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Counterparty Credit Risk
Efficieny of Central Clearing

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Counterparty Credit Risk: Efficiency of Central Clearing

ABSTRACT

In this thesis, we aim to show effects of centrally clearing OTC derivatives on counterparty exposures. Central clearing is the process of replacing bilateral exposures from transactions with a network of multilateral exposures. In all transactions, a central counterparty (CCP) is the intermediary, acting as a buyer to each seller and vice versa. Central clearing has been mandated by regulators to mitigate counterparty credit risk which was a major factor in the 2008/09 financial crisis. We emphasize credit default swaps which are a financial instrument that compensates in the case of a credit event and follow the model by Duffie and Zhu (2011) as well as part of the model extension by Cont and Kokholm (2014). Their research is related to clearing efficiency when clearing a single derivative class. We show how netting efficiency is measured and that splitting counterparty clearing over several clearing facilities can have adverse effects to the intended purpose of the clearing system in some settings. We show that there is a tradeoff between bilateral netting between counterparties across asset classes and multilateral netting between several counterparties for single asset classes, e.g. credit derivatives.

Keywords: *Risk Management, Counterparty Credit Risk, OTC Derivatives, Central Clearing, Multilateral Netting, Bilateral Netting*

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LIST OF ABBREVIATIONS

AIF - Alternative Investment Fund
AIG - American International Group
BCBS - Basel Committee on Banking Supervision
BIS - Bank for International Settlements
CAD - Canadian Dollars
CCR - Counterparty Credit Risk
CCP - Central Clearing Counterparty
CDO - Collateralized Debt Obligation
CDS - Credit Default Swap
CSA - Credit Support Annex
CVA - Credit Value Adjustment
c.p. - Ceteris paribus (all else equal)
EMIR - European Markets Infrastructure Regulation
FC - Financial Counterparty
FX - Foreign Exchange
IR - Interest Rate
ISDA - International Swaps and Derivatives Association
JPY - Japanese Yen
MTM - Mark-to-market (value)
NFC - Non-financial Counterparty
OTC - Over-the-counter
SEC - Securities and Exchange Commission
SEF - Swap Execution Facility
SEK - Swedish Kronor
USD - U.S. Dollars
VaR - Value at Risk

1. INTRODUCTION

"Credit risk, and in particular counterparty credit risk, is probably the single most important variable in determining whether and with what speed financial disturbances become financial shocks with potential systemic traits"

- Counterparty Risk Management Policy Group, 2005

Financial regulation has been a major source of concern for governments, financial institutions and investors among others, not only since the most recent financial crisis from 2008. An area that has received special attention more recently is the market for over-the-counter (OTC) derivatives. Other than exchange traded derivatives, OTC derivative contracts are privately negotiated and not traded on organized exchanges. Their value depends on the performance of underlying market factors, e.g. credit prices and interest rates for credit and interest rate derivatives, respectively (Office of the Comptroller of the Currency, 2017).

The OTC derivatives market is vast and opaque, can spin a transaction web that is hard to see through and has the means to cause massive turmoil as was seen during the financial crisis of 2008-09. In today's highly connected financial system, the complexity and bilateral relation of derivative securities has the means to affect more than only the financial sector. To exemplify, Lehman Brothers had a notional amount of USD 800 billion from OTC derivatives in its books at the point when the bank declared the largest Chapter 11 ("reorganization") bankruptcy in the history of the United States and defaulted on its debt and swap obligations on September 16, 2008. Approximately one million derivatives trades had to be unwound in a legally complex and lengthy process with intense consequences for the economy worldwide, leading to the issue of the "Troubled Asset Relief Program" by the U.S. government, capitalized with close to USD 1 trillion to purchase distressed assets and support failing banks (Gregory, 2015). The assumption of institutions being "too big to fail" has been severely challenged and under public scrutiny since then, an assumption among other factors driven by risks not being properly assessed and counterparties that were assigned a default probability of zero percent or close to it.

Consequentially, ratings of securities did not reflect the actual risk involved due to that entity's previous rating and overall importance to the financial system.
To mitigate counterparty risk, which is the risk that a counterparty defaults in a derivatives contract, policy-makers have mandated central counterparty (CCP) clearing for eligible OTC derivatives, e.g. under the Dodd-Frank Act in the United States and Basel Regulation in the European Union (U.S. Congress, 2010; European Commission, 2010; Loon and Zhaodong, 2014). CCPs are financial intermediaries that severely alter the market structure for OTC derivatives: obligations between CCP members are handled by the CCP, and its members transfer liquidity to the CCP through a guarantee fund contribution (Amini et al., 2013). In return, central counterparties ensure the fulfillment of contractual obligations, even in case of default.
Generally, central counterparties are associated with higher netting efficiency, lower likelihood of default contagion through the financial system and an overall reduction in counterparty risk (Chande et al., 2010). However, many aspects of how to implement a clearing intermediary that actually mitigates systemic risk as opposed to concentrating even more of it in one place remains controversial among academics.

The objective of this thesis is to examine the impact of central clearing. We closely follow the methodology of Duffie and Zhu (2011) and show on the example of credit default swaps, which are instruments that compensate in case of a credit event, that the threshold in terms of required clearing members for a CCP which clears only one class of derivatives has been very high during the time of their writing and has increased immensely since then. We implement the model by the forementioned authors in a numerical computing environment and show in a number of different scenarios that efficiency gains can be achieved with CCP clearing. We revise the model framework with current data and give an outlook in terms of what we believe is a realistic scenario for clearing in Europe at a point of time when the current regulation, which has been undergoing massive changes and currently still is, will be fully implemented. We further show why we disagree with Cont and Kokholm (2014) who extend the Duffie and Zhu model by correlating assets in an argument towards single-asset clearing.
The structure of the thesis is as follows. In Section 2 we give a short overview over relevant developments in academic literature. In Section 3 we present a non-technical background of counterparty credit risk and aspects that are related to it. In Section 4 we model counterparty exposures as in Duffie and Zhu (2011) and set their results

in relation to current data. We give an outlook for a potential future scenario. A summary of our results concludes in Section 5.

2. RELATED LITERATURE

The interest of academics in counterparty risk has seen a huge increase since it has been identified as a key driver of the 2007-08 financial crisis; research spans quantitative as well as qualitative aspects of recent developments in clearing. In this section, we briefly revise some of the recent academic developments in counterparty risk.

In an early publication, Bliss and Steigerwald (2006) argue for co-existence of bi- and multilateral netting, i.e. settlement of net positions between two entities directly and between multiple dealers with a central intermediary, as both methods have their merits depending on the cleared derivative. With implementation of the European Market Infrastructure Regulation, however, bilateral netting in OTC derivative transactions will be a thing of the past for some derivative classes (henceforth also called asset classes).

Jackson and Manning (2007) quantitatively analyze relative cost and risk implications of different clearing methods, including bilateral, ring and CCP clearing. In bilateral clearing, agents post security based on the net obligation to their counterparty. Ring clearing is an informal netting agreement between three or more members. It eliminates bilateral exposures and reallocates multilateral exposures among the members of the ring. Ring clearing is nowadays replaced by CCP clearing in which a clearing house acts as counterparty for all market participants (see Section 3.1). Their research shows that margin pooling, the benefit achieved on margins, which are also called collateral, from pooling a number of positions is an important effect in cross-asset clearing and that merging CCPs has the potential to significantly reduce members' risks and costs. Margins are essentially safety deposits and will be further explained in Section 3.1.

The Bank for International Settlements (2010) similarly identifies consolidation and increased competition among CCPs as a trend, potentially increasing the number of outstanding contracts and fragmenting total clearing volume, leading to a decrease in netting efficiency. It warns of interdependencies between systems and markets but is unable to identify what effect different structures might have on systemic risk.

Cox et al. (2014) compare CCPs clearing separately and in a linked arrangement with each other, meaning that participants can clear positions in any linked CCP without multiple memberships. They find that links in CCP clearing carries the opposing effects of cost from inter-CCP exposure and benefits from inter-CCP netting. Their research suggests that under a number of different assumptions, the benefits of netting prevails and aggregate exposure decreases in a linked setting.
Amini et al. (2013) emphasize the need for CCPs to specifically be designed to reduce systemic risk as exposures are concentrated in one facility while at the same time accounting for participants' perspectives and incentives. Hull (2012*a*) goes so far to suggest an addendum for derivatives contracts to protect CCP equity as these institutions potentially pose a systemic risk and fall into the "too-big-to-fail" category. This addendum would induce a mechanism to offset transactions of a defaulting member with transactions by other clearing members, closing out at most recent pre-default prices. The transactions would be chosen by the CCP on a prorata basis by risk and size of position for all members with offsetting transactions, protecting the CCP from default in case of one or more members defaulting. The non-defaulting members could be at least partially compensated by the defaulter's initial margins and fund contributions. In a similar case, described in Section 3.2, LCH Swapclear fully resolved Lehhman Brothers' USD 9 trillion interest rate swap default comprising over 65,000 trades with its default management process within the margins held and without loss to other market participants (LCH SwapClear, 2008).
Certainly, the Lehman Brothers crisis has shown that there is a lack of transparency in the market of OTC derivatives. Cecchetti et al. (2009) then note that clearing houses serve as a central reference for repositories to increase transparency which has also been addressed in the European Markets Infrastructure Regulation (EMIR). Also, central counterparties improve system safety by compressing outstanding derivatives in a small number of CCPs by netting dealers' positions against each other, introducing standardizing risk management procedures and implementing a more efficient management of collaterals. Even further, CCPs alleviate counterparties of monitoring creditworthiness, determination of collateral requirements, compliance and more functions, effectively reducing the members' costs while homogenizing credit risk through margining and capital requirements (Bliss and Steigerwald, 2006).
Amini et al. (2013) consider network effects in multilateral clearing and emphasize a focus on mitigation of systemic risk in clearing house design, taking all partici-

pants incentives and interests into account through CCP equity, fee and contribution structure.

Pirrong (2009) is skeptical towards the concept of central clearing and argues that credit risk is redistributed rather than reduced due to firm interconnections not only in derivatives but also in other contracts. Reducing default losses on derivatives for big dealers does not necessarily reduce systemic risk, it is rather that CCPs can incentivize further trading activity thus potentially increasing default losses. Besides, it is likely that default risk is mis-priced because of existing information asymmetries and institutional constraints which lead to excessive risk-taking and thus potential problems in terms of systemic risk. Pirrong also makes the argument that institutions have used traditional risk sharing persistently, either because "collective action problems, strategic behavior, or some other transaction cost that could be mitigated by government action is preventing the implementation of the more efficient alternative, or that the existing default mechanism offers lower costs and/or higher benefits than the alternatives". Cont (2010) takes a similar line and emphasizes that systemic risk and default contagion can be channeled through OTC derivatives markets: credit default swaps' (CDS) mark-to-market values absent a default may represent only a small share of a position's notional, yet, in case of default of the reference entity, the generated exposure may be a large fraction of the notional. This risk of default ("jump-to-default risk") must be accounted for as it can yield a huge payment obligation for the CDS protection seller. The stability of the market and its participants thus depends crucially on requirements imposed on large protection sellers regarding capitalization and liquidity as well as clearing mechanisms. Yet, participation of all large protection sellers in a well capitalized central clearing house with adequate risk management can contribute to a reduction in systemic risk. Similarly, Cont and Minca (2016) show that under-capitalized CDS protection sellers increase default contagion and systemic risk.

3. COUNTERPARTY CREDIT RISK

3.1 Background

This section presents more basic concepts of Counterparty Credit Risk and provides the foundation for the models in Section 4. The presentation below is to a large extent based on Gregory (2015).
Credit risk is the risk of an entity, with whom one has entered into a financial contract, failing to fulfill its contractual obligation at due date because it is either unwilling or unable to do so. Counterparty credit risk (CCR or counterparty risk) is typically regarded as credit risk between counterparties in OTC derivative transactions. In OTC markets, interest rate (IR) swaps are the largest asset class.
Differentiating counterparty risk from credit risk is the fact that the future value of derivatives contracts is uncertain. The derivatives' mark-to-market (MTM) value depends on the net value of all future cash-flows scheduled under the contract up to a possible default or maturity date. Since this value can be either positive or negative, every entity has risk exposure to each of its counterparties which means that the risk is bilateral. Hence, the MTM value is closely related to the net position with a particular counterparty and thus potential losses in case this counterparty were to default today. To exemplify, if the value of a derivative is positive to a dealer when its counterparty, which has not posted any collateral, defaults, the dealer will incur a loss. When the derivative's value on the other hand is negative to the dealer, it will not incur a loss (Hull, 2012*b*). The dealer's exposure to the counterparty at all times is thus $\max(\text{MTM}, 0)$.

In similar measure as credit risk in commercial loans such as mortgages, counterparty risk is managed by a collateral requirement. In derivatives trading, collateral requirements are typically referred to as margins. A dealer, whether it be a bank or another institution, that enters a contract in which money can possibly be owed at a future point in time will have to post a so called initial margin at the time of contract formation, covering potential future obligations with high certainty. The additional requirement to post a so called variation margin follows a margin call after adverse

market movements against the dealer of a derivative security. If variation margin is not posted promptly, the owing counterparty defaults and its positions are unwound as described in Subsection 3.3.
For each OTC derivatives contract, institutions calculate a credit value adjustment (CVA) which is equal to the difference between the actual and the risk-free value of the portfolio. It takes the default probability of the counterparty into account and effectively reflects the market value of counterparty credit risk (Hull, 2015).

3.2 Central Counterparties and Clearing

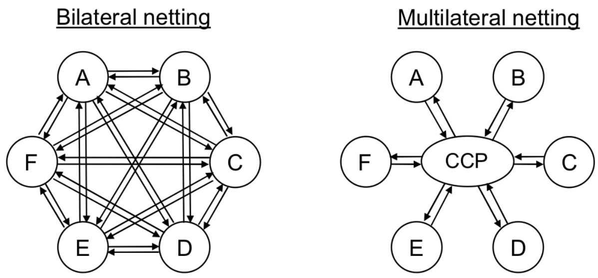
In this subsection we outline the function of central counterparties and the two predominant ways to clear derivatives.
Counterparty credit risk is an inherent component of systemic risk analysis. Due to network effects, the default of one institution on its obligations can lead to the default of one of its counterparties and potentially threaten financial stability through a knock-on effect (Lando, 2004; Bliss and Papathanassiou, 2006). Central clearing has been proposed by regulators as mechanism to mitigate counterparty credit risk. A central clearing counterparty (CCP) is a financial intermediary for buyers and sellers of OTC derivative securities. As legally independent institution it modifies the market structure by taking the part of the contractual counterparty in each transaction for either counterparty as can be seen in Figure 3.1. It guarantees for counterparty risk while at the same time being free of market risk absent defaults due to a matched book (Jackson and Manning, 2007; Duffie and Zhu, 2011). The likelihood for knock-on failures decreases through mutualization of losses over all members while market risk remains with the original trading counterparties. Credit crisk, however, is centralized in the CCP.
Examples for central clearing counterparties are LCH.Clearnet based in the United Kingdom which cleared about 50% of interest swaps globally in 2012[1], ICE Clear Credit based in the U.S. clearing credit default swaps (CDSs) and energy derivatives, and Eurex Clearing based in Germany, clearing mainly equity derivatives.

In the over-the-counter market, transactions are traditionally cleared in two ways: either bilaterally between two counterparties directly or multilaterally through a

[1] http://www.risk.net/infrastructure/clearing/2181788/swapclear-hits-1-trillion-mark-buy-side-otc-clearing

CCP. The different clearing mechanisms are shown schematically in Figure 3.1.

Figure 3.1: Bilateral vs. multilateral netting (Gregory, 2009)



3.2.1 Bilateral Clearing

In bilateral clearing, the two contracting parties typically close an International Swaps and Derivatives Association (ISDA) master agreement which determines the clearing of all outstanding transactions between them and the procedures in case of a default. The ISDA Master Agreement also includes the provision of netting an arbitrary number of derivative transactions which can then be considered as a single transaction. This process can decrease counterparty risk as the potential exposure between two dealers changes from

$$\sum_{i=1}^{N} \max\left(\mathrm{MTM}_i, 0\right)$$

to

$$\max\left(\sum_{i=1}^{N} \mathrm{MTM}_i, 0\right)$$

where N is the number of derivative contracts between the two counterparties and MTM_i is the mark-to-market value of the i-th contract.

In addition to the Master Agreement, a credit support annex (CSA) defines the amount and type of collateral to be made, haircuts to securities, which is the difference between an asset's market value when used as loan collateral and the actual amount of a loan, as well as further contractual details (Hull, 2012*b*).

Regulation under current implementation as explained in Subsection 3.4 requires initial and variation margins for bilaterally cleared transactions between financial institutions or between a financial institution and a systemically important non-financial entity. For bilaterally netted transactions, the initial margin typically has to cover a 10-day 99% Value at Risk in stressed market conditions and has to be maintained by a fiduciary or in a similar way separated from other assets . In transactions with non-systemically important non-financial institutions, there is larger contractual freedom in terms of the master agreement and required margins (Hull, 2015).

3.2.2 Multilateral Clearing

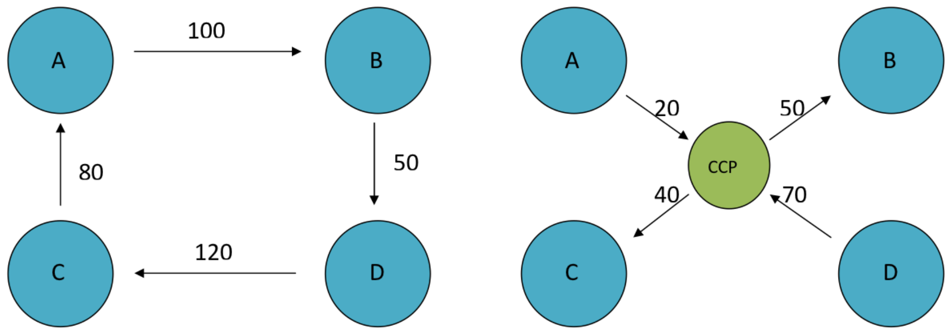
In multilateral clearing, a CCP takes care of the clearing process. When two entities agree on an OTC derivative contract and a CCP accepts it upon presentation, then the clearing house becomes a transaction-offsetting intermediary in between the two affiliates. The transaction is offset in that sense that in case of credit default swaps for example, the CCP buys protection from the protection seller and sells it to the protection buyer. At the same time, the two entities which are members of that CCP pay an initial and potential variation margins for the clearing of their transaction. If an entity is not a clearing member, it can access transactions through an other member of a CCP (Hull, 2015).

The initial margin is typically set so that five-day market movements are covered with 99% confidence (5-day 99% Value at Risk) and margin calls may follow for the respective counterparty depending on market movements of the underlying index or security (Hull, 2015). To calculate the margins for each clearing member, all transactions with the clearing facility are netted. As Hull (2012*a*) points out, CCP clearing induces lower margin requirements than bilateral transactions: assume dealer i has two derivative contracts through a CCP with dealer j. To dealer i, one of the contracts is worth USD 6 million, the other -USD 5 million. For the sake of calculating initial margins, the contracts are netted against each other and thus equivalent to a single contract worth USD 1 million. The same theory applies for a scenario with an arbitrary number of contracts and more than two dealers trading with each other through the CCP. The initial margin demanded from a clearing member also reflects the volatility of its total position with the CCP. On the other hand, margins have to be posted in cash or government securities which likely affects dealers' liquidity negatively as margin calls have to be met immediately, otherwise the position will be unwound.

Cecchetti et al. (2009) shortly highlights three advantages that come with multilateral clearing: firstly, management of counterparty risk is simplified and partly

transferred to the CCP. Secondly, a CCP can net exposures as well as payments, and thirdly, transparency is increased by the CCP which is able to provide information on exposures and market activities to regulators as well as the public.
Figure 3.2 demonstrates the difference in exposure from bilateral to multilateral clearing. While the total exposure in a bilaterally cleared market in this example is 350, it is compressed to 180 in a centrally cleared market where exposures are netted for all market participants and not only direct trading partners.

Figure 3.2: Left: bilateral market. Right: centrally cleared market



Source: Cont and Kokholm (2014)

3.3 Handling of Defaults

As soon as a clearing member fails to meet its margin requirements, it is in default and all its positions with the CCP are closed out. In this case, a CCP often sustains a loss and what is commonly known as the "default waterfall" comes into effect. This mechanism distributes defaults caused losses and is further described by Ghamami (2015). The order of loss funding generally starts at the margin account of the defaulter. Once the account is exhausted, the defaulter's default fund contributions cover further losses, followed by other members' default fund contributions and finally by the CCP's equity. In the case of ICE Clear Credit for example, the CCP's total margin on deposit (initial margins) is USD 22.5 billion and the Guaranty Fund Contributions total USD 1.9 billion. ICE Clear Credit's own "skin in the game" which is the CCP's own guaranty fund contribution totals USD 50 million[2]. Furthermore, the ISDA master agreement typically gives an option to the counterparty

[2] stated by ICE on https://www.theice.com/clear-credit/regulation (20.4.2017)

of the defaulting entity to early terminate all transactions with the defaulter (Hull, 2015). This option can, but does not have to, be exercised. Naturally, early termination is only considered if the dealer with the option to terminate is in-the-money.

3.4 Regulatory Basis

Since the 2008-09 crisis, regulators have increasingly directed their focus towards credit risk. This subsection gives an overview over the regulatory framework under current implementation.

3.4.1 Dodd-Frank Wall Street Reform and Consumer Protection Act

In the United States, the Dodd-Frank Wall Street Reform and Consumer Protection Act (U.S. Congress, 2010, henceforth "Dodd-Frank Act") became effective in 2010. It aims to prevent further bailouts of financial institutions and to protect consumers. The Financial Stability Oversight Council and the Office of Financial Research were instated to monitor systemic risk, report on the state of the economy and identify potential threats to financial stability.

While OTC derivatives in the U.S. had previously not been subjected to governmental oversight, this was changed in Title VII of the Dodd-Frank Act, giving authority to the Securities and Exchange Commission (SEC), the Commodity Futures Trading Commission, and "the prudential regulators for the purposes of assuring regulatory consistency and comparability, to the extent possible"(U.S. Congress, 2010, Title VII, Sec. 712a(2)).

The new legislation also specifically aims to increase transparency in markets for derivatives by setting up records for all transactions, requiring standardized OTC trades to be traded on swap execution facilities (SEFs) or be cleared by CCPs for transactions between financial institutions. Also, CCPs and SEFs are monitored by the Commodity Futures Trading Association. The reasoning behind the striving for transparency was the default of American International Group in 2008. AIG received a credit rating downgrade due to heavy losses incurred as default protection seller on CDOs that contained tranches of subprime mortgages and could not provide variation margins (Financial Crisis Inquiry Commission, 2011). The magnitude of AIG's derivative positions was unknown by U.S. authorities and took them by surprise. The Dodd-Frank regulation aims to keep such events from happening again (Hull, 2015).

3.4.2 The Basel Accords

The Basel Accords were initiated by the Basel Committee on Banking Supervision under the Bank for International Settlements (BIS) in 1988. Initially, what is today best know as Basel I was published: a recommendation for capital requirements in banks that was footed on a risk-based standard and, though only a non-mandatory recommendation, was signed into law by all 12 Basel Committee member countries. In 1996, an amendment was added that included a capital reserve requirement for market risk in trading activities (Basel Committee on Banking Supervision, 1988, 1998).
With Basel II, the committee proposed improved regulation around the year 2004 to address risk- and capital management concerns. The new capital requirements were implemented for internationally active banks in the U.S. as well as for all banks in Europe, and were based on three pillars. The first pillar is regulatory capital, concerning three major risk components of banks: credit, operational and market risk. The other pillars are supervisory review and market disclosure which enhance risk management processes and increase transparency (Basel Committee on Banking Supervision, 2006).
The Basel Committee realized in the wake of the 2008/09 crisis that the existing capital requirement framework needed revisions and finalized its Basel III proposals in 2010. The implemented measures include a drastic increase in equity and liquidity requirements for banks (Basel Committee on Banking Supervision, 2010, 2011*a*). Counterparty credit risk is regarded in the sense that CVA risk from changing credit spreads needs to be considered as part of market risk capital and needs to be included in calculating market risk VaR (Hull, 2015). This is due to the fact that only one third of the CCR losses were due to actual defaults while residual losses can be attributed to CVA losses (Basel Committee on Banking Supervision, 2011*b*).

3.4.3 European Market Infrastructure Regulation (EMIR)

Mandatory clearing for OTC derivatives is currently under implementation and is regulated in the European Market Infrastructure Regulation (EMIR) which came into effect in the European Union in 2013. The regulation specifically addresses central counterparties, OTC derivatives and trade repositories. The directive aims to mitigate operational, counterparty and systemic risk(BaFin Federal Financial Supervisory Authority, 2016). For certain interest rate and credit derivatives, central clearing has been mandatory for so called Category 1 Financial Counterparties, i.e. financial counterparties that are already clearing members, since June 2016 and February 2017, respectively. It is going to become mandatory for certain counterparty categories in several steps until May 2019. The category definitions and their respective dates to go into effect are presented in Table 3.1.

Table 3.1: Schedule for the introduction of mandatory central clearing

	Start of the clearing obligation	
Status of the counterparties	IRS	CDS
Category 1: FCs (Financial Counterparties) that are already a clearing member for one of the IRS subject to the clearing obligation	21.06.2016	09.02.2017
Category 2: FCs or AIF (Alternative Investment Funds) not in category 1, member of a group whose aggregated average of the gross nominal value of all held OTC derivatives is greater than 8 billion for all three of the months following the RTS (regulatory technical standards) coming into force	21.12.2016	09.08.2017
Category 3: FC / AIF and not in category 1 or 2	21.06.2017	09.02.2018
Category 4: NFC (Non-financial counterparties) if not in categories 1 to 3	21.12.2018	09.05.2019

Source: BaFin Federal Financial Supervisory Authority (2017)

4. DUFFIE AND ZHU MODEL OF NETTING EFFICIENCY

From this sections onwards we aim to show effects of OTC derivatives central clearing on counterparty exposure. In order to do so, we follow the methodology of Duffie and Zhu (2011) as well as part of the model extension by Cont and Kokholm (2014). We show how netting efficiency is measured and that in the Duffie and Zhu model, splitting counterparty clearing over several clearing facilities has adverse effects to the intended purpose of the clearing system in some settings and exemplify with credit default swaps. We show that there is a tradeoff between bilateral netting between counterparties across asset classes and multilateral netting between several counterparties for single asset classes, e.g. credit derivatives.

Below we present the model and notation of Duffie and Zhu (2011). We also provide complementary detailed calculations that are not included in the original article.

4.1 Measure of Netting Efficiency

Suppose there are N market participants (henceforth also called "entities" or "dealers") , all of which are exposed to each other in K classes of derivative positions, each class being defined by its type of underlying asset, i.e. interest rates, foreign exchange rates, credit quality, equities or commodities (henceforth called derivative or asset classes). The amount that an entity j will owe an other entity i in all positions of a single class k before netting benefits, payment of collateral and default recovery is denoted as X_{ij}^k. The exposure of dealer i to j in class k is the amount entity i is at risk of losing in case of entity j defaulting. It can be written as $\max(X_{ij}^k, 0)$. The exposure X_{ij}^k before a CCP is set up is uncertain because future positions between the parties are undetermined at the time of observation, and so are price volatilities and correlations. Exposure uncertainty also comprises risks of mark to market values in the time passing between request for and reception of additional collateral.

Assuming initially for simplicity that all X_{ij}^k are i.i.d. and have same variance, i.e. the same degree of risk, expected exposure as measure for netting efficiency prior to CCP introduction is calculated as

$$\phi_{N,K} = \sum_{j \neq i} E\left[\max\left(\sum_{k=1}^{K} X_{ij}^k, 0\right)\right]. \tag{4.1.1}$$

It is further assumed that X_{ij}^k is normally distributed with zero mean and unit variance ($X_{ij}^k \sim N(0, \sigma^2)$) and is symmetric across all *N-1* counterparties from the perspective of *N*, that is entity *i*. With these assumptions in place, the expected exposure $\phi_{N,K}$ of entity i in Equation (4.1.1) to a single counterparty can be calculated as follows. First note that

$$Var\left(\sum_{k=1}^{K} X_{ij}^K\right) = \sum_{k=1}^{K} Var\left(X_{ij}^K\right) = K\sigma^2$$

since X_{ij}^k are i.i.d. and the variance σ^2 is the same for each class K. Hence, from standard statistics we know that $\sum_{k=1}^{K} X_{ij}^k$ is normally distributed with zero mean and variance $K\sigma^2$, that is $\sum_{k=1}^{K} X_{ij} \sim N(0, K\sigma^2)$. This in turn implies that

$$\sum_{k=1}^{K} X_{ij}^K \sim \sqrt{K}\sigma X \quad \text{where } X \sim N(0,1).$$

Hence, we get that

$$\begin{aligned} E\left[\max\left(\sum_{k=1}^{K} X_{ij}^K, 0\right)\right] &= E\left[\max\left(\sqrt{K}\sigma X, 0\right)\right] = \\ &= \sqrt{K}\sigma E\left[\max\left(X, 0\right)\right] = \sqrt{K}\sigma E\left[X \mathbb{1}_{\{X>0\}}\right] \end{aligned} \tag{4.1.2}$$

where $\mathbb{1}_{\{X>0\}}$ is an indicator function for X being larger than unity, i.e.

$$\mathbb{1}_{\{X>0\}} = \begin{cases} 1 & \text{if} \quad X > 0 \\ 0 & \text{else.} \end{cases}$$

Hence, to determine netting efficiency $\phi_{N,K}$ we need to compute $E\left[X\mathbb{1}_{\{X>0\}}\right]$ for $X \sim N(0,1)$. Note that

$$\begin{aligned} E\left[X\mathbb{1}_{\{X>0\}}\right] &= \int_{-\infty}^{\infty} x\mathbb{1}_{\{x>0\}}\ \varphi(x)dx = \int_{0}^{\infty} x\ \varphi(x)dx = \\ &= \int_{0}^{\infty} -\frac{d}{dx}(\varphi(x))dx = \left(-\left[\varphi(x)\right]_{0}^{\infty}\right) = \\ &= \left(-\lim_{x\to\infty}\varphi(x) + \varphi(0)\right) = \frac{1}{\sqrt{2\pi}} \end{aligned} \tag{4.1.3}$$

where the lower bound is set to 0 as expected exposure is conditional on positive values and φ is the probability density function of a standard normal random variable, that is

$$\varphi(x) = \frac{1}{\sqrt{2\pi}}e^{-\frac{x^2}{2}}.$$

So plugging the result (4.1.3) into Equation (4.1.2) yields

$$\phi_{N,K} = E\left[\max\left(\sum_{k=1}^{K} X_{ij}^{K}, 0\right)\right] = \sigma\sqrt{\frac{K}{2\pi}}. \tag{4.1.4}$$

According to the symmetry assumption, dealer i's exposure to its (*N-1*) counterparties is thus

$$\phi_{N,K} = (N-1)\sigma\sqrt{\frac{K}{2\pi}} \tag{4.1.5}$$

with σ being the standard deviation of X_{ij}^{k}.

Now suppose that all derivatives positions in class K are novated to a single CCP. Novation is the process of terminating an existing bilateral contract and replacing it with one contract for each counterparty with a CCP. The expected exposure of clearing member i to the CCP under the same assumptions as applied previously is analog to the calculation of Equation (4.1.4) and thus given by

$$\gamma_N = E\left[\max\left(\sum_{j\neq i} X_{ij}^{K}, 0\right)\right] = \sqrt{\frac{N-1}{2\pi}}\,\sigma. \tag{4.1.6}$$

The expected exposure γ_N of clearing member i to the other *N-1* clearing members of the CCP for the residual bilaterally cleared *K-1* derivative classes is $\phi_{N,K-1}$.

Thus, the average total expected exposure of dealer i with dedicated clearing for single derivative class K is

$$\phi_{N,K-1} + \gamma_N. \tag{4.1.7}$$

This means that central clearing of a single derivative class improves netting efficiency if and only if the expected exposure with one centrally cleared asset class and *K-1* bilaterally cleared asset classes is lower than the expected exposure in the case of K bilaterally netted asset classes, i.e. $\gamma_N + \phi_{N,K-1} < \phi_{N,K}$.

From plugging Equations (4.1.5) and (4.1.6) into Inequality (4.1.7) we get that

$$\sqrt{\frac{N-1}{2\pi}}\sigma + \sqrt{\frac{K-1}{2\pi}}(N-1)\sigma < \sqrt{\frac{K}{2\pi}}(N-1)\sigma.$$

The inequality then simplifies to

$$\left[\sqrt{N-1} + \sqrt{K-1}\,(N-1)\right] < \sqrt{K}\,(N-1)$$

from where we can simplify further and square on both sides to get

$$(1 + \sqrt{K-1}\sqrt{N-1})^2 < K(N-1).$$

Dividing by $2\sqrt{N-1}$ gives that

$$2\sqrt{K-1}\sqrt{N-1} < (N-2)$$

and squaring on both sides to remove the square roots yields

$$K - 1 < \frac{(N-2)^2}{4(N-1)} \iff K < \frac{N^2 - 4N + 4 + 4N - 4}{4(N-1)}$$

that is

$$K < \frac{N^2}{4(N-1)}. \tag{4.1.8}$$

So according to the inequality in (4.1.8), clearing of a particular derivative class is dependent on the number of clearing members and improves netting efficiency in case of K symmetric uncleared classes of derivatives if and only if the number of clearing members N is at least as high as in Table 4.1.

Table 4.1: Required clearing members N to increase netting efficiency with K symmetric uncleared derivative classes

K	2	3	4	5	6	7	8	9	10
N	7	11	15	19	23	27	31	35	39

To exemplify, based on the inequality in (4.1.8), central clearing of a single class of derivatives while *K=3* symmetric classes are uncleared improves overall netting efficiency, i.e. decreases expected exposure when at least 11 members are clearing on the CCP.

4.2 Risk Differentiation

All else equal, the setting now changes in the way that the symmetry assumption is dropped and thus expected exposure of class k can differ among asset classes. Thus, $E\left[\max\left(X_{ij}^k, 0\right)\right]$ can be different for different k according to Equation (4.1.2) which means that the volatility of X_{ij}^k can differ across classes.

When class K is considered for clearing, the expected exposure of entity i to a specific counterparty in specific derivative class K in relation to all other *K-1* asset classes combined can be expressed as a ratio R, given by

$$R = \frac{E\left[\max\left(X_{ij}^K, 0\right)\right]}{E\left[\max\left(\sum_{k<K} X_{ij}^k, 0\right)\right]} = \frac{E\left[\max\left(X_{ij}^K, 0\right)\right]}{E\left[\max\left(\sum_{k=1}^{K-1} X_{ij}^k, 0\right)\right]}. \tag{4.2.1}$$

Since expected exposures are proportional to standard deviations as derived in Appendix A we get:

$$R = \frac{E\left[\max\left(X_{ij}^K, 0\right)\right]}{\sqrt{K-1}\, E\left[\max\left(X_{ij}^K, 0\right)\right]} = \frac{\sigma_K}{\sigma\sqrt{K-1}}. \tag{4.2.2}$$

That is, ratio R is accordingly scaled by expected exposure of K, i.e. if the volatility of class K is 3 times as large as the volatility of each of the other *K-1* classes, then

$$R = \frac{\sigma_K}{\sigma} \frac{1}{\sqrt{K-1}} = \frac{3}{\sqrt{K-1}}.$$

Duffie and Zhu (2011) propose that novating a single class of derivatives to a CCP only decreases average expected counterparty exposure if and only if

$$R > \frac{2\sqrt{N-1}}{N-2} \tag{4.2.3}$$

where R is the ratio of expected entity-to-entity exposure prior to the introduction of CCPs in a single derivative class in relation to that of all other classes combined. The derivation of Equation (4.2.3) can be found in Appendix A.

Table 4.2: Efficiency threshold R at N clearing members based on the inequality in (4.2.3)

N	10	12	14	16	18	20	22	24	26	28	30
R	0.750	0.663	0.601	0.553	0.516	0.484	0.458	0.436	0.417	0.400	0.3850
N	50	52	100	150	200	400	460	500	1000	5000	8192
R	0.292	0.286	0.203	0.165	0.143	0.100	0.0936	0.090	0.063	0.028	0.0221

Note: The scaling of the table is deliberately not symmetrical to facilitate showing the results of our forthcoming calculations

Hence, using the inequality in (4.2.3), for N=20, i.e. when 20 members clear a specific class through a CCP, netting efficiency improves if and only if a fraction R of a bank's expected exposure of at least 48.5% of total expected exposure of the residual bilaterally netted derivative classes can be attributed to the centrally cleared class. At 100 clearing members for example, this efficiency threshold would be at 20.4% as can be seen in Table 4.2. This relationship is additionally pictured in Figure 4.1, displaying the inverse relationship between ratio R and the number of clearing members.

Figure 4.1: Efficiency threshold R at N clearing members based on Table 4.2, scaled to max(N)=1000 (top) and at full scale (bottom)

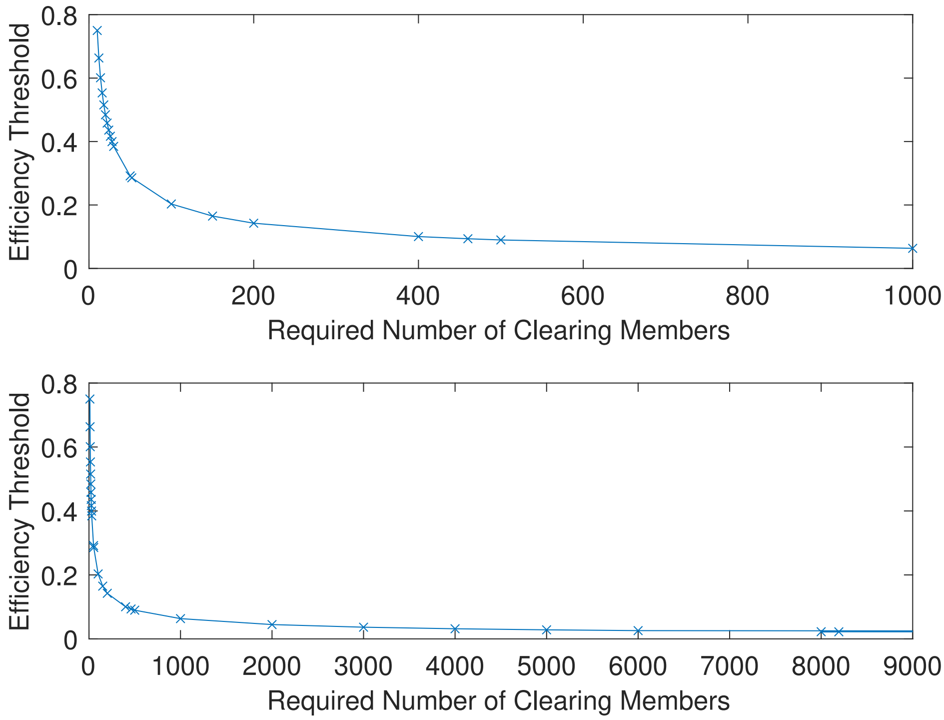


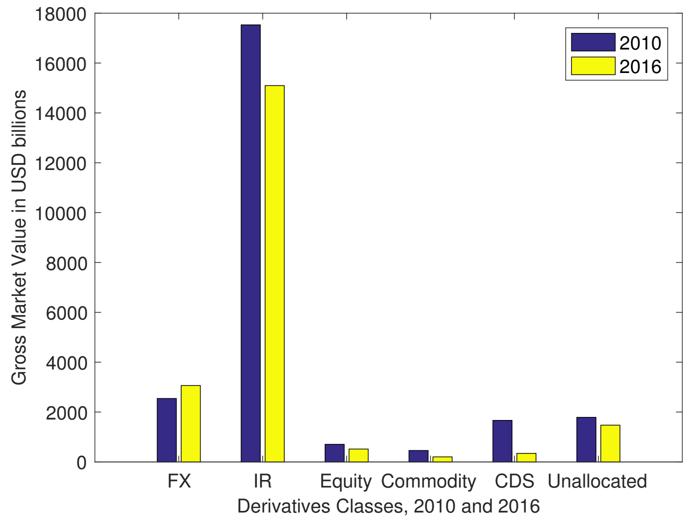
Table 4.3: Global OTC derivatives market (gross value) in USD billions

Class	6/2010	6/2016
Foreign Exchange	2544	3063
Interest Rate	17533	15096
Equity linked	706	515
Commodity	458	202
CDS	1666	342
Unallocated	1788	1473
Total	24695	20701

Source: Bank of International Settlements: BIS Statistics Explorer

As can be seen in Table 4.3 and the corresponding Figure 4.2, OTC derivative exposure has been declining in total size and in each asset class except foreign ex-

Figure 4.2: Global OTC derivatives market (gross value)



change products between the years 2010 and 2016.

ICE Clear Credit is one of the premier CCPs for credit derivatives. It has 30 clearing members in the United States and 22 members in Europe, totaling 52 clearing members as of February 2017[3]. Considering the information in Table 4.2 that would mean that for the combined N=*52* clearing members, the efficiency threshold is at R=28.6%. Hence, the CDS market which constitutes 1.65% of the notional market size would have to represent at least 28.6% of a dealer's expected exposure in all other uncleared OTC asset classes combined to decrease average expected counterparty exposure. Separating ICE Clear Credit's members by Europe and U.S. branch leads to an increase of the R-value to 45.8% for 22 European members and 38.5% for 30 U.S. members, respectively. In light of these results, a case for dedicated CDS central clearing seems to be difficult to make.

The implied ratio R is calculated assuming class-by-class proportionality and independent exposures X_{ij}^{k} across asset classes. The ratio calculates the expected exposure on one asset class, in this case that of CDSs to to exposures on all other asset classes, and is given by

[3] List of clearing members: cf. https://www.theice.com/clear-credit (21.2.2017)

$$R = \frac{\sqrt{(\alpha Z_K)^2}}{\alpha\sqrt{\sum_{k=1}^{K-1} Z_k^2}} = \frac{Z_K}{\sqrt{\sum_{k=1}^{K-1} Z_k^2}} \tag{4.2.4}$$

where Z_k is total gross exposure on asset class $k = 1, 2, ..., K$, and expected counterparty exposure is assumed to be a fixed fraction α of Z_k. The derivation of the implied ratio R can be found in Appendix B.
When the implied ratios R have been calculated for 2010 and 2016, the required members N for the respective year can be backed out and are as follows.

Table 4.4: Implied Ratios for CDS exposures based on Equation (4.2.4) and Table 4.3

Implied Ratio	2010	2016
R	0.0935	0.0221
required N	460	8192

According to the implied ratio R in Table 4.4, dedicated CDS central clearing reduces average expected exposure if more than N=460 members are clearing together for R=0.0935, which is the implied ratio for 2010. For 2016 the implied ratio as threshold is R=0.0221 due to an overall decrease in OTC derivatives, and especially a relatively high decrease in CDSs. From 2010 to 2016, global outstanding notionals of credit default swaps decreased by approximately 60% and gross market values by about 80%, while the overall OTC derivatives market only shrunk by about 16% in gross market values and by less than 7% in outstanding notional amounts according to BIS Statistics Explorer (2016) data. Hence, the required number of clearing members in 2016 which was calculated using the model by Duffie and Zhu (2011) is far higher than in 2010 to justify a dedicated CDS CCP according to the implied ratio: it is at 8192. Of course, this number is far from realistic. Even when accounting for volatility in CDS mark-to-market values justifying a downward correction of the implied ratio, a realistic number of clearing members is out of reach. Furthermore, this framework suggests that if a dedicated CCP is set up for another asset class than CDSs, e.g. for interest rate derivatives as the largest asset class, and a significant portion of that class is being cleared through the CCP, the critical number of clearing members N needed for a dedicated CDS CCP would decrease (i.e. the denominator in Equation (4.2.4) decreases). This is a logical consequence of the implied ratio R increasing as can be seen in Table 4.2.

4.3 Reducing Exposure

In this section we provide examples of changes in exposure under various clearing scenarios further following the setup of Duffie and Zhu (2011). We expect central clearing to have a positive, i.e. decreasing effect on exposures as the opposite effect would be counterproductive from a risk-management perspective and question the purpose of central clearing overall.

Exposure reduction is calculated using OTC derivative notional data for the six largest U.S. dealers from the Office of the Comptroller of the Currency (2017, 2009). The calculations are based on a scenario with N=12 dealers: six dealers in Table 4.5 and six more dealers with identical notional amounts for each class k, as in the original study. This can be thought of as one group of six entities for the U.S. and one identical group for Europe as the same dataset does not exist for European banks. As the inputs for the second group of banks are identical, so are its results.

Table 4.5: Notional amount of OTC derivative contracts in USD billions, Top 6 U.S. dealers in 2009 and 2016

	Q3 2009					Q3 2016				
	Forwards	Swaps	Options	Credit	Total	Forwards	Swaps	Options	Credit	Total
Bank 1	8177	51203	10059	6376	75815	6698	27362	7512	2100	43672
Bank 2	8984	49478	5918	5590	69970	9725	27269	7629	2544	47167
Bank 3	1651	31521	6980	5762	45914	6131	24103	7683	1638	39555
Bank 4	5718	24367	4064	5482	39631	8529	19535	3457	1690	33211
Bank 5	5536	16375	6384	2764	31059	2833	15487	5258	1098	24676
Bank 6	1198	2192	477	268	4135	671	9580	368	145	10764
Total	31264	175136	33882	26242	266524	34587	123336	31907	9215	188281

Source: Office of the Comptroller of the Currency (2017, Table 2)
Note: The banks are listed by derivative notional size, i.e. banks are not necessarily the same in 2009 and 2016. They rather have the highest notional in their books in the respective year, including exchange traded derivatives which are not listed in this table.

While in Q3 2009 the top 6 derivative dealers accounted for 97.11% of derivative notionals, they "only" accounted for 80.32% in Q3 2016 and at the same time, derivative notionals (including exchange trades ones that are not included in Table 4.5) for the 25 largest holding companies in derivatives decreased from USD 293 billion to USD 243 billion or by approximately 17% (Office of the Comptroller of the

Currency, 2017).
To evaluate the clearing scenarios, let S_i^k be bank i's notional position is derivative class k. Let m_k further be a scaling of the notional position S_i^k, which is the standard deviation of exposures caused by derivative class k. This scaling m_k composes from BIS (2010) data which highlights that the market value of IR swaps equals 3.5% of its notional amounts. For the other three classes combined, this value is at approximately 5.9% so that the ratio of market value over notional value of non-IR-swaps is approximately 1.67 times as high as for IR-swaps. As do Duffie and Zhu (2011), we arbitrarily scale this number up from 1.67 to 3, to allow for volatility in market value changes between time of valuation and additional collateral collection prior to a potential default (Duffie and Zhu, 2011).
Furthermore, assume that dealer i to dealer j exposure for a single class k is proportional to S_j^k, hence the standard deviation of dealer i's pre-collateral, pre-clearing class k exposure to j is

$$\sigma_{ij}^k = m_k S_i^k \frac{S_j^k}{\sum_{i \neq j} S_j^k}. \tag{4.3.1}$$

Hence, assuming the exposures X_{ij}^k are independent and normally distributed, we have that

$$X_{ij}^k \sim N\left(0, \left(m_k S_i^k \frac{S_j^k}{\sum_{i \neq j} S_j^k}\right)^2\right) \tag{4.3.2}$$

and as before, X_{ij}^k are independent across different asset classes and entities. With α^k being the ratio of centrally cleared notionals in asset class k, the expected exposure of entity i to a CCP that is dedicated to clearing class-k derivatives is

$$\begin{aligned} E\left[\max\left(\sum_{i \neq j} \alpha^k X_{ij}^k, 0\right)\right] &= \frac{1}{\sqrt{2\pi}} \alpha^k \sigma_i^k \\ &= \frac{1}{\sqrt{2\pi}} \alpha^k \sqrt{\left(m_k S_i^k\right)^2 \frac{\sum_{i \neq j} \left(S_j^k\right)^2}{\left(\sum_{i \neq j} S_j^k\right)^2}} \\ &= \frac{1}{\sqrt{2\pi}} \alpha^k m_k S_i^k \frac{\sqrt{\sum_{i \neq j} \left(S_j^k\right)^2}}{\sqrt{\left(\sum_{i \neq j} (S_j^k)\right)^2}} \end{aligned} \tag{4.3.3}$$

When a single CCP is not dedicated but clears all $k = 1, ..., K$ classes of derivatives, expected exposure of bank i to the CCP is calculated by

$$E\left[\max\left(\sum_{i\neq j}\sum_{k=1}^{K}\alpha^k X_{ij}^k, 0\right)\right] = \frac{1}{\sqrt{2\pi}}\sqrt{\sum_{i\neq j}\sum_{k=1}^{K}\left(\alpha^k \sigma_{ij}^k\right)^2}$$
$$= \frac{1}{\sqrt{2\pi}}\sqrt{\sum_{i\neq j}\sum_{k=1}^{K}\left(\alpha^k m_k S_i^k \frac{S_j^k}{\sum_{i\neq j} S_j^k}\right)^2} \qquad (4.3.4)$$

while dealer i to j expected exposure on all uncleared positions is

$$E\left[\max\left(\sum_{k=1}^{K}(1-\alpha^k)X_{ij}, 0\right)\right] = \frac{1}{\sqrt{2\pi}}\sqrt{\sum_{k=1}^{K}\left((1-\alpha^k)m_k S_i^k \frac{S_j^k}{\sum_{i\neq j} S_j^k}\right)^2} \qquad (4.3.5)$$

Table 4.6 shows expected pre-collateral exposures under clearing Scenarios 0 to 8 as multiples of Scenario 0. In Scenario 0, total expected exposure is calculated when all classes are bilaterally netted. That is, we calculate scenarios in which (partial) multilateral netting occurs and express the exposures in the corresponding scenario as a ratio of this and a scenario in which there is only bilateral netting, i.e. Scenario 0. In this way, it is fast to tell whether exposures and thus netting efficiency in- or decreases when part of the derivatives classes are being cleared centrally.
In Table 4.6, the description "Same CCP" implies that the respective share α^k of each class is being cleared by a single CCP clearing *all* derivative classes. The notation "Mult." on the other hand refers to those scenarios in which there is a dedicated CCP for each class, i.e. one CCP for each class with $\alpha^k > 0$ clearing only class k derivatives. The top part of Table 4.6 shows what fraction $\alpha^k > 0$ of each class is being cleared centrally.

The results are the following: if a single CCP is introduced that clears 75% of credit default swaps, market-wide expected exposures increase by 2.6% in 2010 and by 2.55% in 2016 compared to the base case with bilateral netting (Scenario 3). This is in line with our expectations: we assume a total of 12 clearing members for all scenarios which is well below the efficiency threshold we established in Subsection 4.2. Increasing the clearing ratio for CDSs ceteris paribus to 100%, the market-wide exposure increases by 4.91% in 2010 and 3.66% in 2016 relative to Scenario 0 which is 1.29 and 1.11 percentage points higher than in the respective previous cases with clearing ratio α^k at 75% (Scenario 1). Scenario 2 is a special case in which there are two clearing houses: one for American and one for European banks. Hence, multilateral clearing takes place for two subsets of six banks each, while all 12 banks continue to clear their share of $(1-\alpha^k)$ derivative notionals bilaterally. In this case, expected exposure increases by 10.28% in 2010 and 5.97% in 2016 compared to the base Scenario 0 and an increase of 5.37 and 2.31 percentage points, respectively, to Case 1 in which one CCP clears both European and American Banks. This result strengthens our theory based on Inequality (4.1.8) that netting efficiency increases with the number of members clearing on the CCP. The situation changes with central clearing of interest rate swaps. With 75% of IR swaps being cleared, market-wide expected exposure reduces by 10.45% in 2010 and 7.79% in 2016 (Scenario 4). This ratio is well within the limits of possibility: In 2010, the Federal Reserve Bank of New York published a commitment of major derivatives dealers to centrally clear 85% of both historical and new credit derivatives trades and 75% of historical as well as 92% of new interest rate derivatives trades. Clearing 75% of credit derivatives in addition to 75% of swaps on dedicated CCPs, i.e. one CCP for each class, decreases market-wide exposure by 10.59% in 2010 and by 5.47% in 2016 relative to Scenario 0. That is, expected exposure increases slightly when there are two CCPs, one for swaps and one for CDSs, compared to when only swaps are cleared on a single CCP (Scenario 5). However, when the same ratio α^k of both classes is being cleared on a single clearing house, expected exposure decreases by 16.91% in 2010 and 8.55% in 2016 (Scenario 6) which is 6.32 and 3.08 respective percentage points less than in Scenario 5 where there is one CCP for each class. Clearing a sizeable fraction of all classes with 75% of CDSs and IR swaps as well as 40% of forwards and option derivatives centrally by dedicated clearing houses, one per class (Scenario 7), total expected exposure decreases by 21.33% in 2010 and by 20.11% in 2016 compared to Scenario 0. With the same fractions being centrally cleared but on the same CCP (Scenario 8), market-wide expected exposures decrease further by 36.86% in 2010 and 33.81% in 2016 compared to the benchmark case with only bilateral netting, representing a significant decrease of 15.53 and 13.7 percentage points relative to the previous case in which there was a dedicated CCP for each derivatives class. The results of the Duffie and Zhu (2011) model hence show that expected exposure can be decreased significantly if several derivatives classes are being cleared on the same CCP. For multiple CCPs, each clearing a single class of derivatives, the exposure reduction is less significant.

The detailed results of our computations for exposure reduction according to the model by Duffie and Zhu (2011) are represented in Table 4.6.

Jackson and Manning (2007) mention in an article published before regulation concerning CCPs for OTC derivatives was brought underway that they believe merging domestic clearing houses into a single cross-market one could bring significant efficiency gains. In light of the results previously calculated, this step appears to be a reasonable one as clearing through cross-asset CCPs leads to consistently reduced exposures.

Table 4.6: Expected exposures under various clearing approaches

Scenario	(0)	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
	α^k - Fraction cleared by CCP								
Forwards	0	0	0	0	0	0	0	0.4	0.4
Swaps	0	0	0	0	0.75	0.75	0.75	0.75	0.75
Options	0	0	0	0	0	0	0	0.4	0.4
Credit	0	1	1	0.75	0	0.75	0.75	0.75	0.75
CCP	-	Same	Mult.	Same	Same	Mult.	Same	Mult.	Same
	Total Exposures as Multiples of Bilateral Exposures for 2010								
Bank 1	1	1.0522	1.0993	1.0313	0.8788	0.8895	0.8310	0.7851	0.6310
Bank 2	1	1.0520	1.0977	1.0318	0.8440	0.8508	0.7935	0.7608	0.6162
Bank 3	1	1.0467	1.1162	1.0183	0.8831	0.8541	0.7732	0.7593	0.6086
Bank 4	1	1.0408	1.1167	1.0106	0.9381	0.9079	0.8285	0.8033	0.9290
Bank 5	1	1.0502	1.0913	1.0309	0.9989	1.0243	0.9745	0.8608	0.6931
Bank 6	1	1.0388	1.0623	1.0259	0.9988	1.0211	0.9863	0.8327	0.6957
Total (ratio)	1	1.0491	1.1028	1.0260	0.8955	0.8941	0.8309	0.7867	0.6314
	Total Exposures as Multiples of Bilateral Exposures for 2016								
Bank 1	1	1.0390	1.0645	1.0271	0.9215	0.9457	0.9124	0.8039	0.6598
Bank 2	1	1.0406	1.0665	1.0280	0.9574	0.9830	0.9486	0.8250	0.6771
Bank 3	1	1.0343	1.0555	1.0242	1.9397	0.9621	0.9338	0.8109	0.6701
Bank 4	1	1.0385	1.0633	1.0268	1.9594	0.9842	0.9521	0.8160	0.6801
Bank 5	1	1.0358	1.0577	1.0251	0.9276	0.9503	0.9201	0.7991	0.6632
Bank 6	1	1.0133	1.0194	1.0098	0.5711	0.5791	0.5686	0.5618	0.5118
Total (ratio)	1	1.0366	1.0597	1.0255	0.9221	0.9453	0.9145	0.7989	0.6619
Ratio Δ	-	-0.0125	-0.0431	-0.0005	0.0266	0.0512	0.0836	0.0122	0.0305

Source: Calculations based on Equations (4.3.3) to (4.3.5) with Office of the Comptroller of the Currency (2009, 2017) data in Table 4.5

The total ratios in the penultimate row of Table 4.6 are the individual exposures weighted by the each bank's relative derivatives holdings in the respective years as in the second column of Table 4.7, that is

$$\text{Weight}_i = \frac{\text{Derivatives}_i}{\sum_{i=1}^{N} \text{Derivatives}_i} \tag{4.3.6}$$

where Derivatives_i is the total notional amount of derivatives contracts bank i is holding and N is the number of banks that are considered. The weights given in Table 4.7 are:

Table 4.7: Derivative notionals and weights

Institution	Total Derivatives 2010	Weight 2010	Total Derivatives 2016	Weight 2016
Bank 1	79,397,765	0.2787	51,789,991	0.2318
Bank 2	75,034,108	0.2633	50,667,476	0.2267
Bank 3	49,830,777	0.1749	45,480,638	0.2035
Bank 4	41,830,926	0.1468	35,602,230	0.1593
Bank 5	34,473,426	0.1210	28,379,530	0.1270
Bank 6	4,356,115	0.0153	11,533,150	0.0516
$\sum$	284,923,117	1.0000	223,453,015	1.0000

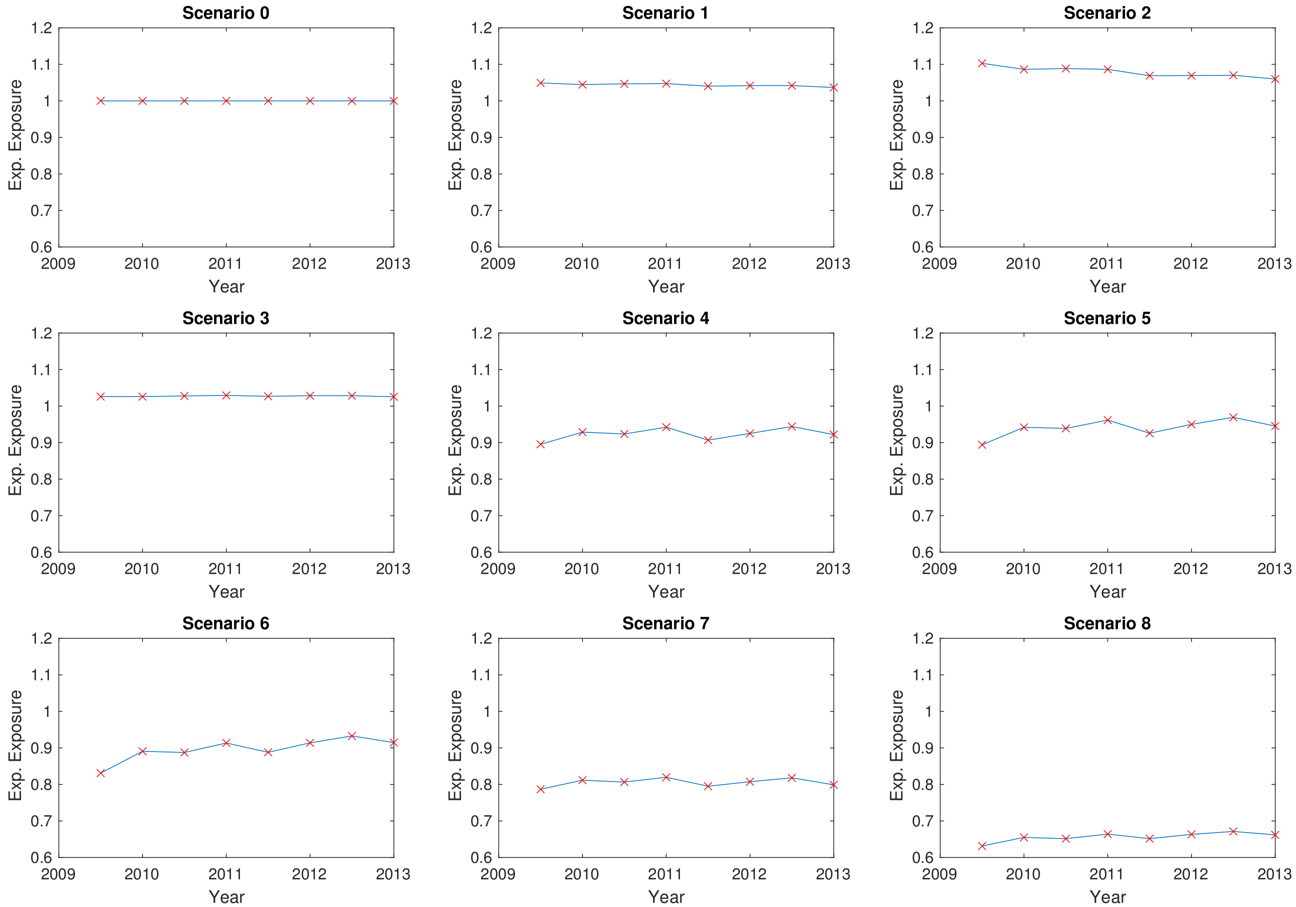
Source: Office of the Comptroller of the Currency (2009, 2017) and Calculations based on Equation (4.3.6)

The ratio-delta Δ in Table 4.6 is the difference between the respective total ratios for 2009 and 2016 and shows that the ratios remain within a 9 percentage point margin, with the maximum decrease being -4.31 percentage points and a maximum increase at 8.36 percentage points.

4.4 Development of Expected Exposures 2009-2016

Figure 4.3 shows expected exposures for each scenario 0 to 8 in Table 4.6 over the last eight years. There is some dispersion visible which follows a common pattern from scenario 4 which is when a large ratio (75%) of interest rate swaps is cleared. As has been shown before, IR swaps make the largest part of total OTC derivatives in both notional and gross-market values. This also is not only the case for the years 2009 and 2016 that have been examined in more detail, but also for the years in between. It can be seen that the dispersion is minimal and remains very stable over time. Again, it shows the large influence of clearing interest rate swaps on exposures of clearing in general. The fact that the volume of IR swaps does not decrease as much as it does in the residual derivatives classes increases the effect.

Figure 4.3: Exposures for each clearing scenario from 2009-2016 computed using Equations (4.3.4) to (4.3.5)



4.5 Estimation of a Realistic Scenario for 2019

This subsection discusses a realistic clearing scenario for end of Q2 in 2019 which is when all counterparty categories are mandated to clear certain derivative classes centrally as presented in Table 4.8. To this end, we assume that the future development of notional sizes follows a historical 5-year average trend.

Table 4.8: Derivative classes affected by mandatory central clearing

Derivative class	Affected products
Foreign Exchange	Non-deliverable forward
Interest Rate	Basis, Fixed-to-float swaps, FRA and IOS in EUR, GBP, JPY, USD; FRA and fixed-to-float swaps in NOK, PLN and SEK
Equity	Lookalike/Flexible equity derivatives and contract for difference
Credit	Index credit default swaps

Source: European Securities and Markets Authority (2016)

As of the second quarter of 2016, 91% of forward rate agreements, 80% of interest rate swaps and close to 0% of interest rate options were cleared centrally. Depending on the currency, the clearing ratios of interest rate derivatives varied between 76% (SEK) and 86% (JPY and CAD). In terms of notional size, the products affected by the clearing obligation make up a sizeable quantity: forward rate agreements and interest rate swaps account for 17.2% and 74.5%, respectively. Interest rate options, which are not affected, account for 8.3% of total interest rate contracts' notional value. It can be assumed that the share of centrally cleared interest rate products will increase further as regulation changes over time towards a target of approximately 90%.

In credit default swaps, central clearing is lagging behind interest rate swaps with 37% being cleared as of mid-2016, up from 10% and 23% in June 2010 and 2013, respectively. The clearing ratio is higher for multi-name products (47%), which usually consist of contracts on indices, than for single name products of which only 29% are currently centrally cleared. A reason for this is higher degree of standardization for multi-name CDSs and concomitant higher volumes to make central clearing worthwhile. As index CDSs currently account for approximately 94% of multi-name CDSs, the simplifying assumption is made that all multi-name CDSs are index contracts which underlie the clearing obligation for all counterparty categories from May 2019. Furthermore, suppose the ratio of centrally cleared single-name CDSs, which is currently 29%, remains constant. Under these assumptions, approximately 60% of credit default swaps can be expected to be centrally cleared by the second quarter of 2019.

In other derivative classes, the share of centrally cleared products is negligible. As of June 2016, less than 2% of foreign exchange and equity derivatives were cleared by CCPs which is due to regulatory differences in regulations as can be seen in Table 4.8. Equity as well as FX derivatives are mostly exempted from regulation. However, margin requirements for many of these products are being increased, incentivizing dealers to novate their trades, according to the Bank for International Settlements (2016). Under these increased collateral demands, we assume that the respective centrally cleared share of foreign exchange and equity derivatives increases to 7.5% until 2019.

A clearing scenario in 2019 could hence look like the following:

Table 4.9: Expected Exposures, realistic scenario 2019

		α^k	
	(0)	(1)	(2)
Forwards	0	0.075	0.075
Swaps	0	0.9	0.9
Options	0	0.075	0.075
Credit	0	0.6	0.6
CCP	-	Mult.	Same
	Exposure Multiple		
Bank 1	1	0.9188	0.8733
Bank 2	1	0.9573	0.9096
Bank 3	1	0.9346	0.8914
Bank 4	1	0.9610	0.9169
Bank 5	1	0.9177	0.8747
Bank 6	1	0.5345	0.5207
Total (ratio)	1	0.9175	0.8741

Calculations based on Equations (4.3.3) to (4.3.5) with modified data from the Office of the Comptroller of the Currency (2017)

Since clearing currently ensues with dedicated clearing houses, the scenario "Mult."

applies. That is, under the previously established assumptions, central clearing in a prognostic scenario could lead to a reduction in expected exposure by 8.25% relative to a scenario in which banks net their positions bilaterally. If the four derivatives classes were to be cleared on a common CCP for all classes, the exposure reduction would be 12.59%.

4.6 Cont and Kokholm Extension

In their 2014 article, Cont and Kokholm extend the previously discussed Duffie and Zhu model by introducing correlation across asset classes. They argue that the number of required participants (at least 461) in a central clearing house is too high to lead to a reduction in expected exposures.
However, they show that this number in highly dependent on the model's assumptions. With introduction of heterogeneous characteristics the results of Duffie and Zhu change significantly. That is, changing the assumed risk per dollar in notionals to $\alpha_K = 3\alpha$ for CDSs, the number of members required falls to at least 54.
Then, Cont and Kokholm (2014) argue that the number of required clearing members drops further to 17 when cross-asset class correlation of 10% is introduced. We disagree with this result and argue that the number of clearing members required to decrease exposure increases in correlation and is inversely related to the relative risk weight of CDSs which has also been noted by Herbertsson (2017). The derivation of this argument based on calculations by Duffie and Zhu (2011) can be found in Appendix C.

Table 4.10: Required number of clearing members with heterogeneous characteristics and cross-asset correlation

Assumptions	2010 C&K-N	2010 N	2016 N
Duffie and Zhu model			
$\rho = 0, \alpha_i = \alpha$	461	461	8192
$\rho = 0, \alpha_K = 3\alpha$			
$\alpha_i = \alpha, i < K$	54	54	913
$\rho = 0.1, \alpha_K = 3\alpha$			
$\alpha_i = \alpha, i < K$	17	58	981
$\rho = 0.2, \alpha_K = 2\alpha$			
$\alpha_i = \alpha, i < K$	11	133	2351

Source:Cont and Kokholm (2014) and own calculations

In Table 4.10, the first column "C&K-N" is the required number of clearing members as calculated by Cont and Kokholm (2014) for 2010 data while the second an third N is the number of clearing members according to our calculations for the 2010 and 2016 case. As can be seen in the third row, introducing cross-asset correlation $\rho = 0.1$ yields a decrease in required clearing members N from 54 to 17 for the original authors. According to our calculations, the effect is adverse and increases the required N to 58. The effect is similar when changing the assumed risk per dollar to $\alpha_K = 2\alpha$ and increasing cross asset correlation to $\rho = 0.2$ (fourth row). Cont and Kokholm calculate a further decrease in N to 11 while we calculate an increase to N=133. There are no calculations from Cont and Kokholm for 2016, but the our calculations for 2016 provide the same conclusions, i.e. an adverse effect to the required clearing members N with the introduction of cross-asset correlation.

5. CONCLUSION

Our thesis is based on the Duffie and Zhu (2011) model for netting efficiency which we implement in a numerical computing environment. We find that a case for dedicated CCPs is hard to make with respect to netting efficiency. According to the model by Duffie and Zhu (2011), the number of clearing members that would have to clear through a dedicated central counterparty for credit default swaps is more than 460 for 2010 data, and this number increases manyfold to far over 8000 for 2016 data. We find however, that under a number of simplifying model assumptions, central clearing can increase netting efficiency. Netting efficiency is measured in terms of expected exposure as a multiple of a multilateral clearing scenario over a scenario in which all derivatives are bilaterally netted. Our calculations show that clearing is most efficient when several derivatives classes, e.g. swaps and credit default swaps, are being cleared by the same central counterparty and efficiency is increasing with the number of a CCP's registered members and with the fraction of each member's cleared position. That is, efficiency is higher when 100% of an entity's swap position is being cleared than when that ratio is 75%. In line with our earlier results we find that a central counterparty dedicated to credit default swaps would lead to an increase in expected exposure compared to netting positions bilaterally.
Furthermore, we examine an extension to the Duffie and Zhu model by Cont and Kokholm (2014) with regards to exposure correlation. Other than Cont and Kokholm, we find that the required number of clearing house members to justify dedicated CCPs increases rather than decreases when we introduce exposure correlation.

APPENDIX

A. DERIVATION OF RATIO R

Let the variance of derivative class K be different from the variance of all other *K-1* derivatives classes:

$$\begin{aligned} X_{ij}^k &\sim N(0,\sigma^2) \quad \forall \quad k<K \\ X_{ij}^K &\sim N(0,\sigma_K^2) \quad \text{assuming independence of } X_{ij}. \end{aligned}$$

The total variance is the sum of the *K-1* symmeric classes' variances and the variance of class K, that is

$$\sum_{k=1}^{K} Var(X_{ij}^k) = \sum_{k=1}^{K-1} Var(X_{ij}^k) + Var(X_{ij}^K) = (K-1)\sigma^2 + \sigma_K^2$$

which yields that

$$\sum_{k=1}^{K} X_{ij}^k \sim \sqrt{\sigma^2(K-1)+\sigma_K^2}\,X, \quad X \sim N(0,1).$$

The notation $\tilde{\phi}_{N,K}$ is for the case in which class K has different standard deviation σ_K than the other *K-1* derivatives classes.

$$\begin{aligned} \tilde{\phi}_{N,K} &= \sum_{i\neq j} E\left[\max\left(\sum_{k=1}^{K} X_{ij}^k, 0\right)\right] \\ E\left[\max\left(\sum_{k=1}^{K} X_{ij}, 0\right)\right] &= E\left[\max\left(\sqrt{\sigma^2(K-1)+\sigma_K^2}\,X, 0\right)\right] \\ &= \sqrt{\sigma^2(K-1)+\sigma_K^2}\,E\left[\max(X,0)\right] = \sqrt{\sigma^2(K-1)+\sigma_K^2}\,\frac{1}{\sqrt{2\pi}} \end{aligned}$$

Hence, netting efficiency prior to introducing a central counterparty is

$$\tilde{\phi}_{N,K} = (N-1)\sqrt{\sigma^2(K-1)+\sigma_K^2}\,\frac{1}{\sqrt{2\pi}}.$$

The expected exposure of entity i to the CCP with novation for class K is analog to the original case but with differing standard deviation σ_K:

$$\tilde{\gamma}_N = \sqrt{\frac{N-1}{2\pi}}\,\sigma_K.$$

Netting efficiency with a CCP remains unchanged as cleared class k is included in $\tilde{\phi}_{N,K}$ and thus

$$\tilde{\phi}_{N,K} = (N-1)\sigma\sqrt{\frac{K}{2\pi}}.$$

Hence, a CCP clearing a single derivative class improves netting efficiency if and only if expected exposure with one centrally cleared class and *K-1* bilaterally cleared classes is lower than expected exposure with K bilaterally cleared classes:

$$\tilde{\gamma}_N + \tilde{\phi}_{N,K-1} < \tilde{\phi}_{N,K}$$

is equivalent to

$$\sqrt{\frac{N-1}{2\pi}}\,\sigma_K + (N-1)\sigma\sqrt{\frac{K-1}{2\pi}} < (N-1)\sqrt{\sigma^2(K-1)+\sigma_K^2}\,\frac{1}{\sqrt{2\pi}}.$$

We can then factor out $\sqrt{\frac{N-1}{2\pi}}$ on both sides of the inequality and reformulate to

$$\sqrt{\frac{N-1}{2\pi}}\left(\sigma_K + \sqrt{N-1}\sqrt{K-1}\,\sigma\right) < \sqrt{\frac{N-1}{2\pi}}\left(\sqrt{N-1}\sqrt{\sigma^2(K-1)+\sigma_K^2}\right)$$

where the factored term cancels out to

$$\sigma_K + \sqrt{N-1}\sqrt{K-1}\,\sigma < \sqrt{N-1}\sqrt{\sigma^2(K-1)+\sigma_K^2}.$$

We then square on both sides to remove most of the square roots to

$$\sigma_K^2 + 2\sigma_K \sqrt{N-1}\sqrt{K-1}\,\sigma + (N-1)(K-1)\sigma^2 < (N-1)(K-1)\sigma^2 + (N-1)\sigma_K^2$$

and can cancel out $(N-1)(K-1)\sigma^2$ on both sides to get

$$\sigma_K^2 + 2\sigma_K \sqrt{N-1}\sqrt{K-1}\,\sigma < N\sigma_K^2 - 2\sigma_K^2.$$

We can the solve for Proposition 1 in Duffie and Zhu (2011) through rearranging:

$$\begin{aligned}\frac{2\sigma_K\sqrt{N-1}\sqrt{K-1}}{N-2} &< \sigma_K^2 \\ \frac{2\sqrt{N-1}}{N-2} &< \frac{\sigma_K^2}{\sigma_K \sigma \sqrt{K-1}} \\ \frac{2\sqrt{N-1}}{N-2} &< \frac{\sigma_K}{\sqrt{K-1}\,\sigma}.\end{aligned}$$

R is defined as the right hand side of the previous inequality, that is:

$$R = \frac{E[\max(X_{ij}^K, 0)]}{E[\max(\sum_{k<K} X_{ij}^K, 0)]} = \frac{\sigma_K \frac{1}{\sqrt{2\pi}}}{\sigma\sqrt{K-1}\frac{1}{\sqrt{2\pi}}} = \frac{\sigma_K}{\sqrt{K-1}\,\sigma}$$

Hence, we get the necessary condition for netting efficiency with one centrally cleared class and *K-1* bilaterally cleared classes, that is

$$\begin{gathered}\tilde{\gamma}_N + \tilde{\phi}_{N,K-1} < \tilde{\phi}_{N,K} \\ \frac{2\sqrt{N-1}}{N-1} < R.\end{gathered}$$

B. ALTERNATIVE DERIVATION OF RATIO R

For the derivation of ratio R which in this case is

$$R = \frac{Z_K}{\sqrt{\sum_{k=1}^{K-1} Z_k^2}}$$

assume once more that Exposures are Gaussian distributed with zero mean and variance σ^2

$$X_{ij}^k \sim N(0, \sigma^2)$$

We have shown in Appendix A that

$$E\left[\max\left(\sum_{k<K} X_{ij}^k, 0\right)\right] = \sqrt{\sum_{k=1}^{K-1} \sigma_k^2}\,\frac{1}{\sqrt{2\pi}}$$

and that

$$E\left[\max\left(X_{ij}^K, 0\right)\right] = \sigma_K \frac{1}{\sqrt{2\pi}}$$

As in Duffie and Zhu we assume class proportionality of dealers to gross credit exposure Z, hence

$$E\left[\max\left(X_{ij}^k, 0\right)\right] = \alpha Z_k$$

Combination of the previous results implies that

$$
\begin{aligned}
\alpha Z_k &= \sigma_k \frac{1}{\sqrt{2\pi}} \\
\Rightarrow \sigma_k &= 2\pi\alpha^2 Z_k^2 \quad \forall \quad k
\end{aligned}
$$

ensues the following:

$$
\begin{aligned}
\sqrt{\sum_{k=1}^{K-1} \sigma_k^2} \, \frac{1}{\sqrt{2\pi}} &= \sqrt{\sum_{k=1}^{K-1} 2\pi\alpha^2 Z_k^2} \, \frac{1}{\sqrt{2\pi}} \\
&= \alpha \sqrt{\sum_{k=1}^{K-1} Z_k^2} \, .
\end{aligned}
$$

Hence,

$$
R = \frac{E\left[\max\left(X_{ij}^K, 0\right)\right]}{E\left[\max\left(\sum_{k<K} X_{ij}^k, 0\right)\right]} = \frac{Z_K}{\sqrt{\sum_{k=1}^{K-1} Z_k^2}}.
$$

C. PROOF OF CORRELATION

This derivation sets the mathematical groundwork for correlated cross-asset exposures as in the last two rows of table 3.1 in the article by Cont and Kokholm (2014) as published by De Gruyter in Statistics & Risk Modeling.

Exposures are normally distributed with zero mean and standard deviation σ

$$X_{ij}^{k} \sim N(0, \sigma^2) \quad \forall \quad k = 1, ..., K$$

For all classes $k, m \in \{1, ..., K\}$ it will hold for all pairs i, j with $i \neq j$ that

$$\rho = Corr(X_{ij}^{k}, X_{ij}^{m})$$

The correlation for two Gaussian random variables X and Y with standard deviations σ_X and σ_Y is of course defined by

$$\rho = Corr(X, Y) = \frac{Cov(X, Y)}{\sqrt{Var(X)}\sqrt{Var(Y)}}$$

$$\rho = \frac{Cov(X, Y)}{\sigma_X \sigma_Y}$$

Rearranging for Covariance we get

$$Cov(X, Y) = \rho \sigma_X \sigma_Y$$

When X and Y have the same variance, the expression reduces to

$$Cov(X, Y) = \rho \sigma^2$$

The next step is calculating netting efficiency $\phi_{N,K}$

$$\phi_{N,K} = \sum_{j \neq i} E\left[\max\left(\sum_{k=1}^{K} X_{ij}^{k}, 0\right)\right]$$

We know that when X and Y are jointly normally distributed variables, then X+Y is still normally distributed and the mean is the sum of means while the standard deviations are not additive, that is

$$\sigma_{X+Y} = \sqrt{\sigma_X^2 + \sigma_Y^2 + 2\rho\sigma_X\sigma_Y}$$

Therefore, $\sum_{k=1}^{K} X_{ij}^{k}$ is still a Gaussian distributed variable with mean $\mu = E\left[\sum_{k=1}^{K} X_{ij}^{k}\right]$ and variance $\tilde{\sigma}^2 = Var\left(\sum_{k=1}^{K} X_{ij}^{k}\right)$. Since it is known that $X_{ij}^{k} \sim N(0, \sigma^2)$, the mean $\mu = 0$. Hence,

$$\sum_{k=1}^{K} X_{ij}^{k} \sim N(0, \tilde{\sigma}^2)$$

Then, using the calculations as in Appendix A,

$$\phi_{N,K} = (N-1)\frac{\tilde{\sigma}}{\sqrt{2\pi}}$$

For standards results of variances for correlated random variables the following holds:

$$Var\left(\sum_{k=1}^{K} X_{ij}^{k}\right) = \sum_{k=1}^{K} Var\left(X_{ij}^{k}\right) + \sum_{k=1}^{K} \sum_{\substack{m=1,\\ m\neq k}}^{K} Cov\left(X_{ij}^{k}, X_{ij}^{m}\right)$$

But since the case here is $Var\left(\sum_{k=1}^{K} X_{ij}^{k}\right) = \sigma^2 \quad \forall \quad k$ and $Cov(X_{ij}^{k}, X_{ij}^{m}) = \rho\sigma^2 \quad \forall \quad$ pairs k, m where $k \neq m$. Hence,

$$\tilde{\sigma}^2 = Var\left(\sum_{k=1}^{K} X_{ij}^{k}\right)$$

$$
\begin{aligned}
&= \sum_{k=1}^{K} Var\left(X_{ij}^{k}\right) + \sum_{k=1}^{K} \sum_{\substack{m=1,\\ m\neq k}}^{K} Cov\left(X_{ij}^{k}, X_{ij}^{m}\right) \\
&= \sum_{k=1}^{K} \sigma^2 + \sum_{k=1}^{K} \sum_{\substack{m=1,\\ m\neq k}}^{K} \rho\sigma^2 \\
\Rightarrow \tilde{\sigma}^2 &= K\sigma^2 + K(K-1)\sigma^2\rho \\
\tilde{\sigma} &= \sqrt{K\sigma^2 + K(K-1)\sigma^2\rho} \\
&= \sigma\sqrt{K + K(K-1)\rho} \\
\tilde{\sigma} &= \sigma\sqrt{K(1+(K-1)\rho}
\end{aligned}
$$

With this result, netting efficiency is equivalent to

$$
\phi_{N,K} = \frac{N-1}{\sqrt{2\pi}}\sigma\sqrt{K(1+(K-1)\rho)}
$$

This result is equivalent to the formula derived in the Appendix of Duffie and Zhu (2011).

Now, utilize that the standard deviation of exposures X_{ij}^{k} is proportional to the credit exposure

$$
\sigma_k = \alpha_k Z_k \quad \forall \quad k = 1, ..., K
$$

where the values for Z_k are given in table 4.3. Let $\alpha_k = \alpha$ for k=1,...,K-1 and $\alpha_K = 3\alpha$. Then for $k \neq m$ and $k, m \in \{1, ..., K-1\}$

$$
Corr\left(X_{ij}^{k}, X_{ij}^{m}\right) = \rho
$$

Hence,

$$
\begin{aligned}
E\left[\max\left(\sum_{k=1}^{K-1} X_{ij}^{k}\right)\right] &= \sqrt{\sum_{k=1}^{K-1} \sigma_k^2 + \sum_{k=1}^{K-1} \sum_{\substack{m=1,\\ m\neq k}}^{K-1} \rho\sigma_k\sigma_m}\ \frac{1}{\sqrt{2\pi}} \\
E\left[\max\left(X_{ij}^{k}\right)\right] &= \sigma_k \frac{1}{\sqrt{2\pi}} \quad \forall \quad k.
\end{aligned}
$$

Given the assumptions as in Cont and Kokholm (2014) that standard deviation is proportional to the credit exposure and risk per dollar notional is 3α for CDSs:

$$\sigma_k = \alpha Z_k,$$
$$\sigma_K = 3\alpha Z_k$$

By plugging these standard deviations in we get

$$\begin{aligned}
E\left[\max\left(\sum_{k=1}^{K-1} X_{ij}^k\right)\right] &= \sqrt{\sum_{k=1}^{K-1}\sigma_k^2 + \sum_{k=1}^{K-1}\sum_{\substack{m=1,\\ m\neq k}}^{K-1}\rho\sigma_k\sigma_m}\,\frac{1}{\sqrt{2\pi}} \\
&= \sqrt{\sum_{k=1}^{K-1}\alpha^2 Z_k^2 + \sum_{k=1}^{K-1}\sum_{\substack{m=1,\\ m\neq k}}^{K-1}\rho\alpha^2 Z_k Z_m}\,\frac{1}{\sqrt{2\pi}} \\
&= \frac{\alpha}{\sqrt{2\pi}}\sqrt{\sum_{k=1}^{K-1} Z_k^2 + \sum_{k=1}^{K-1}\sum_{\substack{m=1,\\ m\neq k}}^{K-1}\rho Z_k Z_m}
\end{aligned}$$

This can be plugged into Formula 4.2.1

$$\begin{aligned}
R &= \frac{E\left[\max\left(X_{ij}^K, 0\right)\right]}{E\left[\max\left(\sum_{k<K} X_{ij}^k, 0\right)\right]} \\
&= \frac{3Z_K\frac{\alpha}{\sqrt{2\pi}}}{\frac{\alpha}{\sqrt{2\pi}}\sqrt{\sum_{k=1}^{K-1} Z_k^2 + \sum_{k=1}^{K-1}\sum_{\substack{m=1,\\ m\neq k}}^{K-1}\rho Z_k Z_m}} \\
R &= \frac{3Z_K}{\sqrt{\sum_{k=1}^{K-1} Z_k^2 + \sum_{k=1}^{K-1}\sum_{\substack{m=1,\\ m\neq k}}^{K-1}\rho Z_k Z_m}}
\end{aligned}$$

The R-value from this formula can then be calculated to solve for N in Inequality 4.2.3 which is

$$R > \frac{2\sqrt{N-1}}{N-2}$$

to calculate the values in Table 4.10 which do not correspond to the values calculated by Cont and Kokholm (2014)

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