

SCHOOL OF BUSINESS, ECONOMICS  
AND LAW AT UNIVERSITY OF  
GOTHENBURG

MASTER OF SCIENCE IN FINANCE

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**Contingent Convertible Bonds and  
the Optimal Default Barrier**

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## *Abstract*

Graduate School

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### **Contingent Convertible Bonds and the Optimal Default Barrier**

by Ludwig ALLARD and Raymond KE

This thesis provides a comprehensive overview of the sensitivity of the optimal default barrier in regard to its input parameters and the use of contingent convertible bonds. Contingent convertible bonds are financial instruments designed to help banks prevent default and absorb losses by converting from debt to equity in times of financial distress. We also study how contingent convertible bonds would have affected the optimal default barriers of the four biggest Swedish banks during the 2007-2009 financial crisis. The results from this thesis suggest that issuing contingent convertible bonds typically increase the optimal default barrier, but that the negative impact on solvency is diminished during the financial crisis. We conclude that the usefulness of contingent convertible bonds is primarily derived from its utility as a bail-in instrument, rather than as a tool to prevent default.

**Keywords:** *Contingent Convertible Bonds, CoCo Bonds, CoCos, Optimal Default Barrier, Swedish Banks, Financial Crisis.*

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## 1. Introduction

In this thesis, we use a structural credit risk model to examine the sensitivity of the optimal default barrier, which indicates the point at which declaring bankruptcy maximizes equity value for shareholders. We also examine how the optimal default barrier changes with the use of contingent convertible bonds. The contingent convertible bond or CoCo bond for short, is a type of subordinated debt product designed to help banks absorb losses in the event of default. We proceed to apply said model on real bank data from year 2005 to 2011 to study how CoCos affect the optimal default barriers of major Swedish banks in the period around the 2007-2009 financial crisis (henceforth simply referred to as the financial crisis). Our results show that the optimal default barrier increases when CoCo bonds are issued, which suggests that the issuance of CoCos has a negative impact on the likelihood of default. When we apply our model to aforementioned bank data, we find that the negative impact that CoCo bonds exert on the optimal default barrier is reduced during the crisis years. The findings of this thesis contribute to the literature on CoCo bonds by questioning the effectiveness of using CoCos as a means to mitigate the default risk of banks in distress.

Central to this thesis is the CoCo bond, which is a type of hybrid security with traits that resemble both debt and equity instruments at different occasions. When issued, a CoCo acts like a debt instrument and pays coupons to the CoCo bond holder; if the CoCo bond is not issued perpetually, the CoCo bond holder also receives a principal payment upon maturity, just like in the case of a vanilla bond. However, certain contingent events will trigger CoCos to either convert to some amount of equity, or have its value written down altogether; hence the name *contingent* convertible bonds. Figure 1.1 shows the main design features of CoCos. As can be seen from the left leg of Figure 1.1, the trigger of a CoCo bond conversion can either be mechanical, discretionary, or both. A mechanical trigger is usually designed to link the CoCo conversion to a financial measurement of book or market value in such a way that conversion takes place as the bank experiences severe financial distress. For example, if a bank's required capital reserves drops below a



certain threshold, a conversion of its CoCos could be triggered. Alternatively, CoCo conversion can be commissioned at the discretion of a relevant authority, such as a banking supervisory authority (BISd, 2017). As an example, the European Central Bank’s banking oversight body, the Single Resolution Body stepped in to write down all of Banco Popular’s outstanding CoCo bonds in 2017 (Smith and Khan, 2017).

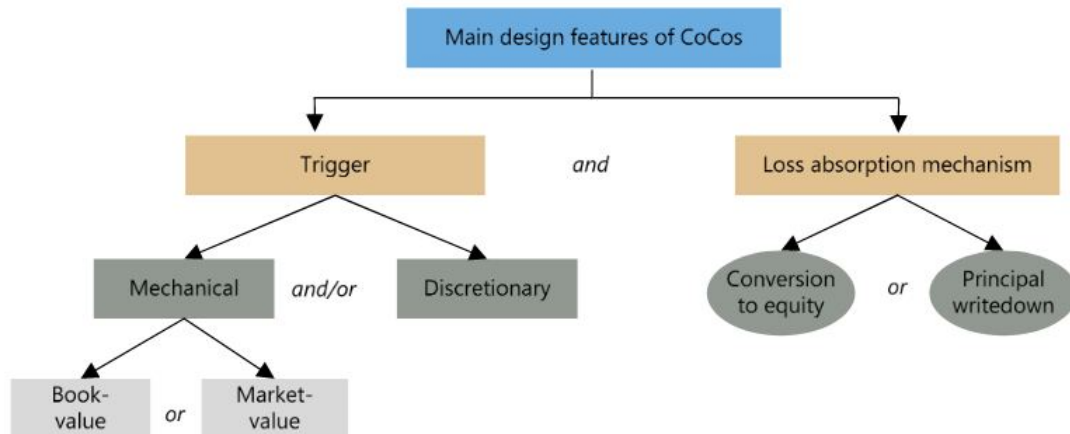


FIGURE 1.1: Main design features of CoCos (Avdjiev et al., 2013).

The purpose of CoCos is to reduce the risk of default and to help banks absorb losses if default occurs through a bail-in scheme. Bail-in refers to how CoCo bond holders help bear the bank’s losses like equity holders in bad times, thus absorbing some of the costs that would otherwise be born by taxpayers via government bail-outs (Perotti and Flannery, 2011). More specifically, the loss absorption process involves one of two things, either converting CoCos to equity stakes or writing down the CoCos altogether, as can be seen from the right leg in Figure 1.1. If the conversion rate is not pre-specified, the amount of shares a CoCo bond holder gets may depend on the financial situation of the company at the time of conversion. The more dire the crisis the bank is in, the fewer the shares might CoCo bond holders reasonably expect to receive upon a conversion; in the most extreme case, all outstanding CoCos may be wiped off the balance sheet, thus leaving the CoCo bond holders empty handed post-conversion. Given the higher risk that is embedded in CoCos compared to ordinary bonds, holders of CoCos typically receive a higher return - in the form of bigger coupon payments - than holders of vanilla debt.

Literature on CoCos date back to 2005, when Flannery (2005) first introduced contingent convertibles. The topic of this thesis however, connects CoCos to the optimal default barrier, which is also the primary topic of research for Chen, Glasserman, Nouri, and Pelger (2017). The difference is that the incentive effects of CoCos are

the focus of Chen et al. (2017), while the focus of this thesis is on the sensitivity of the optimal default barrier to CoCo bonds. Moreover, Chen et al. (2017) find that CoCos have at least three features that create strong incentives for equity shareholders to invest in a firm to avoid conversion. Firstly, reductions in rollover costs give equity holders some potential to benefit from an increase in investment. Rollover costs may be reduced for various reasons, improved company solvency and interest rate cuts by the central bank are two such examples. Secondly, the dilutive effect of CoCos create an incentive for shareholders to prevent conversion. Thirdly, if CoCo coupons are tax deductible, shareholders have more incentive to invest in the firm before conversion triggers to avoid the loss of this tax benefit (Chen et al. (2017)). Using a different structural model, Hilscher and Raviv (2014) decompose bank liabilities into sets of barrier options to study the incentive effects of CoCos, and find that setting an appropriate conversion ratio can significantly reduce the likelihood of default. Their positive results are in line with the results of Chen et al. (2017). In another study, Jaworski, Liberadzki, and Liberadzki (2017) compare issuing CoCos to issuing conventional bonds to examine how bank solvency changes using a value-at-risk and expected shortfall approach. Their results show that CoCos only improve the issuer's solvency if the probability of conversion is greater than the significance level of the value at risk. In other words, Jaworski et al. (2017) concludes that it only makes sense to issue CoCos if the probability of default is high enough above a certain point, which in turn is derived from the value at risk. Meanwhile, Schmidt and Azarmi (2015) study the link between CoCos and default using regressions to test how markets react to the announcement by Lloyds Banking Group (LBG) to issue CoCos, by looking at the share price and Credit Default Swaps (CDS) spreads of LBG before and after the announcement. They find a drop in share price and a rise in the CDS spread, indicating that CoCos had a negative effect on both the share price and the default likelihood.

This thesis has two objectives. The first objective is to provide a more comprehensive understanding of the impact that CoCos have on the likelihood of default, approximated through the optimal default barrier. This objective is achieved by studying the sensitivity of the optimal default barrier to CoCos and changes in its input parameters. The second objective is to study the impact that CoCos would have had on the solvency of Swedish banks for the period around the financial crisis. The second objective is achieved by comparing the optimal default barrier of banks during the years around the financial crisis to the optimal default barrier that the banks would have had if they also had CoCos in their debt mix. Furthermore, we limit the scope of testing to the four biggest Swedish banks: Nordea, SEB, Swedbank

and Handelsbanken. The reasons are that these four banks are fairly comparable in terms of business models and size, and their data is more accessible because they are publicly traded on the Stockholm Stock Exchange. Moreover, our thesis contributes to the research on how CoCos impact the likelihood of default by using bank data outside the US, which is different from Chen et al. (2017). This is interesting because the US was at the epicenter of the financial crisis, whereas Sweden was not. In contrast to the US and most of the developed world, Sweden's economy recovered relatively quickly from the crisis (Ekici, Guibourg, and Åsberg-Sommar, 2009). According to Jaworski et al. (2017), issuing CoCos only makes sense if the financial distress is grave enough, which implies that the positive impact of CoCos on bank solvency may be exaggerated for banks in non-US countries, perhaps even absent. Furthermore, we use a different model from that of Jaworski et al. (2017) to examine how CoCos impact the solvency of banks, and explore the relationship between CoCos and the optimal default barrier while shifting the underlying input parameters to observe a number of different scenarios. Studying these different scenarios also help us explain the relationship between the likelihood of default and key parameters, such as the risk-free interest rate, volatility and debt maturity, which we will see in Chapter 4.

The remainder of this thesis is structured as follows: Chapter 2 contains a discussion about the relevance of CoCos from a regulatory standpoint. Chapter 3 outlines the asset and credit risk models that we apply in the thesis. In Chapter 4, we discuss how valuation models are adapted to accommodate the inclusion of CoCos and its implications. We present a result comparison and sensitivity analyses in Chapter 5 to examine the relationship between the optimal default barrier, its input parameters and the use of CoCo bonds. Then, in Chapter 6, we show the results of our model application on real bank data. Finally, Chapter 7, contains a summary of our findings and our concluding remarks.

## 2. Banking Regulation

The surge in popularity for CoCos since the financial crisis is largely driven by upcoming regulatory requirements. In this chapter, we discuss the Basel Accords, which is the primary regulatory framework that banks must comply with in terms of capital requirements. The purpose of clarifying the connection between CoCos and banking regulations is to explicate the impact that CoCos have on banks on a microeconomic level.

In response to the financial crisis, the Basel Committee of Banking Supervision (BCBS) set out to address the key issues that had caused the global economy to derail (BISb, 2018). BCBS identified insufficient liquidity in banks as one of the major drivers behind the crisis and sought to tighten capital requirements through the third Basel Accords. The Basel Accords are a set of recommendations for the banking industry that are typically enforced on a national level, scheduled to be fully implemented in 2019 (BISc, 2013). The third Basel Accords or Basel III, expands on the "three pillars" concept used in Basel II by enhancing the contents of Basel II with more stringent regulatory requirements (BISd, 2017). The three pillars of Basel III are designed with the intention of addressing the biggest issues in the banking industry and is categorized as follows:

- Pillar 1: Regulatory Capital
- Pillar 2: Supervisory Review
- Pillar 3: Market Disclosure

The aspect of Basel III that is relevant to the topic of this thesis are the capital requirements - the first pillar of Basel III. The regulatory capital requirement recommends that banks hold 6 percent of all risk-weighted assets as so called "Tier One capital", which is the core measure of a bank's financial strength (BISd, 2017). Tier One capital is a core measure because the asset types under Tier One are the most secure kind of assets from the bank's perspective; Tier One capital comprises either pure equity or near-equity equivalents. Out of its 6 percent, the Tier One Capital is

further segmented into 4.5 percent Common Equity Tier One (CET1) capital and 1.5 percent Additional Tier 1 (AT1) capital. CET1 is simply shareholder's equity that the bank holds onto to counterbalance any risk-weighted assets it has outstanding, rather than investing; this makes CET1 the most expensive type of capital as it cannot be invested to improve a bank's turnover. The 1.5 percent AT1 can be made up of either common equity or subordinated debt instruments. Issuing CoCos, which is the most popular subordinated debt instrument for banks to use to remain compliant with AT1 requirements, is often preferred to holding common equity, because CoCos can achieve the goal of reducing default risk more cheaply (Thompson, 2014). Thus, we have shown that the impact of CoCos is derived from the regulatory environment for banks, which encourages its use.

Due to the fact that Basel III requires AT1 instruments to be perpetual, CoCos are typically issued without maturity dates (Avdjiev, Kartasheva, and Bogdanova, 2013). In our model however, we are assuming that new CoCos are issued once old CoCos mature. The reason for making this assumption, rather than assuming that CoCos are perpetual is for modeling purposes; by assuming that straight debt and CoCos are rolled over upon maturity, we are able to find closed-form solutions. More details on assumptions and the formulas are found in Chapter 4. Moreover, this debt environment bears some resemblance to existing revolving credit agreements and is consistent with that of Leland (1994), Chen and Kou (2009) and Chen et al. (2017).

## 3. Credit Risk Modeling

The purpose of this chapter is to justify the selection of our credit risk model by providing an overview of relevant credit risk models, since we use one to calculate the optimal default barrier. We do this by developing our model from its simplest version in Section 3.1, followed by a discussion about the relationship between endogenous default and the optimal default barrier in Section 3.2.

### 3.1 Model Background and Development

One could make the broad distinction that credit risk can be modeled using one of two approaches: the reduced-form approach or the structural approach. The reduced-form approach of modeling credit risk aims to provide a simple framework to fit a variety of credit spreads by abstracting from the firm-value process and postulating default as a single jump time; for examples of the reduced-form approach see Jarrow and Turnbull (1995); Jarrow, Lando, and Turnbull (1997); Duffie and Singleton (1999); Collin-Dufresne, Goldstein, and Hugonnier (2004); Madan and Unal (1998). The structural approach aims to provide an intuitive understanding of credit risk by specifying a firm value process and modeling equity and bonds as contingent claims on the firm value. The structural models that are most frequently applied are the models by Black and Scholes (1973) and Merton (1974). The popularity and widespread application of the classic Black-Scholes model stems partly from their analytical framework and ease of use in practice. The main criticism towards Black-Scholes has been directed towards its inconsistencies with empirical data; in particular, two empirical phenomena that the model does not account for. The first issue is concerned with the leftward skew of the return distribution in the Black-Scholes model and the fact that it has a higher peak and two heavier tails than those observed in normal distributions. The second problem is directed towards the constant volatility assumption, whereas in practice, implied volatility is a convex curve of the strike price, thus forming a "volatility smile" (Kou, 2002).

For the goals of this thesis, we use a type of first-passage time model, which is a class of models that belong to the structural credit risk model family. Hence, we can define default as the first time the firm value falls below an optimal default barrier level. Moreover, the model we use is an extension of the basic Merton model, which assumes that the assets of a firm follows a geometric Brownian motion (GBM), according to:

$$dV_t = \mu V_t dt + \sigma V_t dW_t,$$

where  $V_t$  is the asset price,  $\mu$  is the drift,  $\sigma$  is the volatility and  $W(t)$  is a Wiener Process. GBM indicates that it is a continuous time stochastic process. Merton's model can be modified to include jumps by simply including a 'jump term'. In fact, general jump diffusion processes are processes of the form:

$$X_t = \sigma W_t + \mu t + \sum_{i=1}^{N(t)} Y_i.$$

Note the addition of the jump term to the right. We use a double exponential jump diffusion model developed by Kou (2002), which expresses the asset price  $V(t)$ , under the physical probability measure  $P$  as:

$$\frac{dV(t)}{V(t-)} = \mu dt + \sigma dW(t) + d\left(\sum_{i=1}^{N(t)} (\bar{Y}_i - 1)\right),$$

where  $N(t)$  is a Poisson process with jump intensity  $\lambda$ , and  $Y_i$  is a sequence of independent and identically distributed (i.i.d.) non-negative random variables such that  $Y_i := \ln(\bar{Y}_i)$  has an asymmetric double exponential distribution, with density:

$$f_y(y) = p\eta_1 e^{-\eta_1 y} \cdot 1_{y \geq 0} + q\eta_2 e^{\eta_2 y} \cdot 1_{y < 0},$$

$$\eta_1 > 1, \eta_2 > 0,$$

where  $p$  and  $q$  represent the respective probabilities for upward and downward jumps and  $p + q = 1$ . We also have  $\eta_1$  and  $\eta_2$  which represents the jump sizes in the model. Since studying the point of default is the focus of this study, the jump probabilities are edited in such a way that  $p = 0$  and  $q = 1$ , so that only downward jumps are considered. Since  $p$  is multiplied by  $\eta_1$ , that term will always be equal to 0, which makes  $\eta_2$  the only relevant part of our model; whenever we refer to  $\eta$  without a subscript henceforth, it is  $\eta_2$  that we refer to. If the possibility of upward jumps is not precluded, our results would not change, but they would be weaker, as it would take longer to observe defaults. The justification for choosing the model by Kou (2002) is as follows: Firstly, the model produces results that are consistent

with the results of the Black-Scholes model and fits stock data better than some other models, e.g. the normal jump-diffusion model. Secondly, Kou's model offers an explanation for the two empirical phenomena that the Black-Scholes model has been unable to explain, mentioned previously in this section. Thirdly, the model is simple enough for computation and offers a closed-form solution. Finally, the model has a psychological interpretation of real world behavior (Kou, 2002).

## 3.2 Optimal Default Barrier

The point of default for any company can either be set exogenously or determined endogenously. Exogenous default refers to when the point of bankruptcy is externally set to be linked to some critical event, such as failure to meet interest payments when due or upon reaching a certain debt principal value. In this thesis, along with Chen et al. (2017) and Leland and Toft (1996), we are assuming that default is instead endogenously determined, which is when the point of bankruptcy is optimally determined by equity owners. This idea transforms declaring bankruptcy from being an event-linked consequence into an optimal decision made by equity holders to surrender company control to bond holders. Since bankruptcy is assumed to be endogenously determined, a point of optimal default must be specified as part of our model. Furthermore, the point is essentially a barrier that tells the shareholders of a company to declare bankruptcy if the firm value hits or falls below the barrier, hence the term *optimal default barrier*. Examining the relationship between CoCos and the optimal default barrier is the first main objective in this thesis.



## 4. Model Implications of CoCo Bonds

Introducing CoCos has several implications on modeling, which is discussed in this chapter. Section 4.1 begins with an overview of the modeling assumption that we make for valuing assets and liabilities, respectively. Section 4.2 follows up with a discussion about the idea of ‘debt-induced collapse’, which is a unique situation where CoCos degenerate to straight debt before conversion. Lastly, Section 4.3 dissects the model by Chen et al. (2017), which we use to model the optimal default barrier.

### 4.1 Valuation of Assets and Liabilities

Chen et al. (2017) assumes that the firm finances its assets using straight debt, contingent convertible debt and equity. In the subsequent subsections, we will briefly discuss how these are modeled and the underlying assumptions made. These models give closed-form expressions, which is important for calculation and purposes of analysis. It is thanks to closed-form expressions that we later can derive the optimal default barrier (Chen et al., 2017).

#### 4.1.1 Valuation of Assets

The valuation of assets in our model builds on the background discussion in Chapter 3. We use the model of Kou (2002):

$$\frac{dV(t)}{V(t-)} = \mu dt + \sigma dW(t) + d\left(\sum_{i=1}^{N(t)} (\bar{Y}_i - 1)\right), \quad (4.1)$$

where we define  $\mu$ , following the syntax of Chen et al. (2017), as  $\mu = r - \delta + \frac{\lambda}{1+\eta}$ . Substituting  $\mu$  in Equation (4.1) gives us:

$$\frac{dV(t)}{V(t-)} = \left(r - \delta + \frac{\lambda}{1+\eta}\right) dt + \sigma dW_t + d\left(\sum_{i=1}^{N_t} (Y_i - 1)\right), \quad (4.2)$$

where  $\delta$  is the payout rate of the company,  $\lambda$  is the jump intensity,  $r$  is the risk-free interest rate and  $\eta$  represents the sizes of the downward jumps.

### 4.1.2 Straight Debt

Straight debt is modeled following Leland et al. (1996) where they assume that a firm issue new debt continuously at par value  $p_1$ . The maturity  $m$  of this debt is exponentially distributed with a mean of  $1/m$  and it pays a coupon  $c_1$  per unit of  $p_1$ . According to Chen et al. (2017) this continuous issue rate and maturity profile gives the following equation for calculating total par value of debt  $P_1$ :

$$P_1 = \int_t^\infty \left( \int_{-\infty}^t p_1 m e^{-m(s-y)} du \right) ds = \frac{p_1}{m}. \quad (4.3)$$

This so called debt rollover where the debt is settled and reissued at a continuous rate lays an important foundation for the model and further analysis regarding incentives for equity holders. Another factor that affects the valuation is the funding benefit  $k_1$ , which is defined in the interval  $(0,1)$ , that arises depending on where the fundings come from. One example is that the cost of debt can decrease when issuing new debt since coupon payments are tax deductible. Another funding benefit could be if the banks have a government that insures some of the deposits, this would give investors a safety net that they are willing to pay extra for (Chen et al., 2017). There are different discussions on how to model this effect. DeAngelo and Stulz (2013) and Sundaresan and Wang (2015) models it as an increase in liquidity which lowers the net cost of deposits while Allen (2015) models funding benefit as a product from market segmentation. In this thesis we follow Chen et al. (2017) where they introduce  $k_1$  such that net cost of coupon payments is  $(1 - k_1)c_1P_1$ .

### 4.1.3 Contingent Convertibles

One of the main differences in the model between Chen et al. (2017) and Chen et al. (2009) is the addition of Cocos. Modeling CoCos follows the similar arguments and structure as the modeling of straight debt in Equation 4.3. The total par value of CoCos will be denoted  $P_2$  instead and pay a continuous coupon rate of  $c_2$ , with a mean maturity of  $1/m$ . As in the case of straight debt there also exist a funding benefit  $k_2$  from issuing CoCos.

#### 4.1.4 Liability Valuation

Equity value is expressed as:

$$\text{Equity } (E) = \text{Firm value } (F) - \text{Straight debt } (P_1) - \text{CoCos}(P_2). \quad (4.4)$$

Depending on whether the goal is to value, post-conversion (PC), before conversion BC or when there are no conversion (NC) we will get different values. When considering value of the firm post-conversion ( $V^{PC}$ ) the equity value will be the residual between firm value and straight debt ( $P_1$ ) since the CoCos would have been converted. If we are considering  $V^{BC}$ ,  $E^{BC}$  will follow Equation (4.4) and if we do not have conversion, considering  $V^{NC}$ ,  $E^{NC}$  will denote the equity when the firm do not convert the CoCos. We refer the interested reader to Chen et al. (2017) for a deeper discussion about these calculations.

## 4.2 Debt-Induced Collapse

While CoCos are designed with the intent to help banks absorb losses, they may fail to do so if the bank undergoes a crisis so severe, that wiping CoCos would still be insufficient to save the bank from bankruptcy. Banks with CoCos could also go bankrupt if equity owners decide to declare bankruptcy before conversion, which is a phenomenon that Chen et al. (2017) call "debt-induced collapse". The term coined by Chen et al. (2017) refers to the increase in a firm's debt load when bankruptcy is declared before CoCo conversion, which causes a sharp increase in the firm's probability of default and a decline in equity value. The sharp increase in debt burden in turn, causes the firm to collapse. The idea of debt-induced collapse is important to understand for this thesis because it introduces the third state that was not considered in Chen et al. (2009). The state where no CoCo conversion occurs, as opposed to if we would only consider the firm values before and after CoCo conversion.

Debt-induced collapse is a consequence of bankruptcy being endogenously determined, since the optimal default barrier might be above the conversion point of the CoCo. In contrast, if default is exogenously determined, the point of CoCo conversion would typically be set to precede bankruptcy. The hazard of debt-induced collapse can be avoided by setting the trigger for CoCo conversion at a sufficiently high level above the optimal default barrier. So, debt-induced collapse occurs if the optimal default barrier,  $V_b^*$ , falls below the conversion threshold,  $V_c$ , which is the

value of  $V$  that triggers the conversion of CoCos. Moreover, the possibility for debt-induced collapse introduces the three cases that must be considered at all times, when valuing the firm:

1. Value of firm before conversion ( $V^{BC}$ )
2. Value of firm post conversion ( $V^{PC}$ )
3. Value of firm with no conversion ( $V^{NC}$ )

The firm value is linked to the value of debt and value of equity at each time point; which is why the same three cases will also have to be considered for these. Endogenous default implies that equity holders choose a bankruptcy policy that maximizes the value of equity. This leads to that the optimal default barrier will vary based on which of the three cases that yields the highest equity value for shareholders, since the default barrier is a function of the firm value. At the same time, the policy must be consistent with limited liability, meaning that equity value does not fall below zero at any point. This is a standard formulation that is in line with Leland (1994), Leland et al. (1996) and Chen et al. (2017).

Upon default, a fraction of a bank's asset value is assumed to be lost to bankruptcy and liquidation costs, represented by  $(1 - \alpha)$ , where  $0 \leq \alpha \leq 1$ . The time when bankruptcy is declared is denoted  $\tau_b$  and  $V_{\tau_b}$  the value of the firm at that moment. Firm value after bankruptcy costs are used to repay the creditors and are denoted  $\alpha V_{\tau_b}$ . In the normal case where default occurs after conversion, only straight debt remains at bankruptcy. On the other hand if debt-induced collapse occurs and default is declared before conversion is triggered, the CoCos degenerate to junior debt and are repaid from any assets that remain after the senior debt is repaid.

### 4.3 Modeling the Optimal Default Barrier

We develop our analysis in a structural model of the type introduced in Leland (1994) and Leland et al. (1996), as extended by Chen et al. (2009) to include jumps. Building on the idea of endogenously triggered default, Chen et al. (2017) proceed to derive the optimal default barrier, denoted as  $V_b^*$ , which is the optimal point for equity holders to declare bankruptcy. Meanwhile, arbitrary default barriers are simply denoted  $V_b$ . According to Chen et al. (2009), the optimal default barrier when issuing only straight debt can be expressed as:

$$V_b^{PC} = P_1 \epsilon_1, \quad (4.5)$$

where  $\epsilon$  works as a weighting factor that maximizes the barrier (Chen et al., 2009) and is defined as:

$$\epsilon_1 = \frac{\frac{c_1+m}{r+m}\gamma_{1,r+m}\gamma_{2,r+m} - \frac{k_1c_1}{r}\gamma_{1,r}\gamma_{2,r}}{(1-\alpha)(\gamma_{1,r}+1)(\gamma_{2,r}+1) + \alpha(\gamma_{1,r+m}+1)(\gamma_{2,r+m}+1)} \frac{\eta+1}{\eta}, \quad (4.6)$$

where  $-\gamma_{1,\beta} > -\eta > -\gamma_{2,\beta}$  are the two negative roots of the following equation:

$$G(x) = \left( r - \delta - \frac{1}{2}\sigma^2 - \lambda\left(\frac{\eta}{\eta+1} - 1\right) \right)x + \frac{1}{2}\sigma^2x^2 + \lambda\left(\frac{\eta}{\eta+x} - 1\right) = \beta. \quad (4.7)$$

$V_b^{PC}$  expressed above represents the post-conversion (PC) firm value, and show that a firm only holds straight debt after CoCos have been converted. We list the method that we use to find the roots to Equation (4.7) Appendix B along with a second method proposed by Kou (2005). For a more elaborate discussion on Equation (4.7) and its derivation, see Appendix A. The variable notation used here is generally consistent with what is used in Chen et al. (2017). The only difference in notation from what Chen et al. (2017) uses is what they call  $\rho$  is instead called  $\beta$  here since  $\rho$  was also used to represent minimum capital ratio, which it represents in this thesis as well. There are also some discrepancies between Equation (4.7) in Chen et al. (2017) and Chen et al. (2009). They state that they use the same formula, but present them slightly differently which we show and discuss further in Appendix A. Chen et al. (2017) extend the models of Chen et al. (2009) and show that the optimal default barrier for a firm with no conversion can be expressed as:

$$V_b^{NC} = P_1\epsilon_1 + P_2\epsilon_2,$$

where  $P_2$  represents the par value of CoCos outstanding and  $\epsilon_2$  quite similar to  $\epsilon_1$  can be expressed as:

$$\epsilon_2 = \frac{\frac{c_2+m}{r+m}\gamma_{1,r+m}\gamma_{2,r+m} - \frac{k_2c_2}{r}\gamma_{1,r}\gamma_{2,r}}{(1-\alpha)(\gamma_{1,r}+1)(\gamma_{2,r}+1) + \alpha(\gamma_{1,r+m}+1)(\gamma_{2,r+m}+1)} \frac{\eta+1}{\eta}. \quad (4.8)$$

## 5. Model Robustness and Sensitivity Analysis

In this chapter, we first attempt to replicate parts of the study by Chen et al. (2017), using an altered version of their original model. Section 5.1 contains the results of our comparison and our comments on noteworthy differences and similarities. Later, in Section 5.2, we examine the sensitivity of the input parameters that are used by testing for small changes in volatility  $\sigma$ , the risk-free rate  $r$ , payout rate  $\delta$ , jump intensity  $\lambda$ , the firm-specific jump component  $\eta$ , maturity  $m$  and recovery rate  $\alpha$ . After assessing the reliability of our model compared to that of Chen et al. (2017), this chapter provides an extensive sensitivity analysis of the optimal default barrier.

Chen et al. (2017) provide a table containing most of the input parameters they use for their calculations on page 3935 in their paper. The same input parameters have been listed here in Table 5.1, and these are the same input parameters that we have used in our models unless otherwise stated.

TABLE 5.1: Base-case input parameters (Chen et al., 2017).

Parameter		Value
Initial asset value	$V_0$	100
Risk-free rate	$r$	6%
Volatility	$\sigma$	8%
Payout rate	$\delta$	1%
Funding benefit	$k_1, k_2$	35%
Jump intensity	$\lambda_f$	0.3
Firm-specific jump exponent	$\eta$	4
Coupon rates	$(c_1, c_2)$	(r+3%, r+3%)
Bankruptcy loss	$(1 - \alpha)$	50%

In addition to the values provided in Table 5.1, we also assign values to  $P_1$ ,  $P_2$ ,  $m$ ,  $\rho$ ,  $p$  and  $q$  in our model when we stress-test individual parameters. As previously mentioned, we use  $p = 0$  and  $q = 1$ , because we are only interested in downwards jumps. Furthermore, we have opted to use  $\rho = 0.05$ ,  $P_2 = 5$  and  $P_1 = 65$ , which is

what Chen et al. (2017) typically have used in their tests, see pages 3940-3944 in Chen et al. (2017). In Section 5.2 we set maturity to a constant of  $m = 1$  in all tests except when we test for maturity in Figure 5.9.

## 5.1 Model Comparisons with Chen et al. (2017)

In this section we discuss our attempts to replicate some of the results by Chen et al. (2017). Attempting to replicate their models serves two purposes: 1. it helps us to verify that our model works and 2. it tells us if our results are consistent with their conclusions.

Chen et al. (2017) do not explicitly list all their parameters used to produce every graphical result. Therefore, it was necessary to make some estimates here, which consequently produces results that differ somewhat from that of Chen et al. (2017). Slight differences, such as shifts in intercepts, maximum and minimum points are expected. Rather, the focus is on the general trends, whether the lines develop in the same directions and have similar curvature. In general, all our results were consistent with that of Chen et al. (2017), in the sense that our replicated versions developed in similar ways and had similar shapes as theirs. However, one difference that persisted in all our graphs was that our trends typically lagged behind their trends. For example, when we tried to replicate Figure 4 in Chen et al. (2017), which is shown in Figure 5.1 here, we came up with Figure 5.2. While the figures imply that the general trend is the same in both models, they display differences in curvature. There are several possible explanations for this. The first is that our input parameters differ, which is due to the fact that we have made estimates where Chen et al. (2017) did not provide values. A second plausible explanation is that the underlying models differ; as mentioned in Section 4.3 and Appendix A. The inconsistency between these formulas could imply that we are not using the same underlying model as Chen et al. (2017).

Since debt-induced collapse arise if  $V_b^{PC} > V_c$ , Chen et al. (2017) reasons that the condition  $(\epsilon_1(1 - \rho) - 1)P_1 > P_2$  must hold, where  $\epsilon_1$  follows Equation 4.6. Using this condition, we evaluate values using average maturity on the  $x$ -axis, as can be seen in Figure 5.2. For the full Matlab code to Figure 5.2, see Appendix D.

## 5.2 Sensitivity Tests and Limitations

In their paper, Chen et al. (2017) do not discuss the sensitivity of their model parameters at length. But since understanding the sensitivity of model parameters

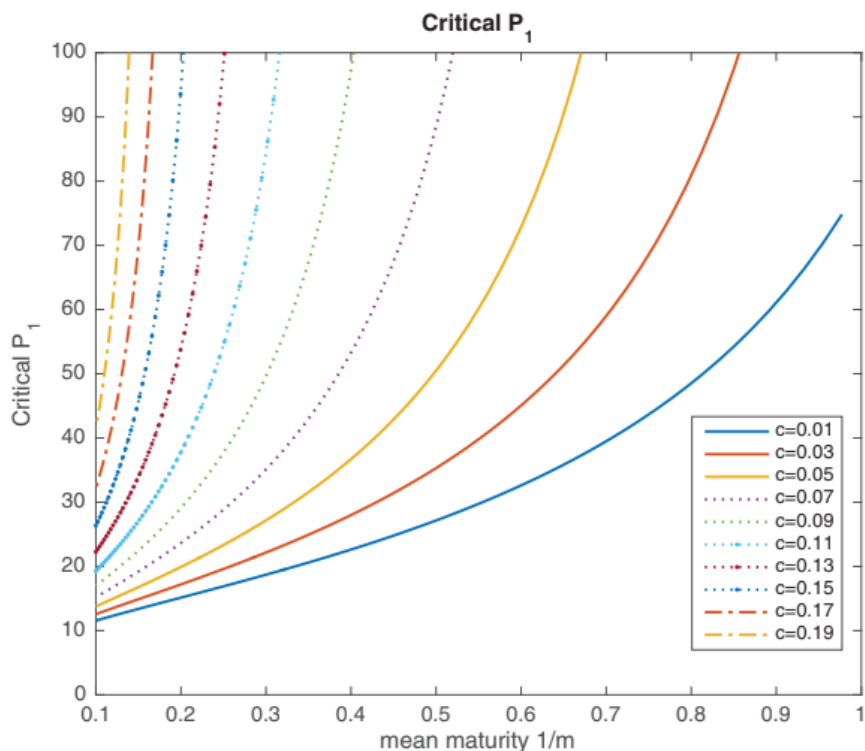


FIGURE 5.1: Figure 4 from page 3940 from Chen et al. (2017). Shows the critical values of straight debt  $P_1$ , over various bond mean maturity rates, ranging from 0.1 to 1.

is important for interpreting the results of any model, we have chosen to dedicate this section to discussing how the optimal default barrier is impacted by changes in the input variables. On the subsequent pages that follow, Figures 5.3 to 5.9 illustrate the relationship between the most important input variables and the optimal default barrier  $V^{NC}$ . In every one of these figures, we have included three different cases, each case representing a different par value for CoCo bonds  $P_2$ , while holding the amount of straight debt constant. Then, by changing the independent variable, we can observe how a specific input parameter and the use of CoCos impact the optimal default barrier.

### 5.2.1 Risk-Free Rate

In Figure 5.3 we can see the relationship between the risk-free rate and the optimal default barrier. The figure shows that the risk-free rate has a non-linear negative correlation with the optimal default barrier, where increases in  $r$  pushes the barrier down. In other words, higher interest rates reduces the optimal default barrier and the likelihood of default. One economic explanation for this relationship is that an



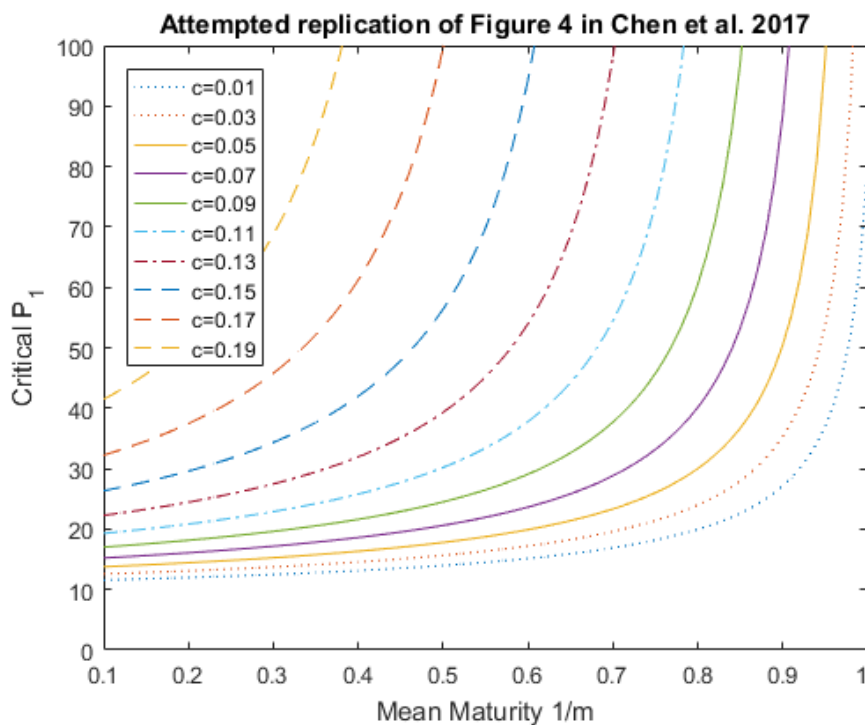


FIGURE 5.2: Attempted replication of figure 4 in Chen et al. (2017). Shows the critical values of straight debt  $P_1$ , over various bond mean maturity rates, ranging from 0.1 to 1.

increase in the risk-free rate reduces the value of outstanding debt, thus lowering the ratio of debt to equity at market value, which in turn reduces the likelihood of default. This economic result is consistent with the results that Leland (1994) has found for financially stable companies. Another explanation for this relationship is that an increase in the risk-free rate would increase interest payments, which would increase the tax shield. A bigger tax shield would contribute towards reducing the default risk of the company. However, the positive effects would be marginal, since the benefits of an increased tax shield are weighed against a higher cost of debt. Furthermore, Figure 5.3 shows that the impact that the risk-free rate has on the optimal default barrier is diminishing in nature. A possible explanation for this tapering effect is that the cost of debt becomes more pronounced the higher the interest rate grows, which counterbalances the ratio of debt to equity by reducing the equity value. However, Leland (1994) also point out that "junk-bonds" exhibit a special relationship that is different from "normal" bonds, where instead increases in the risk-free rate lead to increases in debt value. Furthermore, the relationship between  $r$  and the optimal default barrier appears indifferent to the par value of CoCos used, which can be inferred from the practically constant distance between the curves across the entire  $x$ -axis. Another interesting observation is that issuing

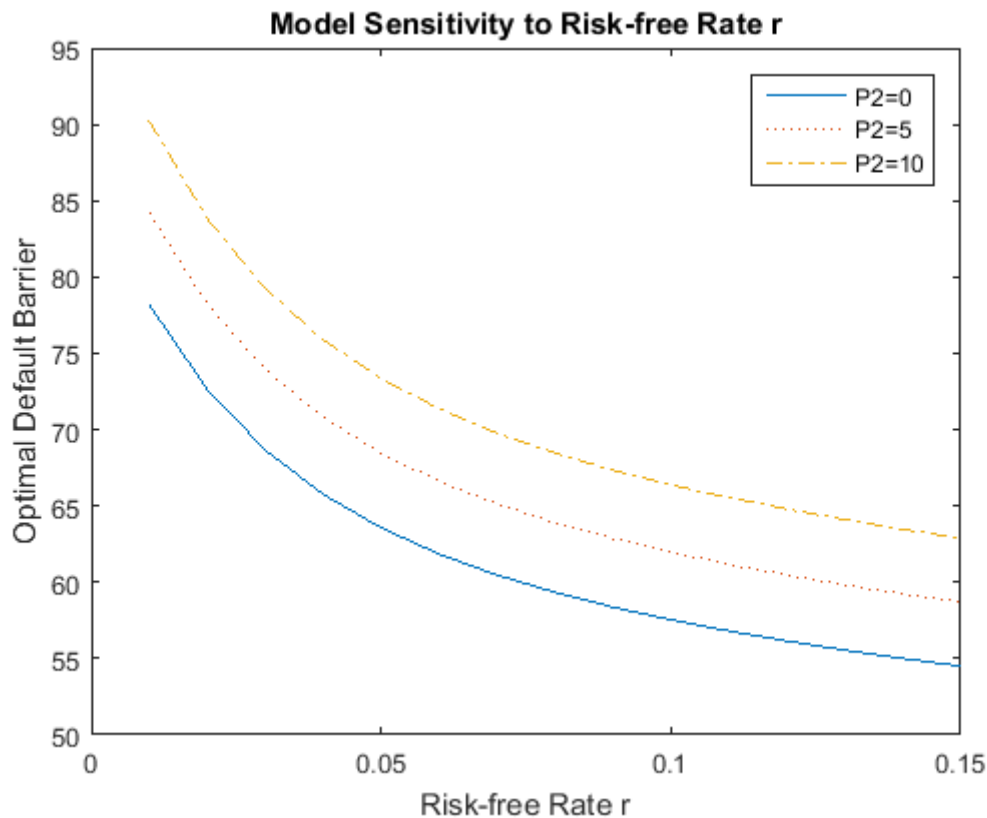


FIGURE 5.3: Shows the impact of risk-free rate  $r$  on the optimal default barrier. Created using an interval ranging from 0.01 to 0.15 for  $r$ , while holding all other variables constant as given in table 5.1. The different lines represent CoCos with different par values.

CoCos has an adverse impact on the optimal default barrier, since more CoCos result in a higher optimal default barrier. A possible explanation for this is that CoCos burden the company in their day-to-day operations, which makes the likelihood of default rise. But this may be a premature conclusion, since we added CoCos without withdrawing any straight debt in exchange for this increased debt load. At the very least however, this suggests that reducing the likelihood of default should not be the primary reason for issuing CoCos.

## 5.2.2 Payout Rate

The payout rate is calculated as the fraction of a company's total assets that goes towards paying all dividends and interest coupons, and is expressed as a weighted percentage of outstanding debt and equity (Leland et al., 1996). We assume that Chen et al. (2017) uses the same definition as Leland et al. (1996), since they claim they are, although their description of the payout rate is very simple; Chen et al. (2017) describe the payout rate as "total dividends and interest payments". If a

company has a high  $\delta$ , it implies that they are spending a greater portion of its funds to finance interest and dividend payments than a company with a low  $\delta$ . Figure 5.4 shows that the correlation between the payout rate,  $\delta$  and the optimal default barrier is strictly positive for the entire range. One explanation for this is that the bigger the proportion of funds that are directed towards paying creditors and shareholders, the less will the company have left to finance its other needs, thus increasing the probability of default, which subsequently raises the optimal default barrier. If we then compare the scenarios with different values for  $P_2$ , we see that CoCos adversely impact the optimal default barrier for the tested range, similar to the case of the risk-free rate. Similar to the risk-free also, is that the par value of CoCos does not have a noteworthy impact on the relationship between the dependent and independent variable in the given range.

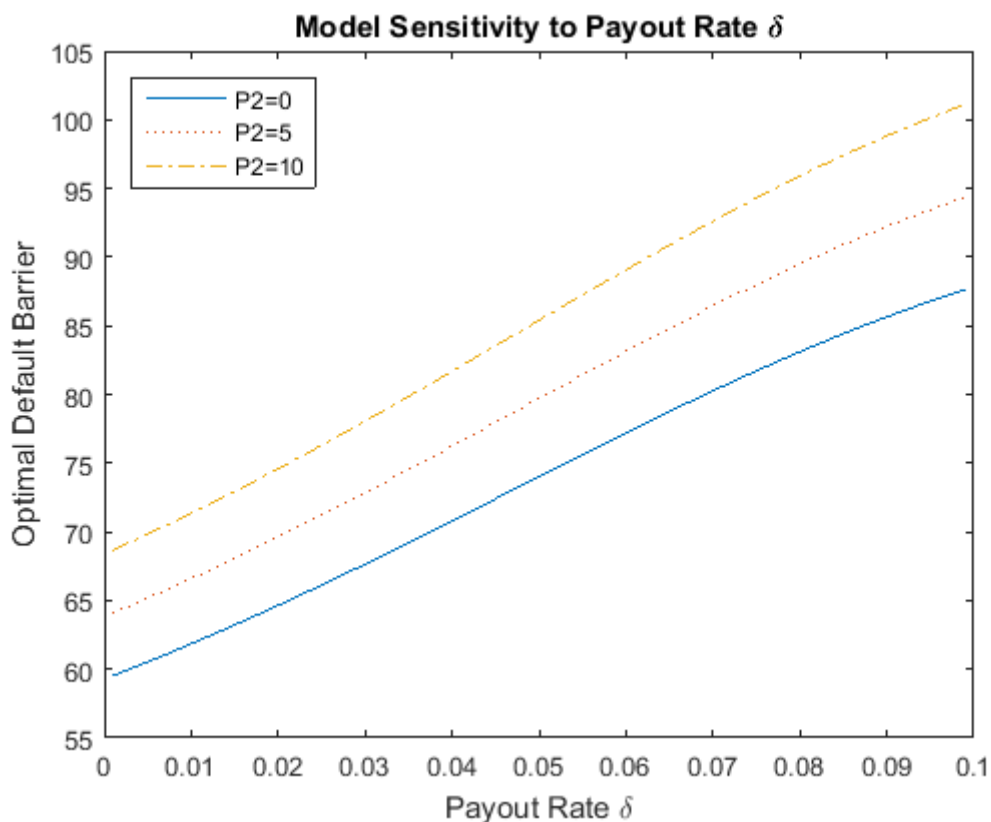


FIGURE 5.4: Shows the impact of payout rate  $\delta$  on the optimal default barrier. Created using an interval ranging from 0.01 to 0.1 for  $\delta$ , while holding all other variables constant as given in table 5.1.

The different lines represent CoCos with different par values.

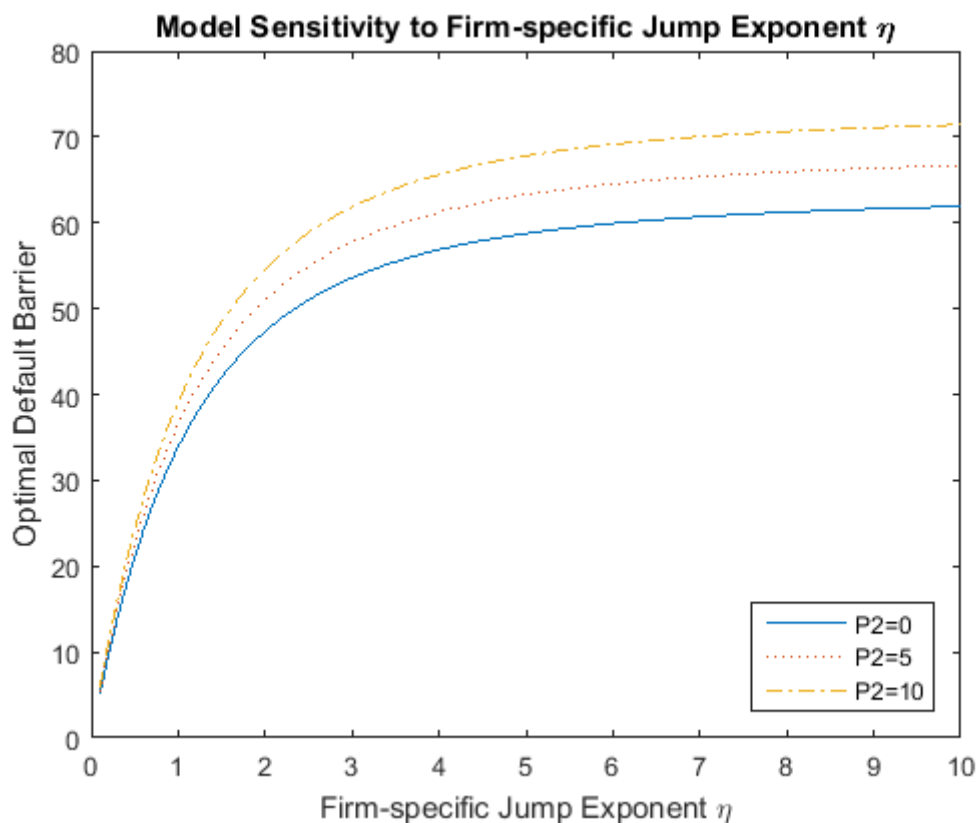


FIGURE 5.5: Shows the impact of jump intensity  $\eta$  on the optimal default barrier. Created using an interval ranging from 0.01 to 10 for  $\eta$ , while holding all other variables constant as given in table 5.1. The different lines represent CoCos with different par values.

### 5.2.3 Firm-Specific Jump Exponent

Figure 5.5 shows the optimal default barrier plotted against changes in  $\eta$ .  $\eta$  denotes the magnitude of jumps in the jump process. Since we only consider negative jumps, a higher  $\eta$  represents a stronger negative jump. In Figure 5.5, we can see that  $\eta$  is positively correlated with the value of the optimal default barrier, where higher values of  $\eta$  results in a preference for declaring bankruptcy earlier and at higher firm values. Moreover, Figure 5.5 also shows that the relationship is non-linear and that the impact of additional units of  $\eta$  have marginally less impact on the optimal default barrier as  $\eta$  grows. This relationship has a solid economic explanation, which is that sharp negative jumps are expected to be more crippling to a firm's livelihood than small jumps. Hence, default is more imminent, the bigger the jumps are. The flattening of the curve can also be plausibly explained. Since the impact of jumps of high orders, such as  $\eta = 7$  or  $\eta = 9$  is already so devastating to the firm, the difference between them become marginalized. In contrast, the difference between

going from a scenario of no jumps,  $\eta = 0$ , to jumps,  $\eta = 2$ , drastically constrains a firm's ability to operate, leading to a sharp increase in the optimal default barrier. In Figure 5.5, we can also see that issuing CoCos has an increasing impact on the optimal default barrier, but that the gravity of this impact depends on the size of  $\eta$ . While issuing CoCos has a sizable impact on the optimal default barrier for high values of  $\eta$ , the impact is practically negligible for small values of  $\eta$ , although it still increases the barrier. Rather than explaining this phenomenon through  $\eta$  however, a more plausible explanation would be that the impact of CoCos depends on the solvency of the firm at the time of issuance. On the one hand, if the firm is highly solvent, the added debt burden of issuing CoCos is not very significant. On the other hand, if the firm is already close to insolvency, any added debt burden from CoCos will be alarming as the company might collapse under the debt.

### 5.2.4 Jump Intensity

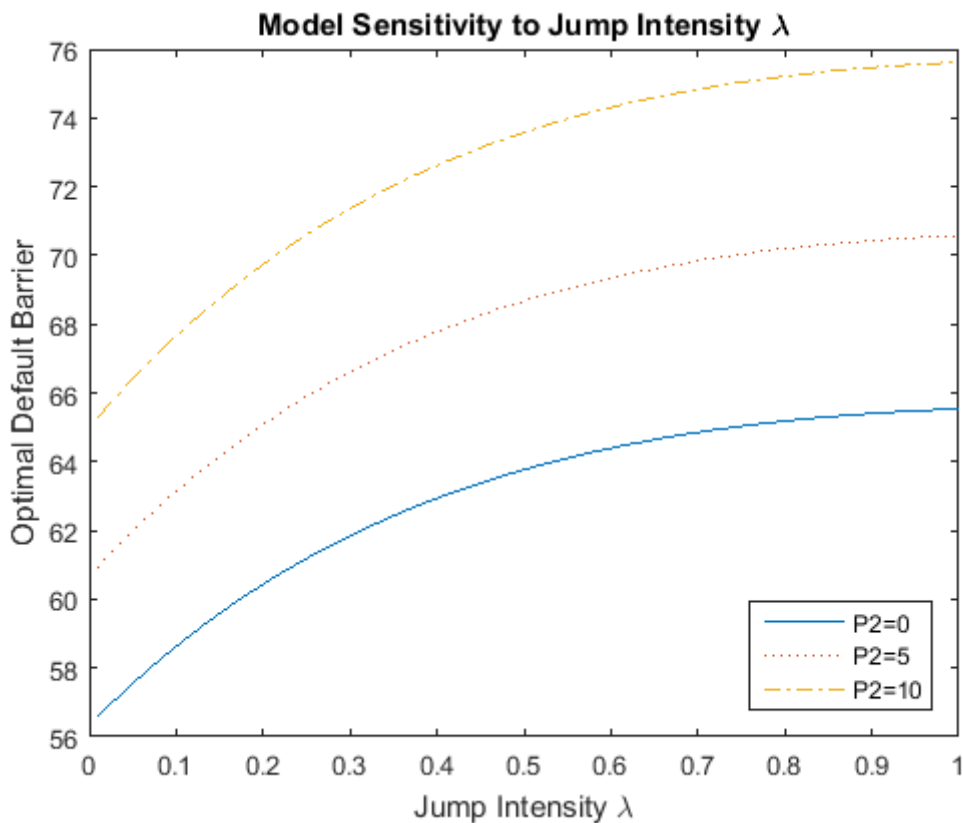


FIGURE 5.6: Shows the impact of jump intensity  $\lambda$  on the optimal default barrier. Created using an interval ranging from 0.01 to 1 for  $\lambda$ , while holding all other variables constant as given in table 5.1.

The different lines represent CoCos with different par values.

Figure 5.6 illustrates the relationship between the jump intensity  $\lambda$  and the optimal default barrier. The jump intensity is the frequency of jumps within a time interval, in this case a year. Thus, a smaller  $\lambda$  signifies fewer jumps per time interval compared to a bigger  $\lambda$ . As can be seen from Figure 5.6, higher  $\lambda$  correlates with a higher optimal default barrier, implying that more frequent jumps lead to a higher optimal default barrier and a more imminent threat of bankruptcy. One possible explanation for this is that more jumps increase the uncertainty and the likelihood for a fatal mishap that bankrupts the company, thus increasing the optimal default barrier for the company. Furthermore, Chen et al. (2009) find a negative correlation between the optimal debt to equity ratio and  $\lambda$ , which would support our findings here, since lowering the debt to equity ratio means to wind down the leverage and is a prudent response to dealing with a more risky environment with a greater likelihood for default. Figure 5.6 further shows that issuing CoCos increases the optimal default barrier and that its potency seems to be linked to the value of the optimal default barrier; if the barrier is low, issuing CoCos have a lesser impact, if the barrier is high, issuing CoCos has a more significant impact. Again, this is consistent with what was observed on the previous figures in this section.

### 5.2.5 Volatility

Figure 5.7 shows how the optimal default barrier changes with changes in volatility  $\sigma$ . From the figure, we can see that  $\sigma$  initially exhibits a positive correlation with the optimal default barrier, but it peaks at around  $\sigma = 0.23$ , after which it turns into a relationship with negative correlation. We could explain the first part of the figure by using a similar argument as when we explained the relationship between  $\lambda$  and the optimal default barrier, namely that increasing volatility is linked to more uncertainty and a higher likelihood of default, thus pushing up the optimal default barrier. Chen et al. (2009) find that increasing the volatility leads to lower optimal debt to equity ratios, i.e. a lower optimal leverage. Lowering the optimal leverage implies that the current the financial situation of the company has worsened, which is consistent with a higher optimal default barrier. However, this only helps explain the part before the peak and not the downward sloping relationship after the peak. Here we concede that there is not always a realistic explanation, simply due to model design; just like how the Black-Scholes formula lacks a sound explanation for the "volatility smile" (Kou, 2002). The impact of issuing CoCos on the other hand exhibits the same negative impact on the optimal default barrier as before. In other words, CoCos increase the optimal default barrier, more if the barrier is high and less if the barrier is low, which could be explained using the same line of reasoning offered for the other parameters; namely that issuing debt when company solvency

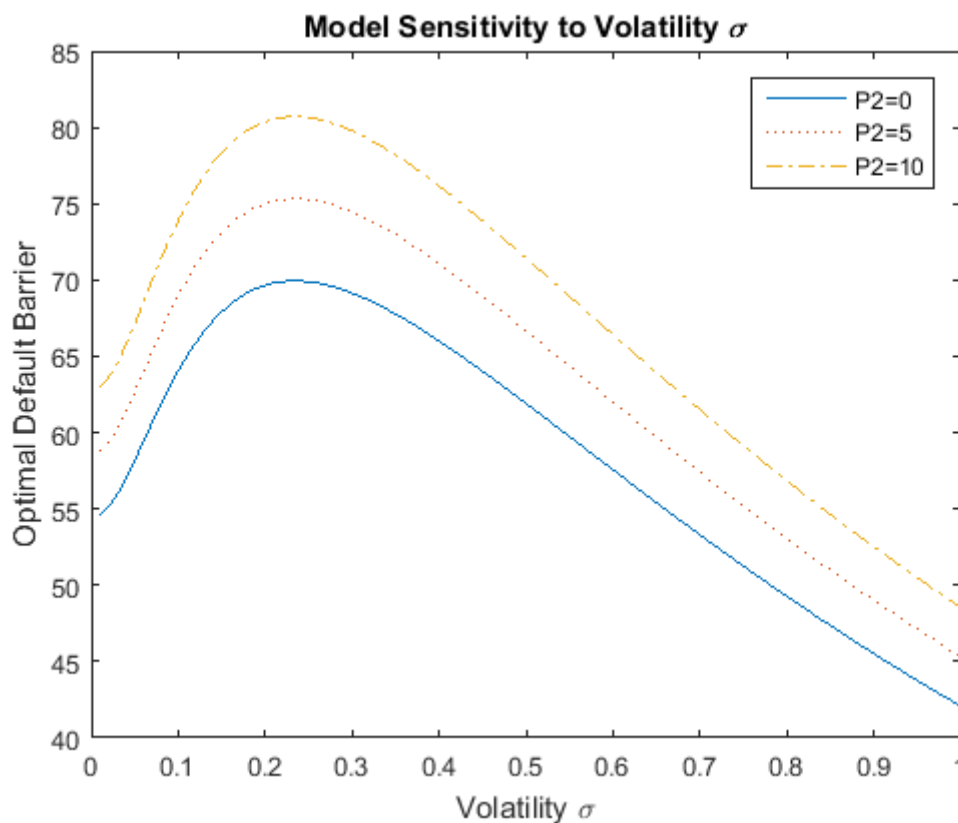


FIGURE 5.7: Shows the impact of volatility  $\sigma$  on the optimal default barrier. Created using an interval ranging from 0.01 to 1 for  $\sigma$ , while holding all other variables constant as given in table 5.1. The different lines represent CoCos with different par values.

is poor has a more severe negative impact on the firm than when company solvency is sound.

### 5.2.6 Recovery Rate, $\alpha$

In the event of bankruptcy,  $\alpha$  represents the recovery rate, which is the portion of a firm's assets that is left after subtracting bankruptcy and liquidation costs. Conversely,  $(1 - \alpha)$  represents the bankruptcy and liquidation costs. Figure 5.8 shows how changes in  $\alpha$  affect the optimal default barrier, everything else unchanged. Since  $\alpha$  is found in the range  $0 \leq \alpha \leq 1$ , the figure tells us that the optimal default barrier always increases with increases in  $\alpha$ . The positive correlation between the recovery rate and the optimal default barrier can be explained by investors having a greater risk appetite when more is recovered upon default. The higher the  $\alpha$ , the better off are creditors and shareholders in the event of a default, which incentivizes shareholders to liquidate the company earlier. Their comfortability with the idea of default suggests that a greater leverage is more optimal, which consequently raises

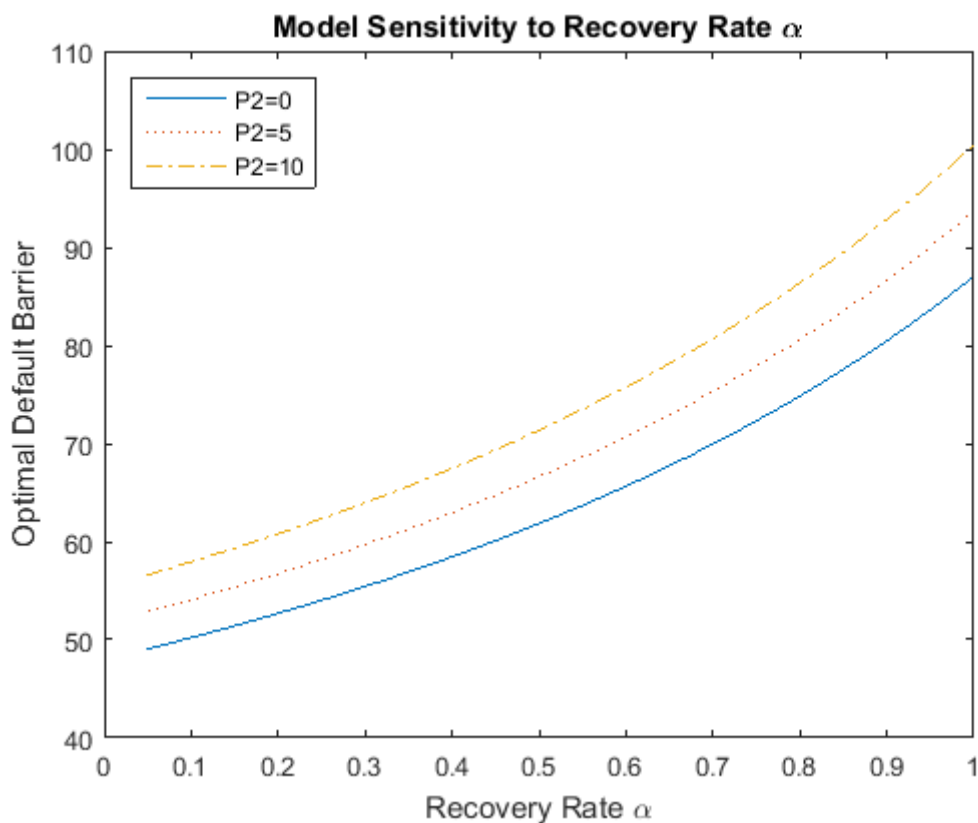


FIGURE 5.8: Shows the impact of recovery rate  $\alpha$  on the optimal default barrier. Created using an interval ranging from 0.01 to 1 for  $\alpha$ , while holding all other variables constant as given in table 5.1. The different lines represent CoCos with different par values.

the stakes and the optimal default barrier. In fact,  $\alpha$  the optimal default barrier increases at an exponential rate with increases in  $\alpha$ , which can be seen from the convex shape of the curve in Figure 5.8. This result is also in line with the results of Chen et al. (2009), who find that the optimal debt to equity ratio is an increasing function of  $\alpha$ , which signals that higher  $\alpha$  incentives a higher leverage and risk. Furthermore, issuing CoCos has an impact on the optimal default barrier, which is consistent with previous results in this section.

### 5.2.7 Maturity, $m$

Figure 5.9 plots the maturity against the optimal default barrier and we can see that there is a positive relationship. This indicates that the use of longer maturity bonds makes declaring bankruptcy sooner a more viable option. One possible explanation for this is that with longer maturity bonds, coupon payments must be made for a longer period time, thus keeping the firm leveraged for a longer period of time. Furthermore, the marginal increase in the optimal default barrier per one more unit



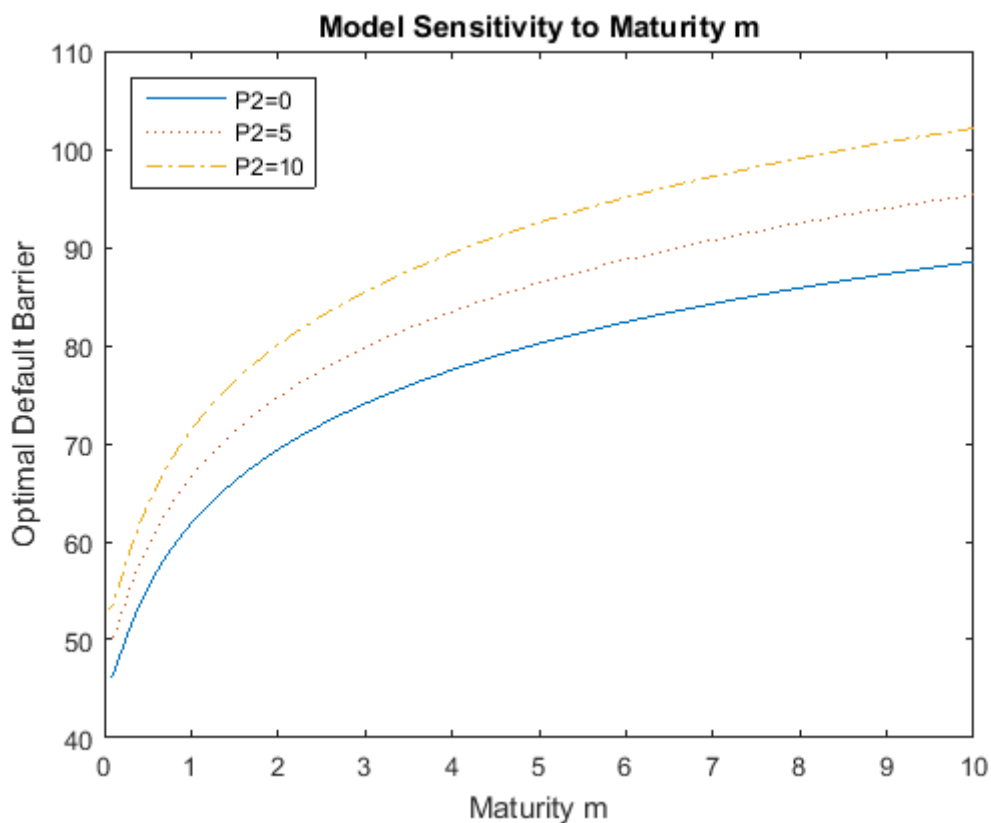


FIGURE 5.9: Shows the impact of average debt maturity  $m$  on the optimal default barrier. Created using an interval ranging from 0.01 and 10 for maturity, while holding all other variables constant as given in table 5.1. The different lines represent CoCos with different par values.

of maturity decreases over time, which can be explained by a limit to the downside of maturity. After all, a bond with a 100-year maturity and a 500-year maturity bond may have different maturities on paper, but in practice, the company is unlikely to outlive either bond. Hence, it makes sense that the additional impact that maturity has on the optimal default barrier tapers off as maturity grows very large. In Figure 5.9, we can also see that the impact of issuing CoCos depends on the value of the optimal default barrier, which consistent with previous results, is explained through the impact of additional debt on the company's current state of solvency.

### 5.2.8 Limitations

We have shown that changing any of the input parameters  $\delta$ ,  $\alpha$ ,  $\lambda$ ,  $\eta$ ,  $r$ ,  $m$  or  $\sigma$  will change the optimal default barrier in a non-linear way. Our results indicate that the optimal default barrier is an increasing function of the recovery rate  $\alpha$ , the firm-specific jump exponent  $\eta$ , the payout rate  $\delta$ , the maturity  $m$ , the jump intensity

$\lambda$  and in some parts of the volatility  $\sigma$ . We have also found that the optimal default barrier is a decreasing function of the risk-free rate  $r$ , and in some parts of the volatility  $\sigma$ . A general trend we have found on the topic of CoCos is that the increasing effect that CoCos has on the optimal default barrier depends on the value of the barrier. The effect is smaller when the optimal default barrier is smaller, and the effect is greater when the optimal default is bigger. In this chapter, we have not compared how CoCos fare in comparison to straight debt in terms of affecting the optimal default barrier, since we have kept the amount of straight debt in our models constant. This was done in order to look at the full impact of CoCos, both as a debt instrument and its conversion mechanic. In the next chapter however, we use a more realistic scenario, which is to replace some straight debt with CoCos, thus focusing on the conversion mechanic's impact on the optimal default barrier. But for now we can at least conclude that the impact that CoCos have on the optimal default barrier is not insignificant and that the potency of using CoCos to prevent default may be overshadowed by its properties as a debt instrument and its fixed coupon payments. Moreover, the values of the input parameters can take a host of different values, as can be seen in Figures 5.3 to 5.9, implying that choosing different values would produce different curves and numerical results. If such changes are made, the general conclusions drawn here may also be subject to change. Since altering any parameter will change the numerical outcome of our models in a non-linear fashion, the key takeaways and contributions of this section of the thesis are on the subject of broad trends between key model parameters and not necessarily exact values.

## 6. Application on Swedish banks

In this chapter, we apply the model that we used in Chapter 5 on real data belonging to the four biggest Swedish banks: Nordea, Swedbank, Handelsbanken and SEB. Since we use a similar process as Chen et al. (2017), we first explain the data collection process and assumptions made by Chen et al. (2017) in Section 6.1. Later, in Section 6.2, we showcase the development of the optimal default barrier between January 2005 and December 2011 for each of the four banks.

### 6.1 Data Collection and Processing

In their paper, Chen et al. (2017) apply their model on 17 major American bank holding companies to estimate how CoCos might have potentially affected the solvency of those banks during the financial crisis. Chen et al. (2017) use quarterly data from 2004 to 2011 and divide the debt of each bank into three categories: deposits, short-term debt and long-term debt, which are sorted in order of seniority. However, they do not distinguish between long-term debt and subordinated debt due to complications in doing so reliably. They then proceed to calculate the average debt maturity for each debt category. The average debt maturity is used to identify the US treasury bill or bond with the closest maturity, which they in turn use as a proxy for the risk-free rate for that debt type. Furthermore, Chen et al. (2017) calculate the total payout rate  $\delta$  each year, based on how much a bank paid in interest and dividends in that year. Jump parameters are not readily available, so Chen et al. (2017) use jump intensities  $\lambda$  of 0.1 and 0.3, and jump sizes  $\eta$  as an integer between 5 and 10. For unavailability reasons also, coupons are set to 3 percentage points over the risk-free rate, while the funding benefits are kept the same as in the base case at 35%. Chen et al. (2017) also linearly interpolate average debt maturity between the annual reports and the other variables between the quarterly reports to obtain weekly observations and argue that weekly observations lead to less abrupt changes at the end of each quarter.

The Swedish financial market is a lot smaller than that of the U.S., and although our sample contains the major players, it still only comprises four banks: Handelsbanken, Nordea, SEB and Swedbank. These four banks were the largest banks in Sweden during the tested time interval, and still are at the time of writing. Our sampling period begins in 2005 and ends in 2011. Our data is primarily retrieved from the Bloomberg terminal and the banks' respective annual reports. For values that are not readily available, such as  $\eta$  and  $\lambda$ , we make the same assumptions as Chen et al. (2017) to make comparison easier. Just like Chen et al. (2017), we start by calculating the average debt maturities for each bank, based on information on outstanding debt found in the annual reports, which we then interpolate to obtain weekly observations. For each week we get four average debt maturities, one for each bank, and we match them with the closest Swedish government bond yield rate among the 2-year, 5-year, 7-year and 10-year Swedish government bonds to use as our risk-free rate. Since our definition of the payout rate follows the definition by Leland et al. (1996), it is given as follows:

$$\text{Payout rate} = \frac{\text{Dividends}}{\text{Equity}} + \frac{\text{Interest expenses}}{\text{Debt}}$$

In the case of the payout rate, we linearly interpolate quarterly data into weekly observations. For volatility, we use the 10-day trailing volatility at the end of every week for each bank to reflect the weekly volatility in any given week - it is the shortest interval option that Bloomberg provides. In the case of CoCo bond coupon rates  $c_2$ , we adopt the same approach as Chen et al. (2017) and use  $r + 3\%$  as the  $c_2$  rate, since no banks had CoCos during the sampled data period. However, for straight debt coupon rates  $c_1$ , we opt to use a weighted average of coupons as presented in Bloomberg, rather than an estimated fixed value like Chen et al. (2017), for higher accuracy. This setup works out well, since it leads to  $c_2 > c_1$ , implying that CoCo bond holders receive bigger coupon payments than senior debt holders, which is plausible. Moreover, we obtain weekly observations for any straight debt outstanding  $P_1$  of each bank. Chen et al. (2017) use two different values of  $\lambda$  to test their model,  $\lambda = [0.1, 0.3]$ . We choose to iterate using  $\lambda = 0.3$  only, firstly because this is the base case value, as shown in Table 5.1, and secondly because using  $\lambda = 0.3$  yields fewer outliers than using  $\lambda = 0.1$ . Appendix C contains a more comprehensive justification for this decision. Finally, we diverge from Chen et al. (2017) in the selection of a value for  $\eta$ ; rather than using a  $\eta$  value in the interval  $\{5, 10\}$  like Chen et al. (2017) do in their application, we use  $\eta = 4$ , which is the base case value of Chen et al. (2017). Increasing  $\eta$  at higher values exerts little additional pressure on the optimal default barrier, due to diminishing returns of  $\eta$ ,

which we showed in Figure 5.5. Thus, choosing  $\eta = 4$  does not incur any drastic changes to our results compared to say  $\eta = 5$ .

Calculating the optimal default barrier is done using Equation (4.7). Equation (4.7) require roots, which we calculate using the first method listed in Appendix B, and insert them into Equation (4.6) and Equation (4.8) to obtain values for  $\epsilon_1$  and  $\epsilon_2$ . Given that the optimal default barrier is ( $V_b^{NC} = \epsilon_1 P_1 + \epsilon_2 P_2$ ), we are able to calculate it using  $\epsilon_1$  and  $\epsilon_2$ . We repeat this procedure for every week from January 2005 to December 2011 using the weekly values for  $r$ ,  $m$ ,  $\delta$ ,  $\sigma$ ,  $c_1$  and  $c_2$ . In order to compare how CoCos impact the optimal default barrier in the debt mix we set  $P_2 \in \{0\%, 5\%, 10\%, 20\%\}$  and reduce the amount of  $P_1$  proportionally whenever we increase  $P_2$  to keep the total debt level unchanged. When  $P_2 = 0\%$  we get  $V_b^{PC}$  from Equation (4.5), while if  $P_2$  assumes any value above 0, for example  $P_2 \in \{5\%, 10\%, 20\%\}$  we get different levels of  $V_b^{NC}$ .

## 6.2 Presentation of Results

Figures 6.1 to 6.4 illustrate how the optimal default barriers of the Swedbank, Handelsbanken, SEB and Nordea changes every week from January 2005 to December 2011. A common feature for all graphs is the general development in each graph; the optimal default barrier typically becomes more volatile during the financial crisis, dips somewhere in early 2009 and peaks sometime in early 2011. The banks in our dataset decrease their debt levels during the financial crisis. Since the optimal default barrier of a bank is calculated based on its own total debt, reductions in either  $P_1$  or  $P_2$  are interpreted as lowered debt burdens, which consequently lowers the value of the optimal barrier. This helps explain the drop in the optimal default barrier during the financial crisis. Another contributing factor is the rise in volatility during the financial crisis, which is shown in Figures 6.5 to 6.8. An increased volatility contributes to lowering the optimal default barrier for high values of  $\sigma$ , which we showed in Figure 5.7. After the financial crisis, we can observe a rise in the optimal default barriers for all banks in Figures 6.1 to 6.4, which begins somewhere around year 2010. A rise in the barrier implies a greater likelihood of default and is primarily explained by an increased debt burden among the banks. During this period, banks scrambled to secure financing through additional borrowing, which subsequently drove up their debt levels. Lower volatility also contributes to increasing the optimal default barrier at lower values of  $\sigma$ , as we established in Subsection 5.2.5 and showed in Figure 5.7.

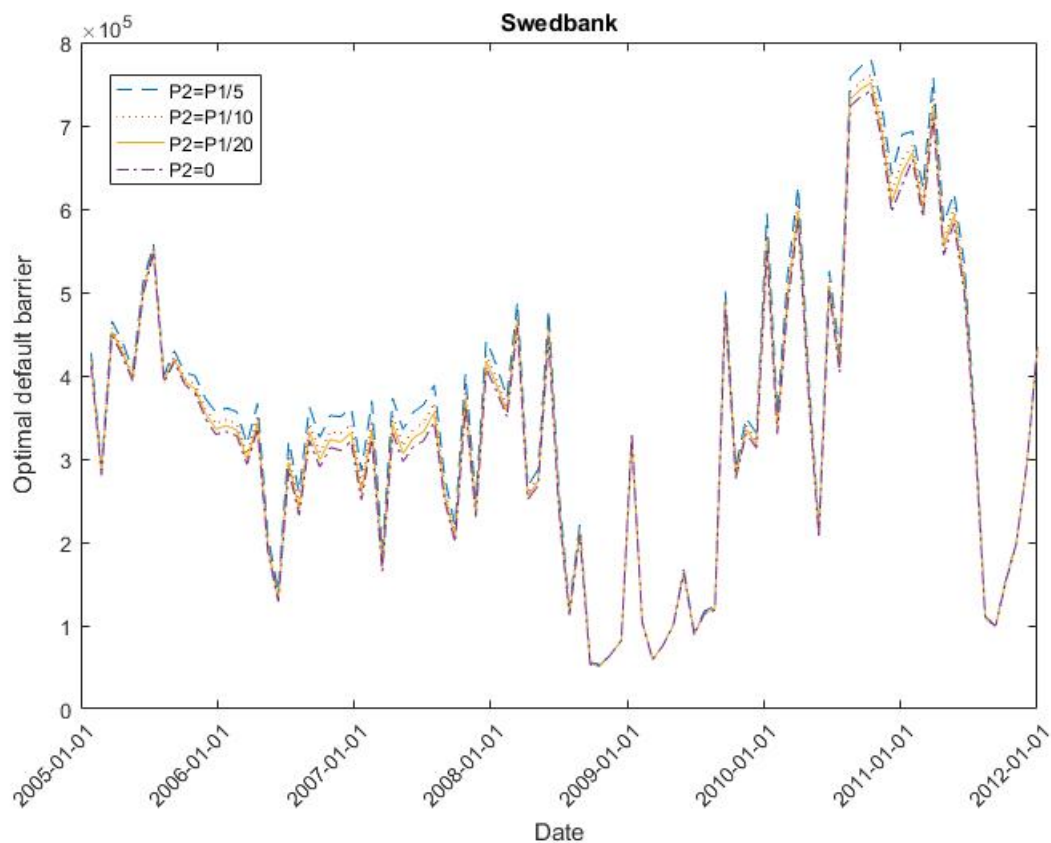


FIGURE 6.1: Shows the weekly changes in the optimal barrier of Swedbank with varying amounts of CoCos in the time interval 2005-2011. A new barrier is calculated every week from the equity holders' point of view. The different lines represent 4 different combinations of debt and CoCos, from using 0% CoCos and up to 20%.  $V_b^{PC}$  shows the optimal default barrier post conversion which implies that  $P_2 = 0$ , while the other lines represent different levels of  $V_b^{NC}$ .

Figures 6.1 to 6.4 also show how the use of CoCos affect the optimal default barrier by changing the debt mix. In all figures, including more CoCos - indicated by a higher portion of  $P_2$  - leads to a higher optimal default barrier. This relationship is particularly visible in the years before the financial crisis. Rather than issuing CoCos anew, like we did in Chapter 5, we replace straight debt with CoCos here. Issuing CoCos without switching out other debt allowed us to study the effects of CoCos by itself in sensitivity analysis. Now we prefer to study the effects of CoCos when they are replaced by straight debt, which is arguably more realistic, since nothing else is added, only the debt type differs. Still, our result here turn out to be consistent with our findings in Chapter 5, which showed that issuing CoCos generated a higher optimal default barrier. We can also see from Figures 6.1 to 6.4

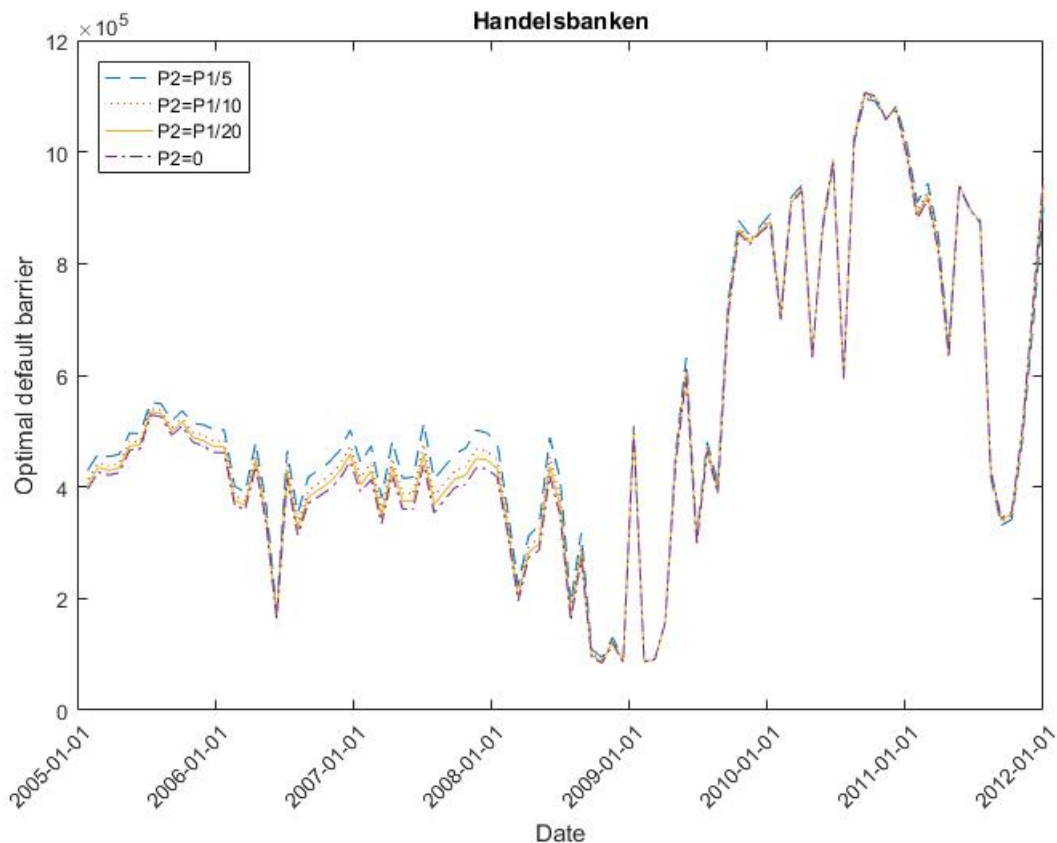


FIGURE 6.2: Shows the weekly changes in the optimal barrier of Handelsbanken with varying amounts of CoCos in the time interval 2005-2011. A new barrier is calculated every week from the equity holders' point of view. The different lines represent 4 different combinations of debt and CoCos, from using 0% CoCos and up to 20%.  $V_b^{PC}$  shows the optimal default barrier post conversion which implies that  $P_2 = 0$ , while the other lines represent different levels of  $V_b^{NC}$ .

that during and following the financial crisis, increasing the proportion CoCos has a significant smaller effect on the optimal default barrier, than it did before. The diminished negative impact of CoCos on the likelihood of default was also observed in Chapter 5 and was then attributed to the greater solvency of banks, which made the added debt burden of CoCos manageable. We use a similar line of reasoning to argue that the lower debt burden the banks had during the crisis allowed them to more easily overcome the bigger coupon payments demanded by CoCo bond holders. Moreover, the option to convert CoCos to equity in the case of financial distress also helps explain the close-to-zero impact that CoCos seemingly has on the optimal default barrier during the crisis.

The results from our model application on real bank data suggests that the use

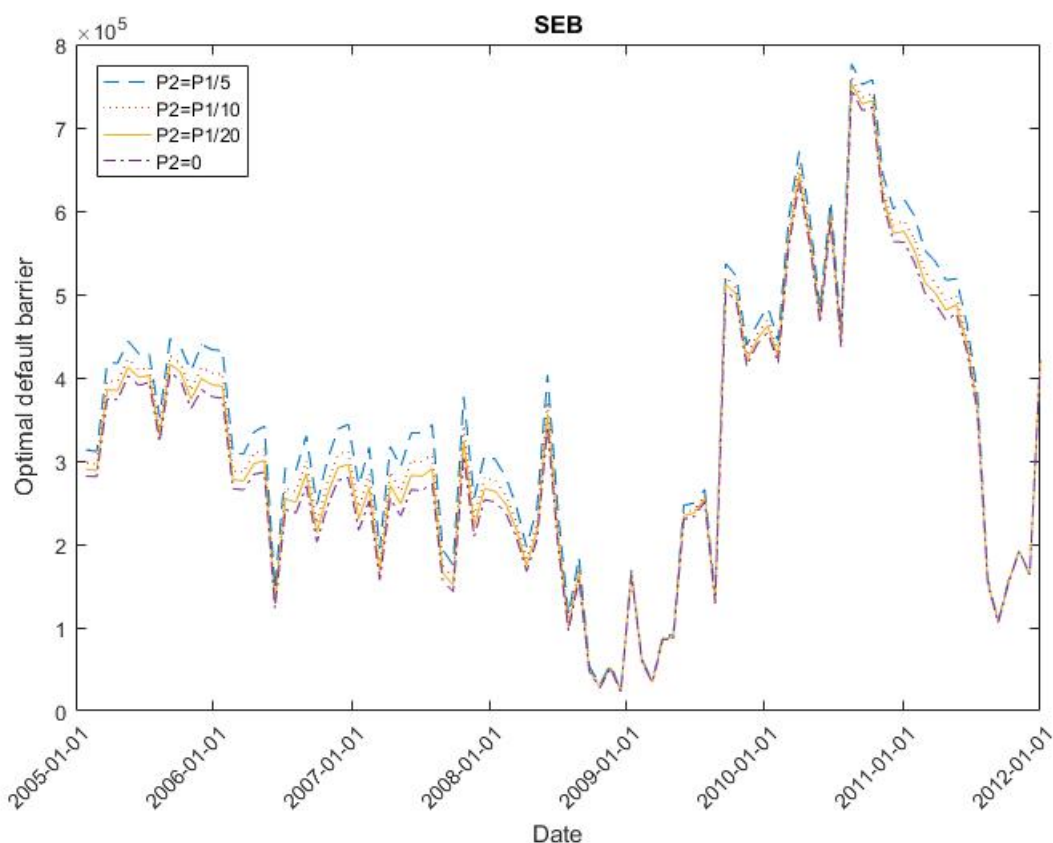


FIGURE 6.3: Shows the weekly changes in the optimal barrier of SEB with varying amounts of CoCos in the time interval 2005-2011. A new barrier is calculated every week from the equity holders' point of view. The different lines represent 4 different combinations of debt and CoCos, from using 0% CoCos and up to 20%.  $V_b^{PC}$  shows the optimal default barrier post conversion which implies that  $P_2 = 0$ , while the other lines represent different levels of  $V_b^{NC}$ .

of CoCos tends to have a negative impact on the optimal default barrier, but that this negative effect diminishes during the financial crisis. In their study, Chen et al. (2017) find that CoCos could have had a significant positive impact on the large bank holding companies during the financial crisis through incentive effects. While our negative results may seem to be at odds with the positive results of Chen et al. (2017), that is not necessarily true. Chen et al. (2017) calculate the reduction in debt overhang costs that result from the banks' increased ability to absorb losses to show that issuing CoCos prior to the crisis could have incentivized shareholders to inject more equity and increased the loss absorption capacity of the banks in question. Debt overhang costs are the costs associated with a company with too much debt and consequently are unable to pursue attractive investment opportunities. Even if a company is profitable, debt overhang can discourage equity holders from



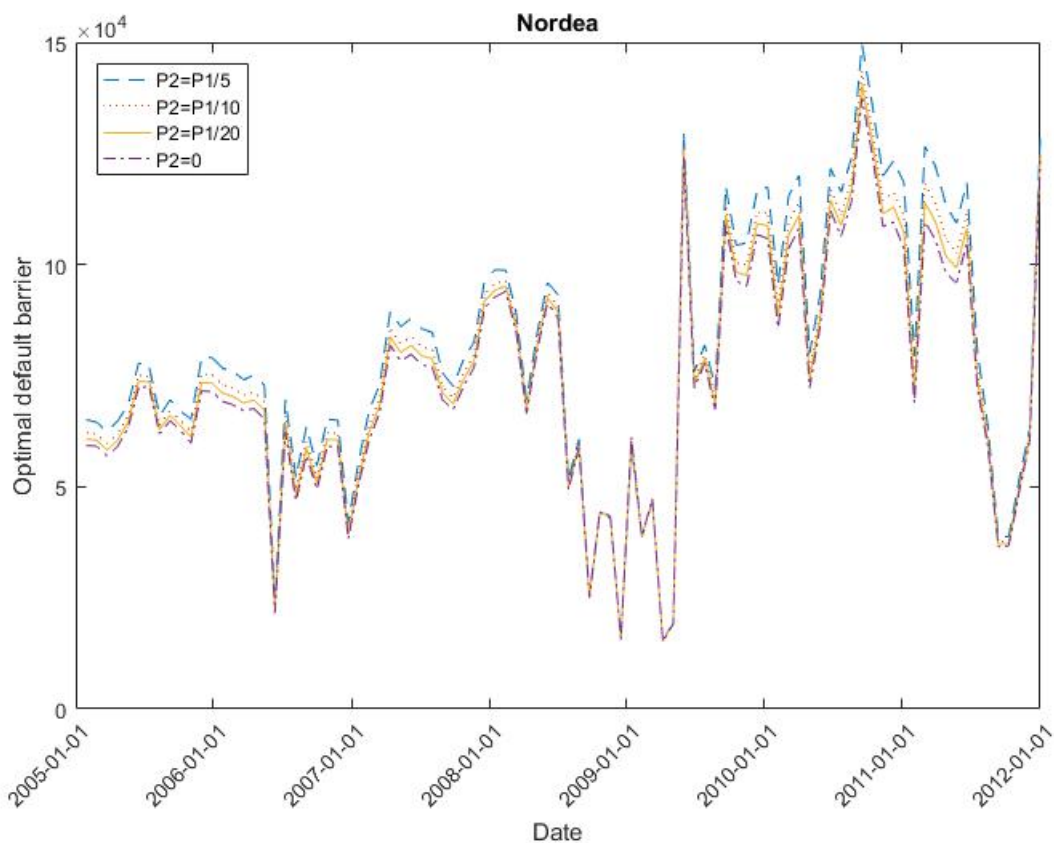


FIGURE 6.4: Shows the weekly changes in the optimal barrier of Nordea with varying amounts of CoCos in the time interval 2005-2011. A new barrier is calculated every week from the equity holders' point of view. The different lines represent 4 different combinations of debt and CoCos, from using 0% CoCos and up to 20%.  $V_b^{PC}$  shows the optimal default barrier post conversion which implies that  $P_2 = 0$ , while the other lines represent different levels of  $V_b^{NC}$ .

taking on projects, since more of the profit ends up at the debt holders table. The reduction in debt overhang that Chen et al. (2017) observe serves as an indication of whether issuing CoCos would have increased the motivation of investors to put in more capital. Although our results show that the contribution from CoCos is at best *no negative impact*, the incentive effects that are gained through reduced overhang may actually make the net impact of CoCos positive, all things considered. But even if the benefits of having CoCos overshadow the drawbacks during a crisis, one must weigh this benefit against the drawback of having to operate with an increased likelihood of default during non-crisis times.

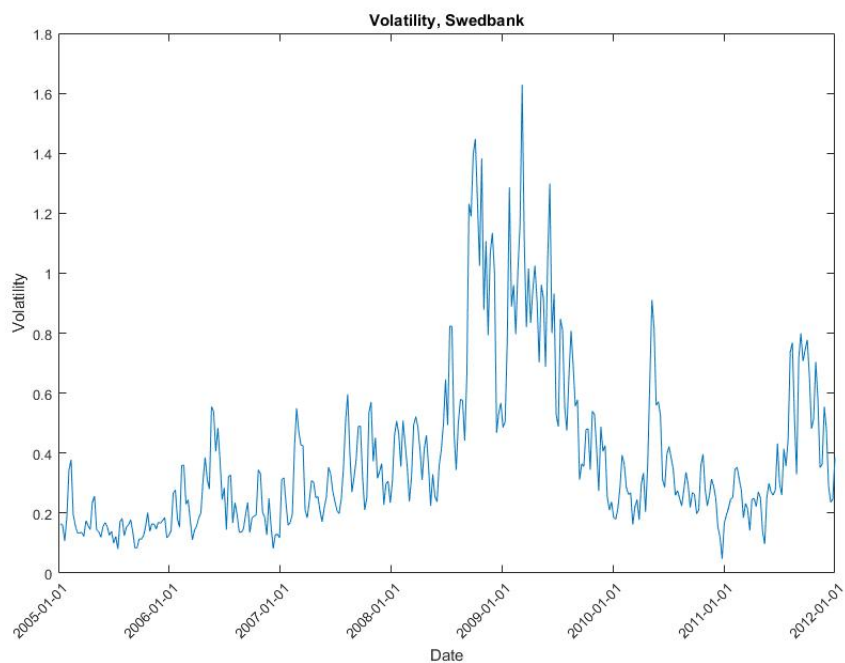


FIGURE 6.5: Shows the 10-day trailing volatility at weekly frequencies between the period 2005-2011 for Swedbank.

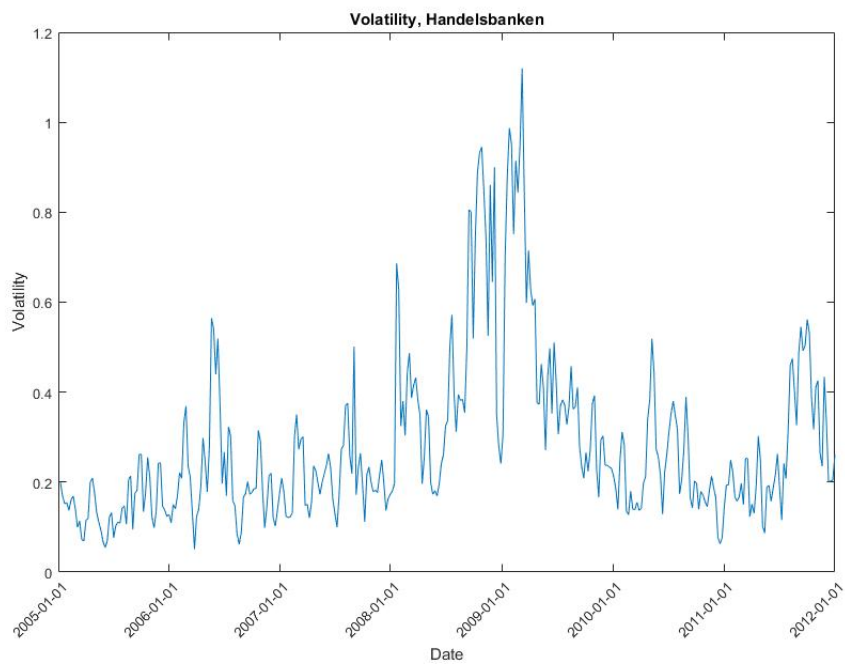


FIGURE 6.6: Shows the 10-day trailing volatility at weekly frequencies between the period 2005-2011 for Handelsbanken.

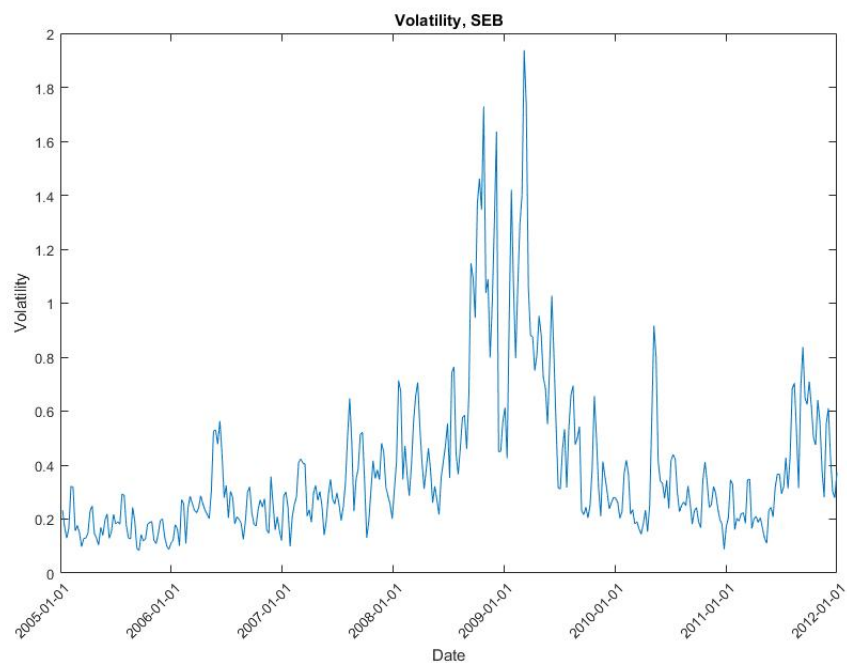


FIGURE 6.7: Shows the 10-day trailing volatility at weekly frequencies between the period 2005-2011 for SEB.

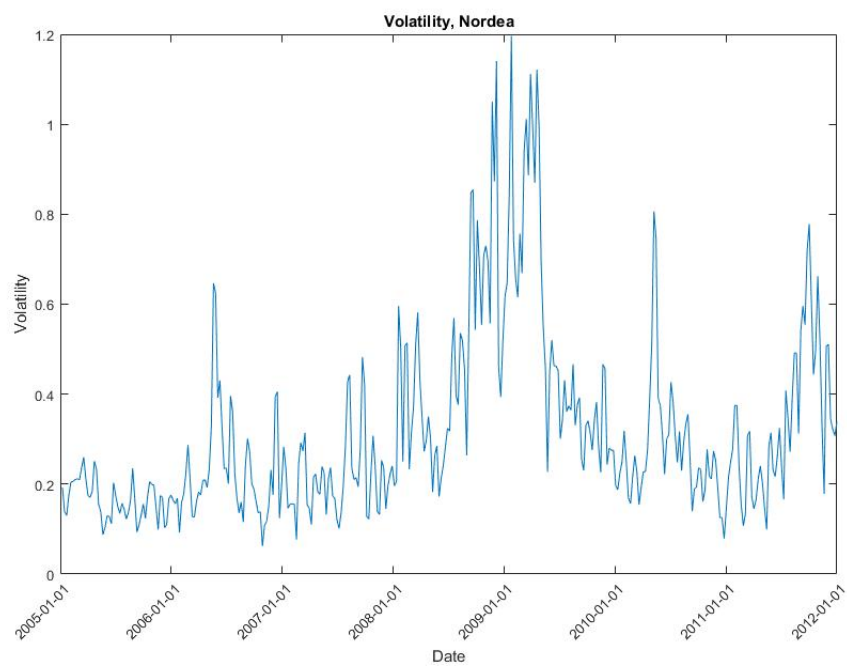


FIGURE 6.8: Shows the 10-day trailing volatility at weekly frequencies between the period 2005-2011 for Nordea.

## 7. Conclusion

In this thesis, we study the sensitivity of the optimal default barrier to changes in its input parameters and issuing CoCos. Our findings suggest that the optimal default barrier may vary greatly depending on the input parameters used. Furthermore, our sensitivity analysis shows that issuing CoCos has an adverse effect on the optimal default barrier and the likelihood of default, but that the potency of this adverse effect depends on the value of the optimal default barrier. We also examine how the solvency of the Swedish banks would have been impacted if the banks included CoCos in their debt mix. Consistent with the results from our sensitivity analysis, we find that including CoCos in the debt mix of the banks would increase their optimal default barriers. Although the optimal default barrier becomes higher, it does not tell us whether or not the new barrier is better or worse for the shareholders. Our model presents a value where the shareholders equity is maximized in an expected sense, so even if the barrier is higher, we can not tell whether the shareholder value is higher or lower than before. In our study, we also find that the adverse effect of CoCos diminishes during the financial crisis, implying that the burden of paying higher coupons is negated by the CoCo bond's conversion mechanic. The findings in our study suggest that the use of CoCos adversely impacts the likelihood of default, which is particularly noteworthy during non-crisis times. This result however, does not discredit the utility of CoCos as a bail-in instrument, nor does it deny the existence of any incentive effects that Chen et al. (2017) proposes.

For future research, it would be interesting to study the sensitivity of the optimal default barrier by changing two or more variables simultaneously to better understand the interaction between input parameters. Since we found the use of CoCos to be detrimental to the default likelihood outside of crises situations, it would also be interesting to make a longitudinal cost-benefit analysis, to compare the costs to society if all banks used CoCos versus if CoCos were not used at all. Finally, a study using market values rather than book values of debt could provide meaningful insights to the results of our study, since the markets lowered the value of all debt during the crisis, as a result of distrust for standing credit ratings.

## Appendix A: Model Discussion

### A.1 The $G(x)$ Equation

The original double jump diffusion model presented in Kou (2005) looks as follows:

$$G(x) = \left( r - \delta - \frac{1}{2}\sigma^2 - \lambda\xi \right)x + \frac{1}{2}\sigma^2x^2 + \lambda \left( \frac{p\eta_1}{\eta_1 - x} + \frac{q\eta_2}{\eta_2 + x} - 1 \right)$$

where  $\xi = \frac{p\eta_1}{\eta_1 - 1} + \frac{q\eta_2}{\eta_2 + 1} - 1$  and  $p$  is the probability for upside jumps and  $q$  is the probability for downside jumps. Since we are only interested in the downside jumps that can potentially cross the optimal default barrier,  $V_b^*$ , we set  $q = 0$  and  $q = 1$ . This yields the equation (4.7), also shown below:

$$G(x) = \left( r - \delta - \frac{1}{2}\sigma^2 - \lambda \left( \frac{\eta}{\eta + 1} - 1 \right) \right)x + \frac{1}{2}\sigma^2x^2 + \lambda \left( \frac{\eta}{\eta + x} - 1 \right) = \beta$$

Where  $\beta$  is either  $\beta = r$  or  $\beta = r + m$  depending on whether we are solving for  $\gamma_r$  or  $\gamma_{r+m}$ . Lemma 2.1 in Kou and Wang (2003) suggest that the above equation has exactly four real roots, two negative and two positive, denoted as follows:  $\gamma_{1,\beta}$ ,  $\gamma_{2,\beta}$ ,  $-\gamma_{3,\beta}$ ,  $-\gamma_{4,\beta}$ . Moreover, the four roots are related to  $\eta_1$  and  $\eta_2$  in the following way:

$$0 < \gamma_{1,\beta} < \eta_d < \gamma_{2,\beta} < \infty, \quad 0 < \gamma_{3,\beta} < \eta_u < \eta_u < \gamma_{4,\beta} < \infty$$

The roots to the equation are inputs to  $\epsilon_1$  and  $\epsilon_2$ , which in turn are inputs to find the optimal default barrier. The method we use to find the roots is outlined in Appendix B.

### A.2 Inconsistency between Chen et al. (2017) and Kou (2005)

Most of the derivations on the  $G(x)$  formula were done in Kou (2005), and Chen et al. (2017) only provide the final formula in its appendix and refer to Kou (2005)

for details. However, the calculations provided in the appendix of Chen et al. (2017) are not exactly the same as the ones that Kou (2005) provide. To clarify, we opted to follow the calculations and formulas provided in Kou (2005). The reason being that developing the formula is a part of the research in Kou (2005), whereas Chen et al. (2017) uses it at one point for some of their research. For this reason, we believe that the formula provided in Kou (2005) is less likely to be wrong.

The difference in question concerns a missing variable,  $x$ ; below is a comparison of the two formulas.

Formula in Chen et al. (2017):

$$G(x) = \left( r - \delta - \frac{1}{2}\sigma^2 - \lambda\left(\frac{\eta}{\eta+1} - 1\right) \right) + \frac{1}{2}\sigma^2 x^2 + \lambda\left(\frac{\eta}{\eta+x} - 1\right) \quad (\text{A.1})$$

Formula in Kou (2005):

$$G(x) = \left( r - \delta - \frac{1}{2}\sigma^2 - \lambda\xi \right) x + \frac{1}{2}\sigma^2 x^2 + \lambda\left(\frac{p\eta_1}{\eta_1 - x} + \frac{q\eta_2}{\eta_2 + x} - 1\right)$$

When we are only interested in down-side jumps we can set  $p = 0$ ,  $q = 1$ ,  $\eta_2 = \eta$ ,  $\eta_1 = 0$  and substitute  $\xi = \frac{p\eta_1}{\eta_1-1} + \frac{q\eta_2}{\eta_2+1} - 1$ , and then we can simplify as follows:

$$\begin{aligned} G(x) &= \left( r - \delta - \frac{1}{2}\sigma^2 - \lambda\left(\frac{p\eta_1}{\eta_1 - 1} + \frac{q\eta_2}{\eta_2 + 1} - 1\right) \right) x + \frac{1}{2}\sigma^2 x^2 + \lambda\left(\frac{p\eta_1}{\eta_1 - x} + \frac{q\eta_2}{\eta_2 + x} - 1\right) \\ G(x) &= \left( r - \delta - \frac{1}{2}\sigma^2 - \lambda\left(0 + \frac{\eta_2}{\eta_2 + 1} - 1\right) \right) x + \frac{1}{2}\sigma^2 x^2 + \lambda\left(0 + \frac{\eta_2}{\eta_2 + x} - 1\right) \\ G(x) &= \left( r - \delta - \frac{1}{2}\sigma^2 - \lambda\left(\frac{\eta}{\eta+1} - 1\right) \right) x + \frac{1}{2}\sigma^2 x^2 + \lambda\left(\frac{\eta}{\eta+x} - 1\right) \quad (\text{A.2}) \end{aligned}$$

After simplifying the equation from Kou (2005), we obtain Equation (A.2), which is identical to Equation (A.1) provided by Chen et al. (2017) in every way except for a missing  $x$ . Since the roots are ultimately used to calculate the optimal default barrier, any error could bear a significant impact on the results.

## Appendix B: Finding Roots

In their appendix, Kou (2005) outlines two analytical methods for finding the roots of Equation (4.7) are provided. We attempted to solve the roots using both methods at first, but were only successful with the first method and chose to proceed with the roots obtained from using the first method. Since the roots of the equation should be the same regardless of which method is used, our results should not have been compromised in any way. For the sake of transparency however, we have listed both methods here; the second method can also be found in the appendix of Kou (2005). Equation (4.7) has again been listed below as Equation (B.1).

$$G(x) = \left( r - \delta - \frac{1}{2}\sigma^2 - \lambda\left(\frac{\eta}{\eta+1} - 1\right) \right)x + \frac{1}{2}\sigma^2x^2 + \lambda\left(\frac{\eta}{\eta+x} - 1\right) = \beta \quad (\text{B.1})$$

### B.1 Method 1

Kou et al. (2003) finds that equation B.1 has four real roots and Kou (2005) provide the method to solve these roots in their Appendix B. Furthermore, they show that since Equation (B.1) is essentially a quartic equation and its roots can be found by rearranging the terms, which then show that the roots of the equation should satisfy:

$$ax^4 + bx^3 + cx^2 + dx + e = 0$$

where

$$a = \sigma^2$$

$$b = 2\mu - \sigma^2(\eta_1 - \eta_2)$$

$$c = -\sigma^2\eta_1\eta_2 - 2\mu(\eta_1 - \eta_2) - 2\lambda - 2\alpha,$$

$$d = -2\mu\eta_1\eta_2 - 2\lambda p(\eta_1 + \eta_2) + 2\lambda\eta_1 + 2\alpha(\eta_1 - \eta_2)$$

$$e = \alpha\eta_1\eta_2$$

and

$$\begin{aligned}\mu &= r - \delta - \frac{1}{2}\sigma^2 - \lambda\xi \\ \xi &= \frac{\eta}{\eta + 1} - 1\end{aligned}$$

## B.2 Method 2

The second method presented by Kou (2005) is derived from the first method and was developed by Lodovico Ferrari back in 1540. A more in-depth discussion about the formulas provided in Kou (2005) can be found in Borwein and Erdelyi (1995) and Boyer and Merzbach (1991). However, we were unsuccessful attempting to use this method to find the roots to (B.1); there is often an imaginary root when reasonable input parameters are used. After evaluating this model over different input variables we found that to avoid imaginary values we would need to use nonsensical values for variables such as  $\sigma$  way above 1. Similar results was obtained for other variables and this made us decide to not use this model even though it, if it worked, would have provided quicker calculations.

The method proposed by Kou (2005) is outlined below in case a reader wants to attempt to apply this method instead. The four roots to equation (B.1) are denoted as  $\beta_1$ ,  $\beta_2$ ,  $\beta_3$  and  $\beta_4$  respectively:

$$\beta_1 = -\frac{b}{4a} + \frac{p_1 - \tilde{p}_2}{2}, \beta_2 = -\frac{b}{4a} + \frac{p_1 + \tilde{p}_2}{2}, \beta_3 = \frac{b}{4a} + \frac{p_1 - p_2}{2}, \beta_4 = \frac{b}{4a} + \frac{p_1 + p_2}{2}$$



where

$$\begin{aligned}
B_0 &= c^2 - 3bd + 12ae \\
B_1 &= 2c^3 - 9bcd + 27ad^2 + 27b^2e - 72ace \\
B_2 &= \sqrt{B_1^2 - 4B_0^3} \\
B_3 &= \frac{b^2}{4a^2} - \frac{2c}{3a} \\
B_4 &= \frac{b^2}{2a^2} - \frac{4c}{3a} \\
B_5 &= d\frac{bc}{a^2} - 8\frac{d}{a} - \left(\frac{b}{d}\right)^3 \\
\tilde{B} &= \sqrt[3]{B_1 + B_2} \\
C_0 &= \frac{\sqrt[3]{2}B_0}{3a\tilde{B}} \\
C_1 &= \frac{\tilde{B}}{3\sqrt[3]{2}a} \\
p_1 &= \sqrt{B_3 + C_0 + C_1} \\
p_2 &= \sqrt{B_4 - C_0 - C_1 - \frac{B_5}{4p_1}} \\
\tilde{p}_2 &= \sqrt{B_4 - C_0 - C_1 + \frac{B_5}{4p_1}}
\end{aligned}$$

Note that the  $\beta$  used to represent roots here are unrelated to the  $\beta$  used in  $G(x) = \beta$  (i.e. in equation (B.1)):

## Appendix C: Selection of Lambda

Figures C.1 to C.4 plots the optimal default barriers of the four Swedish banks from the beginning of 2005 to the end of 2011 using  $\lambda = 0.1$  and  $\lambda = 0.3$ . A persistent characteristic in the graphs, is that  $\lambda = 0.1$  produces more volatile barriers than  $\lambda = 0.3$ . Moreover, there are occasions when using  $\lambda = 0.1$  produces nonsensical results that lack causal explanations; such as the sudden spike in Figure C.1 around late 2005, which does not coincide with any evidence of heightened financial stress or likelihood of default for Swedbank. Less evident but similar jumps in the optimal default barrier can be observed in the other figures around that time. We can also see that the general trend maintains a fairly similar shape for both values of  $\lambda$  in all graphs, and thus argue that it is more appropriate to use  $\lambda = 0.3$  in Section 6.2.

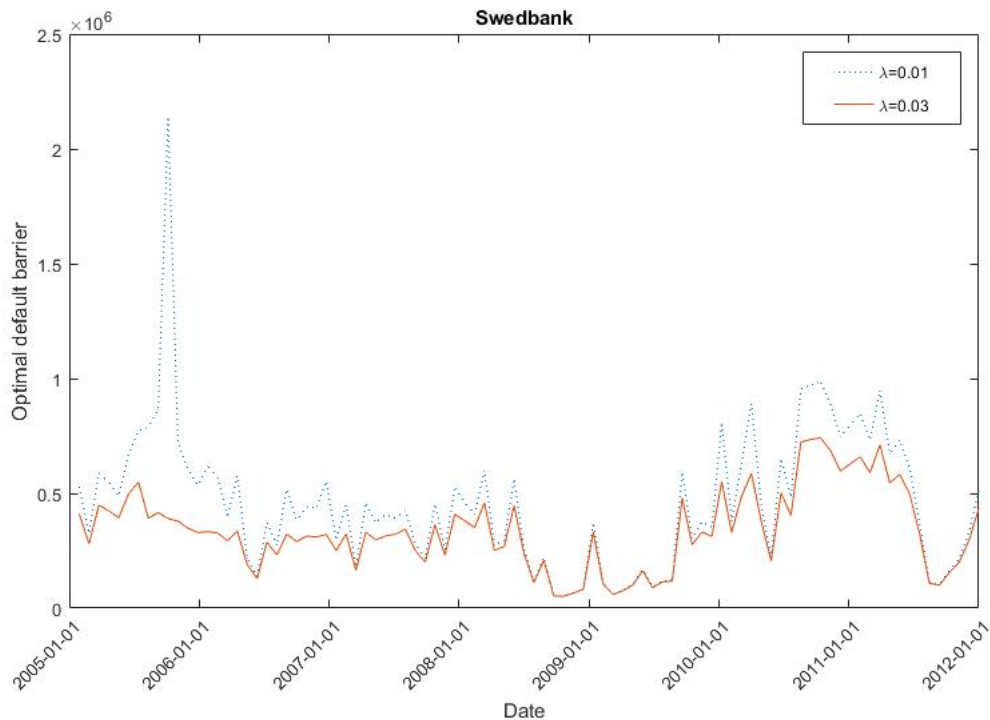


FIGURE C.1: Displays the monthly change in the optimal default barrier of Swedbank between 2005 and 2011, for  $\lambda = 0.1$  and  $\lambda = 0.3$ .

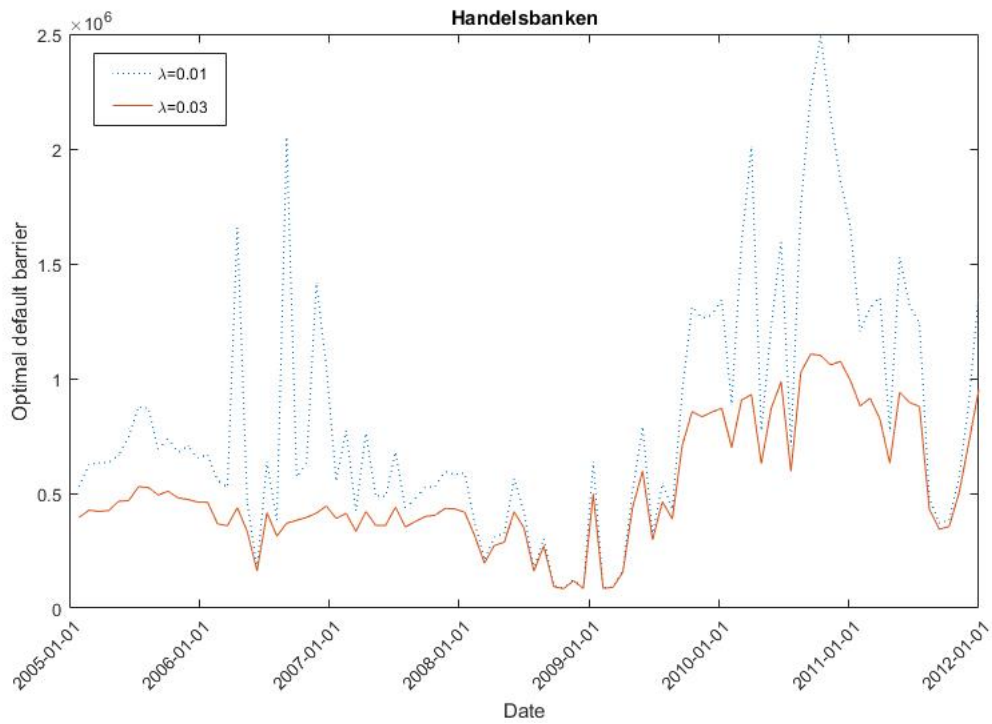


FIGURE C.2: Displays the monthly change in the optimal default barrier of Handelsbanken between 2005 and 2011, for  $\lambda = 0.1$  and  $\lambda = 0.3$ .

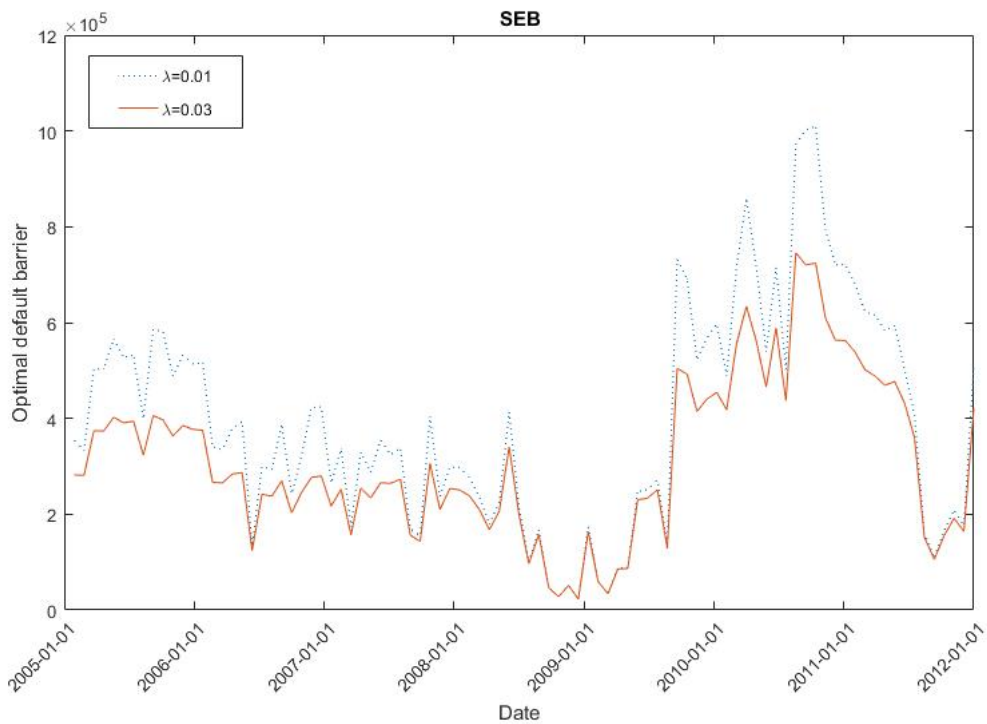


FIGURE C.3: Displays the monthly change in the optimal default barrier of SEB between 2005 and 2011, for  $\lambda = 0.1$  and  $\lambda = 0.3$ .

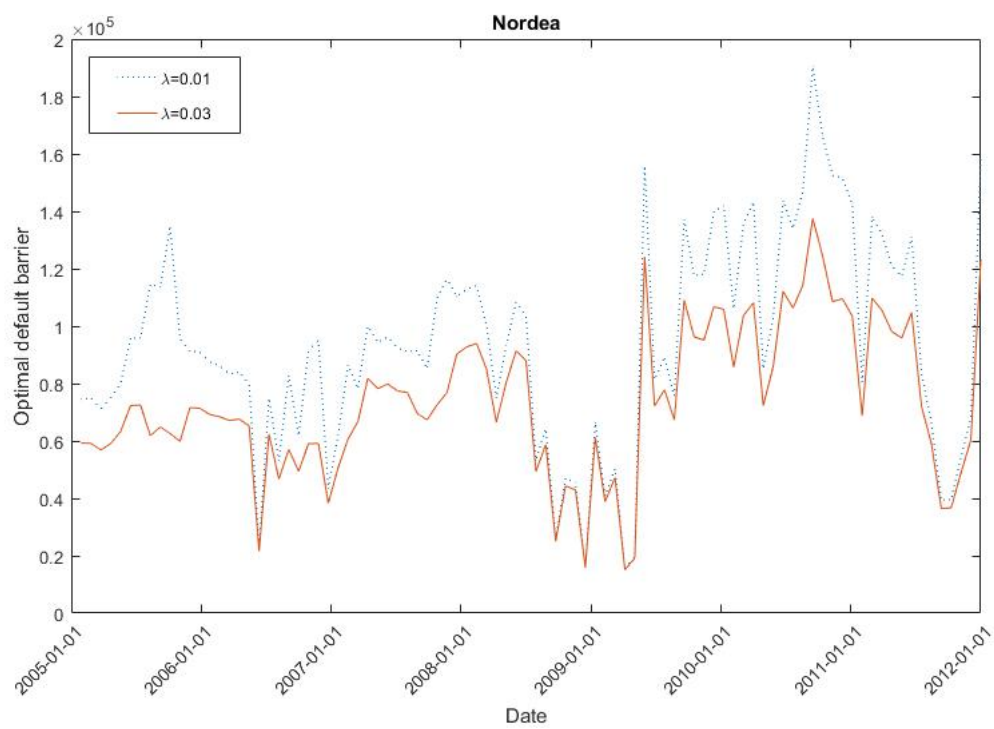


FIGURE C.4: Displays the monthly change in the optimal default barrier of Nordea between 2005 and 2011, for  $\lambda = 0.1$  and  $\lambda = 0.3$ .

## Appendix D: Matlab Code

The following Matlab-code is used to produce Figure 5.2:

```

%% Fig4
% Base case parameters
rho=0.05;
P2=5;
deltau=0.01;
lambda=0.3;
r=0.06;
sigma=0.08;
eta=4;
eta1=0;
eta2=4;
k1=0.35;
alfa=0.5;
m=[10:-0.1:1];
c1=0.09;
p=0;
q=1;

c1=[0.01:0.02:0.19];
for j=1:1:length(c1)
    for i=1:1:length(m);

        %Finding roots "gamma_r"
        xi=p*eta1/(eta1-1)+q*eta2/(eta2+1)-1;
        mu=r-deltau-0.5*sigma.^2-lambda*xi;
        a=(sigma.^2);
        b=2*mu-(sigma.^2)*(eta1-eta2);
    end
end

```

```

c=-(sigma.^2)*eta1*eta2-2*mu*(eta1-eta2)-2*lambda-2*r;
d=-2*mu*eta1*eta2-2*lambda*p*(eta1+eta2)+2*lambda*eta1+2*r*(eta1-
e=2*r*eta2*eta1;
func=[a b c d e];
soltest_r=roots(func);
solution_r=-1*sort(soltest_r);

%Finding roots "gamma_rm"
mu2=r-deltau-0.5*sigma.^2-lambda*xi;
a2=(sigma.^2);
b2=2*mu2-(sigma.^2)*(eta1-eta2);
c2=-(sigma.^2)*eta1*eta2-2*mu2*(eta1-eta2)-2*lambda-2*(r+m(i));
d2=-2*mu2*eta1*eta2-2*lambda*p*(eta1+eta2)+2*lambda*eta1+2*(r+m(i)
e2=2*(r+m(i))*eta2*eta1;
func=[a2 b2 c2 d2 e2];
soltest_rm=roots(func);
solution_rm=-1*sort(soltest_rm);

%Epsilon from appendix of Chen et al. (2017)
epsilon1=(((c1(j)+m(i))/(r+m(i)))*solution_rm(1)*solution_rm(2)-
((1-alfa)*(solution_r(1)+1)*(solution_r(2)+1)+(alfa)*(solution

P1(j,i)=P2/(epsilon1*(1-rho)-1);

%We are only interested in numbers in the range 0 to 100.
if P1(j,i) < 0;
    P1(j,i)=nan;
end
end
end

plot(m,P1(1,:),':',m,P1(2,:),':',m,P1(3,:),'-',m,P1(4,:),'-',m,P1(5,:),'-
axis([1 10 0 100])
xlabel('Mean Maturity 1/m')
ylabel('Critical P_1')
title('Attempted replication of Figure 4 in Chen et al. 2017')
set(gca,'Xdir','reverse','XTickLabel',[1 0.9 0.8 0.7 0.6 0.5 0.4 0.3 0.2
legend('c=0.01', 'c=0.03', 'c=0.05', 'c=0.07', 'c=0.09', 'c=0.11', 'c=0.1

```

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