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# Competitive Neutrality and the Cost and Quality of Welfare Services

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# Competitive Neutrality and the Cost and Quality of Welfare Services\*

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## Abstract

Competition between private and public firms can increase service quality and reduce public costs in markets for tax-financed welfare services with non-contractible quality. Synergies arise from combining high-powered incentives for quality provision (emanating from private firms) with low rents (public firms). However, sometimes, the optimal regulation requires the government to provide public firms with better funding than private competitors, e.g. by paying them higher prices or covering their deficits. This additional compensation is not tied to any additional verifiable quality obligations and may therefore violate competitive neutrality rules incorporated to various areas of legislation.

JEL: H44; L33; L44

Keywords: public-private competition; competitive neutrality; mixed markets; public option; ownership; competition; incomplete contracts; strategic ambiguity; merit goods; SGEI

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# 1 Introduction

Commercial companies can often not compete with government businesses on an equal footing. The commercial media companies struggle to persuade viewers to pay for contents as a result of the many programs provided for free by tax- and license fee-funded public service broadcasting companies such as the BBC in the U.K. In recent times also printed-media companies must struggle to compete with the websites hosted by public service companies.<sup>1</sup> In Australia, private educators are worried about differentiated public funding of private and public schools and discriminatory risk assessments of private providers of higher education.<sup>2</sup> In Sweden, public entities and third sector organizations have been exempted from taxes and charges in sports markets.<sup>3</sup> Other controversial practices include absence of yield requirements, as well as explicit or implicit government guarantees on debts, which may permit government businesses to operate at a loss and with freedom from the threat of insolvency. A recent E.U. case concerned the practice by the local authorities in Brussels to cover the systematic deficits of public hospitals, without offering similar subsidies to the competing private hospitals in the region.<sup>4</sup> Similarly, members of the U.S. business community were concerned that, while the Trans-Pacific Partnership was meant to level the playing field between state-owned enterprises and private businesses, in a growing part of the world with many state owned titans, it only applied to companies where a state (excluding local governments) controls 50 percent of voting rights and also contained exemptions for state-run sovereign wealth funds.<sup>5</sup>

Worries about such unequal treatments and the possible market distortions that may follow, raises the first question of this paper: *Why should public entities and private producers co-exist and compete in the same market?* If there are no specific gains to public-private competition, then the simplest solution is probably to reserve markets exclusively for either public or private producers, thereby eliminating the issues of fair competition all together. A case in point is the debate about the “public option” in American health insurance markets. While the proponents hope that public companies would both restore local competition and rectify that private companies deny coverage for some people, the critiques fear that a public option would allow the government to hide its inefficiencies and draw consumers away from private insurance, despite offering an inferior product.<sup>6</sup> As discussed below, it is possible find limited support for both sides in the previous literature.

On the other hand, if there are specific gains to public-private competition, it is important to understand what they might be. Only then can the second question of

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<sup>1</sup>Dept. for Culture, Media and Sport (2015); European Publishers Council (2009).

<sup>2</sup>ACPET (2012); ISCA (2014).

<sup>3</sup>Förvaltningsrätten (2013).

<sup>4</sup>European Commission (2016).

<sup>5</sup>Lawson (2015).

<sup>6</sup>See e.g. Krugman (2009) and Mankiw (2009).

this paper be asked: *How should mixed markets be organized to secure the gains from public-private competition?* The key regulatory issue is often perceived to be competitive neutrality: ensuring that government business activities compete on their merits and that they do not enjoy a net competitive advantage simply by virtue of their public sector ownership.<sup>7</sup> Achieving such competitive neutrality has motivated adjustments to many areas of public policy, including tax law, trade agreements, competition policy, public procurement regulation, as well as guidelines for corporate governance of state-owned entities.<sup>8</sup> There is, however, not much research to guide how the idea of competitive neutrality should be turned into law and practice.

The answers to these two questions are likely to vary from market to market and to depend on the political goals of the government. Over the past decades there has been a rise in the number of markets with mixed ownership.<sup>9</sup> In some sectors, public firms start to explore commercial opportunities, competing with existing private firms. The OECD (2016) estimates that 22% of the world's largest firms are now effectively under state control and they are active in finance (banking and insurance), energy, communication as well as in e.g. manufacturing. In the welfare sectors, it is the other way around. These sectors are being opened up to greater provision by private firms and third sector organizations in an attempt to reduce costs or increase quality. This paper focuses on the latter only: tax-financed welfare services provided by local governments. Examples may include child care, education, health care and long-term elderly care. The analysis presumes that the local government acts as a representative citizen, aiming to provide high quality for the users at a low cost for the tax-payers. Producer rents (both in the form of profits or supra-competitive wages) are regarded as costs for the taxpayers.

Following the literature on incomplete contracts (discussed below), my model includes two reasons why both markets and regulations fail to deliver the first best in the welfare service sector. The most basic problem is that many important quality dimensions can usually not be verified in a court of law. They are therefore not amenable for contracting or regulation by the government. Examples include teachers and doctors exercising considerable control over quality, e.g. through their choice of educational methods and patient treatments. In many cases, however, the users have at least limited ability to observe various aspects of non-verifiable quality even before the service is used. Families may e.g. visit different schools before selecting one and they may learn from their friends' previous experiences. Thus, while people traditionally were assigned to schools and hospitals based on proximity, today they are often given the right to select their own supplier of welfare services. The service providers then have to offer also non-verifiable quality to attract customers. The second problem is that such quality competition is limited.

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<sup>7</sup>Australian Governments (1996).

<sup>8</sup>European Union (2015); OECD (2015, 2016).

<sup>9</sup>Office of Fair Trading (2010).

One reason is that welfare services are provided on local markets, making them natural oligopolies. And the users choose among the local service providers not only based on the quality they provide but also based on the exact location of their premises. Another reason is that users typically find it difficult to compare the qualities offered by different producers. Thus, as producers can only poach a share of the rivals' customers by offering superior quality, their incentives to do so are limited. In such an environment, the allocation of decision rights matter. Here, it is assumed that the government can choose to either administrate production directly or to contract it out to private firms. With direct administration, the government sets production levels, provides the necessary resources (which may include both appropriations and physical resources such as school or hospital buildings suitable for the intended production volume), hires a manager and sets a wage. The wage is will be fixed since output is already fixed. The managers decide on their non-verifiable qualities, in competition with one another, to attract sufficiently many users to fill their production targets. With contracting, the government only sets the voucher price. In this case, the managers not only choose the non-verifiable qualities but also how much to produce. Ownership is thus associated with the right to decide on the production volume as well as the right to keep surpluses and the obligation to cover deficits. As elaborated further below, the difference between public and private ownership is also a difference in the "power of incentives" and a difference in "contractual completeness."

**Main results** The first main result is that a mix of public and private ownership can ease the tension between incentives and rents and may lead to higher service quality and lower public cost, compared to both private-only and public-only competition. If all producers are public, they will provide low non-contractible quality. The reason is that the managers have a common interest in any concerted reduction in quality that keeps volumes fixed at the levels decided by the owner. The advantage of public ownership is that costs are kept low (given any level of quality). It is sufficient for the government to pay the public managers a fixed compensation, equal to the cost of effort. If all producers are private, any level of quality can be induced. The reason is that a voucher system provides the owners/managers with high-powered incentives to offer high quality to attract customers. The drawback is that, if the government is confined to pay a fixed voucher price per user, then the private producers must be allowed to earn a positive profit. It follows that the choice between pure public ownership and pure private ownership is an instance of the classical trade-off between incentives and rents. A mix of ownership may then be preferred. In particular: when competing with private producers with high-powered incentives, public entities must provide high quality to fill ambitious production plans despite earning no rents. Overall costs are kept low since the public producers earn no rents and since they capture large market shares as a result of their superior quality.

But there is a caveat. The second main result is that the optimal regulation of a mixed market may be construed as a deviation from the principle of competitive neutrality. The government compensates the private producers with a price larger than their unit cost but which is lower than the public competitor's cost (which is higher as a result of a higher quality). Thus, the public producer must either be allowed to run a deficit that the owner covers with tax revenues, or the government must set the per-customer remuneration higher for public producers. In either case, one may speak of a tax-financed subsidy, which is available to public producers but not to their private rivals. The subsidy is reminiscent of predatory pricing. It enables the public producers to provide a quality that would cause a loss for an equally efficient private producer.

Therefore, I also examine the case when the (local) government's activities are constrained by a competitive neutrality regulation. Such a regulation may be warranted for reasons outside of the model considered here. Selective subsidies may *e.g.* be considered unfair or reduce the public managers' incentives to contain costs. As intuition suggests, competitive neutrality regulation benefits the private firms. They will get higher voucher prices, larger market shares and higher profits. But competitive neutrality regulation also makes mixed ownership less attractive for the (local) government. My third main result shows that this effect is particularly strong in case the users are immobile (due to difficulties comparing quality or high transportation costs), in which case pure public ownership is the second best alternative. Thus, the long run effect of a competitive neutrality regulation may be that the government switches from mixed ownership to pure public ownership. In the present model, there is never a switch to pure private ownership.

Clearly the model of this paper does not provide a comprehensive analysis of the costs and benefits of neither mixed markets nor competitive neutrality regulation. Many important issues have been left out, including "cream skimming" and "peer effects" just to mention a few. It follows that the above results should not be taken to prove that mixed ownership should be the preferred ownership model in most markets, not even in most welfare service markets, nor that competitive neutrality regulation would be harmful in most mixed markets. The results do indicate, however, that there can be specific gains to public-private competition (relaxing the tradeoff between incentives and rents) and that competitive neutrality regulation may thwart these gains under some circumstances. The results therefore suggest that competitive neutrality regulations might best be implemented selectively. An example of such selective intervention is the European regulation of "state aid." These rules provide the Member States with considerable discretion with regard to so-called services of general economic interest. However, a precondition for exemption is that the selective compensation is limited to cover the extra costs borne by producers with well-defined extra service obligations.<sup>10</sup> In contrast, this paper provides a rationale for public-private competition and subsidies to public entities exactly in the

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<sup>10</sup>Altmark judgement of the European Court of Justice, C-280/00, 24.7.03.

case when it is impossible to verify quality.

## 2 Related literature

A first rationale for mixed markets is simply that public and private firms are different, that the benefits of different ownership models may vary from sector to sector, and that the best solution is unknown. Therefore, the two ownership models should compete to provide the best solution in all sectors of the economy (Allais, 1948). This argument is, however, weak regarding tax-financed welfare services, since users base their choices on quality only and need not pay the price themselves. Later studies of mixed oligopoly assume that the main difference between private and public firms is what goals they pursue and how productive they are. One strand of the literature focuses on whether a public producer can reduce the welfare losses due to market power in an otherwise private oligopoly. The first papers argue that public producers may indeed fulfill such a role, assuming that public managers set prices to maximize social welfare (see Crémer et al., 1989 with references). Later studies provide less favorable results for mixed markets. With free (but costly) entry, the presence of a welfare (or output) maximizing public producer is irrelevant to overall welfare (Bennett and La Manna, 2012). Public producers may reduce market efficiency by increasing total production costs (De Fraja and Delbono, 1989). Another strand of the literature (Sappington and Sidak, 2003a, 2003b) focus on public firms' incentives for predation. Also they argue that public firms typically are instructed to pursue goals other than profit maximization. However, they emphasize goals such as increasing local employment or to ensure that affordable service is provided to low-income families. They also argue that managers (public or private) often are intrinsically motivated to expand the scale or scope of their operations in part, because a manager's abilities may be inferred from the size of the operations that he or she oversees. Public managers often have considerable discretion to pursue their own objectives. This discretion arises in part because public firms are not subject to takeover threats and are generally less subject to the discipline of capital markets than are private enterprises. As a result of such goals and lack of discipline, public firms may have stronger incentives than profit-maximizing firms to pursue activities that disadvantage competitors. The reason is that such goals make public firms value an expanded operating scale. Thus, allowing public producers to compete with private firms may cause inefficiencies.

A shortcoming of the previous studies of mixed markets is that they are not based on the idea of optimal regulation subject to information constraints. In sharp contrast, the literature on privatization (which, however, mostly focuses natural monopolies) has made use of incomplete contract theory (see Schmidt, 1996, and Hart, Shleifer and Vishny, 1997). The present paper follows this tradition. It describes the government's choice of ownership as a choice of regulatory (or contractual) completeness. Either the government

uses an “as-complete-as-possible regulation” including a specification of the production levels or it uses a “more-incomplete-than-necessary regulation” delegating the right to decide on quantity to the manager. The result that the government may prefer private to public ownership is thus an instance of so-called strategic ambiguity. As noted by Bernheim and Whinston (1998), once some aspects of performance (here: quality) are unverifiable, it might be optimal to leave other verifiable aspects (here: quantity) of performance unspecified. Wolinsky (1997) makes a related point in a context similar to mine. He studies the relative merits of price-regulated monopolies and price-regulated duopoly (called managed competition), when product quality is non-verifiable, using a Hotelling model. The question is thus whether the government should allow the customers to choose service provider (duopoly) or if it should assign exclusive territories (monopoly) which in effect means that not only price but also quantity is regulated. The main difference between my model and Wolinsky’s is that in my model the producers have to compete to fill their production plans, also when the government regulates each producer’s total quantity. In addition, I can also consider the asymmetric case, when one producer is under price regulation only and the other is under both price and quantity regulation.

A shortcoming of the present paper is that I simply equate ownership of a producer with the right to decide on the firm’s output. An alternative interpretation of my model is that the government keeps the ownership of both producers, but that it may delegate authority over quantity to the managers and rely on performance (piece rate) pay. Thus, the government could be described as simply choosing the power of incentives within the public sector. It either uses variable pay (a.k.a. high-powered incentives) to induce effort or fixed pay (a.k.a. low-powered incentives). Moreover, as Lazear (2000) notes, when a firm (here: the government) uses a fixed wage schedule, it typically couples the payment with some minimum level of output. Such a “new public management” interpretation presumes, however, that the owner actually can commit not interfere in the decision making process. Moreover, when discussing the mixed regime, it seems less reasonable to interpret private ownership as delegation within the organization. The reason is that employing two different management styles in parallels within the same organization may cause both confusion and opposition from the employees. Furthermore, despite the reforms during the last decades, introducing elements of competition and performance measurement, incentives are still mainly implicit, e.g. relying on individuals’ desire to attract users rather than on financial rewards (see Grout and Stevens, 2003). For these reasons, I will stick to the ownership interpretation throughout this paper. See Halonen-Akatwijuka and Propper (2008) for a model explicitly designed to study the delegation of decision rights, performance pay and competition, within the public sector.



### 3 Model

Consider some tax-financed welfare service, provided to the citizens free of charge by the (local) government. There are two producers, denoted by  $i = 1, 2$ . Each producer selects its own level of non-verifiable quality, denoted by  $z_i \geq 0$ .<sup>11</sup> There is a unit mass of users and everyone is asking for one unit of service. Before selecting a service provider, an individual user perceives a difference in quality if, and only if, this difference is larger than some threshold value,  $|z_i - z_j| \geq \theta$ . This threshold varies in the population. While some people are able to detect also small differences in quality, other people may not even detect quite substantial differences. In particular, it is assumed that  $\theta$  is uniformly distributed on the interval  $[0, t]$ , where  $t > 0$ . Thus if  $0 \leq z_i - z_j \leq t$ , the share  $s = \frac{z_i - z_j}{t}$  of the population observes which service provider has the highest quality. The informed users will all select that producer. The uninformed users choose a service provider at random, with equal probabilities.<sup>12</sup> Thus, the residual demand for the producer with higher quality is given by  $q_i = \frac{1}{2} \cdot (1 - s) + s = \frac{1}{2} \cdot (1 + s)$ . The producer with lower quality receives only  $q_j = \frac{1}{2} \cdot (1 - s)$  customers. Thus:

**Lemma 1.** *The residual demand for producer  $i$  is given by*

$$q_i = \frac{1}{2} + \frac{z_i - z_j}{2 \cdot t} \quad (1)$$

*if the difference in quality is not too large ( $|z_i - z_j| \leq t$ ). Otherwise the producer with higher quality serves the whole market.*

Note that equation 1 is simply the standard Hotelling demand model, but here used to model the users limited perception of quality differences. It follows that  $t$  can be interpreted either as the usual Hotelling transport cost, or as the highest perception threshold in the population. In either case,  $t$  captures some reason why competition may be limited, *viz.* geographical immobility or a limitation on the users' ability to gauge quality.

The government's valuation of (willingness to pay for) a unit of producer  $i$ 's service is

$$v^0 + v \cdot z_i,$$

where  $v^0$  is the value of a service with contractible quality only and where  $v > 0$  is the value of a unit of non-contractible quality. Note that these are the government's valuations and that they may differ from, and typically will be greater than, the users'

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<sup>11</sup>The assumption that the managers must decide on quality could alternatively be interpreted as a situation where managers would be left with real authority over quality, also in case formal authority resides with the government e.g. because it would be too costly for the government to gather sufficient information over overrule the managers' recommendations (*cf.* Aghion and Tirole, 1997).

<sup>12</sup>It is assumed that the uninformed users do not use market share or ownership as a signal of quality.

own valuations.<sup>13 14</sup>

Production also requires resources. The higher is the quality produced, the higher these production costs are likely to be. In particular, I assume that the producers' production cost per unit of output is increasing in non-contractible quality and given by

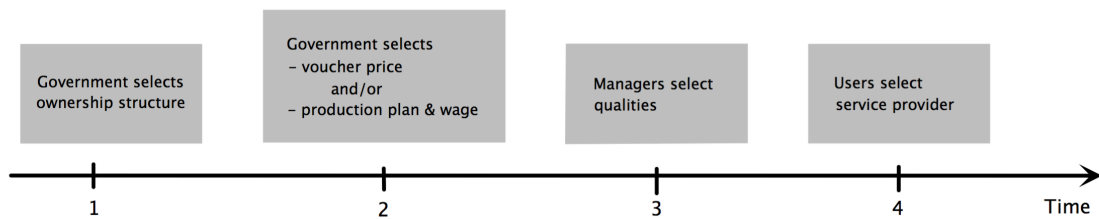
$$c^0 + c \cdot z_i,$$

where  $c^0$  is the cost associated with the provision of contractible quality and  $c \cdot z_i$  is the cost associated with non-contractible quality, including the manager's efforts. For the moment, it is assumed that this cost is not contractible.

Producers often have intrinsic preferences for offering high quality to their customers. Such preferences may be particularly important when it comes to the provision of services, where producers and customers actually meet in person. And they may be even more pronounced when it comes to welfare services such as education and health care, which are of great importance to the users. I assume that a manager's own intrinsic valuation of providing high quality services to the customers is given by  $b \cdot z_i \cdot q_i$ . However, to study the critical issues, I will focus on such dimensions of quality that are not voluntarily offered by the producers, i.e. the net private cost of offering quality is positive,  $c - b > 0$ . I also focus on the case when the value of quality is higher than the net private cost of producing it,  $v > c - b$ . I also assume that if a monetary compensation "crowds out" part of intrinsic motivation, then the effect is the same in case of a fixed compensation and incentive pay. That is,  $b$  is independent of ownership.

The timing is as follows.

Figure 1: Timeline



First, the government uses its regulatory powers to decide on the ownership structure.

<sup>13</sup>In fact, the reason why welfare services in many countries are financed through taxes and provided to the users free of charge is often that the government has a higher willingness to pay. This difference may e.g. arise from positive externalities, specific egalitarianism (i.e. the wish to make the certain goods available to all citizens independent of their ability to pay (Tobin, 1970)), or that the services are so-called merit goods (i.e. goods that yield long-term private benefits that the users tend to undervalue (Ng, 1983)), or a combination of these reasons.

<sup>14</sup>A condition for a voucher system to fulfill the political goals is that the users value the same qualities as the government (think of grade inflation in schools) and that the threat of over-use (which is a risk in health care) can be controlled. I will neglect these problems here and instead focus on another problem, namely the lack of competition.

The producers may all be publicly owned, privately owned or a mixture of both.<sup>15</sup> Second, the government sets the voucher price  $p$ , which must be the same for all producers independent of ownership.<sup>16</sup> The government also sets the compensation schedule (wage and production plan) for public managers. Third, the managers simultaneously decide on their qualities. Finally, the users select service providers, based on their qualities.

### 3.1 First best

The local government acts as a representative citizen in their capacities as users of welfare services and taxpayers. Producer rents in the form of profits or supra-competitive wages are only considered as costs for the taxpayers. The reason may be that the firms are owned by people outside the municipality, that producer rents conflict with the government's concerns for distribution, or that such rents reduce the probability of reelection. The government's objective function, also to be called the social welfare function, has three components. To reduce clutter, I set  $v^0 = c^0 = 0$ . The social benefits is the sum of the value of all services,  $B = \sum_i v \cdot z_i \cdot q_i$ . If the government's expenditures are  $E$ , the social cost is  $C = E + \frac{\lambda}{2} \cdot E^2$  where  $\lambda > 0$ . The reason why social cost is convex in expenditures is that the expenditures for social services such as education and health care are large enough to affect the tax rate and thus the cost of public funds.<sup>17</sup> Finally the government may care about equity. That is, the government may wish to avoid situations where some citizens, as a result of their geographical location or their inability to spot quality differences, use welfare services of lower quality than other citizens. The government's disutility from inequity is given by  $-\alpha \cdot I$  where  $\alpha \geq 0$  is the strength of the government's inequity aversion and  $I = \frac{1}{2} \cdot [(z_i - z_j) + \frac{1}{t} \cdot (z_i - z_j)^2]$  is a quadratic function of the difference in quality,  $z_i - z_j > 0$ .<sup>18</sup> The government's objective function (social welfare) is thus given by

$$W = B - \left[ E + \frac{\lambda}{2} \cdot E^2 \right] - \alpha \cdot I.$$

Before studying the different ownership arrangements, it is instructive to compute the first best as a benchmark. The first best would be achieved if the government could chose the two producers' qualities directly and simply pay the corresponding net private cost of production, *i.e.*  $E = \sum_i (c - b) \cdot z_i \cdot q_i$ .

**Lemma 2.** *If the government could decide on non-contractible quality directly, it would*

<sup>15</sup>I focus on public entities and private for-profit firms, leaving third sector organizations out for now.

<sup>16</sup>Providers of tax-financed services cannot be allowed to set their own prices since the users have no reason to search for cheaper deals. Some form of price regulation, such as a voucher system, is needed.

<sup>17</sup>This assumption is made in order to "convexify" the government's optimization problem. An alternative strategy would have been to assume that the cost of producing quality is strictly convex or that the value of increased quality is strictly concave. The chosen strategy produces less clutter.

<sup>18</sup>The factor  $\frac{1}{t}$  multiplying the quadratic term is included for algebraic convenience. The inequity term could alternatively be written as  $-\alpha \cdot I = -\alpha \cdot (z_i - z_j) \cdot q_i$ .

set the same quality

$$z^* = \frac{v + b - c}{\lambda \cdot (c - b)^2} > 0,$$

for both producers and social welfare would be

$$W^* = \frac{1}{2 \cdot \lambda} \cdot \left( \frac{v + b - c}{c - b} \right)^2 > 0.$$

Proofs are relegated to the Appendix. The first best, thus, requires the producers to provide positive levels of non-contractible quality.

## 4 Public vs. private ownership

Before studying mixed ownership it is instructive to compare pure public and pure private ownership, as this has been the main focus of the previous literature.

### 4.1 Public ownership

I will start with pure public ownership since there is far less analysis of competition between public entities than private firms. In the present paper public ownership is characterized by the following assumptions. First, it is the (local) government that decides how many services that each entity should produce. Traditionally, people were assigned to schools and hospitals based on proximity. Today users are allowed more choice and the providers must compete for users to deliver the number of services expected from them. It is this competition that is the focus of this section. Second, the government also provides the resources for filling the plans. Traditionally, most resources were provided in-kind. Common examples include school and hospital buildings with dimensions embodying the production plan. Politicians even decided on staffing levels of these schools and hospitals. Today it is more common that local governments allow their managers more autonomy by providing appropriations to cover the necessary costs. (This difference has little consequence in the current model, since I assume that the government has perfect information about the cost function.) Third, the government hires a manager for each service provider and sets their wages. The wages are fixed since output is decided by the government and quality is not contractible. In fact, financial incentives still appear to be of relatively small importance in the public sector (Grout and Stevens, 2003).

**Quality competition** While the government cannot instruct a public manager to provide a certain level of quality, it has some ability to influence the managers' choice of quality indirectly by setting the appropriate production plans. Let  $\underline{q}_i \geq 0$  denote the required production level for producer  $i$ . (For the production plans to be meaningful it

is required that both producers could fill their plans at the same time, *i.e.*  $\underline{q}_1 + \underline{q}_2 \leq 1$ . Without loss of generality, producers are numbered such that  $\underline{q}_2 \geq \underline{q}_1$ .) If a public entity succeeds to fill its plan,  $q_i(z_i, z_j) \geq \underline{q}_i$ , the manager receives the compensation  $w_i$  and enjoys utility  $u_i(z_i, z_j) = w_i - (c - b) \cdot z_i \cdot q_i(z_i, z_j)$ , where  $q_i(z_i, z_j) = \frac{1}{2} + \frac{z_i - z_j}{2 \cdot t}$  describes the users' choices of service provider.<sup>19</sup> If not, the manager is swiftly replaced by somebody else. The manager then receives the reservation utility, normalized to zero. The manager will thus choose either to supply no effort at all or the minimum effort for reaching the required production level, which is given by

$$\underline{Z}_i(z_j) = \max \left\{ 2 \cdot t \cdot \left( \underline{q}_i - \frac{1}{2} \right) + z_j, 0 \right\}.$$

Conforming with the production plan is then a best reply if the wage is higher than the necessary net effort cost, *i.e.*  $w_i \geq (c - b) \cdot \underline{Z}_i(z_j) \cdot \underline{q}_i$ .<sup>20</sup> Thus, a public manager will produce a higher quality, the higher quality produced by the other producer and the higher the required production level is.

A few minor clarifications are warranted. First, notice that the manager's compensation is not contingent on output in any other way than to reflect whether the government's production target has been reached. This provides the manager with the maximum incentives to fill the plan (but not necessarily to provide quality). Second, there are two versions of the incomplete contracting problem. Either the government cannot *observe* quality or it cannot fire a public manager based on a too low (observable but) *non-verifiable* quality. Third, notice that if one of the public managers does not meet the required production level, the demand for the other public entity's services could exceed its required production level (which would require that the second producer receives more resources). This will not happen in equilibrium. It will also not happen out of equilibrium since the failing manager is swiftly replaced.

As it turns out, we may confine attention to production plans that sum to one ( $\underline{q}_1 + \underline{q}_2 = 1$ ) without loss of generality.<sup>21</sup>

**Lemma 3.** *If  $\underline{q}_1 + \underline{q}_2 = 1$ , any  $(z_1, z_2)$  with  $z_1 \geq 0$  and  $z_2 = 2 \cdot t \cdot \left( \underline{q}_2 - \frac{1}{2} \right) + z_1$  is a Nash equilibrium, given that the wages cover the managers' net costs, *i.e.*  $w_i \geq (c - b) \cdot z_i \cdot \underline{q}_i$ .*

<sup>19</sup>I disregard the fact that production plans may sometimes also not be exceeded. Rationing is *e.g.* common in higher education.

<sup>20</sup>More precisely, if  $z_i \geq \underline{Z}_i(z_j)$ , the manager stays on the job during the full period and receives utility  $w_i - (c - b) \cdot \underline{Z}_i(z_j) \cdot \underline{q}_i$ . If  $z_i = 0$ , the manager receives utility  $w_i \cdot \Delta$  where  $\Delta$  is the *share* of the time left until being replaced. The manager exerts the minimum necessary effort if  $w_i \geq (1 - \Delta)^{-1} \cdot (c - b) \cdot \underline{Z}_i(z_j) \cdot \underline{q}_i$ . If  $\Delta \approx 0$ , the necessary "bonus" is negligible. And to reduce clutter, I omit it in the calculations. If the manager is replaced, the new manager faces exactly the same tradeoff since  $\Delta$  is the *share* of the time left.

<sup>21</sup>The reason is that this restriction does not reduce the set of outcomes that the government can induce. However, the full set of Nash equilibria, including the case  $\underline{q}_1 + \underline{q}_2 < 1$ , can be found in Lemma 9 in the Appendix, together with all proofs.

Notice that by setting  $\underline{q}_1 = \underline{q}_2 = \frac{1}{2}$  any  $z_1 = z_2$  is a Nash equilibrium. Thus, if the government has the ability coordinate the managers' equilibrium expectations, it could implement any qualities without leaving the managers with any rents. That is, the government could implement the first best. However, such an outcome would be vulnerable to coordinated deviations from the two managers. To select a reasonable equilibrium, one may use the notion of coalition proof equilibrium. Recall that in a game with two players, an equilibrium is coalition proof only if there does not exist any other Nash equilibrium which both prefer. Thus:

**Lemma 4.** *If  $\underline{q}_1 + \underline{q}_2 = 1$ , then  $z_1 = 0$  and  $z_2 = 2 \cdot t \cdot \left(\underline{q}_2 - \frac{1}{2}\right) \leq t$  is the unique coalition proof equilibrium.*

The argument is straight forward. If  $z_1 > 0$ , the two managers could agree to simultaneously reduce the two qualities by the same amounts without any loss of wage. Thus only  $z_1 = 0$  is coalition proof.<sup>22</sup>

It should be noted that there is nothing generic about the number  $\frac{1}{2}$  in the Lemma. It is simply the *natural customer* base of the producer, i.e. the producer's market share when both producers offer the same quality ( $z_1 = z_2$ ). If one producer is better located than the other, its natural customer base would be larger, say  $\frac{3}{4}$ . Inducing such a better located producer to produce a higher quality than the competitor would require setting that firm's production plan higher than its natural customer base, i.e. higher than  $\frac{3}{4}$ .

**Corollary 1.** *When both producers are publicly owned, the government can only induce quality above the contractible level by requiring one producer to attract customers beyond its natural customer base implying that the citizens get access to unequal levels of quality.*

A possible example of such a public policy with asymmetric quality is the Swedish higher education system, which is essentially populated by public universities only. Some of these universities receive better funding and are supposed to provide a higher quality education than other "regional" universities. It appears likely that the bigger universities preferential treatment is contingent on their ability to recruit students from the whole country and not just their surrounding areas.

**Government policy** Before the two public entities start to compete, the government designs an incentive structure, by setting the production plans and wages, to maximize social welfare. To describe government's choice, let  $t^G = \frac{v+b-c-\alpha}{\lambda \cdot (c-b)^2}$  and  $\alpha^G = v + b - c$ .

**Lemma 5.** *If the government is very concerned with equality ( $\alpha \geq \alpha^G$ ), it sets  $\underline{q}_2 = \frac{1}{2}$ . Then both producers provide zero non-verifiable quality. If the government is less*

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<sup>22</sup>The result that public managers provide zero non-verifiable quality (when  $\underline{q}_i = \frac{1}{2}$ ) is due to the assumption that their private benefits from providing quality (intrinsic motivation) is described by the term  $b \cdot z_i \cdot q_i$ . If the model would include a term such as  $b_z \cdot z_i$  or  $b_{dz} \cdot (z_i - z_j)$  or  $b_q \cdot q_i$  (since  $q_i$  is a public signal of quality), the coalition proof equilibrium would entail positive levels.

concerned with equality, it sets  $\underline{q}_2 = \frac{1}{4} + \sqrt{\frac{1}{16} + \frac{v+b-c-\alpha}{2 \cdot t \cdot \lambda \cdot (c-b)^2}} \in (\frac{1}{2}, 1)$  if competition is lax ( $t > t^G$ ) and  $\underline{q}_2 = 1$  if competition is intense. In equilibrium, social welfare is given by

$$W^G = \begin{cases} 0 & \alpha \geq \alpha^G, \\ \frac{1}{2 \cdot \lambda} \cdot \left[ \frac{v+b-c-\alpha}{c-b} \right]^2 > 0, & \alpha < \alpha^G, t > t^G, \\ \left[ (v+b-c-\alpha) - \frac{\lambda}{2} \cdot (c-b)^2 \cdot t \right] \cdot t > 0, & \alpha < \alpha^G, t \leq t^G. \end{cases}$$

An unexpected consequence of the Lemma is that  $W^G$  is increasing in  $t$ . In words:

**Corollary 2.** *When both producers are publicly owned, social welfare is increasing in the users' immobility (geographical immobility or their inability to perceive quality differences).*

To obtain intuition for this result, one should inspect the best reply function for the manager with the ambitious production plan ( $\underline{q}_i > \frac{1}{2}$ ). The less mobile the users are, the higher quality the manager must offer to reach the production plan. This result even suggests that under a system of pure public ownership, the government may not wish to make non-verifiable information about quality differences between service providers public.

## 4.2 Private ownership

Private firms select quality to maximize their profits.<sup>23</sup> The profit of producer  $i$  is given by  $\pi_i = (p - (c - b) \cdot z_i) \cdot \left( \frac{1}{2} + \frac{z_i - z_j}{2 \cdot t} \right)$  and it's best-reply function is  $z_i = \max \left\{ \frac{p - t \cdot (c - b)}{2 \cdot (c - b)} + \frac{1}{2} \cdot z_j, 0 \right\}$ . Qualities are thus strategic complements also under private ownership: the higher quality produced by one producer, the higher quality the competitor wishes to provide.

**Lemma 6.** *When both producers are privately owned, they produce the same quality, given by  $z^P = \max \left\{ \frac{p}{c - b} - t, 0 \right\}$ . The equilibrium profit is given by  $\pi^P = \frac{(c - b) \cdot t}{2}$ , which is strictly positive whenever quality is strictly positive.*

Notice that the government can induce the private firms to produce whatever quality it desires, by setting a sufficiently high price tag. To implement  $z > 0$ , the government must set  $p = (c - b) \cdot z + (c - b) \cdot t$  where  $(c - b) \cdot z$  is the marginal and average net private cost of producing one unit of service and  $(c - b) \cdot t$  is a necessary information rent.

As in the standard Hotelling model, private competition is less efficient the higher the users' transportation cost are. The producers' thus earn higher rents the less mobile the users are. A more surprising part of the result is that:

**Corollary 3.** *When both producers are privately owned, the equilibrium producer rents (profit) are higher the higher is the (net) private cost of quality,  $c - b$ .*

<sup>23</sup>I use the term "profit" despite including the private benefits of quality.

To obtain some intuition, notice that by the Envelope Theorem, cost has two effects on equilibrium profits (at a given voucher price). Increasing the cost of quality clearly has a negative direct effect on profits. But there is also a positive indirect effect: a higher cost of quality implies that the competitor produces a lower quality, which increases a firm's residual demand. As it turns out, the positive indirect effect dominates the negative direct effect.

Before the producers start to compete, the government sets the voucher price to maximize welfare. To describe the government's choice, let  $t^P \equiv \frac{v+b-c}{2 \cdot v} \cdot \frac{v+b-c}{\lambda \cdot (c-b)^2} < \frac{v+b-c}{\lambda \cdot (c-b)^2}$ .

**Lemma 7.** *The government sets a positive voucher price  $p = \frac{v+b-c}{\lambda \cdot (c-b)} > t \cdot (c-b) > 0$ , implying positive non-contractible quality, if users are sufficiently mobile ( $t \leq t^P$ ). Otherwise the voucher price and non-contractible qualities are set to zero. Social welfare is given by:*

$$W^P = \begin{cases} \frac{1}{2 \cdot \lambda} \cdot \left[ \frac{v+b-c}{c-b} \right]^2 - v \cdot t > 0, & t \leq t^P, \\ 0, & \text{otherwise.} \end{cases}$$

When the users are mobile ( $t \leq t^P$ ), the government induces a quality that is lower than the first best and it leaves the producers with a rent. The reason why the government does not offer a positive voucher price when the users are immobile is that quality competition is rather ineffective in that case. Finally, for future reference, note that:

**Corollary 4.** *When both producers are privately owned, the optimal voucher price is higher, the higher is the government's valuation of quality ( $v$ ). As a result, both producers increase their qualities.*

### 4.3 Pure private vs. pure public ownership

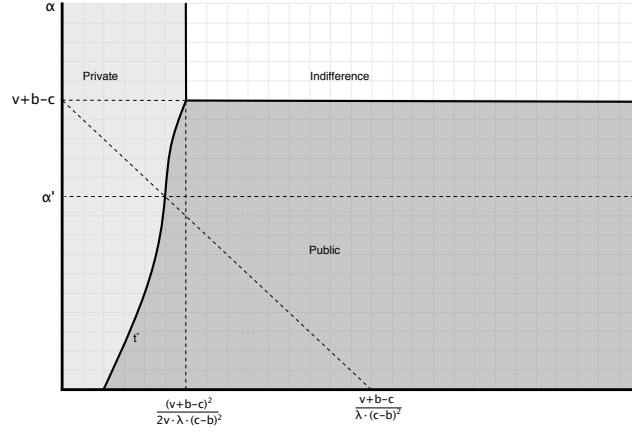
I will say that a certain property A is *more likely* than another property B if the subset of the parameter space where A is true contains the subset in which B is true. Similarly, I will say that a higher  $v$  makes property A more likely if a higher  $v$  enlarges the subset of the parameter space in which A is true.

The following proposition characterizes the government's preferences over (pure) private and public ownership, when a mixed ownership model is not feasible.

**Proposition 1.** *The government is more likely to prefer private ownership to public ownership the more mobile the users are (lower  $t$ ), and the more averse the government is to inequality (higher  $\alpha$ ). If users are sufficiently mobile ( $t \leq t^P$ ) and inequality-aversion is sufficiently low ( $\alpha \leq \alpha^G$ ), the government is more likely to prefer private ownership to public ownership the higher the government's own valuation of quality ( $v$ ) is, the lower the effort cost of quality ( $c$ ) is, and the higher the managers' intrinsic motivation to provide quality ( $b$ ) is.*



Figure 2: Pure private vs pure public ownership



The first part of the proposition is also illustrated in figure 2.

The reason why inequality aversion reduces the attractiveness of pure public ownership is that the government can only induce quality above the contractible level by requiring one producer to attract customers beyond its natural customer base thereby accepting that the citizens get access to unequal levels of quality (Corollary 1).

The reason why a low ability to observe quality differences and low geographical user mobility (*i.e.* a high  $t$ ) increases the relative strength of pure public ownership is that a high  $t$  both weakens quality competition between private producers and actually (Corollary 2) improves quality provision when both producers are public.

The reason why a government with a high valuation of non-verifiable quality is more likely to prefer pure private ownership is that the government then can set a high voucher price and induce both producers to provide high quality (Corollary 4). With public ownership at least one producer provides zero quality, independent of its value.

The role of the net private cost of quality ( $c - b$ ) is less obvious, since a higher cost reduces welfare with both types of ownership. But part of the reason why a high net cost of quality increases the relative strength of public ownership is that the government must leave private producers with higher rents, the higher is their net cost of quality (Corollary 3).

## 5 Mixed ownership

Consider now the case when one producer is private and the other is public. The two producers' best reply functions are the same as above. Given any government policy ( $p$ ,  $q_2$ ,  $w_2$ ), with a sufficiently high wage, there is a unique equilibrium in the quality competition game. The details of the equilibrium are described by Lemma 11 in the Appendix. Here, I will simply compare this outcome with quality competition under pure public and pure private ownership. The first Corollary demonstrates that all outcomes that can be

achieved under pure public ownership can be replicated under mixed ownership.

**Corollary 5.** *If the government does not offer the private firm any margins above the cost of providing verifiable quality, i.e.  $p = 0$ , the private producer does not supply any non-verifiable quality. Then, the public producer supplies the same non-verifiable quality as the high-quality producer under pure public ownership, that is  $z_2 = 2 \cdot t \cdot \left(\underline{q}_2 - \frac{1}{2}\right)$ .*

The second Corollary demonstrates that the government can replicate any outcome under pure private ownership.

**Corollary 6.** *If the public producer is ordered to serve half the market,  $\underline{q}_2 = \frac{1}{2}$ , the public producer will simply match the quality offered by the private producer. As a result, both producers will produce the same quality as under pure private ownership, that is  $z = \max\left\{\frac{p}{c-b} - t, 0\right\}$ .*

A difference is that the public firm's surplus is recouped by the government. Inducing high quality is therefore cheaper under mixed ownership than under pure private ownership.<sup>24</sup> The final Corollary demonstrates that the government also can achieve other outcomes than under pure public and pure private competition:

**Corollary 7.** *Increasing the voucher price above the cost of contractible quality ( $p > 0$ ) increases the private firm's willingness to conquer market shares, and also forces the public producer to respond. In fact, the public producer has to increase its quality by the same amount as the private producer, to be able to defend its assigned market share. Increasing the production plan for the public producer (above  $\underline{q}_2 = \frac{1}{2}$ ) increases the public firm's quality and indirectly also the private firm's quality (if  $p > 0$ ). However, the private producer increases its quality by a smaller amount than the public producer. Thus, by increasing the public firm's production plan beyond a half, inequality is increased.*

Thus, under mixed ownership, the government can either increase the private firm's voucher price or the the public firm's production plan to induce both producers to supply higher non-verifiable quality. Since a higher voucher price leaves the private firm with higher rents and a more ambitious production plan for the public firm increases inequality, the government has to balance these negative effects.

**Government policy** The government's optimal policy is described by the following proposition.

**Proposition 2.** *Consider mixed ownership. The government offers a voucher price above the cost of providing contractible quality (i.e.  $p > 0$ ) to give the private producer high-powered incentives to conquer market shares by offering positive non-contractible quality if,*

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<sup>24</sup>Thus, mixed ownership is a partial remedy to the government's inability to complement the voucher system with fixed fees (see below) under pure private ownership.

and only if, the users are sufficiently mobile. The government assigns an ambitious market share for the public producer to defend ( $\underline{q}_2 > \frac{1}{2}$ ), inducing it to produce an even higher non-contractible quality despite its low-powered incentives if, and only if, it is modestly inequality avert.

The proof, which is relegated to the Appendix, demonstrates that there exists a continuous function  $t^*(\alpha)$  such that (generically)  $p > 0$  if  $t \leq t^*(\alpha)$ . The condition for  $\underline{q}_2 > \frac{1}{2}$  is  $\alpha < 2 \cdot v$  if  $t \leq t^*(\alpha)$  and  $\alpha < v + b - c$  otherwise.

One may think of the government as designing a “Conquest & Defense Game” between the private and the public producer with the purpose to elevate the level of non-contractible quality.

**Choice of ownership structure** When there is pure public ownership, the government can influence the producers’ choices of qualities with one independent instrument, namely the production target  $\underline{q}_2$ . Also, when there is pure private ownership, the government has one instrument, namely the price  $p$ . With mixed ownership, the government can use both these instruments. Thus, the first main result of this paper is that:

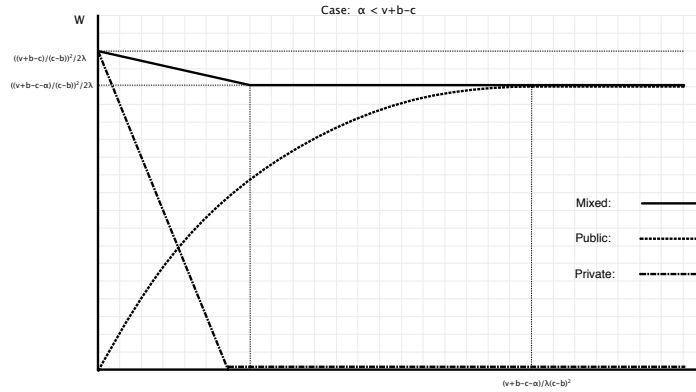
**Proposition 3.** *Mixed ownership (weakly) dominates both pure private and pure public ownership. In particular,*

- Social welfare is higher under mixed ownership than under pure public ownership if users are sufficiently mobile ( $t \leq t^*(\alpha)$ ). The difference is that the government can use  $p > 0$  to induce higher quality from both producers, without causing inequality. Otherwise the two ownership modes yield the same welfare.
- Social welfare is higher under mixed ownership than under pure private ownership if the government’s inequality aversion is sufficiently mild ( $\alpha < v + b - c$ ) or if users are sufficiently mobile ( $t \leq t^*(\alpha)$ ). The difference is that the government can use  $\underline{q}_2 > \frac{1}{2}$  to induce higher quality from both producers, at lower cost for the tax-payers. Otherwise the two ownership modes yield the same welfare.

The essence of the proof is a straight-forward replication argument. By setting  $p = 0$ , the government can implement the same  $(\underline{q}_2, z)$ -combinations as under pure public ownership (Corollary 5). Thus, whenever the government sets  $p > 0$ , mixed ownership must be preferred. By setting  $\underline{q}_2 = \frac{1}{2}$ , the government can implement the same  $(p, z)$ -combinations as under private ownership (Corollary 6). Thus, whenever the government sets  $\underline{q}_2 > \frac{1}{2}$ , mixed ownership must be preferred. And even when the same  $(p, z)$ -combinations are implemented, this is cheaper under mixed ownership, since only one producer keeps the rent.

The welfare levels under the different ownership regimes are also described by figure 3 for the case when the government’s inequality aversion is modest ( $\alpha < v + b - c$ ).

Figure 3: Welfare under different ownership structures



It is instructive to consider the case when unequal qualities is not perceived as a problem (i.e.  $\alpha \approx 0$ ). Then, the mixed market comes close to first best. In particular, the public producer can be induced to produce a quality close to the first best quality and to serve most of the market. The rents paid to the private producer are consequently small. But even if the private producer would serve only a trivial fraction of the population, *i.e.* even if  $q_1 = \frac{3}{4} \cdot \frac{\alpha}{v+\alpha} \approx 0$ , the presence of the private producer is necessary to discipline the public producer to provide high quality services to everyone else. An agreement between the managers to lower quality, keeping market shares fixed, would not be self-enforcing as a private producer always has an interest in poaching customers from the its rival, regardless of market shares. (The private producer is then used in a manner similar to a pace setter - a “rabbit” - in a track race. It is not supposed to win, but it is necessary to get the others going.)

**Competitive neutrality** To maximize social welfare under mixed ownership, the government induces the public producer to produce a higher quality than the private producer (unless inequality aversion is very high). The public producer consequently also has a larger market share than the private producer. Clearly, the government must offer a voucher price that is high enough to cover the private firm’s cost. But, to leave the private producer with the lowest possible rents, the voucher price is sometimes set lower than the cost of the public competitor. Thus, a public producer given the same voucher price as the private producer must be allowed to run a deficit that the owner covers with tax revenues. An alternative is that the government sets the per-customer remuneration higher for the public producer than for the private producer. In either case, one may speak of a tax-financed subsidy, which is available to public producers but not to their private rivals. The second main result of the paper is:

**Proposition 4.** *To maximize social welfare under mixed ownership, either the public producer must be allowed to run a deficit or the compensation must be set higher for the*

public producer than for the private producer (assuming that  $t < t^*(\alpha)$  and  $\alpha < \frac{v}{2}$  or  $t \geq t^*(\alpha)$  and  $\alpha < (v + b - c) - \frac{3}{8} \cdot t \cdot \lambda \cdot (c - b)^2$ ).

Such a subsidy may be construed as deviation from the principle of competitive neutrality. The suggested scheme even resembles predatory pricing.

**Corollary 8.** *The public producer provides a quality, so high that the costs cannot be covered by the revenues from the market (under the same conditions as in Proposition 4). Any equally efficient private producer providing the same quality would consequently make a loss and would be forced into bankruptcy.*

A possible example of such public “predatory quality” is the the public service broadcasting companies such as the BBC and SVT/SR in Sweden.

## 6 Competitive neutrality regulation

Subsidies to public producers are more problematic than described above. I have neglected the potential problems associated with so-called soft budget constraints (see e.g. Megginson and Jeffry, 2001). I have also neglected that subsidized public producers may be expected to engage in predatory pricing, even squeezing private rivals out of the market, under some circumstances (Sappington and Sidak, 2003a and 2003b).

Thus, to reap the full benefits of public-private competition, it might be necessary to decide on the legality of subsidies on a case-by-case basis (cf. De Fraja, 2009). However, such a flexible approach to competitive neutrality may not be feasible due to information problems. Then, it may be necessary to implement competitive neutrality regulation also in the market analyzed above.

I should also point out that my previous analysis builds on the assumption that competitive neutrality is primarily a means for promoting efficiency and less of a goal of procedural fairness in it self. That is, competitive neutrality is not meant to protect private producers at the expense of the interests of the users or to upset broader political goals for social welfare and equity.<sup>25</sup> In contrast, private competitors often argue that public subsidies only available to public producers are *unfair* and unwarrantable for that reason.

Thus, for both reasons of efficiency and procedural fairness, it is important to study what consequences a prohibition of subsidies to public entities in competition with private producers would have in the current model. Any negative consequences would have to be counted as a cost of such a regulation.

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<sup>25</sup>Such an interpretation is supported by the policy statement on competitive neutrality by the Government of Western Australia (1996), the Swedish Competition Act (paragraph 27) and the fact that EU state aid rules on services of general economic interest allow Member States to grant compensation for the fulfillment of clearly defined public service obligations under certain conditions.

A competitive neutrality regulation requires that  $p \geq (c - b) \cdot z_2$  to ensure that the private producer receives the same remuneration per unit of output as the public producer. (Setting  $p > (c - b) \cdot z_2$  is not a problem since, then, the government simply collects the surplus created within the public producer as a profit.)

Note that if the competitive neutrality restriction binds when  $t \geq t^*(\alpha)$ , the government prefers to switch from mixed to public ownership. The reason is that the government is indifferent between the two ownership models absent the competitive neutrality restriction. Thus, I will only consider the case when  $t < t^*(\alpha)$ .

**Lemma 8.** *Assume that  $t < t^*(\alpha)$  and that one producer is publicly owned and the other privately owned. A competitive neutrality regulation is binding if, and only if,  $\alpha < \frac{v}{2}$ . When a binding competitive neutrality regulation is imposed, the government increases the voucher price and lowers the production plan for the public producer. As a result, the private firm's profit is increased. Social welfare is then given by*

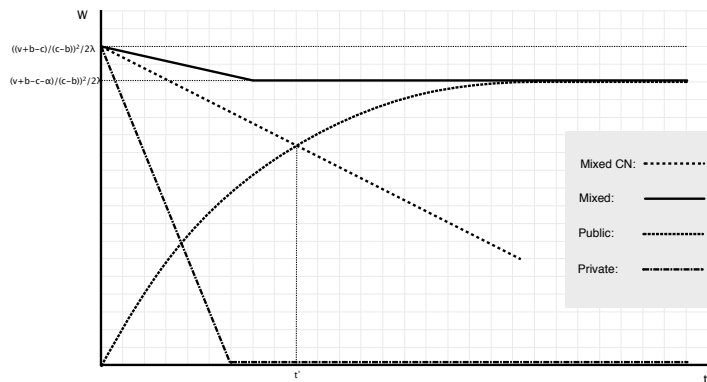
$$W^{CN} = \frac{1}{2 \cdot \lambda} \cdot \left( \frac{v + b - c}{c - b} \right)^2 - \frac{v + 3 \cdot \alpha}{8} \cdot t.$$

However, under a “constitutional ban” on public subsidies of public producers in competition with private producers, the government would clearly be less inclined to promote mixed markets, and rather select one of the two pure-ownership models.

**Proposition 5.** *Under a binding competitive neutrality regulation, the government prefers pure public ownership to mixed ownership if the users are sufficiently immobile (and  $\alpha < \alpha^G$ ).*

The proof of the proposition follows from a simple comparison of the welfare levels under the different regimes, as in figure 4.

Figure 4: Welfare under different ownership structures



The key point is that a competitive neutrality regulation tends to weaken the mixed-ownership model more in case pure-public ownership is the second best alternative and less when pure-private ownership is the alternative.

## 7 Extensions

### 7.1 Complex compensation

It is sometimes possible to achieve a better outcome with private ownership (i.e. assuming the manager to decide on quantity) if a more complex compensation model can be used. If the government could charge the private producers a fixed fee for the right to operate in the market, it could implement the first best. The voucher price could then be set high to provide the producers with strong incentives to provide quality and the fixed fee could be used to recoup all the rents.

In the present model, the government could achieve the first best also without fixed fees, in case all users can distinguish between the first-best quality and no quality (i.e.  $t \leq z^*$ ). To see this, assume that both producers are private and that the compensation schedule is quadratic and given by

$$T(q_i) = (c - b) \cdot [(z^* - t) + 2 \cdot t \cdot q_i] \cdot q_i + r \cdot I_{q_i=\frac{1}{2}}$$

where  $r \geq 0$  and  $I_{q_i=\frac{1}{2}}$  is the indicator function. Notice that  $T(q_i) \geq 0$  for all  $q_i$  since  $z^* \geq t$ . Then, all  $z_1 = z_2 \geq z^* - \frac{r}{c-b}$  are Nash equilibria.<sup>26</sup> Moreover, since all equilibria give rise to the same utility  $r$  for the managers, they are all coalition-proof. Thus, by coordinating expectations on  $z_1 = z_2 = z^*$  the government implements the first best.

However, as fixed fees (and quadratic terms) are rare in reality, I disregard them in the main part of the paper. A possible reason for this is that the government does not know the producers' costs (for more on this, see Wolinsky, 1997). Paying a high fixed fee would also aggravate uncertainty in the market, *e.g.* if the producers are not fully informed about the demand for their services.

### 7.2 Regulation of inputs

It is sometimes argued that the government should impose minimum requirements on the use of certain inputs that can be measured and that are believed to be associated with high quality. Examples in the school sector include the number of qualified teachers per pupil and various facilities such as libraries and gyms. An alternative is that the government would provide also private schools with *e.g.* gyms. The question is if such

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<sup>26</sup>The manager's utility is given by  $u_i(q_i) = T(q_i) - (c - b) \cdot z_i \cdot q_i$ , or, since  $q_i = \frac{1}{2} + \frac{z_i - z_j}{2t}$ ,  $u_i(q_i) = (c - b) \cdot (z^* - z_j) \cdot q_i + r \cdot I_{q_i=\frac{1}{2}}$ . Next, I derive firm  $i$ 's best reply function. If  $z_j = z^*$ , then it is a unique best reply for  $i$  to set  $z_i = z_j = z^*$ , implying  $q_i = \frac{1}{2}$ , giving utility  $r$ . All other quantities yield utility zero. If  $z_j > z^*$ , then  $i$  either sets  $z_i = 0$  implying  $q_i = 0$  (since  $z_j > z^* > t$ ) and thus  $u_i = 0$  or  $z_i = z_j$  implying  $q_i = \frac{1}{2}$  and thus  $u_i = r$ . If  $z_j < z^*$ , then  $i$  either induces  $q_i = 1$  (requiring  $z_i = t + z_j$ ) yielding  $u_i = (c - b) \cdot (z^* - z_j)$  or  $q_i = \frac{1}{2}$  (requiring  $z_i = z_j$ ) yielding  $u_i = r$ . The previous choice is better iff  $z_j \leq z^* - \frac{r}{c-b}$ . Finally, plotting the best reply functions reveals that  $z_1 = z_2 \geq z^* - \frac{r}{c-b}$  are Nash equilibria.

regulation of inputs could improve the provision of non-contractible quality?

To study this issue I assume that non-contractible quality requires both managerial effort and the use of a specialized input. By “specialized” I mean that it can not be used instead of other resources in the production of contractible quality. In particular, consider a Leontief technology for the production of quality adjusted output,  $z_i \cdot q_i$ . A producer must use one unit of the (observable) specialized input and  $e$  units of (unobservable) managerial effort for every unit of  $z_i \cdot q_i$ . The price of the specialized input is  $r$  and the price of managerial effort is one. Cost minimization then requires that the producer uses in total  $e \cdot z_i \cdot q_i$  units of managerial effort and  $z_i \cdot q_i$  units of the specialized input. The total cost is thus  $(e + r) \cdot z_i \cdot q_i$ . Therefore, by defining  $c = e + r$ , the analysis above is still valid for the case with no input regulation.

Assume now that the government regulates the use of the specialized input, by requiring  $\kappa_i \geq \kappa$  per unit of output,  $q_i$ . Notice that such an input regulation does not entail a precise regulation of quality since the producer may well provide a smaller amount of non-contractible quality  $z_i < \kappa_i$  and simply dispose of the superfluous specialized inputs. In fact, it is easy to see that input regulation does not affect the outcome in the case of pure public ownership assuming that  $e > b$ .

In contrast, an input regulation can be used to increase private producers’ incentives to provide quality. Here I will assume that the government simply acquires the resource, say gym facilities, and provides it to the producers free of charge (in-kind provision). But an alternative would be to require the producers to acquire a certain amount of the resource, say qualified teachers, themselves and to compensate the producers for their expenses by use of a fixed transfer.

**Proposition 6.** *Assume that the government can provide private producers with specialized resources in-kind or regulate the use of such resources. Then, the government will do so, but it will optimally induce the same non-contractible quality as otherwise. The welfare gain is that the government can reduce the voucher price and the total voucher cost by more than the additional cost of acquiring the specialized resources. The reduction in public expenditure is given by  $r \cdot t > 0$ .*

The intuition for this result comes from the fact that the rents paid to private producers are increasing in the producers’ cost of providing non-contractible quality. A similar argument would show that also the outcome under mixed ownership could be improved by regulating the inputs used by private producers. Doing so would make mixed ownership even more attractive relative to pure public ownership, which may be crucial in case of a binding competitive neutrality regulation.

Measuring input may be a good option if quality cannot be measured and if it is known that a certain high quality input leads to higher quality. Then, publicizing the input measures may be used by users as a signal of quality and help them to choose



their preferred service provider, thereby increasing quality competition. The point here is different. Measuring input is not a signal of quality, but a way to change the firms' cost functions. The gain is not to increase equilibrium quality, but to reduce the rents payed to induce a given amount of quality.

## 8 Concluding remarks

This paper studies the relative merits of public and private ownership, including the possibility to mix the two ownership models, in sectors characterized by both incomplete contracting problems and (limited) competition. As far as I know, it is the first paper to do so. There are, however, several important issues that I have not been able to deal with here.

One of the main results of the present model is that government subsidies to public firm may sometimes be warranted and that competitive neutrality rules therefore may conflict with the optimal regulation of mixed markets. Only the negative effects of competitive neutrality are visible. It seems reasonable, however, that a government covering systematic deficits in its own firms may create various incentive problems that are associated with weak budget constraints. Such problems have not been addressed in the current paper. To study these issues, some restrictive assumptions must be replaced. Examples include the assumption that the government has complete information about the parameters of the cost function or that the managers do not need to invest efforts to keep costs low. In such a more general model, both the pros and the cons of competitive neutrality regulation could be studied at the same time, allowing also for an analysis of how these effects could be balanced against one another.

In the present model, public ownership is associated with a higher level of inequality than private ownership. Under pure public ownership, the government can only induce positive levels of non-verifiable quality by accepting that one producer offers higher quality than the other. This result is probably, however, due to the restrictive assumptions of the model, e.g. that the two producers are identical. Identical producers offer the same qualities in equilibrium under pure private ownership. If, however, one producer has a lower cost of providing quality or a stronger intrinsic motivation to do so, the two producers would offer different qualities under pure private ownership. In contrast, under pure public ownership, the government could use its right to decide on quantities to reduce the difference in non-verifiable quality, simply by requiring the disadvantaged firm to produce a larger quantity. Then, it may be conjectured, governments would be more inclined to select pure public ownership over pure private ownership, the more inequality averse they are.<sup>27</sup>

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<sup>27</sup>Another possibility to reduce inequality would be to require producers offering low non-verifiable quality in equilibrium to offer higher levels of verifiable qualities.

The model studied here is meant to focus on some common themes associated with most welfare sectors, while abstracting from the many idiosyncratic complications associated with individual welfare sectors. One example is the presence of network effects (a.k.a. peer effects) in schooling. But there are also common themes that have been left out. An example is that governments and users may differ in their valuations of qualities, which may result in over-treatment in the health sector and grade inflation in the school system.

Another limitation is that the present paper is only focusing on public ownership and private for-profit firms, leaving out not-for-profit firms which are relatively common in the welfare sectors.

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## A First best

To prove Lemma 2, recall that social welfare absent inequality aversion ( $\alpha = 0$ ) is given by  $W = \sum_i v \cdot z_i \cdot q_i - [E + \frac{\lambda}{2} \cdot E^2]$  where the government's expenditures are given by  $E = \sum_i (c - b) \cdot z_i \cdot q_i$ . Thus, we can write  $W = (v + b - c) \cdot Z - \frac{\lambda}{2} \cdot (c - b)^2 \cdot Z^2$  where  $Z = \sum_i z_i \cdot q_i$ . Notice that  $\frac{\partial W}{\partial Z} = (v + b - c) - \lambda \cdot (c - b)^2 \cdot Z$  is strictly positive at  $Z = 0$  and that  $\frac{\partial^2 W}{\partial Z^2} = -\lambda \cdot (c - b)^2 < 0$ . Solving the first order condition implies that  $Z^* = \frac{v+b-c}{\lambda \cdot (c-b)^2} > 0$ . This optimum is achieved for any combination of qualities and quantities such that  $\sum_i z_i \cdot q_i = Z^*$ . To minimize inequity, it is sufficient to set  $z_1 = z_2 = Z^*$ . Expressed differently,  $z_1 = z_2 = \frac{v+b-c}{\lambda \cdot (c-b)^2}$ .

## B Public ownership

### B.1 Quality competition

The full Nash equilibrium structure is:

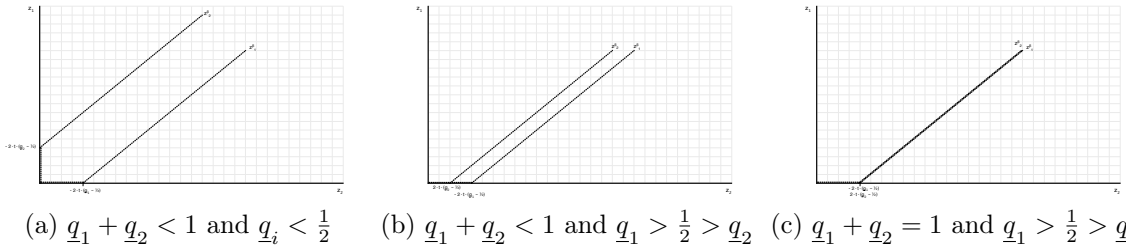
**Lemma 9.** *Assume that both producers are publicly owned. For  $(w_1, w_2)$  sufficiently high:*

- if  $\underline{q}_1 + \underline{q}_2 < 1$  and  $\underline{q}_i < \frac{1}{2}$ , the unique Nash equilibrium prescribes  $z_1 = z_2 = 0$ , and
- if  $\underline{q}_1 + \underline{q}_2 < 1$  and  $\underline{q}_2 > \frac{1}{2} > \underline{q}_1$ , the unique Nash equilibrium prescribes  $z_1 = 0$  and  $z_2 = 2 \cdot t \cdot \left(\underline{q}_2 - \frac{1}{2}\right) < t$ ,
- if  $\underline{q}_1 + \underline{q}_2 = 1$ , any  $(z_1, z_2)$  with  $z_1 \geq 0$  and  $z_2 = 2 \cdot t \cdot \left(\underline{q}_2 - \frac{1}{2}\right) + z_1$  is a Nash equilibrium.

To prove Lemma 9, I will first assume that  $(w_1, w_2)$  are sufficiently high for  $Z_i^\beta(z_j) = \underline{Z}_i(z_j)$  and then derive the necessary conditions for this to be the case.

First, consider the case when  $\underline{q}_1 + \underline{q}_2 < 1$  and  $\underline{q}_i < \frac{1}{2}$ , displayed in figure 6a. Then,

Figure 5: Quality competition under public ownership



manager 1's best-reply function requires that  $z_1 = \underline{Z}_1(z_2) < z_2$  or  $z_1 = 0$ . Similarly, manager 2's best-reply requires that  $z_2 = \underline{Z}_2(z_1) < z_1$  or  $z_2 = 0$ . Both conditions are

fulfilled only when both qualities are equal to zero. In this case,  $w_1 = w_2 = 0$  are sufficiently high.

Second, consider the case when  $\underline{q}_1 + \underline{q}_2 < 1$  and  $\underline{q}_2 > \frac{1}{2} > \underline{q}_1$ . Then  $0 < \left(\underline{q}_2 - \frac{1}{2}\right) < -\left(\underline{q}_1 - \frac{1}{2}\right)$ . This equilibrium is displayed in figure 6b. Clearly,  $z_1 = 0$  and  $z_2 = 2 \cdot t \cdot \left(\underline{q}_2 - \frac{1}{2}\right)$ , implying that  $z_2$  is increasing in  $\underline{q}_2 < 1 - \underline{q}_1$ . In this case,  $w_1 = 0$  and  $w_2 = (c - b) \cdot 2 \cdot t \cdot \left(\underline{q}_2 - \frac{1}{2}\right) \cdot \underline{q}_2$  are sufficiently high.

Third, consider the case when  $\underline{q}_1 + \underline{q}_2 = 1$  and  $\underline{q}_2 \geq \frac{1}{2}$ . This equilibrium is displayed in figure 6c. The two best-reply functions lie on top of each other along the part with positive slope. In case of the coalition proof equilibrium,  $w_1 = 0$  and  $w_2 = (c - b) \cdot 2 \cdot t \cdot \left(\underline{q}_2 - \frac{1}{2}\right) \cdot \underline{q}_2$  are sufficiently high.

## B.2 Social welfare

To prove Lemma 5, I will first rewrite the social welfare function. Since  $z_1 = 0$ ,  $w_1 = 0$  is sufficient. Total public expenditure is  $E = w_2 = (c - b) \cdot z_2 \cdot \underline{q}_2$ . Inequality aversion is given by  $\alpha \cdot I = \alpha \cdot z_2 \cdot \underline{q}_2$ . Thus  $W = (v + b - c) \cdot z_2 \cdot \underline{q}_2 - \frac{\lambda}{2} \cdot (c - b)^2 \cdot \left[z_2 \cdot \underline{q}_2\right]^2 - \alpha \cdot z_2 \cdot \underline{q}_2$ . Moreover, in equilibrium  $z_2 = 2 \cdot t \cdot \left(\underline{q}_2 - \frac{1}{2}\right)$ . Thus:

$$W = (v + b - c - \alpha) \cdot Z - \frac{\lambda}{2} \cdot (c - b)^2 \cdot Z^2$$

where  $Z = 2 \cdot t \cdot \left(\underline{q}_2 - \frac{1}{2}\right) \cdot \underline{q}_2 \in [0, t]$  is strictly increasing in  $\underline{q}_2 \in \left[\frac{1}{2}, 1\right]$ . Notice that  $W$  is strictly concave in  $Z$  and that

$$\frac{\partial W}{\partial Z} = (v + b - c - \alpha) - \lambda \cdot (c - b)^2 \cdot Z$$

is strictly positive at  $Z = 0$  if, and only if  $v + b - c > \alpha$ . In this case, the optimum is characterized by

$$Z^* = \min \left\{ \frac{v + b - c - \alpha}{\lambda \cdot (c - b)^2}, t \right\} > 0,$$

and otherwise  $Z^* = 0$ . The following conclusions are immediate:<sup>28</sup>

If  $\alpha \geq v + b - c$ , the government sets  $\underline{q}_2 = \frac{1}{2}$  and obtains

$$W^G = W^1 = 0.$$

If  $\alpha < v + b - c$  and  $t \leq \frac{v + b - c - \alpha}{\lambda \cdot (c - b)^2}$ ,  $Z^* = t$  and  $1 = 2 \cdot \left(\underline{q}_2 - \frac{1}{2}\right) \cdot \underline{q}_2$ . Thus, the government

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<sup>28</sup>That is, the production plan must be set to satisfy  $\left(\underline{q}_2 - \frac{1}{2}\right) \cdot \underline{q}_2 = \frac{1 - k - \alpha}{2 \cdot t \cdot \lambda \cdot k^2}$ . Notice that for  $\underline{q}_2 = 1$  it is required that  $\frac{1 - k - \alpha}{2 \cdot t \cdot \lambda \cdot k^2} \geq \frac{1}{2}$  that is  $t \leq (1 - k - \alpha) / \lambda \cdot k^2$ .

sets  $\underline{q}_2 = 1$  and obtains

$$W^G = W^3 = \left[ (v + b - c - \alpha) - \frac{\lambda}{2} \cdot (c - b)^2 \cdot t \right] \cdot t > 0.$$

If  $\alpha < v + b - c$  and  $t > \frac{v+b-c-\alpha}{\lambda \cdot (c-b)^2}$ ,  $Z^* < t$ . Thus, the government sets  $\underline{q}_2 \in (\frac{1}{2}, 1)$  and obtains

$$W^G = W^5 = \frac{1}{2 \cdot \lambda} \cdot \left[ \frac{v + b - c - \alpha}{c - b} \right]^2 > 0.$$

In particular, since  $\frac{v+b-c-\alpha}{\lambda \cdot (c-b)^2} = 2 \cdot t \cdot \left( \underline{q}_2 - \frac{1}{2} \right) \cdot \underline{q}_2$  it follows that

$$\underline{q}_2 = \frac{1}{4} + \sqrt{\frac{1}{16} + \frac{v + b - c - \alpha}{2 \cdot t \cdot \lambda \cdot (c - b)^2}}$$

where the positive root is selected since  $\underline{q}_2 > \frac{1}{2}$ .

In sum, the welfare function is continuous and given by:

$$W^G = \begin{cases} 0 & \alpha \geq \alpha^G, \\ \frac{1}{2 \cdot \lambda} \cdot \left[ \frac{v+b-c-\alpha}{c-b} \right]^2 > 0, & \alpha < \alpha^G, t > t^G, \\ \left[ (v + b - c - \alpha) - \frac{\lambda}{2} \cdot (c - b)^2 \cdot t \right] \cdot t > 0, & \alpha < \alpha^G, t \leq t^G. \end{cases}$$

where  $t^G = \frac{v+b-c-\alpha}{\lambda \cdot (c-b)^2}$  and  $\alpha^G = v + b - c$ .

### B.3 Welfare increasing in t

To prove Corollary 2 note that if  $\alpha < v + b - c$ :

$$\frac{\partial W^G}{\partial t} = \begin{cases} 0 & t > \frac{v+b-c-\alpha}{\lambda \cdot (c-b)^2} \\ (v + b - c - \alpha) - \lambda \cdot (c - b)^2 \cdot t > 0 & t < \frac{v+b-c-\alpha}{\lambda \cdot (c-b)^2} \end{cases}$$

If  $\alpha \geq v + b - c$  welfare does not depend on  $t$ .

## C Private ownership

The profit of producer  $i$  is given by  $\pi_i = (p - (c - b) \cdot z_i) \cdot \left( \frac{1}{2} + \frac{z_i - z_j}{2 \cdot t} \right)$ . Thus:

$$\frac{\partial \pi_i}{\partial z_i} \cdot 2 \cdot t = p - 2 \cdot (c - b) \cdot z_i - (c - b) \cdot (t - z_j)$$

and

$$\frac{\partial^2 \pi_i}{(\partial z_i)^2} \cdot 2 \cdot t = -2 \cdot (c - b) < 0.$$

Clearly, if  $p = 0$ , a private producer sets  $z_i = 0$ . If  $p > 0$ , the firm's first order condition is given by  $-(c - b) \cdot \left( \frac{1}{2} + \frac{z_i - z_j}{2 \cdot t} \right) + (p - (c - b) \cdot z_i) \cdot \frac{1}{2 \cdot t} = 0$  and its best-reply function is

$$z_i = \max \left\{ \frac{p-t \cdot (c-b)}{2 \cdot (c-b)} + \frac{1}{2} \cdot z_j, 0 \right\}.$$

If  $p \geq t \cdot (c-b)$ , there exists a unique equilibrium with

$$z = \frac{p - t \cdot (c-b)}{c-b}$$

If  $p < t \cdot (c-b)$ , there exists a unique equilibrium with  $z_1 = z_2 = 0$ .

## C.1 Social welfare

To prove Lemma 7, recall that both producers receive the same price and produce the same quality. Thus, the users' benefits are given by  $v \cdot \max \left\{ \frac{p}{c-b} - t, 0 \right\}$ , there is no inequality, and the total public expenditure is  $E = \sum_i p \cdot q_i = p$ . Thus social welfare is given by  $W = v \cdot \max \left\{ \frac{p}{c-b} - t, 0 \right\} - p - \frac{\lambda}{2} \cdot p^2$ . Since any  $p \in (0, t \cdot (c-b))$  leads to zero quality, the government prefers  $p = 0$  to any such price. For  $p \geq t \cdot (c-b)$ ,

$$W = \frac{v+b-c}{c-b} \cdot p - \frac{\lambda}{2} \cdot p^2 - v \cdot t$$

which is strictly concave in price. The first derivative

$$\frac{\partial W}{\partial p} = \frac{v+b-c}{c-b} - \lambda \cdot p$$

is strictly positive at  $p = t \cdot (c-b)$  if and only if  $t < \frac{v+b-c}{\lambda \cdot (c-b)^2}$ . The interior solution  $p = \frac{v+b-c}{\lambda \cdot (c-b)} > t \cdot (c-b) > 0$  yields  $z^{interior} = \frac{v+b-c}{\lambda \cdot (c-b)^2} - t = z^* - t$  and  $W^{interior} = \frac{1}{2 \cdot \lambda} \cdot \left[ \frac{v+b-c}{c-b} \right]^2 - v \cdot t = W^* - v \cdot t > 0$ . The corner solution  $p = 0$  implies  $z^{corner} = 0$  and  $W^{corner} = 0$ . The interior solution is strictly preferred if and only if  $t < \frac{v+b-c}{2 \cdot v} \cdot \frac{v+b-c}{\lambda \cdot (c-b)^2}$ .

## C.2 Input regulation

To prove Proposition 6, note that a private firm's profit is given by

$$\pi_i = (p - (e-b) \cdot z_i - r \cdot \kappa_i) \cdot \left( \frac{1}{2} + \frac{z_i - z_j}{2 \cdot t} \right),$$

where  $z_i \leq \kappa_i$ . Absent regulation producers set  $\kappa_i = z_i$  and in symmetric equilibrium

$$z' = \frac{p}{e+r-b} - t,$$

which is the same as before since  $e+r=c$ . Recall that the optimal voucher price is given by

$$p' = \frac{v+b-c}{\lambda \cdot (c-b)}.$$



Assume now that the government imposes a minimum requirement  $\kappa$  to be defined below. Then, the first derivative is given by

$$\frac{\partial \pi_i}{\partial z_i} \cdot 2 \cdot t = p - (c - b) \cdot (t + 2 \cdot z_i - z_j)$$

for  $z_i < \kappa$ . Thus, the symmetric equilibrium is given by

$$z^* = \frac{p}{e - b} - t$$

if the government sets  $\kappa = z^*$  or larger. Consider now the government's choice of voucher price. Since both producers produce the same quality and receive the same price, the users' benefits are given by  $v \cdot \left[ \frac{p}{e-b} - t \right]$ , there is no inequality, and the total public expenditure is  $E = \sum_i p \cdot q_i + r \cdot \left[ \frac{p}{e-b} - t \right] = p + r \cdot \left[ \frac{p}{e-b} - t \right]$ . Recall that  $e = c - r$ , it is straight forward to verify that the optimal voucher price is given by

$$p^* = \left[ 1 - \frac{r}{c - b} \right] \cdot \frac{v + b - c}{\lambda \cdot (c - b)} < p'.$$

The equilibrium quality is given by

$$z^* = \frac{v + b - c}{\lambda \cdot (c - b)^2} - t$$

which, however, is the same as absent regulation. Since the benefits are the same, with and without input regulation, we only need to compare the government's expenditures. With input regulation expenditures are given by  $E^* = p^* + r \cdot z^*$  and without expenditures are given by  $E' = p'$ . Substituting, reveals that the difference in expenditures as a result of input regulation is negative

$$\Delta E = p^* + r \cdot z^* - p' = -r \cdot t < 0.$$

## D Public vs private ownership

Recall that Proposition 1 is conveniently summarized in figure 2. To prove the proposition, I first rewrite it as the following Lemma:

**Lemma 10.** *If  $\alpha \geq \alpha^G$ ,  $W^G(t) \leq W^P(t)$  for  $t \leq t^P(v, c - b) = \frac{v+b-c}{2 \cdot v} \cdot \frac{v+b-c}{\lambda \cdot (c-b)^2}$ , with  $\frac{\partial t^P(v, c-b)}{\partial (c-b)} < 0$  and  $\frac{\partial t^P(v, c-b)}{\partial v} > 0$ . If  $\alpha < \alpha^G$ , there exists a  $t^\circ(\alpha, v, c - b) > 0$  such that*

$W^G(t^\circ) \stackrel{\leq}{\geq} W^P(t^\circ)$  for  $t \stackrel{\leq}{\geq} t^\circ$ . The threshold is given by

$$t^\circ(\alpha, v, c-b) = \begin{cases} \frac{(v+b-c)+(v-\alpha)-\sqrt{(v-\alpha)\cdot[(2\cdot(v+b-c)+(v-\alpha))]} }{\lambda\cdot(c-b)^2}, & \alpha \leq \alpha', \\ \frac{\alpha}{v} \cdot \frac{(v+b-c)-\frac{1}{2}\cdot\alpha}{\lambda\cdot(c-b)^2}, & \alpha \geq \alpha', \end{cases}$$

where

$$\alpha' = (v+b-c) - \left[ \sqrt{v^2 + [v - (c-b)]^2} - v \right] \in (0, v+b-c).$$

The threshold  $t^\circ(\alpha, v, c-b)$  is continuous in  $\alpha$ . Moreover

$$\frac{\partial t^\circ(\alpha, v, c-b)}{\partial \alpha} > 0$$

$$\frac{\partial t^\circ(\alpha, v, c-b)}{\partial (c-b)} < 0$$

$$\frac{\partial t^\circ(\alpha, v, c-b)}{\partial v} > 0.$$

To prove this Lemma, recall that welfare under public ownership is given by

$$W^G(\alpha, t) = \begin{cases} 0 & \alpha \geq \alpha^G, \\ [(v+b-c-\alpha) - \frac{\lambda}{2} \cdot (c-b)^2 \cdot t] \cdot t > 0, & \alpha < \alpha^G, t \leq t^G, \\ \frac{1}{2\lambda} \cdot \left[ \frac{v+b-c-\alpha}{c-b} \right]^2 > 0, & \alpha < \alpha^G, t > t^G. \end{cases}$$

where  $t^G = \frac{v+b-c-\alpha}{\lambda\cdot(c-b)^2}$  and  $\alpha^G = v+b-c$ , and, under private ownership, by

$$W^P(t) = \begin{cases} \frac{1}{2\lambda} \cdot \left[ \frac{v+b-c}{c-b} \right]^2 - v \cdot t & t \leq t^P \\ 0 & \text{otherwise} \end{cases}$$

where  $t^P = \frac{v+b-c}{2\cdot v} \cdot \frac{v+b-c}{\lambda\cdot(c-b)^2} < \frac{v+b-c}{\lambda\cdot(c-b)^2}$ , where the inequality follows from  $b-c < v$ .

First, it is immediately clear that:

*Claim 1.* If  $\alpha \geq \alpha^G$ ,  $W^G(t) \leq W^P(t)$  for  $t \leq t^P$ ,

and that

*Claim 2.* If  $t \geq t^P$ ,  $W^G(t) \geq W^P(t)$  for  $\alpha \leq \alpha^G$ .

Second, I show that:

*Claim 3.* If  $\alpha < \alpha^G$ , there exists a  $t^\circ > 0$  such that  $W^G(t^\circ) \stackrel{\leq}{\geq} W^P(t^\circ)$  for  $t \stackrel{\leq}{\geq} t^\circ$ .

To see this, first note that welfare is higher under private ownership if users are very mobile,  $W^G(0) = 0 < W^P(0)$ . However, under public ownership, welfare is increasing in user immobility,  $W_t^G(0) > 0$  and  $W_t^G(t) \geq 0$  for all  $t$ . In contrast, welfare is falling in user

immobility under private ownership,  $W_t^P(0) < 0$ ,  $W_t^P(t) \leq 0$  for all  $t$  and  $W^P(t) = 0$  for  $t$  large enough.

Third, I show that:

*Claim 4.* Assume that  $\alpha < \alpha^G$ . The threshold is given by

$$t^\circ(\alpha, v, c - b) = \begin{cases} \frac{(v+b-c)+(v-\alpha)-\sqrt{(v-\alpha) \cdot [(2 \cdot (v+b-c)+(v-\alpha))]}]{\lambda \cdot (c-b)^2}, & \alpha \leq \alpha', \\ \frac{\alpha}{v} \cdot \frac{(v+b-c)-\frac{1}{2} \cdot \alpha}{\lambda \cdot (c-b)^2}, & \alpha \geq \alpha', \end{cases}$$

To prove this claim I need to solve  $W^G(t^\circ) = \frac{1}{2 \cdot \lambda} \cdot \left[\frac{v+b-c}{c-b}\right]^2 - v \cdot t^\circ$  to find  $t^\circ$ . Thus, I need to consider two cases correspond to the two pieces of  $W^G(t)$ . Also notice that these two pieces meet at the point  $(t, W) = \left(\frac{v+b-c-\alpha}{\lambda \cdot (c-b)^2}, \frac{1}{2 \cdot \lambda} \cdot \left[\frac{v+b-c-\alpha}{c-b}\right]^2\right)$ .

**First**, consider the case when

$$\left[(v+b-c-\alpha) - \frac{\lambda}{2} \cdot (c-b)^2 \cdot t^\circ\right] \cdot t^\circ = \frac{1}{2 \cdot \lambda} \cdot \left[\frac{v+b-c}{c-b}\right]^2 - v \cdot t^\circ$$

or

$$\left[(2v+b-c-\alpha) - \frac{\lambda}{2} \cdot (c-b)^2 \cdot t^\circ\right] \cdot t^\circ = \frac{1}{2 \cdot \lambda} \cdot \left[\frac{v+b-c}{c-b}\right]^2$$

Note that the right hand side is strictly positive. The left hand side (LHS) is zero at  $t = 0$  but increasing in  $t$  since  $2v+b-c-\alpha > 0$ . The LHS is maximized over  $t$  at

$$t = \frac{2v+b-c-\alpha}{\lambda \cdot (c-b)^2}$$

giving

$$LHS = \frac{1}{2 \cdot \lambda} \cdot \left(\frac{(v+b-c)+(v-\alpha)}{c-b}\right)^2$$

Thus, there exists a  $t$  solving the equation if  $\alpha \leq v$ . Rewriting the equation as,

$$\left[t - \left(\frac{2v+b-c-\alpha}{\lambda \cdot (c-b)^2}\right)\right]^2 = \frac{1}{\lambda \cdot (c-b)^2} \cdot \frac{1}{\lambda \cdot (c-b)^2} \cdot (v-\alpha) \cdot [(3v+2(b-c)-\alpha)]$$

and solving it gives

$$t^\circ = \frac{(v+b-c-\alpha) + v - \sqrt{(v-\alpha) \cdot [(3v+2(b-c)-\alpha)]}}{\lambda \cdot (c-b)^2}.$$

Notice that I have selected the lower root, which corresponds to the relevant (increasing) part of  $W^G(t^\circ)$ . This solution is valid only if it occurs below the meeting point of the two parts of  $W^G(t^\circ)$  which occurs at  $\frac{v+b-c-\alpha}{\lambda \cdot (c-b)^2}$ . Thus it is necessary that

$$v \leq \sqrt{(v-\alpha) \cdot [(3v+2(b-c)-\alpha)]}$$

i.e.

$$v^2 \leq (v - \alpha) \cdot [(3v + 2(b - c) - \alpha)]$$

i.e.

$$(2v + (b - c))^2 - 2v \cdot (v + (b - c)) \leq [(2v + (b - c)) - \alpha]^2$$

$$\alpha \leq (2v + (b - c)) - \sqrt{v^2 + (v + (b - c))^2}.$$

Notice that I have chosen the lower root since  $\alpha - v < 0$ . Thus, it is necessary that  $\alpha \leq \alpha'$ , where

$$\alpha' = (v + b - c) - \left[ \sqrt{v^2 + [v - (c - b)]^2} - v \right] \in (0, v + b - c).$$

The two inequalities follow from the fact that  $v^2 + [v - (c - b)]^2 > v^2$  and  $v^2 + [v - (c - b)]^2 < (v + (v + b - c))^2$ .

**Second,** consider the case when

$$\frac{1}{2 \cdot \lambda} \cdot \left[ \frac{v + b - c - \alpha}{c - b} \right]^2 = \frac{1}{2 \cdot \lambda} \cdot \left[ \frac{v + b - c}{c - b} \right]^2 - v \cdot t^\circ$$

$$t^\circ = \frac{\alpha}{v} \cdot \frac{(v + b - c) - \frac{1}{2} \cdot \alpha}{\lambda \cdot (c - b)^2}$$

This solution is valid only if it occurs above the meeting point of the two parts of  $W^G(t^\circ)$  which occurs at  $\frac{v + b - c - \alpha}{\lambda \cdot (c - b)^2}$ . Thus it is necessary that

$$\frac{\alpha}{v} \cdot \frac{v + b - c - \frac{1}{2} \alpha}{\lambda \cdot (c - b)^2} \geq \frac{v + b - c - \alpha}{\lambda \cdot (c - b)^2}$$

i.e.

$$\frac{\alpha}{v} (v + b - c - \frac{1}{2} \alpha) \geq v + b - c - \alpha$$

This inequality is fulfilled if  $\alpha \geq v$ . But it could also be fulfilled when  $\alpha < v$ . Consider this case. Then the inequality can be written as

$$\alpha \geq [2v + (b - c)] - \sqrt{v^2 + [v + (b - c)]^2} = \alpha'.$$

Notice that I have chosen the negative root since  $\alpha < v + b - c$ . Thus, it is required that  $\alpha \geq v$  or  $\alpha \geq \alpha'$ . Finally, notice that  $\alpha' < v$  since  $(v + b - c) - \left[ \sqrt{v^2 + [v + (b - c)]^2} - v \right] < v$  which reduces to  $0 < v^2$ .

Next:

*Claim 5.* Assume that  $\alpha < \alpha^G$ . The threshold  $t^\circ(\alpha, v, c - b)$  is continuous in  $\alpha$ . Moreover

$$\frac{\partial t^\circ(\alpha, v, c - b)}{\partial \alpha} > 0$$

$$\frac{\partial t^\circ(\alpha, v, c - b)}{\partial (c - b)} < 0$$

$$\frac{\partial t^\circ(\alpha, v, c - b)}{\partial v} > 0.$$

**First,** recall that

$$t^\circ(\alpha, v, c - b) = \begin{cases} \frac{(v+b-c)+(v-\alpha)-\sqrt{(v-\alpha)\cdot[(2\cdot(v+b-c)+(v-\alpha))]}]{\lambda\cdot(c-b)^2}, & \alpha \leq \alpha', \\ \frac{\alpha}{v} \cdot \frac{(v+b-c)-\frac{1}{2}\cdot\alpha}{\lambda\cdot(c-b)^2}, & \alpha \geq \alpha', \end{cases}$$

where

$$\alpha' = (v + b - c) - \left[ \sqrt{v^2 + [v - (c - b)]^2} - v \right] \in (0, v + b - c).$$

**Second,** notice that  $t^\circ$  is continuous in  $\alpha$ . When  $\alpha \leq \alpha'$ :

$$t^\circ(\alpha, v, c - b) = \frac{(v + b - c) + (v - \alpha) - \sqrt{(v - \alpha) \cdot [(2 \cdot (v + b - c) + (v - \alpha))]}]{\lambda \cdot (c - b)^2}$$

then:

$$t^\circ(\alpha', v, c - b) = \frac{\sqrt{v^2 + [v - (c - b)]^2} - v}{\lambda \cdot (c - b)^2}.$$

When  $\alpha \geq \alpha'$ :

$$t^\circ(\alpha, v, c - b) = \frac{\alpha}{v} \cdot \frac{(v + b - c) - \frac{1}{2} \cdot \alpha}{\lambda \cdot (c - b)^2}$$

then:

$$t^\circ(\alpha', v, c - b) = \frac{\sqrt{v^2 + [v - (c - b)]^2} - v}{\lambda \cdot (c - b)^2}$$

**Third,** notice that

$$\frac{\partial t^\circ(\alpha, v, c - b)}{\partial \alpha} \cdot \lambda \cdot (c - b)^2 = \begin{cases} -1 + \frac{1}{2} \cdot \left[ \frac{2 \cdot (v+b-c) + (v-\alpha)}{(v-\alpha)} \right]^{\frac{1}{2}} + \frac{1}{2} \cdot \left[ \frac{(v-\alpha)}{2 \cdot (v+b-c) + (v-\alpha)} \right]^{\frac{1}{2}} > 0, & \alpha \leq \alpha', \\ \frac{1}{v} \cdot [(v + b - c) - \alpha] > 0, & \alpha \geq \alpha', \end{cases}$$

since for any  $a$  and  $b$

$$\begin{aligned} -1 + \frac{1}{2} \cdot \left(\frac{a}{b}\right)^{\frac{1}{2}} + \frac{1}{2} \cdot \left(\frac{b}{a}\right)^{\frac{1}{2}} &> 0 \\ (a+b)^2 &> 4a \cdot b \\ (a-b)^2 &> 0. \end{aligned}$$

**Fourth,** Recall that

$$t^\circ(\alpha, v, c-b) = \begin{cases} \frac{[v-(c-b)]+(v-\alpha)-\sqrt{(v-\alpha)\cdot[2\cdot[v-(c-b)]+(v-\alpha)]}}{\lambda\cdot(c-b)^2}, & \alpha \leq \alpha', \\ \frac{\alpha}{v} \cdot \frac{[v-(c-b)]-\frac{1}{2}\cdot\alpha}{\lambda\cdot(c-b)^2}, & \alpha \geq \alpha', \end{cases}$$

Thus

$$\frac{\partial t^\circ(\alpha, v, c-b)}{\partial(c-b)} < 0$$

for  $\alpha \geq \alpha'$ . Moreover, this is also true for  $\alpha \leq \alpha'$ . To see this, note that the nominator is decreasing in  $c-b$  since

$$-1 + \left[ \frac{(v-\alpha)}{2 \cdot [v-(c-b)] + (v-\alpha)} \right]^{\frac{1}{2}} < 0$$

since

$$0 < [v-(c-b)].$$

**Fifth,**

$$\frac{\partial t^\circ(\alpha, v, c-b)}{\partial v} \cdot \lambda \cdot (c-b)^2 = \begin{cases} 2 - \frac{1}{2} \cdot \left[ \frac{2 \cdot [v-(c-b)] + (v-\alpha)}{(v-\alpha)} \right]^{\frac{1}{2}} - \frac{3}{2} \left( \frac{v-\alpha}{2 \cdot [v-(c-b)] + (v-\alpha)} \right)^{\frac{1}{2}} > 0, & \alpha \leq \alpha', \\ \frac{\alpha}{v^2} \cdot \left[ (c-b) + \frac{1}{2} \cdot \alpha \right] > 0, & \alpha \geq \alpha', \end{cases}$$

since

$$\begin{aligned} 2 - \frac{1}{2} \cdot \left[ \frac{a+b}{b} \right]^{\frac{1}{2}} - \frac{3}{2} \left( \frac{b}{a+b} \right)^{\frac{1}{2}} &> 0 \\ 8 \cdot b &> a \end{aligned}$$

$$8 \cdot (v-\alpha) > 2 \cdot [v-(c-b)]$$

$$\alpha < \frac{3}{4} \cdot v + \frac{1}{4} \cdot (c-b)$$

recalling that  $\alpha < \alpha' = (v+b-c) - \left[ \sqrt{v^2 + [v-(c-b)]^2} - v \right]$  and noting

$$\alpha' = (v+b-c) - \left[ \sqrt{v^2 + [v-(c-b)]^2} - v \right] < \frac{3}{4} \cdot v + \frac{1}{4} \cdot (c-b)$$

since

$$5v - 5(c - b) - 4\sqrt{v^2 + [v - (c - b)]^2} < 0.$$

Finally I consider the case when  $\alpha \geq \alpha^G$ . Then,  $W^G(t^\circ) \leq W^P(t)$  for  $t \leq t'(v, c - b) = \frac{v+b-c}{2 \cdot v} \cdot \frac{v+b-c}{\lambda \cdot (c-b)^2}$ . Notice that  $\frac{\partial t'(v, c-b)}{\partial v} = \frac{2 \cdot v}{2 \cdot v^2} \cdot \frac{(v+b-c)}{\lambda \cdot (c-b)^2} - \frac{1}{2 \cdot v^2} \cdot \frac{(v+b-c)^2}{\lambda \cdot (c-b)^2} = \frac{1}{2 \cdot v^2} \cdot \frac{(v+b-c)}{\lambda \cdot (c-b)^2} \cdot [v + (c - b)] > 0$ . Also  $\frac{\partial t'(v, c-b)}{\partial (c-b)} = -\frac{2}{2 \cdot v} \cdot \frac{(v+b-c)}{\lambda \cdot (c-b)^2} - \frac{2}{2 \cdot v} \cdot \frac{(v+b-c)^2}{\lambda \cdot (c-b)^3} < 0$ .

## E Mixed ownership

### E.1 Quality competition

**Lemma 11.** *Given  $p$  and  $\underline{q}_2$ , there is a unique equilibrium  $(z_1, z_2)$  in the quality competition game, assuming that  $w_2 \geq (c - b) \cdot z_2 \cdot q_2$ . If the government sets  $p \geq 2 \cdot t \cdot (c - b) \left(1 - \underline{q}_2\right) \geq 0$ , the public firm's equilibrium quality is given by  $z_2 = 4 \cdot t \cdot \left(\underline{q}_2 - \frac{1}{2}\right) + \frac{p-t \cdot (c-b)}{(c-b)}$  and the private firm's quality is given by  $z_1 = 2 \cdot t \cdot \left(\underline{q}_2 - \frac{1}{2}\right) + \frac{p-t \cdot (c-b)}{(c-b)}$ . If the government sets  $p < 2 \cdot t \cdot (c - b) \left(1 - \underline{q}_2\right)$ , the equilibrium qualities are given by  $z_1 = 0$  and  $z_2 = 2 \cdot t \cdot \left(\underline{q}_2 - \frac{1}{2}\right)$ .*

To prove Lemma 11, notice that the public producer has the same best reply function, independent of how the competitor is owned. In particular, it is given by

$$z_2 = 2 \cdot t \cdot \left(\underline{q}_2 - \frac{1}{2}\right) + z_1,$$

where  $\underline{q}_2 \geq \frac{1}{2}$  is the production plan, assuming that the manager's wage covers his effort cost. The private firm's best reply function is the same, independent of how the competitor is owned. In particular, it is given by

$$z_1 = \max \left\{ \frac{p - t \cdot (c - b)}{2 \cdot (c - b)} + \frac{1}{2} \cdot z_2, 0 \right\}$$

whenever  $p > 0$  and zero otherwise. Assuming  $z_1 > 0$ , the equilibrium is defined by a system of two linear equations. The solution is

$$z_2 = 4 \cdot t \cdot \left(\underline{q}_2 - \frac{1}{2}\right) + \frac{p - t \cdot (c - b)}{(c - b)}, \quad (2)$$

and

$$z_1 = 2 \cdot t \cdot \left(\underline{q}_2 - \frac{1}{2}\right) + \frac{p - t \cdot (c - b)}{(c - b)}. \quad (3)$$

Notice that  $z_1 \geq 0$ , if and only if:

$$p \geq 2 \cdot t \cdot (c - b) \left(1 - \underline{q}_2\right) \geq 0. \quad (4)$$

Otherwise, the equilibrium must entail  $z_1 = 0$  and thus  $z_2 = 2 \cdot t \cdot \left( \underline{q}_2 - \frac{1}{2} \right)$ .

Whether or not (the price and) the wage will be set high enough for this solution to be implemented is for the government to decide.

## E.2 Government policy

To prove Proposition 2, I first rewrite it as the following Lemma:

**Lemma 12.** *Consider the case of mixed ownership. There exists a continuous function  $t^*(\alpha)$  such that the government sets:*

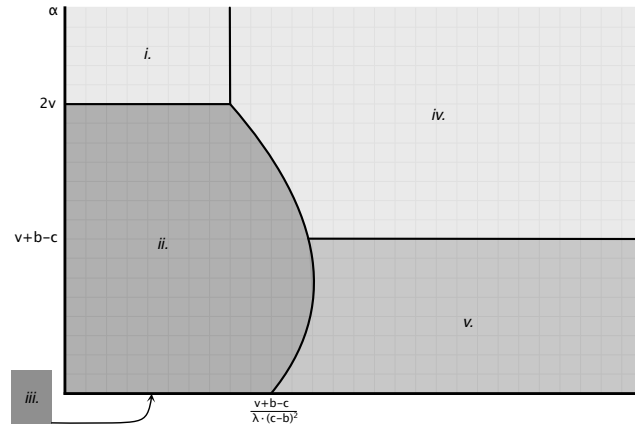
- i.*  $p > 0$  and  $\underline{q}_2 = \frac{1}{2}$  if  $t \leq t^*(\alpha)$  and  $\alpha \geq 2 \cdot v$
- ii.*  $p > 0$  and  $\underline{q}_2 = 1 - \frac{3}{4} \cdot \frac{\alpha}{v+\alpha}$  if  $t \leq t^*(\alpha)$  and  $\alpha \in (0, 2 \cdot v)$
- iii.*  $p > 0$  and  $\underline{q}_2 = 1$  if  $t < t^*(\alpha)$  and  $\alpha = 0$
- iv.*  $p = 0$  and  $\underline{q}_2 = \frac{1}{2}$  if  $t > t^*(\alpha)$  and  $\alpha \geq v + b - c$
- v.*  $p = 0$  and  $\underline{q}_2 \in \left( \frac{1}{2}, 1 \right)$  if  $t \geq t^*(\alpha)$  and  $\alpha < v + b - c$ .

The corresponding levels of welfare are given by:

- i.*  $W = \frac{1}{2 \cdot \lambda} \cdot \left( \frac{v+b-c}{c-b} \right)^2 - \frac{1}{2} \cdot v \cdot t$
- ii.*  $W = \frac{1}{2 \cdot \lambda} \cdot \left( \frac{v+b-c}{c-b} \right)^2 - \alpha \cdot t \cdot \left( 1 - \frac{9}{8} \cdot \frac{\alpha}{v+\alpha} \right)$
- iii.*  $W = \frac{1}{2 \cdot \lambda} \cdot \left( \frac{v+b-c}{c-b} \right)^2 - \alpha \cdot t$
- iv.*  $W = 0$
- v.*  $W = \frac{1}{2 \cdot \lambda} \cdot \left[ \frac{v+b-c-\alpha}{c-b} \right]^2$

This different regions are also illustrated by figure 6.

Figure 6: Optimal regulation under mixed ownership

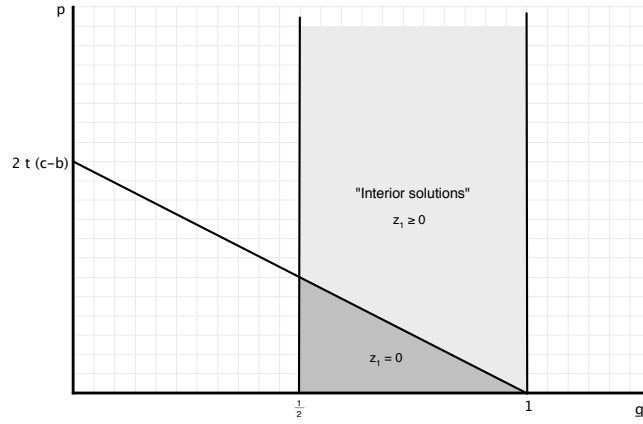




### E.2.1 Case 1: Corner solutions ( $z_1 = 0$ )

To prove Lemma 12, recall that producer 2 is the public producer. The government sets  $p$  and  $\underline{q}_2$  to maximize social welfare,  $W = B - (E + \frac{\lambda}{2} \cdot E^2) - \alpha \cdot I$ . The government's choice set is described by figure 7. I will first consider the case when condition 4 is violated and

Figure 7: Government's choice set



the government implements  $z_1 = 0$ . I will refer to this as the “corner solution” and the case when condition 4 is not binding as the “interior solution.”

When condition 4 is violated, the equilibrium qualities are given by  $z_1 = 0$  and  $z_2 = 2 \cdot t \cdot (\underline{q}_2 - \frac{1}{2})$ . Since the private producer is not meant to provide any non-verifiable quality, the government sets  $p = 0$ . The analysis of the government's choice of the production plan is perfectly analogous to the analysis in the case when both producers are public. Thus:

$$W = \begin{cases} W^1 = 0 & \alpha \geq \alpha^G, \\ W^5 = \frac{1}{2 \cdot \lambda} \cdot \left[ \frac{v+b-c-\alpha}{c-b} \right]^2 > 0, & \alpha < \alpha^G, t > t^G, \\ W^3 = \left[ (v+b-c-\alpha) - \frac{\lambda}{2} \cdot (c-b)^2 \cdot t \right] \cdot t > 0, & \alpha < \alpha^G, t \leq t^G. \end{cases}$$

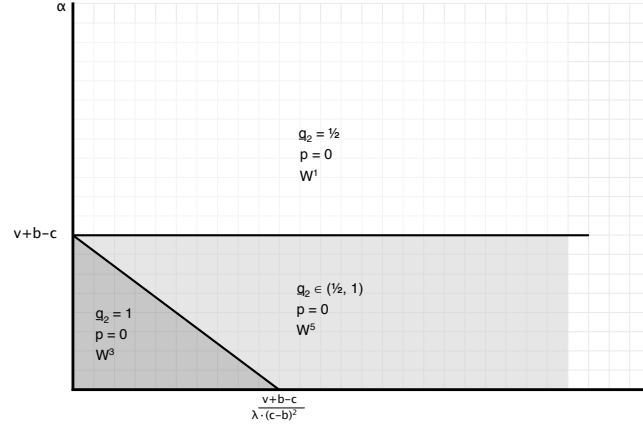
where  $t^G = \frac{(v+b-c)-\alpha}{\lambda \cdot (c-b)^2}$  and  $\alpha^G = v + b - c$ . The solutions are described in figure 8.

### E.2.2 Case 2: Interior solutions ( $z_1 \geq 0$ )

Recall that (by 4) if  $p \geq 2 \cdot t \cdot (c-b) \left(1 - \underline{q}_2\right)$ , the two equilibrium quantities are given by equation 3 and 2. Then, the social benefits are given by

$$B = v \cdot \sum_i z_i \cdot q_i = v \cdot \left[ 2 \cdot t \cdot \left( \underline{q}_2 - \frac{1}{2} \right) + \frac{p}{(c-b)} - t \right] + v \cdot 2 \cdot t \cdot \left( \underline{q}_2 - \frac{1}{2} \right) \cdot \underline{q}_2,$$

Figure 8: Corner solution



the public expenditures by

$$E = p \cdot q_1 + (c - b) \cdot z_2 \cdot q_2 = p + 4 \cdot t \cdot (c - b) \cdot \left( q_2 - \frac{3}{4} \right) \cdot q_2,$$

and the inequality by

$$I = (z_2 - z_1) \cdot q_2 = 2 \cdot t \cdot \left( q_2 - \frac{1}{2} \right) \cdot q_2.$$

The first derivatives of the profit function are given by

$$\frac{\partial W}{\partial p} = \frac{v + b - c}{c - b} - \lambda \cdot E,$$

and

$$\frac{\partial W}{\partial q_2} = v \cdot 4 \cdot t \cdot \left( q_2 + \frac{1}{4} \right) - [1 + \lambda \cdot E] \cdot 8 \cdot t \cdot (c - b) \cdot \left( q_2 - \frac{3}{8} \right) - \alpha \cdot 4 \cdot t \cdot \left( q_2 - \frac{1}{4} \right).$$

The second derivatives are given by

$$\frac{\partial^2 W}{\partial p^2} = -\lambda < 0,$$

and

$$\frac{\partial^2 W}{\partial p \partial q_2} = -\lambda \cdot 8 \cdot t \cdot (c - b) \cdot \left( q_2 - \frac{3}{8} \right) < 0,$$

and

$$\frac{\partial^2 W}{\partial q_2^2} = (v - \alpha) \cdot 4 \cdot t - [1 + \lambda \cdot E] \cdot 8 \cdot t \cdot (c - b) - \lambda \cdot \left[ 8 \cdot t \cdot (c - b) \cdot \left( q_2 - \frac{3}{8} \right) \right]^2,$$

and

$$\Delta^2 = \frac{\partial^2 W}{\partial p^2} \cdot \frac{\partial^2 W}{\partial q_2^2} - \left( \frac{\partial^2 W}{\partial p \partial q_2} \right)^2 = -\lambda \cdot [(v - \alpha) \cdot 4 \cdot t - [1 + \lambda \cdot E] \cdot 8 \cdot t \cdot (c - b)].$$

Assuming the price to be determined by the first-order condition, we have

$$p = \frac{v + b - c}{\lambda \cdot (c - b)} - 4 \cdot t \cdot (c - b) \cdot \left( \underline{q}_2 - \frac{3}{4} \right) \cdot \underline{q}_2,$$

and

$$E = \frac{v + b - c}{\lambda \cdot (c - b)}.$$

Then

$$\frac{\partial W}{\partial \underline{q}_2} = 4 \cdot (v + \alpha) \cdot t \cdot \left( \left[ 1 - \frac{3}{4} \frac{\alpha}{v + \alpha} \right] - \underline{q}_2 \right),$$

and

$$\frac{\partial^2 W}{\partial \underline{q}_2^2} = -(\alpha + v) \cdot 4 \cdot t - \lambda \cdot \left[ 8 \cdot t \cdot (c - b) \cdot \left( \underline{q}_2 - \frac{3}{8} \right) \right]^2 < 0$$

and

$$\Delta^2 = \lambda \cdot (\alpha + v) \cdot 4 \cdot t > 0.$$

Thus, any solution to the two first-order conditions is a local maximum.

Moreover, notice that

$$\frac{\partial W}{\partial \underline{q}_2} \left( \frac{1}{2} \right) = t \cdot (2 \cdot v - \alpha),$$

and

$$\frac{\partial W}{\partial \underline{q}_2} (1) = -t \cdot 3 \cdot \alpha,$$

which means that I need to consider three different cases (including two ‘‘corner solutions’’) regarding the public firm’s production plan.

First, if  $\alpha \in (0, 2 \cdot v)$ , the first-order condition for an interior solution gives

$$\underline{q}_2 = 1 - \frac{3}{4} \cdot \frac{\alpha}{v + \alpha} \in \left( \frac{1}{2}, 1 \right).$$

Recall that condition 4 is fulfilled if  $p \geq 2 \cdot t \cdot (c - b) \left( 1 - \underline{q}_2 \right)$ , or

$$\frac{v + b - c}{\lambda \cdot (c - b)^2} \geq t \cdot \left[ 1 - \frac{9}{4} \cdot \frac{\alpha \cdot v}{(v + \alpha)^2} \right]$$

and since the expression in square brackets is positive, it is required that

$$t \leq t^z = \frac{v + b - c}{\lambda \cdot (c - b)^2} \cdot \left[ 1 - \frac{9}{4} \cdot \frac{v \cdot \alpha}{(v + \alpha)^2} \right]^{-1}.$$

If this condition is satisfied,  $p > 0$  and social welfare is equal to

$$W^4 = \frac{1}{2 \cdot \lambda} \cdot \left( \frac{v+b-c}{c-b} \right)^2 - \alpha \cdot t \cdot \left( 1 - \frac{9}{8} \cdot \frac{\alpha}{v+\alpha} \right).$$

Second, if  $\alpha \geq 2 \cdot v$ ,  $\frac{\partial W(\frac{1}{2})}{\partial q_2} \leq 0$  and the government sets  $q_2 = \frac{1}{2}$ . Then:

$$p = \frac{v+b-c}{\lambda \cdot (c-b)} + t \cdot (c-b) \cdot \frac{1}{2},$$

and condition 4 is fulfilled if

$$t \leq t^{za} = 2 \cdot \frac{v+b-c}{\lambda \cdot (c-b)^2}.$$

If this condition is satisfied,  $p > 0$  and social welfare is equal to

$$W^2 = \frac{1}{2 \cdot \lambda} \left( \frac{v+b-c}{c-b} \right)^2 - \frac{1}{2} \cdot v \cdot t.$$

Third, if  $\alpha = 0$ ,  $\frac{\partial W(1)}{\partial q_2} \geq 0$  and the government sets  $q_2 = 1$ . Then

$$p = \frac{v+b-c}{\lambda \cdot (c-b)} - t \cdot (c-b),$$

and condition 4 is fulfilled if

$$t \leq t^{zb} = \frac{v+b-c}{\lambda \cdot (c-b)^2}.$$

Notice that  $t^{zb} \leq t^z$ . If this condition is satisfied,  $p \geq 0$  (with strict inequality as long as  $t < t^{zb}$ ) and social welfare is equal to

$$W^6 = \frac{1}{2 \cdot \lambda} \cdot \left( \frac{v+b-c}{c-b} \right)^2 - (\alpha \cdot t).$$

Thinking of  $t^z$  as a function of  $\alpha$ , it may be noted that  $t^z(0) = \frac{v+b-c}{\lambda \cdot (c-b)^2} = t^{zb}$  and  $t^z(2 \cdot v) = \frac{v+b-c}{\lambda \cdot (c-b)^2} \cdot 2 = t^{za}$ . The equilibrium structure is displayed in figure 9, where the curved line is  $t^z(\alpha)$ .

### E.2.3 Comparison of solutions

For large  $t$ , there does not exist any interior solution. Then the corner solution is chosen. For lower  $t$ , when both solutions exist, I need to compare the ‘‘interior solution’’ satisfying condition 4 and the ‘‘corner solution’’ with  $z_1 = 0$ . The reason is that the interior solutions may give rise to  $z_1 \approx 0$  despite  $p \gg 0$ . Then, it may be better to set  $p = 0$  entailing  $z_1 = 0$ . The relevant comparisons are displayed in figure 10.

Figure 9: Interior solution

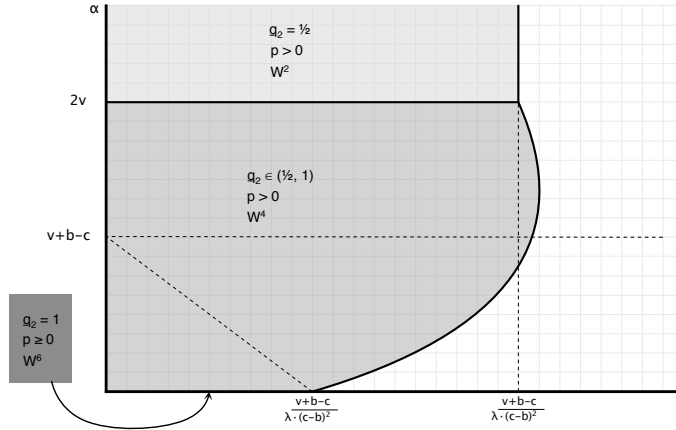
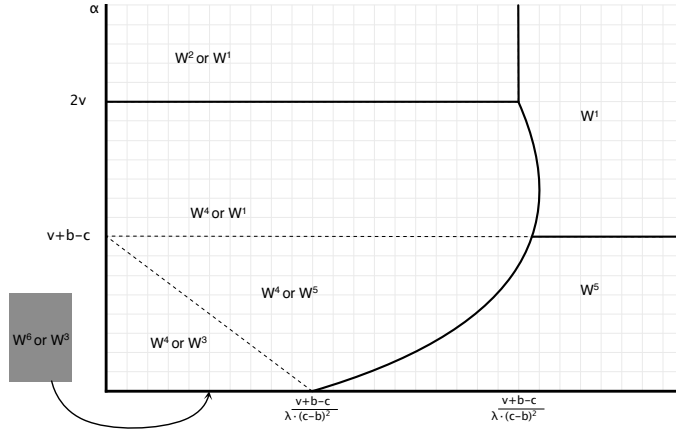


Figure 10: Comparisons



**First**, consider the case when  $\alpha \geq 2 \cdot v$ . Notice that  $W^2 \geq W^1$  if and only if

$$\frac{1}{2 \cdot \lambda} \left( \frac{v+b-c}{c-b} \right)^2 - \frac{1}{2} \cdot v \cdot t \geq 0$$

i.e.

$$t \leq t^*(\alpha) = \frac{1}{v \cdot \lambda} \left( \frac{v+b-c}{c-b} \right)^2.$$

Note that

$$t^*(\alpha) = \left( \frac{v+b-c}{v} \right) \cdot \frac{v+b-c}{\lambda \cdot (c-b)^2} < 2 \cdot \frac{v+b-c}{\lambda \cdot (c-b)^2} = t^{za},$$

since  $b-c < v$ . That is, this condition is more strict, than the condition for  $W^2$  to be valid.

**Second**, consider the case when  $\alpha \in [v+b-c, 2 \cdot v]$ . Notice that  $W^4 \geq W^1$  if and only

if

$$\frac{1}{2 \cdot \lambda} \cdot \left( \frac{v+b-c}{c-b} \right)^2 - \alpha \cdot t \cdot \left( 1 - \frac{9}{8} \cdot \frac{\alpha}{v+\alpha} \right) \geq 0$$

i.e.

$$t \leq t^*(\alpha) = \frac{1}{2 \cdot \lambda} \cdot \left( \frac{v+b-c}{c-b} \right)^2 \cdot \left[ \alpha \cdot \left( 1 - \frac{9}{8} \cdot \frac{\alpha}{v+\alpha} \right) \right]^{-1}.$$

Note that  $t^*(\alpha) \leq t^z$  when  $\alpha \in [v+b-c, 2 \cdot v]$  since

$$\frac{1}{2 \cdot \lambda} \cdot \left( \frac{v+b-c}{c-b} \right)^2 \cdot \left[ \alpha \cdot \left( 1 - \frac{9}{8} \cdot \frac{\alpha}{v+\alpha} \right) \right]^{-1} \leq \frac{v+b-c}{\lambda \cdot (c-b)^2} \cdot \left[ 1 - \frac{9}{4} \cdot \frac{v \cdot \alpha}{(v+\alpha)^2} \right]^{-1}$$

or

$$(v+b-c) \cdot [4 \cdot (v+\alpha)^2 - 9 \cdot v \cdot \alpha] \leq \alpha \cdot (8 \cdot (v+\alpha)^2 - 9 \cdot \alpha(v+\alpha)).$$

This inequality follows from the fact that  $\alpha \geq v+b-c$  in the relevant region and that

$$4 \cdot (v+\alpha)^2 - 9 \cdot v \cdot \alpha \leq 8 \cdot (v+\alpha)^2 - 9 \cdot \alpha(v+\alpha)$$

i.e.  $\alpha \leq 2 \cdot v$  which is true in the relevant region.

**Third**, consider the case when  $\alpha \leq v+b-c$ . Notice that  $W^4 \geq W^5$  if and only if

$$\frac{1}{2 \cdot \lambda} \cdot \left( \frac{v+b-c}{c-b} \right)^2 - \alpha \cdot t \cdot \left( 1 - \frac{9}{8} \cdot \frac{\alpha}{v+\alpha} \right) \geq \frac{1}{2 \cdot \lambda} \cdot \left[ \frac{v+b-c-\alpha}{c-b} \right]^2$$

i.e.

$$t \leq t^*(\alpha) = \frac{1}{\lambda} \cdot \frac{(v+b-c) - \frac{1}{2} \cdot \alpha}{(c-b)^2} \cdot \left( 1 - \frac{9}{8} \cdot \frac{\alpha}{v+\alpha} \right)^{-1}.$$

Recall that  $W^4$  is valid if

$$t^z = \frac{v+b-c}{\lambda \cdot (c-b)^2} \cdot \left[ 1 - \frac{9}{4} \cdot \frac{v \cdot \alpha}{(v+\alpha)^2} \right]^{-1}$$

$$\frac{1}{\lambda} \cdot \frac{(v+b-c) - \frac{1}{2} \cdot \alpha}{(c-b)^2} \cdot \left( 1 - \frac{9}{8} \cdot \frac{\alpha}{v+\alpha} \right)^{-1} \leq \frac{v+b-c}{\lambda \cdot (c-b)^2} \cdot \left[ 1 - \frac{9}{4} \cdot \frac{v \cdot \alpha}{(v+\alpha)^2} \right]^{-1}$$

i.e.

$$\left[ (v+b-c) - \frac{1}{2} \cdot \alpha \right] \cdot \left[ 1 - \frac{9}{8} \cdot \frac{\alpha}{(v+\alpha)} \cdot \frac{2 \cdot v}{(v+\alpha)} \right] \leq [v+b-c] \cdot \left( 1 - \frac{9}{8} \cdot \frac{\alpha}{v+\alpha} \right)$$

which is true, comparing factor by factor, as long as  $\alpha > 0$ .

**Fourth**, consider the case when  $\alpha \leq v + b - c$ . Notice that  $W^4 \geq W^3$  if and only if

$$\frac{1}{2 \cdot \lambda} \cdot \left( \frac{v+b-c}{c-b} \right)^2 - \alpha \cdot t \cdot \left( 1 - \frac{9}{8} \cdot \frac{\alpha}{v+\alpha} \right) \geq \left[ (v+b-c-\alpha) - \frac{\lambda}{2} \cdot (c-b)^2 \cdot t \right] \cdot t$$

i.e.

$$\frac{1}{2 \cdot \lambda} \cdot \left( \frac{v+b-c}{c-b} \right)^2 \geq \left[ (v+b-c) - \frac{9}{8} \cdot \frac{\alpha^2}{v+\alpha} - \frac{\lambda}{2} \cdot (c-b)^2 \cdot t \right] \cdot t.$$

This inequality is always fulfilled, since maximum value of the right hand side (maximizing over  $t$ ) is lower than the left and side.

**Fifth**, consider the case when  $\alpha = 0$ . Notice that  $W^6 \geq W^3$  if and only if

$$\frac{1}{2 \cdot \lambda} \cdot \left( \frac{v+b-c}{c-b} \right)^2 - (\alpha \cdot t) \geq \left[ (v+b-c-\alpha) - \frac{\lambda}{2} \cdot (c-b)^2 \cdot t \right] \cdot t$$

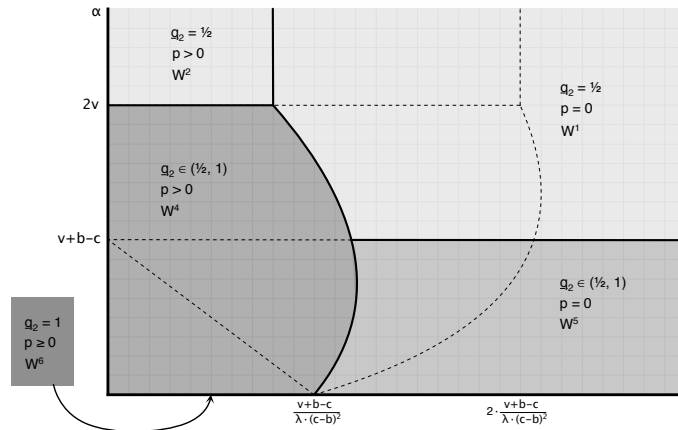
i.e.

$$\frac{1}{2 \cdot \lambda} \cdot \left( \frac{v+b-c}{c-b} \right)^2 \geq \left[ (v+b-c) - \frac{\lambda}{2} \cdot (c-b)^2 \cdot t \right] \cdot t.$$

This inequality is always fulfilled, since maximum value of the right hand side (maximizing over  $t$ ) is equal to the left and side.

The solution is summarized by figure 11. The, border between the interior and the corner

Figure 11: Mixed ownership



solution, the  $t^*(\alpha)$ -function, is given by:

$$t^*(\alpha) = \begin{cases} \frac{1}{v \cdot \lambda} \cdot \left( \frac{v+b-c}{c-b} \right)^2 & \alpha \geq 2 \cdot v \\ \frac{1}{2 \cdot \alpha \cdot \lambda} \cdot \left( \frac{v+b-c}{c-b} \right)^2 \cdot \left( 1 - \frac{9}{8} \cdot \frac{\alpha}{v+\alpha} \right)^{-1} & \alpha \in [v+b-c, 2 \cdot v] \\ \frac{1}{\lambda} \cdot \frac{(v+b-c) - \frac{1}{2} \cdot \alpha}{(c-b)^2} \cdot \left( 1 - \frac{9}{8} \cdot \frac{\alpha}{v+\alpha} \right)^{-1} & \alpha \leq v+b-c \end{cases}$$

It is straight-forward to demonstrate that this function is continuous in  $\alpha$ .

## F Comparison of ownership models

To prove Proposition 3, the main argument is that mixed ownership allows the government to always replicate the outcome of the pure models, but also to do more.

### F.1 Mixed vs private ownership

With mixed ownership the government can always implement the same (symmetric) equilibrium qualities as under private ownership, *i.e.*

$$z = \begin{cases} \frac{p-t \cdot (c-b)}{c-b} > 0, & p > t \cdot (c-b) \\ 0, & p = 0 \end{cases}$$

by setting  $q_2 = \frac{1}{2}$ . With private ownership, the producers always receive rents whenever quality is strictly positive (since  $p = (c-b) \cdot z + t \cdot (c-b)$ ). With mixed ownership, only the private firm keeps the rents. Thus, under the conditions when the government sets  $q_2 = \frac{1}{2}$  and  $p > 0$  under mixed ownership, mixed ownership is preferred. Under the conditions when the government sets  $q_2 = \frac{1}{2}$  and  $p = 0$  under mixed ownership, the two ownership models give exactly the same outcome. Finally, under any conditions when the government sets  $q_2 > \frac{1}{2}$ , mixed ownership is preferred (by revealed preference). In sum: mixed ownership (weakly) dominates private ownership. The government is indifferent only if  $\alpha \geq v + b - c$  and  $t > t^*(\alpha)$ .

### F.2 Mixed vs public ownership

With mixed ownership the government can always implement the same outcome as under public ownership by setting  $p = 0$ . Thus, under any condition when the government sets  $p > 0$ , the mixed model yields a strictly better outcome. Thus, mixed ownership (weakly) dominates public ownership. The government is indifferent only if  $t > t^*(\alpha)$ .

### F.3 Breach of competitive neutrality

To prove Proposition 4, recall that in equilibrium the public producer offers quality

$$z_2 = \frac{p - t \cdot (c-b)}{(c-b)} + 4 \cdot t \cdot \left( q_2 - \frac{1}{2} \right)$$

which leads to cost

$$(c-b) \cdot z_2 = p - t \cdot (c-b) + (c-b) \cdot 4 \cdot t \cdot \left( q_2 - \frac{1}{2} \right)$$



or

$$(c - b) \cdot z_2 = p - 4 \cdot t \cdot (c - b) \cdot \left( \frac{3}{4} - \underline{q}_2 \right).$$

Thus, the voucher does not cover the public producer's cost, i.e.  $p < (c - b) \cdot z_2$ , if  $\underline{q}_2 > \frac{3}{4}$ . (The private producer's cost is always covered in equilibrium.) Recall the government's choice of  $\underline{q}_2$  reported in Lemma 12. When  $t < t^*(\alpha)$ , the government sets  $\underline{q}_2 = 1 - \frac{3}{4} \cdot \frac{\alpha}{v + \alpha}$  in breach of competitive neutrality if  $\alpha < \frac{v}{2}$ . When  $t \geq t^*(\alpha)$ , the government sets  $\underline{q}_2 = \frac{1}{4} + \sqrt{\frac{1}{16} + \frac{v + b - c - \alpha}{2 \cdot t \cdot \lambda \cdot (c - b)^2}}$  in breach of competitive neutrality if  $\alpha < (v + b - c) - \frac{3}{8} \cdot \lambda \cdot (c - b)^2 \cdot t$ .

## G Competitive neutrality

To prove Lemma 8, recall that producer 2 is the public producer. The government sets  $p$  and  $\underline{q}_2$  to maximize social welfare,  $W = B - (E + \frac{\lambda}{2} \cdot E^2) - \alpha \cdot I$ . The government's choice set is described by figure 7.

I will first consider the case when condition 4 is not violated and the government implements  $z_1 > 0$ . I will refer to this as the “interior solution” and the case when condition 4 is not binding as the “corner solution.”

### G.1 Case 1: “interior solution” with $z_1 > 0$

Recall that the two best-reply functions are given by

$$z_1 = \frac{p - t \cdot (c - b)}{2 \cdot (c - b)} + \frac{1}{2} \cdot z_2,$$

$$z_2 = 2 \cdot t \cdot \left( \underline{q}_2 - \frac{1}{2} \right) + z_1.$$

When competitive neutrality is binding the voucher price must be set at  $p = (c - b) \cdot z_2$  to ensure that the private producer receives the same conditions as the public producer. I will use the competitive neutrality restriction to substitute for the government's choice variable  $p$ . Thus, I will formally allow the government to set  $z_2$  instead. It follows immediately from the private firm's best-reply function that  $z_1 = z_2 - \frac{1}{2} \cdot t$  as long as  $z_2 \geq \frac{1}{2} \cdot t$  and from the public firm's best-reply function that the government must set  $\underline{q}_2 = \frac{3}{4}$ .

In equilibrium, the social benefits are given by

$$B = v \cdot z_2 - \frac{1}{8} \cdot v \cdot t,$$

and the public expenditures are given by

$$E = (c - b) \cdot z_2,$$

and the inequality is given by

$$I = (z_2 - z_1) \cdot q_2 = \frac{3}{8} \cdot t.$$

The government sets  $z_2$  to maximize social welfare,

$$W = (v + b - c) \cdot z_2 - \frac{\lambda}{2} \cdot (c - b)^2 \cdot z_2^2 - \frac{1}{8} \cdot [v + 3 \cdot \alpha] \cdot t$$

The first-order condition is

$$\frac{\partial W}{\partial z_2} = (v + b - c) - \lambda \cdot (c - b)^2 \cdot z_2 = 0.$$

Thus  $z_2 = \frac{v+b-c}{\lambda \cdot (c-b)^2}$  and  $z_1 = \frac{v+b-c}{\lambda \cdot (c-b)^2} - \frac{1}{2} \cdot t$ . This solution is valid, i.e.,  $z_1 \geq 0$ , if  $t \leq 2 \cdot \frac{v+b-c}{\lambda \cdot (c-b)^2}$ . Then:

$$W^{CN} = \frac{1}{2 \cdot \lambda} \cdot \left( \frac{v + b - c}{c - b} \right)^2 - \frac{v + 3 \cdot \alpha}{8} \cdot t.$$

Note that since  $q_2 = \frac{3}{4}$ , it follows that  $q_2 \in [\frac{1}{2}, 1]$ .

## G.2 Case 2: “Corner solution” with $z_1 = 0$

When condition 4 is violated, the equilibrium qualities are given by  $z_1 = 0$  and  $z_2 = 2 \cdot t \cdot \left( q_2 - \frac{1}{2} \right)$ . When competitive neutrality is binding the voucher price must be set at  $p = (c - b) \cdot z_2$  to ensure that the private producer receives the same conditions as the public producer. It is, however, immediately clear that pure public ownership is better than mixed oligopoly under a binding competitive neutrality constraint, in case there is a corner solution. The reason is that with a pure public solution, the government does not need to pay any rents to the firm producing zero non-verifiable quality. I will therefore not analyze this solution any further.

## G.3 Effect of a binding competitive neutrality regulation on the private producer

Note that the public producer has a lower quantity under competitive neutrality regulation, i.e.  $\frac{3}{4} < 1 - \frac{3}{4} \cdot \frac{\alpha}{v+\alpha}$  if  $\alpha < \frac{v}{2}$ . Also, the voucher price is lower under competitive neutrality regulation if

$$(c-b) \cdot \frac{v+b-c}{\lambda \cdot (c-b)^2} > \frac{v+b-c}{\lambda \cdot (c-b)} - t \cdot (c-b) \cdot \frac{1}{4} \cdot \left(1 - 3 \cdot \frac{\alpha}{v+\alpha}\right) \cdot \left(4 - 3 \cdot \frac{\alpha}{v+\alpha}\right)$$

$$0 < \left(\frac{1}{3} - \frac{\alpha}{v+\alpha}\right) \cdot \left(\frac{4}{3} - \frac{\alpha}{v+\alpha}\right)$$

i.e if  $\frac{1}{3} > \frac{\alpha}{v+\alpha}$  i.e  $\frac{v}{2} > \alpha$ . Private profit absent competitive neutrality regulation is

$$\pi = (p - (c-b) \cdot z_1) \cdot q_1$$

$$\pi = t \cdot (c-b) \cdot \frac{9}{8} \cdot \left(\frac{\alpha}{v+\alpha}\right)^2$$

Under competitive neutrality regulation it is

$$\pi = (c-b) \cdot \frac{1}{8} \cdot t$$

Thus profit is larger under regulation if  $9 \cdot \left(\frac{\alpha}{v+\alpha}\right)^2 < 1$  i.e.  $\alpha < \frac{v}{2}$ .

## G.4 Choice of ownership

Recall that under mixed ownership and binding competitive neutrality, welfare is given by

$$W^{CN}(t) = \frac{1}{2 \cdot \lambda} \cdot \left(\frac{v+b-c}{c-b}\right)^2 - \frac{v+3 \cdot \alpha}{8} \cdot t.$$

Under private ownership welfare is given by

$$W^P = \begin{cases} \frac{1}{2 \cdot \lambda} \cdot \left[\frac{v+b-c}{c-b}\right]^2 - v \cdot t > 0 & t \leq \frac{v+b-c}{2 \cdot v} \cdot \frac{v+b-c}{\lambda \cdot (c-b)^2} \\ 0 & \text{otherwise} \end{cases}$$

Thus  $W^{CN} \geq W^P$  since  $\frac{v+3 \cdot \alpha}{8} < v$  i.e.  $\alpha < \frac{7}{3} \cdot v$  which is fulfilled when the regulation is binding.

Also recall that

$$W^G(t) = \begin{cases} 0 & \alpha \geq \alpha^G, \\ \frac{1}{2 \cdot \lambda} \cdot \left[\frac{v+b-c-\alpha}{c-b}\right]^2 > 0, & \alpha < \alpha^G, t > t^G, \\ \left[(v+b-c-\alpha) - \frac{\lambda}{2} \cdot (c-b)^2 \cdot t\right] \cdot t > 0, & \alpha < \alpha^G, t \leq t^G. \end{cases}$$

Notice that when  $\alpha < \alpha^G$ ,  $W^G(t)$  is continuous and increasing from  $W^G(0) = 0$  to  $W^G(t) = \frac{1}{2 \cdot \lambda} \cdot \left[\frac{v+b-c-\alpha}{c-b}\right]^2 > 0$  for large  $t$ , and that  $W^{CN}(t)$  is falling in  $t$ . In fact,

$$W^{CN}(t) = \frac{1}{2 \cdot \lambda} \cdot \left(\frac{v+b-c}{c-b}\right)^2 - \frac{v+3 \cdot \alpha}{8} \cdot t < \frac{1}{2 \cdot \lambda} \cdot \left[\frac{v+b-c-\alpha}{c-b}\right]^2$$

when

$$t > \frac{8 \cdot \alpha}{v + 3 \cdot \alpha} \cdot \frac{(v + b - c) - \frac{1}{2} \cdot \alpha}{\lambda \cdot (c - b)^2}.$$

Thus for  $t$  large enough, the government prefers pure public ownership to mixed ownership under competitive neutrality regulation.