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Peer-to-Peer Lending from a CDO Perspective

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ABSTRACT

In this thesis, we will attempt to model a peer-to-peer lending intermediary according to a CDO. A CDO is a credit risk protection product that distributes credit risk among investors. The business of a peer-to-peer lending intermediary is to connect individuals who want to borrow money with individuals who want to lend. With the increasing popularity of peer-to-peer lending, it is of interest to study the portfolio credit risk that is inherent to such a business, not the least in anticipation of a possible downturn in the economy that is likely to follow once interest rates rise again. To the best of our knowledge, this is the first study that makes a rigorous attempt to examine peer-to-peer lending from a credit risk portfolio point of view. In particular, the CDO perspective seems to fit nicely into the peer-to-peer lending framework, and also gives us answers to, for instance, what a fair interest rate should be for lenders. We find that the CDO-structure can be a viable way to profitably structure the business of peer-to-peer lending given the assumptions and the inputs that we use in our model.

Keywords: *Credit risk management, Credit risk modeling, Collateralized debt obligations (CDO), Peer-to-peer lending*

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1 Introduction

Online peer to peer (P2P) lending has increased in popularity since its first commercial use in 2005 (Bachmann et al. 2011). The idea is to facilitate a liquid lending and borrowing market without having a bank as a financial intermediary and thus reducing associated transaction costs and enabling otherwise unqualified borrowers to borrow at a high yield from interested counter parties. Companies such as Lendify in Sweden act as intermediaries between peers and connect people who want to borrow with people who are willing to take the risk of lending to these individuals.

Thus far, many of the largest P2P intermediaries in Sweden[1] have no (or very limited) safety buffers for their loans and do not manage credit risk in the way that a large bank would. This is not that strange since they are essentially offering a platform for risky investments into the equity of people. However, with increasing usage of P2P lending (Eisenberg 2015) and with actual defaults that have happened within the industry (Trustbuddy for instance[2]), some credit risk management from P2P intermediaries might be appropriate, especially if there is a downturn in the economy, in order to maintain clientèle and reputation. At the present, few P2P intermediaries set aside capital to cover losses, as can be expected. Lendify, for instance, has a credit loss fund, but it is only required to be 0.1% of the total lended capital (Lendify 2018). Some P2P intermediaries (Lendify and Sparlån for instance) offer an insurance for the borrower in case of unforeseen unemployment or otherwise (because of unforeseen circumstances) are unable to make monthly payments during a period of time. While this reduces default risk, it is entirely voluntary[3], and can be canceled after the initially free three-month period.

The purpose of this thesis is to establish tools for a peer-to-peer lending intermediary to manage its credit risk. One might say that this is not necessary, after all, both the borrowers and lenders know what they are getting into and should be prepared to make losses. However, with the increasing use of peer-to-peer lending, more

[1]Lendify, Saveland, Toborrow and Sparlån for instance.

[2]This default was a result of hazardous actions by Trustbuddy (lending out repayments) and not something inherent to P2P lending, but it shows that there could be unforeseen risks within the industry that might not entirely be possible to avoid with regulation. (Carlsson 2016)

[3]The first three months are free, after that, there is a monthly fee based on the monthly cost of the loan.

capital is at stake and some credit risk management could thus be warranted. If not for the safety of the borrowers and lenders, then at least for the trustworthiness and systematic safety of the peer-to-peer lending sector going forward. Considering the economic boom environment that has prevailed during the most recent years, the question of how the peer-to-peer market will sustain a recession is also an interesting one (Finopti n.d.). It is not unreasonable to believe that peer-to-peer lending intermediaries will have to take on more rigorous credit risk management in order to continue to operate and maintain their customer base. Another purpose of this thesis is to use the risk management tools in order to quantify what a ”fair” interest rate for lenders should be. To this end we will argue that modeling the peer-to-peer lending business in the framework of a transparent collateralized debt obligation (CDO) will let the intermediaries contain their risk in a responsible way while making their product more safe to invest in for lenders. A CDO is a financial product for managing portfolio credit risk in a way that spreads out the risk exposure among several investors. A CDO is thus a good tool to use for the portfolio credit risk management in peer-to-peer lending since there are plenty of investors (lenders) whose investments are generally spread out across several loans. Our results indicate the CDO structure can be a viable approach to peer-to-peer lending, by comparing the calculated ”fair” interest rates to lenders to the expected interest income from borrowers. We also find that the correlation among loans have a large impact on the risk profile of the loan portfolio and thus subsequently the appropriate fair interest rates paid to the investors (the lenders) in the CDO-peer-to-peer lending structure.

The rest of this thesis is organized as follows. First, Section 2 gives a literature review of work related to credit risk within peer-to-peer lending. Section 3 will give a brief presentation of two of the currently largest peer-to-peer lending intermediaries in Sweden and a relevant law that will soon come to pass. Section 4 will address the theory that is relevant to the thesis, including pricing equations and general technical knowledge regarding CDOs. In Section 5, we will analyze the theory in the context of peer-to-peer lending intermediaries and in Section 6 we draw conclusions based on our findings.

2 Literature Review

In this section, we will give a brief overview of earlier studies concerning credit risk within peer-to-peer lending. While the research mentioned here does not directly connect to our thesis[4], it covers the basis of what is the current credit risk management at peer-to-peer lending firms.

Earlier studies on the subject of credit risk within peer-to-peer lending include studies about credit evaluation of loan takers and market inefficiencies (Serrano-Cinca & Gutiérrez-Nieto 2016), credit risk assessment of Chinese peer-to-peer online lending (Chen 2017), and default risk based on borrower characteristics in peer-to-peer lending in China (Lin et al. 2017).

Serrano-Conca and Gutiérrez-Nieto study the profitability of investing in peer-to-peer loans and use the expected profitability (measured by the internal rate of return) instead of focusing on default probabilities. They propose a profit scoring system (rather than a credit scoring system) in order to evaluate potential loan offers. They find a lack of efficiency in the peer-to-peer lending market since their profit scoring system was able to beat the market and thus outperform traditional credit scoring.

Chen (2017) analyzes a data sample from the peer-to-peer lending platform Paiai Lending in China. The author used this data to screen out variables that could indicate the level of credit risk in a peer-to-peer loan, that is, the article evaluated the default rate of the borrowers. Not surprisingly, the study found that variables such as income and rate of repayment were significant indicators of default risk and this was evaluated using a 0.1 percent confidence level.

Lin, Li, and Zheng (2017) also evaluates borrower default risk in peer-to-peer lending. They highlight, among other things, the significance of information asymmetry between borrowers and lenders in the peer-to-peer lending market. In contrast to a bank that can alleviate the information asymmetry by in place institutions (such as financial reporting, bank guarantees, and certified accounts), the same procedure will be harder to implement in an online peer-to-peer lending situation. The authors

[4] In fact, we could not find any such papers.

point out that large transaction costs and the fact that borrowers and lenders never actually meet face to face have a negative effect on the ease of alleviating information asymmetry in peer-to-peer setting. The purpose of the study in Lin et al. (2017) is to find borrower characteristics that impact the default rate and discovers that individuals with low default rate are, on average; young adult women with stable jobs in large companies, high education, stable marital status, low loan amounts, monthly payments and debt levels (relative to income), and no default history.

Ma and Wang (2016) examine credit risk in the peer-to-peer online lending market in China. They look at peer-to-peer lending from three aspects: the platform, the borrowers, and the environment. The authors state that the defaults in the Chinese peer-to-peer lending market are becoming more serious and that limiting the credit risk of peer-to-peer lending is one of the key problems in the financial market of China. The purpose of the paper is to identify factors that have an influence on the credit risk of the Chinese peer-to-peer lending market and possible ties to relevant Chinese policies. The paper identifies eight influential factors from the three aspects stated above; audit mechanisms, credit rating mechanisms, and information disclosure mechanisms for the peer-to-peer lending platforms; borrower's moral level, social network situation, and job stability for the borrowers; and big data and policy for the environmental factors. The authors also find connections between the different factors, suggesting that they are not independent of one another.

Our research will take a different approach than the papers described above. Instead of focusing on the screening of potential borrowers, we will attempt to model a peer-to-peer lending intermediary in a way that fairly distributes the burden of defaults when defaults occur. In particular, we will take a credit portfolio approach in order to quantify certain core quantities for the peer-to-peer lending intermediary, such as "fair" interest rates, loss distributions, and value at risk for different levels of confidence.

3 Background

In this section, we will give a brief presentation of two of the currently largest peer-to-peer lending intermediaries in Sweden and a description of a relevant law that will soon come to pass.

3.1 A Brief Industry Overview

We will examine two of the largest peer-to-peer lending intermediaries on the Swedish market, which are Lendify and Savelend. Both of these companies offer peer-to-peer loan intermediation between consumers that can be entered into either on a loan to loan basis or by depositing money into an account which is then spread out over available loans according to the investor's risk preferences. In their latest financial report, Lendify state that 90% of their investors invest using an *autoinvest* account, which is an account that investors deposit money into which is then subsequently distributed among available borrowers (Lendify 2017). In order to apply a CDO-framework, we are assuming that this is an indication of the industry standard.

3.1.1 Savelend

Savelend is currently the largest peer-to-peer lending intermediary in Sweden seen to net turnover with a massive increase from around four MSEK in 2015 to about 14.5 MSEK in 2016 according to their annual report 2016. The company focuses mainly on small loans as their average loan amount is about 4000 SEK with approximately 12 500 loans. The maturity of these loans varied between 61 days and 60 months. It is stated in their annual report that they offer loans between 1000-50 000 SEK. According to their website, they also offer loans in the 15 000-100 000 SEK segment, but as of the annual report 2016, such loans seem to be scarce. Savelend gained permit from Finansinspektionen during 2016 meaning that it is allowed to mediate loans between lenders and borrowers (Leijonhufvud 2016).

Savelend has their own credit evaluation process which they use to evaluate potential borrowers. The process itself is not stated in detail, but applicants have to verify themselves using BANKID[5] and calculations are made based on expected future income to determine if the borrower will be able to repay the loan. How rigorous this process is is hard to tell by just looking at the annual report. Saveland made a profit as of 2016 (which they did not during the previous three years) and has a total of 14 615 511 SEK in assets. Additionally, Savelend has intermediated approximately 50 MSEK in loans during 2016. (Savelend 2016)

3.1.2 Lendify

Compared to Savelend, Lendify's assets are worth substantially more (133 759 888 SEK as of may 2017). A large part of these assets, however, were financed using a loan of 72 MSEK which was taken during the year. They also have a total of 146 MSEK in lended capital as of 2016 and an average loan size of 110 000 SEK. The average loan maturity was seven years. This puts Lendify in a different segment compared to Savelend.

Lendify's total mediated lended capital has increased dramatically between 2015 and 2016 (the cumulative lending expanded from 10 MSEK to 146 MSEK) and this has required investments in the infrastructure of the company and increased its operational costs. Lendify, like Savelend, received its permit from Finansinspektionen during 2016. Lendify has received large amounts of capital through equity issues, raking in 70 MSEK in August of 2016 (Ekström 2016) and 111.5 MSEK in January of 2018 at a pre-money valuation of 650 MSEK (Eliasson 2018). Lendify also raised 21 MSEK in June 2017 in order to gain a certain (not named) investor (Boström 2017). In this regard, Savelend pales in comparison with an equity issue of 22.7 MSEK before the summer of 2016 (Canoilas 2016). Another P2P-lending intermediary Sparlån, which is a competitor to Lendify and Savelend, had a similar equity issue of 20 MSEK in may of 2017 (Leijonhufvud 2017). Hence, at least in Sweden, there seems to be a large interest from investors in the peer-to-peer lending industry, which indicates that it is a growing market that is relevant to do research on, not the least within risk management.

Like Savelend, Lendify also performs its own credit evaluation of its borrowers.

[5] This is a type of electronic identification in Sweden.

The process also requires verification via BANKID and the credit evaluation process seems very similar between the two companies, judging by what is presented in the reports. The process incorporates both automatic credit scoring procedures and manual checks of the potential borrower. Approximately ten percent of borrowing-applicants pass the credit evaluation process. (Lendify 2017)

3.2 PSD2

The PSD2 (or payment services directive 2) is an EU law that is expected to be implemented in Sweden in May of 2018 (Swedbank n.d.). The law states that banks can no longer choose to withhold information about their customers from third party payment services, given that the customers want the banks to share this information (Nexusgroup 2017). This becomes interesting in the peer-to-peer lending industry since individuals that want to borrow money through a peer-to-peer lending intermediary can now share information directly through their bank, which decreases asymmetric information and makes for a more robust and credible credit scoring evaluation of possible borrowers. This might increase the credibility of the peer-to-peer lending industry and make it a more legitimate competitor to the large banks that as of right now mediate most of the loans in the market place.

The aim of the directive, as stated by EUR-Lex (2017), is to, within the EU, better enable an integrated internal market for electronic payment services. The definition of a payment service, according to the directive, is "*services enabling cash to be deposited in or withdrawn from, for example, a bank account, as well as all the operations required to operate the account. This can include transfers of funds, direct debits, credit transfers and card payments. Paper transactions are not covered by the directive.*". It is difficult to say if the peer-to-peer lending intermediaries legally fit this description, but intermediaries such as Lendify certainly transfer funds and manage accounts through the use of their *autoinvest* service described above. It is thus reasonable to assume that PSD2 covers the realm of peer-to-peer lending intermediaries given the characteristics of their operations. For a more in depth description of the legislation, we refer to EUR-Lex (2017).

4 Theoretical Framework for CDOs

This section will outline the theoretical and mathematical foundation for the thesis going forward. We will briefly introduce the credit default swap before going into the collateralized debt obligation.

4.1 The Credit Default Swap (CDS)

In this subsection, we will give a short non-technical introduction to a credit default swap. A CDS is a credit derivative that protects against losses resulting from a default of a reference security or against the default of an issuer. The protection seller receives a premium from the protection buyer[6], and in return the protection seller compensates the buyer for the loss incurred by the buyer in the case of a default on the reference security or credit. If a default happens within the maturity of the CDS (i.e. the protection period), the CDS ceases to exist; the protection seller pays for the loss and the protection buyer stops making premium payments, see Figure 1. A CDS is set up in a way that makes the expected discounted premium payments equal to the expected discounted payment from the protection seller to the protection buyer, where the payment from the protection seller to the protection buyer only occurs if there is a default on the underlying security. If traded over-the-counter, a CDS is thus ”free” to enter into, i.e. at the start of the contract, the NPV (net present value) for both parties is zero.

4.2 Collateralized Debt Obligations

In the following subsection, we will follow the frameworks of Lando (2004) and O’Kane (2008). When introducing collateralized debt obligations (CDOs), the following reasoning can be helpful. Equity and debt (with junior and senior claims on the assets of a company, respectively)) can be seen as claims on the value of the underlying assets of a firm. Given that a firm defaults when the asset value of the firm is below the value of the debt of the firm, equity can be seen as a call option on the firm’s assets, using total debt as the strike price. Similarly, the value of

[6]Normally on a quarterly basis.

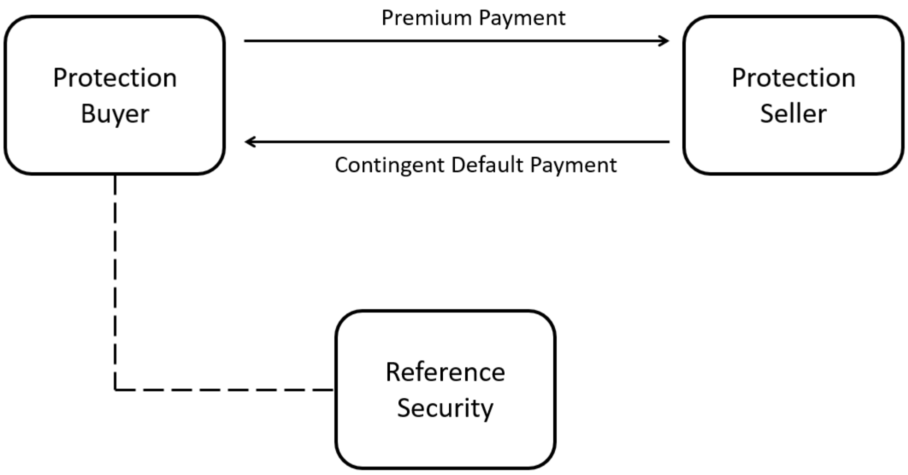


Figure 1: Illustration of a CDS. The protection buyer makes periodic premium payments to the protection seller. The protection seller's payment to the protection buyer is contingent on the reference security defaulting before the maturity of the CDS.

senior debt at the firm can be viewed as the value of the firm's assets minus a call option on the firm's assets using the senior debt as the strike price. This essentially translates into taking the value of the firm's assets and subtracting everything but the senior debt, which then equals the senior debt. Finally, junior debt can be seen as a call option on the firm's assets using the senior debt as the strike price, minus the value of a call option on the firm's assets using the total debt as the strike price. This basically translates into taking the value of everything above the value of the senior debt and subtracting the value of equity; what remains is then the value of the junior debt of the firm (Lando 2004).

The above reasoning can be used to explain how a CDO works.[7] Instead of a firm and its assets, we look at a portfolio of loans. In order to simplify, we can assume that the portfolio contains 60 loans (issued by 60 different obligors) with the same maturity and zero recovery in the case of a default. By securitization, the loan portfolio can be divided into three categories (or tranches) for the sake of the example: Equity, mezzanine, and senior. These categories have the values (sizes) of 10, 30, and 20 respectively (see Figure 2).

[7] A CDO is a type of financial instrument that contains assets that are backed by collateral. A CDO can for instance be a portfolio of loans, which in that case sometimes in called a CLO (Collateralized Loan Obligation). The idea behind a CDO is to provide protection against a specific proportion of the portfolio's total credit loss.

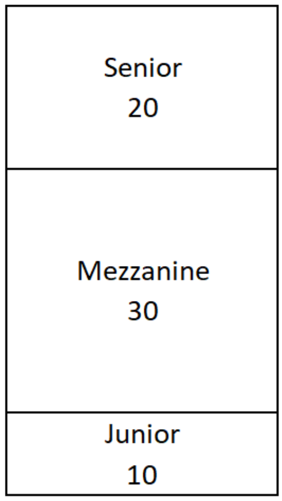


Figure 2: Illustration of different claims on a firm's assets. Senior is the most senior claim and junior is the most junior claim.

This means that the senior debt will be repaid in full if there are no more than 40 defaults (60-20=40), the mezzanine debt will be repaid (up to 30) if there are less than 40 defaults (if there are no more than 10 defaults, both the mezzanine and senior debt will be paid in full). The equity will be repaid using the capital that remains after the senior and mezzanine debt have been repaid, receiving up to 10 in the case when there are no defaults. This is simple illustration of what a CDO is. In practice, additional factors come into play, but the basic idea as presented above remains (O'Kane 2008).

More specifically, a CDO is a security that is built up by a portfolio of credits with possibly varying risk profiles. The cash flow from the different CDO tranches is linked to the health of the underlying portfolio of defaultable loans, i.e., it is linked to default events. A traditional CDO functions in the following way; the securities of the CDO are sold to investors, the proceeds from these sales are used to buy the collateral portfolio of risky credit assets (these could be loans or bonds for instance). These assets are then sold to a special purpose vehicle (SPV), which then issues the CDO securities.[8] The CDO securities are typically divided into different risk profiles, usually a senior, a mezzanine and an equity category. The coupon payments stemming from the CDO are paid in a falling order from senior to equity.

[8]Observe that CDO can refer to the entire structure of the SPV and the securities, but also to a security that is issued by a CDO. A CDO can issue a collection of CDOs for instance.

The rules determining the payments are sometimes referred to as the *waterfall*, and can vary in complexity. Generally the coupon payments will be a function of if the underlying credits have defaulted or not. If there is a default in the portfolio, the holders of the equity tranche will start to see their payments decrease, since the payments are based on how much is left in the tranche of interest (here, the equity tranche). Additional defaults keep eating into the payments of the equity holders until they no longer receive payments, at which point the owners of the mezzanine portion of the portfolio will start to receive reduced payments. This continues until the owners of the senior portion can no longer be paid. The holders of the senior portion are thus the investors in the CDO that are the least exposed to the credit risk of the underlying portfolio and the holders of the equity portion of the CDO are the most exposed to the credit risk of the CDO. The coupons on the securities are set accordingly, with the equity holders receiving the largest coupons and the senior holders receiving the lowest coupons (O'Kane 2008).

After the financial crisis of 2007-2009 it has become important among regulators that CDO pricing has to be one with the out-most transparency regarding the underlying portfolio of credit assets in the CDO. Lack of transparency of CDOs using sub-prime mortgages as collateral was a significant reason for why the financial crisis was so severe (Reuters 2007). Because of this, CDOs have received a perhaps undeservedly bad reputation.

There are different types of CDOs that function as described above. That is, the credit portfolio, and its accompanying credit risk, is entirely held by an SPV and is then sold to various investors through the issued securities. This type of CDO is referred to as a *full capital structure deal* because every CDO tranche security is sold, which results in the issuer having no credit risk (O'Kane 2008). There is a workaround to this, which entails that the issuer buys and holds the equity portion of the CDO. This way, the issuer will be punished first if there are defaults in the CDO, which should help alleviate problems stemming from asymmetric information regarding the quality of the collateral portfolio. This is a good way of signaling credibility and monitoring quality to outsiders. If there is a loss, the issuer will lose money and is thus incentivized to keep an extra eye on the securities in the portfolio. Since it is likely that the most risky security of the portfolio will be the ones to default first, the issuer is incentivized to increase the quality of these securities and thus the overall quality of the portfolio.

A shortcoming of the traditional CDO is that it is often a complex process to set up the contract so that the various investors are satisfied. Matching investment requirements with the market views of the investors in the CDO securities will often require compromise, leaving an individual investor with limited control over the deal in regards to the waterfall and selection of credits for instance. Because of its complexity, setting up a CDO can also be a tedious process that can result in large administrative and legal costs. There are, however, alternatives to the traditional CDO structure.

4.2.1 The Single-Tranche Synthetic CDO

An alternative to the above stated CDO structure is the single-tranche synthetic CDO (or an STCDO). This instrument is an OTC (over the counter) derivative variant of a CDO. The STCDO varies from the traditional CDO in several ways (O'Kane 2008). First, the credit risk is synthetic. The reference portfolio is connected to a pool of 50 to 150 entities (usually equally weighted), where each entity is equal to a CDS position with that entity as the underlying asset. Second, in an STCDO, there is no SPV. Instead, the STCDO is a contract that is entered into between two parties, an investor and a dealer. The contract is also unfunded, meaning that it is generally free to enter into and both parties have zero NPV at the start. Third, only a single CDO tranche, or security, has to be issued, and the issuance is generally much faster than that of a regular CDO because of standardized documentation. Fourth, the payment structure (sometimes referred to as the *waterfall*) is different from that of a regular CDO, which will be discussed below. An example of STCDOs in the real world are the iTraxx portfolios which contain the most liquid CDSs on companies in a specified market (Europe for instance) (Markit 2018). Single tranches on these indices are liquidly traded for tranches between 0 and 22% (Herbertsson 2017).

An important difference between a STCDO and a CDO is that the issuer (or dealer) is exposed to the STCDO's credit risk. The credit risk a dealer retains is the same type of risk that stems from buying protection on a CDS, but the dealer is exposed to a CDO tranche, so it maintains the exposure to the underlying credits in the reference portfolio and the correlation between defaults in that portfolio.

4.2.2 STCDO Waterfall

The cash flows for a STCDO is different from that of a traditional CDO. Consider a portfolio consisting of m different and equally weighted obligors with default times $\tau_1 ... \tau_m$, where τ_i is the default time for obligor i. The payoff is dependent on the cumulative percentage loss L_t of the underlying portfolio, and looks as follows:

$$L_t = \frac{1}{m} \sum_{i=1}^{m} (1 - \delta_i) \mathbf{1}_{\{\tau_i \leq t\}} \tag{1}$$

The loss L_t is a sum of all of the credits in the portfolio at time t, each weighted by its individual loss given default $(1 - \delta_i)$, where δ_i is the recovery rate[9] in percent for obligor i, and the indicator function $\mathbf{1}_{\tau_i \leq t}$, which is one if the default time τ_i for obligor i in the CDO portfolio happens before T and zero otherwise. The tranche loss is defined as:

$$L_t^{a,b} = \max(L_t - a, 0) - \max(L_t - b, 0) \tag{2}$$

In Equation 2, $L_t^{a,b}$ is the fractional loss of the tranche $[a, b]$ at time t. See Figure 3 for a visualization. Here, a is equal to the lower bound percentage loss of a particular tranche of the portfolio, indicating the border level where, if the loss exceeds this level, the cash flow to the STCDO is reduced. Similarly, b is equal to the upper bound where if the loss exceeds this percentage, the STCDO receives zero payment. Hence, from Equation 2, we see that $L_T^{(a,b)} \in [0, b - a]$ with 100% probability. The width of the tranche is $b - a$. We have illustrated a CDO in figure 4. The tranche is riskier if a and b are closer to zero than if a and b are closer to 1 (or 100%). Between the border levels a and b, the loss of the tranche is linear in L_t.

[9]The recovery rate is equal to one minus the percentage loss of a particular security if it were to default.

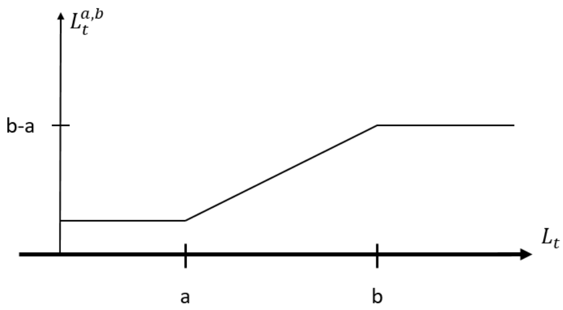


Figure 3: Visualization of a CDO tranche. L_t is the portfolio loss, $L_t^{a,b}$ is the tranche loss and a and b are the borders of the tranche.

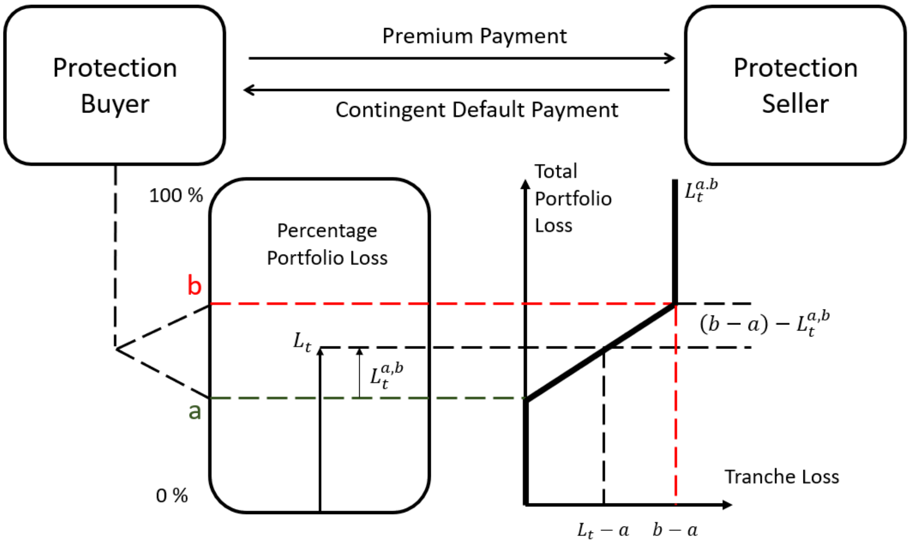


Figure 4: Illustration of a CDO. L_t is the portfolio percentage loss and $L_t^{(a,b)}$ is the loss within the $[a, b]$ tranche. The default payment from the protection seller to the protection buyer is contingent on defaults occurring within the $[a, b]$ tranche. The premium payment from the protection buyer to the protection seller is made periodically with regular intervals. (Herbertsson 2017)

We can thus think of the loss function as a combination of a long call position on the reference portfolio with strike a and a short put position on the reference portfolio with strike b. We will now look at the premium and default legs of the STCDO.

4.2.3 The Premium Leg

The premium leg of a STCDO is the premium payment made to the tranche protection seller by the tranche protection buyer. The tranche spread (which can be viewed as an interest rate) is a function of the two strikes and is denoted $S_{(a,b)}(T)$. The payments from the protection seller to the protection buyer are dependent on the total percentage loss of the portfolio, as discussed above, and can be (per $1 face value) written in the following way:

$$S_{(a,b)}(T)((b-a) - L_{t_i}^{a,b})\Delta$$

where t_i are the time points between 0 and T where premium payments are made,[10] Δ is the time interval in years between two payments, and T is the insurance time period, also called the CDO maturity. So, the premium payment decreases with the total loss given that it is between the upper and lower bounds of the tranche, and if $L_t > b$ then we see from 2 that $L_t^{(a,b)} = b - a$ so there will be no further payments because the entire $[a, b]$ tranche has been wiped out.

4.2.4 The Default Leg

The default leg represents the payments made from the investor to the dealer if the total percentage portfolio loss, L_t, exceeds the lower bound of the tranche. The loss size is determined by the $L_t^{a,b}$ function, which, as previously mentioned, only alters if $a \leq L_t \leq b$. If there are N credits in the reference portfolio, that for simplicity have the same recovery rate δ and the same face value, each loss (or default) in the portfolio will result in a $(1-\delta)/N$ percentage loss in the portfolio. We denote this u. The number of defaults required before the losses start eating into the tranche is $n_1 = ceil(a/u)$, where *ceil* denotes the first number that is equal to or larger than the number within the parenthesis. If there are $n_2 = ceil(b/u)$ losses in the portfolio, the tranche is completely depleted and no further payments will be made. (O'Kane 2008)

In a STCDO, the buyer of protection does not have to keep the reference portfolio on its books. The reference portfolio is just that, a reference (or a virtual portfolio), and is only used to determine the payments of the STCDO through the waterfall.

[10] This could be quarterly or monthly for instance.

4.2.5 CDOs and Correlation

By introducing the loss distribution of the portfolio, we can see that the CDO is a so called credit ”correlation” product. The loss distribution ($P[L_t \leq x]$) tells us the probability of future losses (at differing levels), i.e. the CDO is in a sense a function of the default dependency among the obligors that constitute the underlying portfolio connected to the CDO. If all of the credits share the same face value, then the expected loss in the portfolio, at any timepoint t, is given by (using Equation 1):

$$\mathbb{E}[L_t] = \frac{1}{m}\sum_{i=1}^{m}(1-\delta_i)(1-P[\tau_i \leq t])$$

where $P[\tau_i \leq t]$ corresponds to the probability that issuer i does not default between time 0 and time T. This will be defined in more detail later in Section 4.4.

4.2.6 Arbitrage Spread Opportunities

Looking at the assets in the collateral pool that is used for a CDO, they are priced on a *single asset basis*. What this means is that no diversification effects are taken into account in their pricing, and the weighted average coupon of the portfolio is essentially just equal to the weighted sum of the risks of the single assets. Individually, and naturally, only the bonds themselves affect their performance. In a CDO, the portfolio risk is essential when it comes to its payoff structure. One can view tranching of notes to be tranching of the loss distribution of the CDO’s collateral pool. This takes diversification effects into account and consequently reduces the risk of the portfolio as opposed to just managing a single loan. This indicates that the price of the risk of the portfolio should be lower than the exposure-weighted price of the combined single risks.

Consequently, the premiums paid to the investors in the notes should be considerably lower than the premiums that are earned from the collateral pool bonds. This is what creates the arbitrage spread; that is, there is a mismatch between the weighted average coupon of the notes in the CDO and the weighted average coupon of the single assets in the collateral pool. The mismatch is caused both by diversification effects and by the structure of the CDO. For instance, because of subordination between tranches in the CDO, the most senior tranches are not affected until almost the entire asset base of the CDO has defaulted, meaning that they are relatively safe in the context of the CDO. Following this, it becomes evident why CDOs sometimes are called *correlation products*. (Bluhm et al. 2003)

4.3 Setup for the peer-to-peer lending CDO framework

In this subsection, we will present how peer-to-peer lending can be structured in a CDO framework. In Figure 5, we have made some modifications to Figure 4 in order to make some observations regarding the peer-to-peer lending CDO framework. In the CDO setting, the peer-to-peer lending intermediary is the protection buyer and the investors (lenders) are the protection sellers. The idea is that when the lenders lend out money through the peer-to-peer lending intermediary, the lenders invest into the loans of the borrowers. The borrowers are screened and selected by the intermediary and make up the loan portfolio. The lenders are then entitled to the *premium leg* payments of the CDO, which vary based on which tranche a lender invests in. These payments are determined by the tranche spread, which can be seen as an interest rate to the lender. The premium leg payment is equal to this interest rate multiplied by the nominal loan value that remains in the tranche. In exchange for receiving the premium payment, the lenders are obliged to make the *default leg* payments to the borrowers, which are contingent on occurring defaults. Here is where our setup varies from the traditional CDO. In our setting, the lenders "pay" the borrowers by lending them capital and expect to get their payment back at the time of maturity of the loan. We can view this as the lenders (protection sellers) making the default leg payment up-front under the promise that it will be paid back. Because of this, the protection leg payments in our setting are made by simply not requesting that the borrower pays back the initial investment. From a practical standpoint, the default leg payments can be handled by making write-downs on the invested capital. The timing of the write-downs is very important since the premium leg payments are dependent on the cumulative percentage loss of the loan portfolio at every time point where the premium payments are made. It therefore becomes imperative to model the loan default times when pricing the

CDO tranche spreads. This will be the objective of Section 4.4.

The peer-to-peer lending intermediary's task in the CDO setting is to screen borrowers, channel funds between borrowers and lenders, and to administrate the lending-platform. When capital is invested (lent) into the CDO, it goes to the intermediary first and is then spread out over loans in the CDO-portfolio. The interest payments that the borrowers make on their loans are used to make the premium leg payments. The interest payments are made to the peer-to-peer lending intermediary and then distributed to the lenders based on the CDO tranche spreads, or tranche interest rates with the peer-to-peer-CDO framework. It is important to note that the peer-to-peer intermediary is at risk of default if the payments from borrowers to the intermediary do not cover the obligated payments from the intermediary to the lenders. Additionally, if there are too many defaults, the lending intermediary will probably go out of business because of the severely damaged reputation that would follow. In Table 1, we have made a summary of some CDO terms and what their interpretation is under the peer-to-peer framework.

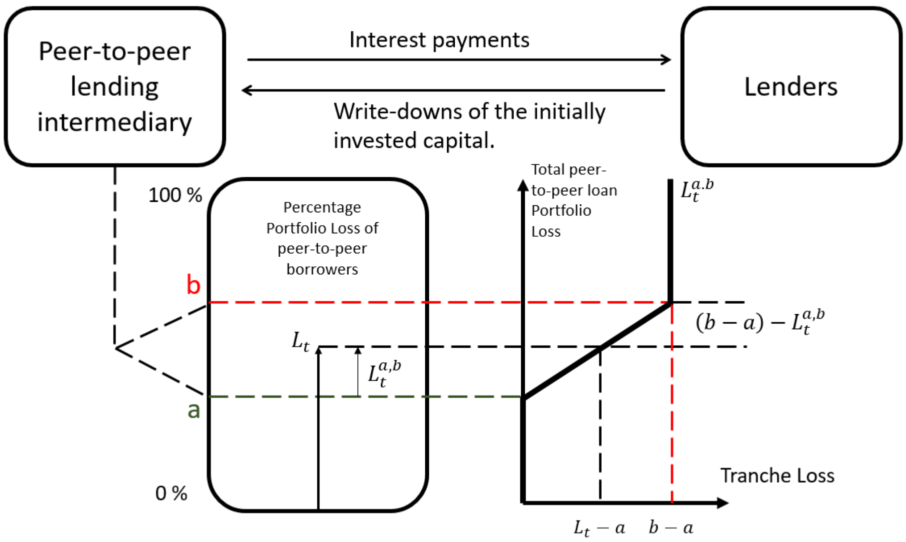


Figure 5: A CDO structure under the peer-to-peer lending framework.

4.4 CDO Pricing

In the following subsection our setup and notation strongly follows the outline of chapters 7 and 8 in Herbertsson (2017) and regards the pricing of a CDO tranche

CDO terms and their peer-to-peer lending equivalent	
CDO	Peer-to-Peer Lending CDO
The protection buyer	The peer-to-peer intermediary.
The protection seller	The lenders.
Tranche Spread	Interest rate for a particular tranche.
Premium payments	Interest payments made to the lenders in the peer-to-peer loan portfolio.
Default payments	Write-downs of capital invested by the lenders in the peer-to-peer loan portfolio.

Table 1: The analogy between a CDO and a peer-to-peer lending intermediary under the CDO framework.

spread. The purpose of this section is to quantify the interest rate $S_{(a,b)}(T)$ for each tranche in the peer-to-peer loan portfolio. Analogously, we can view this as finding the CDO tranche spread for a traditional synthetic CDO. We will start by reiterating the structure of a CDO tranche.

4.4.1 The CDO tranche spread

A tranche of a CDO is used as the basis for a contract between protection buyer and seller, where the protection buyer pays the protection seller a periodic fee and the protection seller reimburses the protection buyer in the case of a default, if the total cumulative loss of the reference portfolio lies within the tranche. This loss is written as $L_t^{(a,b)}$, where a and b represent the lower and upper limits of the tranche. The premium payments from the protection buyer to the protection seller, can be written as:

$$S_{(a,b)}(T)((b-a) - L_t^{(a,b)})\Delta_n$$

where $S_{(a,b)}(T)$ is the spread (or interest rate) that is weighted by what is left of the tranche and $\Delta_n = t_n - t_{n-1}$, that is the time interval between the premium payments in years.[11] The contract terminates if the entire tranche has been wiped out since there is then nothing left to insure. We denote the expected payments done by the protection seller as the protection leg $V_{(a,b)}(T)$ and the expected payments done by the protection buyers as the premium leg $W_{(a,b)}(T)$. The value of the protection seller's payments to the protection buyer (discounted to the present value) up until

[11](b-a) equals the entire tranche and $L_t^{(a,b)}$ is the tranche loss.

time T can be written as

$$\sum_{\tau_i:\Delta L^{(a,b)}_{\tau_i}>0} B_{\tau_i}\Delta L_{\tau_i^{(a,b)}} 1_{\{\tau_i\le T\}}$$

where τ_i are the default times in the portfolio, B is the discount factor, and $\Delta L^{(a,b)}_{\tau_i}$ is the increase in $L^{(a,b)}_t$ that results from τ_i, that is:

$$\Delta L^{(a,b)}_{\tau_i} = L^{(a,b)}_{\tau_i} - L^{(a,b)}_{\tau_{i-}}.$$

Mathematically, we can rewrite $\sum_{\tau_i:\Delta L_{\tau_i^{(a,b)}}>0} B_{\tau_i}\Delta L_{\tau_i^{(a,b)}} 1_{\{\tau_i\le T\}}$ as $\int_0^T B_t dL^{(a,b)}_t$. The expected value of $\int_0^T B_t dL^{(a,b)}_t$ is equal to the protection leg of the CDO-tranche. If r_t (the interest rate) is deterministic, then the protection leg $V_{(a,b)}(T)$ can be written as[12]:

$$V_{(a,b)}(T) = \mathbb{E}[\int_0^T B_t dL^{(a,b)}_t] = B_T\mathbb{E}[L^{(a,b)}_T] + \int_0^T r_t B_t \mathbb{E}[L^{(a,b)}_t]dt$$

Additionally, we can write the premium leg $W_{(a,b)}(T)$ as follows:

$$W_{(a,b)}(T) = S_{(a,b)}(T)\sum_{n=1}^{n_T} B_{t_n}(b-a-\mathbb{E}[L^{(a,b)}_{t_n}])\Delta_n$$

where n_T is defined as the number of premium payments until time T. So if premium payments are quarterly and T is one year, then n_T is equal to four.

The spread (or interest rate) $S_{(a,b)}(T)$ is set so that the expected value of the protection leg equals the expected value of the premium leg at the time of the initiation of the contract. Because of this, the spread can be written as

$$S_{(a,b)}(T) = \frac{B_T\mathbb{E}[L^{(a,b)}_T] + \int_0^T r_t B_t \mathbb{E}[L^{(a,b)}_t]dt}{\sum_{n=1}^{n_T} B_{t_n}(b-a-\mathbb{E}[L^{(a,b)}_{t_n}])\Delta_n}. \tag{3}$$

The numerator in the expression is the default leg of the tranche, meaning that it is the present value of the *expected* default payment of the tranche given a \$1 notional value. The denominator in the expression is the premium leg of the tranche, which

[12]Using integration by parts for Lebesgue-Stieltjes measures together with Fubini-Tonelli. See for instance page 22 of Frey & Herbertsson (2016) and page 107 of Folland (1999).

is essentially equal to the discounted *expected* total payments from the protection buyer to the protection seller, if each yearly payment was \$1. The spread $S_{(a,b)}(T)$, which can be seen as an interest payment made by the protection buyer to the protection seller, is then set so that the spread multiplied by the premium leg equals the default leg. This way, the contract has zero NPV upfront for both parties and is thus free to enter into. To get the actual premium payments, the spread is multiplied by the notional amount money that the contract is written on.

The presented way of calculating the CDO tranche spread is widely used (see for instance Cousin & Laurent (2008), Mortensen (2006), Gibson (2004), and Herbertsson (2009)), but the way that the expected portfolio losses is calculated differs based on the model of choice. Now that the general framework has been established, we will describe the procedure used for estimating the expected portfolio tranche losses $\mathbb{E}[L_t^{(a,b)}]$ (which is the only unknown in Equation 3) that we will use in this thesis. For this purpose, we start by introducing the one-factor Gaussian copula model.

4.4.2 The one-factor Gaussian copula model

Up until the recent financial crisis, the *one-factor Gaussian copula model* has been an industry standard model for modeling probabilities (e.g. O'Kane (2008) and Herbertsson (2017)). To set up this model, we state the following . There are m obligors with individual default times τ_i (where $i = 1, 2, ..., m$). We define their default distributions as $F_i(t) = \mathbb{P}[\tau_i \leq t]$, which can be extracted from each obligor's individual CDS spread. What this means is that we assume that the individual default probability distributions are given by the market through the market's pricing of the CDSs. In the peer-to-peer lending setting, however, we will set $F_i(t) = \mathbb{P}[\tau_i \leq t] = 1 - e^{-\lambda_i t}$, where λ_i is the default intensity[13] of τ_i, calibrated to a time period t (one year for instance), defined as

$$\lambda_i = -\log(1 - F_i(t))/t.$$

We do this because the default probability $F_i(t)$ will be taken from available loan

[13] λ can be seen as the *instantaneous* default probability conditional on not having already defaulted. Intuitively, this is the default probability in the limit when the time period approaches instantaneous.

statistics or be otherwise assumed for scenario analysis in the analysis section (Section 5). Next, we define Y_i as an i.i.d (independently and identically distributed) variable with standard normal distribution. This could refer to individual characteristics of obligor i. We also define Z as a random standard normal variable that is independent of Y_i. This can be seen as a market background factor. We let X_i be defined in the following way:

$$X_i = \sqrt{\rho_i} Z + \sqrt{1-\rho_i} Y_i \tag{4}$$

In this expression, ρ_i is the correlation between obligor i and the background factor Z, and ρ_i is limited to be between zero and one. We also define a so called "threshold" for each obligor and call it $D_i(t) = N^{-1}(F_i(t))$, where $F_i(t)$ has the same definition as described above, i.e. $F_i(t) = \mathbb{P}[\tau_i \leq t]$. We can define the default times $\tau_i ... \tau_m$ as,

$$\tau_i = \inf\{t > 0 : X_i \leq D_i(t)\} \tag{5}$$

which means that τ_i (i.e. the individual default time for obligor i) is defined as the first time (after 0, we assume that there has not already been a default) the variable X_i decreases under $D_i(t)$, which is the threshold level described above. This threshold level could be seen as the value of debt at a company and once the assets of the company is worth less than its total debt, the company defaults. We can thus write that

$$\mathbb{P}[\tau_i \leq t] = \mathbb{P}[X_i \leq D_i(t)]$$

since there will only be a default before time t if X_i reaches or falls below $D_i(t)$, which depends on time.

By definition, X_i, which is built up by the two standard normal variables Z and Y_i, is also standard normal. This means that we can rewrite $\mathbb{P}[\tau_i \leq t] = \mathbb{P}[X_i \leq D_i(t)]$ as

$$\mathbb{P}[\tau_i \leq t] = N(D_i(t)) = N(N^{-1}(F_i(t))) = F_i(t) \tag{6}$$

which shows that the construction of τ_i in Equation 5 is consistent with the exogenously given distribution $F_i(t) = \mathbb{P}[\tau_i \leq t]$. Additionally, since we have the definition of X_i above (Equation 4), we can say that $\tau_i \leq t$ if (and only if) $\sqrt{\rho}Z + \sqrt{1-\rho}Y_i \leq D_i(t)$. That is, we switch out X_i for the definition of it. We can thus say that, τ_i (the individual default time) is calculated by a process that is driven by the random individual variable Y_i for each obligor and the random common variable Z, which, as stated earlier, corresponds to the economic environment. So, we can say that Z is creating default dependence because it is present for all obligors. That is, it is not indexed by i (by obligor) like Y_i is. Another interesting result is that the default times τ_i of the obligors are independent if we condition on Z:

$$\mathbb{P}[\tau_1 \leq t, \tau_2 \leq t, ..., \tau_m \leq t|Z]$$

We want to find $\mathbb{P}[\tau_i t|Z]$, which is the probability of default conditional on Z, which is then independent from the other default probabilities. We can thus write

$$\mathbb{P}[\tau_1 \leq t, \tau_2 \leq t, ..., \tau_m \leq t|Z] = \prod_{i=1}^{m} \mathbb{P}[\tau_i \leq t|Z].$$

From Equations 4 and 5, we get that:

$$\tau_i \leq t \quad \text{if (and only if)} \quad Y_i \leq \frac{D_i(t) - \sqrt{\rho}Z}{\sqrt{1-\rho}}$$

This means that we can write the default times as:

$$\mathbb{P}[\tau_i \leq t|Z] = \mathbb{P}\left[Y_i \leq \frac{D_i(t) - \sqrt{\rho}Z}{\sqrt{1-\rho}}|Z\right] = N\left(\frac{D_i(t) - \sqrt{\rho}Z}{\sqrt{1-\rho}}\right)$$

Because we know that Y_i is normally distributed with zero mean and unit variance, we can write $\mathbb{P}\left[Y_i \leq \frac{D_i(t) - \sqrt{\rho}Z}{\sqrt{1-\rho}}|Z\right] = N\left(\frac{D_i(t) - \sqrt{\rho}Z}{\sqrt{1-\rho}}\right)$.[14]

Next, we define $p_{t,i}(Z)$ in the following way

$$p_{t,i}(Z) = \mathbb{P}[\tau_i \leq t|Z] = N\left(\frac{D_i(t) - \sqrt{\rho}Z}{\sqrt{1-\rho}}\right)$$

[14] We are using the fact that if X and Y are random variables, and $F_X(x) = \mathbb{P}[X \leq x]$, and X is independent of the σ-algebra $\tilde{F}$ and Y is $\tilde{F}-measurable$, then $\mathbb{P}[X \leq Y|\tilde{F}] = F_X(Y)$ (Herbertsson 2017).

In order to simplify the expression, we assume that the threshold $D_i(t)$ is the same for all obligors in the portfolio, that is, $D_i(t) = D(t)$. This also means that

$$p_{t,i}(Z) = p_t(Z) = \mathbb{P}[\tau \leq t|Z] = N\left(\frac{N^{-1}(F(t)) - \sqrt{\rho}Z}{\sqrt{1-\rho}}\right). \tag{7}$$

Note that we here use the fact that $D(t)$ is equal to $N^{-1}(F(t))$, with $F(t)$ being the probability of default up until time t. A result of the simplification is that the default probability is now the same for all obligors, so the probability $\mathbb{P}[\tau_1 \leq t]$ is equal to the probability $\mathbb{P}[\tau_2 \leq t]$, and so on.

To calculate the expected tranche loss of a certain tranche in a CDO, we want to calculate the probability $\mathbb{P}[N_t = k]$ for all k up to m, in which N_t is defined as

$$N_t = \sum_{i=1}^{m} 1_{\{\tau_i \leq t\}}$$

where $1_{\{T_k \leq t\}}$ is an indicator function indicating if there has been a default or not before time t for each of the obligors. N_t will thus denote the number of defaults up until time t. Using the conditional probability of default $p_t(Z)$ (as defined in Equation (7)) for a fixed t, which is the same for each of the obligors, and that the $1_{\{\tau_i \leq t\}}$ variable, given Z, is conditionally independent, we can state the following:

$$\mathbb{P}[N_t = k|Z] = \binom{m}{k} p_t(Z)^k (1 - p_t(Z))^{m-k}$$

So, N_t (which is a random variable), for a fixed t and conditional on Z, has a binomial distribution with the probability $p_t(Z)$.

Furthermore, because $\mathbb{P}[N_t = k] = \mathbb{E}[\mathbb{P}[N_m = k|Z]] = \mathbb{E}[\binom{m}{k} p_t(Z)^k (1 - p_t(Z))^k]$, we have that

$$\mathbb{P}[N_t = k] = \int_{-\infty}^{\infty} \binom{m}{k} p_t(z)^k (1 - p_t(z))^{m-k} \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}} dz \tag{8}$$

since Z is standard normal and $p_t(u)$ is given by

$$p_t(u) = N\left(\frac{D(t) - \sqrt{\rho}u}{\sqrt{1-\rho}}\right).$$

We can calculate $\mathbb{P}[N_t = k]$ (as defined in Equation (8)) for portfolios that are sufficiently small, that is, at least if m is somewhere below 20. For larger values of m, say 120, we will run into problems. For instance, $\binom{m}{k}$ will become too large to be stored accurately if m becomes too large. Additionally, p^k might become small enough so that it is recognized as zero since p is less than one. Luckily, there are approximations that can be made, which will be described in the next section.

4.4.3 The one-factor Gaussian copula model with Large Portfolio Approximation (LPA) for CDO pricing

This section will focus on the earlier described one-factor Gaussian copula model and its implementation on CDOs.

Using the law of large numbers, we can bypass the problem of managing large portfolios that is present when using Equation (8). We start with observing that by the law of large numbers, we have that, in a homogeneous portfolio, $\frac{N_t}{m} \to p_t(z)$ as $m \to \alpha$ and thus:

$$\mathbb{P}[\frac{N_t}{m} \leq x] \to \mathbb{P}[p_t(Z) \leq x] = F_{p_t}(x) \text{ as m } \to \infty$$

that is, $F_{p_t}(x)$ is equal to $\mathbb{P}[p_t(Z) \leq x]$, which is the distribution function for the random variable $p_t(Z)$. Additionally, we define $L_t = \frac{1-\delta}{m} N_t$ as the percentage loss of the portfolio, and we thus also have:

$$\mathbb{P}[L_t \leq x] = \mathbb{P}\left[\frac{N_t}{m} \leq \frac{x}{1-\delta}\right] \to F_{p_t}\left(\frac{x}{1-\delta}\right) \text{ as m } \to \infty \tag{9}$$

This means that, for a homogeneous portfolio with constant recovery δ, we can make the following approximation:

$$\mathbb{P}[L_t \leq x] \approx F_{p_t}\left(\frac{x}{1-\delta}\right)$$

Additionally, because $\mathbb{P}[N_t = k] = \mathbb{P}[\frac{N_t}{m} = \frac{k}{m}]$ and since

$$\mathbb{P}\left[\frac{N_t}{m} = \frac{k}{m}\right] \approx \mathbb{P}\left[\frac{k-1}{m} < \frac{N_t}{m} \leq \frac{k}{m}\right] =$$
$$\mathbb{P}\left[\frac{N_t}{m} \leq \frac{k}{m}\right] - \mathbb{P}\left[\frac{N_t}{m} \leq \frac{k-1}{m}\right] \approx F_{p_t}\left(\frac{k}{m}\right) - F_{p_t}\left(\frac{k-1}{m}\right)$$

we have the following if m is large enough (say, above 50):

$$\mathbb{P}[N_t = k] \approx F_{p_t}\left(\frac{k}{m}\right) - F_{p_t}\left(\frac{k-1}{m}\right)$$

Recall that $F_{p_t}(x) = \mathbb{P}[p_t(Z) \leq x]$. We now want to figure out an explicit expression of $F_{p_t}(x) = \mathbb{P}[p_t(Z) \leq x]$. We already know from before (Equation 7), that $p_t(Z) = N(\frac{N^{-1}(F(t)) - \sqrt{\rho}Z}{\sqrt{1-\rho}})$. Also Recall that $F(t)$ corresponds to the default probability up until time t. We can then state the following:

$$\mathbb{P}[p_t(Z) \leq x] = \mathbb{P}\left[N(\frac{N^{-1}(F(t)) - \sqrt{\rho}Z}{\sqrt{1-\rho}}) \leq x\right]$$
$$= \mathbb{P}\left[\frac{N^{-1}(F(t)) - \sqrt{\rho}Z}{\sqrt{1-\rho}} \leq N^{-1}(x)\right] = \mathbb{P}\left[-Z \leq \frac{1}{\sqrt{\rho}}\left(\sqrt{1-\rho}N^{-1}(x) - N^{-1}(F(t))\right)\right]$$
$$= \mathbb{P}\left[Z \geq \frac{1}{\sqrt{\rho}}\left(N^{-1}(F(t)) - \sqrt{1-\rho}N^{-1}(x)\right)\right]$$
$$= 1 - \mathbb{P}\left[Z \leq \frac{1}{\sqrt{\rho}}\left(N^{-1}(F(t)) - \sqrt{1-\rho}N^{-1}(x)\right)\right]$$
$$= 1 - N\left[\frac{1}{\sqrt{\rho}}\left(N^{-1}(F(t)) - \sqrt{1-\rho}N^{-1}(x)\right)\right] = N\left[\frac{1}{\sqrt{\rho}}\left(\sqrt{1-\rho}N^{-1}(x)N^{-1}(F(t))\right)\right].$$

The final equality here uses the fact that $N(-x) = 1 - N(x)$. Concluding, we can state that $F_{p_t}(x) = \mathbb{P}[p_t(Z) \leq x]$, in which $\mathbb{P}[p_t(Z) \leq x]$ is stated as:

$$\mathbb{P}[p_t(Z) \leq x] = N\left[\frac{1}{\sqrt{\rho}}\left(\sqrt{1-\rho}N^{-1}(x)N^{-1}(F(t))\right)\right].$$

So, if m is large enough, $\mathbb{P}[N_t = k]$ can be stated as[15]:

[15] Using $\mathbb{P}[N_t = k] \approx F_{p_t}(\frac{k}{m}) - F_{p_t}(\frac{k-1}{m})$

$$\begin{aligned}\mathbb{P}[N_t = k] \approx N\left[\frac{1}{\sqrt{\rho}}\left(\sqrt{1-\rho}N^{-1}\left(\frac{k}{m}\right)N^{-1}(F(t))\right)\right] \\ - N\left[\frac{1}{\sqrt{\rho}}\left(\sqrt{1-\rho}N^{-1}\left(\frac{k-1}{m}\right)N^{-1}(F(t))\right)\right]\end{aligned} \tag{10}$$

This formula can be used to calculate the spreads of CDO tranches, here denoted $S_{(a,b)}(T)$, where a and b are the lower and upper thresholds of the tranche.

We remember that the loss of a tranche ($L_t^{a,b}$) is stated as the following (if $F_{L_t}(x) = \mathbb{P}[L_t \leq x]$)

$$L_t^{(a,b)} = (L_t - a)1_{\{L_t \in [a,b]\}} + (b-a)1_{\{L_t > b\}} \tag{11}$$

And the spread for a tranche of a CDO is stated as

$$S_{(a,b)}(T) = \frac{B_T \mathbb{E}[L_T^{(a,b)}] + \int_0^T r_t Bt \mathbb{E}[L_t^{(a,b)}]dt}{\sum_{n=1}^{n_T} B_{T_n}(b - a - \mathbb{E}[L_{t_n}^{(a,b)}])\Delta_n}.$$

So, to find the tranche spreads for a CDO, we need to first find $\mathbb{E}[L_t^{(a,b)}]$ (the expected tranche loss). We note that $L_t^{(a,b)}$, as defined in Equation (11), implies the following:

$$\mathbb{E}[L_t^{(a,b)}] = \mathbb{E}[(L_t - a)1_{\{L_t \in [a,b]\}}] + \mathbb{E}[(b-a)1_{\{L_t > b\}}]$$

$$= (b-a)\mathbb{P}[L_t > b] + \int_a^b (x-a)dF_{L_t}(x)$$

$$= (b-a)\mathbb{P}[L_t > b] - a(\mathbb{P}[L_t \leq b] - \mathbb{P}[L_t \leq a]) + \int_a^b x dF_{L_t}(x)$$

$$= b\mathbb{P}[L_t > b] + a\mathbb{P}[L_t \leq a] - a + \int_a^b x dF_{L_t}(x)$$

This means that

$$\mathbb{E}[L_t^{(a,b)}] = b\mathbb{P}[L_t > b] + a\mathbb{P}[L_t \leq a] - a + \int_a^b x dF_{L_t}(x).$$

Recall that $F_{L_t}(x) = \mathbb{P}[L_t \leq x]$. If we define $dF_{L_t}(x) = f_{L_t}(x)dx$ (that is, the derivative of $dF_{L_t}(x)$, assuming that it is differentiable, where $f_{L_t}(x)$ can be seen as the density of L_t), then:

$$\mathbb{E}[L_t^{(a,b)}] = b\mathbb{P}[L_t > b] + a\mathbb{P}[L_t \leq a] - a + \int_a^b x f_{L_t}(x)dx.$$

We know from before (Equation 9) that

$$F_{L_t}(x) = \mathbb{P}[L_t \leq x] \approx F_{p_t}(\frac{x}{1-\delta}) \text{ as m } \to \infty$$

Recall that $F_{p_t}(x) = \mathbb{P}[p_t(Z) \leq x] = N\left[\frac{1}{\sqrt{\rho}}\left(\sqrt{1-\rho}N^{-1}(x)N^{-1}(F(t))\right)\right]$, so

$$\mathbb{P}[L_t \leq x] = F_{L_t}(x) \approx N\left[\frac{1}{\sqrt{\rho}}\left(\sqrt{1-\rho}N^{-1}(\frac{x}{1-\delta})N^{-1}(F(t))\right)\right]$$

Additionally, we can approximate $f_{L_t}(x)$ as $\frac{dF_{p_t}(\frac{x}{1-\delta})}{dx}$, which can be shown to be equal to: (Herbertsson 2017)

$$\frac{dF_{p_t}(\frac{x}{1-\delta})}{dx} = \sqrt{\frac{1-\rho}{\rho}} \exp\left(\frac{1}{2}(N^{-1}(x))^2 - \frac{1}{2\rho}(N^{-1}(F(t)) - \sqrt{1-\rho}N^{-1}(x))^2\right).$$

Because $\frac{dF_{p_t}(\frac{x}{1-\delta})}{dx} = \frac{1}{1-\delta}F'_{p_t}(\frac{x}{1-\delta})$, we can finally conclude that

$$f_{L_t}(x) \approx \frac{1}{1-\delta}\sqrt{\frac{1-\rho}{\rho}} \exp\left(\frac{1}{2}(N^{-1}(\frac{x}{1-\delta}))^2 - \frac{1}{2\rho}(N^{-1}(F(t)) - \sqrt{1-\rho}N^{-1}(\frac{x}{1-\delta}))^2\right)$$

So, by plugging in the expression for $f_{L_t}(x)$ into $\mathbb{E}[L_t^{(a,b)}] = b\mathbb{P}[L_t > b] + a\mathbb{P}[L_t \leq a] - a + \int_a^b x f_{L_t}(x)dx$, we can calculate the expected tranche loss $\mathbb{E}[L_t^{(a,b)}]$ using the LPA (large portfolio approximation) approach.

Before going further, we want to demonstrate the effect that the correlation parameter has on the calculated loss distribution, using the one-factor Gaussian copula model. Using this model, the default probabilities of individual loans in the portfolio are conditionally independent of one another. This, however, does not say anything about the distribution of the losses. Plotting the loss distribution for different levels of correlation (ρ), we can see that the higher correlation we use, the larger will the tails of the distribution be (see Figure 6). What this means is that, as we increase the correlation among defaults, the probability of suffering very large losses or very small losses increases even though the expected loss remains the same.

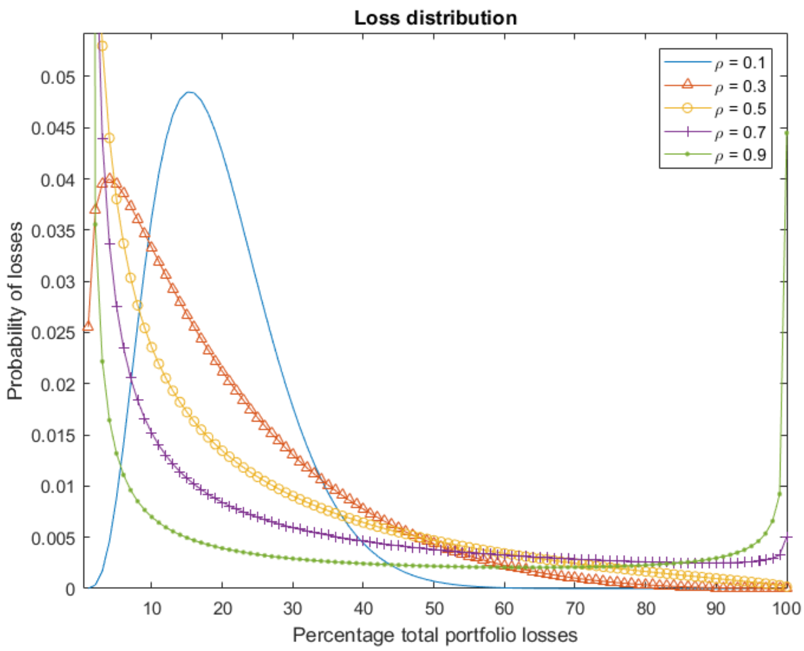


Figure 6: Loss distribution for a homogeneous loan portfolio generated by the one-factor Gaussian copula model using LPA, a probability of default of 10%, and different values of correlation ρ.

So far our analysis has been focused on portfolios of loans that are homogeneous. That is, they share the same characteristics with regards to probability of default and correlation to the background and so on. This is not necessarily a huge deal when it comes to pricing if one can observe that the loans in a portfolio are similar to each other and that the time saved by applying the LPA-approach outweighs potential errors in the calculations. However, if the loans in the portfolio are not similar to each other, we can no longer confidently assume that the LPA-approach will yield desirable results with regards to the reference portfolio. In this case we need a procedure for dealing with heterogeneity[16] among the loans.

[16]That is, the loans (or borrowers) are different from each other in respect of probability of default or correlation for instance.

4.4.4 Monte-Carlo Simulation

There are different techniques that can be utilized for dealing with the heterogeneity problem and one of them is performing so-called Monte-Carlo simulations. Monte-Carlo simulation is very flexible and enables modeling defaults (for instance) if there are no closed formulas for handling the loans, or can be used to complement other methods of modeling. Monte-Carlo simulation is based on generating a large quantity of possible outcomes (for instance number of defaults) with regards to (for instance) an underlying portfolio of loans. In our case, we are interested in knowing how many of the loans that will default given a certain period of time. From Equation (7), we have the conditional probability of default, defined as:

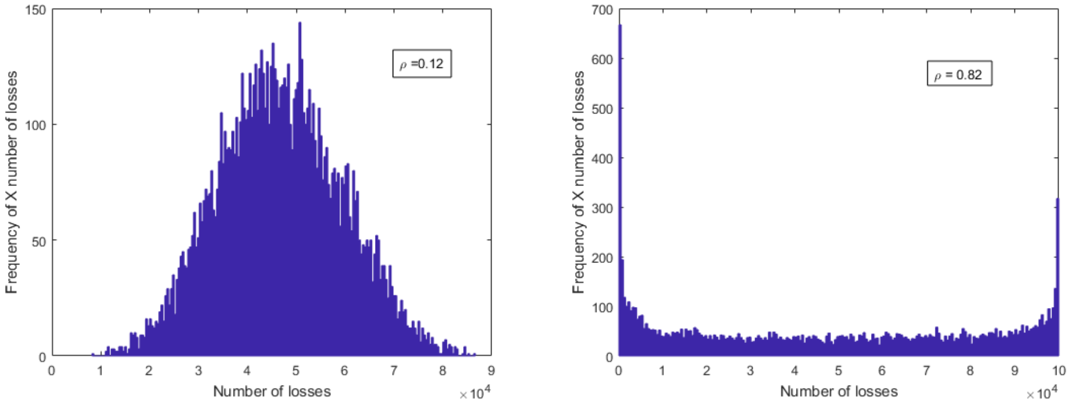
$$\mathbb{P}[\tau_i \leq t|Z] = N\left(\frac{N^{-1}(F(t)) - \sqrt{\rho}Z}{\sqrt{1-\rho}}\right)$$

We then assumed that the default probabilities $F(t)$ are the same for all loans and that all loans have the same correlation ρ with the background factor Z. In a heterogeneous portfolio, $F(t)$ and ρ can vary between loans, so we add a subscript i to the variables ($F_i(t)$ and ρ_i). What we then want to do is simulate the standard normal random variable Z and the i.i.d random standard normal variables Y_i. The Z and the Y_i's are then used, for a fixed t, to find $X_i = 1_{\{\tau_i \leq t\}}$ as follows from Equations 4 and 5. It can be shown that $X_i = 1$ is equivalent with $Y_i \leq \frac{N^{-1}(F_i(t)) - \sqrt{\rho_i}Z}{\sqrt{1-\rho_i}}$ (Herbertsson 2012), so by using this formula, we can simulate defaults while varying the input parameters p and ρ. The result is stored in the variable X_i as demonstrated below (Herbertsson 2012):

$$X_i = \begin{cases} 1 & \text{if } Y_i \leq \frac{N^{-1}(F_i(t)) - \sqrt{\rho_i}Z}{\sqrt{1-\rho_i}} \\ 0 & \text{if } Y_i > \frac{N^{-1}(F_i(t)) - \sqrt{\rho_i}Z}{\sqrt{1-\rho_i}}. \end{cases}$$

Once we have simulated our X_i's, we can get a value for N_t and thus also L_t. The results from Monte-Carlo simulation increase in accuracy as the number of simulation increase. However, Monte-Carlo simulations can take a long time to process so a trade-off has to be made between accuracy and time consumption. For demonstration, we have run the simulation with two fixed values of ρ (one at

12% and one at 82%) and varying levels of $F_i(t)$ in order to compare to the earlier calculated loss distribution using the LPA technique. In the simulation we used a loan pool of 100 000 loans and the simulation was run 10 000 times.



(a) Loss distribution with a low correlation.

(b) Loss distribution with a large correlation.

Figure 7: Loss distributions for low and high correlation.

As we can see in figure 7, the results are similar to what we got with the LPA approach in Figure 6, but Monte-Carlo simulation can be run with the actual estimated parameters of each individual loan in order to arrive at a loss distribution that is tailored to a specific portfolio of loans. We want to use the output from the simulation when pricing our CDO-tranche spreads. In order to do this, we can use the fact that the tranche loss can be written in the following way:

$$L_t^{a,b} = \max(L_t - a, 0) - \max(L_t - b, 0)$$

In this expression, a and b are the tranche borders and L_t is the portfolio loss at time t. L_t can be simulated n times, as stated above, and expected tranche losses can be estimated by taking the average tranche loss of, say, 100 000 simulated tranche losses. This can then be used in the CDO-tranche spread formula in place of $\mathbb{E}[L_t^{(a,b)}]$ when calculating the spreads.

5 Numerical Studies and Analysis

In this section we will use the previously discussed tools in order to analyze a peer-to-peer lending intermediary under the CDO framework. Knowing how to analytically calculate the expected tranche loss, we can calculate appropriate interest rates that should be given to investors (lenders) in a CDO. One can view the lenders in a P2P-setting as investors into a CDO consisting of available loans to borrowers with varying credit risk.

5.1 Tranche spreads and loss distributions

We can calculate the CDO-tranche spread (o, equivalently, interest rate in our case) $S_{(a,b)}(T)$ as defined in Equation (3), that is:[17]

$$S_{(a,b)}(T) = \frac{B_T \mathbb{E}[L_T^{(a,b)}] + \int_0^T r_t B t \mathbb{E}[L_t^{(a,b)}] dt}{\Sigma_{n=1}^{n_T} B_{T_n}(b - a - \mathbb{E}[L_{t_n}^{(a,b)}])\Delta_n}$$

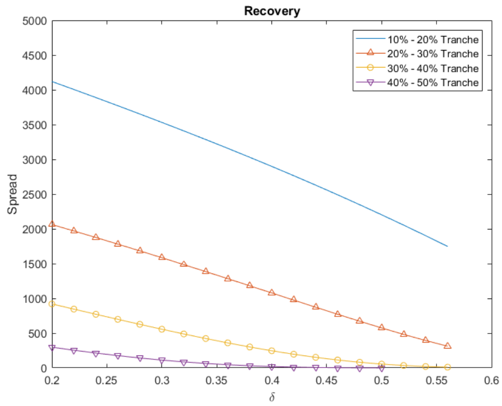
Moving forward, we will assume that $\Delta_n = \frac{1}{12}$. This assumes monthly payments, which is reasonable given that we are dealing with interest rates on loans. The variables that we can adjust regarding this equation are the probability of default p, the recovery rate δ, the correlation ρ, and the tranche-borders a and b. We assume that the risk free rate of interest r_f and the time to maturity of the loans T are given exogenously. Regarding the risk free rate of interest, using the 7 year Swedish government bond as a proxy[18] we arrive at a rate of about 0.3% (Riksbanken 2018). We believe that this is unreasonably low going several years into the future, but at the same time we do not want to over-estimate the risk free rate of interest. In the remainder of the analysis, we will therefore use a proxy of 1% for the risk free rate of interest.

First, we want to conduct an analysis of how the input-parameters affect the spread. Holding everything fixed except for the parameter of choice (p, ρ, or δ) and letting the selected parameter vary, we can distinguish the relative effect of that

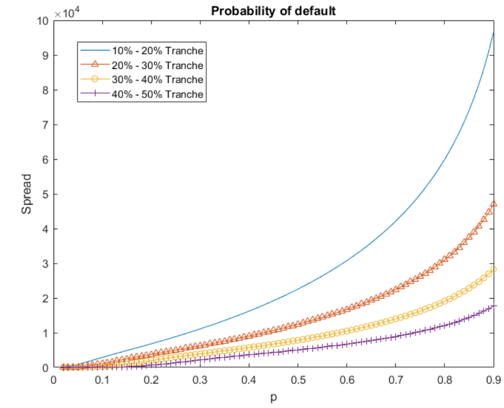
[17]Initially, we will assume homogeneous loans since we will then be able to use the LPA-formula in our calculations.

[18]Because this lines up with the average loan maturity at Lendify (Lendify 2017).

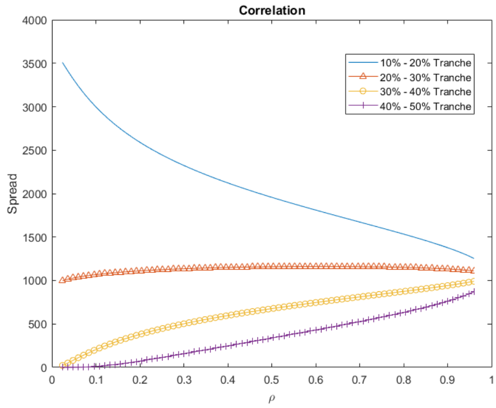
parameter. Figure 8 displays three graphs showing the effect that the parameters have on the spread. We have also included the figure of the loss distribution for different valued of ρ, which is generated using the LPA approximation detailed in section 4.4.3 (equation (10)).



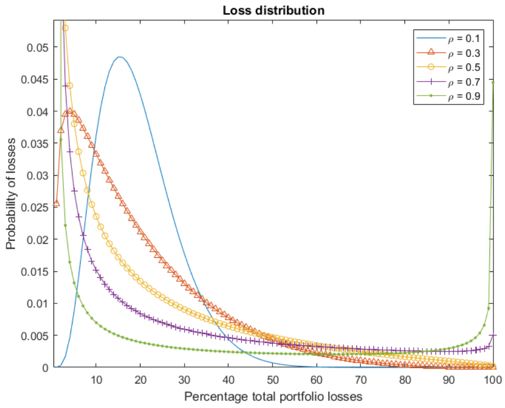
(a) The effect of the recovery rate on the spread.



(b) The effect of the default probability on the spread.



(c) The effect of the correlation on the spread.



(d) The loss distribution for different values of ρ

Figure 8: Plots of the tranche spreads based on different values of the parameters δ (a), p (b), and ρ (c), accompanied by a graph of the loss distribution for different values of ρ (d) (this is the same figure as figure 6). The base values for the parameters are as follows: $\delta = 0.4$, $p = 0.1$, $\rho = 0.12$.

What we can see from our quick analysis is that, as might be expected, the spread increases with the probability of default and decreases with the amount of recovery given default. What might be less obvious is how the correlation affects the spread. The intuition behind the correlation parameter is that if $\rho = 0.05$, for instance, then $100\% - 5\% = 95\%$ of your income is affected by your own performance alone and 5% is due to other factors such as the background economy. We can see that the spreads behave differently depending on which tranche is considered. This can be explained by the fact that a high correlation will likely result in a ”all or nothing case” (see e.g. in Figure 6 when $\rho = 0.9$ or in Figure 7 (b)), where the high correlation has either resulted in a large amount of defaults or a low amount of defaults. The spread (or interest rate in the peer-to-peer lending setting) will then be affected by this phenomenon and decrease based on expected tranche losses. So for the lowest tranche in our example (10%-20%) we can see that the interest rate decreases with ρ because there is a large chance that there will be no or very few defaults, which would benefit the investors of this tranche. With low correlation, the risk of having more than 10 percent loss in the portfolio is fairly large (depending on the selected default probability) and thus, the interest rate is higher with lower correlation. For the higher tranches we can see a different pattern. The 20%-30% tranche first increases and then decreases because of the correlation, and the 30%-40% tranche has a marginally decreasing increase in the spread as ρ increases.[19] This can be explained by the same phenomenon as for the lowest tranche. Since these tranches are more in the middle of the total portfolio loss, an increasing correlation will benefit them as long as it is high enough so that the probability of having losses that lie in the middle of the spectrum decreases. Finally, looking at the 40%-50% tranche, we can see that it is continually increasing with ρ since it becomes more and more likely to have a large amount of losses in the portfolio, i.e., losses that exceed the 40 percent mark. Since the total portfolio losses in this example are capped at 60% (that is, $\delta = 0.4$), losses between 40% and 50% are at the end of the spectrum. Additionally, we observe that as the correlation nears 100%, the tranche spreads converge toward the same value. This is because when the correlation is 100%, all obligors will be extremely dependent on the background factor, meaning that all obligors will ”behave” in the same way in regards to defaulting or not.

So far, the analysis has been based on results from a homogeneous portfolio since we have been using the LPA approach to model losses and relationships between the input parameters. In order to make the analysis more realistic, we will also use the results from Monte-Carlo simulation to complement the LPA results.

[19]In fact, the spread for the 20%-30% tranche actually starts decreasing at around 50% correlation.

Before we go into simulating heterogeneous portfolios, we want to complement the analysis in Figure 8 with simulated results, using the available information about the actual portfolio of the peer-to-peer lending intermediary Lendify. In their annual financial report of 2016-2017, it is stated that the average credit losses during the previous 12 months had been 1% (Lendify 2017). Using this as the input for probability of default (we are here assuming that all loans have the same nominal value 1), we can calculate possible loss distributions where we adjust ρ in our simulations to generate different scenarios. Note that, since the portfolios here are homogeneous, we do not actually need to simulate in order to reach the results in Figure 9, but we want to show that the simulation generates similar default distributions as the LPA method. Since we are using a lower default probability here, 1% instead of 10%, the high number of defaults that are expected from a large correlation are there (in the 90% correlation case in Figure 9 there are up to 100 defaults in the portfolio for instance) but the effect is not as explicit.

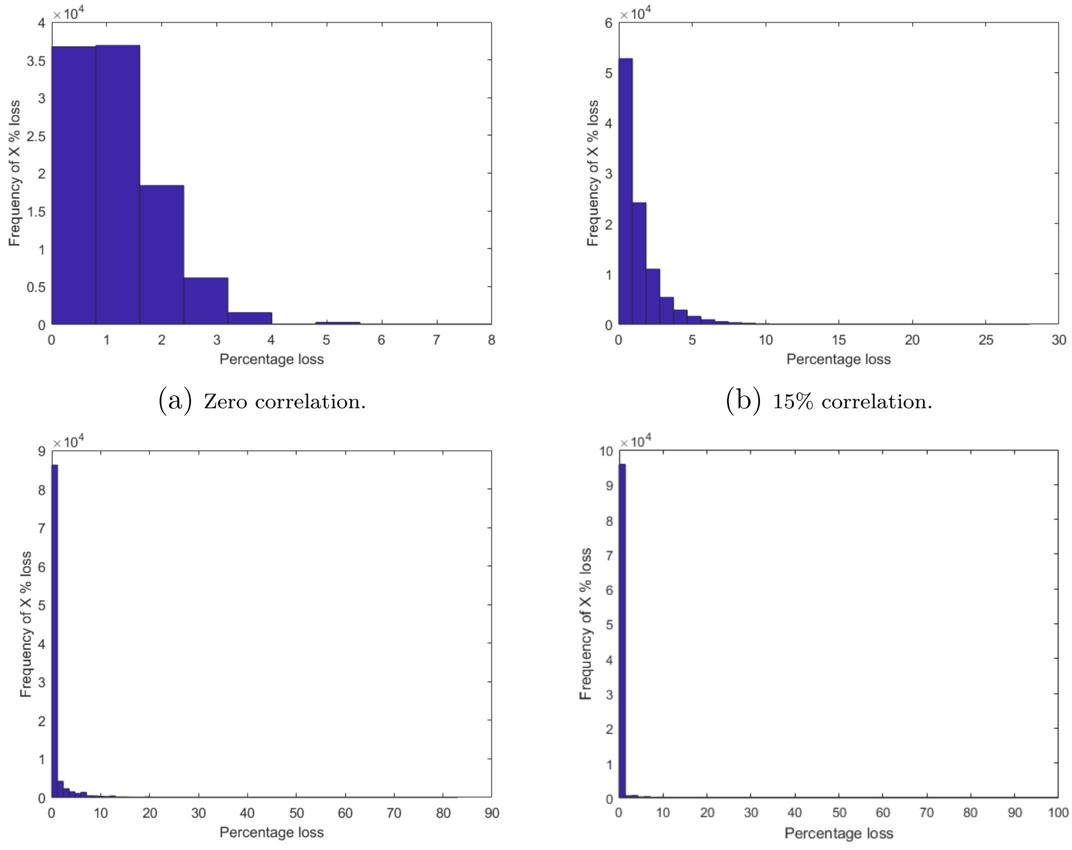
In the loss distributions used in Figure 9, we have assumed a fixed default probability of 0.01 and varied the correlation to see its effects. In Figure 9 observe that, as expected, a larger correlation will result in there being a risk of having large amounts of losses in the portfolio, even if the individual default probability is only one percent. We can use the distributions from the simulations in order to estimate the value-at-risk in the different scenarios. To do this, we take the α% quantile (95 and 99 percent) of the simulated losses. We also calculate the respective expected shortfall[20] of the distributions. The results are presented in Table 2 below.

ρ	$VaR_{0.95}$	$VaR_{0.99}$	$ES_{0.95}$	$ES_{0.99}$
0	2.5	3	2.71	3.25
0.15	4	7	6.11	9.75
0.5	5	17	12.89	27.87
0.9	1	36	19.93	67.31

Table 2: Value at Risk and Expected Shortfall for for a homogeneous portfolio using different levels of correlation and a default probability of 1%.

Table 2 demonstrates that even though the mean losses are similar across the dif-

[20]The expected shortfall is the expected loss given that the value at risk is surpassed.



(a) Zero correlation.

(b) 15% correlation.

(c) 50% correlation.

(d) 90% correlation

Figure 9: Loss distributions for different levels of correlation. The simulations use a loan portfolio of 100 loans, a time to maturity of 7 years, and a default probability of 1%. The average loss is approximately 1% for all correlations.

ferent correlations, the possible losses have a large variation based on the correlation parameter ρ. The correlation is thus very important to monitor for a peer-to-peer intermediary. In Table 3 we display the respective tranche spreads that results from varying the correlation. We observe that a larger correlation would reduce the payments (fair value) made to the investors (lenders) in the lower tranches (however, the over-all spread remains about the same), but as can be seen in table 2 above, increasing the correlation also increases the risk of serious credit events. Before proceeding, we define one Basis Point (bp) as a one hundredth of a percent, which means that 100 bp is equal to one percent. We will use the *bp* notation for the tranche spreads (interest rates) going forward.

	$p = 0.01$			
Tranche	Spread	Spread	Spread	Spread
	$\rho = 0$	$\rho = 0.15$	$\rho = 0.5$	$\rho = 0.9$
$0\% - 5\%$	2241.95 bp	2111.62 bp	1284.94 bp	398.87 bp
$5\% - 10\%$	1.34 bp	113.32 bp	356.61 bp	249.88 bp
$10\% - 100\%$	0 bp	0.95 bp	24.38 bp	76.44 bp
$0\% - 100\%$	100.32 bp	100.48 bp	101.09 bp	104.14 bp

Table 3: Tranche spreads (peer-to-peer interest rates) for different levels of correlation ρ.

5.2 Earnings and possible returns

As discussed earlier, the peer-to-peer lending intermediary should be able to make a gain on the spread between what is paid to the lenders and what is gained from the borrowers. If the peer-to-peer lending intermediary collects money from the lenders and then lends it out to the borrowers, the borrowers will make interest payments to the intermediary which then pays the tranche interest rates (spreads) to the lenders. This type of structure suggests that the interest rate that is paid from the borrowers to the intermediary should (at least in theory) be larger than the corresponding tranche interest rates paid to lenders. This is because of the waterfall-cash-flow structure of a CDO. Since the lenders essentially have invested in a diversified portfolio where they, based on their tranche, take losses in a falling order (starting with the equity tranche), they are not explicitly exposed to specific loans. Instead, they are affected by the number of losses (or total cumulative losses) within their tranche. The same cannot be said about the relationship between the borrowers and the peer-to-peer lending intermediary, since the exposure is here on a "per-loan-basis". That is, the interest rates charged by the peer-to-peer lending intermediary from the borrowers do not take diversification effects into account in the same way as the CDO-relationship between the lenders and the peer-to-peer lending intermediary does. Then, in theory, the interest rates that can be charged from the borrowers should be higher than the required spread payments to the lenders.

We can test this by benchmarking our model to the average interest rate at Lendify, which is 10% according to their financial report of 2016-2017. If the intermediary receives 10% in interest (on average) from the borrowers and (in the CDO-framework) makes CDO-spread payments to investors (lenders), we want to examine if our model can generate scenarios where there is an "arbitrage spread" that can be earned by the intermediary, as described earlier. Since there seems to be very limited data available on the loans of peer-to-peer lending intermediaries, it is difficult to estimate the expected tranche losses (and the resulting tranche interest rates) to accurately make the simulation reflect the situation at Lendify for instance. However, if we base our analysis on a 1% default rate, we could attempt to generate a realistic outcome by adjusting the correlation among loans. It is hard to make an accurate assumption about the correlation parameter, but there should be some correlation between the economy of households; borrowers could be working at similar companies or be invested in similar ventures for instance. Furthermore, it is probably safe to assume that peer-to-peer lending targets people who cannot get the same loan at a traditional bank. These individuals are probably worse off economically and therefore more exposed to economic downturns than someone with more assets and savings. If the 1% default probability is the "actual" default probability, then finding a reasonable correlation should yield fairly reasonable results regarding default distributions when simulating. Individuals who are worse off probably have a larger correlation with the background economy. For instance, someone who is living paycheck to paycheck might be very correlated with the background economy since a shock to their expenses (for instance if interest rates were to rise) could put such a person in a very serious negative situation. On the other hand, individuals who have capital buffers (money on savings accounts or invested in capital markets) should not be as affected by a negative shock to the background economy since they can still cover their expenses and pay back their loans. Based on this, we will run three of simulations where we vary the input parameters ρ and the probability of default, in particular, letting ρ be in a higher range.

5.2.1 Simulation 1 - 1% default probability & correlation between 5% and 15%

In the first simulation, we let ρ run linearly from 0.05 to 0.15 in order to capture a spectrum of possible borrowers and use a probability of default of 1% with no recovery. We are thus considering a heterogeneous loan portfolio with regards to the correlation ρ. The loss distribution is presented in Figure 10 and the interest rates along with value at risk and expected shortfall are presented in Table 4 below. We have here assumed that the peer-to-peer intermediary covers the initial 1% of the losses in the portfolio. We also use a maturity of seven years, since this is the average term of the loans at Lendify.

$p = 0.01$, $\rho =$ From 0.05 to 0.15	
Tranche	Interest rate
$1\% - 5\%$	1932.39 bp
$5\% - 9\%$	673.12 bp
$9\% - 16\%$	200.58 bp

$VaR_{0.95}$	$VaR_{0.99}$	$ES_{0.95}$	$ES_{0.99}$
16	23	20.50	26.75

Table 4: Tranche interest rates, VaR, and ES for the loss distribution in Figure 10. The Value at Risk and Expected Shortfall are denoted in number of losses, out of a total of 100 loans in the simulation. The time horizon is seven years.

Because the interest rates in our model in this case are very low once we move past the 20% mark, there is no real reason for anyone to invest money in those tranches. In our analysis, we have used a risk-free rate interest rate of 1% so if a tranche-spread is lower than this, there is no reason to invest into it.

In practice, the lenders would here insert money into the peer-to-peer lending intermediary (i.e. invest in the tranche of interest), which would be spread out among the available borrowers. Then, based on the number of defaults in the portfolio, investors receive reduced payments based on their selected tranche spread(s).[21] One way this could work is if the invested capital only yields interest as long as the selected tranche has not been completely depleted. So if there are two investors, where one has invested \$60 and the other has invested \$40 (so that the loans are fully

[21] That is, we assume that the lenders can select their level of risky compensation regardless of where their capital is allocated.

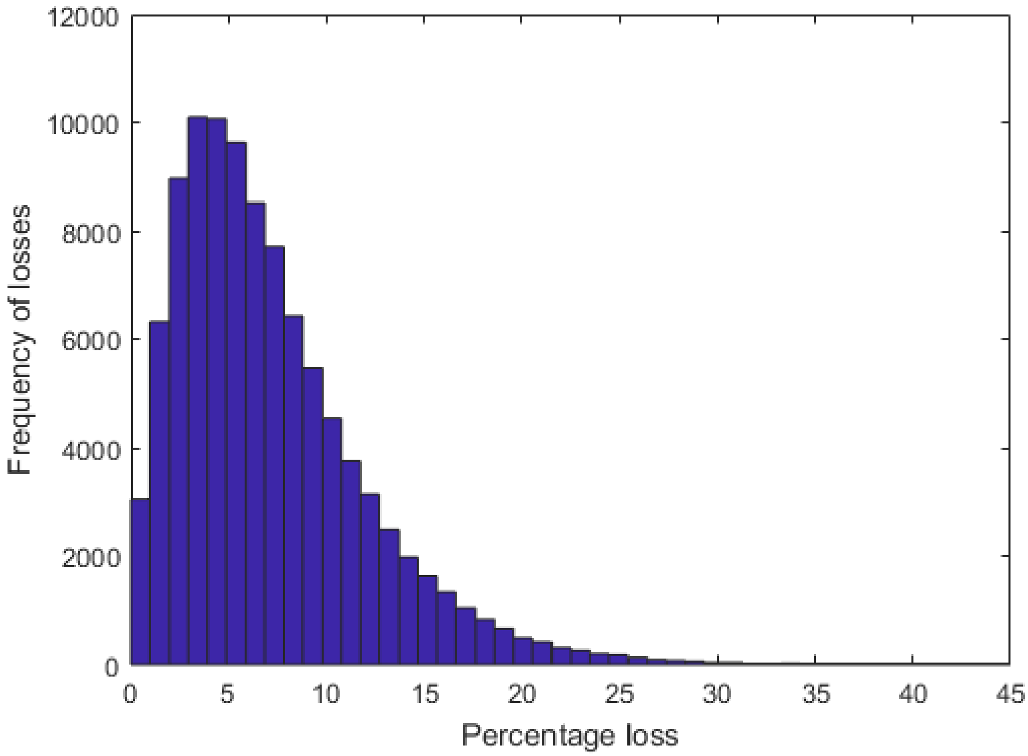


Figure 10: Example of a loss distribution using a default probability of 1% and letting the correlation (ρ) run linearly from 0.05 to 0.15 in steps of 0.001. The time to maturity is seven years, the portfolio consists of 100 loans, and 100 000 simulations were run. The time horizon is seven years.

funded given 100 loans with \$1 face value), and both select the 1%-16% tranches displayed above, the investors will split the default payments until the tranche is fully depleted and then not gain any further interest on their invested capital. For this to work, someone else needs to cover the losses above 16%. Here, the peer-to-peer intermediary could step in and cover losses that exceed 16%. The expected loss past 16% is equal to 20.5% and will be realized in 5% of cases, since it lines up with the 95% value at risk. The intermediary should insure itself against that exposure, which will cost at least $0.05 \cdot 0.205 = 1.025\%$ over the period. Note that this thought experiment is not necessarily representative of how the peer-to-peer CDO structure would look in practice[22], but it presents a general idea of how the problem can be approached. The reason for not including tranches above 16% is that the spread

[22]For instance, the peer-to-peer lending intermediary might not accept a scenario where it has to cover losses above 16%. In addition, this example builds on the assumption that nobody wants to invest in tranches above 16%.

becomes very low due to the unlikely nature of such default events in our model.

The distribution of the losses will naturally depend on the default rate and the correlation used in the simulation, as we have seen in the earlier simulations. The conclusions drawn here assume that the 1% default rate is plausible going forward, but the analysis at its core still holds if the default probability increases. Assuming that the average interest rate from the borrowers remains at 10%, we can check to see if the spreads above are plausible from an economic standpoint. If all loans have a face value of \$1, and there are 100 loans in the portfolio, the peer-to-peer lending intermediary will, on average, receive \$10 per year.[23] Out of this, \$1 will be used to cover the initial 1% of portfolio losses. Then the lenders have to be paid their tranche spreads. Based on the selection of tranches, the aggregate premium payments made to the lenders will vary.

In our example (displayed in Table 4), investors in the 1% to 5% tranche will receive an interest rate of about 19.3%, the investors in the 5% to 9% tranche will receive about 6.7% in interest payments. The final tranche in our example will yield an interest rate of about 2%. If we assume that there are 100 investors (lenders) and that they are split up evenly among the tranches, the average premium paid will be about 935.36 bp. In Figure 11 we plot the mean losses ($\mathbb{E}[L_t]$) during the 7 year period. In expectation, the portfolio losses will not exceed 7 during this time. Thus, only the lowest and second lowest tranches (in expectation) should be affected by losses during the time period. The effective interest rate will on average look as displayed in Figure 12 for these tranches. We define the effective interest rate as the original interest rate weighted by 1 minus the expected tranche loss at every time period t. The average effective tranche interest rate is then calculated by taking the mean of the weighted interest rates.

$$\text{Average effective tranche interest rate } = \frac{1}{n}\sum_{n=1}^{n_T} S_{(a,b)}(T)(1-\mathbb{E}[L_{t_n}^{(a,b)}])$$

[23]In Lendify's financial report 2016-2017, the interest rate paid by borrowers is made up by three parts. First, there had been an average loss of 1% during the previous 12 months. Second, Lendify has a service fee of 3% that it charges the lenders. The third part is what is left after the service fee and the expected credit losses have been taken into account, that is 0.1-0.03-0.01 = 6%, which had been the average return to lenders during the previous 12 months. So the interest rate is 10%, but the lenders pay a 3% service fee and lose on average 1%.

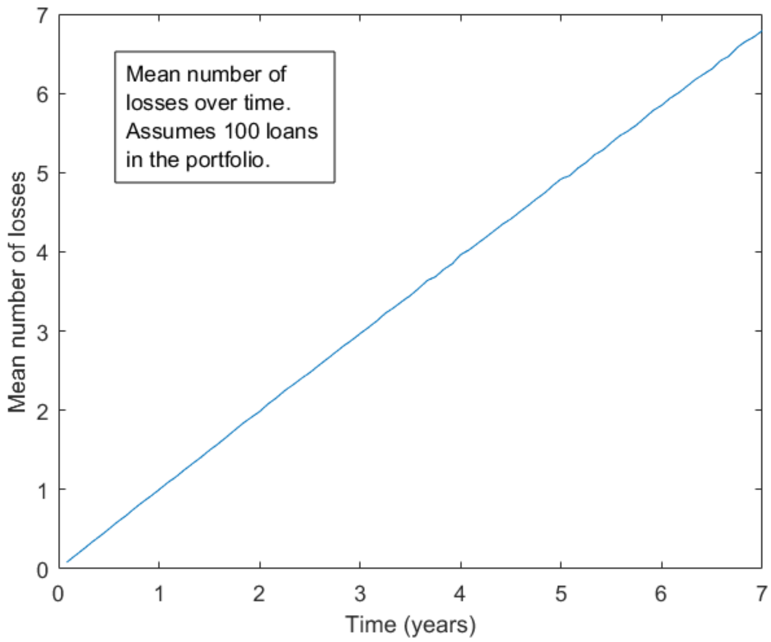
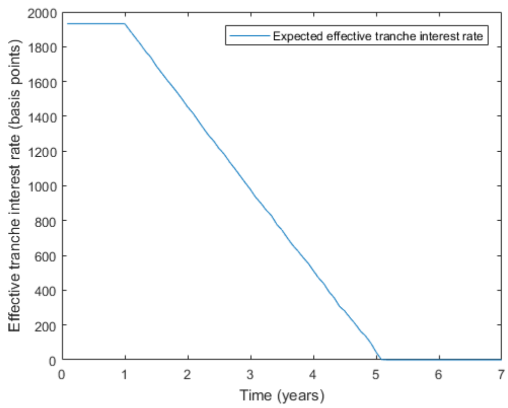
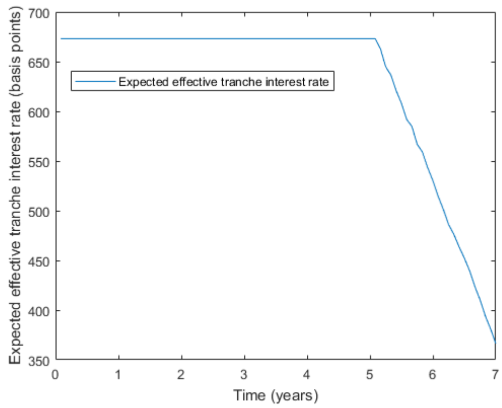


Figure 11: Average losses ($\mathbb{E}[L_t]$) in the portfolio over time (t). Corresponding to Figure 10.



(a) Expected effective 1%-5% tranche spread (interest rate).



(b) Expected effective 5%-9% tranche spread (interest rate).

Figure 12: Expected effective spreads (interest rates) for the 1%-5% and 5%-9% tranches. Corresponding to Figure 10.

The average effective spreads to the lenders in the 1%-5% and 5%-9% tranches will be 827.02 bp and 630 bp respectively over the period. The average premium payments will thus be (8.27%+6.3%+2%)/3=5.52%. The portfolio losses will, however, also affect the income from the loans on the aggregate since defaulted loans do not yield interest. The average interest income for the peer-to-peer lending intermediary over the period will be 9.65% in expectation , and is graphed in Figure 13. The average interest income is defined as follows (since we assume an interest income of 10% per loan and there are 100 loans in the portfolio):

$$\text{Average interest income } = \frac{1}{n}\sum_{n=1}^{n_T} 0.1(1-\mathbb{E}[L_{t_n}])$$

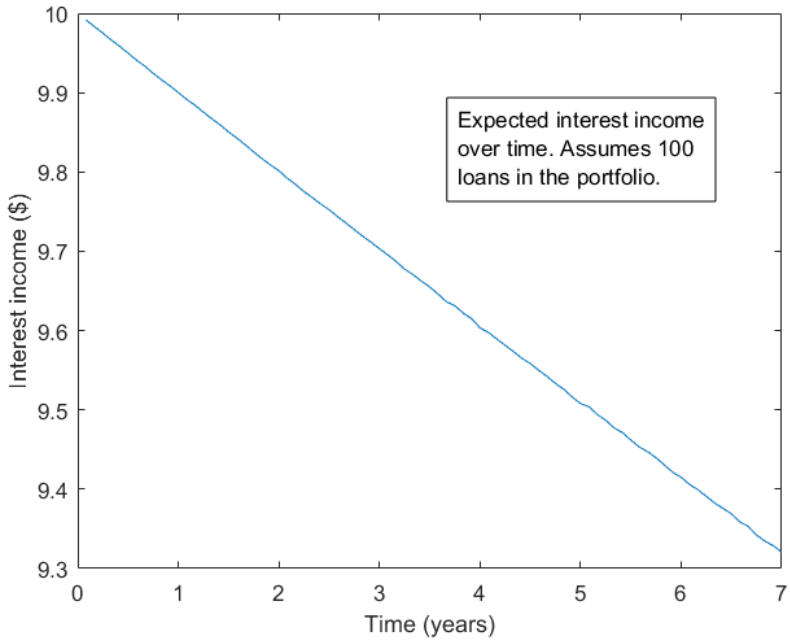


Figure 13: Average interest income over time. Corresponding to Figure 10.

The net result of this is that the peer-to-peer intermediary will gain, on average, 9.65% - 1% - 5.52% - 0.15% = 2,98% per year.[24] This is of course dependent on the distribution of lenders among tranches.[25] There is, however, a risk that investors gain more than the average spread, which would be bad for the peer-to-peer intermediary, and a chance that more loans default than was expected and the margin for the intermediary increases. This increases the risk for moral hazard on the part of the intermediary. Figure 14 below shows the value-at-risk for different percentiles. We can see that in five percent of cases, both tranches will gain the full tranche spread over the seven year period. In this case, the intermediary would have

[24]The 0.15% refers to the assumed yearly insurance payment, which is the insurance payment for seven years divided by seven.

[25]For this analysis we assume an even distribution among tranches in order to keep it more concise.

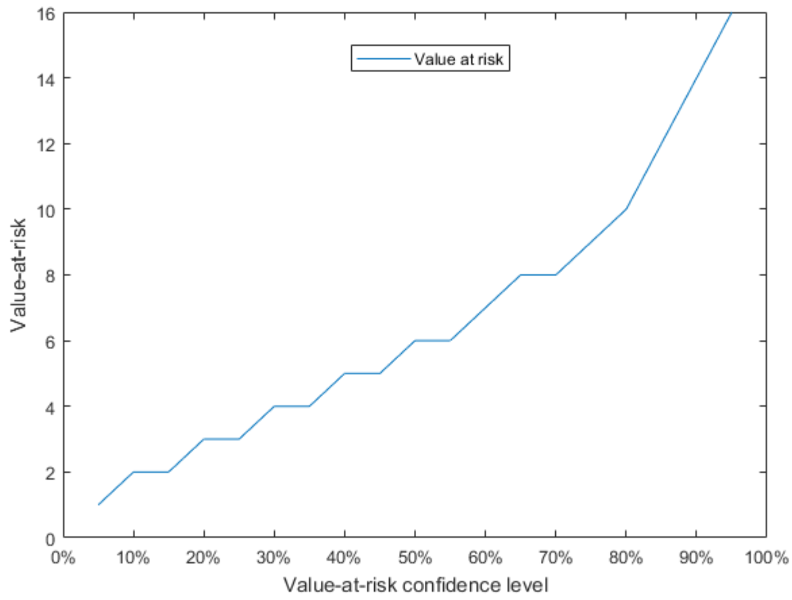


Figure 14: Value at risk for different confidence levels. Corresponding to Figure 10.

a result of 9.95%-1%-9.35%-0.15%=-0,52% each year[26] and probably go bankrupt. On the contrary, in 60% of cases, the lowest tranche will be entirely depleted over the course of the 7 years[27], resulting in a gain of 9.5%-1%-(0%+6.73%+2%)/3-0.15%=5.44% for the intermediary.

5.2.2 Simulation 2 - 1% default probability & correlation between 5% and 75%

In our next simulation, we will look at a scenario where the correlation ρ runs linearly from 5% to 75% instead linearly of from 5% to 15%, and keep the default probability of 1% with zero recovery. We are thus again considering a heterogeneous portfolio with regards to the correlation ρ. A larger correlation means that we assume that the borrowers have a larger sensitivity to the background economy and thus have a larger chance of defaulting if there is a downturn. Since the analysis will be similar to what we have just done, we will be more brief this time. The loss distribution is presented in Figure 15 and Table 5 displays the tranche spreads (tranche interest rates) and

[26] Assuming a one percent loss, leading to an average interest income of 9.95% over the period if the loss occurs half way to maturity. The 9.35% is the average spread with no more than 1% credit losses, (19,32%+6.73%+2%)/3=9.35%.

[27] The 40% value-at-risk is 5, so there is a 100%-40%=60% risk that losses will exceed 5.

value-at-risk with accompanying expected shortfall at 95% and 99% confidence.

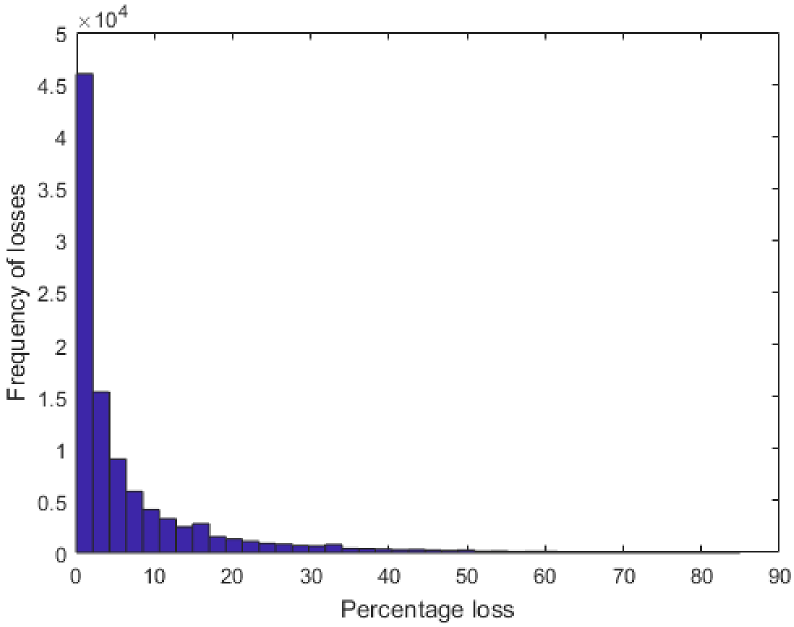


Figure 15: Example of a loss distribution using a default probability of 1% and letting the correlation (ρ) run linearly from 0.05 to 0.75 in steps of 0.007. The time to maturity is seven years, the portfolio consists of 100 loans, and 100 000 simulations were run. The time horizon is seven years.

$p = 0.01$, $\rho =$ From 0.05 to 0.75		
Tranche	Spread	Average effective spread
$1\% - 5\%$	1026.36 bp	439 bp
$5\% - 9\%$	468.47 bp	439 bp
$9\% - 16\%$	256.90 bp	257 bp
$16\% - 29\%$	117,62 bp	118 bp

$VaR_{0.95}$	$VaR_{0.99}$	$ES_{0.95}$	$ES_{0.99}$
29	50	41.66	59.14

Table 5: Tranche spreads (interest rates), VaR, and ES (stated in amount of losses out of 100) for the loss distribution in Figure 15, where ρ runs linearly from 5% to 75% in steps of 0.007. The average effective spreads are rounded. The time horizon is seven years.

The average premium payments (using the average effective spreads (interest rates)) during the period (using seven years again) will be $(4.39\% + 4.39\% + 2.57\% + 1.18\%)/4 = 3.13\%$, assuming that the tranches are distributed evenly among lenders. The expected interest income during the time period will be 9.65%, and the insurance premium (fair value) should be $0.05 \cdot 0.4166 = 2.083\%$, using the fact that we match the upper attachment point of the final tranche to the 95% value at risk. The projected gain for the peer-to-peer lending intermediary could therefore be 9.65%-1%-3.13%-0.3%=5.22%, where 0.3% corresponds to yearly insurance payments and the 3.13% correspond to the average interest rate to lenders assuming equal distribution among the tranches.[28] In this example, we can see that increasing the correlation leads to the tranche interest rates lowering for the initial tranches, and in expectation increasing what the peer-to-peer lending intermediary can earn if it employs fair spreads (interest rates). Plotting the value at risk for different confidence levels we can get an overview of the risk profile of the portfolio. We can see

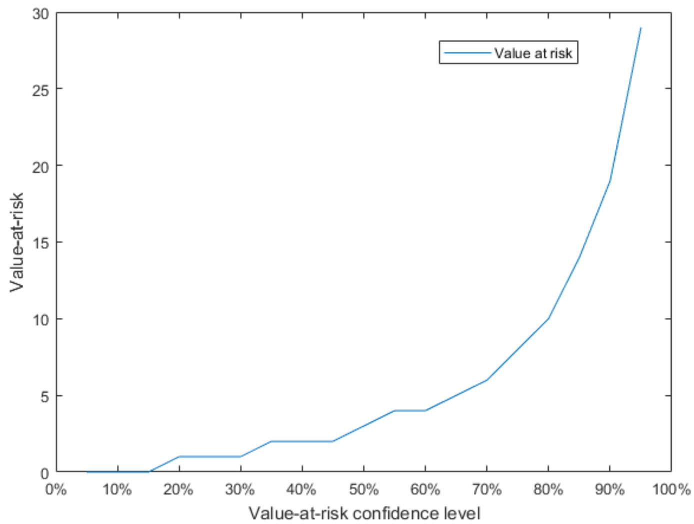


Figure 16: Value at risk for confidence levels. Corresponding to Figure 15.

in Figure 16 that in about 35% of cases, the 1%-5% tranche will be completely depleted over the time period.[29] Additionally, since the correlation is relatively large, the value at risk is a marginally increasing function of the α percentile that is used. This further demonstrates the increased risk of large losses that come with increased portfolio correlation.

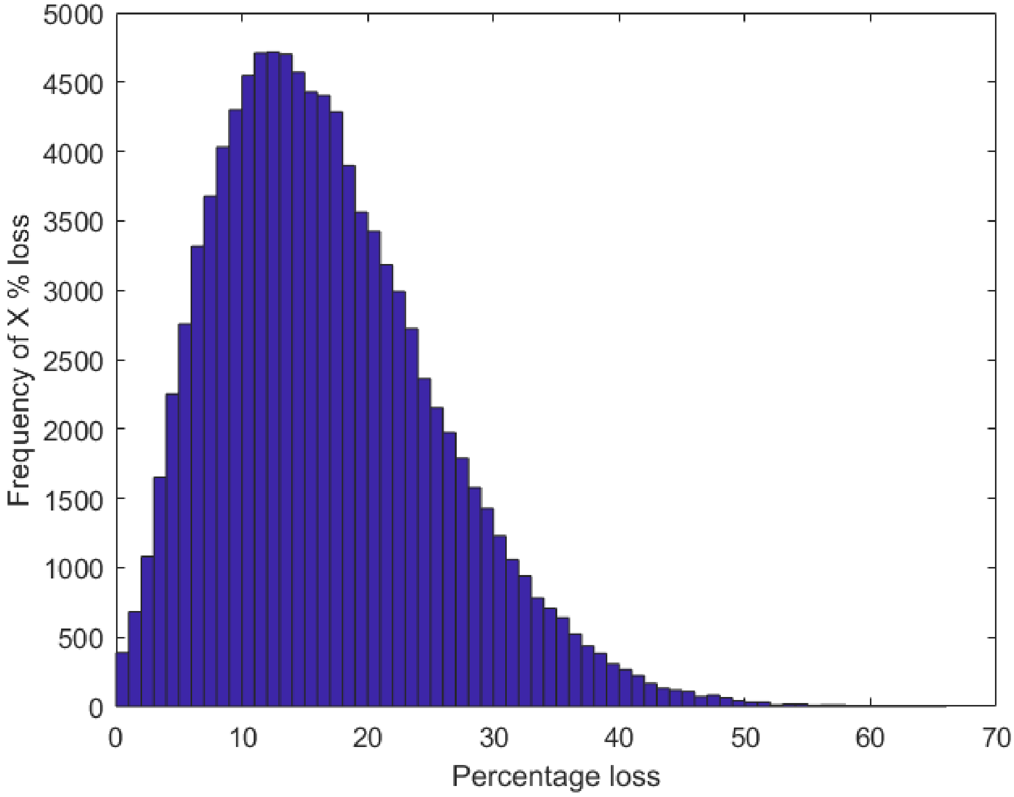
[28] $(4.39\% + 4.39\% + 2.57\% + 1.18\%)/4 \approx 3.13\%$.

[29] The 65% value-at-risk is 5, so there is a 100%-65%=35% risk that losses exceed 5.

5.2.3 Simulation 3 - Default probability between 0.5% and 5% & correlation between 5% and 15%

We also want to look at a setting where we assume that the borrowers have increased in riskiness, i.e., their default probabilities have gone up. To do this, we change the default probability variable in our simulation to run linearly from 0.5% to 5%. What this is supposed to represent is a setting where more risky individuals have been attracted to borrowing through peer-to-peer channels, assuming that they are also accepted to borrow by the intermediary. This could be the effect of the average default probability among borrowers going up because of an economic downturn for example. We will keep the correlation factor the same as in Figure 10, running linearly from 5% to 15%. We are thus considering a heterogeneous loan portfolio with regards to both the correlation ρ and the probability of default. The loss distribution is presented in Figure 17 below. In this setting, it becomes viable to assume that there is only one tranche in the CDO structure, the 0 to 1 tranche. This is because there is now a larger probability of a high number of defaults, which increases the fair interest rate to reasonable levels, even if there is only a single tranche. We believe that using a single tranche instead of several tranches would increase the simplicity of the product and make it easier to implement, both on an intuitive level and by requiring less administration from the peer-to-peer lending intermediary. This, however, also removes the risk category aspect of the CDO approach, which is why we also include separate tranches in Table 6. Using a single tranche under this scenario, we reach a spread of 272 bp with an average of 247 bp over the period of 7 years. Using the same assumptions as before regarding interest income (10% interest on average from the loans), the average interest income over the period is about 9%. This means that, on average, and using our assumptions, the intermediary stands to gain 9%-2.47%=6.53% from the loans.

Using the individual tranches in Table 6, we can calculate the average spreads and the resulting expected income for the intermediary, assuming equal distribution among the tranches. Using the average effective spreads, we reach an average premium payment of 497.22 bp. Again, assuming that the intermediary insures against losses above the 95% value at risk, and that the insurance is fairly priced, we reach



$VaR_{0.95}$	$VaR_{0.99}$	$ES_{0.95}$	$ES_{0.99}$
34	42	39	47

Figure 17: Loss distribution where the default probability runs from 0.005 to 0.05 and the correlation runs from 0.05 to 0.15. The maturity time is seven years, there are 100 loans in the portfolio, and 100 000 simulations were run. We are also displaying the value at risk and expected shortfall (in number of losses out of 100) at 95% and 99% confidence.

$p = 0.005 - 0.05, \rho = 0.05 - 0.15$		
Tranche	Spread	Average effective spread
$2\% - 16\%$	2021.42 bp	993.70 bp
$16\% - 22\%$	608.64 bp	602.25 bp
$22\% - 28\%$	278.81 bp	278.81 bp
$28\% - 34\%$	114.12 bp	114.12 bp

Table 6: Example of tranches and their corresponding spreads (interest rates) for Figure 17.

an insurance payment of $0.05 \cdot 0.39 = 0.0195$ over the period. Spreading this payment out over the seven years yields a yearly payment of 0.28%. Using these inputs, and the fact that the intermediary here covers two percent of the initial portfolio losses, we reach an average revenue of 9%-2%-4.97%-0.28%=1.75% per year.[30] While this is considerably lower than the 6.53% with the single tranche, it presents a more attractive investment opportunity for lenders. We should remind ourselves, however, that the tranches are arbitrarily chosen and that the revenue displayed above is just an example displaying that the setup can work. Finding optimal tranches that maximize revenue while retaining a large enough customer base is not within the scope of this thesis, but could be interesting for future research.

5.2.4 A brief scenario analysis and LPA benchmarking

Additionally, we have run a scenario analysis where the probability of default runs from 0.002 to 0.2 where we plot the 0-1 tranche spread for every default probability. As we have seen before (Table 3), the level of the correlation has little impact on the spread if we look at the 0-1 tranche. The result is presented in Figure 18 below where we, for the sake of transparency, have used 15% correlation.

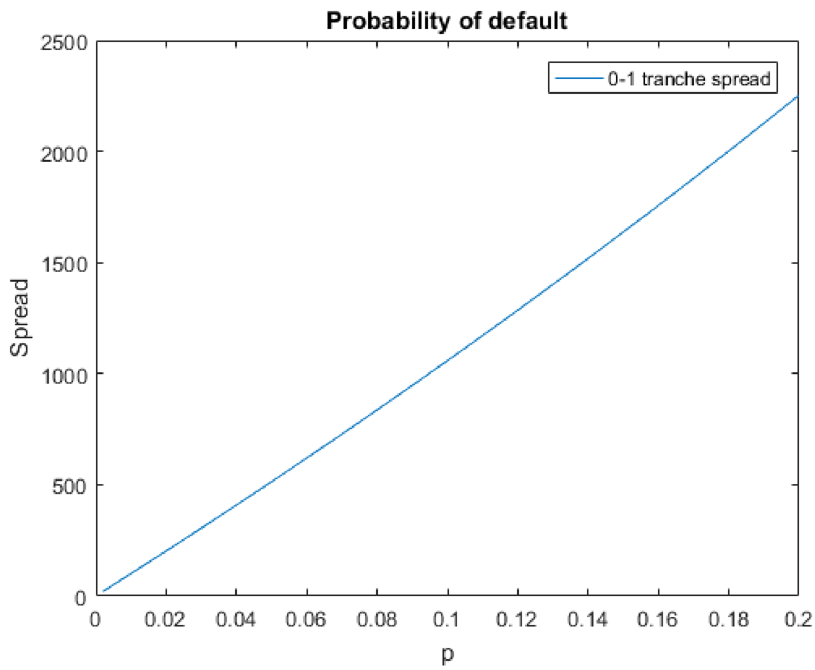


Figure 18: The 0-1 tranche spread as a function of the probability of default, using a time horizon of seven years and $\rho = 15\%$

The result was generated using the LPA approach. Keeping the assumption of

[30] The average interest payments are calculated as: $(9.94\% + 6.02\% + 2.79\% + 1.14\%)/4 \approx 4.97\%$.

10% interest income per loan, the peer-to-peer lending intermediary appears to be able to sustain default probabilities up to about 9.5% in order to avoid taking losses in expectation. Here it becomes interesting to check if the LPA approach can be a sufficient approximation for a heterogeneous portfolio. We can check this inputting the average values p and ρ of heterogeneous portfolios into the LPA formula and compare the resulting spreads. Since we already have simulated spreads (Tables 4 and 5 for instance) we will benchmark against these. Additionally, we can benchmark against the 0-1 tranche that was calculated using the loss distribution in Figure 17. The result is presented in Table 7.

$p = 0.01, \rho = (0.05 + 0.15)/2 = 0.1$			
Tranche	Spread	LPA Spread	LPA deviation
$1\% - 5\%$	1932.39 bp	2100.21 bp	+8.68%
$5\% - 9\%$	673.12 bp	649.17 bp	-3.56%
$9\% - 16\%$	200.58 bp	168.07 bp	-16.21%
$p = 0.01, \rho = (0.05 + 0.75)/2 = 0.4$			
Tranche	Spread	LPA Spread	LPA deviation
$1\% - 5\%$	1026.36 bp	987.50 bp	-3.79%
$5\% - 9\%$	468.47 bp	491.52 bp	+4.92%
$9\% - 16\%$	256.90 bp	269.75 bp	+5%
$16\% - 29\%$	117.62 bp	116.42 bp	-1.02%
$p = (0.005 + 0.05)/2 = 0.0275,\ \rho = (0.05 + 0.15)/2 = 0.1$			
Tranche	Spread	LPA Spread	LPA deviation
$0\% - 100\%$	272 bp	279.29 bp	+2.68%

Table 7: Comparison between homogeneous LPA spreads using average values and the simulated heterogeneous spreads.

From the short analysis in Table 7, we can see that the LPA approximation appears to be a fairly good predictor of the "actual" spread of a heterogeneous portfolio if we use the averages as inputs. Apart from two deviations, the LPA spreads stay within 5% of the simulated spreads. This gives more credibility to the analysis of Figure 18 if we assume that the default probabilities are averages of the underlying loan portfolio.

6 Conclusion

In this section, we will conclude our findings and suggest future research within the area.

In this thesis, we have attempted to model a peer-to-peer lending intermediary in a CDO setting. Based on our analysis, a couple of points can be made. First, when modeling the peer-to-peer lending business according to a CDO, we made some deviations from the setting of a standard CDO. The lending of capital in our model is viewed as the investment into the CDO portfolio and default payments are payments in the sense that the lenders do not require their capital back, as opposed to there being a transfer of capital in the event of a default. We thus view default payments as write-downs of the capital that was initially invested by the lenders. Additionally, when looking at the results from Section 5.2.1 for instance, we see that our modeled defaults were insufficiently large to warrant feasible spreads past the event of a 16% loss in the portfolio. We then suggest that the peer-to-peer lending intermediary itself takes on responsibility for the defaults in the event that losses exceed that level by buying insurance on that credit event. Second, when dealing with a correlation product such as a CDO, it is important to consider the effects of correlation in the portfolio. When the correlation in the underlying loan-portfolio is low, the loss distribution will be approximately normally distributed (because we use a Gaussian model) around the average amount of defaults in the portfolio. Using a higher correlation coefficient, we observe that the risk of experiencing large losses increases (Section 5.2.2). As a peer-to-peer lending intermediary, it thus becomes central to manage the correlation in their loan portfolio in order to limit credit risk. Regarding the other input parameters, we have seen that the recovery rate and default probability have predictable effects on the tranche spreads in that the spreads decline with increasing recovery and increase with increased default probability. Third, under some circumstances, and using the one-factor Gaussian model to model the defaults, there is an opportunity for a peer-to-peer lending intermediary to earn revenue by applying a CDO-framework to its business model. Using the inputs that were given in the annual report of Lendify, we were able to construct scenarios where the average tranche spread payments are outweighed by the average interest income from the loans in expectation. We gave four examples where the lenders receive above risk-free rate of return and where the peer-to-peer lending intermediary stands to make a profit, at least on average. We also made a brief analysis displaying that when using the 0-1 tranche approach (Section 5.2.4), a peer-to-peer lending intermediary should (according to our assumptions and using our model) be able to sustain average portfolio default probabilities of up to about 9.5% before making a loss. It is of interest to point out that as of right now, Lendify makes on average 3% on every loan in service fees. Comparing this to our projected margins of 2,98%, 5,22%, 6.28%, and 1.75% respectively, we can see that our model can be better from a revenue standpoint compared to what the intermediary is currently doing. There is also the added benefit of having a system in place that very clearly distributes the responsibility of default protection. One difference between the current setting and the CDO-setting is also that the invested capital is not tied to certain specific loans, but instead to the over-all health of the loan portfolio. We have also seen that an increasing correlation to the background economy among the loans in the portfolio leads to a lowering of the lower tranches of the CDO. What this means is that it could be of interest for a peer-to-peer lending intermediary under the CDO-setting to attempt to increase the correlation of its loan portfolio. Increasing the correlation does, however, increase the risk of suffering large losses which could cause the intermediary to go out of business even if it is insured. This is because the trust in the intermediary almost certainly would be damaged from such an event so that investors (lenders) will not return to that intermediary in the future. If the economy goes into a downturn, as we attempted to simulate in Section 5.2.3, we have demonstrated that using only a single tranche that covers the entire portfolio becomes viable from an investment standpoint. This makes it a viably safer investment than being invested in individual loans.

While our results show promise, one has to remember that this research is based on simulated results with low access to real data on the loans and loss distributions of actual peer-to-peer lending intermediaries. We believe that the intermediaries will become more transparent in the years to come and that loan data thus will be more readily available. For future research we therefore propose running a similar analysis based on the actual loan data. Additionally, research into which tranches to use in order to gain the optimal amount of revenue while at the same time attracting new customers would be interesting. We also suggest using different models for the loss distributions in order to more rigorously, and more realistically, examine whether modeling a peer-to-peer lender according to a CDO is a viable option in the real world, and whether it produces a safer way to invest into the lives and ventures of individuals. Based on our research, however, we believe that the outlook is promising, especially if the recent economic boom ceases and instead gives rise to negative trends.

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