UNIVERSITY OF GOTHENBURG
SCHOOL OF BUSINESS, ECONOMICS AND LAW

Master Degree Project in Finance

Backtesting the Magic Formula in the Brazilian Stock Market

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*To achieve the results presenting in this thesis, several rigorous decisions were made. If any person encounters this paper and wishes to replicate the results do not hesitate to contact me through my email: *alexander.gjfp@gmail.com*.

Abstract

This thesis performs a backtest of the *Magic Formula* for the Brazilian stock market in the period between 2006 and 2015. The *Magic Formula* methodology is a stock picking strategy that combines *Return on Capital* and *Earnings Yield* factors in a ranking system aiming to outperform the market average. We found that after correcting for survivorship bias and look-ahead bias, all the *Magic Formula* portfolios outperformed both *Ibovespa* and *IBrX-100* benchmarks during the period analyzed. However, when using a asset price framework through the *CAPM* to find positive and significant *alpha*, only the 0.98% *alpha* of top 5 portfolio resorted every 12 months regressed on *Ibovespa* could achieve a 90% level of significance while all other portfolios *alphas* were not significant at any meaningful level. Additionally, we perform the same exercise for portfolios constructed based on each isolated factor. We conclude that despite the fact that the *Magic Formula* portfolios outperformed the benchmarks we could not assure with a high level of certainty that the strategy is *alpha* generator, and that our results were not due to randomness.

Keywords: Magic Formula, Value Investing, Joel Greenblatt, Efficient Market Hypothesis (*EMH*), Capital Asset Pricing Model (*CAPM*).

JEL Classifications: G11, G12, G14

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1 Introduction

Value investing philosophy has its beginning as a discipline with the publication of *Security Analysis* (Graham and Dodd, 1934) and, afterward, gained a further contribution with the first publication of *The Intelligent Investor* in 1949. The value investing motto is: buy a high-quality asset for less than it is worth. Even though it is an intuitive and almost obvious statement, doing so is a much more complex skill to master. Value investing approach differs significantly from standard finance theory regarding several key variables and more particularly, the definition of risk. While value investing defines risk as a permanent loss of capital (Graham and Zweig, 2003), standard finance theory assumes that risk is the volatility of an asset's market value. Furthermore, for the value investing community, markets are frequently efficient but not always and, in fact, it is often described as irrational as in the *Mr. Market* allegory (Graham and Zweig, 2003). On the other hand, in mainstream finance theory, the markets are efficient and, any investor's attempt to yielding abnormal returns in a consistent style is doomed to fail ((Fama, 1970) and (Jensen, 1978)). For these reasons and to some degree, it is hard to integrate both perspectives and test in a definitive manner, if value investing is an *alpha* generator method of investing.

The *Magic Formula* created by Joel Greenblatt was first published in the book *The little book that beats the market* in 2006. The formula is an attempt to provide to non-professional investors, guidance and insights on how to beat the market using the value investing philosophy. In order to incorporate elements of value investing, Greenblatt (2006) uses *return on capital* (ROC), which measure the quality of a company or an asset, and the *earnings yield* (EY), which quantify how much cheap a business or asset is in relation to its capacity of generating earnings. What makes Greenblatt's formula interesting and unique is how these two accounting measures are calculated. Greenblatt (2006) changes substantially the standard way of calculating them[1] to compare different business in different industries with different capital structures.

For these reasons, the *Magic Formula* is a desirable method to use for the ones trying to incorporate both, value investing and standard finance perspectives and tests if the value philosophy is indeed a superior approach on how to invest.

The fundamental hypothesis we ought to test is: can the *Magic Formula* beat a Brazilian market *proxy* in a risk-adjusted manner and partially contradict the *EMH*? Our contribution is twofold.

First, to expand the literature that tries to integrate the academic approach to value investing practitioners' methods. Secondly, examine if a value investing strategy holds for

[1]Which will be explained in detail in the methodology section.

a emerging market that has a developing capital market. Brazil is a interesting market to investigate because, despite the fact that has the 7th largest GDP in the world by purchasing power parity[2] its stock market is characterized by first, a very low participation of individual investors[3], and secondly, Brazilian public companies have concentrated capital structures with a significant participation of the central government through the Brazilian Development Bank (*BNDES*) and public pension funds.

To the best of our knowledge, even though there are many papers about the *Magic Formula*, none so far have approached the Brazilian stock market.

This work is organized in 6 sections being this introduction the first one. The second section is a comprehensive literature review covering from efficient market hypothesis (*EMH*) through the findings contradicting the *EMH* to *Magic Formula* empirical evidence. The 3rd section is the hypothesis section. The 4th section is composed by the explanation of the *Magic Formula* methodology as well as other models and methods used in the analysis, data description and possible sources of biases. The 5th segment shows the results and findings, and the sixth and last section briefly discusses and concludes.

2 Literature Review

This section review the efficient market hypothesis (*EMH*), the collection of empirical evidence opposing the *EMH* developed during the last 40 years and, finally, the *Magic Formula* empirical evidence in different markets and time frames.

2.1 Efficient Market Hypothesis - *EMH*

Fama (1970) defines an efficient market as a marketplace where prices consistently reflect all the available information. To derive this result, Fama (1970) set three necessary conditions: (i) there must be no transaction costs (no costs to acquire information or to reach the marketplace, for instance); (ii) all the relevant information must be publicly available and (iii) the economic agents must freely agree on prices given the available information and trade on them. Jensen (1978) afterward defined an efficient market as one that is impossible to generate economic profits[4] with a given up to date set of information. Considering the financial markets, if all investors (*e.g.* private, institutional, etc.) had

[2] According to the World Bank: `http://databank.worldbank.org/data/download/GDP_PPP.pdf`.

[3] Only 0.3% of the Brazilian population is actively participating in the stock market through BM&FBovespa. `http://www.bmfbovespa.com.br/pt_br/servicos/market-data/consultas/historico-pessoas-fisicas/`.

[4] By economic profits Jensen means: *risk-adjusted returns net of all costs.*

the same set of information, they would trade and achieve a market equilibrium where abnormal profits would not be possible. Consequently, the expected return on any asset would be solely a function of its underlying risk (Malkiel, 2003). Consistent with *EMH*, Fama (1965) and Malkiel (1973) formulate the random walk hypothesis which states that past prices and historical information are already incorporated into the market. Thus, they are irrelevant to predict future prices.

Fama (1970) developed three forms of *EMH* known as the weak, semi-strong and strong form. In the weak form, future prices of assets cannot be forecasted by utilizing historical price or data. In that sense, asset prices have no patterns and methods such as technical analysis are not able to achieve excess returns in the long-run. Regarding the semi-strong form of *EMH*, both technical analysis and fundamental analysis are unable to yield abnormal returns. According to Fama (1970), this happens because the market assimilates all new public available information in a rapid and unbiased manner adjusting all asset prices. Lastly, in the strong version of *EMH* market prices adequately reflect all (both private and public) information in the present and in the foreseeable future through rational expectations and; therefore, it is impossible to produce any risk-adjusted excess return based on any information.

Concluding, Fama (1970) and Malkiel (2003) points that there is enough empirical evidence that supports both the weak and semi-strong form of *EMH*. Furthermore, Jensen (1978) states that the extensive amount of evidence makes *EMH* the most reliable empirical assertion of economic science. Next section introduces the arguments and empirical evidence that criticize the *EMH*.

2.2 Evidence Opposing the *EMH*

This section presents and walk through several studies that show findings contradicting the *EMH*. Supporters of the *EMH* argue that these findings are neglectable deviations from the theory and in the long-run the market mechanisms would correct them making the market efficient once again (Malkiel, 2003). However, researchers who criticize the *EMH* and, especially, supporters of the behavioral approach argue that individuals suffer from systemic biases making decisions (Kahneman and Tversky, 1979) (Tversky and Kahneman, 1974) and (Thaler, 2015).

2.2.1 *Firm's Market Value and Size Effect*

Banz (1981) analyses the possible causal relation between firm's market value and average returns and conclude what is known as the size effect. Using the Capital Asset Price Model (*CAPM*) to calculate the risk-adjusted returns, Banz (1981) investigated this relationship from 1936 to 1975 and found that small market value firms have higher average returns than the high market value ones. Additionally, Banz (1981) concluded that the size effect is not linear and stronger in smaller firms. Klein and Bawa (1977) found one possible explanation to the size effect. They argue that small companies are not well known by the public and investors in general and, therefore, the lack of information imposes a risk-premium for investors.

In addition to Banz (1981), Chan and Chen (1991) also show that size (*i.e.* market value of firm) has sound explanatory power on returns distribution among portfolios sorted by size. Nevertheless, when adjusting for new additional factors related to stock's responsiveness to, for instance, value-weighted index the size factor loses explanatory power. These findings suggest that the *CAPM* is not one size fits all model because there are possible hidden risks that could be captured by the size factor. However, in a more recent study Horowitz *et. al.* (2000) uses data from 1982 to 1997 and could not find evidence supporting the causal relationship between small market value companies (size) and risk-adjusted average returns.

2.2.2 *The January Effect*

The January effect is a known calendar phenomenon in the finance industry since the study of Wachtel (1942) when he investigated the returns in the Dow Jones Industrial Average (*DJIA*) during the period of 1927 to 1942. Confirming Wachtel (1942) findings, Rozeff and Kinney (1976) carried an empirical research about seasonality in the US stock market for the period of 1904 to 1974 and found that the average return for the month of January was 3.48% while for the remaining months was 0.42%. Furthermore, Keim (1983), Reinganum (1983), Roll (1983) and Haug and Hirschey (2005) studies confirm this calendar anomaly and found that the January effect is more prominent for small market capitalization firms. Additionally, Keim (1983) discovered that almost 50% of the excess returns observed in small market capitalization companies were obtained at the beginning of January.

There are three lines of arguments that try to explain the January effect. Researches such as Wachtel (1942), Branch (1977) and Keim (1983) support that the tax-loss strategy is a possible explanation. This procedure consists of selling losing positions during the end of the fiscal year to generate tax credits for winning positions. If the sales volume is enough to depress the stocks prices, this will create a premium to be explored in January making the returns in January higher. Thaler (1987) shows that there were a January effect and a tax-loss effect in both UK and Australia. However, those countries use different fiscal years than other countries meaning that the January anomaly cannot be explained entirely by the tax-loss strategy. Another possible explanation is the window-dressing hypothesis. According to Lakonishok *et. al.* (1991), since institutional investors are assessed by their annual performances there is a tendency of selling poorly performing positions at the end of the year making room for higher returns in January. A third possible explanation is the information hypothesis developed by Keim (1983). He claims that January and beginning of year periods have the characteristic of being highly uncertain because of significant data releases, and this effect is more visible on small businesses. On the other hand, it is important to highlight a recent study made by Lindley *et. al.* (2004) which exhibited that during 1962 until 2000 the January phenomenon is insignificant for many years and also negative for several other years.

2.2.3 *The Weekend Effect*

According to Cross (1973) and French (1980), the weekend phenomenon is the observational evidence that stocks tend to have a higher performance on Fridays when compared with Mondays. Rubenstein (2001) and Keim and Stambaugh (1984) show that majority of the average negative return on the *S&P* occurs during the non-trading period of Friday-close to Monday-open market in spite of the upwards long-run trend of stock markets. One possible explanation according to Dyl and Marbely (1988) is that companies tend to realize negative information on Friday and, moreover, with more news throughout the weekend the market cannot correct for it making Monday an adjustment day. In addition to these findings, Rogalski (1984) exhibits empirical evidence that the weekend effect is linked to the January and size events saying that small firms have higher returns on Mondays.

Nevertheless, in more recent studies, the weekend effect has been questioned. Schwert (2003) estimate that the weekend anomaly is not significantly different from other days of the week since 1978. Sullivan *et. al.* (2001) argues that the demonstration of weekend phenomenon suffers from a data mining problem.

2.2.4 *Contrarian and Momentum Strategies*

The contrarian strategy consists of acquiring stocks that have underperformed in a previous period and short-sell the ones that have outperformed in the past to make excess returns compared to the market (Lakonishok *et. al.* 1994). This approach is based on two behavioral claims. First, Lakonishok *et. al.* (1994) argues that investors tend to be overconfident extrapolating past performance into the future, hence, making stocks with good past performance be overpriced. Second, De Bondt and Thaler (1985) support that investors suffer from an over-reaction bias towards new market information and, consequently, selling loser (making them underpriced) and buying winners (making them overpriced). In fact, De Bondt and Thaler (1985) examine the contrarian procedure using monthly data from 1926 to 1982 forming winners and losers portfolios based on long-term past performance. They conclude that the losers' portfolio yielded 24.6% more than the winners for the subsequent three years and 31.9% for the five years period whilst undertaking less risk.

According to Jegadeesh and Titman (1993), the momentum anomaly states quite the opposite when comparing to the contrarian strategy. The momentum effect says that winning position (*i.e.* stocks that have had an excellent performance for a particular period in the past) will maintain a superior rate of return in the short-run future. Jegadeesh and Titman (1993) carried an experiment consisting of purchasing winner stocks (stocks that had performed well in the previous 3 to 12 months) and selling losers (stocks that had performed poorly during the same time frame) for the period from 1965 to 1989. They found evidence that the strategy was capable of delivering an abnormal return for the period of 6 to 12 month after the portfolio formation. Adding to this evidence, Lo and MacKinlay (1988), using weekly and monthly data returns for the US stock market for the 1962-1987 period, obtained a significant and positive serial correlation of the returns for weekly and monthly holding periods.

Additional researches such as Moskowitz and Grinblatt (1999) and Grinblatt and Moskowitz (2004) confirm the momentum phenomenon. Moskowitz and Grinblatt (1999) noticed that the momentum effect could be perceived and was stronger when analyzing industry by industry. Grinblatt and Moskowitz (2004) found that the return premium between top deciles of the momentum strategy could be twice as big of the losers' portfolio.

2.2.5 *Price-Earnings Ratio (P/E)*

Basu (1977) demonstrates that stocks with low *P/E* ratios were able to outperform high *P/E* stocks for the period from 1957 to 1971. Using measures such as Jensen's *alpha*, Treynor and Sharpe ratios, Basu (1977) also shows that portfolios with low *P/E* were capable of outperforming random portfolios with same risk level whereas stocks with high *P/E* ratio were not. Basu (1977) argues that his conclusions are not consistent with the semi-strong form of *EMH*. Also, Lakonishok *et al.* (1994) exhibited that value strategies such as the low *P/E* portfolio seem to be no riskier when compared to other approaches and, moreover, that holding more fundamental risk does not explain higher average returns. Lakonishok *et al.* (1994) conclude low *P/E* portfolios and other value strategies yielded risk-adjusted excess returns from 1968 to 1990 period. Additionally, Jaffe, Keim, and Westerfield (1989) using data from 1951 to 1986 found significance for *P/E* as an explanatory variable of the expected returns. Furthermore, Basu (1983) using *P/E* ratios along with size and market *beta* could explain the cross-section of the average expected returns.

On the contrary, many other authors suggest flaws in the low *P/E* portfolio strategy as one that can gain abnormal returns. First, Fama and French (1992) argue that low *P/E* stocks are essentially riskier, and the above average returns are due to this. Ball (1978) claims that the *P/E* variable is a *proxy* for unknown risk factors and states that low *P/E* portfolios have higher expected returns because of its higher exposure to risk. Reinganum (1981) suggests that the *P/E* variable as an attempt to explain expected returns suffers the same misspecification problem as the *CAPM* and loses its explanatory power when analyzed jointly with the size effect.

2.2.6 *Book-to-Market Ratio (B/M)*

The book-to-market ratio (B/M) effect is one of the most abundantly studied market anomaly phenomenon by academia and is known as the value effect. The idea is that, *B/M* ratio is a *proxy* for risk and hence has explanatory power of the cross-sectional expected returns. Fama and French (1993) and Fama and French (1992) studied the US Stock market during the period from 1963 to 1991 and discovered that the *B/M* factor along with size could explain and predict the average market return. Additional evidence for the US stock market found by Stattman (1980) and Rosenberg *et al.* (1985) confirms that high book values of equity to equity market values of firms yielded a premium return. Moreover, Chan, Hamao, and Lakonishok (1991) show a positive relationship between *B/M* and average market return for the Japanese market. There are two possible explanations for this effect. First, De Bondt and Thaler (1987), Lakonishok *et al.* (1994), and Haugen (1995) argue that investors have bias expectations because they tend to extrapolate good past performance to the future and, consequently, underestimate value stocks prices' (high B/M firms) and overprice growth stocks (low B/M firms). Secondly, Daniel and Titman (1997) argues that in fact, the B/M effect is an indication of investors' preference for growth companies over firms with value characteristics.

2.3 The *Magic Formula* Empirical Evidence

The recent trend of investigating value investing strategies by academia has yielded several studies about its effectiveness. The backtest of Greenblatt's formula is no exception and, in fact, because of its simplicity has been one of the most tested value investing strategy in academia. Several studies have performed empirical tests not only to validate and question Greenblatt's results but also to investigate its efficiency on other stock markets in different economies and also to test the stock market efficiency itself.

Larkin (2011) tested several value investing strategies including Greenblatt's *Magic Formula* from 1998 until 2006 for the US market. All the value investing strategies outperformed the market's average by a significant margin. For instance, the average returns of the combined book-to-market value (B/M) and size strategy yielded 26.19% while the market's average return was 7% per annum. Larkin (2011) highlighted that even though this combined book-to-market and size strategy outperformed Greenblatt's formula average return (23.48%), Greenblatt's methodology exhibited less volatility and also was the only one that did not show negative returns during the entire period analyzed.

Abbey and Larkin (2012) improved the analysis of Larkin (2011) by extending the study period from 1981 until 2010. They found that not only did value strategies consistently outperform the market portfolio weighted by market capitalization, but also without increasing risk. Abbey and Larkin (2012) results show that among value investing strategies, Greenblatt's magic formula had the best performance, yielding an annualized return of 23.1% meanwhile the market annual rate of return was 12.1%. Furthermore, Abbey and Larkin (2012) exposed that, despite the fact that value investing strategies are riskier (note that risk here is measured by volatility, *i.e.* value investing strategies are, according to them more volatile), the *Magic Formula* methodology shows few episodes of negative returns over 3 to 5 years periods.

Blij (2011) examined the magic formula for the US stock market from 1988 until 2009 using both equally weighted (EW) and market capitalization weighted (CW) portfolios.

Blij (2011) found that both the *EW* and *CW* portfolios yielded excess risk-adjusted returns when compared to the market version of the *CW* and *EW* portfolios. More precisely, the *Magic Formula* implemented in a *CW* portfolio obtained 23.3% per annum and in *EW* case earned 21.8% annually whereas Greenblatt's (2010) made 23.8% on an annual basis. The *EW* and *CW* market portfolios, according to Blij (2011), were 10.6% *p.a.* and 12.3% *p.a.* (similar to Greenblatt's (2010) market portfolio of 12.3% annual average return) respectively.

It is important to notice that the period analyzed by Blij (2011) and Abbey and Larkin (2012) covers the period studied in Greenblatt's (2010) and Greenblatt's (2006) and work as tests of Greenblatt's results. Furthermore, other studies such as, Alpert (2006), ClariFI[5] and Montier and Lancetti (2006), have replicated Greenblatt's formula for the US stock market to validate his findings. They found similar results but lower returns on average mainly due to different databases and differences in accounting measures.

Persson and Selander (2009) backtest Greenblatt's *Magic Formula* for the Nordic region during 1998 to 2008. The portfolio had a 14.68% compounded annual growth rate (*CAGR*) and was significantly higher than the MSCI Nordic for the same period (9.28%) and the *S&P500* (4.23%). In addition, the Sharpe ratio of different portfolios constructed (including the portfolio with intangible assets) was greater than the market portfolio. One key contribution made by Persson and Selander (2009) was to insert the *Magic Formula* into an asset price theory perspective using both the *CAPM* and the Three-Factor model of Fama and French (1992). Using these models and adjusting for risk, Greenblatt's formula could not statistically beat the market.

Olin (2011) provides additional empirical evidence from a Nordic country. Olin (2011) researched the consistency of *Magic Formula* in the Finnish stock market during 2000 to 2009. Olin (2011) showed that the value portfolios formed had a performance between 9.4% and 20% while the market portfolio, which was the OMXH Cap index, had an average annual yield of 3.5%. In other words, the best value combination portfolio outperforms the market by 16.5% with less risk, thus, with a higher Sharpe ratio.

Sareewiwatthana (2011) demonstrates a further evidence of Greenblatt's formula success in the Thailand stock market during the period from 1996 to 2010. After testing various value investing strategies, Sareewiwatthana (2011) shows that the *Magic Formula* was the best performing strategy with an outstanding annual yield of 66.2% while the market annual return was 2.4% for the period.

Howard (2015) did a similar exercise to Sareewiwatthana (2011) and tested several value investing strategies for the Johannesburg stock exchange from 1998 until 2013. Howard

[5] A provider of software and services to quantitative portfolio managers acquired by Capital IQ.

(2015) found that the magic formula had only a 1.0% excess return when comparing to the JSE ALSI TRI as the benchmark. For the period, value strategies such as B/M, Dividend Yield (DY) and Earnings Yield (EY) performed better than the *Magic Formula*. Moreover, Howard (2015) points that, although all value strategies outperformed the JSE ALSI TRI, they were not statistically significant.

Chun and Si (2015) backtested Greenblatt's method for the Hong Kong stock market during the period from 2001 to June the 30*th* of 2014. Chun and Si (2015) concluded that the top 10% of the stock sample sorted according to the *Magic Formula* had a 2.53% average monthly return. On the other hand, the bottom 10% of the sample performed only 1.30% monthly. Additionally, they demonstrated that the new factor created by the *Magic Formula* had further explanatory power in describing the variation on the expected stock returns in addition to the Fama-French three-factor model.

3 Hypotheses

Based on the literature review discussion presented previously it is possible to state few assertions. As evidence shows, the efficient market hypothesis might not hold true for several reasons and therefore, it may be possible for investors to achieve abnormal returns. Moreover, as the *Magic Formula* empirical evidence shows, Greenblatt's methodology have achieved superior returns for different time frames in different economies which, may signal that the *Magic Formula* is a *alpha* generator strategy. For these reasons, the research question is: can the *Magic Formula* method deliver higher risk-adjusted returns than a market *proxy* of the Brazilian market average and oppose partially the efficient market hypothesis?

The hypotheses that will be tasted are:

- *Hypothesis I*: *The Magic Formula outperforms the market proxy.*
- *Hypothesis II*: *The Magic Formula strategy generates positive and significant betas with values below 1.*
- *Hypothesis III*: *The Magic Formula generates positive and significant alphas.*

If the hypothesis *I* holds true and the other two are false the *Magic Formula* outperforms the market because bears more risk than the market average. If the hypothesis *I* and *II* are true and the hypothesis *III* is false then the *Magic Formula* strategy outperforms the market average, however it is not possible to state if it was due to a model misspecification or due to pure and simple luck. If all the hypothesis hold true then, there is evidence to affirm that the *Magic Formula* is a market anomaly. Next section will present the model and methods used to test these hypotheses.

4 Methodology

This section describes meticulously the techniques and methods used to perform the research and replicate Joel's formula. It presents also procedures and approaches to deal with database limitations, data unavailability, and constraints faced throughout the investigation.

4.1 Joel Greenblatt and The *Magic Formula* Ranking System

Joel Greenblatt is known in the financial community as a successful value investor with a track record of 40% annualized returns during 20 years (1985 to 2006) as Gotham Capital Manager (Alpert, 2006)[6]. Greenblatt's investment philosophy and the *Magic Formula* method were presented in his book *The little book that beats the market*. Joel states that to be able to beat the market (*i.e.* have returns above a benchmark that represents the market average or a broad market portfolio) and build wealth the intelligent investor should buy securities of good companies at bargain prices (Greenblatt, 2006). To assess what is a good company and at which level a stock is a bargain Joel Greenblatt developed a replicable method that he calls *Magic Formula*. The formula combines in a ranking system companies with both good past performance of returns on capital (ROC, which is the "good company"[7] measurement) and high earnings yield (EY, which in turn is how he evaluates if a security is a bargain).

The step by step process is displayed in Greenblatt (2006) in two different approaches, and it can be summarized as follow:

1. Set a minimum market capitalization;[8]
2. Exclude all public utilities and financial stocks[9];
3. Exclude foreign companies;

[6]Depending on the source there are divergent numbers. For instance, according to the book *Excess Returns: A comparative study of the methods of the world's greatest investors* Joel Greenblatt performance is approximately 31% annualized returns during the same 20 years period.

[7]Throughout the thesis good companies, quality companies or quality strategy are used interchangeably

[8]Joel recommends for individual investors a minimum of US$50 million.

[9]Including mutual funds, banks, clearing houses, brokerage firms, and insurance companies.

4. Calculate earnings yield and return on capital;
5. Ranking all companies taking into consideration the market capitalization threshold by highest earnings yield and highest return on capital.[10];
6. Invest in 20 to 30 highest ranked companies by accumulating 2 to 3 positions per month over a year period;
7. Rebalance the portfolio once a year by selling the losers before the end of the year and the winners after one year and redo the steps according to the rank criteria above;[11]
8. Proceed with this process over the long-term and a minimum of 3 to 5 years.

It is important to highlight that Greenblatt's calculation of the two building blocks of the *Magic Formula* (ROC and EY) are unique and, therefore, different from the standard definitions found in accounting books or valuation ones such as in Koller *et. al.* (2010).

4.1.1 The *Magic Formula*

Return on capital (ROC) is defined by Greenblatt (2006) as:

$$ROC = \frac{EBIT}{Net\ Working\ Capital + Net\ Fixed\ Assets} \tag{1}$$

Greenblatt (2006) calculates the ROC ratio with the 12-month trailing operating earnings before interest and taxes ($EBIT$) divided by the tangible capital employed [12] reported on the most recent balance sheet available. The ROC is used as an alternative for the most common metrics such as return on equity (ROE) and return on assets (ROA) for the reasons presented below.

The $EBIT$ measures the operating earnings produced by a company with a given amount of assets, which in our case, is the tangible capital employed. When the $EBIT$ substitutes the reported earnings, it allows investors to compare companies' performance from the operational level in which different tax rates and capital structures (*i.e.* debt levels) exists. In other words, comparisons are made possible between the various companies

[10] It is important to stress that the earnings yield is more relevant to the ranking system since the *Magic Formula* is based on value investing approach and, therefore, the price paid is crucial.
[11] Joel advises this strategy for tax purposes.
[12] Tangible capital employed is defined as net working capital + net fixed assets.

in various industries without distortions. It is important to stress out that, by Greenblatt, the depreciation and amortization expenses are approximately equal to the maintenance capital spending required to sustain the business activity and hence, the *EBIT* is simple the *EBITDA* (earnings before interest, taxes, depreciation and amortization) minus the maintenance capital expenditures.

According to Greenblatt (2006), the tangible capital employed is a better estimate of the amount of assets needed to generate a company's earnings instead of total equity or total assets as used in metrics likewise *ROE* and *ROA*. Among the reasons for using tangible capital employed is because *ROE* and *ROA* measurements suffer from several distortions since they are a function of the reported earnings, hence, functions of different tax and debt levels. Additionally, for Greenblatt (2006), intangible assets are excluded since they do not need to be replaced for companies to generate earnings, meanwhile intangible assets are counted in *ROE* and *ROA* metrics because reported equity contains *goodwill*. The net working capital represents the necessity that every company has to fund its receivables and inventory[13] but, at the same time, the benefit of being financed through its payables which, for Greenblatt, works as interest-free loans[14]. The net fixed assets denote the assets such as PP&E (Property, Plant, and Equipment) that are essential for every business. Ultimately, in Joel's words *in most cases, return on tangible capital alone (excluding goodwill) will be a more accurate reflection of a business's return on capital going forward* (Greenblatt 2006).

The earnings yield (EY) is defined by Greenblatt (2006) as follow:

$$EY = \frac{EBIT}{Enterprise\ Value} \tag{2}$$

Greenblatt (2006) calculates the EY as the ratio between the $EBIT$ as explained previously and the enterprise value (EV) instead of using the most common metrics such as price/earnings ratio (P/E) or earnings/price ratio (E/P). The Enterprise value is the sum of the market value of equity and the net interest-bearing debt. Again, the ratio is defined and used by Greenblatt (2006) to avoid distortions caused by different tax and debt levels and hence, enabling investors to make direct comparisons between companies in different industries.

According to Lancetti and Montier (2006), Greenblatt's approach of calculating the EY takes a private equity assessment of potential companies to invest through the EV. Lancetti and Montier (2006) argues that this is because Greenblatt takes into consideration

[13] Excess cash not used on business operations is excluded.
[14] Short-term interest-bearing debt is excluded from current liabilities.

the full acquisition price of the business *i.e.* the price paid by the investor includes not only the equity price but also the debt price which is the price to finance the company's operating activities.

4.2 Performance Measurements and the Sharpe Ratio

To determine securities returns, portfolios returns, cumulative returns and the risk measurement it was used the methods presented in this section. Portfolios performance during a holding period is the weighted sum of the returns of each security, and it can be calculated as below:

$$R_p = \sum_{i=1}^{n} w_{i,t\text{-}1} \left(\frac{P_{i,t}}{P_{i,t\text{-}1}} - 1 \right) \tag{3}$$

$$w_{i,t\text{-}1} = \frac{MV_{i,t\text{-}1}}{\sum_{n=1}^{n} MV_{i,t\text{-}1}} \tag{4}$$

Where:

- n = Number of securities that composes the portfolio;
- t = Current period;
- t − 1 = Previous period;
- $w_{i,t\text{-}1}$ = Weight of the i^{th} security in the previous period;
- $P_{i,t}$ = Adjusted market price of the i^{th} security at the current period;
- $P_{i,t\text{-}1}$ = Adjusted market price of the i^{th} security at the previous period;
- $MV_{i,t}$ = Market value of i^{th} security's company at the current period;
- $MV_{i,t\text{-}1}$ = Market value of i^{th} security's company at the previous period;

To calculate the cumulative returns of the portfolios we simply compound the returns of each holding period. It is important to emphasize two points. First, in our study we are not considering any taxes or transaction costs, and second, it was adopted equally-weighted portfolio as in Greenblatt (2006)[15] and consequently each stock contributes equally to the

[15]The argument for using equally-weighted portfolio is that, according to Greenblatt (2006) and Greenblatt (2010), market capitalization weighted index or portfolio will systematically invest in overpriced or

portfolios' returns. Furthermore, after measuring the cumulative returns it was calculated the geometric mean return or also known as compound annual growth rate (CAGR) of the portfolios using the following formula:

$$R_{t_0,t_i} = \left(\frac{V_{t_i}}{V_{t_0}}\right)^{\left(\frac{1}{t_n - t_0}\right)} - 1 \quad (5)$$

Where:

- R_{t_0,t_i} = Geometric mean return or $CAGR$ of the portfolio for the period;
- V_{t_i} = Value of the portfolio in local currency in the end of the holding period;
- V_{t_0} = Value of the portfolio in local currency in the beginning of the holding period;
- $t_n - t_0$ = Difference of the last period and the first period;

One way of calculating the risk-adjusted return of a portfolio is using the ex-post Sharpe ratio also known as the reward-to-variability ratio. It was introduced by Sharpe (1966) afterward revised in Sharpe (1994), and it is broadly used in the fund industry because of its simplicity. The ratio measures the excess return of a fund or portfolio per unit of risk or volatility taken. It is defined as:

$$S_p = \frac{r_p - r_f}{\sigma_p} \quad (6)$$

$$\sigma_{p,i} = \left(\sqrt{\frac{\sum_{n=1}^{n}(r_i - \overline{r})^2}{n-1}}\right) \quad (7)$$

Where:

- S_p = Sharpe ratio of the portfolio;
- r_p = Geometric mean return of the portfolio;
- r_f = Geometric mean return of the risk-free rate;
- σ_p = Annual standard deviation or annual volatility of the portfolio;
- $\sigma_{p,i}$ = Standard deviation or volatility of the portfolio in the i period;

expensive stocks because the allocations on securities increases with the increase of market prices making difficult to an investor buying and holding stocks at bargain prices.

- $\frac{\sum_{n=1}^{n}(r_i-\bar{r})^2}{n-1}$ = The unbiased sample variance of portfolio returns;

4.3 The *CAPM* Model and the Joint Hypothesis Problem

As a mean to analyze the *Magic Formula* methodology in a risk-adjusted framework, it was used the Capital Asset Pricing Model (*CAPM*) described and explained as in Cochrane (1999) and specified below.

$$r_{pt} = r_{ft} + \beta_{pt}(r_{mt} - r_{ft}) \tag{8}$$

$$r_{pt} - r_{ft} = \alpha_p + \beta_{pt}(r_{mt} - r_{ft}) \tag{9}$$

- r_{pt} = Return of the portfolio in period t;
- r_{ft} = Return of the risk-free rate in period t;
- β_{pt} = The coefficient of the explanatory variable or the beta of the portfolio;
- r_{mt} = Market Return in period t;
- α_p = Jensen's *alpha* or the portfolio return that cannot be explained by the independent variables;
- $r_{pt} - r_{ft}$ = Portfolio excess returns in period t;
- $r_{mt} - r_{ft}$ = Market excess returns in period t;

The risk-free rate used in the Sharpe ratio calculation as well as in the *CAPM* regressions is the average annualized SELIC rate[16] which is an overnight rate set by the Brazilian Central Bank to pursue the monetary policy. About the market return we have decided to use two different *proxies* for the market portfolio, the Ibovespa[17] and the IBrX-100[18]

[16]Converted to monthly rates to match with the monthly returns of the portfolios.

[17]The *Ibovespa* index is composed of 59 stocks according to market capitalization and traded volume criteria. Since its creation in 1968, it has been the benchmark for the Brazilian stock market and has had several changes in its methodology. The definition and methodology can be found on `http://www.bmfbovespa.com.br/pt_br/products/indices/indices-amplos/indice-ibovespa-ibovespa.htm`

[18]The *IBrX-100* index is composed of 100 stocks, and it is a representation of Brazilian economy. The criteria to include securities are stricter when compared to the *Ibovespa* and

indexes.

In the attempt to test the efficient market hypothesis (*EMH*) using a value investing stock picking methodology through the *CAPM* (such as in this research with the *Magic Formula*) a researcher faces a fundamental problem of modern finance called the joint hypothesis problem. The joint hypothesis arises from the fact that if you consider that the *CAPM* is the true model, then the beta of the portfolio should be the only source of variation in the cross-sectional average returns and, therefore, the value premium ought to be attributed to different *betas*. However, from Fama (1991), Fama and French (1992) and Jensen (1978), it is known that the data shows quite the opposite, in other words, the *betas* of value portfolios are lower than expected, and the *CAPM* suggests lower returns than the data presents[19]. The remaining question is: is the value premium an anomaly to profit from or is the *CAPM* the wrong model[20]? Simply, the definition of the joint hypothesis problem is that: when a value investing strategy delivers abnormal returns (*i.e.* positive *alpha*), is the model used miss-specified and mismeasuring risk or is the strategy truly finding value[21].

4.4 Data, Sources of Bias and Procedures

This section presents the data and variables. Additionally, it shows the step by step procedures adopted so any person or researcher can reproduce the results achieved in this study. Furthermore, we explain possible sources of biases and how we address them to achieve robust results.

4.4.1 Data and Sources of Bias

All the data used in our research was gathered from *Bloomberg*. The variables and the respective *Bloomberg* functions are presented in table I in the appendix.

All companies that didn't have all the variables available was automatically excluded and since the reasons for the lack of data is unknown we assume that they are random and, therefore, will not impact or bias the study. With the purpose of understanding the distribution of the *Magic Formula* variables and to recognize possible outliers we present

it can be found on: `http://www.bmfbovespa.com.br/pt_br/produtos/indices/indices-amplos/indice-brasil-100-ibrx-100-1.htm`.

[19]This is also known as the value premium puzzle.

[20]From Fama and French (2004) we have an extensive revision of the theory and empirical evidence about the *CAPM* model and its flaws.

[21]For a short but excellent description of the joint hypothesis problem see:`https://www8.gsb.columbia.edu/ideas-at-work/publication/764`.

the descriptive statistics table in appendix II and III.

As in any method used in the financial research field, this study also faces few obstacles regarding biases. Since the present study performs a backtest using the *Magic Formula* for the Brazilian stock market it is important to identify possible sources of biases as well as assure to correct them properly to achieve robust results. Both the literature about backtests and performance studies using historical data emphasize four distinct sources of biases.

The look-ahead bias is essentially a timing problem. It arises from the use of data or public information that would not have been available to the general public during the backtest or research period (Banz and Breen, 1986). For instance, an investor who wants to calculate the earnings yield for a certain company at the end of the year ought to wait a few months to have access to the audited financial statements. Several authors have acknowledged this bias such as Fama and French (1992), Asness *et. al.* (2015), Basu (1977) and Banz and Breen (1986). The latter is one of the most cited investigations regarding the recognition and the effect of both look-ahead and survivorship bias. They conclude that look-ahead bias creates higher returns for the portfolios, which in turn, biases upwards any backtest that does not take it into consideration. Banz and Breen (1986) suggests that to solve for it, one must match the portfolio formulation with the date of availability of accounting information *i.e.* adding a lag or gap between the two events. In accordance with both the Banz and Breen (1986) and the *Comissão de Valores Mobiliários* (CVM)[22] rule number 202 from 1993[23] we have applied a lag between the expected date of availability of information and the actual calculation as a measure to deal with the look-ahead bias. It was used three months for both annual and quarterly releases.

The ex-post selection bias or more known as the survivorship bias occurs because the database contains only those companies that have survived. Hence, companies that have merged, bankrupt or have exited their activities throughout the time are excluded from the sample (Banz and Breen, 1986). Also, another possible source of survivorship bias emerges when a company is inserted in the database with historical information before the point in time where the company was included in the database. In our case, since we are dealing with public companies, there is also the possibility of delisting or IPOs during the time frame analyzed. Numerous researches, including but not limited to Greenblatt and Titman (1989), Elton, Gruber and Blake (1996), Brown *et. al.* (1992) and Banz and Breen (1986), have tried to measure the impact of the survivorship bias and concluded that it is

[22]The Brazilian exchange regulatory body and the correspondent of the American Security Exchange Commission (SEC).

[23]All public companies has 3 months to disclose their annual accounting statements and 45 days to disclose their quarterly accounting statements.

clearly positive, nonetheless with different results and estimations comprising small effects and significant ones. As a way to correct this bias, Greenblatt (2006) used the Standard & Poor's Compustat database called *Point in Time* which contains the exact information available to customers at each point in time in the past. In our research, we have adopted the same reasoning; nevertheless, we perform the adjustment manually using the function *as if* in *Bloomberg*.

Selection bias or data mining in the case of a backtesting exercise is when a researcher considers many signals, as accounting measures or ratios, test them and only reports the one that performs the best (Novy Marx, 2016). In our case, we are performing a backtest of a known and given formula (*Magic Formula*) and, furthermore not comparing to any other factor or signal in the attempt to conclude which one is better. Furthermore, Greenblatt (2006) and Greenblatt (2010) states that there was no data mining in the original testing because it was simple the first two factors tested.

The overfitting bias is a case where the researcher takes into consideration many signals and optimize them through a combination or weighting of them to overfit the data available (Novy Marx, 2016). This thesis case, since there is a unique and precise methodology to follow, there is no concern with the overfitting bias.

4.4.2 Procedures

The procedures adopted had the aim to reproduce as thoroughly as possible the behavior of an investor or hedge fund manager that would have attempted to test the *Magic Formula* to achieve positive and significant *alpha*.

The first step taken into consideration was the time frame to be analyzed. Since the first edition of the book *The Little Book that Beats the Market* was published in late 2005 the period chosen was from the first day of 2006 until the last day of 2015 *i.e.* 01/01/2006 to 12/31/2015. Greenblatt (2006) and Greenblatt (2010) states that to work the strategy[24] must be employed during a minimum time of 3 years[25], thus, we can expect that the replicable method in the short-term can fail and underperform the market average.

During the replication process of the methodology, we have encountered an obstacle regarding step number 6 of the ranking system. As Greenblatt (2006) advises, the investor should accumulate 2 to 3 positions per month throughout the year until a maximum between 20 and 30 securities for diversification purposes. However, month after month we end up having the same securities to add to our portfolio and hence, replicating

[24]As pointed above in the step 8 of the ranking system.

[25]In fact, according to Greenblatt (2006): *following the formula for any three-year period in a row, the magic formula beat the market averages 95 percent of the time (160 out of 169 three-year periods tested)!*

the same method of Greenblatt would reach heavily concentrated portfolios and not diversified enough according to Greenblatt (2006) standards. To overcome this obstacle and still capture the essence and objectives aimed to be the *Magic Formula*, it was adopted a similar methodology as in Olin (2011). There are three different size of portfolios (5, 10 and 15 securities), and the resampling process is done every six months and 12 months[26]. It is vital to highlight that this is the major difference between the methodology and my replication.

To be able to filter potential stocks to be selected it was used the Equity Screen (*EQS*) function in *Bloomberg*. The first criteria used was the *BM&FBOVESPA* which filters all available financial instruments traded in the major stock exchange in Brazil[27]. The second criteria is the *Trading Status Active*. This *Bloomberg* filter takes into account only those instruments that were traded in the last 30 days. This liquidity filter was used for obvious reasons *i.e.* an investor could only buy and build a portfolio if there was enough liquidity to do so. The next criteria is also related to the liquidity issue. It was used the *Show Primary Security of company only* which discard all second class securities *i.e.* the investor would focus on the main security of a company which is, most probably, the most liquid one. To execute the fourth criteria and implement step number two of the ranking system and remove all the financial and utility companies it was used the Industry Classification Benchmark (*ICB*). It was taken into consideration that the energy and telecom were utility sectors. Also, all financial companies such as mutual funds, banks, insurance companies and clearing houses were excluded. In addition to the two liquidity measures performed and explained above, it was additionally considered (after still facing a numerous amount of illiquid stocks) the first and the second quartile of the market capitalization in the screening selection to comply with the first step of the ranking system[28].

5 Results

This section presents and analyzes the results found in the replication of the *Magic Formula* stock picking methodology applied to the Brazilian market. It shows different performance measurements, particularly, cumulative returns, compound annual growth rate (CAGR), volatility, Sharpe ratio and the regression analyses in other to find Jensen's *alpha* and to test the *EMH*. All the measurements are be calculated for six portfolios composed of three size types (5, 10 and 15 securities) and two rebalancing periods (6

[26]As mention in the *Performance Measurements and the Sharpe Ration* section all of the portfolios are equally-weighted.

[27]Among them are securities, ETF's, Real State funds, funds of funds, etc.

[28]In tables numbers IV and V in the appendix is possible to see all the security sample for each period.

months and 12 months). Furthermore, we will perform an additional exercise which consists of constructing portfolios based on each and isolated *Magic Formula* factors as a way to quantify if the buying "quality" companies or buying "cheap" stocks can outperform the combination of the factors. This added 12 portfolios more to the analysis.

5.1 Portfolios Performances, Volatility and Sharpe Ratios

5.1.1 Portfolios Performances

The portfolio returns have shown an outstanding pattern, especially after the 2008 crises. All of 18 portfolios outperformed both *Ibovespa* and *IBrX-100* benchmarks regarding cumulative returns and, hence, compound annual growth rate ($CAGR$)[29]. We start with the analysis of *Magic Formula* portfolios.

As it can be seen in *Figure 1* in the appendix an investor who invests $1 in the top 5 *Magic Formula* ranking would have $5.04 at the beginning of 2016 gross of costs and taxes. This is a cumulative return of 404.32% in 10 years period and, as expected, is the best performance compared to all portfolios since it is the most concentrated in the best ranking securities with only once a year rebalancing. The top 10 experienced an 185.61% cumulative return while the top 15 had a 128.53% cumulative returns. Even though the cumulative returns were high, the pattern to it was quite unstable. Both the top 15 and top 10 portfolios encounter four periods of underperformance which count for 40% throughout the ten years period. Meanwhile, the top 5 portfolio faced underperformance during 30% of the time. When taking into consideration the *Magic Formula* portfolios rebalanced every six months the top 5 portfolio is the best in class once again as shown in *Figure 2* in the appendix. Nevertheless, one observant investor can point that the top 15 portfolio have outperformed the top 5 and the top 10 portfolios for a considerable amount of time. The top 15 portfolio had a cumulative return of 176.03% while the top 5 had a 256.04% cumulative return. Not only was the total cumulative return of the top 10 portfolio the lowest (118.70%) but also it faced more uncertain periods and underperformed the *IBrX-100* benchmark 45% of the time during the 20 periods of 6 months through the ten years cover by this study. At the same time, both top 15 and top 5 portfolios experienced underperformance during 35% of the period.

Isolating *Magic Formula* factors and constructing portfolios according to the same idea of the ranking system we can build the "quality" companies (ranking based on ROC) and "cheap companies" (ranking according to EY) portfolios and examine if the buying quality and timing the market combination is better than strategies where you consider them

[29] The $CAGR$ for each portfolio can be seen in the table number VI in the appendix.

separately.

As *Figure 3* and *Figure 4* in the appendix show the quality portfolios have performed in a "nice" fashion, *i.e.*, there is an order where the top 5 portfolio outperformed the top 10, and the top 10 outperformed top 15. This should be the expected behavior among the portfolios since the more concentrated a portfolio is, the higher the expected risk and higher the expected returns if one considers that risk is the only explanatory variable of expected returns. As we can see, this pattern only occurs with the "quality" portfolios. The top 5, top 10 and top 15 quality portfolios resorted every 12 months had cumulative returns of 311.54%, 282.47%, and 159.95% respectively. Throughout the ten periods of 1 year, the top 5 portfolio underperformed the benchmark 30% of the time while the top 10 and top 15 performed below the *IBrX-100* 20% and 40% of the time respectively.

Meanwhile, the top 5, 10 and 15 "quality" companies portfolios that was reassembled every six months demonstrated 242.58%, 229.91% and 189.51% of cumulative returns respectively. When comparing with the benchmark that performed the best the top 15 faced a worse performance during 25% of the time while the top 10 was 35% of the time, and the top 5 portfolio was 30% of the time during the 20 periods of 6 months from of 2006 until 2015.

The portfolios formed by the *EY* ranking demonstrated an unusual behavior. The top 10 portfolio in both the 6 and 12 month rebalancing periods had the best returns suggesting that in this contrarian strategy there might be an optimal portfolio size to achieve higher returns[30]. However, when compared to the quality portfolios of the same size all "cheap" portfolios underperformed besides the top 10 portfolio resorted every six months. This insinuates that the *ROC* factor contributes in a greater degree for the *Magic Formula* portfolios performance, although we have to call attention to the fact that the *Magic Formula* ranking it is not a simple linear combination of the two separate ranking factors.

As one can see in the *Figure 5* and *Figure 6* in the appendix, the cumulative returns of the top 15, top 10 and top 5 "cheap" portfolios rebalanced every 12 months were 115.62%, 230.78%, and 182.74% respectively whereas the same portfolios resorted every 6 months were 135.39%, 254.73%, and 89.95%. They have experienced a similar amount of underperformance periods when compared to other portfolios. The top 15 portfolio reassembled every 12 months underperformed during 30% of the period as the top 10 and top 5 portfolio underperformed during 40% of the time. Concurrently, the top 15 and top 10 6 months portfolios performed below the benchmark 40% of the time and the top 5 35%.

[30]More research, including *out-of-sample* studies, ought to be done in order to conclude if there is any relation among contrarian strategy, portfolio size, and abnormal returns.

5.1.2 Volatility and Sharpe Ratios

As demonstrated above all the portfolios have outperformed both benchmarks. The question is: was this due to skill and winning strategies, or was it due to more risk? To assess and answer this question we have calculated the annual volatilities of each portfolio to comprehend the risks involved in those strategies and also its respective Sharpe ratios as a simple way to measure a risk-adjusted performance.

As *Figure 7* shows in the appendix, 14 of the portfolios are concentrated in the upper left side of the quadrant. This means that majority of the portfolios not only outperformed both the *Ibovespa* and *IBrX-100* but also with less underlying risk[31]. The remaining 4 portfolios are composed of the ones shown on in the same *Figure 7* in the appendix. The top 5 *Magic Formula* portfolio case (resorted every 12 months) the annualized volatility is 23.70% and 1.57% more than the *Ibovespa* and 2.53% more than the *IBrX-100.* Albeit the top 5 *Magic Formula* portfolio had more risk the degree of outperformance was quite significant with a *CAGR* of 19.7% while the *Ibovespa* and *IBrX-100* had a 2.92% and 6.03% *CAGR* respectively. The worst portfolio is the top 5 *EY* (rebalanced every 6 months) with a performance of 6.81% which is close to *IBrX-100* performance, nonetheless bearing a risk of 24.01% and being only less risky than the top 5 *Magic Formula* portfolio (resorted every 6 months) with an annualized standard deviation of 24.36%[32].

When taking into consideration not only the nominal returns but also the excess returns[33] through the *ex-post* Sharpe ratios we can observe a different picture from the previous graph[34]. As *Figure 8* in the appendix shows 9 or half of all portfolios have presented negative Sharpe ratios with ratios ranging between *-0.01* and *-0.24*. The worst performance according to the Sharpe ratio was the top 5 "cheap" portfolio built every six months while the best risk-adjusted returns by the reward-to-variability ratio were presented by the top 5 *Magic Formula* portfolio resorted every 12 months. It is interesting to point out that the all top 15 and top 10 *Magic Formula* portfolio sizes displayed negative Sharpe ratios while the top 5 presented positive numbers. That could suggest that in a risk-adjusted excess returns environment the intelligent investor should pick more concentrated portfolios. Whereas only the top 10 portfolios of the *EY* strategy show positive Sharpe ratios. At the same time, we can notice that the *ROC* portfolios are the most consistent ones with only the top 15 portfolio resorted every 12 months showing negative Sharpe ratio.

[31]Measure by the annualized standard deviation.

[32]For the detailed data points see appendix tables number VI and VII.

[33]The returns above the annualized risk-free rate.

[34]It is indispensable to stress the fact that Brazil has one of the highest interest rates in the world, and, therefore, one of the highest theoretically risk-free rate.

5.2 Linear Regression Results

An additional and more robust manner to evaluate the risk-adjusted performance of the *Magic Formula* methodology and the two separate factors portfolios is to use a traditional asset pricing model framework. To do so, it is employed *CAPM* and regress the excess market returns using both the *IBrX-100* and the *Ibovespa* indices with the excess returns of all portfolios. As a result, we find all portfolios' *betas* and *alphas*.

As expected, all the *betas* of the *Magic Formula* portfolios were statistically significant at 99% level and below 1, more precisely, between 0.60 and 0.77[35]. This shows that even after outperforming the benchmarks with quite substantial margin the portfolios, according to the *CAPM*, were less exposed to market risk. Regarding the *alphas*, as expected, they were all positive, however, only the top 5 portfolio resorted every 12 months had a statistically significant *alpha* at 90% level[36].

All the "quality" portfolios which consist of ranking according to *ROC* factor presented *betas* below 1 and statistically significance at 99% level. The range was between *0.57* and *0.79*[37]. Again, this shows that even outperforming the benchmarks with a wide margin the portfolios were less exposed to market risk. The expected outcome for the *alphas* was met because all of them were positive, even though only two portfolios were statistically significant. The top 10 portfolio resorted every 12 months achieve the highest significance level of all portfolios of all methodologies with 95% level of significance when regressed against the *Ibovespa* and its *alpha* was 0.74%. Additionally, the top 10 portfolio resorted every 6 months showed an *alpha* of 0.62% at 90% level of significance[38].

The last set of portfolios are the ones formed by the *EY* factor which we call the "cheap" portfolios. Again, all the *betas* were positive, less than 1 and statistically significant at 99% level ranging between 0.64 and 0.79[39].Regarding the *alphas* all were positive but without statistical significance at any meaningful level. The only portfolio that showed statistical significance at 90% level was the top 10 rebalanced every 6 months and regressed against the *Ibovespa*[40].

[35]For the complete table of all *Magic Formula betas* see the table number IX in the appendix.

[36]All of the other *alphas* were not statistically significant at any meaningful level as it can be seen in the table VIII in the appendix.

[37]For the complete table of all *ROC* portfolios *betas* see the table number XI in the appendix.

[38]For all *alphas* see table number X in the appendix.

[39]For the complete table of all *EY* portfolios *betas* see the table number XIII in the appendix.

[40]For the all *alphas* see table number XII in the appendix.

6 Discussion and Conclusion

As demonstrated above we could show that the backtest of the *Magic Formula* methodology for the Brazilian stock market has outperformed both *Ibovespa* and *IBrX-100* benchmarks during the period from 2006 until 2015. In fact and more precisely not only all the *Magic Formula* portfolios have outperformed in cumulative and, hence, in $CARG$ terms but also our additional exercise with the ROC and EY separately factors portfolios did show the same qualitative results despite different performance numbers. Furthermore, all the portfolios' *betas* were below 1 and statistically significant at 99% level showing that the portfolios did not bear more systematic risk despite outperforming our benchmarks as standard finance theory would suggest. It is important to emphasize though that the *Magic Formula* portfolios have experienced underperformance periods that comprehend 30% to 45% of the total period analyzed demonstrating that there are risks involved in such strategy, and there might be potential losses at least in opportunity cost terms as well as in absolute terms in more severe scenarios such as in the financial crises of 2008-2009.

Comparing our findings with the literature presented we can state that our results are in accordance with both Joel Greenblatt conclusions (Greenblatt (2006) and (2010)) and the empirical literature that shows that the *Magic Formula* presents positive cumulative returns and outperformed a market portfolio *proxy*. More precisely, the replication of the *Magic Formula* in different stock markets and different time periods seems to demonstrate that Joel's methodology is, in fact, a stock picking strategy that works, and it's not due to pure luck or randomness.

Notwithstanding, when taking into consideration an asset price framework through the $CAPM$ model, one cannot state with a high level of certainty that the *Magic Formula* methodology will consistently deliver above risk-adjusted returns, *i.e.* positive and significant *alpha*. The regression results of the *Magic Formula* portfolios demonstrate that, although all *alphas* were positive, only the 0.98% *alpha* of the top 5 portfolio resorted every 12 months regressed on the *Ibovespa* presented a 90% level of significance while all the others didn't present any meaningful level of significance. Moreover, one can argue in spite of the fact that the top 5 *Magic Formula* portfolio resorted every 12 months has a positive and significant *alpha* the regression was on *Ibovespa* which is an imperfect market portfolio *proxy* because it is not broadly diversified and it could be easily replaced by the *IBrX-100 proxy* yielding a not significant and robust results as demonstrated in this study. Furthermore, the previous argument could also be used for ROC and EY portfolios that achieve some level of significance such as the ROC top 10 resorted every 12 months regressed on *Ibovespa* and the EY top 10 resorted every 6 months regressed on *Ibovespa*.

To conclude, from an *EMH* standpoint, there is more risk associated to the *Magic Formula* strategy than the *CAPM' betas* are showing and that the model cannot capture. Albeit the results presented here, we believe that the *Magic Formula* methodology add invaluable insights to any stock picking strategy due to its value investing components which are complex to convey with standard finance theory. Future researches that would be relevant to examine could be the addition of transaction costs to investigate if there is any qualitative change in the results and the utilization of more contemporary models such as the Fama and French tree-factors model or the Carhart (1997) four-factor model.

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8 Appendix

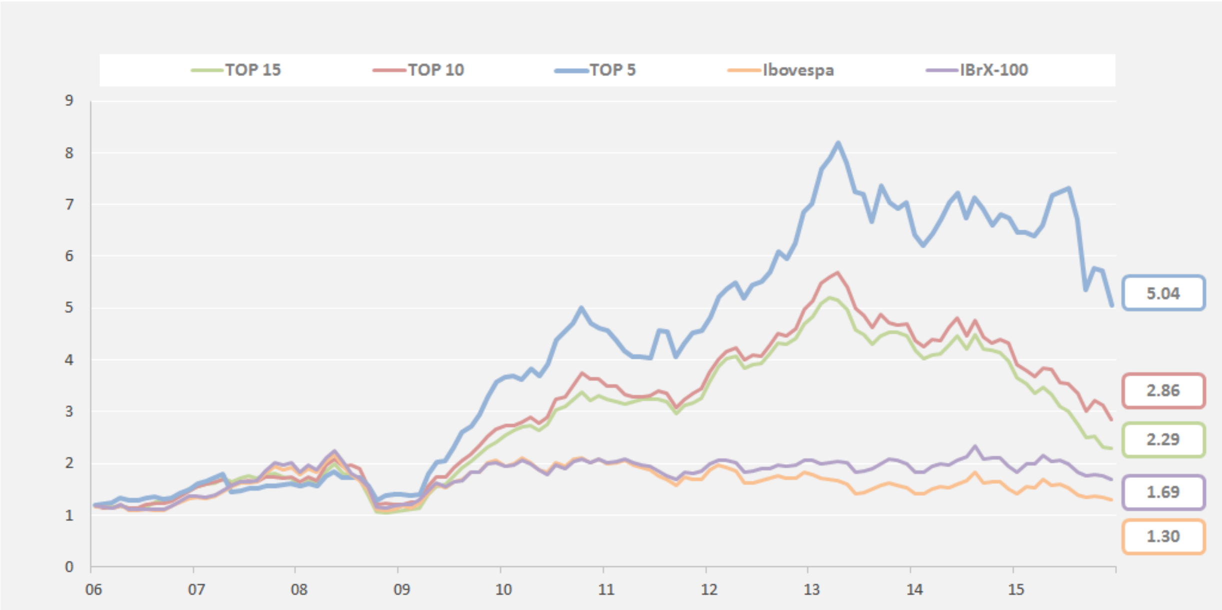

Figure 1: *Cumulative Returns of Magic Formula Strategy for 12 Months Rebalancing Portfolios.*

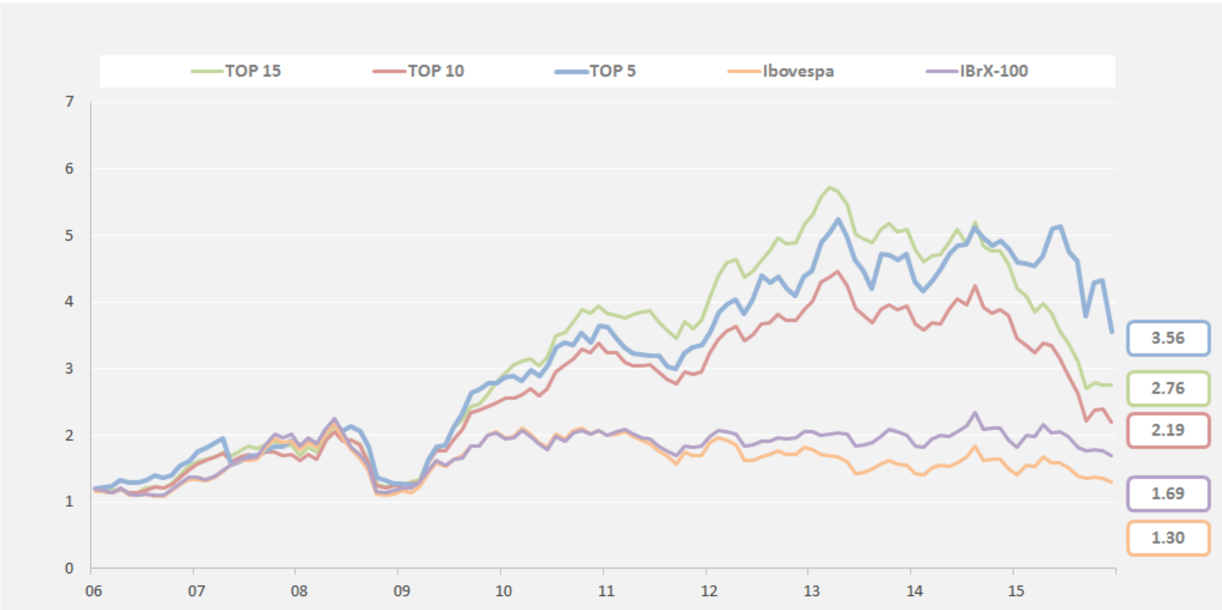

Figure 2: *Cumulative Returns of Magic Formula Strategy for 6 Months Rebalancing Portfolios.*

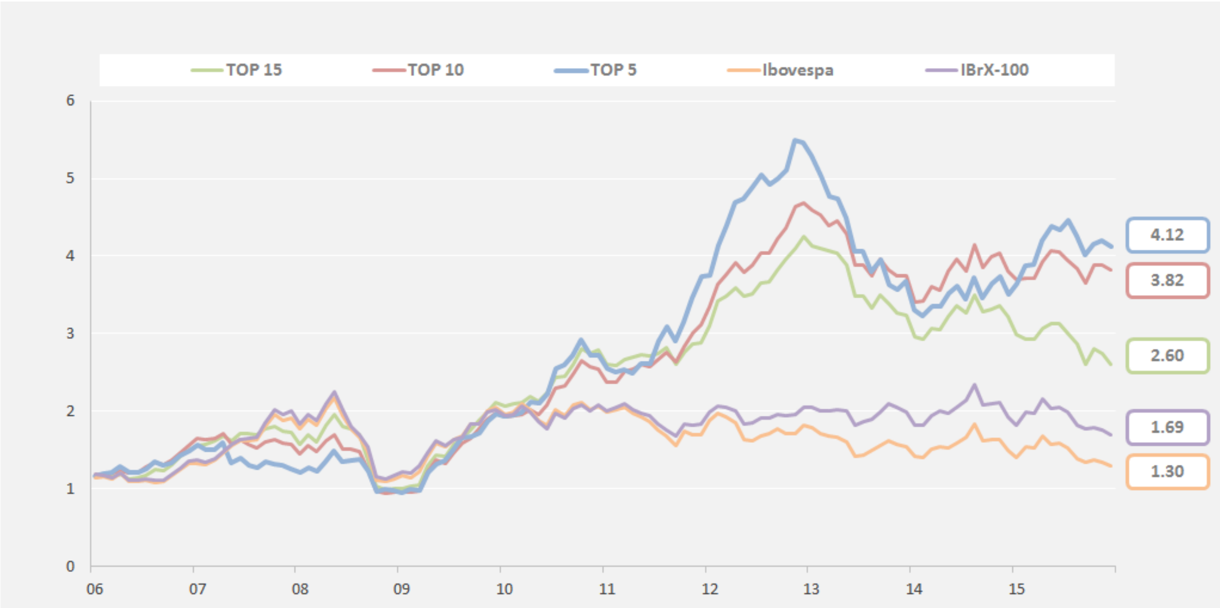

Figure 3: *Cumulative Returns of Quality Strategy for 12 Months Rebalancing Portfolios.*

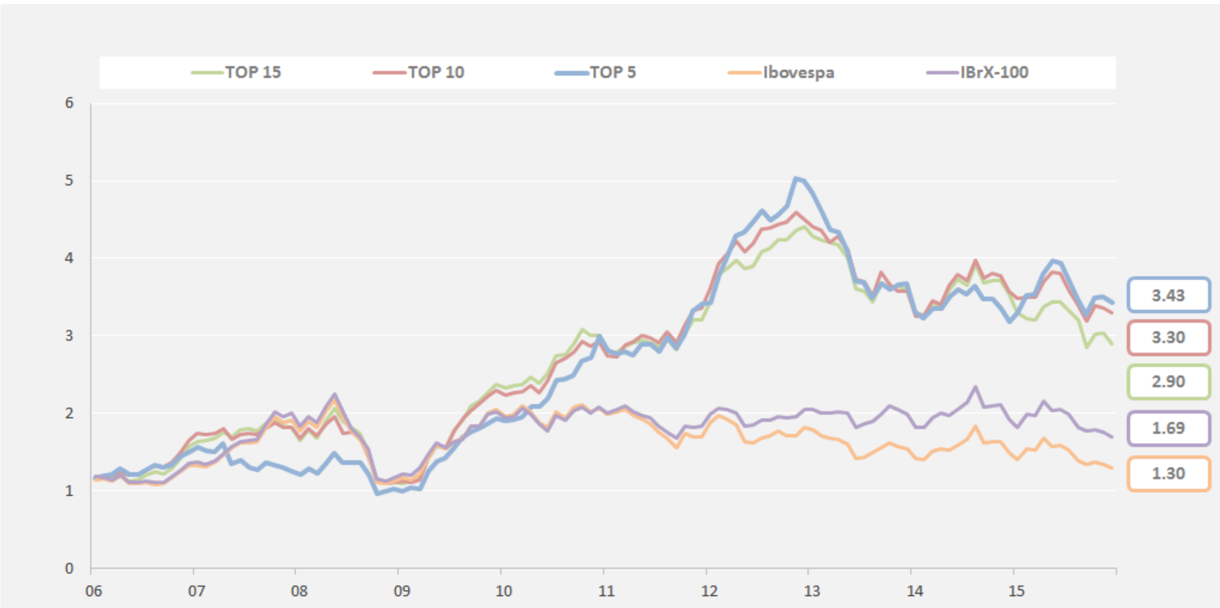

Figure 4: *Cumulative Returns of Quality Strategy for 6 Months Rebalancing Portfolios.*

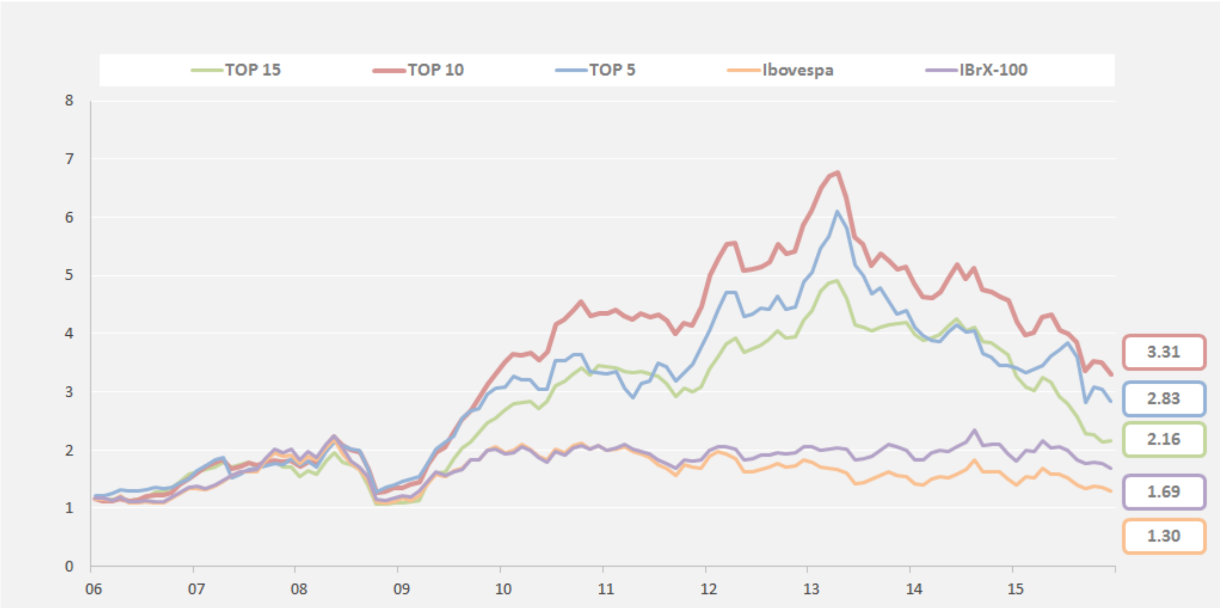

Figure 5: *Cumulative Returns of Cheap Strategy for 12 Months Rebalancing Portfolios.*

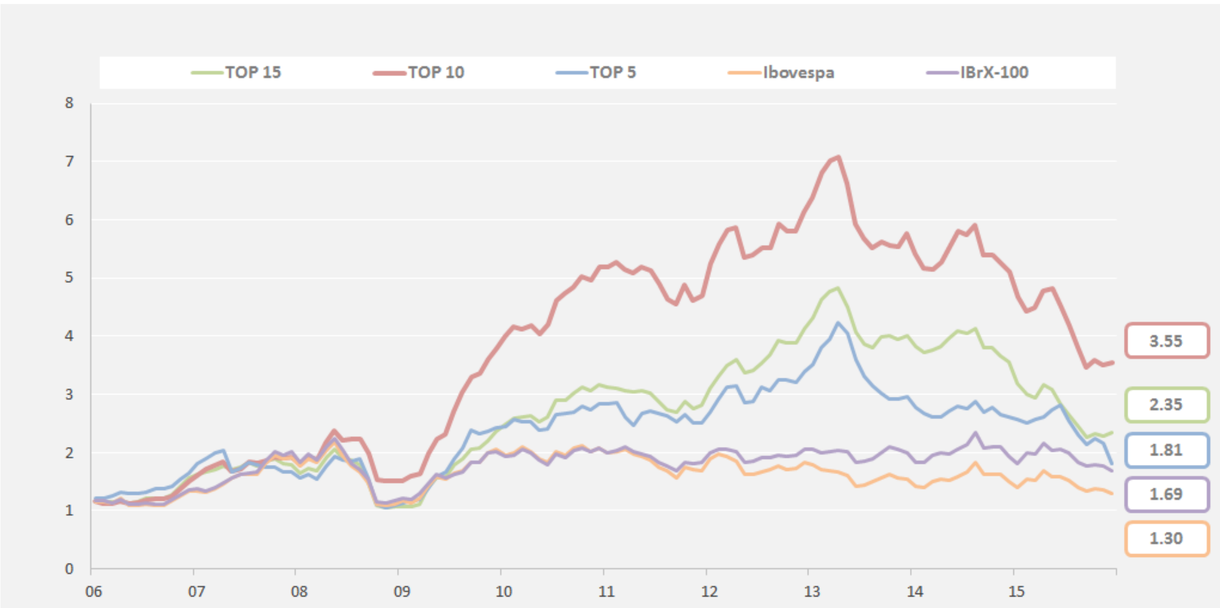

Figure 6: *Cumulative Returns of Cheap Strategy for 6 Months Rebalancing Portfolios.*

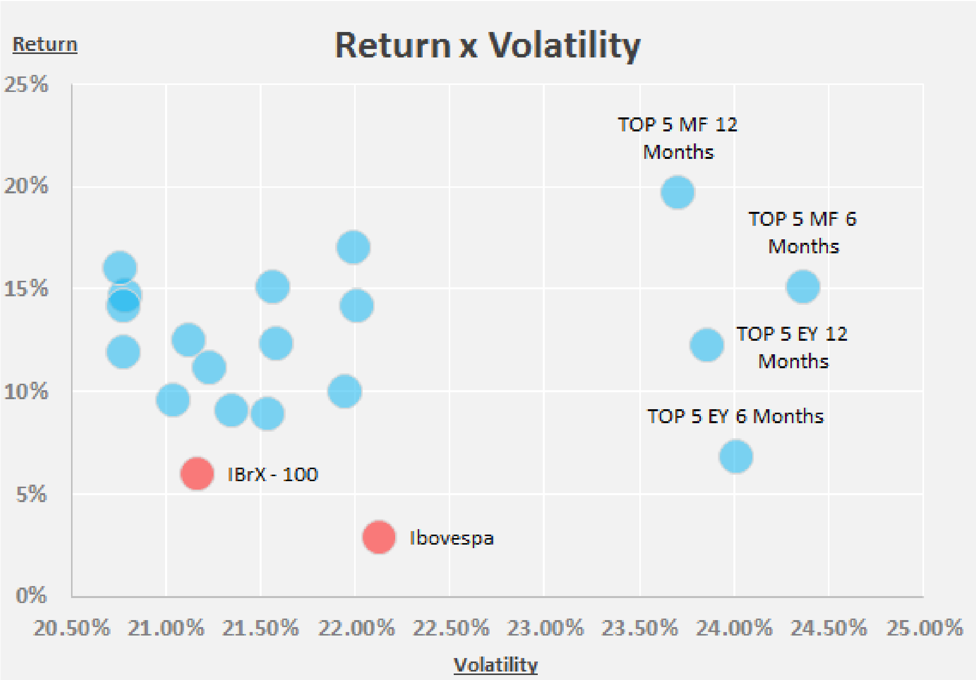

Figure 7: *Scatter Plot of Portfolios' Return x Volatility.*

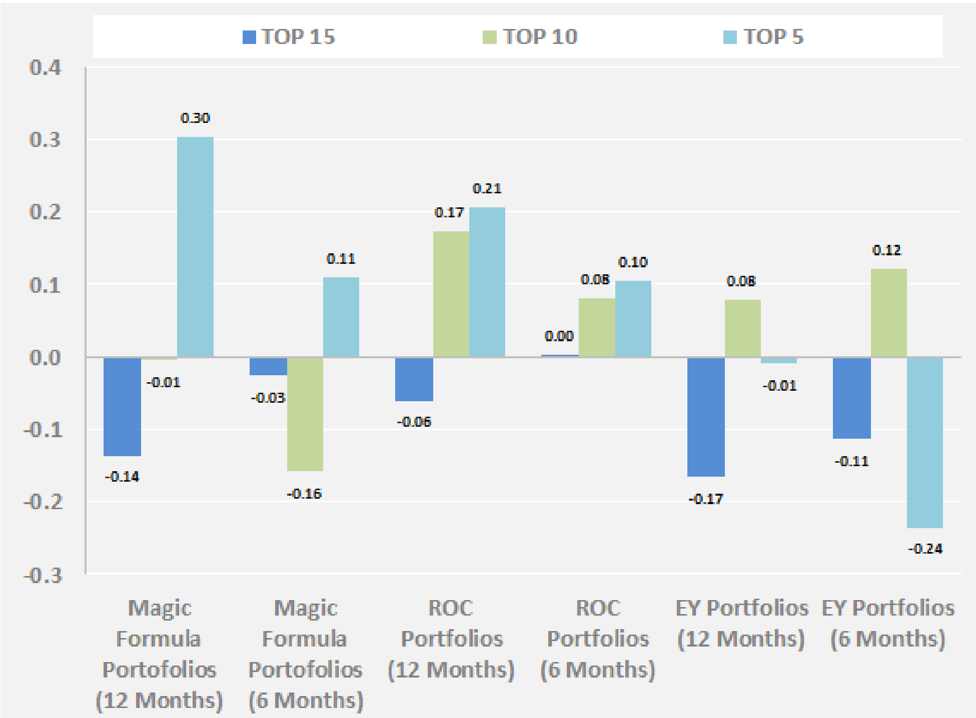

Figure 8: *Portfolios' Sharpe Ratios.*

Table I
Variables and its Correspondent *Bloomberg* Function

Variable	**Bloomberg Function**
Adjusted Price	*TOT_RETURN_INDEX_GROSS_DVDS*
Current Assets	*BS_CUR_ASSET_REPORT*
Current Liabilities	*BS_CUR_LIAB*
Current Market Capitalization	*CUR_MKT_CAP*
Earnings before Interest and Taxes (*EBIT*)	*EBIT*
Goodwill	*BS_GOODWILL*
Net Debt	*NET_DEBT*
Risk-free Rate	*BZSELICA Index*
Short Term Borrowing	*BS_ST_BORROW*
Total Assets	*BS_TOT_ASSET*

The table reports the variables applied to measure the return on capital and the earnings yield used to ranking and formulating the portfolios as well as other variables such as adjusted price and risk-free rate to calculate returns, excess returns and to be used in the regression analyses. The Adjusted Price is the price adjusted for stock splits, dividends and other distributions and corporate events and is calculated by Bloomberg as a total return index. Current assets are the summation of Cash & Cash Equivalents, Marketable Securities & other short-term investments, accounts & notes receivable, inventories, and other current assets reported in the last balance sheet and it includes accrued income. Current liabilities is the summation of accounts payable, short-term borrowings, and other short-term liabilities reported in the last balance sheet. The current market capitalization is the total current market value of all of a company's outstanding shares stated in the pricing currency (in our case the local currency Brazilian Real. The *EBIT* is the earnings before interest expenses and income taxes and for industrials companies is the same as operating income (losses) according to Bloomberg. The goodwill is the excess price paid over the fair market value of assets in an acquisition accounted for by the purchase method. This amount is included in other intangible assets on the balance sheet and is net of accumulated amortization. Metric that shows a company's overall debt situation by netting the value of a company's liabilities and debts with its cash and other similar liquid assets. Calculated as: (Total Debt - Financial Subsidiary Debt) - (Cash & Marketable Securities + Collaterals for Debt - Financial Subsidiary Cash and Cash Equivalents - Financial Subsidiary Marketable Securities). The risk-free rate is the average annualized monthly SELIC rate.The short-term borrowing includes bank overdrafts, short-term debts and borrowings, repurchase agreements (repos) and reverse repos, short-term portion of long-term borrowings, current obligations under capital (finance)leases, current portion of hire purchase creditors, trust receipts, bills payable, bills of exchange, bankers acceptances, interest bearing loans, and short term mandatory redeemable preferred stock. Net with unamortized premium or discount on debt and may include fair value adjustments of embedded derivative. Total assets are the total of all short and long-term assets as reported on the Balance Sheet. Source: Bloomberg (2016).

Table II

Descriptive Statistics of the $\boldsymbol{EY}$ Variable

Period	Min	Max	Average	Standard Deviation
01/01/2006	0.82%	31.26%	9.92%	8.02%
07/01/2006	-1.47%	19.56%	7.83%	6.13%
01/01/2007	-0.51%	18.85%	6.60%	4.97%
07/01/2007	-2.51%	16.75%	4.84%	4.07%
01/01/2008	-5.77%	14.16%	4.26%	4.29%
07/01/2008	-11.41%	15.67%	4.26%	10.75%
01/01/2009	-11.84%	40.94%	9.15%	9.96%
07/01/2009	-6.08%	65.45%	8.42%	10.61%
01/01/2010	-15.18%	18.23%	4.24%	5.75%
07/01/2010	-19.42%	16.15%	4.96%	5.54%
01/01/2011	-96.55%	33.42%	6.15%	11.44%
07/01/2011	-18.48%	68.68%	8.05%	7.95%
01/01/2012	-33.63%	24.85%	7.10%	7.18%
07/01/2012	-94.38%	26.20%	5.43%	11.33%
01/01/2013	-269.72%	23.71%	3.33%	26.46%
07/01/2013	-25.22%	624.54%	10.99%	56.84%
01/01/2014	-174.31%	30.81%	3.18%	23.16%
07/01/2014	-497.43%	43.48%	-0.36%	54.59%
01/01/2015	-1062.61%	42.72%	-1.13%	96.55%
07/01/2015	-48.03%	553.65%	13.95%	54.85%

The table shows the minimum, maximum, average and standard deviation of the sample composed by all stocks that had available information in order to calculate the EY before the formation of the *Magic Formula* portfolios.

Table III

Descriptive Statistics of the *ROC* Variable

Period	Min	Max	Average	Standard Deviation
01/01/2006	2.88%	55.57%	19.77%	13.68%
07/01/2006	-4.72%	59.89%	16.66%	14.28%
01/01/2007	-1.59%	60.51%	16.07%	13.09%
07/01/2007	-8.55%	63.98%	13.79%	14.77%
01/01/2008	-11.08%	62.68%	12.76%	14.66%
07/01/2008	-14.32%	54.63%	10.75%	14.12%
01/01/2009	-15.67%	59.25%	12.17%	14.15%
07/01/2009	-26.57%	70.93%	12.68%	16.32%
01/01/2010	-49.75%	85.61%	11.29%	20.72%
07/01/2010	-28.59%	69.14%	13.74%	17.67%
01/01/2011	-73.97%	180.97%	14.03%	23.84%
07/01/2011	-27.07%	172.56%	14.93%	20.55%
01/01/2012	-11.03%	99.29%	13.11%	15.72%
07/01/2012	-23.88%	177.18%	13.11%	21.10%
01/01/2013	-16.21%	369.15%	15.30%	36.64%
07/01/2013	-13.72%	166.78%	11.93%	19.15%
01/01/2014	-534.44%	160.66%	6.68%	54.32%
07/01/2014	-2614.02%	4156.86%	25.5%	439.08%
01/01/2015	-2874.66%	153.68%	-12.11%	260.56%
07/01/2015	-878.42%	838.38%	12.96%	111.98%

The table shows the minimum, maximum, average and standard deviation of the sample composed by all stocks that had available information in order to calculate the *ROC* before the formation of the *Magic Formula* portfolios.

Table IV

Securities Sample and Filtering Process for each Period

	01/01/2006	07/01/2006	01/01/2007	07/01/2007	01/01/2008	07/01/2008	01/01/2009	07/01/2009	01/01/2010	07/01/2010
Screening Criteria										
Exchange: *BM&FBOVESPA*	240	241	267	320	345	334	333	343	362	344
Trading Status: *Active*	240	241	267	320	344	334	333	343	362	344
Security Attributes:										
Show Primary Security of Company Only	91	103	115	147	182	173	172	178	196	189
Sector (*ICB*):										
Financials, Telecommunications, Utilities	50	61	69	84	95	97	96	97	104	105

The screening criteria are *Bloomberg* functions that can be replicated directly. The numbers represent the amount of public companies available as a function of the filtering process. For instance, in the first period of this table and after the filtering process the amount of companies available to invest was 50 while the largest portfolios were composed by 15 securities which in turn represents 30%. On the other hand, when taking the last period of this table into consideration one can see that the amount of options available for an investor, after filtering, went up to 105 and the biggest portfolios only represents 14.28% of the sample (*i.e.* 15 out of 105). It is important to notice that in order to achieve the same results as in this study one needs to use the *Bloomberg* function "as if" and select the date as the table shows.

Table V
Securities Sample and Filtering Process for each Period

	01/01/2011	07/01/2011	01/01/2012	07/01/2012	01/01/2013	07/01/2013	01/01/2014	07/01/2014	01/01/2015	07/01/2015
Screening Criteria										
Exchange: *BM&FBOVESPA*	844	837	762	777	782	797	855	850	834	823
Trading Status: *Active*	820	811	729	738	743	750	802	795	788	785
Security Attributes:										
Show Primary Security of Company Only	409	411	393	394	421	424	429	427	424	420
Sector (*ICB*):										
Financials, Telecommunications, Utilities	242	238	225	230	240	250	252	251	250	247

The screening criteria are *Bloomberg* functions that can be replicated directly. The numbers represent the amount of public companies available as a function of the filtering process. For instance, in the first period of this table and after the filtering process the amount of companies available to invest was 242 while the largest portfolios were composed by 15 securities which in turn represents 6.2%. On the other hand, when taking the last period of this table into consideration one can see that the amount of options available for an investor, after filtering, went up to 247 and the biggest portfolios only represent 6.07% of the sample (*i.e.* 15 out of 2047). It is important to notice that in order to achieve the same results as in this study one needs to use the *Bloomberg* function "as if" and select the date as the table shows.

Table VI
Compound Annual Growth Rate (*CAGR*) of each Portfolio

Portfolios	**TOP 15**	**TOP 10**	**TOP 5**
Magic Formula (12 Month)	9.62%	12.37%	19.7%
Magic Formula (6 Month)	11.94%	9.08%	15.15%
ROC (12 Months)	11.20%	16.07%	17.02%
ROC (6 Months)	12.54%	14.18%	14.66%
EY (12 Months)	8.91%	14.22%	12.24%
EY (6 Months)	9.98%	15.11%	6.81%

The amount of months in parenthesis indicates the rebalancing period for each portfolio.

Table VII
Annual Volatilities of each Portfolio

Portfolios	**TOP 15**	**TOP 10**	**TOP 5**
Magic Formula (12 Month)	21.04%	21.58%	23.70%
Magic Formula (6 Month)	20.77%	21.35%	24.36%
ROC (12 Months)	21.23%	20.75%	21.99%
ROC (6 Months)	21.12%	20.78%	20.79%
EY (12 Months)	21.54%	22.01%	23.86%
EY (6 Months)	21.94%	21.57%	24.01%

The amount of months in parenthesis indicates the rebalancing period for each portfolio. All numbers are annualized standard deviations.

Table VIII
***Alphas* of *Magic Formula* Portfolios**

Regression on	**TOP 15**	**TOP 10**	**TOP 5**
IBrX-100 (12 Months)	0.19%	0.37%	0.86%
Ibovespa (12 Months)	0.33%	0.51%	0.98%*
IBrX-100 (6 Months)	0.34%	0.15%	0.61%
Ibovespa (6 Months)	0.48%	0.29%	0.74%

Jensen's *alpha* is a measure of risk-adjusted returns in relation to market returns. The amount of months in parenthesis indicates the rebalancing period for each portfolio. The * is the level of significance of *alpha* where * represents 90% level of significance, ** represents the 95% level of significance and *** represents the 99% level of significance.

Table IX
***Betas* of *Magic Formula* Portfolios**

Regression on	**TOP 15**	**TOP 10**	**TOP 5**
IBrX-100 (12 Months)	0.76***	0.73***	0.62***
Ibovespa (12 Months)	0.73***	0.71***	0.60***
IBrX-100 (6 Months)	0.77***	0.75***	0.71***
Ibovespa (6 Months)	0.74***	0.72***	0.68***

The *betas* measure the correlated relative volatility and capture the sensitivity of portfolios' returns to market returns. The amount of months in parenthesis indicates the rebalancing period for each portfolio. The * is the level of significance of *beta* where * represents the 90% level of significance, ** represents the the 95% level of significance and *** represents the the 99% level of significance.

Table X
***Alphas* of *ROC* Portfolios**

Regression on	TOP 15	TOP 10	TOP 5
IBrX-100 (12 Months)	0.30%	0.60%	0.65%
Ibovespa (12 Months)	0.45%	0.74%**	0.77%
IBrX-100 (6 Months)	0.39%	0.48%	0.48%
Ibovespa (6 Months)	0.54%	0.62%*	0.59%

Jensen's *alpha* is a measure of risk-adjusted returns in relation to market returns. The amount of months in parenthesis indicates the rebalancing period for each portfolio. The * is the level of significance of *alpha* where * represents 90% level of significance, ** represents the 95% level of significance and *** represents the 99% level of significance.

Table XI
***Betas* of *ROC* Portfolios**

Regression on	TOP 15	TOP 10	TOP 5
IBrX-100 (12 Months)	0.77***	0.72***	0.60***
Ibovespa (12 Months)	0.74***	0.70***	0.58***
IBrX-100 (6 Months)	0.79***	0.73***	0.59***
Ibovespa (6 Months)	0.75***	0.70***	0.57***

The *betas* measure the correlated relative volatility and capture the sensitivity of the portfolios' returns to market returns. The amount of months in parenthesis indicates the rebalancing period for each portfolio. The * is the level of significance of *beta* where * represents the 90% level of significance, ** represents the the 95% level of significance and *** represents the the 99% level of significance.

Table XII
***Alphas* of *EY* Portfolios**

Regression on	**TOP 15**	**TOP 10**	**TOP 5**
IBrX-100 (12 Months)	0.14%	0.51%	0.40%
Ibovespa (12 Months)	0.28%	0.65%	0.52%
IBrX-100 (6 Months)	0.23%	0.55%	0.04%
Ibovespa (6 Months)	0.39%	0.69%*	0.17%

Jensen's *alpha* is a measure of risk-adjusted returns in relation to market returns. The amount of months in parenthesis indicates the rebalancing period for each portfolio. The * is the level of significance of *alpha* where * represents 90% level of significance, ** represents the 95% level of significance and *** represents the 99% level of significance.

Table XIII
***Betas* of *EY* Portfolios**

Regression on	**TOP 15**	**TOP 10**	**TOP 5**
IBrX-100 (12 Months)	0.74***	0.74***	0.67***
Ibovespa (12 Months)	0.72***	0.71***	0.64***
IBrX-100 (6 Months)	0.79***	0.72***	0.71***
Ibovespa (6 Months)	0.76***	0.70***	0.67***

The *betas* measure the correlated relative volatility and capture the sensitivity of the portfolios' returns to market returns. The amount of months in parenthesis indicates the rebalancing period for each portfolio. The * is the level of significance of *beta* where * represents the 90% level of significance, ** represents the the 95% level of significance and *** represents the the 99% level of significance.