompanied by the fact that some charities publish the names of the donors (and sometimes the size of the donations), may speak in favor of it, however. When charitable giving is subsidized, rather than taxed, each individual also has incentives to make the giving observable to the government. This assumption is also quite natural in our setting, where charitable giving partly reflects conspicuous consumption. An interesting avenue for future research would be to extend the analysis to a case where charitable giving is only observable in part.
A. Welfarist Government

The welfarist government maximizes the social welfare function (5) subject to (6), (7), and the non-negativity constraint $G_{Gov} \geq 0$. The non-negativity constraint on government contributions plays an important role for the marginal tax treatment of charitable giving. Let $\varepsilon^{lc}_l = (\partial MRS_{l,c} / \partial l)(1 / MRS_{l,c})$ represent the elasticity of $MRS_{l,c}$ with respect to $l$ and let $\delta_m = \psi'(u_w)u_l^{(w)} / \hat{\lambda}$ denote the welfare weight attached to type $w$, where $\hat{\lambda}$ is the Lagrange multiplier on the resource constraint. Following Diamond’s (1998) ABC formulation, we can then define

$$A_w = 1 + \varepsilon^{lc}_{l(w)}, \quad (8a)$$

$$B_w = \int_w^\infty \left(1 - \delta_w\right) \exp\left(-\int_w^\infty \frac{\partial MRS_{c,m}^{(m)}}{\partial c} dy_{m,m}\right) \exp\left(-\int_w^\infty \frac{\partial MRS_{c,c}^{(m)}}{\partial c} dg_{m,m}\right) \left(1 - F(w)\right) ds, \quad (8b)$$

$$C_w = \frac{1 - F(w)}{w f(w)}. \quad (8c)$$

Equations (8a)–(8c) are the building blocks of the policy rule for marginal income taxation. The variable $A$ is interpretable as an efficiency mechanism based on behavioral responses, $B$ reflects the desire for redistribution, and $C$ reflects the thickness of the upper tail of the ability distribution. Under leisure separability, we can express $A$ in the same way as Saez (2001) such that $A_w = 1 + \varepsilon^{lc}_{l(w)} = (1 + \varepsilon^u_{l(w)}/\varepsilon^c_{l(w)})$, where $\varepsilon^u$ and $\varepsilon^c$ are the uncompensated and compensated labor supply elasticity, respectively, with respect to the marginal wage rate derived under a linearized budget constraint. Under quasi-linearity, as in Diamond (1998), (8b) simplifies to $B_w = \int_w^\infty \left(1 - \delta_w\right) f(s) / (1 - F(w)) ds$.

To simplify the presentation, let $\varepsilon^{lc}_{l,c} = (\partial MRS_{l,c} / \partial l)(1 / MRS_{l,c})$ and $\varepsilon^{lc}_{c,c} = (\partial MRS_{c,c} / \partial c)(1 / MRS_{c,c})$ represent the elasticities of $MRS_{l,c}$ and $MRS_{c,c}$ with respect to labor. While our main concern is the optimal tax treatment of charitable giving, this policy is

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14 We choose not to include non-negativity constraints on charitable giving here. If individuals do not engage in social comparisons with respect to charitable giving, such non-negativity constraints do not affect the policy rule underlying the optimal tax treatment of charitable giving. We introduce non-negativity constraints in Section IV, where individuals derive well-being from their relative charitable giving.

15 It is natural to base these welfare weights on the social welfare function (our assumed social objective). However, many results can be generalized to a broader set of social objectives where the welfare weights are replaced with the corresponding (to the social objective) generalized welfare weights, using the terminology of Saez and Stancheva (2016).
closely related to the marginal income tax policy and the governmental public good provision. We start by presenting the marginal income tax policy and governmental public good provision in Proposition 1 and analyze the marginal tax treatment of charitable giving in Proposition 2.\footnote{16}

**Proposition 1.** For a welfarist government, the optimal policy rules for marginal income taxation and governmental public good provision can be written as

(i) \[ \frac{T^{(w)}_y}{1 - T^{(w)}_y} = A_w B_w C_w. \]

(ii) If \( G^{\text{Gov}} > 0 \), then

\[
\int_0^w \text{MRS}^{(w)}_{G,c} \left(1 + \epsilon^{Gc}_{l(w)} B_w C_w \right) f(w)dw = \int_0^w \text{MRS}^{(w)}_{G,c} \left(1 + \epsilon^{Gc}_{l(w)} \frac{T^{(w)}_y}{1 - T^{(w)}_y} \right) f(w)dw = 1.
\]

(iii) If \( \mu = 0 \) and \( \epsilon^{Gc}_{l(w)} \geq 0 \) for some \( w \) for which \( g_w > 0 \), then \( G^{\text{Gov}} = 0 \).

The ABC rule for marginal income taxation in result (i) takes the same general form as in the absence of any charitable giving, although the \( B \)-term is modified with an additional income effect in the \( g \)-dimension. To interpret result (ii), which refers to the optimal governmental provision of the public good, consider first the special case where the utility functions are leisure separable (according to the definition presented above). This special case means that the marginal willingness to pay for the public good does not vary with leisure time, i.e., \( \epsilon^{Gc}_{l(w)} = 0 \). The policy rule for public good provision then reduces to the standard Samuelson condition, \( \int_0^w \text{MRS}^{(w)}_{G,c} f(w)dw = 1 \), as in the seminal contributions by Christiansen (1981) and Boadway and Keen (1993), both of which are based on models without private contributions to the public good. The logic is similar to the one underlying the Atkinson-Stiglitz (1976) theorem: under leisure separability, and if the government has access to a general, nonlinear income tax, the use of other policy instruments will be guided solely by concerns for economic efficiency.

In the general case, where the preferences are not necessarily leisure separable, there is an additional term proportional to the behavioral elasticity \( \epsilon^{Gc}_{l(w)} \) in the policy rule for public good provision. This additional term arises because the public good now supplements the income tax as an instrument for income redistribution, i.e., we should no longer rely solely on

The marginal income tax rates in Proposition 1 apply for those supplying labor (as in, e.g., Saez 2001), and the marginal subsidies/taxes on charitable giving in Proposition 2 apply for those contributing to charity. In the numerical simulations, we address zero-bunching in terms of both work hours and charitable giving, and also illustrate the income ranges where people contribute to charity and where they do not.
efficiency concerns when providing the public good. This explains the interaction between the behavioral elasticities and the product $BC$. To further emphasize how $G^{Gov}$ interacts with the marginal income tax policy, in the expression after the second equality we present another variant where the behavioral elasticity is proportional to $T_y/(1-T_y)$. Thus, the absolute value of this redistributive component increases with the marginal income tax rate, ceteris paribus. This is intuitive: the higher the marginal income tax rates, the more costly the redistributive income tax policy, and the greater the need to use public good provision as a supplemental instrument for income redistribution. We can also see that result (ii) implies under-provision of the public good relative to the standard Samuelson condition if the marginal willingness to pay for the public good increases in leisure time (such that $\varepsilon_i^{Ge} < 0$), and over-provision if the opposite applies. While it is well known (from models without charitable giving) that public good provision ought to supplement the income tax for purposes of redistribution, we are not aware of any previous formulation of the policy rule for public good provision expressed in terms of either the $BC$ factor or the optimal marginal income tax. We show below that the marginal tax treatment of charitable giving plays a similar redistributive role (through similar mechanisms).

So far, we have assumed that the optimal governmental provision is positive, but in the presence of voluntary private contributions this is far from obvious. Indeed, as shown in result (iii), without transaction costs and under leisure separability, where the marginal willingness to pay for warm glow does not depend on leisure (such that $\varepsilon_i^{Glw} = 0$), it is optimal for the government not to contribute at all to the public good. The intuition is that charitable giving provides an additional welfare benefit due to the warm glow of giving that does not follow from governmental provision. In the non-separable case, and if $\mu = 0$, a sufficient condition for $G^{Gov} = 0$ is that $\varepsilon_{i(w)}^{Glw} \geq 0$ for some $w$ for which $g_w > 0$; hence, $\varepsilon_{i(w)}^{Glw} \geq 0$ need not hold for all $w$. The intuition is that if individuals for whom $\varepsilon_{i(w)}^{Glw} \geq 0$ can be made to contribute one more unit of the public good, there will be an additional social net benefit compared with a one unit contribution by the government.\footnote{Note that increased charitable giving by type $w$ leads to a relaxation (tightening) of the incentive compatibility constraint for this type if $\varepsilon_{i(w)}^{Glw} > 0$ ($< 0$). Thus, it would actually suffice to assume that there are contributing individuals for whom $\varepsilon_{i(w)}^{Glw}$ is either non-negative or sufficiently small in absolute value, such that the welfare effect through the incentive compatibility constraint, if negative, is not large enough to outweigh the positive welfare effect through the warm glow of giving. Corresponding remarks can be made on subsequent results concerning zero public provision, i.e., result (i) of Proposition 3 and result (iii) of Proposition 4.} This social net benefit of

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voluntary contributions comprises both warm glow and a relaxation of the incentive compatibility constraints.

We are now ready to characterize the marginal tax treatment of charitable giving, our main concern. Let \( S^0 \) denote society’s net marginal cost of public good provision in the absence of any direct contribution to the public good by the government (to be formalized below), and consider Proposition 2.

**Proposition 2.** For a welfarist government, the optimal marginal subsidies/taxes on charitable contributions satisfy:

(i) If \( G^{Gov} > 0 \), then

\[
T_g^{(w)} = -1 + \mu - \varepsilon_{l(w)}^{gc} \cdot MRS_{g,c}^{(w)} B_u C_w = -1 + \mu - MRS_{g,c}^{(w)} \cdot \frac{\varepsilon_{l(w)}^{gc}}{1 + \varepsilon_{l(w)}^{gc}} \cdot \frac{T_y^{(w)}}{1 - T_y^{(w)}}.
\]

(ii) If \( G^{Gov} = 0 \), then

\[
T_g^{(w)} = -1 + \mu + (1 - \mu)S^0 - \varepsilon_{l(w)}^{gc} \cdot MRS_{g,c}^{(w)} B_u C_w = -1 + \mu + (1 - \mu)S^0 - MRS_{g,c}^{(w)} \cdot \frac{\varepsilon_{l(w)}^{gc}}{1 + \varepsilon_{l(w)}^{gc}} \cdot \frac{T_y^{(w)}}{1 - T_y^{(w)}}.
\]

(iii) For \( g_w > 0 \), and where \( \varepsilon_{l(w)}^{gc} \) and \( \varepsilon_{l(w)}^{lc} \) are constant, \( \partial T_g^{(w)} / \partial y_w > (<) 0 \) iff \( \varepsilon_{l(w)}^{gc} T_y^{(w)} < (>) 0 \).

With reference to policy rule (iii) in Proposition 1, if \( \mu > 0 \) and sufficiently large, the government contribution to the public good is positive and the marginal subsidy attached to charitable giving is based on policy rule (i) in Proposition 2. Let us again start with the special case where the utility functions are leisure separable, implying that policy rule (i) reduces to \( T_g^{(w)} = -1 + \mu \). This rule reflects pure efficiency concerns, again by analogy to the Atkinson-Stiglitz (1976) theorem, where the marginal cost arising from private contributions compared to government contributions is due to the transaction cost \( \mu \).

Going back to the non-separable case in Proposition 2, the final term on the right hand side of the policy rule (i) arises because the marginal subsidy on charitable giving supplements the income tax as an instrument for redistribution, which explains the interaction between the elasticity of the marginal private value of warm glow with respect to labor, \( \varepsilon_{l(w)}^{gc} \), and the marginal income tax rate. This component (which vanishes under leisure separability) takes the same form as the corresponding redistributive element in the formula for governmental public good provision in Proposition 1. If \( \varepsilon_{l(w)}^{gc} < 0 \), charitable giving becomes more valuable relative to consumption when leisure increases (labor decreases). Thus, the government can relax the incentive compatibility constraints by subsidizing charitable giving at lower marginal rates, which opens up for more redistribution. There is a corresponding
incentive to relax the incentive compatibility constraints through higher marginal subsidies if $\varepsilon^C_l > 0$. Here too, the strength of this effect depends on the marginal income tax wedge.

Next, consider result (ii), i.e., the case where $G^{Gov} = 0$ at the social optimum. Under leisure separability, we obtain $T_g^{(w)} = -1 + \mu + (1 - \mu)S^0$, which reduces to $T_g^{(w)} = -1 + S^0$ in the absence of transaction costs. Thus, the marginal subsidy is still less than 100%, which is logical since utility increases monotonically in charitable giving. The variable $S^0$ reflects the discrepancy between society’s marginal cost and marginal benefit of governmental provision evaluated at $G^{Gov} = 0$ (or technically the Lagrange multiplier of the non-negativity constraint for $G^{Gov}$ over the Lagrange multiplier of the resource constraint),

$$S^0 = 1 - \left( \int_0^\infty MRS^{(w)}_{G^c} f(w) dw \right) \geq 0.$$  

The government reduces the marginal subsidy in order to avoid that voluntary contributions lead to overprovision of the public good relative to the Samuelson condition, by choosing $T_g^{(w)} = -1 + S^0$ for all $w$ when $\mu = 0$. If the preferences are not leisure separable, the same reasoning applies, except that the formula for $S^0$ extends to reflect the redistributive role of public good provision.\(^{18}\)

It is interesting to compare this policy rule with a corresponding result by Saez (2004), who derives optimal linear income and charitable giving taxation. Such a comparison is complicated by the fact that the linearity restriction (in addition to the other constraints) typically requires additional simplifying assumptions to obtain policy rules comparable with those derived under nonlinear taxation. When Saez makes a number of additional assumptions,\(^{19}\) he finds that the optimal subsidy rate for charitable giving is 100%; thus, there is no additional $S^0$ term, as in our case. Yet, we can solve this puzzle by noting that Saez does not explicitly assume utility to be monotonically increasing (as we do), but only non-decreasing, in charitable giving. Therefore, it is possible to interpret the extreme 100% subsidy as resulting from a situation where the marginal utility of charitable giving equals

\(^{18}\) Specifically: i) there are no income effects on earnings, ii) the aggregate gross income is independent of both the public good and the subsidy on charitable giving, iii) the compensated supply of contributions does not depend on the income tax, and iv) the aggregate voluntary contribution is reduced by exactly one unit for each additional unit of governmental provision. The results in the present paper, in contrast, hold regardless of whether these assumptions are fulfilled.
zero beyond a certain level of giving. This opens up for the possibility that the voluntary contributions are small enough to imply that direct governmental provision would still be optimal. Consequently, the non-negativity constraint for governmental provision does not bind in this case, and a 100% subsidy rate would be perfectly logical. The same result would follow in our model. Moreover, if utility would be monotonically increasing in charitable giving in the model by Saez, the policy rule $-1 + S^0$ would result there as well.

If the preferences are leisure separable ($\frac{\partial u}{\partial l} = 0$), result (iii) implies that individuals contributing to charity should face a flat rate marginal subsidy on charitable giving regardless of whether the government contributes directly to the public good. In other words, although the level of this subsidy rate depends on whether the government contributes directly, it is always desirable with a flat rate under leisure separability. Thus, contrary to the marginal income tax rate that typically varies with gross income, the optimal marginal subsidy on charitable giving is in this case independent of both gross income and the individual’s contribution. Another interpretation of the policy rules in (i) and (ii) is in terms of an income tax credit proportional to the level of charitable giving, i.e., individuals can reduce their tax payment by a fixed amount, $-1 + \mu$ or $1 + \mu + S^0$, per unit of their charitable contribution.

If the preferences are not leisure separable, result (iii) shows that the marginal subsidy on charitable giving typically varies with the before-tax income. This also applies for the natural benchmark case of constant behavioral elasticities, which we focus on for simplicity. To see the intuition behind this result, suppose that charitable giving and leisure are complementary in the sense that $\frac{\partial u}{\partial l} < 0$. Then, if the marginal income tax increases with income (as for high income levels in Saez, 2001, or our simulations in Section VI), this mechanism makes it more attractive for the adjacent type with higher productivity to mimic type $w$ in order to reduce the marginal income tax rate, ceteris paribus, which tightens the incentive compatibility constraint imposed on type $w$. To make such mimicking less attractive,

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20 However, when individuals are indifferent about whether or not to contribute more to the public good (despite a 100% subsidy), the individual optimization problem does not have a unique solution. Thus, to reach a unique solution, we would also need an additional assumption that all individuals prefer not to contribute more in this situation.

21 The first-order condition for governmental provision of the public good is given by (5) in Saez (2004), which assumes that this contribution is positive. By adding the assumption that utility is monotonically increasing (instead of non-decreasing) in charitable giving, and explicitly recognizing the non-negativity constraint on government contributions, it is straightforward to use Saez’s model to show that the subsidy on charitable giving becomes $-1 + S^0$, where $S^0$ is given by the Lagrange multiplier of the non-negativity constraint over the Lagrange multiplier of the resource constraint.
the government reduces the marginal subsidy on charitable giving implemented for type \( w \), which hurts the potential mimicker more than the true type \( w \) due to the complementarity property. Therefore, the marginal subsidy on charitable giving decreases in ability and hence in gross income.\(^{22}\) Correspondingly, if the marginal income tax rate decreases in income (as would be the case for high income levels based on an ability distribution with a thin upper tail), the marginal subsidy on charitable giving increases in ability and gross income. Naturally, qualitative results opposite to those just described follow if charitable giving and leisure are instead substitutable in the sense that \( \epsilon_{i(w)}^{GC} > 0 \). Note also that result (iii) implies that a (locally) constant marginal income tax rate \( (T_{wy}^{(w)} = 0) \) is sufficient for a (locally) constant marginal subsidy rate on charitable giving regardless of the sign and magnitude of \( \epsilon_{i(w)}^{GC} \).

\[ B. \text{Non-Welfarist Government} \]

The non-welfarist government would like individuals to behave as if they do not derive well-being from the warm glow of giving. Following an approach developed in another context by Aronsson and Johansson-Stenman (2018), the government imposes a “laundered” utility function on each individual of any ability \( w \),

\[ u^n_w = u(c_w, l_w, \overline{g}_w, G) = \nu(c_w, l_w, G), \quad (9) \]

where the non-welfarist government treats charitable giving as exogenous and attaches no social value to changes in warm glow; yet, in equilibrium we have \( \overline{g}_w = g_w \), meaning that (2) and (9) take the same value.\(^{23}\) Therefore, the social welfare function in (5) is now replaced with

\[ W^n = \int_0^\infty \psi(u^n_w) f(w) dw. \quad (10) \]

Compared with the welfarist model examined above, the optimal control problem is modified in the sense that (10) replaces (5), and (9) appears as an additional Lagrange constraint. Thus,

\[ ^{22} \text{Since charitable giving is a normal good, charitable giving increases monotonically in the before-tax income among individuals contributing to charity. Thus, the marginal subsidy on charitable giving increases or decreases in the contribution level, } g_w, \text{ under the same general conditions as it increases or decreases in the before-tax income.} \]

\[ ^{23} \text{If the utility functions were additively separable in charitable giving, an obvious and simpler alternative would be to define the laundered utility function just by subtracting the utility of charitable giving. In that case, the resulting social first-order conditions would take the same form as those following the procedure presented above. In our study, charitable giving obviously affects social welfare indirectly via the non-separable utility functions and the nonlinear social welfare function.} \]
\( u^w \) is treated as an additional state variable here (see the Appendix). The other constraints are the same as in the welfarist model, as individuals’ actual utility functions still drive their behavior.

The policy rules for marginal income taxation and governmental public good provision turn out to be the same as in the welfarist case. The marginal tax treatment of charitable giving changes as follows:

**Proposition 3.** For a non-welfarist government, (i)–(iv) hold:

(i) If \( \mu = 0 \) and \( \epsilon_{l(w)}^c \geq 0 \) for some \( w \) for which \( g_w > 0 \), then \( G^G = 0 \).

(ii) If \( G^G > 0 \), then

\[
T^G = -1 + \mu + \delta MRS_{\delta,e}^c - \epsilon_{l(w)}^c MRS_{\delta,e}^c B_w C_w
\]

(iii) If \( G^G = 0 \), then

\[
T^G = -1 + \mu + (1 - \mu) S^0 + \delta MRS_{\delta,e}^c - \epsilon_{l(w)}^c MRS_{\delta,e}^c B_w C_w
\]

(iv) For \( g_w > 0 \), and where \( \epsilon_{l(w)}^c \) and \( \epsilon_{l(w)}^l \) are constant, \( \partial T^G / \partial y_w < 0 \) iff

\[
\frac{\partial \delta}{\partial y_w} - \frac{\epsilon_{l(w)}^c}{1 + \epsilon_{l(w)}^l} \frac{T^G}{(1 - T^G)^2} < 0.
\]

Note first that the condition for when it is optimal with zero governmental provision (result i) is identical to the one under welfarism. Thus, in the absence of transaction costs, the leisure separable case implies complete crowding out of governmental provision. This may seem surprising, since the non-welfarist government attaches no social value to warm glow. The underlying mechanism behind this result is therefore different from the one in the welfarist case. The non-welfarist government prefers charitable giving to public contributions since the former reduces the social cost of redistribution. Here, there is a social net cost if people with low net incomes (for whom \( \delta_w > 1 \)) contribute and a social net benefit if people with high net incomes (where \( \delta_w < 1 \)) do. By combining the marginal subsidy rule for charitable giving (results ii and iii) with the private first-order condition, it is easy to see that only individuals for whom \( \delta_w < 1 \) will contribute at the optimum and thus generate welfare gains in terms of a lower social cost of redistribution, while there are no corresponding benefits from governmental provision.

The above mechanism also explains the role of the welfare weight in the policy rules.
in results (ii) and (iii). Note that these rules differ from the corresponding welfarist rules in Proposition 2 solely by the term \( \delta_u MRS_{g,c}^{(w)} \). This corrective component, which is positive and contributes to reduce the marginal subsidy, is present because individuals value the warm glow of giving (reflected in their marginal rates of substitution between charitable giving and private consumption) while the government does not, meaning that the government will adjust the incentive structure accordingly. The lower the type-specific welfare weight, \( \delta_u \), the smaller this corrective component is, and the less inclined the government will be to tax away the individual’s marginal warm glow benefit, ceteris paribus. The appearance of the welfare weight in the policy rule for \( T_g \) thus reflects that redistribution is costly, and that it is socially preferable for this reason that individuals with high net incomes rather than individuals with low net incomes contribute to charity.

Consider finally result (iv), describing how the marginal subsidy varies with the before-tax income. Under leisure separability (where \( \varepsilon_{l(u)} = 0 \)), we see that for income-decreasing welfare weights \( \delta_u \), the marginal subsidy on charitable giving increases in before-tax income, as also illustrated numerically in Figure 1 of Section VI. The intuition is that the social benefit from redistribution by letting the rich pay for the public good through charitable giving increases as \( \delta_u \) decreases. In the general, non-separable case, this mechanism is combined with the mechanism underlying result (iii) in Proposition 2, for which we saw that the sign was undetermined and that the magnitude is small unless the curvature of the marginal income tax schedule is large.

IV. Incorporating Preferences for Relative Giving and Relative Consumption

In Section III, we made the conventional assumption that utility depends only on the individual’s own consumption and charitable giving (in addition to leisure time and the public good). We will now assume that individuals also derive well-being from their relative consumption and relative charitable giving. This means that individuals impose positional externalities on one another. Following the bulk of earlier research on optimal taxation and relative consumption, we start with the most common comparison form, the mean value comparison, which in our case means that individuals compare their own consumption with the average consumption and their own charitable giving with the average charitable giving. At the end of this section, we discuss some alternative comparison forms and the implications thereof.
A. The model with social comparisons

Extending the utility function to accommodate relative consumption and relative charitable giving, (2) is now replaced with

\[ u_w = v(c_w, l_w, g_w, \Delta c_w, \Delta g_w, G) = u(c_w, l_w, g_w, \bar{c}, \bar{g}, G), \]

where \( \Delta c_w = c_w - \bar{c} \) denotes the relative consumption and \( \Delta g_w = g_w - \bar{g} \) the relative charitable giving of an individual of type \( w \), while \( \bar{c} \) denotes the average consumption and \( \bar{g} \) the average charitable giving in the economy as a whole, i.e.,

\[ \bar{c} = \int_0^\infty c_w f(w)dw \quad \text{and} \quad \bar{g} = \int_0^\infty g_w f(w)dw. \]

The function \( v() \) in (11) is increasing in \( c, g, \) and \( G \), non-decreasing in \( \Delta c \) and \( \Delta g \), decreasing in \( l \), and strictly concave, while the function \( u() \) is now interpretable as a reduced form, which will be used in some of the calculations presented below. We summarize the relationships between \( v() \) and \( u() \) as follows:

\[ u_c = v_c + v_{\Delta c}, \quad u_l = v_l, \quad u_g = v_g + v_{\Delta g}, \quad u_G = v_G, \]

\[ u_c = -v_{\Delta c}, \quad \text{and} \quad u_g = -v_{\Delta g}, \]

where subscripts denote partial derivatives.\(^{24}\) Note that the conditions for implementability are the same here as in the model without social comparisons, since each individual will treat the reference levels of consumption and charitable giving as constants. This also implies that charitable giving is a normal good here, too. Each individual behaves as an atomistic agent and treats \( G, \bar{c}, \) and \( \bar{g} \) as exogenous. The individual first-order conditions in (4) continue to hold, where the MRS expressions are defined in terms of the function \( u() \) in (11). Leisure separability is correspondingly defined such that the utility function can be written

\[ u_w = V(k(c_w, g_w, \Delta c_w, \Delta g_w, G), l_w). \]

Let us now introduce measures of the importance of relative consumption and relative charitable giving. Following Johansson-Stenman et al. (2002), the degree of consumption positionality, \( \alpha \equiv v_{\Delta c} / (v_c + v_{\Delta c}) \in [0,1) \), reflects the share of the marginal utility of consumption arising from an increase in \( \Delta c \). Similarly, the degree of positionality in charitable giving, \( \beta \equiv v_{\Delta g} / (v_g + v_{\Delta g}) \in [0,1) \), is the share of the marginal utility of charitable giving that arises from an increase in \( \Delta g \). In general, these measures vary across individuals and the corresponding average degrees of positionality are given by

\[ \bar{\alpha} = \int_0^\infty \alpha_w f(w)dw \in [0,1) \quad \text{and} \quad \bar{\beta} = \int_0^\infty \beta_w f(w)dw \in [0,1), \]

\(^{24}\) Using the language of Dupor and Liu (2003), the properties \( u_c = -v_{\Delta c} < 0 \) and \( u_g = -v_{\Delta g} < 0 \) can be referred to as “jealousy.”
We can interpret each such average degree as the sum of all individuals’ marginal willingness to pay to avoid the corresponding externality.\(^{25}\)

In addition to the average degrees of positionality, the relationship between each degree of positionality and the labor supply is important for tax policy and public good provision. This is because the government can exploit these relationships in order to relax the incentive compatibility constraints. Let \( e_\alpha^\prime = (\partial \alpha / \partial l) / (\alpha / l) \) denote the elasticity of the degree of consumption positionality with respect to labor supply, and let \( e_\beta^\prime = (\partial \beta / \partial l) / (\beta / l) \) denote the corresponding labor supply elasticity of the degree of positionality in charitable giving. We can then define the following indicators of how the concerns for relative consumption and relative charitable giving affect the incentive compatibility constraints, which will be part of the policy rules presented below:

\[
\alpha^d = \int_0^\infty e_\alpha^\prime u_B C_w f(w) dw
\]

\[
\beta^d = \int_0^\infty e_\beta^\prime MRS^{(m)}_\delta B_w C_w f(w) dw.
\]

Note the type-specific \( BC \) component in (14a) and (14b), connecting the redistributive aspects of positional externalities to the \( ABC \) rule for optimal taxation. The variables \( A \) and \( C \) are defined as in (8a) and (8c), whereas the definition of \( B \) changes slightly as the positional consumption externality affects the cost of redistribution,

\[
B_w = \left[ \frac{1 + \alpha^d}{1 - \beta^d} - \delta_w \right] \exp \left( -\int_0^\infty \frac{\partial MRS^{(m)}_\delta}{\partial c} \frac{dy_m}{m} \right) \exp \left( -\int_0^\infty \frac{\partial MRS^{(m)}_\delta}{\partial c} \frac{dg_m}{C_w} \right) \frac{f(s)}{1 - F(w)} ds.
\]

The welfarist government maximizes social welfare function (5), where \( u_w \) is now given by (11), subject to resource and incentive compatibility constraints analogous to equations (6) and (7), the externality constraints (12), and the non-negativity constraint \( G^{Gov} \geq 0 \) on direct public contribution. We also impose non-negativity constraints on charitable giving, \( g_w \geq 0 \) for all \( w \). Although these constraints played no role for tax policy in Section III (and were consequently omitted), they are important here. The reason is that concerns for relative charitable giving among the non-contributors will influence the marginal tax treatment of charitable giving. For later use, let \( \phi_w \) denote the Lagrange multiplier attached to the non-negativity constraint for \( g \) on type \( w \).

\(^{25}\) Quasi-experimental research estimates \( \bar{\alpha} \) to be in the 0.2–0.6 range (see, e.g., Johansson-Stenman et al., 2002; Clark and Senik, 2010; Carlsson et al., 2007; and the overview by Wendner and Goulder, 2008). We are not aware of any empirical estimate of \( \bar{\beta} \). However, clearly visible goods are characterized by higher degrees of positionality than less visible goods (e.g., Alpizar et al., 2005; Carlsson et al., 2007).
The non-welfarist government solves a similar decision problem, albeit with two important modifications. First, the non-welfarist government does not respect the individual preferences for charitable giving, neither in absolute nor in relative terms, and therefore imposes a laundered utility function on each individual, which is given as follows for any type $w$:

$$u^n_w = v(c_w, l_w, g_w, \Delta c_w, \Delta g_w, G) = v(c_w, l_w, \bar{c}, G).$$  \tag{16}$$

During optimization, the non-welfarist government thus treats the absolute and relative charitable giving as exogenous. Yet, in equilibrium we have $\bar{g}_w = g_w$ and $\Delta g_w = \Delta g_w$, meaning that equations (11) and (16) take the same value. Second, under non-welfarism, $u^n_w$ replaces $u_w$ for all $w$ in the social welfare function.

The social decision problems faced by the welfarist and non-welfarist governments are solved in the same way as in Section II with the modification that $\bar{c}$ and $\bar{g}$ are now added to the set of control variables (which also includes $l_w$, $g_w$, and $G$). Thus, individual consumption is now defined by the inverse of the $u(\cdot)$ function in (11) instead of in (2), such that $c_w = h(l_w, g_w, G, \bar{c}, \bar{g}, u_w)$, where the properties with respect to $l_w$, $g_w$, $G$, and $u_w$ are analogous to those described in Section III, while the properties with respect to $\bar{c}$ and $\bar{g}$ are summarized by $h^{(w)}_c = -u^{(w)}_c / u^{(w)}_c = \alpha_w$ and $h^{(w)}_g = -u^{(w)}_g / u^{(w)}_c = \beta_w MRS^{(w)}_{g,c}$.

**B. The optimal policy rules under social comparisons**

As we indicated above, bunching at zero charitable giving affects the optimal tax treatment of voluntary contributions and thus the optimal distribution between voluntary contributions and direct government contributions to the public good. This is because bunching at zero charitable giving reduces the positional gifts externality. Let

$$\beta^{\text{non}} = \int_0^{w_0} (\beta_w \phi_w / \lambda) dw > 0$$

denote the decrease in the positional gifts externality caused by bunching at zero charitable giving, where $w_0$ represents the type with the highest productivity that does not contribute to the public good. The role of $\beta^{\text{non}}$ will be made more precise in the context of Proposition 5 below.

We begin with marginal income taxation and governmental provision of the public good in Proposition 4 and continue with the marginal tax treatment of charitable giving in Proposition 5.
Proposition 4. Under social comparisons, and regardless of whether the government is welfarist or non-welfarist, the optimal marginal income tax policy and governmental public good provision satisfy:

(i) \[
\frac{T_{y}^{(w)}}{1-T_{y}^{(w)}} = A_{w} B_{w} C_{w} + \bar{\alpha} + \alpha^{d}.
\]

(ii) If \(G_{Gov}^{\ast} > 0\), then \[
\int_{0}^{\infty} MRS_{G,c}^{(w)}(1+\alpha^{d}+\varepsilon_{(w)}^{Gc} B_{w} C_{w}) f(w)dw = 1.
\]

(iii) If \(\mu = 0\), \(\beta^{m} \geq \beta^{d}\), and \(\alpha^{d} > -1\), and if \(\varepsilon_{(w)}^{Gc} \geq 0\) for some \(w\) for which \(g_{w} > 0\), then \(G_{Gov}^{\ast} = 0\).

Results (i) and (ii) generalize the policy rules presented by Aronsson and Johansson-Stenman (2008) for a two-type model without charitable giving. Under leisure separability, where \(\alpha^{d} = 0\) and \(\varepsilon_{(w)}^{Gc} = 0\) for all \(w\), these policy rules reduce to read

\[
\frac{T_{y}^{(w)}}{1-T_{y}^{(w)}} = A_{w} B_{w} C_{w} + \bar{\alpha},
\]

\[
\int_{0}^{\infty} MRS_{G,c}^{(w)} f(w)dw = 1.
\]

Compared with the model examined in Section III, the average degree of consumption positionality, \(\bar{\alpha}\), now enters the policy rules. In a first-best setting where individual productivity is observable, which is equivalent to the special case of our model where the incentive compatibility constraints do not bind, the policy rule in (i) reduces even further to \(T_{y}^{(w)} = \bar{\alpha}\), which is a conventional Pigouvian tax reflecting the marginal willingness to pay to avoid the positional consumption externality. In a second-best world with information asymmetries, therefore, this Pigouvian element is combined with the \(ABC\) component. Correspondingly, the public good formula is modified to reflect that the private and social marginal willingness to pay measures for public goods differ under positional consumption externalities. Specifically, the private marginal willingness to pay, \(MRS_{G,c}\), underestimates the social marginal willingness to pay if \(\bar{\alpha} > 0\).

In the general, non-separable case, the additional variable \(\alpha^{d}\) reflects that correction for the positional consumption externality will now have a role in redistribution, again relating to the logic underlying the Atkinson-Stiglitz theorem. If \(\varepsilon_{(w)}^{Gc} < 0\), individuals with more leisure time suffer more from the positional consumption externality than individuals who spend less time on leisure, ceteris paribus. This means from (14a) that \(\alpha^{d} < 0\), such that the government can relax the incentive compatibility constraints by a policy-induced increase.
in \( \bar{c} \), which motivates lower marginal income taxes and a smaller government contribution to the public good. Policy implications opposite to those just described arise if \( \varepsilon_i^\alpha > 0 \). The variable \( \varepsilon_i^{GC} \) in result (ii) plays the same role for public good provision here as it did in the model examined in Section III.

Result (iii) shows that the complete crowding out of governmental public good provision under zero transaction costs and leisure separability (where \( \alpha^d = \beta^d = \varepsilon_i^{GC} = 0 \)) holds here as well, regardless of type of government, based on the same logic as before. In the non-separable case we must, in addition to the conditions in the corresponding result (iii) in Proposition 1, add that the effects of the consumption and charitable giving externalities through the incentive compatibility constraint are not extreme, in the sense that \( \beta^{non} \geq \beta^d \), and \( \alpha^d > -1 \) are fulfilled. 26

We now turn to the marginal tax treatment of charitable giving. While these rules will not be identical between welfarist and non-welfarist governments, they are still sufficiently similar to be presented in the same proposition. Let \( N \) be an indicator variable, such that \( N = 0 \) under welfarism and \( N = 1 \) under non-welfarism. The results are summarized in Proposition 5:

**Proposition 5.** Under social comparisons, the marginal subsidies/taxes on charitable contributions satisfy:

(i) If \( G_{Gov}^{(w)} > 0 \), then

\[
T_{g}^{(w)} = -1 + \frac{1 - \bar{\alpha}}{1 + \alpha^d} \left( N \delta_w - e_i^c B_w C_w \right) MRS_{g,c}^{(w)} + \frac{1 - \bar{\alpha}}{1 - \bar{\beta}} \mu + \frac{\beta^d - \beta^{non}}{1 + \alpha^d}.
\]

(ii) If \( G_{Gov}^{(w)} = 0 \), then

\[
T_{g}^{(w)} = -1 + \frac{1 - \bar{\alpha}}{1 + \alpha^d} \left( N \delta_w - e_i^c B_w C_w \right) MRS_{g,c}^{(w)} + \frac{1 - \bar{\alpha}}{1 - \bar{\beta}} \mu + (1 - \mu)S^0 + \frac{\beta^d - \beta^{non}}{1 + \alpha^d}.
\]

(iii) For \( g_w > 0 \), and where \( e_i^{GC} \) and \( e_i^{lc} \) are constant, \( \partial T_{g}^{(w)} / \partial y_w < (>) 0 \) under welfarism iff \( e_i^{GC} T_{yy}^{(w)} < (>) 0 \), while \( \partial T_{g}^{(w)} / \partial y_w < 0 \) under non-welfarism iff

\[
\frac{\partial \delta_w}{\partial y_w} - \frac{e_i^{GC}}{1 + \alpha^d} \frac{T_{yy}^{(w)}}{\left(1 - T_{g}^{(w)}\right)^2} < 0.
\]

26 As we show in the Appendix, the even less restrictive assumption that there exists some \( w \) such that

\[
e_i^{GC} - \frac{1 - \bar{\alpha}}{1 + \alpha^d} B_w C_w MRS_{g,c}^{(w)} + \frac{1 - \bar{\alpha}}{1 - \bar{\beta}} \left( \beta^{non} - \beta^d \right) > 0
\]

in equilibrium is sufficient in the welfarist case. In the non-welfarist case, sufficiency follows from

\[
MRS_{g,c}^{(w)} \left(1 - \delta_w - \frac{1 - \bar{\alpha}}{1 + \alpha^d} + e_i^{c} B_w C_w MRS_{g,c}^{(w)} + \frac{1 - \bar{\alpha}}{1 - \bar{\beta}} \left( \beta^{non} - \beta^d \right) > 0 \right)
\]

for some \( w \).
Consider first the leisure-separable case, where result (i) simplifies to

\[ T_g^{(w)} = -1 + \frac{1-\bar{\alpha}}{1-\bar{\beta}}(\mu - \beta^{\text{non}}) + (1-\bar{\alpha})N \delta_w MRS_{g,c}^{(w)}. \]

There are three important differences between this policy rule and the corresponding ones in Propositions 2 and 3. First, the transaction cost, \( \mu \), is now multiplied by the factor \((1-\bar{\alpha})/(1-\bar{\beta})\), reflecting the ratio of the degrees of non-positionality between private consumption and charitable giving. \( \mu(1-\bar{\alpha})/(1-\bar{\beta}) \) is then interpretable in terms of a corresponding social transaction cost.

Second, if the optimal resource allocation implies bunching in that some individuals should not contribute to charity (which is typically the case), the positional externality attributable to charitable giving will be smaller than otherwise. The intuition is that a small decrease in \( \bar{g} \) cannot be accompanied by decreased charitable giving among the non-contributors (this adjustment is only feasible for the contributors), meaning that the average degree of positionality in charitable giving, \( \bar{\beta} \), overestimates the corresponding welfare benefit. The variable \( \beta^{\text{non}} > 0 \) reflects society’s valuation of this discrepancy. This adjustment thus works to increase the marginal subsidy on charitable giving. It also contributes to modify the sufficient condition for complete crowding out of governmental provision discussed above. While we focused on the case without transaction costs in Proposition 4, the absence of such costs no longer plays the same critical role. Indeed, it is sufficient that \( \mu + \beta^d - \beta^{\text{non}} \leq 0 \) for \( G^{\text{Gov}} = 0 \) to hold, which in the leisure separable case reduces to \( \mu \leq \beta^{\text{non}} \).

Third, in the policy rules for \( T_g \) in the non-welfarist case, the term

\[ (1-\bar{\alpha})\delta_w MRS_{g,c}^{(w)} \equiv (1-\bar{\alpha})\delta_w \frac{u_g^{(w)}}{u_c^{(w)}} = (1-\bar{\alpha})\delta_w \frac{v_g^{(w)}}{v_c^{(w)}} + v_{Ac}^{(w)}, \]

now reflects the warm glow for the individuals of both absolute and relative charitable giving. The reason is that the individual values both kinds, while the government values neither of them. Hence, the correction terms include both the absolute and relative components of the warm glow of giving. Note also that the factor \( 1-\bar{\alpha} \) serves to take into account that decreased charitable giving comes at the cost of an increase in the positional consumption externality. The latter motivates a smaller increase in \( T_g \) to correct for the behavioral failure than in the absence of any consumer preference for relative consumption, ceteris paribus.
The modifications due to non-separability largely follow the same logic as before. The usefulness of $\bar{c}$ and $\bar{g}$ as means of relaxing the incentive compatibility constraints, and consequently the importance of $\alpha^d$ and $\beta^d$ for the marginal tax treatment of charitable giving, depend on how positional people are, how the degrees of positionality vary with the labor supply, and on the product $BC$. If the degree of consumption positionality increases with the time spent on leisure (such that $\alpha^d < 0$), we saw above that the government responds through a policy-induced increase in $\bar{c}$, which includes lower marginal income tax rates and less direct public good provision. In alignment with this policy response, we can see from Proposition 5 that $\alpha^d < 0$ also leads to lower marginal subsidies (or higher marginal taxes) on charitable giving. By analogy, $\alpha^d > 0$ would motivate a lower $\bar{c}$ than otherwise, which leads to higher marginal subsidies (or lower marginal taxes) on charitable giving.

The effect of $\beta^d$ – see (14b) – in the policy rule for $T_g$ can be interpreted in a similar way. If the degree of positionality in charitable giving increases with the time spent on leisure, i.e., if $\epsilon_l^d < 0$, individuals with more leisure time suffer more from the positional gifts externality compared with individuals with less leisure time, ceteris paribus, implying that $\beta^d < 0$. This means that the government can relax the incentive compatibility constraints through a policy-induced increase in $\bar{g}$, which motivates a higher marginal subsidy (or lower marginal tax) on charitable giving than otherwise. The opposite policy incentive arises if $\beta^d > 0$, in which case a policy-induced decrease in $\bar{g}$ leads to a relaxation of the incentive compatibility constraints.

The product $BC$ is now interpretable in terms of the non-corrective component of the marginal income tax, which follows directly from (i), i.e.,

$$B_w^C = \frac{1}{1 + \epsilon_l^d(\bar{w})} \left( \frac{T_y^{(w)}}{1-T_y^{(w)}} - \frac{\bar{\alpha} + \alpha^d}{1-\bar{\alpha}} \right).$$

The $BC$ component is proportional to the difference between the marginal income tax wedge and the marginal social value of the positional consumption externality. As such, the product $BC$ is still interpretable in terms of a tax distortion (as in the model without relative concerns examined in Section III), since the corrective tax component is subtracted away. Thus, the more distortive the income tax, the more important the other channels of redistribution will be.

Next, note that whether or not the government directly contributes to the public good plays the same role for the marginal tax treatment of charitable giving as it did in Section III.
The variable $S^0$, which adjusts the policy rule for $T_g$ when $G^{Gov} = 0$ in result (ii), is now slightly modified to reflect the positional consumption externality, as follows:

$$S^0 = 1 - \left( \int_0^\infty MRS_{G,c}^{(w)} \left( \frac{1 + \alpha^d}{1 - \alpha} + \varepsilon_{G,c}^B B_w C_w \right) f(w) dw \right)_{G^{Gov} = 0} \geq 0.$$ 

Finally, note that result (iii) on the relationship between the marginal subsidy/tax on charitable giving and the before-tax income is driven by the same mechanisms as in Section III, meaning that the corresponding results presented in Proposition 2 (under welfarism) and Proposition 3 (under non-welfarism) are the same here as well, as are the provided intuitions. The reason is that the component reflecting the positional externalities, i.e.,

$$\frac{1 - \bar{\alpha} \mu + \beta^d - \beta^{mon}}{1 - \bar{\beta} 1 + \alpha^d},$$

enters the policy rule for $T_g$ additively through a term common for all contributors, and does not qualitatively affect the relationship between the marginal subsidy and the before-tax income.

### C. Briefly on alternative measures of reference consumption and reference giving

The mean value comparison form examined above is the standard assumption in research on optimal taxation and relative consumption. We have examined the policy implications of two alternative comparison forms, both of which result in non-atmospheric externalities. One is the within-type comparison, where each individual compares their own consumption and charitable giving with those of individuals of the same ability type. Choosing it can be justified based on the idea that individuals may in particular compare their own behavior with that of similar others (e.g., Runciman, 1966). Equations (12) can then be replaced with the following type-specific externality constraints:

$$\bar{c}_w = c_w \text{ and } \bar{g}_w = g_w.$$  \tag{17}

Let $\alpha^d_w = \varepsilon_{G,c}^d B_w C_w$ and $\beta^d_w = \varepsilon_{G,c}^B B_w C_w$ be type-specific indicators of how the degrees of consumption positionality and positionality in charitable giving vary with labor supply (or leisure time), ceteris paribus. These variables will now replace $\alpha^d$ and $\beta^d$ in Propositions 4 and 5, which are defined in equations (14a) and (14b). As above, $N$ is an indicator such that $N = 0$ under welfarism and $N = 1$ under non-welfarism. Irrespective of whether the

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27 The calculations are available from the authors upon request.
government is welfarist or non-welfarist, the policy rules for marginal income taxation and governmental provision in Proposition 4 change to read
\[
\frac{T^{(w)}_y}{1-T^{(w)}_y} = A_w B_w C_w + \frac{\alpha_w + \alpha_d}{1-\alpha_w}, \quad (18a)
\]
\[
\int_0^\infty MRS^{(w)}_{G,C} \left( \frac{1 + \alpha_d}{1-\alpha_w} + \epsilon^{G}_i B_w C_w \right) f(w)dw = 1, \text{ if } G^{Gov} > 0. \quad (18b)
\]
Similarly, the marginal tax treatment of charitable giving obeys the following policy rules for \( G^{Gov} > 0 \) and \( G^{Gov} = 0 \), respectively, at the social optimum:
\[
T^{(w)}_g = -1 + \frac{1-\alpha_w}{1+\alpha_w} \left( N \delta_w - \epsilon^{G}_i B_w C_w \right) MRS^{(w)}_{G,C} + \frac{1-\alpha_w}{1-\alpha_d} \epsilon^{G}_i B_w C_w, \quad (18c)
\]
\[
T^{(w)}_g = -1 + \frac{1}{1+\alpha_w} \left( N \delta_w - \epsilon^{G}_i B_w C_w \right) MRS^{(w)}_{G,C} + \frac{1}{1+\alpha_w} \epsilon^{G}_i B_w C_w, \quad (18d)
\]
where \( S^0 \) is now related to the generalized Samuelson condition in (18b), i.e.,
\[
S^0 = 1 - \int_0^\infty MRS^{(w)}_{G,C} \left( \frac{1 + \alpha_d}{1-\alpha_w} + \epsilon^{G}_i B_w C_w \right) f(w)dw \bigg|_{G^{Gov}>0} .
\]
Therefore, the policy rules derived under within-type comparisons are very similar to those presented in Proposition 5, with the only exceptions being that the average degrees of positionality are replaced with type-specific degrees and that zero-bunching in charitable giving has no direct consequences for the policy rule underlying the marginal subsidy/tax on charitable giving. The reason is, in both cases, that the externalities generated by within-type comparisons are type specific, such that the reference measures for consumption and charitable giving differ between types. Thus, (18c) and (18d) imply type-specific marginal taxation of charitable giving under welfarism regardless of whether the preferences are leisure separable. All other interpretations are the same as in the previous subsection.

The other comparison form is a generalized mean value comparison, implying that \( \overline{c} \) and \( \overline{g} \) are replaced with general weighted averages, \( c^R \) and \( g^R \), such that the externality constraints become
\[
c^R = \int_0^\infty c_w \kappa_w f(w)dw \text{ and } g^R = \int_0^\infty g_w m_w f(w)dw, \quad (19)
\]
where \( \kappa_w \) and \( m_w \) reflect the relative weights of type \( w \) in the reference measures. Each such relative weight sums to unity over the population as a whole,
\[
\int_0^\infty \kappa_w f(w)dw = \int_0^\infty m_w f(w)dw = 1.
\]
Thus, if the marginal contribution to the externality by each type is proportional to the number
of persons of this type, then \( \kappa_w = m_w = 1 \) for all \( w \). In general, however, \( \kappa_w \) and \( m_w \) will vary among types. This comparison form encompasses the case where most individuals compare upward, as suggested already by Veblen (1899), which in our case means that \( \kappa_w \) and \( m_w \) increase in ability. Although the generalized mean value comparison implies slightly more complex policy rules than mean value comparisons, most qualitative conclusions and interpretations presented above continue to hold here as well. Let

\[
\hat{\alpha} = \int_0^{\infty} \alpha_w f(w) \, dw \quad \text{and} \quad \hat{\beta} = \int_0^{\infty} \beta_w f(w) \, dw
\]

denote “contribution-weighted averages” of the degrees of consumption positionality and positionality in charitable giving, respectively. The following policy rules for marginal income taxation and governmental provision will now replace the policy rules in Proposition 4 (regardless of whether the government is welfarist or non-welfarist):

\[
\frac{T_y}{1-T_y} = A_y B_y C_y + \frac{\overline{\kappa}_w}{1+\overline{\alpha}} + \frac{\overline{\alpha}}{1+\overline{\alpha}} + \frac{\overline{\beta}}{1+\overline{\beta}} \quad \text{if } G^{\text{Gov}} > 0.
\]

There are two differences between these policy rules in (20) and those in Proposition 4. First, type \( w \)’s relative weight in the reference measure is now proportional to the externality component in (20a), emphasizing that non-atmospheric externalities necessitate type-specific corrections. Second, the variable \( \hat{\alpha} \) appears because an increase or decrease in \( \epsilon^R \) feeds back into the consumption behavior. In turn, this leads to additional effects on the externality, which depend on how the associated changes in consumption interact with the relative weights in the reference measure. In the special case examined in the previous subsection where the externality is atmospheric, \( \hat{\alpha} = \overline{\alpha} \).

The marginal tax treatment of charitable giving obeys the following policy rule under generalized mean value comparisons if \( G^{\text{Gov}} > 0 \) at the social optimum:

\[
T_y = 1 + \frac{1-\hat{\alpha}}{1+\overline{\alpha}} \left( N \delta_w MRS^{(w)} - \frac{\delta_{(w)}}{MRS^{(w)}} B_w C_w \right)
+ \frac{1-\hat{\beta}}{1+\overline{\beta}} \frac{\mu \beta + \beta^d - \beta^{\text{mon}}}{(1+\hat{\beta})} m_w + \mu
\]

(20c) coincides with the policy rule for \( T_y \) in (i) in Proposition 5 in the special case where the externalities are atmospheric, which means that \( \kappa_w = m_w = 1 \) for all \( w \), \( \hat{\alpha} = \overline{\alpha} \), and \( \hat{\beta} = \overline{\beta} \). In a way similar to within-type comparisons, albeit in contrast to mean value comparisons, (20c)
implies that the marginal subsidy/tax on charitable giving varies among types under welfarism even if the preferences are leisure separable. This is because the positional externalities associated with generalized mean value comparisons are non-atmospheric. By analogy to the analyses above, if $G^{\text{Gov}} = 0$ at the social optimum, a term proportional to $S^0$ will be added to the right-hand side of (20c), where $S^0$ now reflects (20b) such that

$$S^0 = 1 - \left( \int_0^\infty \text{MRS}_{G,C} \left( \frac{1 + \alpha - \hat{\alpha} - \hat{\alpha}^d}{1 - \hat{\alpha}} + \hat{\gamma} B \mathcal{C} \right) f(w) dw \right)_{G^{\text{Gov}} = 0}.$$

In summary, the policy implications of within-type comparisons and generalized mean value comparisons are reminiscent of those following from mean value comparisons. The structure of the policy rules is qualitatively very similar in all three cases. Therefore, the conclusion is that the main policy implications of relative concerns presented in Propositions 4 and 5 carry over to the other two comparison forms. The most important exception arises in the special case where the preferences are leisure separable, where a welfarist government would subsidize charitable giving at a flat rate under mean value comparisons, while the marginal subsidy rate varies among types under the other two comparison forms.

**V. A Dual Screening Approach to Optimal Taxation and Charitable Giving**

So far, we have assumed that individuals differ in productivity, measured by the before-tax wage rate, and that the preferences may vary between types but are identical for individuals of the same productivity. Drawing on Cremer et al. (2001), we will as an extension here present a model in which individuals differ in two dimensions: the before-tax wage rate ($w$), as above, and exogenous wealth ($b$), where wealth is also unobservable to the government and independent of the before-tax wage rate. We assume that charitable giving is a normal good here too, implying that we are able to use it as a second screening device in order to redistribute from individuals with higher ability and higher wealth. Ability and wealth then follow a joint distribution with density $f(w,b)$. 

The analytical approach is largely based on Lehmann et al. (2020). As is typically the case in multiple screening, the implementability conditions become more complex. To simplify the comparison with the policy rules derived in the single-screening model in Section IV, we assume that sufficient conditions are fulfilled, such that there is no “interior bunching.” Thus, the government can distinguish between all types working and giving to charity, respectively, implying that bunching only occurs at 0 (as before).

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28 For important earlier contributions to the literature on optimal taxation under multiple screening, see Mirrlees (1986), Kleven et al. (2009), and Golosov et al. (2014).
The utility function facing any individual of type \((w,b)\) can then be written as
\[
u_{w,b} = v(c_{w,b}, l_{w,b}, g_{w,b}, c_{w,b} - \bar{c}, g_{w,b} - \bar{g}, G),
\]
where \(\Delta c_{w,b} = c_{w,b} - \bar{c}\) and \(\Delta g_{w,b} = g_{w,b} - \bar{g}\). (21) has the same properties as (11). In addition, we assume that the marginal willingness to pay for the public good increases in income. The special case of leisure separability discussed below is the same as before. This means that (21) simplifies to
\[
u_{w,b} = V(k(c_{w,b}, g_{w,b}, c_{w,b} - \bar{c}, g_{w,b} - \bar{g}, G), l_{w,b}).
\]
As in most of Section IV, we assume that the relative concerns are driven by mean value comparisons, where the average consumption and average charitable giving can now be written as
\[
\bar{c} = \int_0^\infty \int_0^\infty c_{w,b} f(w,b) dw db \quad \text{and} \quad \bar{g} = \int_0^\infty \int_0^\infty g_{w,b} f(w,b) dw db,
\]
respectively. The individual budget constraint is now
\[
w_l_{w,b} - T(w_l_{w,b}, g_{w,b}) + b = c_{w,b} + g_{w,b},
\]
where a wealth term, \(b\), is added to net labor income. Note that since wealth is unobserved by the government, so is consumption for a given net income. Yet, the individual first-order conditions for labor and charitable giving are still given by (4).

The welfarist government still maximizes a generalized utilitarian social welfare function
\[
W = \int_0^\infty \int_0^\infty \psi(u_{w,b}) f(w,b) dw db,
\]
subject to a resource constraint
\[
\int_0^\infty \int_0^\infty w_{l,b} f(w,b) dw db = \int_0^\infty \int_0^\infty (c_{w,b} + g_{w,b}) f(w,b) dw db + G^{Gov},
\]
and, now a two-dimensional, incentive compatibility constraint
\[
\frac{du_{w,b}}{dw} = -\frac{l_{w,b} u_{(w,b)}}{w}, \quad \text{and} \quad \frac{du_{w,b}}{db} = u_{(w,b)}.
\]
Thus, the government maximizes social welfare function (24), where \(u_{w,b}\) is given by (21), subject to resource constraint (25), incentive compatibility constraints (26), externality constraints (22), and non-negativity constraints \(G^{Gov} \geq 0\) and \(g_{w,b} \geq 0\) for all \(w\) and \(b\).

In the non-welfarist case we add the “laundered” utility function that the government attaches to each type, which takes the same form as (16), i.e.,
\[
u_{w,b} = v(c_{w,b}, l_{w,b}, g_{w,b}, \Delta c_{w,b}, \Delta g_{w,b}, G) = v(c_{w,b}, l_{w,b}, \bar{c}, G).
\]
By analogy to Section IV, \( \bar{g}_{w,b} = g_{w,b} \) and \( \bar{\Delta}g_{w,b} = \Delta g_{w,b} \) in equilibrium, meaning that equations (21) and (27) take the same value. During optimization, the non-welfarist government treats the absolute and relative charitable giving as exogenous. Also, \( u_{w,b}^w \) replaces \( u_{w,b} \) in the social welfare function such that (24) is now replaced with

\[
W^w = \int_0^\infty \int_0^\infty \psi(u_{w,b}^w) f(w,b) dw \, db.
\]  

(28)

In addition, (27) is added to the set of constraints and \( u_{w,b}^w \) is treated as an additional state variable for all \( w \) and \( b \) under non-welfarism. As pointed out in earlier research, e.g., Lehmann et al. (2020), it is typically not possible to fully identify the optimal policy rules in multi-screening problems algebraically. Yet, in our case, we can solve the two-dimensional differential equation for the multipliers of the incentive compatibility constraint in a form such that, for each type, the Lagrange multiplier associated with the incentive compatibility constraint in the \( w \) dimension is proportional to what we can interpret as a weight factor in the \( w \) dimension, \( \Gamma_{w,b}^w \). Simultaneously, the Lagrange multiplier associated with the incentive compatibility constraint in the \( b \) dimension is proportional to the residual of the weight factor in the \( w \) dimension, \( 1 - \Gamma_{w,b}^w \). While this solution is clearly not unique (but depends on the weight factors), it provides considerable structure to the optimal policy rules. Knowledge about these weights can in principle (though not algebraically) be gained in simulations through the symmetry properties of the tax function. In particular, since both the marginal income tax and the marginal subsidy on charitable giving are functions of both gross income and charitable giving, we have \( T_{w}(w, b) = T_{b}(w, b) \) for each type (given interior solutions).

This approach allows us to define ABC factors associated with screening in the \( w \) dimension identical to the ones in Section IV, with the only difference being that each variable is a function of both \( w \) and \( b \) (instead of just \( w \)). Let us denote these factors \( A_{w,b}^w \), \( B_{w,b}^w \), and \( C_{w,b}^w \). We can correspondingly define for the screening problem in the unobserved wealth dimension:

\[ B_{w,b}^w = \int_0^1 \left[ 1 + \frac{\partial^0}{1 - \delta} \right] \exp \left[ -2 \left( \frac{u_{w,b}^{(w)}}{1 - \delta} - \int \frac{\partial \text{MRS}_{w,b}^{(w)}}{\text{MRS}_{w,b}} \, dm \right) \exp \left( -\frac{\partial \text{MRS}_{w,b}^{(w,m)}}{\text{MRS}_{w,b}} \, d\tau_{w,b} \right) \exp \left( -\frac{\partial \text{MRS}_{w,b}^{(w,m)}}{\text{MRS}_{w,b}} \, d\sigma_{w,b} \right) \right] \frac{f(w,b)}{1 - \bar{F}(w,b)} \, dw \, db. \]

29 Jacobs and Boadway (2014) use a similar formulation as a function of the ratio of marginal utilities of consumption in a (single screening) problem of optimal linear consumption taxation under nonlinear income taxation. The expression for \( B_{w,b}^w \) can alternatively be written as
\[ A_{w,b}^D = \varepsilon_{c(w,b)}^c, \]
\[ B_{w,b}^D = \int_{x}^{\infty} \left( \frac{1 + \alpha^D}{1 - \delta_{w,b}} - \delta_{w,b} \right) \frac{u_{(w,b)}}{u_e} \exp \left[ -\int_{x}^{\infty} \frac{u_{(w,b)}}{m} f(s,b) \right] \frac{f(s,b)}{1 - F(w,b)} \, ds, \]
\[ C_{w,b}^D = \frac{1 - F(w,b)}{cf(w,b)}, \]

where \( \varepsilon_{c}^c = (\partial MRS_{1,c} / \partial c)(c / MRS_{1,c}) \). Finally, we define two-dimensional analogues of \( \alpha^d \) and \( \beta^d \) in Section IV, showing how the degrees of positionality vary with respect to both labor (as before) and consumption:

\[ \alpha^D = \int_{0}^{\infty} \left( \Gamma_w^w B_w^w C_{w,b} e_{I_{(w,b)}}^c - (1 - \Gamma_w^w) B_{w,b}^c C_{w,b} e_{c_{(w,b)}}^c \right) \alpha_{w,b} f(w,b) \, dw \, db, \quad (29a) \]
\[ \beta^D = \int_{0}^{\infty} \left( \Gamma_w^w B_w^w C_{w,b} e_{G_{(w,b)}}^G - (1 - \Gamma_w^w) B_{w,b}^c C_{w,b} e_{c_{(w,b)}}^c \right) \beta_{w,b} MRS_{G,c}^G f(w,b) \, dw \, db, \quad (29b) \]

where we have introduced \( \varepsilon_{c}^c = (\partial \alpha / \partial c)(c / \alpha) \) and \( \varepsilon_{c}^d = (\partial \beta / \partial c)(c / \beta) \). The main results are presented in Proposition 6 where we again use \( N \) as an indicator variable, such that \( N = 0 \) under welfarism and \( N = 1 \) under non-welfarism:

**Proposition 6.** Under dual screening and social comparisons, we obtain:

(i) \[ T_{x}^{(w,b)} = \frac{1 - T_{y}^{(w,b)}}{\Gamma_{w,b}^w A_{w,b}^w B_{w,b}^w C_{w,b}^w - (1 - \Gamma_{w,b}^w) A_{w,b}^w B_{w,b}^w C_{w,b}^w - \alpha^D + \frac{\alpha^D}{1 - \alpha}}, \]

(ii) If \( G_{x}^{Gov} > 0 \), then \[ \int_{0}^{\infty} MRS_{G,c}^{(w,b)} \left( \frac{1 + \alpha^D}{1 - \alpha} + \Gamma_{w,b}^w B_{w,b}^w C_{w,b}^w e_{I_{(w,b)}}^c - (1 - \Gamma_{w,b}^w) B_{w,b}^c C_{w,b}^c e_{c_{(w,b)}}^c \right) f(w,b) \, dw \, db = 1. \]

(iii) If \( G_{x}^{Gov} > 0 \), then \[ T_{y}^{(w,b)} = -1 + \frac{1 - \alpha^D}{1 + \alpha^D} \left( \Gamma_{w,b}^w B_{w,b}^w C_{w,b}^w e_{I_{(w,b)}}^c - (1 - \Gamma_{w,b}^w) B_{w,b}^c C_{w,b}^c e_{c_{(w,b)}}^c + N \delta_{w,b} MRS_{G,c}^{(w,b)} \right) + \frac{1 - \alpha^D + \mu + \beta^D - \beta_{nom}^D}{1 + \alpha^D} \]

(iv) If \( G_{x}^{Gov} = 0 \), then \[ T_{y}^{(w,b)} = -1 + \frac{1 - \alpha^D}{1 + \alpha^D} \left( \Gamma_{w,b}^w B_{w,b}^w C_{w,b}^w e_{I_{(w,b)}}^c - (1 - \Gamma_{w,b}^w) B_{w,b}^c C_{w,b}^c e_{c_{(w,b)}}^c + N \delta_{w,b} MRS_{G,c}^{(w,b)} \right) + \frac{1 - \alpha^D + (1 - \mu) S_{D}^D + \beta^D - \beta_{nom}^D}{1 + \alpha^D} \]

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Starting with the marginal income tax rates, we note that the $\Gamma^x_{w,b} A^x_{w,b} B^x_{w,b} C^x_{w,b}$ component carries over from the single screening case with the same interpretation, although we have a weight factor here, and the marginal income tax will vary with both income and charitable giving. The second component, $-(1-\Gamma^x_{w,b}) A^b_{w,b} B^b_{w,b} C^b_{w,b}$, is novel and contributes to decrease the marginal income tax, ceteris paribus, since $\epsilon^{x,c} \epsilon > 0$ due to the assumption that leisure is a normal good. Here too, the A term reflects labor-related efficiency concerns, whereas the B term reflects the desire for redistribution and the C term the fatness of the upper tail of the wealth distribution. The intuition is that a lower marginal income tax rate leads to higher consumption, which makes mimicking in the $b$ dimension less attractive.

The other policy rules are extended in a similar way due to screening in the $b$ dimension. In particular, note that the properties of the utility function imply $\epsilon^{x,c} > 0$ and $\epsilon^{x,c} > 0$. Taken together, this means that terms proportional to $B^b_{w,b} C^b_{w,b}$ that are not due to concerns regarding relative consumption and relative charitable giving consistently work to i) decrease the marginal income taxes, ii) decrease the governmental provision of the public good, and iii) increase the marginal subsidy (or decrease the marginal tax) attached to charitable giving, ceteris paribus. The intuition is that higher income (due to lower marginal income taxation) increases the individuals’ marginal willingness to pay for voluntary contributions. To relax the incentive compatibility constraints, the government contributions to the public good are, therefore, partly replaced with charitable giving.

The final term in each policy rule reflects the positional externalities. It is striking that the structure of these effects is very similar to that under the one-dimensional heterogeneity. Yet, note that $\alpha^D$ also reflects a relationship between the degree of consumption positionality and private consumption, measured with the labor supply held constant. If $\epsilon^{x} > 0$, individuals with a higher level of consumption will suffer more from the positional consumption externality than those with a lower level of consumption, ceteris paribus. In the policy rules described above, this works to reduce $\alpha^D$. To keep individuals with a higher level of wealth from mimicking those with a lower level, the government decreases the marginal income tax rate (which leads to an increase in $\tilde{c}$), decreases the provision of the public good, and decreases the marginal subsidy on charitable giving, and vice versa if $\epsilon^{x} < 0$. Similarly, if $\epsilon^{x} > 0$, individuals with a high consumption level will suffer more from the positional gifts externality than individuals with a lower level of consumption, ceteris paribus. This effect works to decrease $\beta^D$ in the policy rules in Proposition 4. The government can then relax the
incentive compatibility constraint through a decrease in the marginal tax (or increase in the marginal subsidy) attached to charitable giving, which contributes to increase $\bar{g}$. The opposite policy incentive emerges if $e_c^d < 0$.

Finally, although $e_i^a = e_i^b = e_G^{d,c} = e_G^{\bar{c},c} = 0$ under leisure separability, $e_c^a$ and $e_c^b$ are typically different from zero, meaning that $\alpha^b$ and $\beta^b$ do not vanish from the policy rules under such separability. Moreover, since $e^{d,c} > 0$ and $e^{\bar{c},c} > 0$, the corresponding terms in the policy rules will not vanish either. This implies that public good provision and the marginal tax treatment of charitable giving are not based on first-best policy rules under welfarism and leisure separability, as they were in the simpler model in Section IV. This is interpretable as an implication of a broader result pointed out in earlier work, namely that the Atkinson-Stiglitz theorem does not apply in general under multiple heterogeneity (e.g., Cremer et al., 2001; Saez, 2002).

VI. Numerical Illustration

In this section, we illustrate the theoretical results with numerical simulations based on simple functional forms. In doing so, we are able to go beyond the policy rules and illustrate how the levels of marginal and average taxation, as well as the overall redistribution policy, vary with key parameters and across policy objectives.

The wage rates ($w$) are distributed according to the Champernowne distribution with density function $f(w) = \zeta (z^{-\zeta} w^{\zeta-1}) / (z^{-\zeta} + w^{\zeta})^2$, where $\zeta$ is the shape parameter and $z$ the scale parameter. Our parameter choices are intended to approximately mimic the U.S. economy with respect to the mean gross income and the corresponding Gini coefficient, as well as allow for a realistic thick upper tail of the resulting income distribution, and are based on Tuomala (2016): $\zeta = 3.3$ and $z = \exp(-1)$. This implies a productivity Gini coefficient of about 0.3.

While empirical realism is an obvious virtue, it is also important that the simulations are presented in a form and build on models that facilitate straightforward interpretations. In this paper, we prioritize the latter over empirical realism, which allows us to examine how the optimal policies vary with key parameters and illustrate key theoretical results. Therefore, we choose a simple leisure-separable utility function, where the degrees of positionality in charitable giving and consumption positionality are constant across types, as follows:

$$u_c = \ln \left( (1-\alpha)c_w + \alpha(c_w-\bar{c}) \right) + \xi \ln(1-l_w) + \sigma \ln \left( \chi + (1-\beta)g_w + \beta(g_w-\bar{g}) \right) + \rho \ln (G), \quad (30)$$

$$= \ln(c_w-\alpha\bar{c}) + \xi \ln(1-l_w) + \sigma \ln(\chi + g_w - \beta\bar{g}) + \rho \ln (G)$$
where $\alpha$ and $\beta$ denote the constant degrees of consumption positionality and positionality in charitable giving, respectively. The parameters $\xi$, $\sigma$, and $\rho$ reflect the strengths of the preferences for leisure, charitable giving, and public consumption, respectively, relative to private consumption. The logarithmic functional form is mathematically convenient, and variants of this utility function have been used in other numerical work on optimal taxation (e.g., Saez, 2001; Kanbur and Tuomala, 2013; Aronsson and Johansson-Stenman, 2018).

In the numerical reference scenario, we set $\xi = 1$, $\sigma = 0.065$, $\rho = 0.2$, $\chi = 0.02$, $\mu = 0.4$, $\alpha = 0.2$, and $\beta = 0.25$. We also present extensive sensitivity analyses with respect to the key parameters $\mu$, $\alpha$, and $\beta$, while the other parameters are held constant. These sensitivity analyses include cases where the individuals are completely non-positional and cases where they are very positional, as well as cases where the transaction cost ranges from zero (the assumption in earlier research) to a relatively high number, allowing us to connect the numerical illustration to the theoretical results. The social welfare function is assumed to be utilitarian, i.e., $\psi'(\cdot) = 1$ for all $w$. In all simulations, the before-tax income increases monotonically in ability, such that the before-tax income and the level of charitable giving are distinguishable among types, if they work and contribute to charity, respectively. We have also verified that all simulations satisfy the second-order conditions for welfare maximization.

We begin by illustrating the optimal tax policy and contributions to the public good in an economy where the consumers are not concerned with their relative consumption and relative charitable giving in Figures 1 and 2. This scenario corresponds to the model examined in Section III and means that $\alpha = \beta = 0$ in (30). Figures 3–7 present the optimal tax policy and contributions to the public good in economies where the individuals are concerned with their relative consumption and relative charitable giving such that $\alpha > 0$ and $\beta > 0$. These simulations correspond to the models examined in Section IV.

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30 The leisure parameter $\xi$ is typically set to 1 in related studies (Mirrlees, 1971; Kanbur and Tuomala, 2013; Tuomala, 2016). The parameters $\sigma$, $\rho$, and $\mu$ are chosen such that there will be a substantial government contribution to the public good in the numerical reference scenario, whereas only those earners with relatively high incomes contribute voluntarily. The degrees of positionality in the numerical reference scenario are set in the lower part of the range of empirical estimates referred to in Footnote 24.
Figure 1. Optimal taxation under varying transaction costs.

Note: The figure plots marginal and average income tax rates and the marginal subsidy on charitable giving for different transaction costs when $\alpha = \beta = 0$. W denotes welfarism and NW non-welfarism.

Figure 1 examines the effects of varying the transaction cost, $\mu$, in a model without any relative concerns ($\alpha = \beta = 0$). We can see that the marginal income tax ($T_y$) schedule follows the well-known U-shaped pattern (e.g., Saez, 2001; Tuomala, 2016), which is due to our relatively fat-tailed ability distribution (compared with the log-normal distribution), and the average tax rate ($T/y$) increases in income. The optimal marginal income tax schedules in the figure reflect a welfarist objective; however, the pattern is very similar under non-welfarism (as we saw in Section III, the two governments implement the same policy rule for marginal income taxation).

Our simulations clearly support the idea that the government ought to subsidize charitable giving at the margin. In accordance with the theoretical results, these subsidies differ considerably between the welfarist and non-welfarist governments. The welfarist government implements a flat subsidy rate, since the utility function (30) is characterized by leisure separability, whereas the marginal subsidy rate implemented by the non-welfarist government increases with the before-tax income. Note that these curves are shown only for the interval where people contribute a positive amount; thus, to the left of each such curve we have bunching at zero contributions (which implies that the optimal marginal subsidy rate is
not unique in the corresponding interval). Consistent with Proposition 2, the marginal subsidy on charitable giving decreases sharply with the transaction cost under both policy objectives and may even turn into a marginal tax in the non-welfarist case.

Figure 2. Public good contributions under varying transaction costs.

![Graph showing public good contributions under varying transaction costs.](image)

*Note:* The figure shows how private and public contributions to the public good vary with the transaction cost when $\alpha = \beta = 0$. W denotes welfarism and NW non-welfarism; $G^{priv}$ = private charitable contribution.

Figure 2 shows, as expected, that the average charitable giving decreases and the public contributions increase with the transaction cost attached to private contributions. When the transaction cost is zero, the entire public good supply comes from voluntary contributions under both the welfarist and the non-welfarist policy objective, consistent with Propositions 1 and 3.\(^{31}\) As indicated above, the intuition is that public provision and voluntary contributions are equally resource efficient in that case, whereas the voluntary contributions come with the additional benefit of warm glow, which the welfarist government recognizes as a welfare gain and the non-welfarist government as a redistributive gain. Consistent with the latter, note from Figure 1 that the average tax rate, and thus the total amount of taxes paid, actually becomes negative for high-income earners when the transaction cost is zero. The explanation

\(^{31}\) As we saw in Section IV, this result continues to apply if we add the assumption that people are concerned with their relative consumption and relative charitable giving. To save space, we therefore refrain from showing the corresponding diagrams for the more general model where $\alpha > 0$ and $\beta > 0$. 

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is that these individuals must receive large enough subsidies in order to contribute sufficiently. In turn, these subsidies are key ingredients of the redistribution policy.

Let us now add concerns about relative consumption and relative charitable giving. Figure 3 shows that the marginal income tax rates implemented by a welfarist government are higher throughout the income distribution than in Figure 1.\(^{32}\) The same qualitative result (albeit not reported) applies under non-welfarism. This is a direct consequence of the relative consumption concerns, leading to negative positional consumption externalities.\(^{33}\) Correspondingly, the tax system is also more redistributive in the sense that the average income tax is lower among low-income earners and higher among high-income earners than in Figure 1. The intuition is that positional consumption externalities allow the government to raise some of its revenue from non-distortive taxation, which in turn opens up for more redistribution.

As expected from Propositions 2 and 3, Figure 3 also shows that the marginal tax treatment of charitable giving differs substantially between the two policy objectives. The marginal subsidy is again constant among givers under welfarism and increases with the before-tax income under non-welfarism, although the levels of the marginal subsidies differ from those in Figure 1. The marginal subsidy on charitable giving decreases with the degree of charitable giving positionality, \(\beta\), under welfarism. In the non-welfarist case, the marginal subsidy is much less sensitive to (and even increases with) \(\beta\). This is because very few individuals contribute to charity under non-welfarism in our model, meaning that \(\beta^{\text{non}}\) becomes large. It also means that an increase in \(\beta\) leads to a relatively large increase in \(\beta^{\text{non}}\), which counteracts (and may even dominate) the incentive to “tax away” a larger positional gifts externality.

\(^{32}\) This is in line with findings in other studies on optimal taxation and relative consumption (e.g., Kanbur and Tuomala, 2013; Aronsson and Johansson-Stenman, 2018).

\(^{33}\) Since the utility function in (30) is additively separable, one can show that both the marginal and average income tax rates are independent of \(\beta\).
Figure 3. Tax policy and variation in $\beta$.

Note: The figure shows the marginal and average income tax rates for the numerical reference scenario and plots the marginal subsidy on charitable giving against the degree of positionality in charitable giving, $\beta$. W denotes welfarism and NW non-welfarism.

Figure 4 shows, correspondingly, that the average charitable giving decreases with $\beta$ and the government contribution increases with $\beta$ under welfarism. These relationships are absent under a non-welfarist objective.
Figure 4. Public good contributions and variation in $\beta$.

![Graph showing public good contributions and variation in $\beta$.](image)

Note: The figure shows how the private and public contributions to the public good vary with the degree of positionality in charitable giving, $\beta$, when the other parameters are chosen according to the numerical reference scenario. $W$ denotes welfarism and NW non-welfarism.

Figure 5 shows how the marginal subsidy on charitable giving varies with the degree of consumption positionality, $\alpha$, when all other parameters reflect the numerical reference scenario. In accordance with the policy rules for $T_\ell$ presented in Proposition 5, we can see that a higher $\alpha$ is always associated with higher marginal subsidies on charitable giving, regardless of whether the government is welfarist or non-welfarist. The intuition is straightforward: The welfarist government aims at internalizing the positional consumption externality by incentivizing a shift in expenses from consumption to charitable giving. A non-welfarist government additionally raises the marginal subsidies to high-income earners. Since redistribution is costly, it is socially preferable that high-income earners contribute more than low-income earners to charity.

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Again for simplicity, we only present one curve each for the marginal and average income tax rates (corresponding to the numerical reference scenario where $\alpha = 0.2$ and $\beta = 0.25$), although these rates will now vary considerably with $\alpha$, as shown by comparing Figures 1 and 3.
Figure 5. Subsidy schedules for charitable giving under different values of $\alpha$

![Graph showing subsidy schedules for charitable giving under different values of $\alpha$.](image)

*Note: $T_y$ and $T/y$ are plotted for the numerical reference scenario, while $T_g$ varies with the degree of consumption positionality, $\alpha$.*

According to Figure 6, $\alpha$ does not affect the levels of charitable giving; instead, the tax system is adjusted in such a way that charitable giving remains constant. Intuitively, an increase in $\alpha$ implies, in addition to higher marginal subsidies on charitable giving, increased marginal income taxes. The latter leads to lower disposable incomes and contributes to reduce the levels of charitable giving, which tends to offset the positive effect of increased subsidization. With the functional form assumptions presented above, the two effects cancel each other out. Still, charitable giving and government contributions to the public good differ between the welfarist and non-welfarist objectives, such that individual contributions are higher under welfarism than non-welfarism and vice versa for public provision.

While the results presented in Figures 3 and 5 are based on mean value comparisons, i.e., individuals compare their own consumption and charitable giving with economy-wide averages, Figure 4 supplements these simulations by showing numerical results based on within-type comparisons (as in Section IV C).
Figure 6. Public good contributions for values of $\alpha$

![Graph showing public good contributions for values of $\alpha$.]

Note: All other parameters are chosen according to the numerical reference scenario.

Figure 7 is similar to Figure 3, which shows the corresponding results under mean value comparisons. Since our functional form assumption for the utility function (30) means that the degrees of positionality are the same for everybody, the marginal subsidy on charitable giving is constant (independent of income) under welfarism here too, while it increases in income under non-welfarism. The structure of marginal and average income taxation is also similar to that under mean value comparisons.
Figure 7. Optimal taxation under within-type comparisons

Note: The marginal and average income tax rates are based on the numerical reference scenario. The marginal subsidies on charitable giving are calculated for different values of $\beta$.

It is worth emphasizing two important differences between mean value comparisons and within-type comparisons for the marginal tax treatment of charitable giving. First, the marginal subsidies on charitable giving decrease with $\beta$ under within-type comparisons regardless of whether the government is welfarist or non-welfarist. Under mean value comparisons, the negative relationship between the marginal subsidy on charitable giving and $\beta$ only applies under welfarism. The intuition is that zero-bunching in terms of charitable giving does not affect the marginal tax treatment of charitable giving under within-type comparisons, as explained in Section IV. In other words, and contrary to the case of mean value comparisons, the variable $\beta^{\text{non}}$ does not affect the underlying policy rules here. Therefore, the welfarist and non-welfarist governments always change the marginal subsidies/taxes on charitable giving in the same qualitative way in response to a change in $\beta$.

Second, the marginal subsidies on charitable giving are much lower under within-type comparisons than under mean value comparisons. Intuitively, as only high-income individuals give to charity, the reference levels, and ceteris paribus the marginal utilities of giving, are higher under within-type than mean value comparisons. Consequently, to reach a given level of the public good, we need less subsidization under within-type comparisons.
VII. Conclusion

Let us return to the question posed initially: What is the optimal tax treatment of charitable giving? While the answer is, naturally, complex, we would like to emphasize nine key results, of which the first seven are based on the single screening models in Sections III and IV.

1) For a welfarist government, and regardless of whether the individuals are concerned with their relative consumption and relative charitable giving, zero transaction costs and leisure separability together imply that charitable giving should be subsidized to an extent that completely crowds out government contribution to the public good.

2) The same result holds for a non-welfarist government that does not value the warm glow of giving.

3) The optimal marginal subsidy on charitable giving decreases strongly with the size of the transaction costs.

4) When these costs are sufficiently high, the marginal subsidy decreases with the degree of positionality in charitable giving, and the marginal subsidy is much lower for a non-welfarist than a welfarist government.

5) A welfarist government would implement a flat rate marginal subsidy on charitable giving under leisure separability, regardless of whether individuals are concerned with their relative consumption and relative charitable giving. Under a non-welfarist government, the marginal subsidy on charitable giving typically increases in before-tax income.

6) In the general non-separable case, we demonstrate how deviation from the simpler results under leisure separability can be expressed in terms of the marginal income tax rate directly (or components thereof).

7) Stronger concerns for relative charitable giving tend to support lower marginal subsidies, whereas concerns for relative consumption work in the other direction.

In the dual screening case, where both individual ability and wealth are unobservable to the government and charitable giving is used as a screening device in the wealth dimension, the policy rules naturally extend to reflect the additional source of heterogeneity.

8) All insights regarding externality correction carry over to the dual screening model with the only modification being that the government now has an incentive to account for relationships between the degrees of positionality and private consumption (not just the relationships between positionality degrees and leisure) in order to make mimicking less attractive.

9) Other tax components that are due to screening in the wealth dimension consistently work in the direction of decreasing the marginal income taxes, decreasing the provision of the public good by the government, and increasing the marginal subsidies on charitable giving. Thus, the role of charitable giving as a means of income redistribution is emphasized even further.
Appendix

Proof of Propositions 4 and 5 (Propositions 1–3 follow as special cases)

We start by proving Proposition 4 for the welfarist case. In the optimal control problems, we treat utility, \( u_w \), as a state variable, while \( l_w, g_w, G^\text{Gov}, \bar{c}, \) and \( \bar{g} \) are control variables. Consumption, \( c_w \), is defined by the inverse of the function \( u(\cdot) \) in (17), i.e.,

\[ c_w = h(l_w, g_w, G, \bar{c}, \bar{g}, u_w; w) \].

The Lagrangean becomes

\[
L = \int_0^\infty w(u_w) f(w) dw + \lambda \left( \int_0^\infty \left( w l_w - h(l_w, g_w, G, c, \bar{g}, u_w) - g_w \right) f(w) dw - G^\text{Gov} \right) + \int_0^\infty \left( \theta_a \frac{u_a(l_w, g_w, G, \bar{g}, \bar{c}, u_w, l_w, g_w, G)}{w} - \eta u_{\bar{c}} \right) dw + \int_0^\infty \eta \left( \bar{c} - \int_0^\infty h(l_w, g_w, G, \bar{g}, \bar{c}, u_w) f(w) dw \right) + \zeta \left( \bar{g} - \int_0^\infty g_w f(w) dw \right) + \int_0^\infty \phi a g Dw + \Upsilon G^\text{Gov}.
\]

To avoid clutter, we suppress the type indicator \( w \) except in integral expressions. By using \( h_l = -u_l / u_c = -MRS_{l,c} \) and \( h_G = -u_G / u_c = -MRS_{G,c} \), the social first-order conditions for \( l \) and \( G^\text{Gov} \), respectively, can be written as follows if \( G^\text{Gov} > 0 \):

\[ w + MRS_{l,c} = -\frac{\theta u_{MRS_{l,c}}}{\lambda f(w)} 1 + \epsilon l^e_i \frac{\eta}{\lambda} MRS_{l,c} = 0 \quad (A2) \]

\[ \int_0^\infty MRS_{G,c}^{(w)} (1 + \frac{\eta}{\lambda}) f(w) dw + \frac{1}{\lambda} \int_0^\infty \theta u_{MRS_{G,c}^{(w)}} MRS_{G,c}^{(w)} dw = 1. \quad (A3) \]

We can derive an expression for \( \eta / \lambda \) from the social first-order condition for \( \bar{c} \). By using \( h_c = -u_c / u_c = -MRS_{c,c} \), the social first-order condition for \( \bar{c} \) can be written as follows:

\[ \frac{1}{\lambda} \int_0^\infty \theta u_{MRS_{c,c}^{(w)}} MRS_{c,c}^{(w)} dw = \frac{1}{w} \int_0^\infty (1 + \frac{\eta}{\lambda}) f(w) dw + \frac{\eta}{\lambda} \left( 1 + \frac{\eta}{\lambda} \int_0^\infty MRS_{c,c}^{(w)} f(w) dw \right) = 0. \quad (A4) \]

Since \( \alpha = -MRS_{c,c}, \bar{\alpha} = \int_0^\infty \alpha f(w) dw, \) and \( \partial MRS_{c,c} / \partial \bar{c} = -\partial \alpha / \partial \bar{c}, \) (A4) implies

\[ \frac{\eta}{\lambda} = \frac{\bar{\alpha}}{1 - \bar{\alpha}} + \frac{1}{1 - \bar{\alpha}} \frac{\eta}{\lambda} \int_0^\infty \frac{\partial \alpha^{(w)}}{\partial \bar{c}} \frac{l_w}{w} dw = \frac{\bar{\alpha} + \alpha^d}{1 - \bar{\alpha}}. \quad (A5) \]

Substitute (A5) into (A2) and (A3) and use the private first-order condition \( w + MRS_{l,c} = wT_y \).

Finally, use \( B_c w = (u_w^{(w)} \theta_a) / (\lambda w f(w)) \) and rearrange to obtain (i) and (ii) in Proposition 4.

Proposition 1 follows as the special case where \( \bar{\alpha} = \alpha^d = 0, \) in which \( \eta = 0. \)

By using \( h_g = -u_g / u_c = -MRS_{g,c} \) and \( h_g = -u_g / u_c = -MRS_{g,c} = \beta MRS_{g,c} \), the social first-order conditions for \( g \) and \( \bar{g} \), respectively, can be written as follows if \( G^\text{Gov} > 0 \):
\[
MRS_{g,c} \left(1 + \frac{\eta}{\lambda}\right) = -\frac{\partial u_c}{\lambda f(w)} - \frac{\partial MRS_{g,c} \frac{l}{w} + \mu + \xi - \phi}{\lambda f(w)} \quad \text{(A6a)}
\]

\[
\frac{\zeta}{\lambda} = \frac{1}{\lambda} \int_0^\infty \theta u_c(w) \left(\frac{\partial \beta_{w}^c}{\partial \lambda} MRS_{g,c}^w + \beta_{w}^c \frac{\partial MRS_{g,c}^w}{\partial \lambda}\right) l_w \, dw + \left(1 + \frac{\eta}{\lambda}\right) \int_0^\infty \beta_{w}^c MRS_{g,c}^w f(w) \, dw \quad \text{(A6b)}
\]

Substituting (A6) into (A7) gives

\[
\frac{\zeta}{\lambda} = \frac{\mu \beta}{1 - \beta} + \frac{1}{1 - \beta} \int_0^\infty \theta u_c(w) \frac{\partial \beta_{w}^c}{\partial \lambda} l_w \, dw - \frac{1}{1 - \beta} \int_0^\infty \beta_{w}^c \phi w \, dw = \frac{\mu \beta + \beta^d - \beta^{non}}{1 - \beta} \quad \text{(A6c)}
\]

Substitute (A6c) into (A6a) and use the private first-order condition \(MRS_{g,c} = 1 + T_g\). Finally, write the first term on the right-hand side of (A6a) in elasticity form and use (A7) to derive (A6c). This gives the policy rule (i) for \(T_g\) in Proposition 5.

If \(G^{Gov} = 0\), as in result (ii), the derivation procedure is the same, except that (A3) does not hold and \((1 - \mu)S^0\) will be added to the right-hand side of (A6), where \(S^0\) is given by

\[
S^0 = \frac{Y}{\lambda} = 1 - \int_0^\infty MRS_{g,c}^w \left(1 + \frac{\alpha^d}{1 - \alpha^d} + \epsilon_{i(w)}^G C_u \right) f(w) \, dw \quad \text{at the social optimum.}
\]

Results (i) and (ii) in Proposition 2 follow as the special cases where \(\alpha = \alpha^d = \beta = \beta^d = 0\), in which \(\eta = 0\) and \(\zeta = 0\).

Let us then turn to result (iii) in Proposition 5. Consider first the case where \(G^{Gov} > 0\) at the social optimum. By using the private first-order condition for charitable giving, \(MRS_{g,c} = 1 + T_g^{(w)}\), the policy rule for \(T_g^{(w)}\) in result (i) in Proposition 5 can be written as follows under welfarism:

\[
T_g^{(w)} = -1 - \frac{1 - \alpha^d}{1 + \alpha^d} \Lambda_w \left(\frac{T_y^{(w)}}{1 - T_y^{(w)}} - \frac{\alpha^d}{1 - \alpha^d} \right) (1 + T_g^{(w)}) + \frac{1 - \alpha^d}{1 - \beta^d} \left(1 + \frac{\alpha^d}{1 + \alpha^d}\right), \quad \text{(A7)}
\]

where \(\Lambda_w = \epsilon_{i(w)}^G \left(1 + \epsilon_{i(w)}^G C_u \right)\) is assumed to be locally constant. Differentiating with respect to \(T_g^{(w)}\) and \(y_w\) gives

\[
\frac{\partial T_g^{(w)}}{\partial y_w} = -\frac{1}{K^{wef}_w} \frac{1 - \alpha^d}{1 + \alpha^d} \Lambda_w T_y^{(w)} \left(1 - T_y^{(w)}\right)^2, \quad \text{(A8)}
\]

where

\[
K^{wef}_w = 1 + \frac{1 - \alpha^d}{1 + \alpha^d} \Lambda_w \left(\frac{T_y^{(w)}}{1 - T_y^{(w)}} - \frac{\alpha^d}{1 - \alpha^d} \right) = 1 + \frac{1 - \alpha^d}{1 + \alpha^d} \Lambda_w A_u B_u C_w > 0.
\]

To show that \(K^{wef}_w > 0\), we can rewrite (A7) to read
\[ T^{(w)}_g = -1 + \frac{1}{K^{wel}_w} \left( 1 - \bar{\alpha} \right) \frac{\mu + \beta^d - \beta^{non}}{1 - \bar{\beta} + (1 + \alpha^d)} \].

Note that \((\mu + \beta^d - \beta^{non}) / (1 + \alpha^d)\) must be positive for the policy rule in (A7) to apply; otherwise, \(G^{Gov} = 0\) and the policy rule for \(T^{(w)}_g\) would be modified as described by result (ii) in Proposition 5. Therefore, \(K^{wel}_w > 0\) implies \(T^{(w)}_g > -1\) for all types \(w\) giving to charity.

If \(G^{Gov} = 0\), the same reasoning applies to the policy rule in result (ii) in Proposition 5. In this case, \((\mu + (1 - \mu) S^0 + \beta^d - \beta^{non}) / (1 + \alpha^d) > 0\) plays the same role as \((\mu + \beta^d - \beta^{non}) / (1 + \alpha^d)\) did in (A7) above. Therefore, \(K^{wel}_w > 0\) ensures that \(T^{(w)}_g > -1\), and (A8) continues to apply. The corresponding special case in Proposition 2 follows if \(\bar{\alpha} = \alpha^d = \bar{\beta} = \beta^d = 0\) in (A8).

In order to show (iii) in Proposition 4 in the welfarist case, note that under zero transaction costs, the social first-order conditions for \(g_w\) and \(G^{Gov}\) can be written as

\[
g_w : MRS_{g,c}^{(w)} + \frac{1 - \bar{\alpha}}{1 + \alpha^d} \epsilon_{(w)}^{gc} B_w C_w MRS_{g,c}^{(w)} + \frac{1 - \bar{\alpha}}{1 - \bar{\beta} + 1 + \alpha^d} \frac{\beta^{non} - \beta^d}{1 - \bar{\beta} + 1 + \alpha^d} \leq 0 \forall w
\]

\[
G^{Gov} : \int_0^y \left( \frac{1 + \alpha^d}{1 - \bar{\alpha}} + \epsilon_{g,c}^{gc} B_w C_w \right) f(w) dw + S^0 - 1 = 0,
\]

where strict equality holds for individuals who contribute to the public good \((g_w > 0)\). Also, recall that \(S^0 = 0\) if \(G^{Gov} > 0\) and \(S^0 > 0\) if \(G^{Gov} = 0\). Assume now that we have an interior solution in the sense that both the government and some individuals contribute. This means that \(S^0 = 0\). However, the social first-order conditions for \(g_w\) are not satisfied in this case. Instead, if \(\beta^d \leq \beta^{non}\), \(\alpha^d > -1\), and \(\epsilon_{(w)}^{gc} \geq 0\), we obtain

\[
MRS_{g,c}^{(w)} + \frac{1 - \bar{\alpha}}{1 + \alpha^d} \epsilon_{(w)}^{gc} B_w C_w MRS_{g,c}^{(w)} + \frac{1 - \bar{\alpha}}{1 - \bar{\beta} + 1 + \alpha^d} \frac{\beta^{non} - \beta^d}{1 - \bar{\beta} + 1 + \alpha^d} > 0,
\]

since all three terms on the left-hand side are positive. Thus, the only way to satisfy the social first-order conditions simultaneously is to choose \(G^{Gov} = 0\), so \(S^0 > 0\). It is also obvious that we do not need all three terms to be positive for this result to hold; it suffices that their sum is positive. Again the corresponding results in Proposition 2 follow as special cases where \(\alpha_w = \beta_w = 0\) for all \(w\), in which \(\eta = \zeta = 0\).

We will next prove Proposition 5 for the non-welfarist case, where the Lagrangean can be written as follows:
There are two state variables here, $u$ and $u^n$, whereas the control variables are the same as above. Note that $\nu_h + \nu_i = 0$, $\nu_h + \nu_e = 0$, $\nu_h + \nu_c = 0$, and $\nu_h + \nu_v = 0$, which means that the social first-order conditions for $l$, $G^\text{gov}$, and $\bar{c}$ coincide with equations (A2), (A3), and (A4), respectively. If $G^\text{gov} > 0$, the social first-order conditions for $g$ and $\bar{g}$, i.e., analogues of equations (A6) and (A7), can now be written as

\[
MRS_{g,c} \left(1 + \frac{\eta}{\lambda}\right) = -\frac{\partial u_c}{\lambda f (w)} \frac{\partial MRS_{g,c}}{\partial l} l w + \mu + \frac{\zeta}{\lambda} f (w) + \frac{\psi (u) \nu_i}{\lambda} MRS_{g,c}
\]

(A10a)

\[
\frac{\zeta}{\lambda} = \frac{1}{\lambda} \left( 1 + \frac{\eta}{\lambda}\right) \int_0^\infty \theta_w u_c (w) \left( -\beta w \frac{\partial MRS_{g,c}}{\partial l} + \beta_w \frac{\partial MRS_{g,c}}{\partial l} \right) l_w \frac{\beta w}{w} f (w) dw + \left(1 + \frac{\eta}{\lambda}\right) \int_0^\infty \beta_w MRS_{g,c}^{(w)} f (w) dw
\]

(A10b)

Combining (A10a) and (A10b) gives (A8) above. Finally, substituting (A8) into (A10a) and using the private first-order condition for charitable giving, we can derive the non-welfarist policy rule for $T_g$ in Proposition 5. Again, the derivation is analogous if $G^\text{gov} = 0$ at the social optimum. The corresponding policy rules in Proposition 3 follow as special cases where $\alpha_w = \beta_w = 0$ for all $w$, in which $\eta = \zeta = 0$.

Result (iii) in Proposition 5 under non-welfarism can be derived in the same way as under welfarism. Consider the case where $G^\text{gov}$. By using the private first-order condition for charitable giving, the policy rule for $T_g^{(w)}$ in result (i) in Proposition 5 can be written as

\[
T_g^{(w)} = -1 + \frac{1 - \bar{\alpha}}{1 + \alpha'^d} \left[ \frac{1}{1 + \alpha'^d} \left( \frac{T_y^{(w)}}{1 - T_y^{(w)}} - \frac{\bar{\alpha} + \alpha'^d}{1 - \bar{\alpha}} \right) \right] (1 + T_g^{(w)})
\]

\[
+ \frac{1 - \bar{\alpha} \mu + \beta'^d - \beta'^d_{\text{nom}}}{1 - \bar{\beta}} \frac{1 + \alpha'^d}{1 + \alpha'^d}
\]

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Suppose that $\Lambda_w = \varepsilon_{i(w)}^{gc}/(1 + \varepsilon_{i(w)}^{le})$ is locally constant and differentiate with respect to $T_{g}^{(w)}$ and $y_w$. This gives

$$\frac{\partial T_{g}^{(w)}}{\partial y_w} = \frac{1}{K_{w}^{non-wel}} \frac{1 - \bar{\alpha}}{1 + \alpha d} \left( \frac{\partial \delta_{w}^{e}}{\partial y_w} - \Lambda_w \frac{T_{y}^{(w)}}{(1 - T_{y}^{(w)})^2} \right), \quad (A11)$$

where

$$K_{w}^{non-wel} = 1 + \frac{1 - \bar{\alpha}}{1 + \alpha d} \left( -\delta_{w}^{e} + \Lambda_w \left( \frac{T_{y}^{(w)}}{1 - T_{y}^{(w)}} - \frac{\bar{\alpha} + \alpha d}{1 - \bar{\alpha}} \right) \right).$$

Note that $K_{w}^{non-wel}$ can be signed in exactly the same way as we signed $K_{w}^{wel}$ in (A8). If $G^{Gov} = 0$, the same reasoning applies to the policy rule in result (ii) in Proposition 5, in which $(\mu + \beta_d - \beta_{non})/(1 + \alpha d)$ in the policy rule for $T_{g}^{(w)}$ above is replaced with $(\mu + (1 - \mu)S^0 + \beta_d - \beta_{non})/(1 + \alpha d) > 0$. Equation (A11) continues to apply also in this case.

Finally, the special case in Proposition 3 follows when $\bar{\alpha} = \alpha d = \bar{\beta} = \beta_d = 0$ in (A11).

To show (iii) in Proposition 4 in the non-welfarist case, note that zero transaction cost implies that the social first-order conditions for $g_w$ and $G^{Gov}$ can be written as

$$g_w : \text{MRS}_{g,c}^{(w)} \left( 1 - \delta_w^{e} \frac{1 - \bar{\alpha}}{1 + \alpha d} \right) + \varepsilon_{i(w)}^{gc} \frac{1 - \bar{\alpha}}{1 + \alpha_d} B_w C_w \text{MRS}_{g,c}^{(w)} + \frac{1 - \bar{\alpha}}{1 - \bar{\beta}} \beta_{non} - \frac{\bar{\alpha} + \alpha d}{1 - \bar{\beta}} \frac{1 - \bar{\alpha}}{1 - \bar{\beta} + \alpha d} S^0 \leq 0 \quad \forall w,$$

$$G^{Gov} : \int_0^{\infty} \text{MRS}_{g,c}^{(w)} \left( \frac{1 + \alpha d}{1 - \bar{\alpha}} + \varepsilon_{i(w)}^{gc} B_w C_w \right) f(w)dw + S^0 - 1 = 0.$$

Based on the same reasoning as in the welfarist case, if $\beta_{non} \geq \beta_d$ and $\alpha_d > -1$, we must have $G^{Gov} = 0$ (such that $S^0 > 0$) for these conditions to hold simultaneously. The corresponding policy rules in Proposition 3 follow as special cases where $\alpha_w = \beta_w = 0$ for all $w$, in which $\eta = \zeta = 0$.

**Proof of Proposition 6**

To shorten the notation, let $\omega = (w,b) \in \Omega$ represent type in two-dimensional space, and let $f(\omega) = f(w,b)$ denote the corresponding density function such that

$$\int_{\Omega} f(\omega)d\omega = \int_0^{\infty} \int_0^{\infty} f(w,b) db dw = 1.$$
In a way similar to the proof of Propositions 3 above, we treat utility, \( u_\omega \), as a state variable, while \( l_\omega \), \( g_\omega \), \( G^{\text{Gov}} \), \( \bar{c} \), and \( \bar{g} \) are control variables. Consumption, \( c_\omega \), is defined by the inverse of the function \( u(\cdot) \) in (27), i.e., \( c_\omega = h(l_\omega, g_\omega, G, \bar{c}, \bar{g}, u_\omega) \).

With a welfarist government, the Lagrangean can be written as follows:

\[
L = \int_\Omega \psi(u_\omega) f(\omega) d\omega \\
+ \lambda \left( \int_\Omega \left( w_\omega l_\omega - h(l_\omega, g_\omega, \bar{c}, \bar{g}, G, u_\omega) - g_\omega \right) f(\omega) d\omega - G^{\text{Gov}} \right) \\
+ \int_\Omega \left( \theta^{\nu}_\omega \frac{du_\omega}{dw_\omega} + \theta^{\phi}_\omega \frac{du_\omega}{db_\omega} \right) d\omega \\
+ \eta \left( \bar{c} - \int_\Omega h(l_\omega, g_\omega, G, \bar{c}, \bar{g}, u_\omega) f(\omega) d\omega \right) \\
+ \zeta \left( \bar{g} - \int_\Omega g_\omega f(\omega) d\omega \right) + \int_\Omega \phi_\omega g_\omega d\omega + Y G^{\text{Gov}} + \Phi
\]

Following, e.g., Mirrlees (1976) and Kleven, Kreiner, and Saez (2009), we can use two-dimensional partial integration to obtain

\[
\int_\Omega \left( \theta^{\nu}_\omega \frac{du_\omega}{dw_\omega} + \theta^{\phi}_\omega \frac{du_\omega}{db_\omega} \right) d\omega = \int_0^\infty \left( \theta^{\nu}_\omega \frac{du_\omega}{dw_\omega} + \theta^{\phi}_\omega \frac{du_\omega}{db_\omega} \right) dw db \\
= \int_0^{\infty} u_\omega \theta^{\nu}_\omega \bigg|_{b=0}^{b=\infty} dw + \int_0^{\infty} u_\omega \theta^{\phi}_\omega \bigg|_{b=0}^{b=\infty} dw - \int_0^{\infty} (\theta^{\nu}_\omega + \theta^{\phi}_\omega) u_\omega dw db \\
= \int_0^{\infty} u_\omega \theta^{\nu}_\omega \bigg|_{b=0}^{b=\infty} dw + \int_0^{\infty} u_\omega \theta^{\phi}_\omega \bigg|_{b=0}^{b=\infty} dw - \int_\Omega (\theta^{\nu}_\omega + \theta^{\phi}_\omega) u_\omega d\omega
\]

Using \( \Phi \) as a short notation for the sum of the first two terms in the final expression and substituting into the Lagrangean give

\[
L = \int_\Omega \psi(u_\omega) f(\omega) d\omega \\
+ \lambda \left( \int_\Omega \left( w_\omega l_\omega - h(l_\omega, g_\omega, \bar{c}, \bar{g}, G, u_\omega) - g_\omega \right) f(\omega) d\omega - G^{\text{Gov}} \right) \\
+ \int_\Omega \left( -\theta^{\nu}_\omega + \theta^{\phi}_\omega \right) u_\omega + \theta^{\nu}_\omega \frac{l_\omega u_\omega}{w_\omega} - \theta^{\phi}_\omega u_\omega d\omega \\
+ \eta \left( \bar{c} - \int_\Omega h(l_\omega, g_\omega, G, \bar{c}, \bar{g}, u_\omega) f(\omega) d\omega \right) \\
+ \zeta \left( \bar{g} - \int_\Omega g_\omega f(\omega) d\omega \right) + \int_\Omega \phi_\omega g_\omega d\omega + Y G^{\text{Gov}} + \Phi
\]

Under non-welfarism, (33) is added to the set of constraints, and \( u^n_\omega \) will be used as an additional state variable. Therefore, the Lagrangean is given by
\[ L = \int_{\omega} \psi(u^n) f(\omega) d\omega \]
\[ + \lambda \left( \int_{\omega} \left( w_m l_m - h(l_m, g_m, \bar{c}, \bar{g}, G, u_m) - g_m \right) f(\omega) d\omega - G_{Gov}^{\text{w}} \right) \]
\[ + \int_{\omega} \left( -\left( \theta_m^{u} + \theta_m^{\bar{c}} \right) u_m + \theta_m^{\bar{c}} l_m^{u_{\omega}} - \theta_m^{\bar{c}} u_m \right) d\omega \]
\[ + \eta \left( \bar{c} - \int_{\omega} h(l_m, g_m, G, \bar{c}, \bar{g}, u_m) f(\omega) d\omega \right) + \zeta \left( \bar{g} - \int_{\omega} g_m f(\omega) d\omega \right) \]
\[ + \int_{\omega} \theta_m \left( v(h(l_m, g_m, G, \bar{c}, \bar{g}, u_m), l_m, \bar{c}, G) - u_m \right) d\omega + \int_{\omega} \phi_m g_m d\omega + Y G_{Gov} + \Phi \]

Note that the “laundered utility function” \( \nu(\cdot) \) in the final row of (A13) satisfies
\[ \nu, h_i + \nu_i = u_i (-u_i / u_i) + u_i = 0, \]
\[ \nu, h_{i_{\bar{c}}} + \nu_{i_{\bar{c}}} = u_{i_{\bar{c}}} (-u_{i_{\bar{c}}} / u_{i_{\bar{c}}}) + u_{i_{\bar{c}}} = 0, \]
\[ \nu, h_G + \nu_G = u_G (-u_G / u_G) + u_G = 0. \]

Therefore, regardless of whether the government is welfarist or non-welfarist, i.e., regardless of whether we maximize (A12) or (A13), the social first-order conditions for \( l_m, \bar{c} \), and \( G_{Gov} \) take the same form. By using \( h_i = -u_i / u_i = -MRS_{i,e} \) and \( h_G = -u_G / u_G = -MRS_{G,e} \), and if we suppress the type indicator \( \omega \) for notational convenience, the social first-order conditions for \( l_m \) and \( G_{Gov} \) (if \( G_{Gov} > 0 \)) can be written as follows:
\[ w + MRS_{i,e} = -\frac{\partial u_i MRS_{i,e}^{\bar{c}}}{\lambda f(w)} \left( 1 + e_i^{\bar{c}} \right) \]
\[ - \frac{\partial u_i MRS_{i,e}^{\bar{c}}}{\lambda f(\omega)} \frac{1 + \eta}{\lambda} MRS_{i,e} = 0 \]  \hspace{1cm} (A14)

\[ \int_{\omega} MRS_{i,e} f(\omega) d\omega \left( 1 + \frac{\eta}{\lambda} \right) + \int_{\omega} \frac{\partial u_i MRS_{i,e}^{\bar{c}}}{\lambda f(\omega)} e_i^{\bar{c}} f(\omega) d\omega - \int_{\omega} \frac{\partial u_i MRS_{i,e}^{\bar{c}}}{\lambda f(\omega)} e_i^{\bar{c}} f(\omega) d\omega = 1. \]  \hspace{1cm} (A15)

As we did in the proof of Propositions 3, we can derive an expression for \( \eta / \lambda \) from the social first-order condition for \( \bar{c} \). By using \( h_\bar{c} = -u_\bar{c} / u_\bar{c} = -MRS_{\bar{c},e} \), the social first-order condition for \( \bar{c} \) can be written as
\[ \frac{\eta}{\lambda} \left( 1 + \int_{\omega} u_\bar{c} f(\omega) d\omega \right) = \frac{1}{\lambda} \int_{\omega} \theta^{u} u \frac{\partial \alpha}{\partial l} \frac{l}{w} d\omega - \frac{1}{\lambda} \int_{\omega} \theta^{\bar{c}} u \frac{\partial \alpha}{\partial c} d\omega - \int_{\omega} u_\bar{c} f(\omega) d\omega. \]  \hspace{1cm} (A16)

Define
\[ \bar{\alpha} = \int_{\omega} u_\bar{c} f(\omega) d\omega = \int_{\omega} \alpha f(\omega) d\omega \]
\[ \alpha^P = \frac{1}{\lambda} \left( \int_{\omega} \theta^{u} u \frac{\partial \alpha}{\partial l} \frac{l}{w} d\omega - \int_{\omega} \theta^{\bar{c}} u \frac{\partial \alpha}{\partial c} d\omega \right) = \int_{\omega} \frac{\theta^{u} u \alpha}{\lambda f(\omega)} e_i f(\omega) d\omega - \int_{\omega} \frac{\theta^{\bar{c}} u \alpha}{\lambda f(\omega)} e_i^{\bar{c}} d\omega. \]

Substituting into (A16) gives
Next, substitute (A17) into equations (A14) and (A15) and use the private first-order condition, \( w + MRS_{i,c} = -wT_c \), in (A14). Then, by using the notations

\[
\begin{align*}
 B_{w,b}^w C_{w,b}^w &= \frac{\theta_{w,b}^u u_{c(w,b)}}{\Gamma_{w,b}^w \lambda w_f (w, b)}, \\
 B_{w,b}^b C_{w,b}^b &= \frac{\theta_{w,b}^b u_{c(w,b)}}{(1 - \Gamma_{w,b}^w) \lambda c_f (w, b)},
\end{align*}
\]

we obtain the policy rules for marginal income taxation and public good provision in (i) and (ii) of Proposition 4.

Let us then turn to the marginal tax treatment of charitable giving under welfarism. Suppose first that \( G^{\text{Gov}} > 0 \) at the social optimum. By using \( h_g = -u_g / u_c = -MRS_{g,c} \) and \( h_g = -u_g / u_c = -MRS_{g,c} = \beta MRS_{g,c} \), we can derive the following social first-order conditions for \( g_w \) and \( \bar{g} \) from (A12):

\[
MRS_{g,c} \left( 1 + \frac{\eta}{\lambda} \right) = -\frac{\partial u_c}{\partial l} \frac{\partial MRS_{g,c}}{\partial l} \frac{l}{w} + \frac{\partial u_c}{\partial c} \frac{\partial MRS_{g,c}}{\partial c} + \mu + \frac{\zeta}{\lambda} - \frac{\phi}{\lambda f(\omega)},
\]

\[
\frac{\zeta}{\lambda} = \frac{1}{\lambda} \int_{\Omega} \theta_w u_c \left( \frac{\partial MRS_{g,c}}{\partial c} + \beta \frac{\partial MRS_{g,c}}{\partial c} \right) \frac{l}{w} d\omega - \frac{1}{\lambda} \int_{\Omega} \theta_w u_c \left( \frac{\partial MRS_{g,c}}{\partial c} \right) d\omega + \left( 1 + \frac{\eta}{\lambda} \right) \int_{\Omega} \beta MRS_{g,c} f(\omega) d\omega,
\]

where we have used

\[
\frac{\partial MRS_{g,c}}{\partial l} = -\frac{\partial \beta}{\partial c} MRS_{g,c} - \beta \frac{\partial MRS_{g,c}}{\partial c} \quad \text{and} \quad \frac{\partial MRS_{g,c}}{\partial c} = -\frac{\partial \beta}{\partial c} MRS_{g,c} - \beta \frac{\partial MRS_{g,c}}{\partial c}.
\]

Substituting (A20) into the final row of (A21) and rearranging gives

\[
\frac{\zeta}{\lambda} = \frac{\bar{\beta} \mu}{1 - \bar{\beta}} + \frac{1}{1 - \bar{\beta}} \frac{1}{\lambda} \left( \int_{\Omega} \theta_w u_c \frac{\partial \beta}{\partial c} \frac{l}{w} d\omega - \int_{\Omega} \theta_w u_g \frac{\partial \beta}{\partial c} \phi d\omega \right) - \frac{1}{1 - \bar{\beta}} \frac{1}{\lambda} \int_{\Omega} \beta \phi d\omega,
\]

where

\[
\bar{\beta} = \int_{\Omega} \beta \phi f(\omega) d\omega.
\]
\[ \beta^D = \frac{1}{\lambda} \left( \int_{\Omega} \theta^w u_g \frac{\partial \beta}{\partial l} w \, d\omega - \int_{\Omega} \theta^h u_g \frac{\partial \beta}{\partial c} d\omega \right) \]
\[ = \int_{\Omega} \frac{\theta^w u_g}{\lambda f(\omega) w} \beta_i(w) f(\omega) d\omega - \int_{\Omega} \frac{\theta^h u_g}{\lambda f(\omega) c} \beta_i(w) f(\omega) d\omega \]
\[ \beta^{mon} = \frac{1}{\lambda} \int_{\Omega} \beta_i \phi \, d\omega. \]

By using the elasticities \( \beta_i^c \) and \( \beta_i^e \), (A20) can be rewritten to read

\[ MRS_{\eta, \lambda} \left( 1 + \frac{\eta}{\lambda} \right) = -\frac{\theta^w u_c MRS_{\eta, \lambda} \epsilon_i^c}{\lambda f(\omega) w} + \frac{\theta^h u_c MRS_{\eta, \lambda} \epsilon_i^e}{\lambda f(\omega) c} + \mu + \frac{\lambda}{\lambda f(\omega)} \phi. \quad (A23) \]

Substitute the expression for \( \zeta / \lambda \) in (A22) into (A23) and use the short notations in equations (A18) and (A19). Finally, using the private first-order condition, \( MRS_{\eta, \lambda} = 1 + T_g \)
gives the policy rule in (iii).

If \( G^Gov = 0 \) at the social optimum, the analogues of equations (A20) and (A22) become

\[ MRS_{\eta, \lambda} \left( 1 + \frac{\eta}{\lambda} \right) = \mu - \frac{\theta^w u_c}{\lambda f(\omega) w} \frac{\partial MRS_{\eta, \lambda}}{\partial l} + \frac{\theta^h u_c}{\lambda f(\omega) c} \frac{\partial MRS_{\eta, \lambda}}{\partial c} + \frac{\lambda}{\lambda f(\omega)} \phi + (1 - \mu) S^0 \quad (A24) \]

\[ \frac{\zeta}{\lambda} = \frac{\beta \left( \mu + (1 - \mu) S^0 \right)}{1 - \beta} + \frac{1}{1 - \beta} \lambda \left( \int_{\Omega} \theta^w u_g \frac{\partial \beta}{\partial l} w \, d\omega - \int_{\Omega} \theta^h u_g \frac{\partial \beta}{\partial c} d\omega \right) - \frac{1}{1 - \beta} \lambda \int_{\Omega} \beta u_c \phi \, d\omega \]
\[ = \frac{\beta \left( \mu + (1 - \mu) S^0 \right) + \phi - 1}{1 - \beta}, \quad (A25) \]

where \( S^0 = \frac{\gamma}{\lambda} = 1 - \left( \int_{\Omega} MRS_G \left( 1 + \epsilon_i^D - \frac{\theta^w u_c \epsilon_i^c}{\lambda f(\omega) w} - \frac{\theta^h u_c \epsilon_i^e}{\lambda f(\omega) c} \right) f(\omega) d\omega \right)_{G^Gov = 0}. \]

Combining equations (A24) and (A25) in the same way as above gives the policy rule in (iv).

Under non-welfarism, the procedure is the same, except that the social first-order conditions are now derived from (A13). We can differentiate (A13) with respect to \( g_{\omega} \) and \( \bar{g} \)
in order to derive the following analogues to equations (A20) and (A22) when \( G^Gov > 0 \) at the social optimum:

\[ MRS_{\eta, \lambda} \left( 1 + \frac{\eta}{\lambda} \right) = \mu - \frac{\theta^w u_c}{\lambda f(\omega) w} \frac{\partial MRS_{\eta, \lambda}}{\partial l} + \frac{\theta^h u_c}{\lambda f(\omega) c} \frac{\partial MRS_{\eta, \lambda}}{\partial c} + \frac{\lambda}{\lambda f(\omega)} \phi \left( u_{\omega} \epsilon_i \right) MRS_{\eta, \lambda} \quad (A26) \]

\[ \frac{\zeta}{\lambda} = \frac{\beta \mu}{1 - \beta} + \frac{1}{1 - \beta} \lambda \left( \int_{\Omega} \theta^w u_g \frac{\partial \beta}{\partial l} w \, d\omega - \int_{\Omega} \theta^h u_g \frac{\partial \beta}{\partial c} d\omega \right) - \frac{1}{1 - \beta} \lambda \int_{\Omega} \beta u_c \phi \, d\omega \]
\[ = \frac{\beta \mu + \beta^D - \beta^{mon}}{1 - \beta}. \quad (A27) \]
By substituting (A27) into (A26), we obtain the policy rule in (v). If $G^{\text{Gov}} = 0$ at the social optimum, the procedure is again analogous to that under welfarism and implies that an additional term proportional to $S^0$ will appear in the social first-order condition for $g_\omega$ and in the expression for $\zeta / \lambda$.

References


