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Pricing contingent convertible bonds: A numerical implementation with the hybrid equity-credit model

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Abstract

The contingent convertible (CoCo) bond is a loss-absorbing instrument which can be converted mandatorily to common equity when a trigger event happens, such as the book-value trigger and the discretionary trigger. The book-value trigger means that once the capital ratio hits the pre-specified threshold, the equity conversion will be activated. The capital ratio is the proportion of capital to risk-weighted assets (RWA), which reflects the financial health of the bank. With the discretionary trigger, the conversion of the CoCo bond will be decided by the regulators according to the financial situation of the issuing bank. In this thesis, we examine the hybrid equity-credit model suggested by Chung and Kwok (2016), which combines the book-value trigger and the discretionary trigger, assuming that the capital ratio has a mean-reversion movement and that the stock price follows a geometric Brownian motion with jumps. Furthermore, we perform a real world implementation of the Credit Suisse CoCo bond by calibrating the parameters of the hybrid model against market data and applying both Monte Carlo simulation and the so-called Fortet algorithm. As an extension, we add a Cox, Ingersoll and Ross (CIR) framework to the equity-credit model to reflect the dynamic of the interest rate. We present the results of the CoCo prices, the calibration errors and the implied conversion probabilities as well as sensitivity analyses and find several interesting numerical results for the Credit Suisse CoCo bond. For example, the data seems to imply that the CoCo will be converted with almost 100% probability within 2 years from April 2014.

Keywords: Contingent Convertible Bonds, Equity-credit Model, CoCos, Fortet Algorithms, Pricing

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1 Introduction

The contingent convertible (CoCo) bond is a hybrid instrument which has gained substantial popularity in recent years. Since the financial crisis during 2007-2009, financial regulators around the world has been increasingly tightening the regulation with regard to the capital of banks. In particular, the Basel Committee on Banking Supervision (BCBS) issued a global financial regulation - Basel III - to fortify the capital requirements for banks in 2010-2011 (Bank for International Settlement, 2017). One of the purposes that banks issue the CoCo bond is to reinforce its ability to fulfill the capital requirements and maintain financial stability during distressed times. (Cheridito and Xu, 2015a)

The contingent convertible bond will be converted into common equity of the issuing bank or cash when a trigger event happens. Depending on the type of the contract, the CoCo bond holder will either lose the bond partially or completely or become a common equity holder. Thus, the coupon rate of the contingent convertible bond is usually higher than the normal bonds to compensate the additional conversion risk (Chung and Kwok, 2016). The CoCo bond functions as an alternative of equity raising when the bank is in financial distress, since it can be converted from debt to equity, often as Common Equity Tier 1 (CET1) capital which is regulatory capital used to absorb potential losses. The loss-absorbing instrument allows banks to infuse equity without raising capital externally. (Wilkens and Bethke, 2014)

Although the contingent convertible bond is designed to be a loss-absorbing solution and plays a role as additional equity in times of distress, its rules and thus the pricing mechanism are complex due to the complicated clauses (Wilkens and Bethke, 2014). Many financial institutions are currently having doubts on the applicability of the CoCo bond and are worried that the complexity will increase the market volatility. In fact, banks are banned from selling the contingent convertible bond to retail investors due to its intricacy, and there is a potential risk of mis-selling if retail investors were allowed to trade the CoCo bond (Arnold and Hale, 2016). In this thesis, we hope to shed some light on the pricing mechanism and contribute to clarifying some complexity.

Flannery (2002) introduced the contingent convertibles already in year 2002. However, there had not been many academic researches on the pricing model of the CoCo bond until the financial crisis in 2008 (Alvemar and Ericson, 2012). In general, the pricing model of the contingent convertible bond can be divided into three categories, including the structural model, the credit derivative model and the equity derivative model (Wilkens and Bethke, 2014). There are also hybrid models that combine different features of the above three models, for example, the equity-credit model presented by Chung and Kwok (2016). One of the main difference between the above mentioned models is the modeling of the trigger event. The trigger event is one of the most important characteristics of the contingent convertible bond, and in general there exist two possible types of trigger. The first one is the mechanical trigger, which is based on the regulatory capital ratio that reflects the financial health of the bank. Once the capital ratio hits the pre-specified level, the trigger event will occur. The regulatory capital ratio plays a major role in the design of the CoCo bond. There are different levels of regulatory capital that consist of mixed liquid or semi-liquid assets. More detail of the capital ratio is presented in Section 2. The second trigger is the discretionary trigger. If a CoCo bond has a discretionary trigger, the trigger events are decided by the financial regulators. When the bank reaches the point-of-non-viability (PONV), the regulators will determine the trigger of the equity conversion according to the financial situation of the issuing bank. (Avdjiev et al., 2013)

The hybrid equity-credit model developed by Chung and Kwok (2016) includes both the book-value mechanical trigger which is based on the ratio of the book value of capital to risk-weighted assets (RWA) and the discretionary trigger, thus this model provides more flexible and complete trigger mechanism to reflect the financial health of banks. In this thesis, we examine and implement the equity-credit pricing model, which assumes that the stock price follows a geometric Brownian motion with jumps and that the capital ratio has a mean-reversion movement. In the equity-credit model, the price of the contingent convertible bond is divided into three components. The first component is the conversion value which is the value of the equity once the conversion happens. The second component is the value of the coupon payments. Since there is a possibility that the CoCo bond will be converted into common equity, the bond holder is not guaranteed to receive all the coupons. The third component is the value of the principal payment. For the same reason as for the coupons, we need to simulate the conversion probability to calculate the principal payment of the CoCo bond.

Besides examining the pricing process of Chung and Kwok (2016), this thesis also contains five main contributions which to the best of our knowledge have not been studied in other papers based on the Chung and Kwok (2016) framework. First, we perform a real world implementation of the Credit Suisse CoCo bond by applying both Monte Carlo simulation and Fortet method as Chung and Kwok (2016). Second, in order to obtain more realistic parameters, we calibrate the parameters of the hybrid model against market data, such as the CoCo prices, the stock prices, the capital ratios, etc., while Chung and Kwok (2016) just assume the values of the parameters fictively. Third, we consider a stochastic interest rate following the Cox, Ingersoll and Ross (CIR) model as an extension of the Chung and Kwok (2016) framework, which only allows for a constant interest rate. Fourth, we present the final prices of the contingent convertible bond, the calibration errors, the conversion probabilities and analyze economical intuitions behind the results, while Chung and Kwok (2016) only show one of the component of the final price, i.e. the conversion value. Finally, we perform a sensitivity analysis to test the sensitivity of the contingent convertible bond price to different parameters. As a result, we conclude that the CoCo prices obtained by applying both the equity-credit model and the extended model with a stochastic interest rate fit the market prices well for the period from 2012 to 2014, except the overestimation in several time points in 2013 because of the impact of the high values of the market CoCo prices, the stock prices and the capital ratios on the calibrated parameters. Meanwhile, the data seems to imply that the CoCo will be converted with almost 100% probability within 2 years from April 2014.

The structure of this thesis is as follows. In Section 2, we present the characteristics of the CoCo bond in more detail. Section 3 gives a literature review. In Section 4, we show the equity-credit pricing model of CoCo bonds suggested by Chung and Kwok (2016) and an extended model where we incorporate the stochastic dynamic of the interest rate. Section 5 presents the implementing process of the pricing models by applying a CoCo bond issued by Credit Suisse. In Section 6 we show both the replication results of Chung and Kwok (2016) and the results of our application. We also perform a sensitivity analysis in Section 7.

2 CoCo introduction

In this section, we introduce several main concepts with regard to the contingent convertible bond, including the capital regulation, the loss absorption mechanism and the triggers of the CoCo bond.

Usually when a financial crisis happens and a bank is faced with insolvency, no investor wants to invest in this bank. Thus, it is hard for the bank in the financial crisis to obtain enough capital and survive. In some situations, governments will provide capital to assist the banks in distress. However, the assistance from governments will cause losses for taxpayers, which is often not a desirable consequence. Therefore the assistance of governments forfeits the popularity and the contingent convertible bond emerges as a bail-in instrument which can raise equity when the banks face insolvency (Chen et al., 2013). When a bank falls into a financial crisis, the CoCo bond can be converted into equity or the principal of this bond can be written down, which will decrease the debt of the issuing bank and increase its capital. (Avdjiev et al., 2013)

2.1 Capital regulation

Since the financial crisis during 2007-2009, financial regulators around the world has been increasing the demand of regulatory capital for banks. The contingent convertible bond has become popular because of its bail-in feature during difficult times. There exists close connection between the CoCo bond and capital regulation. On one hand, the regulatory capital ratio plays an important role in the design of the trigger event that will activate the equity conversion of CoCos. The conversion trigger is usually based on the ratio of the book-value capital to the risk-weighted assets (RWA). The RWA is a capital adequacy measurement listed in the capital regulation, which measures the riskiness of the assets. The larger the RWA is, the risker the asset is (Alvemar and Ericson, 2012). On the other hand, when the bank is in financial distress, the contingent convertibles can be converted from debt to equity, often as Common Equity Tier 1 (CET1) capital which is an important capital item listed in Basel III (Wilkens and Bethke, 2014). CoCos have the ability to improve the balance sheet of the bank. Here we mainly introduce the global financial regulation Basel III, which is issued by the Basel Committee to improve the risk management and the supervision of banks. Basel III is currently applied to banks widely, which introduces the definition of capital and its different measures (Bank for International Settlement, 2017). According to Basel Committee on Banking Supervision (2010), the total regulatory capital contains the following categories:

1. Tier 1 Capital (going-concern capital), including:
    - Common Equity Tier 1
    - Additional Tier 1
2. Tier 2 Capital (going-concern capital)

Common Equity Tier 1 capital mainly includes the sum of common shares, stock surplus and retained earnings. Additional Tier 1 consists of instruments issued by the bank and its consolidated subsidiaries or held by third parties which fulfill the criteria for inclusion in Additional Tier 1 capital and relevant stock surplus. The instruments issued by the bank and its consolidated subsidiaries or held by third parties which fulfill the criteria for inclusion in Tier 2 capital and relevant stock surplus are included in Tier 2 capital.

For all the three elements that constitute the total regulatory capital, there are minimum requirements as follows:

- Common Equity Tier 1 must be at least 4.5% of risk-weighted assets (RWA)
- Tier 1 Capital must be at least 6.0% of RWA
- Total regulatory capital must be at least 8.0% of RWA

2.2 Loss absorption mechanism

The loss absorption mechanism is one of the main characteristics of contingent convertible bonds. There are two routes for implementing the loss absorption mechanism: (1) the equity conversion; (2) the principal writedown. First, the equity conversion means that when the contractual trigger event happens, the contingent convertible bond will be mandatorily converted to the common equity. Thus the bank's Common Equity Tier 1 capital will increase. Second, the principal writedown means that the principal of the CoCo bond will be written down as soon as the trigger event occurs. Once the value of the principal is written down, the equity of the bank will grow. The principal of the CoCo bond can be partially or wholly written down (Avdjiev et al., 2013). In this thesis, we only consider the feature of the equity conversion.

2.3 Trigger

The trigger activates the loss absorption mechanism in the contingent convertible bond, such as the equity conversion and the principal writedown, and is therefore one of the main characteristics of this type of bonds. There are two types of triggers: (1) the mechanical trigger; (2) the discretionary trigger. The mechanical trigger is based on the ratio of the capital to the risk-weighted assets (RWA) of the banks which issue CoCo bonds. This ratio can be called the capital ratio. Once the capital ratio is lower than a predetermined level, the mechanical trigger is activated. Because the capital can be measured by book values or market values, the mechanical trigger includes the book-value trigger and the market-value trigger. The book-value trigger is also called the accounting-value trigger. The ratio of Common Equity Tier 1 capital book value to the risk-weighted assets is a very common book-value capital ratio that the book-value trigger is based on. The other type of trigger is the discretionary trigger or point of non-viability (PONV) trigger. Under the situation of the discretionary trigger or PONV trigger, whether the conversion should be activated is based on the regulator's or the supervisor's decision. In a contract of CoCo bond, multiple triggers can exist. When one of the trigger events is activated, the equity conversion or the principal writedown will happen (Avdjiev et al., 2013). In the CoCo pricing models of this thesis, a combination of the book-value trigger and the discretionary trigger is applied.

3 Literature review

Below we will present and discuss earlier studies on the subject of CoCo pricing. Earlier pricing models of CoCo bonds can generally be divided into three different approaches. The first approach (Subsection 3.1) is the firm value approach which is originated from Merton (1974) and later evolved into different variations of firm value pricing models. The second approach, discussed in Subsection 3.2, is the credit derivative model which is presented by De Spiegeleer and Schoutens (2011). Subsection 3.3 shows the third approach which is the equity derivative model also suggested by De Spiegeleer and Schoutens (2011). Additionally, in Subsection 3.4 we outline a hybrid model that combines the stochastic movement of the equity and the credit derivative model, developed by Chung and Kwok (2016). Finally, we discuss the interest rate model of Cox et al. (1985) in Subsection 3.5. In this thesis, we will refer heavily to the equity-credit model by Chung and Kwok (2016).

3.1 Structural models - the firm value approach

The structural models that we introduce in this subsection are essentially variations of the famous firm-value approach developed by Merton (1974). Merton (1974) assumes that the firm value follows a classic Gaussian-Wiener process. Based on the non-arbitrage theory, a risk-free portfolio is created in order to derive the value of the risky bond. The risk-free portfolio consists of the firm asset, a claim on the firm asset and a risk-free bond. The intuition is that a portfolio without any risk should only gain the risk-free interest rate. In a perfect market, there should exist no arbitrage opportunity. Thus, by Ito's lemma, the value of the risky bond is derived by satisfying the non-arbitrage theory.

The ordinary contingent convertible claim pricing model was developed long before CoCo bonds became popular. Ingersoll (1977) and Brennan and Schwartz (1977) extend the corporate debt pricing model of Merton (1974) and price the voluntary contingent convertible bonds. However, there is a significant difference between the voluntary contingent convertible bond and the CoCo bond that we investigate in this thesis. One major difference is that the CoCo bond is much riskier compared with the voluntary contingent convertible, since investors will be reluctant to become an equity holder when the company is in distress.

Following the study of Merton (1974), the structural models use values on the balance sheet as the indicator of the trigger. According to the predetermined trigger level, the structural model prices CoCo bonds corresponding to the asset value movement. Leland (1994) provides a pricing model for the value of a risky debt with a perpetual constant coupon payment based on Merton (1974). Albul et al. (2015) extend the model of Leland (1994) and consider a firm with a balance sheet consisting of straight bonds, shareholder's equity, and contingent convertible bonds. Both straight bonds and contingent bonds are assumed to be perpetual (Albul et al., 2015). Based on Leland's endogenous default and dynamic aspect of the contract, they develop a closed-form solution for valuation of contingent convertible bonds.

Pennacchi (2010) introduces the pricing model of the contingent capital with jump diffusion. The model reflects the feature of CoCo bond in form of full equity conversion. Moreover, the model allows jump diffusion of company's asset value and therefore takes into account that the market value usually moves dramatically in the time of crisis. The jump-diffusion of the asset is expressed as the Poisson process. Pennacchi (2010) further assumes that a bank balance sheet consists of deposit, shareholder's equity, and contingent capital. The trigger level is determined by the ratio between the asset and the deposit. Once the bank's deposit is below the value of the asset, the bank is insolvent and the CoCo bonds will be fully converted to the equity.

Among different structure models that are based on the firm value theory, the assumption on the conversion trigger and the timing also differ between scholars. In Cheridito and Xu (2015b), the authors assume that there is a probability that the default and the conversion would happen at the same time. This assumption of simultaneous default and the conversion differ from Chung and Kwok (2016) who assumes that the conversion always comes before the default, since the purpose of CoCo bond conversion is to prevent the event of the default.

3.2 Credit derivative model

De Spiegeleer and Schoutens (2011) introduce the credit derivative approach for pricing CoCo bonds, which uses fixed-income theory when valuating the CoCo bond as debt. In this approach, De Spiegeleer and Schoutens (2011) apply a market trigger instead of the book-value trigger, which they set as the event when the share price decreases below a certain level. Furthermore, the credit derivative approach introduces the so called CoCo-spread which is the extra return above the risk free rate and represents the compensation of the potential losses when the trigger event happens. According to De Spiegeleer and Schoutens (2011), the value of CoCo's credit spread can be derived by utilizing the barrier option pricing equation under the assumptions of Black-Scholes. The CoCo's credit spread depends on the trigger level and the conversion price, however, in the credit derivative models, De Spiegeleer and Schoutens (2011) do not consider the losses from the cancellation of the coupon flows.

3.3 Equity derivative model

De Spiegeleer and Schoutens (2011) also introduce the equity derivative approach for pricing the CoCos. In this approach, the trigger event occurs when the share price hits the low-barrier level. Different from the credit derivative model, the equity derivative model derives the theoretical value of the contingent convertible bond. De Spiegeleer and Schoutens (2011) conclude that the CoCo bond is a combination of a corporate bond, a knock-in option package and a sum of binary down-and-in options. Furthermore, De Spiegeleer and Schoutens (2011) derive the equation of the theoretical value of CoCos by directly applying the existing equations of pricing the corporate bond, the knock-in option package and the sum of binary down-and-in options.

3.4 Equity-credit model

Chung and Kwok (2016) combine some of the models mentioned above and propose an equity-credit model, assuming that the stock price follows a geometric Brownian motion with jumps and that the capital ratio follows a mean reversion process. The capital ratio in the equity-credit model is the ratio of the book value of capital to risk-weighted assets (RWA). In the model of Chung and Kwok (2016), two types of triggers are applied simultaneously. One of the triggers is the book-value trigger, which will be activated when the book-value capital ratio meets the pre-specified trigger threshold and the other trigger is the discretionary trigger. Chung and Kwok (2016) let the equity conversion be triggered by the judgment of the financial regulator. The discretion of the financial regulator is indicated by the first jump of the Poisson process in the stock price model. The equity-credit model provides a more flexible approach to decide the conversion trigger. For example, a bank can still keep a good level of the capital ratio even when the bank is in financial distress. With an additional discretionary trigger or a jump-to-non-viability (JtNV) trigger, the regulator can control the risk more flexible and efficient. Chung and Kwok (2016) decompose the CoCo price into three components, including the value of the coupon payments, the value of the principal payment and the conversion value, however in their paper only one of these components are considered in their numerical studies. We will consider all of them in our numerical implementation. Chung and Kwok (2016) also introduce the Fortet algorithms for pricing the conversion value of the CoCo bond in their study.

3.5 Interest rate model

Cox et al. (1985) introduce the theory of term structure that describes the dynamic of interest rate, also known as CIR mean-reversion movement. The term structure describes the relationship between different yields and different corresponding maturities. In general, the yield will increase with the maturities up to a point where the maturities are too far into the future (Cox et al., 1985). Empirically, one can calibrate the parameters from the actual market prices of bonds with different maturities. Brown and Dybvig (1986) study the implication of one factor CIR model and the calibration of the model. Episcopos (2000) presents the term structure of interbank interest rate across European countries and they use the Maximum Likelihood method to estimate different one-factor interest rate models including CIR model. In this thesis, we will consider a stochastic interest rate following a CIR model where the CIR-parameters are estimated as in Episcopos (2000). We thus use these parameters to simulate the interest rate model as an extension of Chung and Kwok (2016) who only allow for a constant interest rate.

4 Model

In this section, we present the CoCo pricing models which will be used in the rest of this thesis. First, in Subsection 4.1 we introduce a typical structure of contingent convertible bonds. Second, Subsection 4.2 outlines the CoCo pricing model developed by Chung and Kwok (2016). Finally, Subsection 4.3 shows an extension of the Chung and Kwok (2016) model which includes a stochastic interest rate.

4.1 The CoCo bond structure

In this subsection, we will introduce a typical structure of contingent convertible bonds. By construction, a CoCo bond will contain a bond component and a stock component. As long as the trigger event has not happened, the owner of a CoCo will get coupon payments c_i at time points t_i, where $i = 1, 2, ..., n$. At the maturity $T = t_n$ of the contingent convertible bond, the principal payment F will also be paid. However, once a trigger event happens, each CoCo bond will be converted into G units of the stock shares which are issued by the same bank.

As Cheridito and Xu (2015b) point out, the price of contingent convertible bond is determined by three sources of risks: (1) the interest rate risk, (2) the conversion risk and (3) the equity risk. The interest rate risk exists since it is used when we discount the CoCo bond's coupon payment and principal payment. The conversion risk is the uncertainty on when or whether the CoCo bond will be mandatorily converted to equity. In other words, the conversion risk is caused by the adverse situation for the equity conversion. For example, when the stock price is low and the equity conversion happens, the investor will face a loss since the value of the contingent convertible bond now partly is erased. The equity risk is due to the movement of the equity value. When the CoCo bond is converted to equity, the value of equity is usually very low, since the company is in financial distress. When the market environment is volatile, the equity risk is also larger (Chung and Kwok, 2016). We will present the models which reflect these three different risks later in this section. In these models, there exist multiple triggers for the equity conversion, i.e. the conversion occurs when the capital ratio falls below the pre-specified level (the book-value trigger) or when financial regulators decide to enforce the conversion (the discretionary trigger). In order to model the discretionary trigger, here the first jump of the Poisson process is set as the signal that activates the conversion. The conversion risk comes from both the capital ratio movement which follows a mean-reversion stochastic process and the regulators' decision. The equity risk will be modeled by the stock price movement which follows a geometric Brownian motion with jumps, since the capital ratio and the stock price are assumed to be correlated. Finally, we model the interest rate as in Cox et al. (1985) and this is an extension of the framework in Chung and Kwok (2016) who only consider a constant interest rate. Note that a CIR-interest-rate process allows for a stochastic interest rate and a non-constant term structure.

4.2 Model setup

In this subsection, we present the CoCo pricing models developed by Chung and Kwok (2016). In Chung and Kwok (2016), a filtered probability space $(\Omega, \mathcal{F}, (\mathcal{F}_t)_{t\geq 0}, Q)$ is used, where Q is a risk-neutral probability measure which always will exist if we rule out arbitrage possibilities (Bjork, 2009). Furthermore, the filtration $(\mathcal{F}_t)_{t\geq 0}$ is in general generated by the underlying process that drives the dynamics of the derivatives in the model.

Chung and Kwok (2016) set S_t = exp (x_t) and H_t = exp(y_t), where S_t is the stock price and H_t is the book-value capital ratio. The stock price movement follows a geometric Brownian motion with jumps and the capital ratio follows a mean-reversion stochastic process under the risk-neutral measure, given by:

$$dx_t = \left[r - q - \frac{\sigma^2}{2} - \gamma\lambda(x_t, y_t)\right] dt + \sigma dW_t^{(1)} + \ln(1+\gamma) dN_t, \tag{1}$$

$$dy_t = \kappa(\theta - y_t)dt + \eta\left(\rho dW_t^{(1)} + \sqrt{1-\rho^2} dW_t^{(2)}\right). \tag{2}$$

Note that Equation (1) and (2) can also be rewritten as:

$$x_t = \tilde{x}_t + N_t \ln(1+\gamma) - \gamma \int_0^t \lambda(x_s, y_s)\, ds, \tag{3}$$

$$y_t = y_0 e^{-\kappa t} + \theta(1 - e^{-\kappa t}) + \int_0^t \eta e^{-\kappa(t-s)} \left(\rho dW_s^{(1)} + \sqrt{1-\rho^2} dW_s^{(2)}\right), \tag{4}$$

where

$$\tilde{x}_t = x_0 + \left(r - q - \frac{\sigma^2}{2}\right)t + \sigma W_t^{(1)}. \tag{5}$$

Here, the interest rate $r \geq 0$, the dividend yield $q \geq 0$, the stock price volatility $\sigma > 0$, the capital ratio volatility $\eta > 0$, the correlation coefficient $-1 \leq \rho \leq 1$, the mean reversion level $\theta > 0$ and the speed of reversion $\kappa > 0$ are all constants. Furthermore, $W_t^{(1)}$ and $W_t^{(2)}$ are two uncorrelated Brownian motions and N_t denotes a Poisson process with γ as the constant jump magnitude of the stock price and λ as the intensity of N_t.

From Equation (3) and (5), we see that the movement of the stock price is the combination of a Brownian motion $W_t^{(1)}$, a drift $\left(r - q - \frac{\sigma^2}{2}\right)$ and a jump process $\left[N_t \ln(1+\gamma) - \gamma \int_0^t \lambda(x_s, y_s)\, ds\right]$, which means that the stock price will allow for jumps. As shown in Equation (2), the capital ratio will move stochastically but around a long-term mean θ. Once the capital ratio deviates from the long-term mean level, it will move back to its mean with the speed κ. There also exists a correlation coefficient between the capital ratio and the stock price movement, because they both contain the same Brownian motion $W_t^{(1)}$. Figure 1 and 2 show four simulated trajectories of the log stock price x_t and the log capital ratio y_t, respectively. As can be seen in Figure 1, the jump of the stock price happens in two of the trajectories.

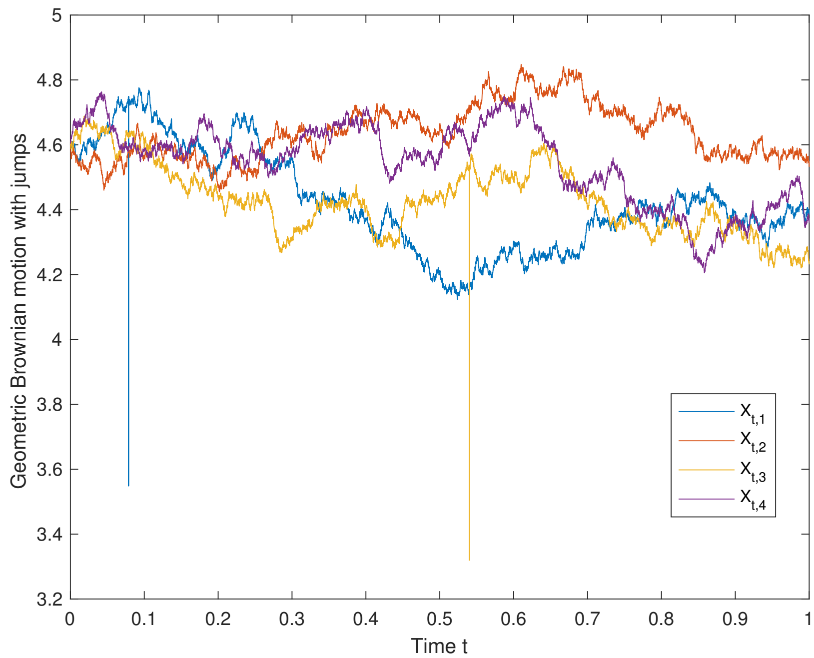

Figure 1: Four Monte Carlo simulation trajectories: the stock price simulation

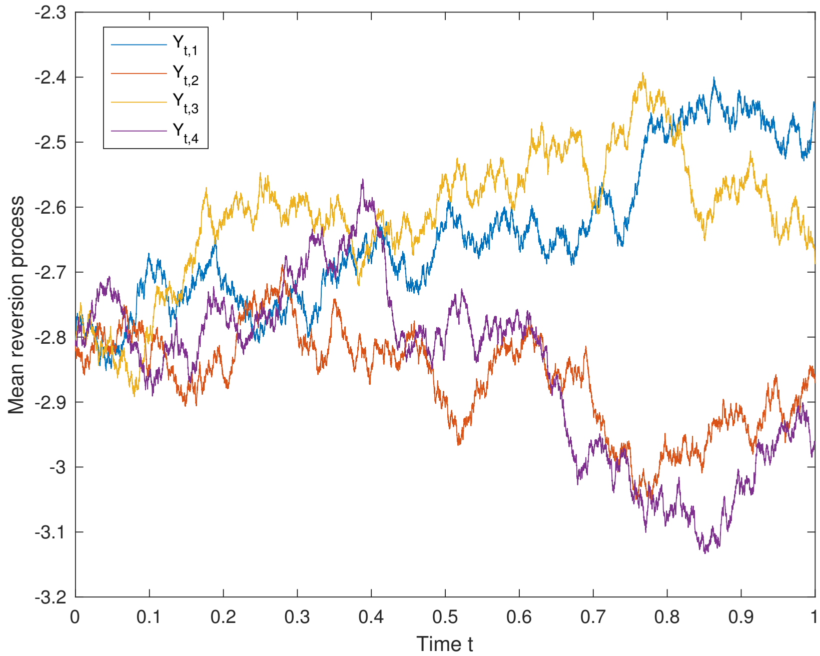

Figure 2: Four Monte Carlo simulation trajectories: the capital ratio simulation

In Chung and Kwok (2016), a combination for the CoCo conversion trigger is used, which includes the book-value trigger and the discretionary trigger. The CoCo conversion time τ is defined as the first time that one of the two triggers happens. The book-value trigger occurs at the random time τ_B, the first time that the capital ratio y_t is equal to or smaller than the natural logarithm of the capital ratio trigger level y_B=$\ln H_B$, given by:

$$\tau_B = \inf\{t \geq 0; y_t = y_B\}.$$

The discretionary trigger occurs at the random time τ_R, which is the time when the regulators decide to activate the equity conversion, given by:

$$\tau_R = \inf\{t \geq 0; N_t = 1\},$$

that is, the first jump of the Poisson process. Note that the stock will also jump right after τ_R as a consequence of the regulators' determination. The conversion time τ is then defined as the first event of these two random variables, i.e.:

$$\tau = \tau_B \wedge \tau_R = \min(\tau_B, \tau_R).$$

According to Chung and Kwok (2016), the price of CoCo bond (P_{CoCo}) can be divided into three parts: the value of coupon cash flows (P_C), the present value of principal (P_F) and the value of conversion (P_E):

$$P_{CoCo} = P_C + P_F + P_E. \tag{6}$$

The value of coupon cash flow P_C is calculated by adding all the discounted coupon payments c_i which are received at time points t_i, where $i = 1, 2, ..., n$, if the trigger event has not yet happened at t_i, that is:

$$P_C = \sum_{i=1}^{n} \mathbb{E}^Q[c_i e^{-rt_i} 1_{\{\tau > t_i\}}] = \sum_{i=1}^{n} c_i e^{-rt_i}[1 - Q(\tau \leq t_i)]. \tag{7}$$

The present value of the principal payment P_F can be obtained only if the conversion does not happen before the maturity T, which therefore is given by:

$$P_F = \mathbb{E}^Q[Fe^{-rT} 1_{\{\tau > T\}}] = Fe^{-rT}[1 - Q(\tau \leq T)] \tag{8}$$

where F is the principal payment of the CoCo bond at the maturity $T = t_n$. The value of conversion P_E is given by:

$$P_E = \mathbb{E}^Q[e^{-r\tau} G S_\tau 1_{\{\tau \leq T\}}] \tag{9}$$

where G is the conversion factor, i.e. once a trigger event happens, one CoCo bond will be converted into G units of the stock shares which are issued by the same company and S_τ is the stock price at the conversion time τ.

4.2.1 Constant intensity

From Equation (7) - (9) we see that we need the distribution of τ and following the derivation in Chung and Kwok (2016), the conversion probability $Q(\tau \leq t)$ in the constant intensity case (i.e. N_t is a Poisson process with constant intensity λ) is then given by:

$$Q(\tau \leq t) = \int_0^t \lambda e^{-\lambda u}[1 - Q(\tau_B \leq u)]du + \mathbb{E}^Q[e^{-\lambda \tau_B} 1_{\{\tau_B \leq t\}}]. \tag{10}$$

In order to calculate the conversion value P_E in Equation (9), Chung and Kwok (2016) change the risk-neutral measure Q to a so called stock price measure Q*. Under the measure Q*, the conversion value P_E is given by:

$$P_E = GS_0 \left\{ \frac{\lambda^*}{\lambda^* + q} \left[1 - e^{-(\lambda^* + q)T} Q^*(\tau_B > T)\right] + \frac{q}{\lambda^* + q} \mathbb{E}^{Q^*}\left[e^{-(\lambda^* + q)\tau_B} 1_{\{\tau_B \leq T\}}\right] \right\}. \tag{11}$$

So to obtain the conversion value P_E, we need to calculate the Q*-probability that the first time of the capital ratio reaching the threshold happens before the maturity date $Q^*(\tau_B \leq T)$ and the Q*-expectation $\mathbb{E}^{Q^*}[e^{-(\lambda^*+q)\tau_B}1_{\{\tau_B \leq T\}}]$.Chung and Kwok (2016) apply the Fortet method, which is a discrete and recursive method, to calculate the above two quantities, as follows:

$$Q^*(\tau_B \leq t_m) = \sum_{j=1}^{m} q_j, \tag{12}$$

$$\mathbb{E}^{Q^*}\left[e^{-(\lambda^*+q)\tau_B}1_{\{\tau_B \leq t_m\}}\right] = \sum_{j=1}^{m} e^{-(\lambda^*+q)t_j} q_j, \tag{13}$$

where

$$q_1 = N(a(t_1)),$$
$$q_j = N(a(t_j)) - \sum_{i=1}^{j-1} q_i N(b(t_j,t_i)), j = 2,3,...,m,$$
$$a(t) = \frac{y_B - \mu(t,0)}{\Sigma(t,0)},$$
$$b(t,s) = \frac{y_B - \mu(t,s)}{\Sigma(t,s)} \mid_{y_s = y_B},$$
$$\mu(t,s) = y_s e^{-\kappa(t-s)} + (\theta + \frac{\rho\sigma\eta}{\kappa})\left[1 - e^{-\kappa(t-s)}\right],$$
$$\Sigma^2(t,s) = \frac{\eta^2}{2\kappa}\left[1 - e^{-2\kappa(t-s)}\right],$$

and $N(\cdot)$ is the cumulative normal distribution function. The time period $[0,T]$ is divided into m equal intervals. The conversion probability of every small interval q_j is calculated and summed up to obtain the total conversion probability $Q^*(\tau_B \leq t_m)$ and $\mathbb{E}^{Q^*}\left[e^{-(\lambda^*+q)\tau_B}1_{\{\tau_B \leq t_m\}}\right]$. Then, the conversion value P_E can be obtained.

4.2.2 State-dependent intensity

Intuitively, it is more likely that the firm has higher conversion or default intensity when the stock price of the firm drops since the stock price has a signal effect on the financial health of the firm. The intensity λ_t of the Poisson process N_t should, therefore, be dependent on the stock price movement. By the same token, the conversion intensity λ_t is also influenced by the capital ratio, that is $\lambda_t = \lambda(x_t, y_t)$. Thus, Chung and Kwok (2016) also present a state-dependent intensity pricing model of the CoCo bond and there are several possible ways to model such a intensity $\lambda(x_t, y_t)$. One example is the case with only a stock price dependent intensity $\lambda(x,y) = \lambda(x)$, given by:

$$\lambda(x) = \exp(a_0 - a_1 x), \quad a_1 > 0 \tag{14}$$

where a_1 denotes the delta risk of the CoCo bond with the subscript of time and a_0 denotes the delta risk of CoCo bond at time 0. We set that the stock price x_0 and the intensity $\lambda(x_0)$ are known at the initial time and a_0 fulfills that $\lambda(x_0) = exp(a_0 - a_1 x_0)$. In this case, the conversion intensity is correlated with the stock price. When the stock price increases, the conversion intensity will decrease.

A second example of state dependent intensity $\lambda(x,y)$ is a capital ratio dependent intensity, given by:

$$\lambda(y) = b_0 1_{\{y \leq y_{RT}\}}, \quad b_0 > 0 \tag{15}$$

where y_{RT} denotes a warning level of the capital ratio, which is close to the predetermined logarithm value of capital ratio stated in the CoCo contract. According to Chung and Kwok (2016), the warning level y_{RT} can also be the indicator where the financial supervisor will potentially initiate the process of Point-of-Non-Viability. As can be seen from Equation (15), the conversion intensity is correlated with the capital ratio. Once the capital ratio y decreases and reaches the warning level y_{RT}, the intensity λ will jump to b_0. Otherwise, the intensity λ will be equal to zero. Finally, a combined dependency of intensity can be specified as follows:

$$\lambda(x,y) = \exp(a_0 - a_1 x) + b_0 1_{\{y \leq y_{RT}\}},$$

$$a_1 > 0, b_0 > 0.$$

The result of the state-dependent model is estimated by applying Monte-Carlo simulation, which will be presented in Section 5.

4.3 Extension - interest rate model

Below we introduce the extended model, in which we relax the previous assumption of constant interest rate and introduce the Cox, Ingersoll and Ross (CIR) model into the equity-credit model to reflect stochastic dynamics of the interest rate movement. Furthermore, we also discuss the correlation between the interest rate and the capital ratio.

4.3.1 CIR model

According to Cox et al. (1985), the CIR term structure theory describes the mean reversion dynamic of the interest rate r_t, which is specified as follows:

$$dr_t = \zeta(\psi - r)dt + \omega\sqrt{r_t}dW_t^{(r)}, \tag{16}$$

where r_t is the instantaneous interest rate at time t, ζ is the speed of mean-reversion, i.e. how fast the instantaneous interest rate moves back to the long-term mean, ψ is the long-term interest rate and ω is the standard deviation of the instantaneous interest rate. Furthermore, $W_t^{(r)}$ is a Wiener process. The stochastic interest rate movement is then assumed to be correlated with the capital ratio movement, which allows us to model the mean-reversion movement of the interest rate as:

$$dr_t = \zeta(\psi - r)dt + \omega\sqrt{r_t}\left(\rho_{cir}dW_t^{(2)} + \sqrt{1-\rho_{cir}^2}dW_t^{(r)}\right), \tag{17}$$

here, ρ_{cir} is the correlation coefficient between the interest rate and the capital ratio. In order to find reasonable parameters for the CIR model, one can calibrate these parameters from the historical data of market prices of treasury bills with a broad range of maturities from two years to thirty years, by minimizing the sum of squared errors between the pricing results from all possible parameters and the actual market prices (Sevcovic and Csajková, 2005). The method is similar to the calibration of the parameters in the equity-credit model that is presented in Section 5.

As mentioned in the literature review, there have been extensive researches and implementations with regard to the CIR model calibration. Thus, in this thesis, we will use the CIR-parameters estimated as in Episcopos (2000) directly to model the stochastic interest rate. Our focus for this extended part of the pricing model will be on the correlation between the interest rate movement and the capital ratio movement.

4.3.2 Correlation between the interest rate and the capital ratio

In this subsection, we analyze the correlation between the interest rate and the capital ratio from the perspective of bank's profitability. In our opinion, when a bank earns a good margin and satisfactory profitability, the profit will in turn result in a robust financial ground and a stable capital structure of the bank. Since capital ratio is an indicator for the financial solidness of a bank, the connection between the interest rate and bank's profitability should imply a direct relationship between the interest rate and the capital ratio. The profit margin of a bank is generated mainly from the spread between the rate of lending and the rate of deposit. Moreover, the net interest margin, which is the relationship between the net interest income and the interest earning asset, plays an important role for the bank's profitability (Stijn Claessens, 2016).

It is clear that the interest rate influences the bank's profitability. When the interest rate is lower, banks will profit the difference in valuation of projects. However, when the bank generates less interest income, the borrowing cost of the bank will also decrease. There are many other factors that in total will offset each other and whether the correlation is positive or negative can be rather ambiguous. Nevertheless, the study conducted by Stijn Claessens (2016) from the U.S. Federal Reserve, shows that, while controlling other variables, the banks have better profitability when the interest rate is higher. When the interest rate is at the bottom, banks have a hard time to adjust the deposit rate due to the competition in the market. For the lending business, the floating rate on the asset side adapts to a lower level much faster than the rate of liabilities does. As a result, the bank will generate less interest income margin. Meanwhile, with a sunken interest rate, the bank will tend to borrow more due to the decreased cost of capital and become more leveraged. In other words, the low interest rate will harm bank's interest rate margin, which results in a less desirable profitability and in turn reduces the retained earning and decreases reinvestment of the profit. In summary, when the profitability increases with the interest rate, the capital ratio would also grow, which leads us to conclude that there is a positive correlation between the capital ratio and the interest rate (Stijn Claessens, 2016).

5 Application

In this section, we introduce the numerical implementation of the pricing models derived in Section 4. The parameters we use in the implementation for the pricing models are calibrated from real market data. Moreover, we perform the numerical implementation with two numerical methods. One is Fortet algorithm which is a recursive calculation method that solves an integral equation and allows us to obtain the pricing result and the conversion probability in a deterministic way. The other method is Monte Carlo simulation, which we use to simulate stock prices, capital ratios and instantaneous interest rates. Furthermore, we also compute the final CoCo bond prices, the calibration errors and the conversion probabilities according to the calibrated parameters with Monte Carlo simulation. Subsection 5.1 introduces the chosen CoCo bond for the implementation. In Subsection 5.2, we estimate the values of three parameters that are needed for the equity-credit model and assume two parameters fictively. Subsection 5.3 presents the procedure and the numerical setting of calibration in which we evaluate the rest four parameters by matching the pricing model against real market prices of the chosen CoCo bond. Subsection 5.4 outlines the statistical procedure of Monte Carlo simulation. Finally, we motivate and show the values of the parameters that are applied to the extended model which adds a stochastic interest rate in the original model by Chung and Kwok (2016) in Subsection 5.5.

5.1 Credit Suisse CoCo bond

In the following, we present the chosen contingent convertible bond for the numerical implementation. In this thesis, we choose the CoCo bond issued on 22 March 2012 by Credit Suisse Group (Guernsey) IV LTD.. In the rest of this thesis, we will call this chosen instrument ”Credit Suisse CoCo bond”. The Credit Suisse CoCo bond matures on 22 March 2022 and has the earliest call-back date on 22 March 2017. For many loss-absorbing instruments, it is predetermined in the contract that the issuer has the right to call back and redeem the bond after a specific period. The call-back of a capital contingent bond may happen when the issuer has obtained an adequate level of the capital on the balance sheet. The earliest call-back date is often set as five years after the bond being issued (European Parliament, 2016). During the period of the model implementation in this thesis, the Credit Suisse CoCo bond has been called back on 22 March 2017. Although intuitively the option of the call-back is incorporated in the market price of the loss-absorbing instrument, we do not model the call-back feature due to the time limitation of this thesis and therefore leave it for future studies. The information of the chosen CoCo is presented in Table 1.

Table 1: Credit Suisse CoCo Bond Description

Credit Suisse CoCo bond	
Maturity	10 years
Issued date	22-Mar-2012
Call-back date	22-Mar-2017
Coupon	7.125%
Trigger	Common Equity Tier 1 Capital Ratio
Trigger level	7%

The trigger of Credit Suisse CoCo bond is the Common Equity Tier 1 (CET1) capital ratio, which means that the equity conversion is activated automatically when the CET1 capital ratio falls below the predetermined trigger level of 7%. On 15 March 2017, the data from Bloomberg terminal showed that there were 349 capital contingent bonds issued with only the book-value trigger. However, we failed to find a contingent convertible bond which contains both a book-value trigger and a discretionary trigger. When we implement the pricing model of Chung and Kwok (2016) and choose a contingent convertible bond, there are several aspects that we need to consider. The chosen bond has a maturity, i.e. the duration of the bond is not perpetual and once the trigger event happens, the contingent convertible bond will be converted to common equity instead of being written down. Since the pricing model for the loss-absorbing bond with a writedown feature is very similar to the pricing model of a defaultable bond, it is more relevant for us to price the convertible bond with a equity conversion feature for the purpose of analyzing the complexity of the CoCo pricing model. Furthermore, the contingent convertible bond should have a book-value trigger. Finally, the instrument has combined conversion triggers including a book-value trigger and a discretionary trigger. The Credit Suisse CoCo bond fulfills all the conditions except that it does not have combined triggers. Here we assume that the Credit Suisse CoCo bond also has a discretionary trigger for the implementation of the pricing model. As discussed earlier, because the dual triggers of a CoCo bond introduced by Chung and Kwok (2016) provides a more flexible framework for regulators to supervise the financial institutions, in our opinion, the discretionary trigger will become increasingly popular and more loss-absorbing instruments will be issued with the combined triggers in the future.

The target implementation period is between April 2012 and October 2014 and we choose to price the CoCo bond on the 27th of every second month. The chosen months are April, June, August, October and December. If the 27th of the chosen month is not a trading day, then the data is replaced with the nearest trading day. The historical data before and during April 2012 to October 2014 is collected from Bloomberg terminal.

5.2 Parameter estimation

Below we give the summary of the parameter estimation for the risk-free interest rate r, the standard deviation of the stock price σ and the dividend yield q. Moreover, we will also discuss the lower bound of the assumed value for the conversion intensity λ and the fictive assumption on the volatility η of the log capital ratio.

For the risk-free interest rate, we use the term structure from European Overnight Index Swap (EUR-OIS) as a proxy. The EUR-OIS is an overnight fund rate that is published on a daily basis. The overnight fund rate can then be compounded for different maturities and constitutes the term structure. We obtain daily yield curves of EUR-OIS from Bloomberg terminal for the target implementation period, presented in Figure 3. Then we calculate the average EUR-OIS of one to ten years maturity for every implementation date and use the results as the estimated interest rates, which is shown in Table 9.

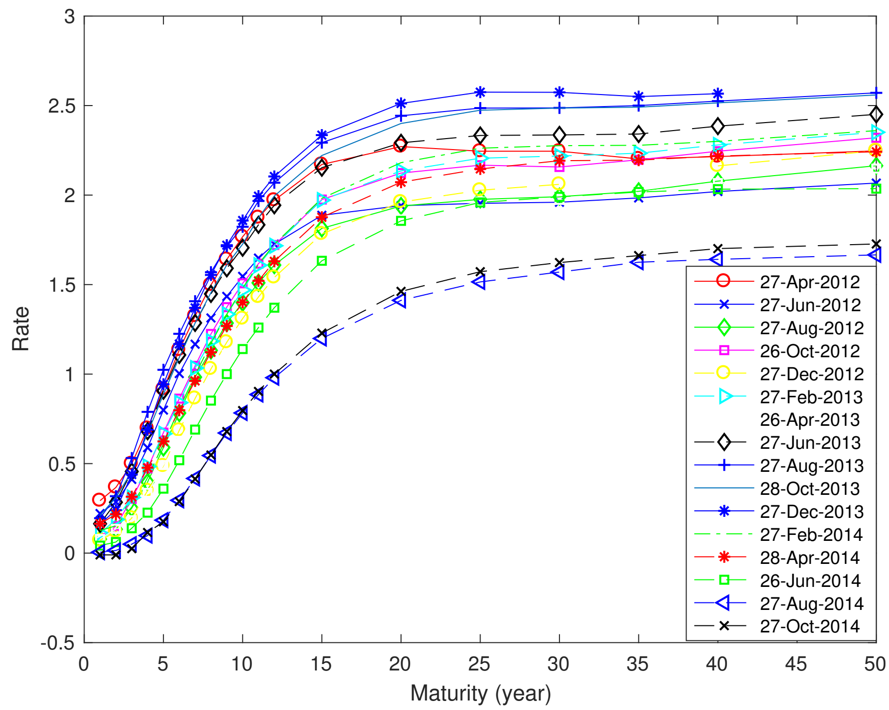

Figure 3: Term Structure

In our numerical implementation, we will first employ a constant conversion intensity λ. As mentioned in the Section 4, the conversion intensity is an arrival intensity which we specify in the Poisson distribution to model the discretionary conversion. In the constant intensity case, we assume that the conversion intensity to be 0.05, which is the same as the assumption in Chung and Kwok (2016). However, before we decide the value for the conversion probability, we examine and make sure that the implied default intensity from credit default swaps (CDS) does not exceed the value we assume for the conversion intensity. As Chung and Kwok (2016) discuss in their numerical implementation, the event of the conversion always happens before the default, since the purpose of a CoCo bond is to prevent the situation where the bank defaults. Intuitively, the default intensity will be lower than the conversion intensity since the discretionary decision of the regulator is more likely to occur than the default does. In order to make sure that the assumed value of 0.05 for the conversion intensity is higher than the actual market implied default intensity, we calibrate the implied default intensity $\lambda_{default}$ from a CDS written on Credit Suisse, which is given by $\lambda_{default} = 4ln[\frac{CDS}{4(1-\phi)} + 1]$ as in Herbertsson and Frey (2016) and Herbertsson (2016), where we set that the recovery rate ϕ is 0.3 as in Chung and Kwok

(2016). During April 2012 to October 2014, the one year CDS is traded at spreads ranging from 115.2 basis points to 8 basis points, which results in very small default intensities. Therefore, we can assume that the conversion intensity to be 0.05, which roughly translates into a spread of 350 basis points.

The standard deviation σ of the stock price is estimated from the historical movement of Credit Suisse stock price. In order to obtain the estimated standard deviation of the stock price for each of the implementation dates, we observe the previous ten years daily stock prices and calculate the daily volatility of the share price. Moreover, we calculate the yearly volatility by scaling up the daily volatility with the number of trading days in a year. The results of the rolling estimation of the standard deviation are around 19% for the period from April 2012 to October 2014, which are shown in Table 9. Figure 4 presents the movement of Credit Suisse stock prices between year 2002 and year 2017.

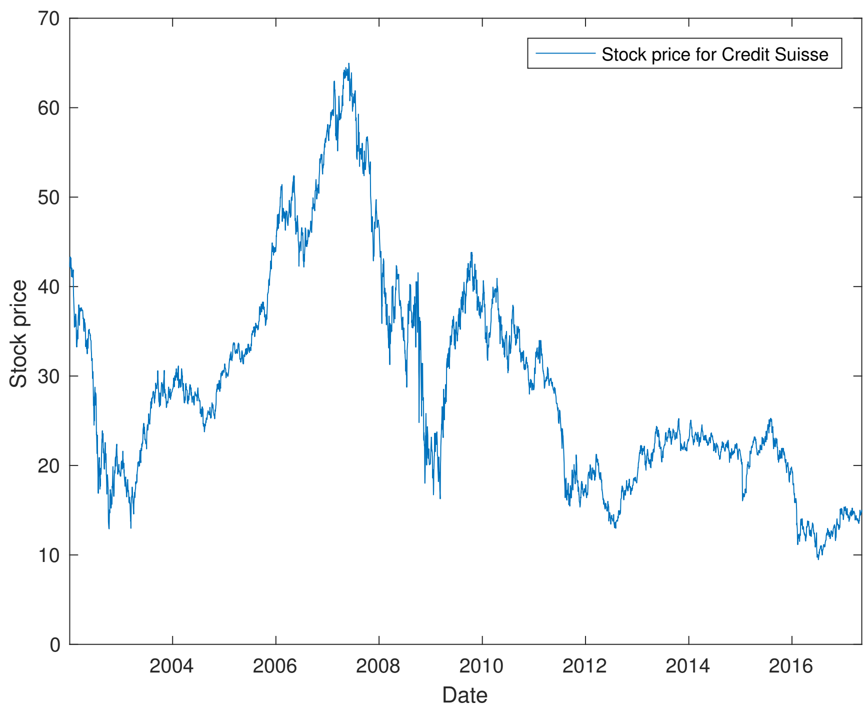

Figure 4: Stock Prices for Credit Suisse

The dividend ratio is collected from the average yearly dividend yield of the year before the observation time. The dividend yields between 2012 and 2014 are presented in Table 9. We assume that the investors only have access to the dividend yield from the previous year.

Since Basel Committee has been tightening the capital regulation during the last decade, the capital ratio is expected to be growing due to the regulation. The historical capital ratio data of Credit Suisse is not available before 2012 on Bloomberg. We use the historical data from 2012 to present time (2017) to estimate the volatility of the log capital ratio. The real data is constant growing from 7% to 13% during this time frame. The standard deviation is rather low since the movement is small and the growing is consistent. The result of the volatility of log capital ratio (η in Equation (2)) during this period is nearly zero. We then assume a reasonable value of 13%. We will discuss more the sensitivity of the volatility of log capital ratio η in Section 16.

5.3 Parameter calibration

There are in total nine parameters we will use for the implementation of the equity-credit model by Chung and Kwok (2016) and we have obtained five values of parameters (r, σ, q, λ and η) in Subsection 5.2. Now we present the procedure of the calibration for the last four unknown parameters, which are the speed of mean reversion κ, the long-term mean capital ratio level θ, the correlation coefficient between the stock price and the capital ratio ρ and the loss proportion of the stock price γ. We will obtain the numerical values of κ, θ, ρ and γ by calibrating the CoCo-model prices against the corresponding market data. We remind the reader that we only use the constant intensity model. The same calibration procedure can also be applied for the state-dependent intensity model and the extended model with a stochastic interest rate.

The calibration is conducted by minimizing the squared error between the model prices and the actual market prices. This estimation is essentially a process of tests for all possible values within a given range, plugging them into the pricing model and looking for the smallest squared error for market prices of CoCo. According to Herbertsson (2016), the calibration procedure is given by:

$$\{\kappa,\theta,\gamma,\rho\} = \underset{\hat{\kappa},\hat{\theta},\hat{\gamma},\hat{\rho}}{\arg\min}\left(\frac{P_{CoCo}(\hat{\kappa},\hat{\theta},\hat{\gamma},\hat{\rho}) - P_M}{P_M}\right)^2, \tag{18}$$

where $P_{CoCo}(\hat{\kappa},\hat{\theta},\hat{\gamma},\hat{\rho})$ is the model price as function of the parameters $\hat{\kappa}$, $\hat{\theta}$, $\hat{\gamma}$, $\hat{\rho}$ and

$P_{CoCo}(\hat{\kappa}, \hat{\theta}, \hat{\gamma}, \hat{\rho})$ is given by Equation (6), while P_M is the market price of the corresponding CoCo. Moreover, the model price $P_{CoCo}(\hat{\kappa}, \hat{\theta}, \hat{\gamma}, \hat{\rho})$ is generated by 1,000 Monte Carlo simulation which will be discussed more in Subsection 5.4

We repeat the calibration procedure 16 times in the period 2012-2014 where the calibration dates are chosen to be either 26^{th}, 27^{th} or 28^{th} bimonthly. The stock prices and capital ratios for Credit Suisse are shown in Table 2, which are used as initial values for the Monte Carlo simulations. Figure 5 presents the historical price movement of Credit Suisse CoCo bond from 2012 to 2017. The market CoCo price is an important factor for our result of parameter calibration. We will discuss the relationship between the model price and the market price of the contingent convertible bond in Section 6. The result of the calibration will also be reported in Section 6.

Table 2: Credit Suisse Description

Date	CoCo price	Stock price	Capital ratio (%)
27-Apr-2012	101.78	17.31	7.50
27-Jun-2012	98.37	13.89	7.50
27-Aug-2012	101.47	14.90	7.50
26-Oct-2012	105.71	17.15	7.50
27-Dec-2012	107.69	18.25	7.50
27-Feb-2013	107.51	20.47	8.00
26-Apr-2013	110.44	21.75	8.60
27-Jun-2013	104.69	21.00	8.60
27-Aug-2013	106.32	23.00	9.30
28-Oct-2013	106.91	23.90	10.20
27-Dec-2013	106.58	22.92	10.20
27-Feb-2014	110.50	22.98	10.00
28-Apr-2014	110.50	22.89	10.00
26-Jun-2014	109.40	21.63	10.00
27-Aug-2014	107.33	22.45	9.50
27-Oct-2014	106.95	20.81	9.80

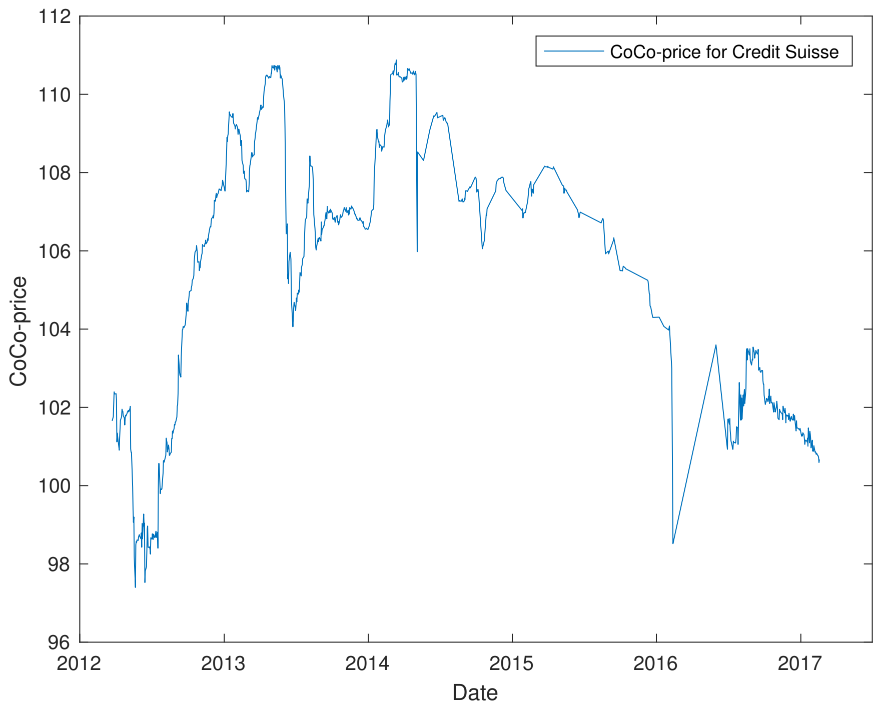

Figure 5: Historical Prices of Credit Suisse CoCo bond

Before the calibration procedure starts, we need to set a start value and a range for every parameter. Here we only decide the start values for the first observation date, while for the rest of the days, we use the market implied values from previous day as the start values and the same ranges as the first day. Table 3 presents the detail of the calibration setting for the first implementation date. For the speed of reversion κ, we allow the values to range from 0.001 to 0.9 and assumes the start value to be 0.2 as in Chung and Kwok (2016), which indicates that it takes five years for the capital ratio to move back to its mean. The start value of the long-term mean of the capital ratio θ is set to be the log value of 13%, which is close to the level of the capital ratio in 2017 and its range is from the log value of 5% to 20%. We assume the start value of correlation coefficient ρ to be 0.5, ranging from -1 to 1. The start value for the proportion of the loss proportion of the stock price γ is set to be -0.7. Both start values of the correlation coefficient ρ and the loss proportion of the stock price γ follow the same assumption as in Chung and Kwok (2016).

Table 3: Calibration Set-up

	Value			
Parameter	κ	θ	ρ	γ
Start Value	0.2	-2,0402	0.5	-0.7
Upper Bound	0.9	-1.6094	1	-0.05
Lower Bound	0.001	-2.9957	-1	-1

5.4 Monte Carlo simulation

Monte Carlo simulation is the major method applied to the numerical implementation in this thesis. The framework of the Monte Carlo simulation used here is very similar to the one we apply when repeating the model of Chung and Kwok (2016) and only the parameters and the values of data regarding the Credit Suisse CoCo bond are changed. In this subsection, we briefly summarize the actual implementation process of the Monte Carlo simulation for the constant conversion intensity model.

The values of the log stock price x_t in Equation (3), the log capital ratio y_t in Equation (4) and the interest rate r_t in Equation (17) are simulated with 10,000 paths, each with a time step of $\frac{1}{365}$, where the time is measured in years, so a step of $\frac{1}{365}$ implies that we simulate the stock price, the capital ratio and the instantaneous interest rate on a daily basis. In the simulation we use the Euler-Maruyama method see e.g. in Kloeden and Platen (1995) applied to Equation (3), (4) and (17), that is, if $\triangle W_{t_n} = W_{t_n} - W_{t_{n-1}}$, $\triangle N_{t_n} = N_{t_n} - N_{t_{n-1}}$ and $\triangle t_n = t_n - t_{n-1}$, then these three processes are approximated with discrete entities according to the following equations:

$$x_{t_n} = x_{t_{n-1}} + (r - q - \frac{\sigma^2}{2} - \gamma\lambda) \triangle t_n + \sigma \triangle W^1_{t_n} + \ln(1+\gamma) \triangle N_{t_n}.$$

$$y_{t_n} = y_{t_{n-1}} + \kappa(\theta - y_{t_{n-1}}) \triangle t_n + \eta \left(\rho \triangle W^1_{t_n} + \sqrt{1-\rho^2} \triangle W^2_{t_n} \right),$$

$$r_{t_n} = r_{t_{n-1}} + \zeta(\psi - r_{t_{n-1}}) \triangle t_n + \omega \sqrt{r_{t_{n-1}}} \left(\rho_{cir} \triangle W^2_t + \sqrt{1-\rho^2_{cir}} \triangle W^{(r)}_t \right).$$

The process N_t is generated by simulating τ_R as $\tau_R = \frac{E^{(1)}}{\lambda}$ where $E^{(1)} \sim Exp(1)$ and thereafter randomly generating the Poisson distributed indicators for each time step, so N_t

is then a series of indicators that are either 1 or 0 and $N_t = 1$ indicates if there has been a discretionary trigger conversion at time t. Furthermore, $N_t = 0$ indicates that there has not been a conversion up time t.

5.4.1 Simulate the conversion time τ

After all daily stock prices, capital ratios and interest rates are simulated, we can find the conversion time for all 10,000 simulations. The purpose of this step is to find out when exactly the conversion happens and calculate the conversion probability according to the simulated conversion time.

When the book-value trigger occurs and the CoCo bond is converted, the time of conversion is τ_B, given by:

$$\tau_B = \inf\{\, t \geq \ 0;\ y_t = y_B\}.$$

Since all the simulated capital ratios in all the time steps are known, τ_B is also known. We can thus record τ_B which is the first time that the capital ratio is equal or less than the trigger level.

The same procedure applies to finding τ_R which is the first time $N_t = 1$, indicating that the discretionary trigger is activated, so τ_R is recorded for each simulation path, given by:

$$\tau_R = \inf\{\, t \geq \ 0; N_t = 1\}.$$

Then we choose the first event of the two random variables τ_R and τ_B and record it as the time of the conversion τ, that is $\tau = \min(\tau_B, \tau_R)$. We identify the time of conversion for each simulation path.

5.4.2 Simulate the conversion value P_E

Let M be the number of MC-simulation and P_{E_1}, P_{E_2}, ..., P_{E_M} be independent stochastic variables, given by:

$$P_{E_i} = e^{-r\tau_i} G S_{\tau_i} 1_{\{\tau_i \leq T\}}$$

where $\tau_i = \min(\tau_{B,i}, \tau_{R,i})$ and $\tau_{B,i}$ is the first time that the capital ratio hits the trigger level in the i_{th} simulation and $\tau_{R,i}$ is the first time that N_t jumps in the i_{th} simulation.

Furthermore, P_{E_i} is the conversion values for one single simulated path. Next, and let $\overline{P}_{E_M}$ be the sample mean of P_{E_1}, P_{E_2}, ..., P_{E_M}, i.e. $\overline{P}$., that is:

$$\overline{P}_{E_M} = \frac{1}{M}\sum_{i=1}^{M} P_{E_i}.$$

Then $\overline{P}_{E_M}$ is an MC-estimator of the conversion value P_E in Equation 9 and by the law of large numbers, $\overline{P}_{E_M}$ will be close to P_E when $M \to \infty$, i.e. $P\Big[\mid \overline{P}_{E_M} - P_{E_i} \mid \to 0$ as $M \to \infty\Big] = 1$.

5.4.3 Simulate values of P_C and P_F

Let $P_C^{(1)}$, $P_C^{(2)}$, ..., $P_C^{(M)}$ be independent stochastic variables and c_j as the j_{th} coupon at time $t_j^{(c)}$, given by:

$$P_C^{(i)} = \sum_{j=1}^{n} e^{-rt_j^{(c)}} c_j 1_{\{\tau_i > t_j^{(c)}\}}.$$

Here, $j = 1,2,3,...,10$ and $P_C^{(i)}$ is the value of all the coupons for the simulated path i. If the conversion time happens after each coupon date, the coupon will be received by the bond holder and be discounted. If the conversion time happens before the coupon date, the coupon is set to be zero. We define $\overline{P}_{C_M}$ as the sample mean value of P_{C_1}, P_{C_2}, ..., P_{C_M}, that is:

$$\overline{P}_{C_M} = \frac{1}{M}\sum_{i=1}^{M} P_C^{(i)}$$

where $\overline{P}_{C_M}$ is the MC-estimator of P_C in Equation (7).

Let P_{F_1}, P_{F_2}, ..., P_{F_M} be independent stochastic variables, given by:

$$P_{F_i} = Fe^{-rT}\ 1_{\{\tau_i > T\}}$$

where P_{F_i} is the value of the principal payment for the simulated conversion time τ_i in simulation number (i) and T is the maturity. Let $\overline{P}_{F_M}$ be the sample mean of P_{F_1}, P_{F_2}, ..., P_{F_M}, that is:

$$\overline{P}_{F_M} = \frac{1}{M}\sum_{i=1}^{M} P_{F_i}.$$

So $\overline{P}_{F_M}$ is an MC-estimator of the conversion value P_F in Equation (8).

Since we also incorporate the stochastic interest rate in our pricing model, the values of P_E, P_C and P_F should be discounted with the simulated interest rate r_t as in Chung and Kwok (2016), given by:

$$P_E = GS_0\mathbb{E}^Q\left[e^{-q\tau}\frac{e^{-\int_0^\tau r_s\,ds}\,e^{q\tau}\,S_\tau}{S_0}\,1_{\{\tau\leq T\}}\right],$$

$$P_C = \sum_{i=1}^{n}\mathbb{E}^Q\left[c_i e^{-\int_0^{t_i} r_s\,ds}\,1_{\{\tau>t_i\}}\right],$$

$$P_F = \mathbb{E}^Q\left[Fe^{-\int_0^T r_s\,ds}\,1_{\{\tau>T\}}\right].$$

After finding the conversion time τ_i using MC-simulation for $i = 1,2,...,M$, we multiply the instantaneous interest rate r_{t_j} by Δt_j in all the time steps up to τ_i and add them up, that is,

$$D_{\tau_i}^{(i)} = \sum_{j=1}^{n} r_{t_j}\Delta t_j 1_{\{\tau_i>t_j\}}$$

where $D_{\tau_i}^{(i)}$ denotes the approximation of the integral $\int_0^{\tau_i} r_s\,ds$.

5.5 Parameters for CIR model

As an extension to the model in Chung and Kwok (2016), we add a stochastic interest rate in the equity-credit model, assuming that the interest rate follows a CIR model and is correlated with the capital ratio. In order to calculate the stochastic interest rates by Equation (17), we need to obtain the parameters in the model, including the speed of mean-reversion ζ, the long-term interest rate ψ, the standard deviation of the interest rate ω and the correlation coefficient between the interest rate and the capital ratio ρ_{cir}. Since there have been extensive researches and implementations with regard to the parameter calibration for the CIR model, in this thesis we will not calibrate the parameters, instead we will use the CIR-parameters estimated as in Episcopos (2000) directly to modelling the stochastic interest rate.

In the study of Episcopos (2000), the term structure in Switzerland has the speed of reversion of 0.0258, which means that it takes around 39 years for the interest rate to be back to the long-term level. The long-term interest rate of Switzerland is 4.26% and the standard deviation of the instantaneous interest rate is 2.65%. We assume the correlation coefficient between the capital ratio and the interest rate to be 0.3, because there are studies that show a strong positive effect of the interest rate on the bank's capital ratio as mentioned in Subsection 4.3.2. A summary of the parameters we employ in the model of the interest rate is shown in Table 4.

Table 4: Parameters for CIR model

Parameter	ζ	ψ	ω	ρ_{cir}
	0.0258	0.0426	0.0265	0.3000

6 Result

In this section we present the results from the implementation of the pricing models discussed in earlier sections. First, Subsection 6.1 reports the results of replicating the pricing process of Chung and Kwok (2016). Then Subsection 6.2 shows the results of our application with Credit Suisse CoCo bond. In Subsection 6.3, we analyze the model prices and calibration errors. Finally, we discuss the conversion probabilities obtained from our pricing process in Subsection 6.4.

6.1 Comparison of our results with results of Chung and Kwok (2016)

Below we present the results of replicating the equity-credit model developed by Chung and Kwok (2016) with both the Fortet method and the Monte Carlo simulation method to confirm our understanding of the model and the implementation.

6.1.1 The Fortet method

We replicate the numerical implementation for calculating the conversion value P_E with the Fortet method in Chung and Kwok (2016). The numerical Fortet method is implemented with a time step of $\Delta t = 0.0001$ for the maturity of one and two years, while for maturity of three, four and five years, the time step is as $\Delta t = 0.001$. We apply this setting of time steps, because the size of the matrices generated during the Fortet calculation is rather large, which results in a significant time consumption during the numerical calculation. We test both the time steps of $\Delta t = 0.0001$ and $\Delta t = 0.001$ for the maturity of one and two years and the results of applying the different time steps are almost the same. Since the focus of this thesis is not to test the exact precision of the numerical implementation, we decide to use a larger time step for prices of the maturity of three, four and five years.

As can be seen in Table 5, by applying the exactly same parameters and the pricing model as Chung and Kwok (2016), we obtain the exactly same results for the conversion values P_E with the maturity of one and two years. For the maturity of three, four and five years, the numerical results are very close to the results from Chung and Kwok (2016), even though the time step is ten times bigger than the size of the time step in the study of

Chung and Kwok (2016).

Table 5: Constant conversion intensity: comparing results of the conversion values P_E of Chung and Kwok (2016) with our results.

Chung and Kwok (2016)			Our results		
Time	Fortet	Monte Carlo	Time	Fortet	Monte Carlo
(i) capital ratio = 6%			(i) capital ratio = 6%		
1	37.76	37.42	1	37.76	37.12
2	46.61	46.11	2	46.61	46.26
3	51.09	50.67	3	51.06	50.31
4	54.08	53.68	4	54.05	54.04
5	56.35	55.87	5	56.32	54.86
(ii) capital ratio=8%			(ii) capital ratio=8%		
1	6.63	6.46	1	6.63	6.66
2	14.53	14.42	2	14.53	14.08
3	20.03	19.88	3	20.01	19.08
4	24.19	24.00	4	24.18	23.02
5	27.57	27.60	5	27.56	26.29
(iii) capital ratio=10%			(iii) capital ratio=10%		
1	2.24	2.23	1	2.24	2.21
2	6.64	6.66	2	6.64	6.27
3	10.95	10.92	3	10.94	10.35
4	14.73	14.73	4	14.72	14.31
5	18.06	17.85	5	18.05	18.25

6.1.2 Monte Carlo simulations

We also employ the same parameters and replicate the pricing model as in Chung and Kwok (2016) with Monte Carlo simulation method. The results are presented in Table 5, Table 6 and Table 7, which only includes one component of the CoCo price, i.e. the conversion value P_E. The simulation is conducted 10,000 times for the maturity of one, two and three years. The results are very close to those presented in Chung and Kwok (2016) and the difference mainly comes from the randomness of the simulation. We also test the results by only applying 1,000 simulation paths. The results are very close to the results with 10,000 times simulation. Thus we set the simulation paths to be 1,000 for the maturity of four and five years to reduce the process time.

The results of the conversion value P_E from the constant intensity pricing model implemented with Monte Carlo simulation are presented in Table 5. Furthermore, Table 6 and Table 7 show the results of the conversion value P_E from the state-dependent intensity model. Table 6 incorporates the intensity dependency on the movement of the stock price. However, the conversion intensity in Table 7 is dependent on both the movement of the stock price and the capital ratio. The complete interpretation and modelling for the state-dependent model is presented in Subsection 4.2.2. We can see that despite the difference in the number of simulation paths, the results are very close to our results obtained with the Fortet method and the corresponding results in Chung and Kwok (2016). The differences are merely due to the randomness of the Monte Carlos simulation and this comparison leads to some confidence that our implementation is the same as in Chung and Kwok (2016).

Table 6: State-dependent intensity: comparing results for the conversion value P_E of Chung and Kwok (2016) with our results. The parameters are the same as in Chung and Kwok (2016) and the state-dependent intensity is specified as $\lambda(x) = \exp(a_0 - a_1 x)$, $a_1 = 0.5$ and a_0 is set so that $\lambda(x_0) = 0.05$.

State- dependent P_E

Chung and Kwok (2016)		Our Results	
Time	Monte Carlo	Time	Monte Carlo
(i) capital ratio = 6%		(i) capital ratio = 6%	
1	37.17	1	37.46
2	46.03	2	45.32
3	50.38	3	49.23
4	53.30	4	52.93
5	55.40	5	55.49
(ii) capital ratio=8%		(ii) capital ratio=8%	
1	6.51	1	6.48
2	14.14	2	13.97
3	19.66	3	18.90
4	23.62	4	22.82
5	26.80	5	26.54
(iii) capital ratio=10%		(iii) capital ratio=10%	
1	2.17	1	2.18
2	6.39	2	5.97
3	10.66	3	10.33
4	14.15	4	13.74
5	17.38	5	17.12

Table 7: State-dependent: comparing results for the conversion value P_E from Chung and Kwok (2016) and our results with state-dependent intensity set as $\lambda(x,y) = \exp(a_0 - a_1 x) + b_0 1_{\{y \leq y_{RT}\}}$. Here, $a_1 = 0.5$, a_0 is fit to $\lambda(x_0) = 0.05, b_0 = 0.1$ and $y_{RT} = 0.07$.

Chung and Kwok (2016)		Our Results	
Time	Monte Carlo	Time	Monte Carlo
(i) capital ratio = 6%		(i) capital ratio = 6%	
1	38.36	1	37.10
2	47.16	2	45.27
3	51.70	3	48.89
4	54.34	4	53.02
5	56.70	5	54.42
(ii) capital ratio=8%		(ii) capital ratio=8%	
1	6.91	1	6.16
2	14.85	2	13.90
3	20.39	3	18.51
4	24.20	4	23.77
5	27.51	5	25.91
(iii) capital ratio=10%		(iii) capital ratio=10%	
1	2.29	1	2.18
2	6.64	2	6.19
3	10.87	3	10.01
4	14.72	4	13.65
5	17.58	5	16.92

6.1.3 Results of the values of P_C, P_F and P_{CoCo}

Here we present the values of coupon payments P_C, the values of the principal payment P_F and the final CoCo bond prices P_{CoCo} from the pricing models with Monte Carlo simulation in Table 8. The parameter values and the model assumptions are the same as Table 1, 2 and 3 in Chung and Kwok (2016). The values of coupon cash flow P_C is increasing with the maturity. The values of the principal payment P_F is decreasing with the maturity, because the longer the maturity is, the larger the conversion probability becomes in general. The numerical results from both the constant intensity and the state-dependent intensity models are presented in Table 8.

Table 8: The results of the values of coupon payments P_C, the values of the principal payment P_F and the final CoCo prices by the Monte Carlo simulation, with the same model settings as in Chung and Kwok (2016). The state-dependent intensity (1) indicates the dependency on the stock price. The state-dependent intensity (2) indicates the dependency on both the stock price and the capital ratio.

Constant intensity-Monte Carlo				State-dependent intensity (1)				State-dependent intensity (2)			
Time	P_C	P_F	P_{CoCo}	Time	P_C	P_F	P_{CoCo}	Time	P_C	P_F	P_{CoCo}
(i) $Y_0 = 6\%$				(i) $Y_0 = 6\%$				(i) $Y_0 = 6\%$			
1	5.30	53.01	95.43	1	5.27	52.68	95.41	1	5.05	50.50	92.65
2	9.46	40.76	95.75	2	9.36	40.53	95.20	2	8.64	36.89	90.80
3	7.38	32.75	90.43	3	12.74	33.40	95.36	3	11.74	30.28	90.91
4	14.85	26.22	95,11	4	15.00	27.23	95.17	4	13.64	23.82	90.48
5	18.23	24.16	97.25	5	17.55	23.44	96.48	5	15.96	21.44	91.83
(ii)$Y_0 = 8\%$				(ii) $Y_0 = 8\%$				(ii) $Y_0 = 8\%$			
1	8.59	85.93	101.19	1	8.60	86.00	101.08	1	8.57	85.73	100.46
2	15.86	71.61	101.48	2	15.72	71.13	100.8	2	15.45	69.50	98.85
3	21.83	60.69	101.59	3	21.65	59.92	100.48	3	21.18	57.91	97.60
4	26.83	51.60	101.45	4	26.63	51.05	100.50	4	25.42	47.45	96.64
5	32.09	45.88	104.25	5	31.84	44.61	102.99	5	30.44	43.43	99.79
(iii) $Y_0 = 10\%$				(iii) $Y_0 = 10\%$				(ii) $Y_0 = 10\%$			
1	9.18	91.83	103.23	1	9.19	91.87	103.25	1	9.18	91.78	103.14
2	17.39	81.55	105.21	2	17.3421	81.30	104.62	2	17.21	80.48	103.88
3	24.38	70.92	105.64	3	24.31	70.37	105.01	3	24.08	69.18	103.27
4	30.14	60.74	105.20	4	30.21	61.02	104.97	4	29.76	58.34	101.76
5	35.41	52.48	106.15	5	35.80	52.03	104.95	5	34.44	50.49	101.84

6.2 Results of applying the Credit Suisse CoCo bond

In the following, we present the pricing results from the implementation of the Credit Suisse CoCo bond. In Subsection 6.2.1, we show the calibrated parameters. Furthermore, we report the final CoCo prices in Subsection 6.2.2. The results of conversion values P_E from both the Fortet algorithm and the Monte Carlo simulation are displayed in Subsection 6.2.3. Meanwhile, Subsection 6.2.4 gives a summary of the values of the coupon payments P_C and the values of the principal payment P_F. Moreover, we report the pricing results for the extended model where a stochastic interest rate is incorporated in the constant intensity model in Subsection 6.2.5.

6.2.1 The result of calibrated parameters

We described the method and the procedure of calibrating the parameters against market CoCo prices in Section 5. Here, we present the result of daily calibration from the constant intensity pricing model. The same calibration procedure should be applied for the state-dependent intensity model and the extended model with a stochastic interest rate. However, in this thesis, we only calibrate the parameters from the constant intensity model and use these parameters for all the models that are interpreted. Table 9 shows the results of the calibrated parameters and Figure 6, 7, 8 and 9 display the time series of implied values of each calibrated parameters. As mentioned in Section 5, we calibrate the parameters through Monte Carlo simulations with a step of $\frac{1}{365}$ and the number of Monte Carlo simulation paths is 1,000 for the calibration.

The implied loss proportions of the stock price γ range from -0.9914 to -0.05. Figure 6 shows that the value of the loss portion shifts to -0.05 and moves back to the normal value during August 2012 and December 2012. By the normal value, we refer to the value of loss proportion around -0.7 as Chung and Kwok (2016) assumed. As can be seen from Figure 7, the implied values of speed of mean-reversion κ range around 0.2 to 0.9, which means that once the capital ratio deviates from the long-term mean value, it takes around five years to one year for the capital ratio to be adjusted back to the long-term mean value. As shown in Figure 8, the implied log values of long-term capital ratio θ range from -2.8497 to -2.0402, which indicates the long-term means of the capital ratio range from 5.79% to 13%. The implied long-term capital ratio starts to decrease suddenly in 2014 to a level below the capital ratio requirement and thus below the trigger level of 7%. In fact, the implied long-term capital ratio matches well to the actual capital ratio. As can be seen from Table 2, the actual capital ratio starts to decrease in February 2014. Moreover, while the capital ratio trigger in the CoCo contract is 7%; the actual capital ratios during this period are not so far from the trigger level in 2012. In this case, the market CoCo price decreases in respect of the capital ratio, which leads to the implied long-term capital ratio being low as well. In Figure 9, we can see that the implied correlation coefficients ρ between the capital ratios and the stock prices range between 0.9975 to -0.9957 and shift in 2014 from being positive correlated to negative correlated. We note in particular that the model and data thus imply that from April 2013 to August 2014 the correlation of stock prices and capital ratios will go from perfect positive correlation (i.e. $\rho \approx 1$) to perfect negative correlation, that is $\rho \approx 0$. Table 9 presents that the implied values of different parameters move and shift almost at the same time in 2014. This shift indicates that there are some major changes in the market stock prices, the market CoCo prices and the capital ratios. As mentioned in Subsection 5.2, the interest rates r, the standard deviation of log stock prices σ and the dividend yield q are obtained by simple estimation based on the real market data.

Table 9: Results of parameter calibration

	Parameters						
Time	γ	κ	θ	ρ	r (%)	σ	q
27-Apr-2012	-0.7000	0.2000	-2.0402	0.5000	1.0100	0.1908	0.0520
27-Jun-2012	-0.0516	0.3468	-2.3104	0.1500	0.8655	0.1913	0.0520
27-Aug-2012	-0.0500	0.4009	-2.3610	0.2558	0.7035	0.1915	0.0520
26-Oct-2012	-0.0500	0.3859	-2.3624	0.1238	0.7723	0.1899	0.0520
27-Dec-2012	-0.0500	0.4260	-2.3410	0.2328	0.6234	0.1840	0.0520
27-Feb-2013	-0.0500	0.4260	-2.3408	0.2323	0.7677	0.1787	0.0407
26-Apr-2013	-0.7516	0.4263	-2.2397	0.9975	0.5957	0.1763	0.0407
27-Jun-2013	-0.7233	0.4231	-2.2436	0.9975	0.9554	0.1750	0.0407
27-Aug-2013	-0.7233	0.4231	-2.2436	0.9975	1.0458	0.1717	0.0407
28-Oct-2013	-0.7223	0.4219	-2.2411	0.9975	0.9501	0.1708	0.0407
27-Dec-2013	-0.7223	0.4219	-2.2411	0.9975	0.9862	0.1708	0.0407
27-Feb-2014	-0.7003	0.7861	-2.4841	0.9973	0.7100	0.1704	0.0306
28-Apr-2014	-0.9398	0.8998	-2.8497	-0.0106	0.7282	0.1701	0.0306
26-Jun-2014	-0.9398	0.8998	-2.8497	-0.0105	0.4987	0.1699	0.0306
27-Aug-2014	-0.9914	0.9000	-2.6374	-0.9957	0.2980	0.1694	0.0306
27-Oct-2014	-0.9914	0.9000	-2.6374	-0.9957	0.2922	0.1694	0.0306

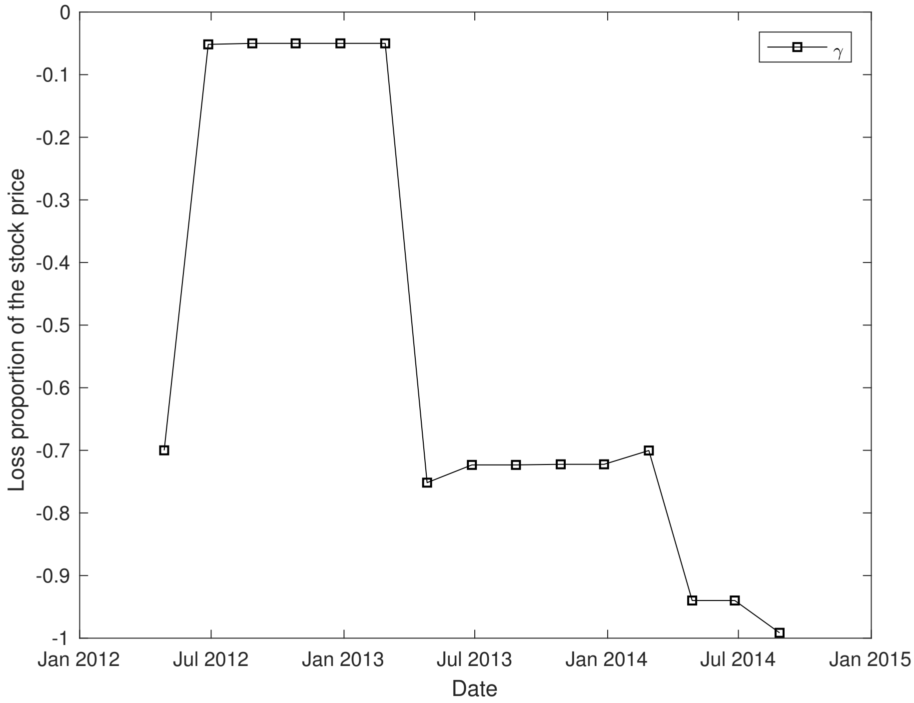

Figure 6: The bimonthly implied loss portion of the stock price γ.

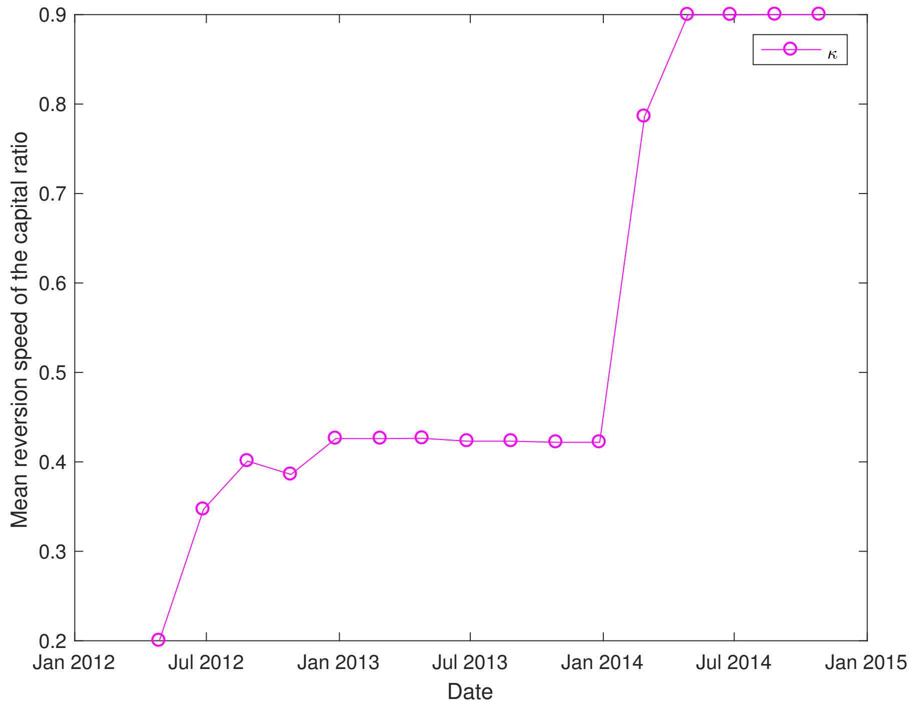

Figure 7: The bimonthly implied speed of reversion of capital ratio κ.

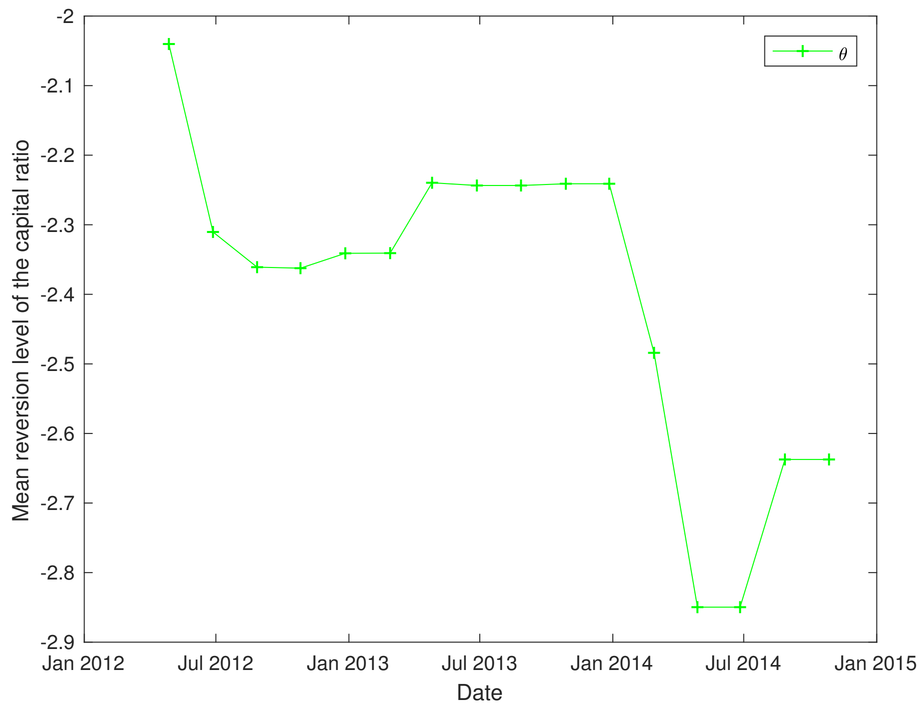

Figure 8: The bimonthly implied mean level of long-term capital ratio θ.

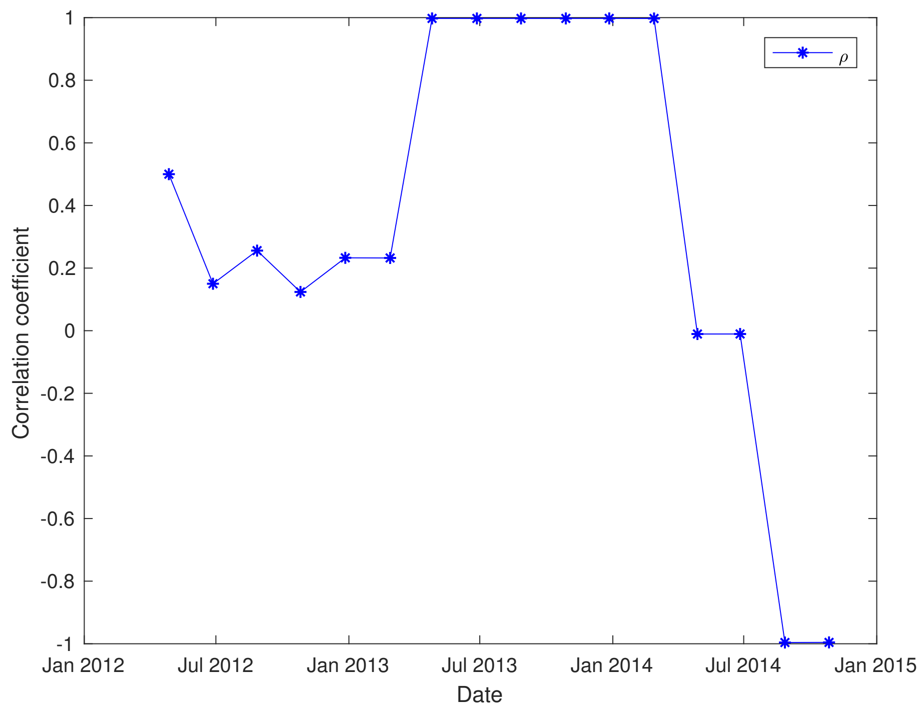

Figure 9: The bimonthly implied correlation coefficient between stock prices and capital ratios ρ.

6.2.2 CoCo prices

In order to compute the Credit Suisse CoCo bond prices, here we implement both the constant intensity model and the state-dependent intensity model with Monte Carlo simulations for each observation date, plugging in all the parameters, the stock prices and the capital ratios of Credit Suisse. The state-dependent model applied here is set as $\lambda(x,y) = \exp(a_0 - a_1 x) + b_0 1_{\{y \le y_{RT}\}}$, where $a_1 = 0.5$, a_0 is fit to $\lambda(x_0) = 0.05$, $b_0 = 0.1$ and $y_{RT} = 7.5\%$. As mentioned in Section 5, we use a time step of $\frac{1}{365}$ for Monte Carlo simulations and the number of simulation paths is 10,000. We then calculate the CoCo prices and compare them with the real market CoCo prices, which is presented in Table 10. We will analyze more about the pricing results in Subsection 6.3.

Table 10: Comparison between the market Credit Suisse CoCo bond prices and the prices derived from the pricing models with calibrated parameters.

Monte Carlo

		Model price	
Time	Market price	Constant intensity	State-dependent intensity
27-Apr-2012	101.78	99.24	96.20
27-Jun-2012	98.37	98.63	94.90
27-Aug-2012	101.47	99.68	97.84
26-Oct-2012	105.71	109.06	106.57
27-Dec-2012	107.69	116.59	114.17
27-Feb-2013	107.51	132.27	131.37
26-Apr-2013	110.44	114.55	112.43
27-Jun-2013	104.69	113.15	109.98
27-Aug-2013	106.32	115.75	113.11
28-Oct-2013	106.91	119.42	116.17
27-Dec-2013	106.58	119.94	116.96
27-Feb-2014	110.50	110.30	109.59
28-Apr-2014	110.50	109.63	108.28
26-Jun-2014	109.40	105.62	104.00
27-Aug-2014	107.33	111.65	108.42
27-Oct-2014	106.95	105.94	102.33

6.2.3 The conversion value P_E

In this subsection, we report the results of the conversion value P_E by applying different pricing models and different methods. In the constant conversion intensity case, we compute P_E with both the Monte Carlo simulation method and the Fortet method. However, in the state-dependent conversion intensity case, we only apply the Monte Carlo simulation method. We realize the fact that we are calibrating the parameters and computing the quantities P_C and P_F via Monte Carlo simulations and of course the fortret method for P_E

could be skipped and simply replaced with Monte Carlo simulations for P_E, since we do this for P_C and P_F. However, there could be many risk-management situations where one only wants to do sensitivity studies of the quantity P_E, just as in Chung and Kwok (2016). Then of course, given the parameters, the Fortret method is much better then Monte Carlo simulations when computing different values of P_E as function of different parameters, because it is certainly much quicker.

As can be seen from Table 11, most of the results from applying different schemes are similar to each other. However, the differences in the results for the conversion values P_E between the Monte Carlo simulation method and the Fortet method are noticeably larger than the differences in Table 5 which likely accounts for the randomness of simulation. There are mainly two reasons. On one hand, in Table 11, we use a time step of $\frac{1}{365}$ for both Fortet algorithm and Monte Carlo simulation, while the time step in the study of Chung and Kwok (2016) is $\frac{1}{10,000}$. The size of the time step in our application is almost 30 times larger than the study conducted by Chung and Kwok (2016). The large time step might have an impact on the Fortet algorithm and Monte Carlo simulation. On the other hand, since both methods are approximations of the real stochastic movements, the differences in results might also due to the difference in approximations between the Fortet algorithm and the Monte Carlo simulation method. Nevertheless, the results in Table 11 are satisfactorily close to each other.

Table 11: Comparison between the conversion prices P_E of the Credit Suisse CoCo bond derived from the constant intensity model with Monte Carlo simulation method and Fortet method and the state-dependent intensity model.

	Conversion value P_E		
Time	Constant intensity (Monte Carlo)	Constant intensity (Fortet)	State-dependent intensity
27-Apr-2012	35.20	38.63	35.51
27-Jun-2012	43.25	44.71	44.79
27-Aug-2012	47.87	49.35	48.86
26-Oct-2012	56.61	58.46	57.79
27-Dec-2012	54.61	56.78	55.67
27-Feb-2013	50.60	51.58	49.58
26-Apr-2013	11.81	14.31	11.73
27-Jun-2013	12.44	14.93	12.57
27-Aug-2013	11.78	14.04	11.82
28-Oct-2013	10.83	13.36	11.39
27-Dec-2013	10.38	12.65	10.54
27-Feb-2014	40.44	48.35	39.34
28-Apr-2014	104.91	112.03	103.63
26-Jun-2014	99.43	105.88	97.84
27-Aug-2014	95.80	106.67	92.85
27-Oct-2014	88.02	98.54	84.53

6.2.4 The value of the coupon payments P_C and the principal payment P_F

Table 12 shows the values of coupon payments P_C and the values of the principal payment P_F derived from the constant intensity model and the state-dependent intensity model with Monte Carlo simulation method, respectively. As can be seen, the values of the principal payment P_F are zero or close to zero and the values of coupon payments P_C are very low for the last four observation dates in 2014, so it is very likely that the bond in a very near future will convert to a stock and therefore there are no expected principal payment left and the expected coupon payments are also much less. This result will be explained later in Figure 14 and 15. When the 2-year conversion probability is almost 100% in 2014, the principal payment and the coupon payments will disappear in the near future.

Table 12: Comparison between values of coupon payments P_C and the principal payment P_F of the Credit Suisse CoCo bond derived from the constant intensity model and the state-dependent intensity model with Monte Carlo simulation method by applying the calibrated parameters.

	Constant intensity		State-dependent intensity	
Time	P_C	P_F	P_C	P_F
27-Apr-2012	32.32	31.71	31.45	29.25
27-Jun-2012	29.45	25.94	27.68	22.44
27-Aug-2012	28.47	23.33	27.55	21.44
26-Oct-2012	29.04	23.41	27.80	20.98
27-Dec-2012	33.27	28.71	32.29	26.21
27-Feb-2013	43.70	37.97	44.16	37.63
26-Apr-2013	47.15	55.59	46.98	53.72
27-Jun-2013	46.87	53.85	46.02	51.39
27-Aug-2013	48.45	55.52	47.83	53.47
28-Oct-2013	50.03	58.56	49.14	55.64
27-Dec-2013	50.59	58.97	49.88	56.53
27-Feb-2014	40.63	29.24	40.77	29.48
28-Apr-2014	4.72	0.00	4.66	0.00
26-Jun-2014	6.19	0.00	6.16	0.00
27-Aug-2014	15.22	0.63	15.09	0.48
27-Oct-2014	17.15	0.77	17.06	0.73

6.2.5 The extension model with a stochastic interest rate

We present the model prices of the Credit Suisse CoCo bond by applying the constant intensity model with a stochastic interest rate in this subsection. As mentioned before, we use the calibrated parameters reported in Table 9 and the data regarding the Credit Suisse

CoCo bond shown in Table 2. Table 13 presents the pricing results of the extended model.

Table 13: Model prices of the Credit Suisse CoCo bond derived from the extended model with Monte Carlo simulation method, applying calibrated parameters.

Constant intensity (Monte Carlo)

Time	P_E	P_C	P_F	P
27-Apr-2012	35.86	31.67	28.99	96.52
27-Jun-2012	43.40	28.96	24.70	97.05
27-Aug-2012	48.71	27.90	22.47	99.08
26-Oct-2012	56.94	28.39	22.85	108.18
27-Dec-2012	54.70	33.25	28.91	116.86
27-Feb-2013	50.40	43.80	37.65	131.85
26-Apr-2013	11.59	47.34	53.45	112.37
27-Jun-2013	12.62	46.54	54.26	113.43
27-Aug-2013	11.84	47.71	54.25	113.80
28-Oct-2013	11.08	49.30	58.16	118.53
27-Dec-2013	10.34	49.98	55.77	116.10
27-Feb-2014	41.19	40.28	27.45	108.92
28-Apr-2014	105.23	4.60	0.00	109.83
26-Jun-2014	99.44	6.16	0.00	105.60
27-Aug-2014	95.69	15.33	0.65	111.68
27-Oct-2014	88.62	16.98	0.81	106.41

6.3 Comparing model prices and calibration errors

Figure 10 displays the comparison of the computed prices of the Credit Suisse CoCo bond from different pricing models with the real market prices, including the constant intensity model, the state-dependent intensity model and the extended model with a stochastic interest rate. We can see that the model CoCo prices and the market CoCo prices are very close to each other for the year 2012 and 2014, while in year 2013 the model prices are overestimated compared to the actual market prices. Figure 11 shows the historical daily prices of Credit Suisse CoCo bond from 2012 to 2014. Comparing Figure 10 with Figure 11, the model CoCo prices follow the same trend as the historical prices of Credit Suisse CoCo bond.

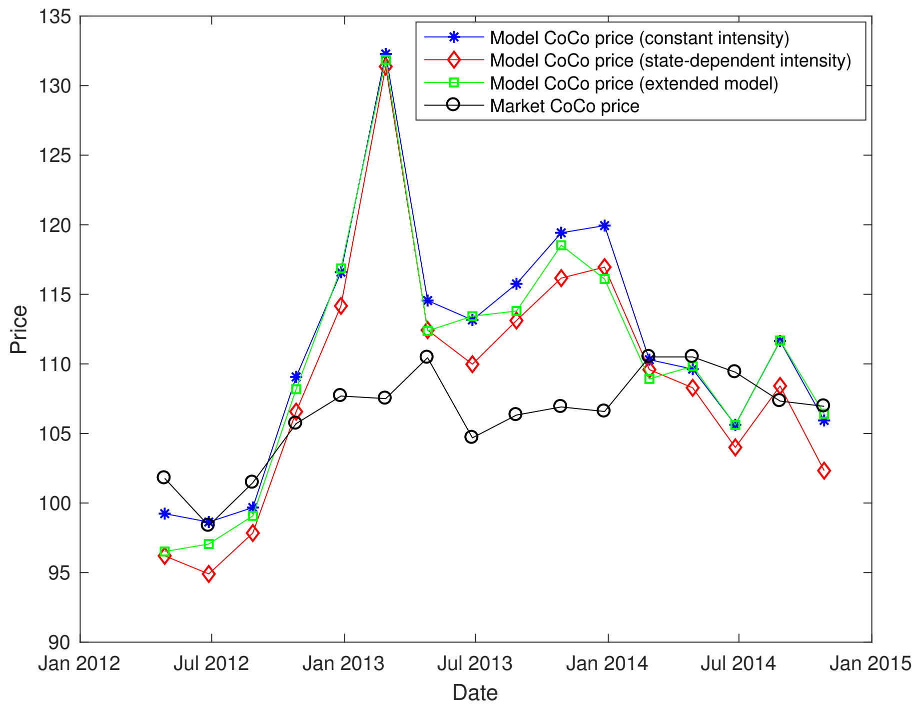

Figure 10: Comparison between the model CoCo prices and the market CoCo prices.

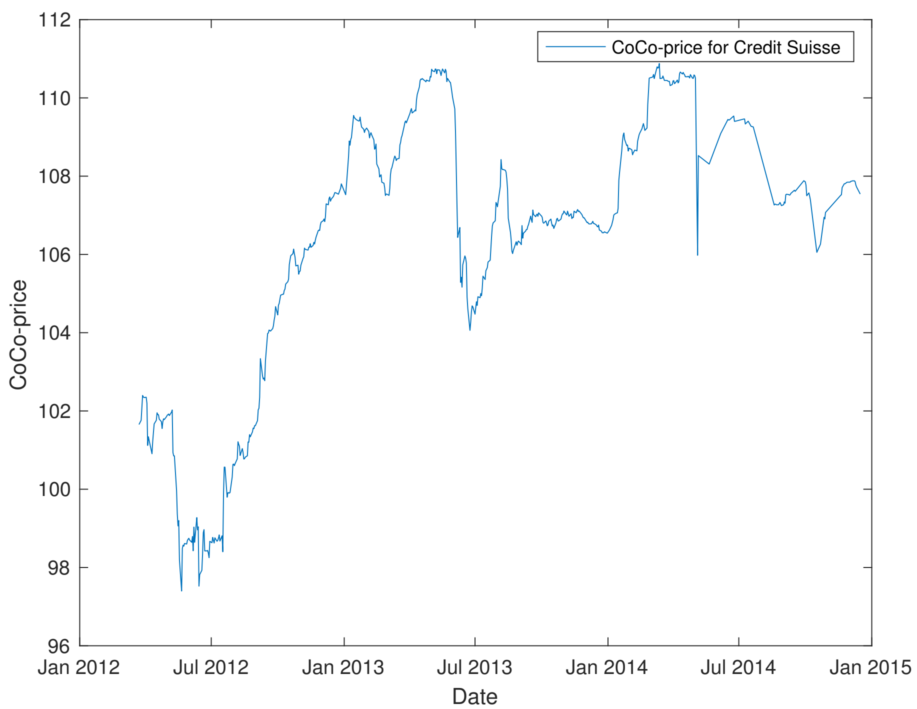

Figure 11: Historical Prices of Credit Suisse CoCo bond

As can be seen from Figure 11, there is a drastic increasing trend on the real market CoCo prices in year 2013. The higher market CoCo prices in year 2013 that reflect on the implied parameters in turn results in higher computed CoCo prices in our model. Figure 4 in Subsection 5.2 shows that the market stock prices were rising sharply in 2013 and Table 2 displays that the real capital ratios in 2013 were at its highest level during the entire implementation period. With all the combining market data trending upwards that intuitively would increase the price of a CoCo, our model might have overestimated the effect of these market factors.

In the following part of this subsection, we compare the calibration error between the three calibrated models. The calibration error is the difference in prices when comparing the model prices with the actual market prices. Figure 12 displays the absolute value of calibration error, while in Figure 13, we present and compare the calibration error in percentage.

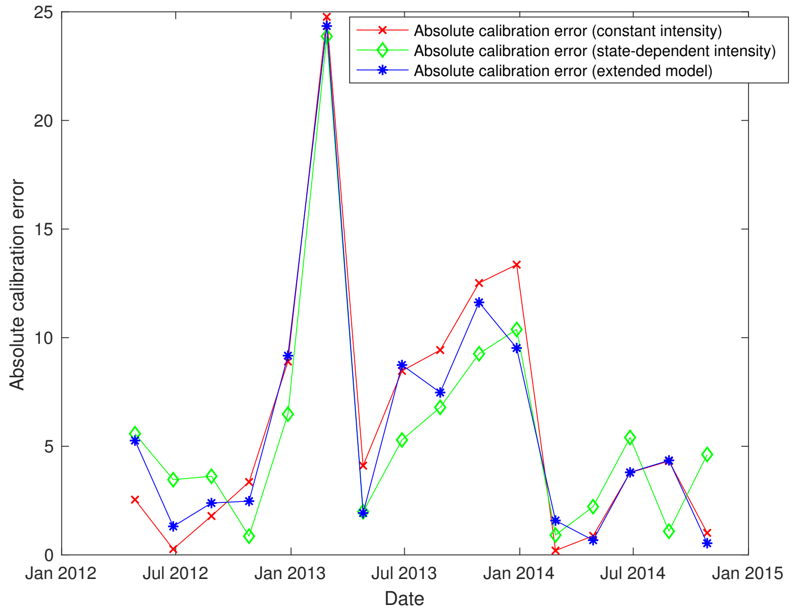

Figure 12: Calibration error from three different models: constant intensity, state-dependent and the extended model.

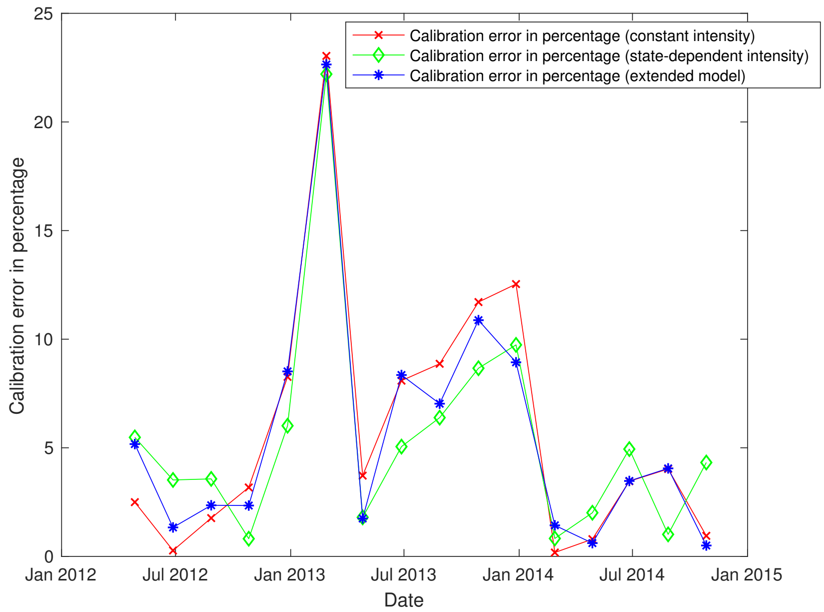

Figure 13: Calibration errors in % from three different models: the constant intensity, the state-dependent intensity and the extended model

6.4 Implied conversion probabilities

In this subsection, we compute the implied probability of a conversion of the CoCo bond into equity in three different pricing models by applying the estimated parameters. The conversion probability presented here is calculated under the risk neutral measure. Note that these probabilities can not directly be observed on the market and is therefore only available via a model. Furthermore, we also believe these conversion probabilities are highly important for CoCo investors when assessing the overall risk of a contingent convertible. Initially, the maturity for calculating the conversion probabilities is shrinking, which is the duration from the observation date to the maturity of the Credit Suisse CoCo bond that is 22 March 2017. Table 14 and Figure 14 displays the results of conversion probabilities with a shrinking maturity, that is, the maturity will be $10-t$ where t is time after issuance (see Table 1). Then, we set the maturity to be constant and equal to two years. The implied conversion probabilities with fixed two-year maturity are shown in Table 15 and Figure 15. In order to observe the effect of the model on the implied conversion probability, we present two lines in Figure 18 and 19, respectively. One shows the difference of the implied conversion probability obtained from the state-dependent intensity model and the constant intensity model, that is $Prob_{sta-dep} - Prob_{con}$. The other line displays the difference of the implied conversion probability computed by the extended model with a stochastic interest rate and the constant intensity model, that is $Prob_{ext} - Prob_{con}$.

For the conversion probability with decreasing maturity, there are three factors that affect its value. First, since the conversion probabilities have shrinking maturities, the larger the maturity $10-t$ is, the larger is the probability, keeping everything else fixed. Second, the probabilities of conversion are impacted by different sets of underlying parameters. Third, the earliest call-back date is within three to five years, which is incorporated in the market CoCo price and therefore reflected in the conversion probabilities via the calibration. Because of the above three factors, it would be misleading to compare the conversion probabilities with each other. Therefore we also report the fixed two-year implied conversion probabilities so that the implied conversion probabilities with different parameters and market data can be compared to each other and there would be no risk of call-back.

The implied conversion probability is an important pricing factor that cannot be directly observed from the market, which might be a crucial tool for the purpose of risk management or pricing other instruments. As can be seen from Figure 15 and 19, the state-dependent intensity model results in a higher conversion probability of two years maturity, compared with the constant intensity model, which is probably because the additional dependency of the conversion intensity on the stock price and the capital ratio increases the probability of conversion. When the stock price increases, the conversion intensity will decrease. When the capital ratio increases, the conversion intensity will not change. However, if the capital ratio falls below the warning level y_{RT} of 7.5%, the conversion intensity will increase with a constant amount.

As can be seen from Figure 15, there is a sharp increase in the conversion probability from around 48% to 100% in the beginning of 2014, which might be caused by the sudden decrease of the stock price in January of 2014 shown in Figure 16 and the sharp drop of the capital ratio during the same period shown in Figure 17. This also explains the almost zero values of the principal payment P_F in Table 12 and 13 for the period April-October

2014. On one hand, the decreasing capital ratio will increase the conversion probability. The lower the capital ratio is, the larger the possibility of hitting the trigger threshold is. On the other hand, since the conversion probabilities in Figure 15 are computed from the pricing models, using the market implied parameters which reflect the market view of the financial situation Credit Sussie, when the stock price and the capital ratio vary, the calibrated parameters will be affected and thus in turn results in implied conversion probabilities that match the movement of the stock prices and the capital ratios.

Table 14: Implied conversion probability derived from the constant intensity pricing model, the state-dependent intensity pricing model and the interest rate extended pricing model.

Implied conversion probability

Time	Constant intensity	State dependent intensity	Interest rate extension
27-Apr-2012	0.6495	0.6767	0.6532
27-Jun-2012	0.7178	0.7559	0.7171
27-Aug-2012	0.7502	0.7706	0.7546
26-Oct-2012	0.7483	0.7744	0.7581
27-Dec-2012	0.6958	0.7223	0.6942
27-Feb-2013	0.5926	0.5965	0.5875
26-Apr-2013	0.4138	0.4335	0.4085
27-Jun-2013	0.4145	0.4411	0.4088
27-Aug-2013	0.3926	0.4150	0.3951
28-Oct-2013	0.3655	0.3974	0.3683
27-Dec-2013	0.3604	0.3866	0.3604
27-Feb-2014	0.6903	0.6877	0.6944
28-Apr-2014	1.0000	1.0000	1.0000
26-Jun-2014	1.0000	1.0000	1.0000
27-Aug-2014	0.9936	0.9951	0.9931
27-Oct-2014	0.9921	0.9925	0.9916

Table 15: The 2 years implied conversion probability derived from the constant intensity pricing model, the state-dependent intensity pricing model and the interest rate extended pricing model.

Implied 2 years conversion probability

Time	Constant intensity	State dependent intensity	Interest rate extension
27-Apr-2012	0.4291	0.4372	0.4252
27-Jun-2012	0.4673	0.4713	0.4721
27-Aug-2012	0.4842	0.4963	0.4861
26-Oct-2012	0.4970	0.4994	0.5035
27-Dec-2012	0.4461	0.4527	0.4481
27-Feb-2013	0.2548	0.2567	0.2598
26-Apr-2013	0.1305	0.1326	0.1329
27-Jun-2013	0.1335	0.1347	0.1316
27-Aug-2013	0.1088	0.1078	0.1090
28-Oct-2013	0.0951	0.0976	0.0973
27-Dec-2013	0.0952	0.1043	0.1000
27-Feb-2014	0.1568	0.1593	0.1566
28-Apr-2014	0.9653	0.9684	0.9632
26-Jun-2014	0.9674	0.9674	0.9682
27-Aug-2014	0.5449	0.5483	0.5607
27-Oct-2014	0.5064	0.5289	0.5097

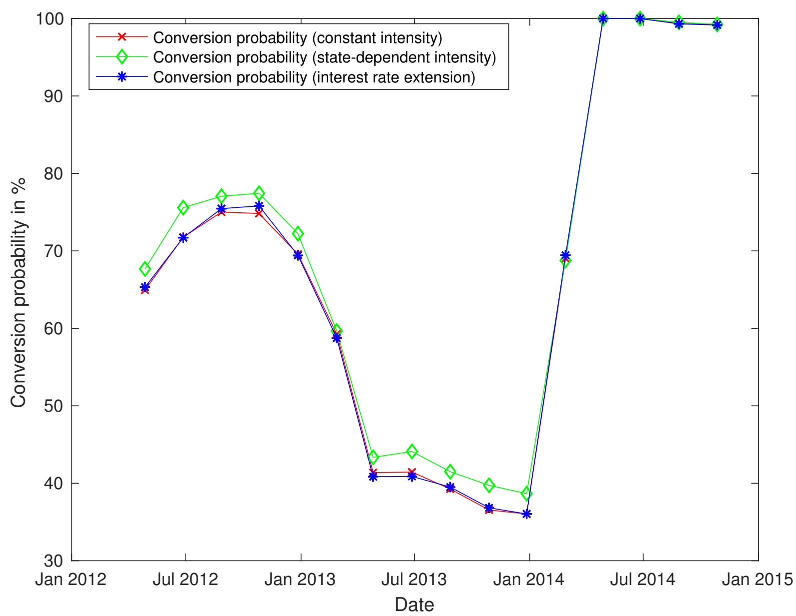

Figure 14: Implied conversion probability in % with the actual time to maturity from three different models: the constant intensity, the state-dependent and the extended interest rate model

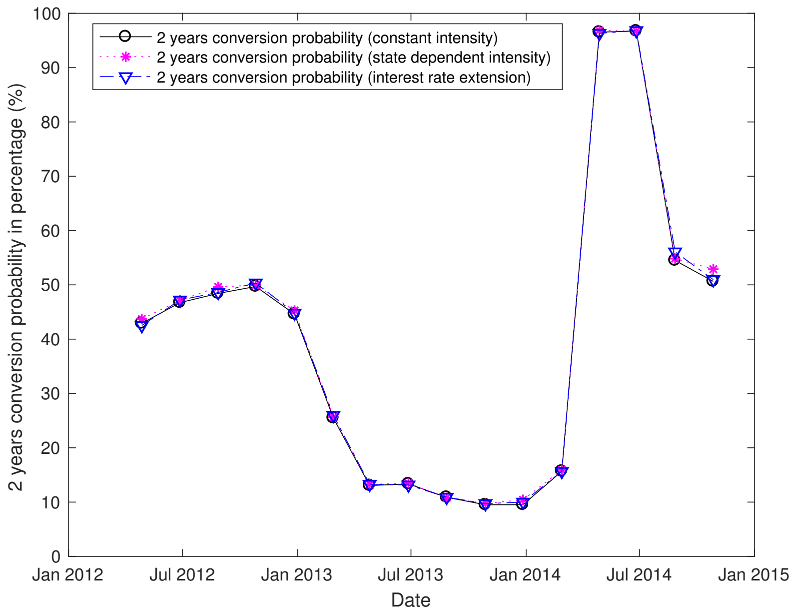

Figure 15: Implied conversion probability in % with maturity of 2 years from three different models: the constant intensity, the state-dependent and the extended interest rate model

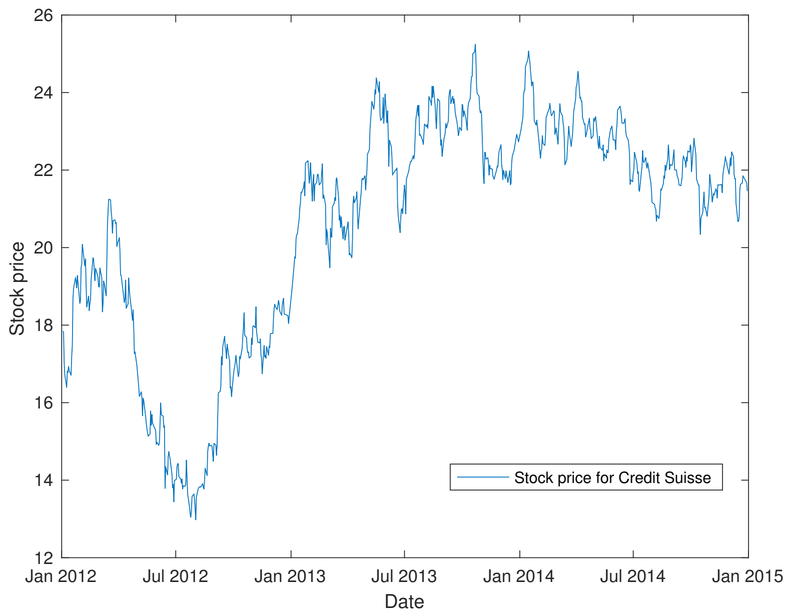

Figure 16: Market stock prices for Credit Suisse during the period from 2012 to 2014

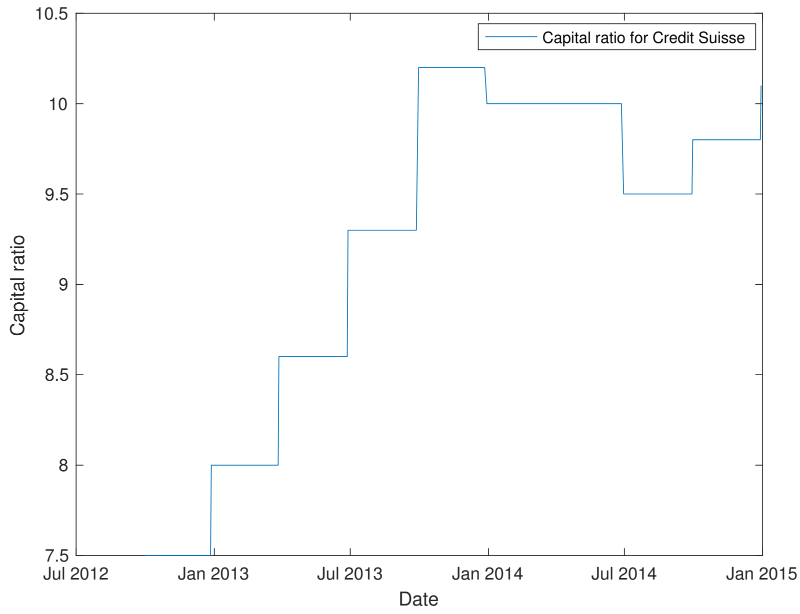

Figure 17: Capital ratios for Credit Suisse

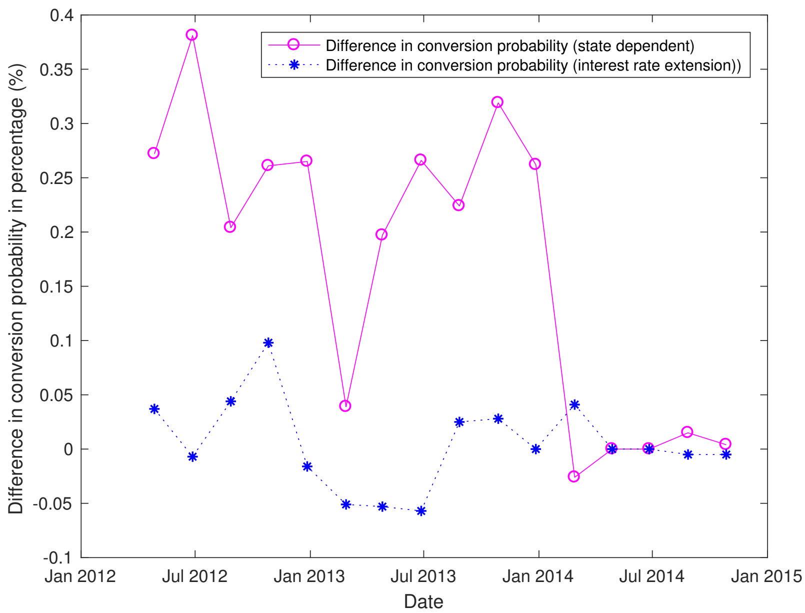

Figure 18: Difference in conversion probability in % with actual time to maturity

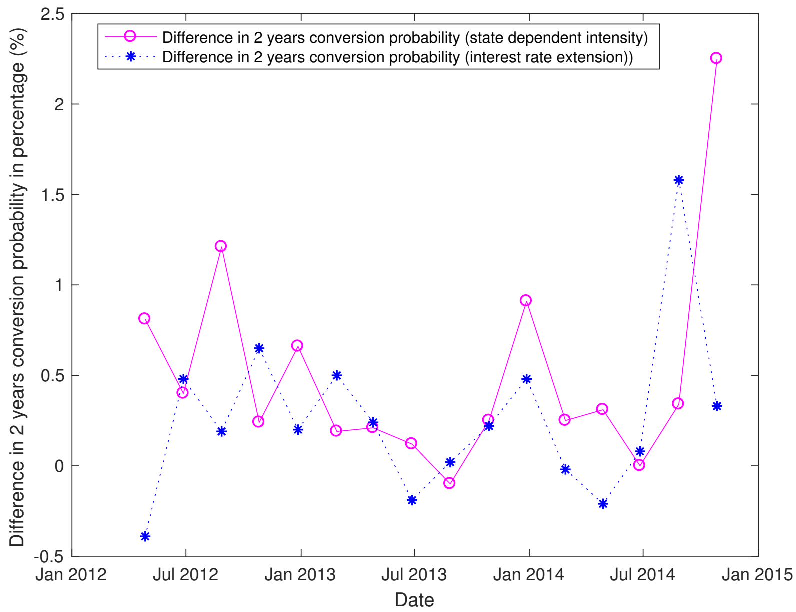

Figure 19: Difference in 2 years conversion probability in %

7 Sensitivity analyses

In this section, we analyze the impacts of different parameters on the Credit Suisse CoCo bond price, including the stock price-capital ratio correlation coefficient ρ, the stock price volatility σ, the conversion intensity λ, the interest rate r, the dividend yield q, the volatility of the log capital ratio η, the speed of reversion κ, the mean reversion level θ and the jump magnitude of the stock price γ.

We implement sensitivity analyses with the constant intensity pricing model, using the Monte Carlo simulation method, the parameters we obtained through calibration shown in Table 9 and the data with regard to the Credit Suisse CoCo bond on 27 April 2012. The values of the data are listed in the Table 16, so in the rest of this section, we use these values to implement the equity-credit model unless otherwise noted.

Table 16: The values of the regarding data used for sensitivity analyses

Date	Current capital ratio	Trigger threshold	Stock price
27-Apr-2012	7.50%	7%	17.31

7.1 Effect of stock price-capital ratio correlation coefficient

As shown in Figure 20 and 21, we test the sensitivity of the CoCo bond price with respect to the stock price-capital ratio correlation coefficient ρ by setting correlation coefficients from -1 to 1, stock price volatilities as 20%, 40%, 60% and capital ratios as 8%, 11%, respectively. The CoCo bond price decreases when the correlation coefficient ρ increases. When the capital ratio is constant, the larger the stock price volatility σ is, the larger is the effect of the correlation coefficient ρ on the CoCo bond price. This effect is expected since the stock price volatility σ influences the movement of the stock price, which leads to larger risk of the contingent convertible bond.

It is intuitive that the CoCo bond price and the correlation coefficient have a negative relationship. When the correlation coefficient ρ is larger, the correlation between the capital ratio and the stock price is stronger, so when the capital ratio decreases and meets the trigger threshold, the stock price will also drop. As a result, the conversion value P_E

will decline, and the CoCo bond price will be lower.

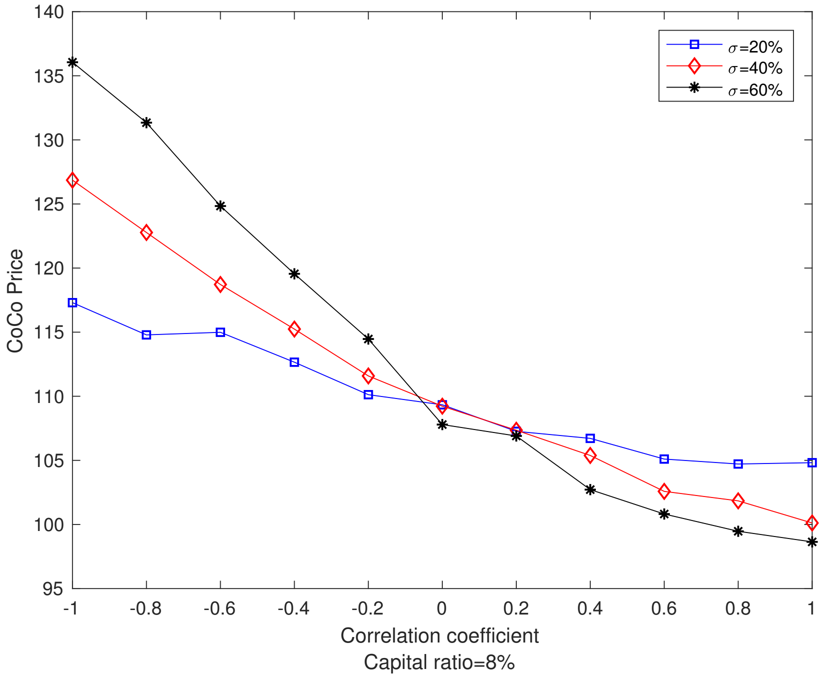

Figure 20: Effect of stock price-capital ratio correlation coefficient ρ on the CoCo price, with different sets of the stock price volatility and the capital ratio of 8%.

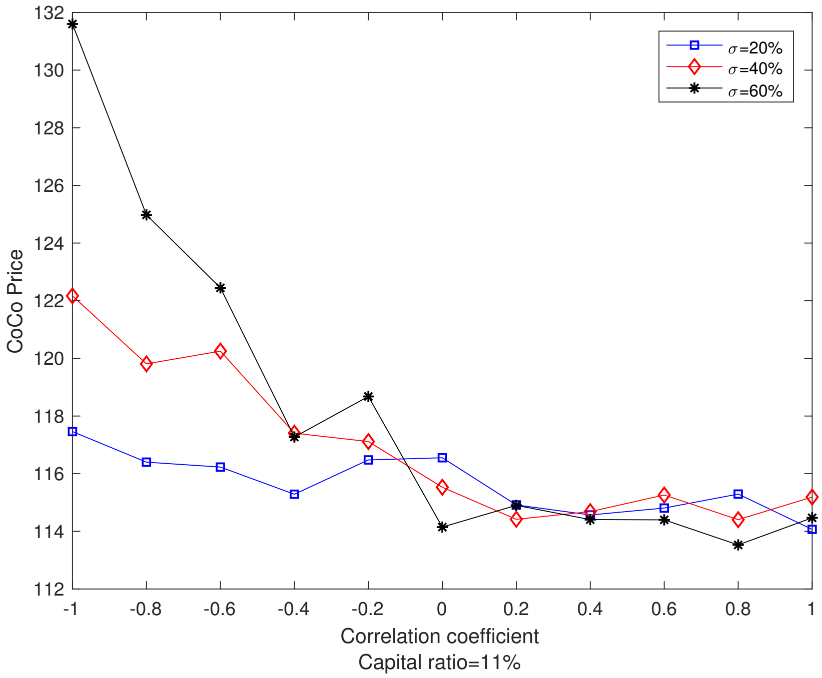

Figure 21: Effect of stock price-capital ratio correlation coefficient ρ on the CoCo price, with different sets of the stock price volatility and the capital ratio of 11%.

7.2 Sensitivity studies of CoCo-price with regard to the stock price volatility

We examine the sensitivity of the CoCo bond price with respect to the stock price volatility σ at different degrees of the correlation coefficient ρ by letting the stock price volatility range from 0 to 0.5 and correlation coefficients as 0, 0.5, 1, respectively. As can be seen from Figure 22, when the correlation coefficient is equal to zero, the stock price volatility has no impact on the CoCo bond price. When the correlation coefficient is larger than zero, the CoCo bond price decreases when the stock price volatility increases. The impact of the stock price volatility on the CoCo bond price will rise with the correlation coefficient.

This result is expected, since when the stock price increases, there is no positive effect on the CoCo bond price. However, when the sock price decreases and is highly and positively correlated with the capital ratio, the capital ratio will also plunge, and thus the CoCo bond price will decline. The larger the stock price volatility is, the greater this negative impact of the stock price is. (Chung and Kwok (2016))

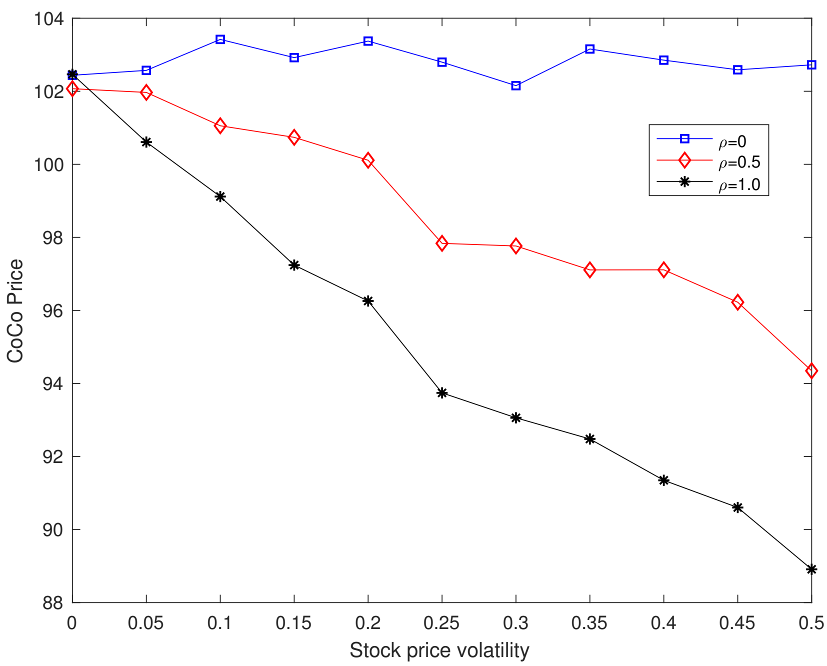

Figure 22: Effect of stock price volatility σ on the CoCo price, with different sets of the stock price volatility.

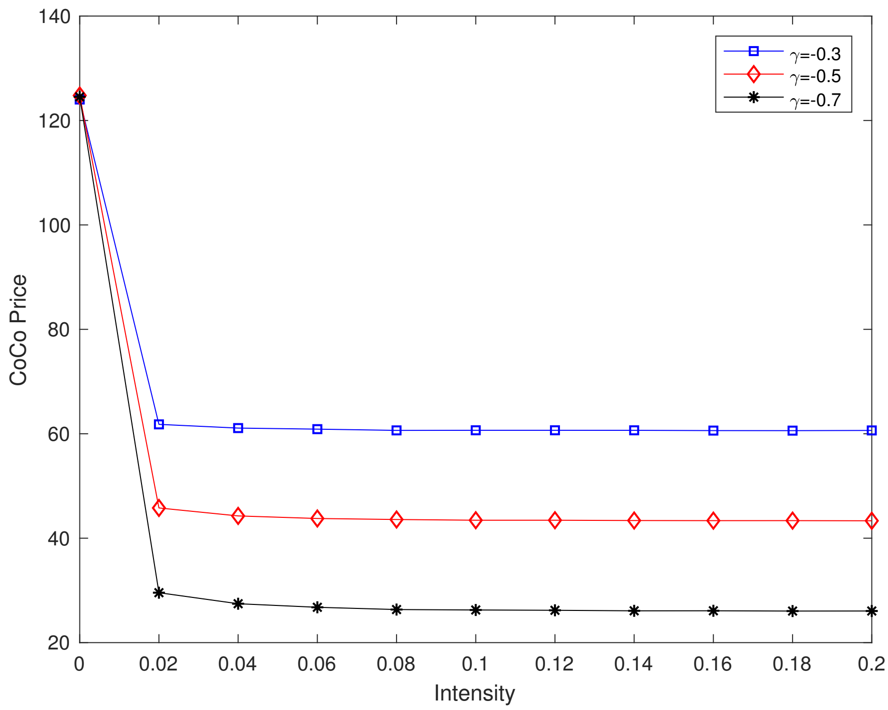

Figure 23: Effect of conversion intensity λ on the CoCo price, with different sets of the asset loss proportion.

7.3 Studying the CoCo-price with respect to the conversion intensity

As shown in Figure 23, the CoCo price increases with the loss proportion of the stock price γ. The lower the absolute value of loss proportion is, the higher the CoCo price is. For the conversion intensity λ, the CoCo price increases when the conversion intensity is lower. The intuition is that when the jump of the Poisson process is less likely, which means that the conversion intensity λ is low, the CoCo bond is less risky and behaves more like a ordinary bond with only a book-value trigger. When the CoCo bond is less risky, the return is lower, which means that the price of the CoCo bond is higher. As we can see in Figure 23, the CoCo price decreases much more when the conversion intensity λ is below 0.02, which means that if we ignore the book-value trigger τ_B ($\tau_B = \infty$) so that $\tau = \tau_R$, then the expected conversion time is after 50 years, given by $E[\tau] = E[\tau_R] = \frac{1}{\lambda}$ since τ_R is exponentially distributed with the parameter λ. For conversion intensity that is higher than 0.02, the simulated jump probability will probably be very high for most of the simulation paths. The CoCo bond price will in these cases reaches the lowest level.

7.4 Effect of interest rate on the CoCo-price

In Figure 24, we present the sensitivity of the CoCo bond price to the interest rate r in the constant interest rate case, setting the interest rate from 0% to 0.2%. The interest rate has a negative impact on the CoCo bond price, since the larger interest rate means that the discounted rate is more significant, which results in a lower CoCo bond price.

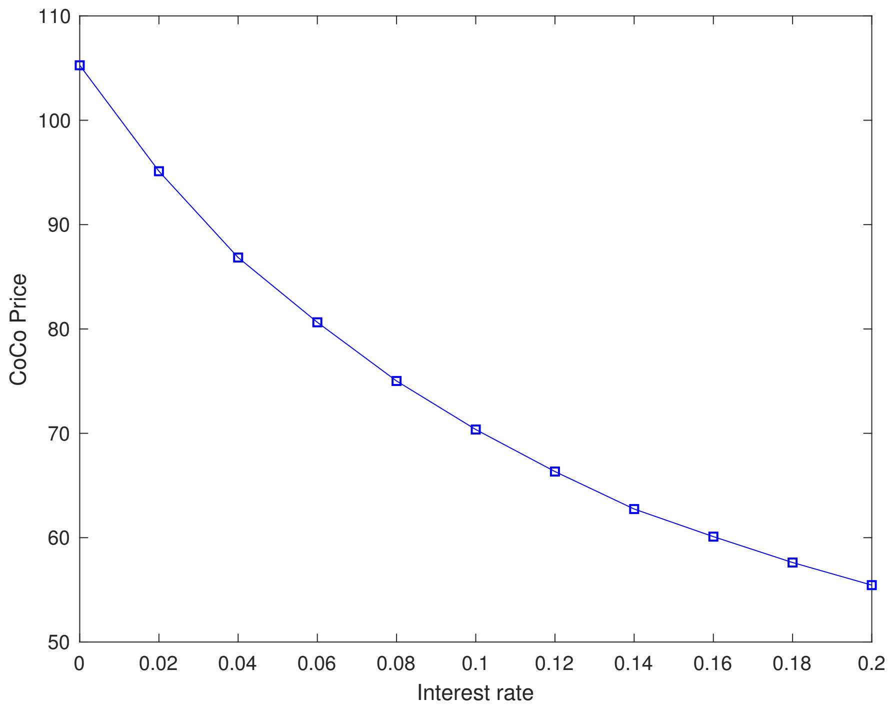

Figure 24: Effect of interest rate r on the CoCo price.

7.5 Effect of stock dividend yield

Figure 25 reveals the sensitivity of the CoCo bond price with respect to the stock dividend yield q. We vary the dividend yield from 0 to 0.5. The CoCo bond price falls when the stock dividend yield increases. According to Equation 3, the stock dividend yield q has a negative effect on the stock price. Thus, when the stock dividend yield increases, the stock price will decrease. Meanwhile, according to Equation (9) the lower stock price results in a lower conversion price P_E, thus a lower CoCo bond price.

In practice, the decreasing stock price will cause reduction for the capital ratio, for example, the Common Equity Tier 1 capital ratio will decline and thus the CoCo bond price will decrease. This relationship is consistent with the sensitivity test results from our pricing model.

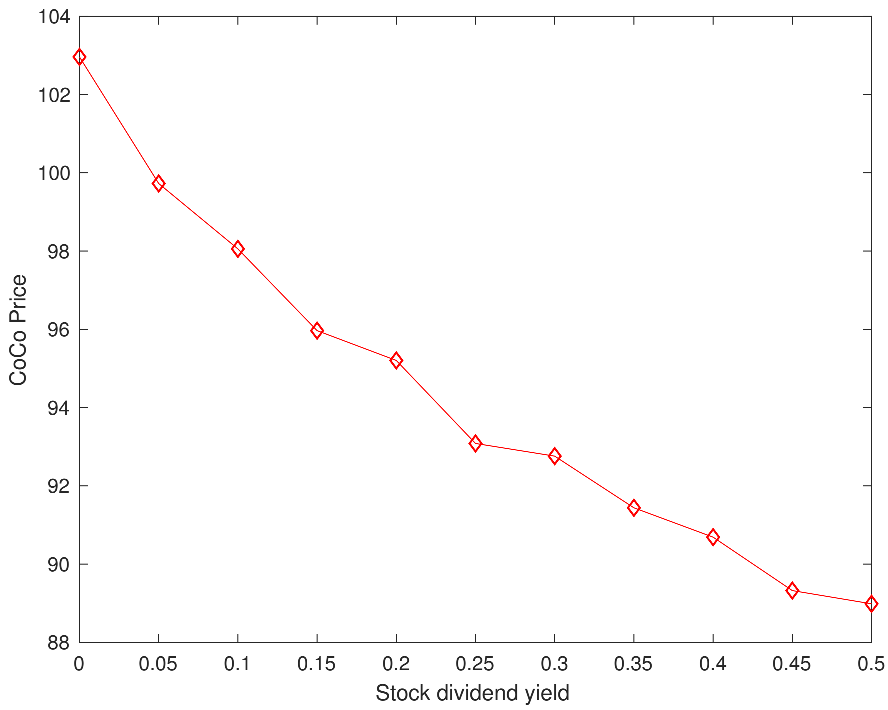

Figure 25: Effect of stock dividend yield q on the CoCo price.

7.6 Effect of log capital ratio volatility

The sensitivity of the CoCo bond price to the volatility of the log capital ratio η is shown in Figure 26. We set the range of the capital ratio volatility from 0 to 0.5. The CoCo bond price decreases significantly when the capital ratio volatility increases from 0 to around 0.2. When the capital ratio volatility rises from 0.2 to 0.5, the CoCo bond price decreases slower.

The negative relationship between the log capital ratio volatility and the CoCo price is expected since the larger capital ratio volatility will result in a higher possibility of hitting the trigger threshold and thus a lower CoCo bond price. Meanwhile, we can observe that this negative effect of capital ratio volatility decreases when the capital ratio volatility is larger than 0.2. We also note that the price decreases very sharply for η between 0 and 0.2.

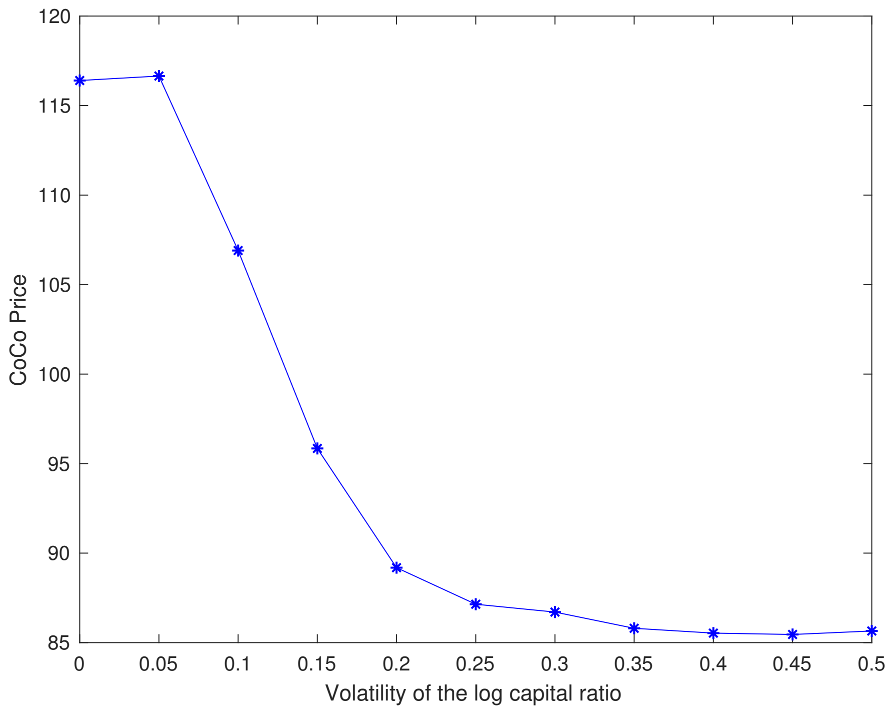

Figure 26: Effect of log capital ratio volatility η on the CoCo price.

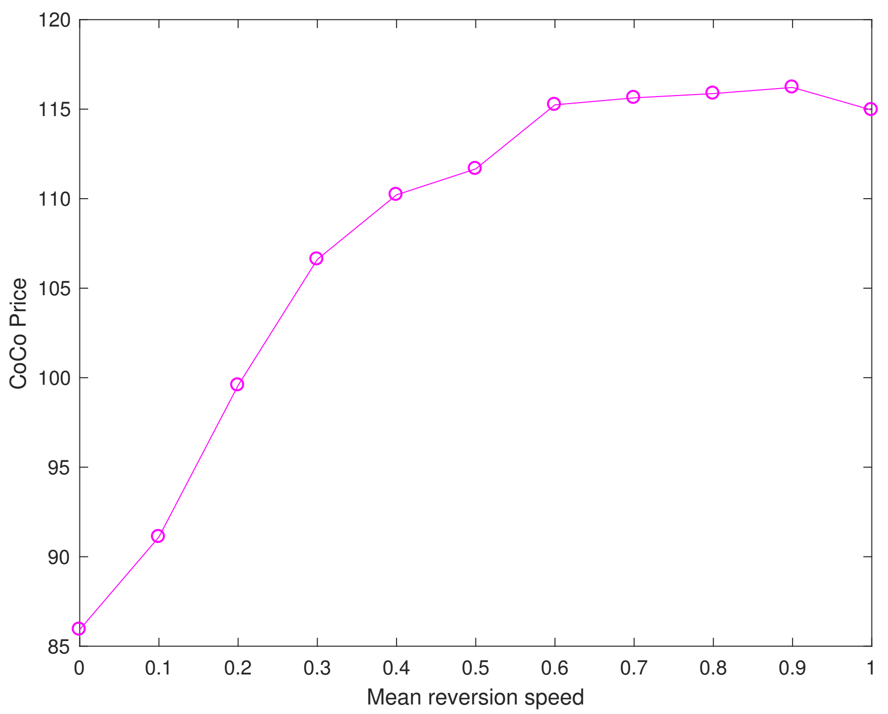

Figure 27: Effect of mean reversion speed κ on the CoCo price.

7.7 Effect of mean reversion speed

We test the sensitivity of the CoCo bond price with respect to the speed of mean reversion κ by setting it from 0 to 1. As can be seen from Figure 27, the price of the contingent convertible bond increases with the mean reversion speed. The larger speed of mean reversion means that the capital ratio will move back to the long-term level faster, which will decrease the possibility of triggering. Thus, the CoCo bond price will also grow.

7.8 Effect of log long-term capital ratio

Figure 28 reveals the sensitivity of the CoCo bond price to the log long-term capital ratio θ. We set the log long-term capital ratio from -2.5 to -1 and observe that the log long-term capital ratio and the CoCo price have a positive relationship. The higher long-term capital ratio reduces the risk of triggering, so the CoCo bond price will rise.

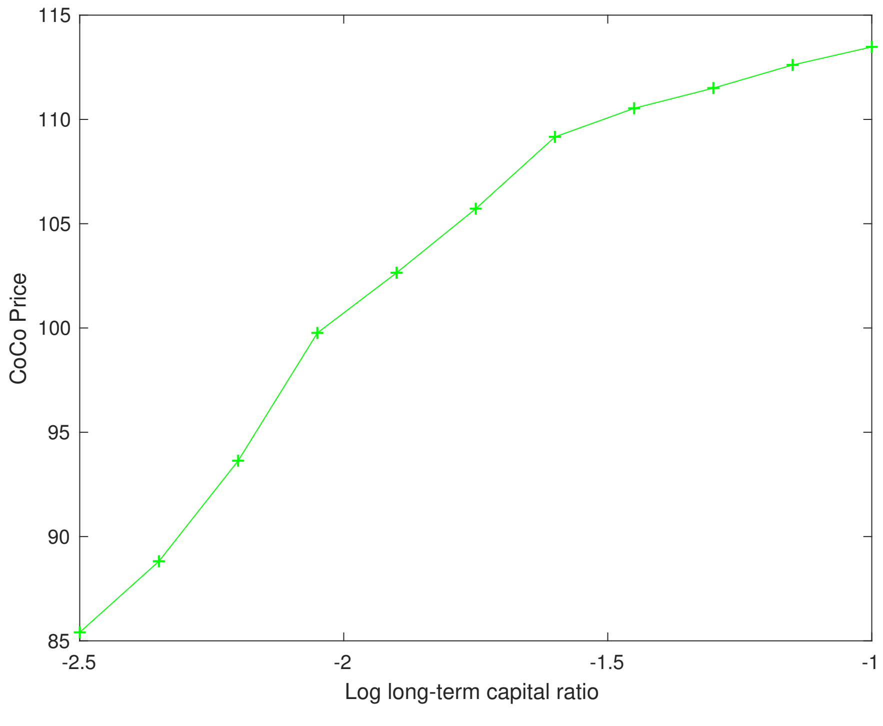

Figure 28: Effect of log long-term capital ratio θ on the CoCo price.

7.9 Effect of jump magnitude

The sensitivity of the CoCo bond price with respect to the jump magnitude of the stock price γ is shown in Figure 29. The range of jump magnitude of the stock price is from

-0.9 to 0. When the absolute value of the jump magnitude of the stock price is decreasing, the CoCo bond price increases. The result is intuitive, because when the jump level of the stock price is larger, the stock price will decrease more once the jump happens, which will result in a smaller conversion value and a lower CoCo bond price.

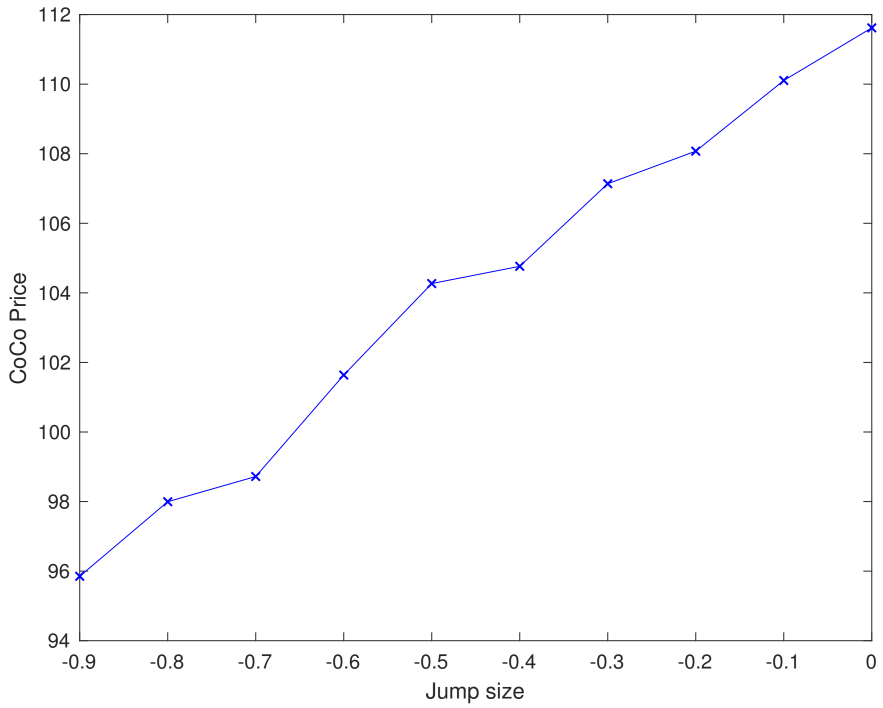

Figure 29: Effect of stock price loss proportion γ on the CoCo price.

8 Conclusion

In this thesis, we have examined and implemented the hybrid equity-credit pricing model suggested by Chung and Kwok (2016). After comprehending and replicating the pricing process in Chung and Kwok (2016), we also implement four major extensions. First, we obtain the parameters of the pricing model by calibration from the market data with the choose Credit Suisse CoCo bond as our research object. Second, we extend the equity-credit model by adding a stochastic interest rate. Third, besides presenting the conversion value of the CoCo price as Chung and Kwok (2016), we also show the results of the final CoCo price, the values of the coupon payments and the values of the principal payments for both the replication of Chung and Kwok (2016) and our application. The calibration errors and the conversion probabilities of our application are also displayed and analyzed. Finally, we implement the sensitivity studies where many of the results are in line with our intuition, but it is difficult to guess the actual sizes of the impact without having a computationally tractable model. In this thesis, we do not consider the principal write-down and the call-back features of the CoCo bond for the simplicity reason. These two features are topics for future studies.

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