UNIVERSITY OF GOTHENBURG
SCHOOL OF BUSINESS, ECONOMICS AND LAW

WORKING PAPERS IN ECONOMICS

No 641

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January 2016

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Genuine Saving and Conspicuous Consumption[**]

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Abstract

Much evidence suggests that people are concerned with their relative consumption, i.e., their own consumption relative to that of others. Yet, conspicuous consumption and the corresponding social costs have so far been ignored in savings-based indicators of sustainable development. The present paper examines the implications of relative consumption concerns for measures of sustainable development by deriving analogues to genuine saving when people are concerned with their relative consumption. Unless the positional externalities have been fully internalized, an indicator of such externalities must be added to genuine saving to arrive at the proper measure of intertemporal welfare change. A numerical example based on U.S. and Swedish data suggests that conventional measures of genuine saving (which do not reflect conspicuous consumption) are likely to largely overestimate this welfare change. We also show how relative consumption concerns affect the way public investment ought to be reflected in genuine saving.

JEL classification: D03, D60, D62, E21, H21, I31, Q56.

Keywords: Welfare change, investment, saving, relative consumption.

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1. Introduction

The concept of genuine saving has gained much attention in literature on welfare measurement in dynamic economies. Genuine saving is an indicator of comprehensive net investment in the sense of summarizing the value of all capital formation undertaken by society over a time period. Earlier research shows that the genuine saving constitutes an exact measure of welfare change over a short time interval if the resource allocation is first best.[1] Furthermore, in the aftermath of the World Commission on Environment and Development, genuine saving has also become an indicator of sustainable development. The World Commission defines the development to be sustainable if it meets "the needs of the present without compromising the ability of future generations to meet their own needs" (Our Common Future, 1987, page 54)[2]. One possible interpretation is that sustainable development requires welfare to be non-declining, meaning that genuine saving becomes an exact indicator of sustainable development over a short time interval.[3] Another is that the instantaneous utility must not exceed its maximum sustainable level, on the basis of which Pezzey (2004) shows that non-positive genuine saving constitutes an indicator of unsustainable development, although positive genuine saving does not necessarily imply that development is sustainable.[4] In either case, genuine saving gives information of clear practical relevance for economic welfare.[5]

Yet, the literature dealing with genuine saving has so far focused on traditional neoclassical textbook models, where people derive utility solely from their own absolute consumption of goods and services (broadly defined). As such, it neglects the possibility

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[1] The seminal contributions are Pearce and Atkinson (1993) and Hamilton (1994, 1996). See also Hamilton (2010) for a recent overview of the literature and van der Ploeg (2010) for a political economy analysis of genuine saving. A public economics approach is taken by Aronsson et al. (2012), who derive a second-best analogue to genuine saving in a representative-agent economy with distortionary taxes and public debt accumulation, while Li and Löfgren (2012) address genuine saving in an economy where growth is stochastic. See also the recent empirical application by Greasley et al. (2014), who test the welfare significance of genuine saving based on historical data from Britain, and Fleurbaey (2015) for an examination of relationships between social welfare and measures of sustainable development (including genuine saving).

[2] This report is often referred to as *The Brundtland Report*.

[3] See, e.g., Arrow et al. (2003).

[4] See also Pezzey (1993) and Asheim (1994).

[5] This is further emphasized by the attention paid to genuine saving by the World Bank, which regularly publishes estimates of genuine saving for a large number of countries.

discussed in behavioral economics literature that people also enjoy consuming more, and dislike consuming less than others – an idea that appeared rather obvious to many leading economists of the past such as Adam Smith, John Stuart Mill, Karl Marx, Alfred Marshall, Thorstein Veblen, and Arthur Pigou, before it became unfashionable in the beginning of the 20th century.

The purpose of the present paper is to examine how relative consumption concerns, through a preference for "keeping-up-with-the-Joneses," affect the principles for measuring welfare change. It will then present a correspondingly adjusted measure of genuine saving. Arguably, such a study is relevant for several reasons. First, there is now a large body of empirical evidence showing that people are concerned with their relative consumption, (and not just their absolute consumption as in standard economic models).[6] Questionnaire-experimental research often concludes that 30-50 percent of an individual's utility gain from increased consumption may actually be due to increased relative consumption (e.g., Alpizar et al., 2005; Solnick and Hemenway, 2005; Carlsson et al., 2007). Similarly, happiness-based studies typically find that a large (or even dominating) share of consumption-induced well-being in industrialized countries is due to relative effects (e.g., Luttmer, 2005; Easterlin, 2001; Easterlin et al., 2010). In turn, this may distort the incentives underlying consumption and capital formation. Second, based on the estimates referred to above, wasteful conspicuous consumption is likely to result in significant welfare costs, which – if not properly internalized – may change the principles for calculating welfare change-equivalent measures of saving. Indeed, we show that the more positional people are on average, the more will the conventional measure of genuine saving (where people are assumed not to have positional preferences) overestimate the true welfare change. Third, recent literature shows that optimal policy rules for public expenditure are modified in response to relative consumption concerns,[7] suggesting that such concerns may also affect the value of public investment in the context of genuine saving. This will be further discussed below.

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[6] See, e.g., Easterlin (2001), Johansson-Stenman et al. (2002), Blanchflower and Oswald (2004), Ferrer-i-Carbonell (2005), Luttmer (2005), Solnick and Hemenway (2005), Carlsson et al. (2007), Clark and Senik (2010), and Corazzini, Esposito, and Majorano (2012). See also Fliessbach et al. (2007) and Dohmen et al. (2011) for evidence based on brain science and Rayo and Becker (2007) for an evolutionary approach.

[7] See Ng (1987), Brekke and Howarth (2002), Aronsson and Johansson-Stenman (2008, 2014b), and Wendner and Goulder (2008), who analyze different aspects of public good provision in economies where people are concerned with their relative consumption.

We develop a dynamic general equilibrium model where each consumer derives utility from his/her own consumption and use of leisure, respectively, and from his/her relative consumption compared with a reference consumption level (reflecting other people's consumption). In the benchmark model, the relative consumption comparisons are of the keeping-up-with-the-Joneses type, meaning that each individual compares his/her current consumption with the current consumption of referent others, which is the case that best corresponds to the empirical evidence discussed above. However, we will also – although briefly – touch upon catching-up-with-the-Joneses comparisons, where the reference measure refers to other people's past consumption, and argue that the associated externalities affect the welfare change measure in the same general way as the externalities following from keeping-up-with-the-Joneses types of comparisons.

Our main contribution is that we show how positional concerns influence the way welfare-change equivalent savings ought to be measured. We distinguish between a social optimum where all externalities are internalized, and imperfect market economies without externality correction. We also distinguish between first-best and second-best social optima by extending the benchmark model to allow for asymmetric information between the consumers and the social planner (or government). Furthermore, by using insights developed in the literature on tax and other policy responses to relative consumption concerns, we are also able to relate genuine saving to empirical measures of "degrees of positionality," i.e., the extent to which relative consumption is important for individual well-being.

The paper closest in spirit to ours is Aronsson and Löfgren (2008). They consider the problem of calculating an analogue to Weitzman's (1976) welfare-equivalent net national product in an economy where the consumers are characterized by habit formation. Their results show that if the habits are fully internalized through consumer choices, habit formation does not change the basic principles for measuring welfare (except that the individual's own past consumption affects his/her current instantaneous utility). However, with external habit formation, i.e., if the habits partly reflect other people's past consumption, the present value of this marginal externality affects the welfare measure through an addition to the comprehensive net national product. Our study differs from Aronsson and Löfgren (2008) in at least four distinct ways: we (i) consider measures of genuine saving (or analogues thereof) instead of net national product measures, (ii) focus attention on the empirically well-established keeping-up-with-Joneses type of comparison, (iii) allow for redistributive aspects

by considering a case with heterogeneous consumers, and (iv) introduce public investments into the study of welfare change-equivalent savings.[8]

The paper is outlined as follows. In Section 2, we present the benchmark model where each individual derives utility from consuming more than other people. We also present useful indicators of the extent to which relative consumption matters for individual well-being. Following earlier literature on genuine saving, we assume in the benchmark model that individuals are identical. In Section 3, we use the benchmark model to analyze economy-wide measures of welfare change. Sections 4 and 5 present two extensions by addressing catching-up-with-the-Joneses comparisons and public investments, respectively. Section 6 examines a more general model with two ability types that differ in productivity, where productivity is private information not observable to the social planner. Such a model allows us to extend the welfare analysis to a second-best model that includes both redistribution and externality correction subject to an incentive constraint. Section 7 presents a numerical example based on data for Sweden and the U.S., while Section 8 concludes the paper.

2. The Benchmark Model and Equilibrium

Consider an economy with a constant population comprising identical individuals, whose number is normalized to one.[9] The assumption of identical individuals is made for purposes of simplification; all qualitative results that we derive for this representative agent model would carry over in a natural way to a framework with heterogeneous consumers, as long as the redistribution policy can be implemented through lump-sum taxation. We will first, in Subsection 2.1, define the instantaneous utility function, which forms the basis for the subsequent measures of welfare change when people care about relative consumption. In Subsection 2.2 we consider the production side of the economy, where output is a function of labor and capital, before dealing with the dynamic optimization problem of individuals as well as the social planner.

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[8] Yamaguchi (2014) uses the representative-agent model developed in Aronsson and Löfgren (2008) to derive an indicator of genuine saving. He examines conditions under which the value of investment in physical capital and the value of investment in the stock of habits jointly contribute to increased welfare in a social optimum. As such, his analysis has a very different focus from ours.

[9] To be able to focus on the implications of relative consumption concerns in a simple way, we abstract from population growth. Genuine saving under population growth is addressed by Pezzey (2004). See also Asheim (2004).

2.1 Instantaneous Utility Function and the Degree of Positionality

Let c denote private consumption and z leisure. Similarly to the models analyzed in Aronsson and Johansson-Stenman (2010), the instantaneous utility function faced by the representative individual takes the form

$$U_t = u(c_t, z_t, \Delta_t) = \upsilon(c_t, z_t, \overline{c}_t), \qquad (1)$$

where the variable $\Delta_t = c_t - \overline{c}_t$ denotes the individual's relative consumption and is defined as the difference between the individual's own consumption and a reference measure, $\overline{c}_t$.[10] Although the reference consumption is an endogenous variable (see below), each individual behaves as an atomistic agent and treats $\overline{c}_t$ as exogenous.

The assumption that the individual's relative consumption reflects a difference comparison is made for technical convenience: all qualitative results derived below will also follow – yet with slightly more complex mathematical expressions – if the difference comparison is replaced with a ratio comparison (in which the relative consumption would become $c_t / \overline{c}_t$). The function $u(\cdot)$ defines the instantaneous utility in terms of the individual's absolute consumption and use of leisure, respectively, as well as in terms of the individual's relative consumption compared to the reference measure, while the function $\upsilon(\cdot)$ is a convenient reduced form allowing us to shorten some of the notation below. We assume that the function $u(\cdot)$ is increasing in each argument and strictly concave, implying that $\upsilon(\cdot)$ is increasing in its first two arguments and decreasing in the third. To be more specific, following equation (1) the relationships between the functions $\upsilon(\cdot)$ and $u(\cdot)$ are $\upsilon_c = u_c + u_\Delta$, $\upsilon_z = u_z$ and $\upsilon_{\overline{c}} = -u_\Delta$, where subscripts denote partial derivatives.

We follow Johansson-Stenman et al. (2002) and define the "degree of positionality" as a measure of the extent to which relative consumption matters for an individual's marginal utility of consumption. To be more specific, the degree of positionality represents the share of the overall instantaneous utility gain from increased consumption that is due to increased relative consumption. By using the function $u(\cdot)$, which distinguishes between absolute and relative consumption, the degree of positionality at time t can be written as

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[10] Note also that leisure is assumed to be completely non-positional. This is of course questionable, yet the limited empirical evidence available suggests that private consumption or income is much more positional than leisure (Solnick and Hemenway, 2005; Carlsson et al., 2007).

$$\alpha_t = \frac{u_\Delta(c_t, z_t, \Delta_t)}{u_c(c_t, z_t, \Delta_t) + u_\Delta(c_t, z_t, \Delta_t)} \in (0,1) \text{ for all } t. \tag{2}$$

Therefore, $1 - \alpha_t$ measures the degree of non-positionality, i.e. the extent to which the instantaneous utility gain of increased consumption is due to increased absolute consumption – an entity that is always set to unity in standard economic models. As indicated in the introduction, empirical evidence suggests that the degree of positionality on average is in the interval 0.3-0.8 for income (which is interpretable as a proxy for overall consumption) in industrialized countries, while it may be even higher for certain visible goods such as houses and cars.

2.2 Production and Intertemporal Maximization

We assume that production is determined by labor and capital. Let l denote the hours of work, defined by a time endowment, $\bar{l}$, less the time spent on leisure, and k denote the physical capital stock. While one may also consider other capital stocks, such as environmental and human capital stocks, the usefulness of genuine saving as a measure of welfare change does not in any way depend on the number of capital stocks in the economy. Therefore, to simplify the benchmark model as much as possible we refrain from considering other types of capital than physical capital. In Section 5, we extend the model by incorporating public investment to show how the treatment of such investment in genuine saving reflects the policy rule for contributions to a state-variable public good.

Output is produced by a constant returns to scale technology with production function $f(l,k)$, which is such that $f_l > 0$, $f_k > 0$, $f_{ll} < 0$ and $f_{kk} \leq 0$.[11] We suppress depreciation of physical capital, as it is of no concern in our context. This means that $f(\cdot)$ is interpretable as net output (or that the depreciation rate is zero). The net investment at time *t* is then written in terms of the resource constraint as

$$\dot{k}_t = f(l_t, k_t) - c_t, \tag{3}$$

where the initial (time zero) capital stock, k_0, is fixed. The terminal condition can be written as $\lim_{t \to \infty} k_t \geq 0$.

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[11] Note that the possibility of $f_{kk} = 0$ means that the model is consistent with an AK structure, such that the economy grows at a constant rate in the steady state.

The objective faced by each consumer is to maximize the present value of future utility. If expressed in terms of the function $\upsilon(\cdot)$, the intertemporal objective function can then be written as (if measured at time 0)

$$\int_0^\infty U_t e^{-\theta t} dt = \int_0^\infty \upsilon(c_t, z_t, \overline{c}_t) e^{-\theta t} dt \,, \tag{4}$$

where θ is the utility discount rate.

The social decision problem is then to choose c_t and l_t for all t to maximize the present value of future utility given in equation (4), subject to the resource constraint in equation (3), the initial capital stock, and the terminal condition. In doing this, and in contrast to individual consumers, the social planner also takes into account changes in the reference consumption $\overline{c}_t$. The corresponding present value Hamiltonian of this problem is given by (if written in terms of the utility formulation $\upsilon(\cdot)$ in equation [1])

$$H_t^p = \upsilon(c_t, z_t, \overline{c}_t) e^{-\theta t} + \lambda_t^p [f(l_t, k_t) - c_t] \,, \tag{5}$$

where λ denotes the costate variable attached to the capital stock and superscript p denotes present value. Note also that we are considering a representative-agent economy, where $\overline{c}_t = c_t$. In addition to equation (3) and the initial and terminal conditions, the social first-order conditions include

$$[\upsilon_c(c_t, z_t, \overline{c}_t) + \upsilon_{\overline{c}}(c_t, z_t, \overline{c}_t)] e^{-\theta t} = \lambda_t^p \tag{6a}$$

$$\upsilon_z(c_t, z_t, \overline{c}_t) e^{-\theta t} = \lambda_t^p f_l(l_t, k_t) \tag{6b}$$

$$\dot{\lambda}_t^p = -\frac{\partial H_t^p}{\partial k_t} = -\lambda_t^p f_k(l_t, k_t) \,, \tag{6c}$$

where subscripts attached to the instantaneous utility and production functions denote partial derivatives. For further use, we also assume that the transversality conditions

$$\lim_{t \to \infty} \lambda_t^p \geq 0 \quad (= 0 \text{ if } \lim_{t \to \infty} k_t > 0) \tag{6d}$$

$$\lim_{t \to \infty} H_t^p = 0 \tag{6e}$$

are fulfilled.[12] Note that the left-hand side of equation (6a) reflects the social marginal utility of consumption, $\upsilon_c + \upsilon_{\overline{c}} = u_c$, since the social planner recognizes that relative consumption is social waste.

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[12] For a more rigorous analysis of transversality conditions in optimal control theory, see Michel (1982) and Seierstad and Sydsaeter (1987).

In an unregulated economy where the consumption externality is uninternalized, the social first-order condition for private consumption given by equation (6a) is not satisfied. Instead of introducing the decision problems faced by consumers and firms in the unregulated economy and then characterizing the general equilibrium, we just note that the outcome of such an economy would be equivalent to the special case of the model set out above where the social planner (erroneously) treats $\bar{c}_t$ as exogenous for all t. The first-order condition for consumption would then change to

$$\upsilon_c(c_t, z_t, \bar{c}_t) e^{-\theta t} = \lambda_t^p , \tag{7}$$

whereas the first-order condition for work hours and the equation of motion for the costate variable remain as in equations (6b) and (6c), respectively.

3. Measuring Welfare Change in the Benchmark Model

This section presents measures of welfare change based on the benchmark model set out above. We begin by considering a welfare change measure under first-best conditions in Subsection 3.1 and continue with the unregulated economy in Subsection 3.2. Some extensions of the benchmark model are discussed in Sections 4 and 5.

3.1 First-Best Resource Allocation

As a point of departure, consider first the problem of measuring welfare change along the first-best optimal path that obeys equations (6a)-(6c), where the externalities associated with relative consumption concerns are fully internalized. This constitutes a natural reference case, although it is presumably not very realistic. We use superscript * to denote the socially optimal resource allocation, such that

$$\left\{ c_t^*, l_t^*, k_t^*, \lambda_t^{p,*} \right\} \forall t$$

satisfy equations (4) and (6a)-(6e) along with the initial and terminal conditions for the capital stock, and then define the corresponding optimal value function at time t as follows:

$$V_t^* = \int_t^\infty \upsilon(c_s^*, z_s^*, \bar{c}_s^*) e^{-\theta(s-t)} ds . \tag{8}$$

The welfare change over the short time interval $(t, t+dt)$ is given by the time derivative of equation (8), i.e.,

$$\frac{dV_t^*}{dt} \equiv \dot{V}_t^* = \theta V_t^* - \upsilon(c_t^*, z_t^*, \bar{c}_t^*) . \tag{9}$$

Defining genuine saving at any time t as $\lambda_t \dot{k}_t$, where $\lambda_t = \lambda_t^p e^{\theta t}$ denotes the current value shadow price of physical capital (which also equals the marginal utility of consumption) our first result is summarized as follows (and formally derived in Appendix):

Observation 1. *In a first-best optimum, genuine saving constitutes an exact measure of welfare change such that*

$$\dot{V}_t^* = \lambda_t^* \dot{k}_t^* . \tag{10}$$

Observation 1 is a standard result, which reproduces the welfare change-equivalence property of genuine saving in the context of the benchmark model. The left-hand side of equation (10) is the welfare change over the short time interval $(t, t+dt)$, while the right-hand side is interpretable as the genuine saving for the model set out above measured in units of utility at the first-best social optimum. While our model for simplicity only contains a one-dimensional capital concept (or state variable), the physical capital stock, a generalization to several capital stocks is straightforward: the right-hand side of equation (10) would then simply be the sum of changes in the value of all relevant capital stocks (see also Section 5 below).

With reference to sustainable development, we make three broad observations based on earlier research. First, as indicated in the introduction, if sustainable development is interpreted to mean non-declining intertemporal welfare, then $\lambda_t^* \dot{k}_t^* \geq 0$ is a necessary and sufficient condition for local sustainable development, i.e., sustainable development over the short time interval $(t, t+dt)$, and $\lambda_t^* \dot{k}_t^* < 0$ is the corresponding necessary and sufficient condition for local unsustainable development.

Second, negative genuine saving means that the instantaneous utility must eventually decline, whereas non-negative genuine saving is less informative about the equilibrium path of future instantaneous utilities (see Aronsson et al., 1997). By using Weitzman's welfare measure, $H_t^* = \theta V_t^*$ (see the Appendix), and then integrating equation (8) by parts, we can rewrite the relationship between genuine saving and welfare as follows:

$$\lambda_t^* \dot{k}_t^* = \int_t^\infty \frac{d\upsilon(c_s^*, z_s^*, \bar{c}_s^*)}{ds} e^{-\theta(s-t)} ds . \tag{10a}$$

Accordingly, $\lambda_t^* \dot{k}_t^* < 0$ means that the instantaneous utility must decline during some future time interval (for the sum on the right hand side to be negative), while $\lambda_t^* \dot{k}_t^* \geq 0$ only means that the future discounted changes in the instantaneous utilities must sum to a non-negative number; it does not require that all these future changes in instantaneous utilities are non-negative. Equation (10a) is particularly interesting if we interpret the model in terms of a continuum of perfectly altruistic generations of consumers (instead of in terms of consumers with infinite planning horizons), in which case the instantaneous utility is interpretable as a measure of generational well-being. Third, and related to the second point, by assuming that the instantaneous utility is non-constant along the optimal path and that this path is unique, Pezzey (2004) shows that non-positive genuine saving at time t means unsustainable development in the sense that the (actual) instantaneous utility at time t exceeds the maximum sustainable instantaneous utility level.[13] Therefore, and irrespective of which perspective we take, negative genuine saving contains a strong message: neither the current instantaneous utility nor the current intertemporal welfare level is sustainable.

3.2 Unregulated Economy

Here we analyze the probably more realistic case where the externalities associated with relative consumption concerns are not internalized, implying that the first-order condition for private consumption is given by equation (7) instead of equation (6a), and that equation (10) is no longer valid. Let

$$\left\{ c_t^0, k_t^0, \lambda_t^{p,0} \right\} \; \forall t$$

denote the resource allocation in the unregulated economy. As indicated above, the basic assumption underlying equation (7) is that consumers maximize the discounted stream of future utilities in a world where the positional externalities are not internalized (since all of

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[13] Following Pezzey (2004), note that $\lambda_t \dot{k}_t \leq 0$ implies $H_t \leq \upsilon_t$. Then, using Weitzman's (1976) welfare measure $H_t = \theta V_t$, we have

$$\frac{1}{\theta} H_t = \int\limits_t^\infty H_t e^{-\theta(s-t)} ds = \int\limits_t^\infty \upsilon_s e^{-\theta(s-t)} ds .$$

Let υ_t^m be the maximum instantaneous utility at time t that can be sustained forever. If the optimal path is unique and non-constant, we have $H_t > \upsilon_t^m$ and, therefore, $\upsilon_t \geq H_t > \upsilon_t^m$.

them treat the reference consumption as exogenous). The corresponding value function at time t be given by

$$V_t^0 = \int_t^\infty \upsilon(c_s^0, z_s^0, \overline{c}_s^0) e^{-\theta(s-t)} ds .$$ (11)

Also, let $r_t = f_k(l_t, k_t)$ denote the interest rate at time t, and $R_{s-t} = \int_t^s r_\tau^0 d\tau$ denote the sum of interest rates from t to s (where $s > t$). Now, by using

$$\dot{V}_t^0 = \theta V_t^0 - \upsilon(c_t^0, z_t^0, \overline{c}_t^0),$$ (12)

we can derive the following result:

Proposition 1. *In an unregulated economy with externalities caused by relative consumption concerns, the measure of welfare change takes the form*

$$\dot{V}_t^0 = \lambda_t^0 \left[\dot{k}_t^0 - \int_t^\infty \alpha_s^0 \exp(-R_{s-t}) \dot{\overline{c}}_s^0 ds \right]$$ (13)

where $\alpha_t^0 = \dfrac{u_\Delta(c_t^0, z_t^0, \Delta_t^0)}{u_c(c_t^0, z_t^0, \Delta_t^0) + u_\Delta(c_t^0, z_t^0, \Delta_t^0)}$.

Proof: See Appendix.

The right-hand side of equation (13) is written as the product of the real welfare change in consumption units (the expression within square brackets) and the marginal utility of consumption. Proposition 1 implies that the conventional measure of genuine saving, $\lambda_t \dot{k}_t$, does not generally constitute an exact measure of welfare change in an unregulated economy.[14] The second term on the right-hand side is interpretable as the value of the change in the marginal positional externality. To see this more clearly, note that α_t measures the instantaneous marginal positional externality at time t, implying that $\alpha_t \dot{\overline{c}}_t$ denotes the change

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[14] The insight that externalities may change the principles of measuring welfare and welfare change is, of course, not new. Earlier research on green national accounting shows that technological change and environmental externalities add additional components to measures of welfare and welfare change in unregulated economies (see Aronsson, Johansson and Löfgren, 1997, and Aronsson and Löfgren, 2010, as well as references therein). The novelties of the present paper are that it (i) shows how conspicuous consumption modifies the principles of measuring welfare-equivalent saving (an issue that to our knowledge has never been addressed) and (ii) by relating the welfare change measure to empirical measures of positional concerns.

in this externality over $(t, t+dt)$. Integrating forwards along the economy's general equilibrium path gives the final component in equation (13). In particular, note that this component is forward looking, since the welfare function at any time t is intertemporal and reflects future utility. It arises because the relationship between c and $\bar{c}$ is not internalized, implying, in turn, that the model is *non-autonomous time-dependent* through the equilibrium path for $\bar{c}$.

Note also that in the economy with identical individuals analyzed so far, $\bar{c}$ is always equal to c, implying that their growth rates will also be the same. Thus, in a growing economy in which $\dot{\bar{c}}$ is predominantly positive, the second term on the right-hand side of equation (13) will be negative and genuine saving will overestimate the welfare change. Also, the more positional the consumers are, i.e., the larger the α, the greater the discrepancy between the conventional measure of genuine saving and the welfare change, ceteris paribus. Therefore, empirical estimates of the degree of positionality, along with estimates of changes in the average consumption, are important for calculating the second term on the right-hand side of equation (13). Such calculations are presented in Section 7 based on data for Sweden and the U.S.

The right hand side of equation (13), i.e.,

$$\lambda_t [\dot{k}_t - \int_t^\infty \alpha_s \exp(-R_{s-t}) \dot{\bar{c}}_s ds],$$

plays exactly the same role here as the conventional genuine saving did in subsection 3.1. As such, it constitutes an exact measure of welfare change as well as gives the same qualitative information on local (un)sustainable development as the conventional genuine saving measure does in a first best resource allocation. However, since the second term on the right hand side of equation (13) is typically unobserved, an important question is that of the remaining informational content of $\lambda_t \dot{k}_t$, i.e., the conventional genuine saving measure. In general, if we erroneously were to apply the genuine saving formula given by equation (10) in an unregulated economy, it gives little information for assessing the welfare change or of relevance for local sustainable development. However, in a growing economy where consumption typically increases along the equilibrium path, such that the second term of the right hand side of equation (13) is negative, $\lambda_t \dot{k}_t$ can still be used for one-sided tests of welfare decline and local unsustainable development. More specifically, $\lambda_t \dot{k}_t < 0$ then implies (i) welfare decline and the corresponding interpretation in terms of local unsustainable

development, and (ii) that the instantaneous utility exceeds the maximum sustainable instantaneous utility level, meaning that it will eventually decline. The latter can be understood from an analogue to equation (10a), which in this setting is given by

$$\lambda_t[\dot{k}_t - \int_t^\infty \alpha_s \exp(-R_{s-t})\dot{\bar{c}}_s ds] = \int_t^\infty \frac{d\upsilon(c_s, z_s, \bar{c}_s)}{ds} e^{-\theta(s-t)} ds ,$$

where $\lambda_t \dot{k}_t < 0$ is a sufficient (yet not necessary) condition for the left hand side to be negative and, as a consequence, that the instantaneous utility will decline sometime in the future.

4 Adding a Catching-up-with-the-Joneses Mechanism

So far, we have assumed that the relative consumption comparisons are atemporal: each individual compares his/her current consumption with other people's current consumption. Although a great deal of empirical evidence on relative consumption concerns is interpretable as mainly reflecting such comparisons, it is easy to argue that intertemporal consumption comparisons may also be relevant to consider. For example, Senik (2010) presents empirical evidence consistent with the catching-up-with-the-Joneses comparison by showing that individual well-being is higher if the individual's standard of living exceeds that of his/her parents 15 years earlier, ceteris paribus. Such comparisons are also consistent with the empirical pattern of various financial puzzles (Constantinides, 1990; Campbell and Cochrane, 1999; and Díaz et al., 2003). It is therefore clearly interesting to investigate whether the qualitative results and interpretations presented in the previous section above carry over to a framework where the individual also derives utility from comparisons with other people's past consumption. As will be shown, they essentially do.

There are at least two ways of modeling such intertemporal consumption comparisons. One is to add a stock reflecting a weighted average of other people's past consumption over the entire history of the model, as Aronsson and Löfgren (2008) did in their study on the comprehensive net national product under external habit formation. Another way is to assume that the individual directly derives utility from comparing his/her time t consumption with others' consumption at time $t - \varepsilon$ (and possibly other points in time as well). We use the latter approach as it requires less modifications of the model set out above. The instantaneous utility at time t is then rewritten to read

$$U_t = u(c_t, z_t, \Delta_t, \delta_t) = \upsilon(c_t, z_t, \bar{c}_t, \bar{c}_{t-\varepsilon}) , \tag{14}$$

where $\delta_t = c_t - \overline{c}_{t-\varepsilon}$ denotes the individual's relative consumption compared with others' past consumption. Thus, the instantaneous utility function is no longer time separable. All other aspects of the model remain as above.

We can then distinguish between the degree of atemporal positionality (which is analogous to the positionality concept discussed above) and the degree of intertemporal positionality.[15] By using equation (14), these two measures are summarized as

$$\alpha_t = \frac{u_\Delta(c_t, z_t, \Delta_t, \delta_t)}{u_c(c_t, z_t, \Delta_t, \delta_t) + u_\Delta(c_t, z_t, \Delta_t, \delta_t) + u_\delta(c_t, z_t, \Delta_t, \delta_t)} \in (0,1) \qquad (15)$$

$$\beta_t = \frac{u_\delta(c_t, z_t, \Delta_t, \delta_t)}{u_c(c_t, z_t, \Delta_t, \delta_t) + u_\Delta(c_t, z_t, \Delta_t, \delta_t) + u_\delta(c_t, z_t, \Delta_t, \delta_t)} \in (0,1) . \qquad (16)$$

As before, α_t denotes the degree of atemporal positionality (with the same interpretation as equation (2) above) and β_t the degree of intertemporal positionality at time t. The latter positionality concept measures the fraction of the utility gain of an additional dollar spent on consumption that is due to increased relative consumption compared with other people's past consumption.

Consider first the socially optimal resource allocation. Here, since the reference consumption measure on which the intertemporal consumption comparison is based enters the instantaneous utility function with a time lag, equation (6a) is replaced with

$$[\upsilon_{c_t}(c_t, z_t, \overline{c}_t, \overline{c}_{t-\varepsilon}) + \upsilon_{\overline{c}_t}(c_t, z_t, \overline{c}_t, \overline{c}_{t-\varepsilon})]e^{-\theta t} + \upsilon_{\overline{c}_t}(c_{t+\varepsilon}, z_{t+\varepsilon}, \overline{c}_{t+\varepsilon}, \overline{c}_t)e^{\theta(t+\varepsilon)} = \lambda_t^p , \qquad (17)$$

where the third term on the right-hand side is due to the delayed response mechanism caused by intertemporal consumption comparisons. The other first-order conditions remain as in equations (6b) and (6c). By using the same procedure as above, it is straightforward to show that equation (10) in Observation 1 still applies, since there are no uninternalized externalities that affect the welfare change measure, i.e., $\dot{V}_t^* = \lambda_t^* \dot{k}_t^*$.

In the unregulated economy, equation (17) is replaced by

$$\upsilon_{c_t}(c_t, z_t, \overline{c}_t, \overline{c}_{t-\varepsilon})e^{-\theta t} = \lambda_t^p ,$$

which is analogous to equation (7), while the other first-order conditions remain unchanged. We can then derive the following analogue to Proposition 1:

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[15] This distinction originates from Aronsson and Johansson-Stenman (2014a), who analyze the simultaneous implications of keeping-up-with-the-Joneses and catching-up-with-the-Joneses types of comparisons for optimal taxation in an OLG model.

Proposition 2. *In an unregulated economy, where the externalities associated with relative consumption concerns are driven by both keeping-up-with-the-Joneses and catching-up-with-the-Joneses comparisons, the measure of welfare change takes the form*

$$\dot{V}_t^0 = \lambda_t^0 \left[\dot{k}_t^0 - \int_t^\infty \alpha_s^0 \exp(-R_{s-t}) \dot{\bar{c}}_s^0 ds - \int_t^\infty \beta_{s+\varepsilon}^0 \exp(-R_{s+\varepsilon-t}) \dot{\bar{c}}_s^0 ds \right]. \qquad (18)$$

The interpretation of equation (18) is analogous to that of equation (13), with the only modification that the value of the marginal externality is divided in two parts in equation (18), which we may refer to as keeping-up and a catching-up with the Joneses externalities, respectively. We can observe that both kinds of externalities affect the welfare change measure in the same general way. Therefore, the interpretations of Proposition 1 carry over to Proposition 2. While this conclusion is interesting *per se*, it also suggests that we may skip the catching-up component in what follows in order to keep the model as simple and transparent as possible.

5. Public Investments

This section extends the benchmark model by analyzing the role of public investments. The implications of positional externalities for the optimal provision of public goods have been addressed in several studies (e.g., Ng, 1987; Aronsson and Johansson-Stenman, 2008, 2014b). Relative concerns for private consumption affect the optimal policy rule for public good provision via two channels: (1) an incentive to internalize positional externalities through increased public provision (which reduces the private consumption) and (2) an (indirect) incentive to reduce the public provision as relative consumption concerns lower the consumers' marginal willingness to pay for public goods, ceteris paribus. In turn, this has a direct bearing on the way public investment ought to be reflected in the context of genuine saving, which motivates the following extension.

We will, consequently, add a public good of stock character, which means that equation (1) changes to

$$U_t = u(c_t, z_t, g_t, \Delta_t) = \upsilon(c_t, z_t, g_t, \bar{c}_t), \qquad (19)$$

where g_t denotes the level of the public good at time t (e.g., the state of the natural environment). The public good accumulates through the following differential equation:[16]

$$\dot{g}_t = q_t - \gamma g_t ,\tag{20}$$

where q_t denotes the flow expenditure directed toward the public good at time t, i.e., the instantaneous contribution, and γ denotes the rate of depreciation. We also impose the initial condition that g_0 is fixed and the terminal condition $\lim_{t\to\infty} g_t \geq 0$. Finally, since part of output is used for contributions to the public good, the resource constraint slightly changes such that

$$\dot{k}_t = f(l_t, k_t) - c_t - \rho q_t ,\tag{21}$$

where ρ is interpretable as the (fixed) marginal rate of transformation between the public good and the private consumption good.

These extensions mean that we have added the control variable q and the state variable g to the benchmark model; otherwise, the model is the same as in Section 2. Thus, the social decision problem is to choose c_t, l_t, and q_t for all t to maximize the present value of future utility,

$$\int_0^\infty U_t e^{-\theta t} dt = \int_0^\infty \upsilon(c_t, z_t, g_t, \overline{c}_t) e^{-\theta t} dt ,$$

subject to equations (20) and (21) along with initial and terminal conditions. The present value Hamiltonian can then be written as

$$H_t^p = \upsilon(c_t, z_t, g_t, \overline{c}_t) e^{-\theta t} + \lambda_t^p [f(l_t, k_t) - \rho q_t - c_t] + \mu_t^p [q_t - \gamma g_t].\tag{22}$$

The variable μ_t^p denotes the present value shadow price of the public good at time t, and its current value counterpart is given by $\mu_t = \mu_t^p e^{\theta t}$. Note that equations (6a)-(6e), if written in terms of the instantaneous utility function examined here (i.e., equation [19]), are necessary conditions also in this extended model. In addition to (the modified) equations (6), the first-order conditions for a social optimum also include an efficiency condition for q, an equation of motion for μ^p, and an additional transversality condition, i.e.,

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[16] Since the public good is interpretable in terms of environmental quality, a possible (and realistic) extension of equation (20) would be to assume that increased output leads to lower environmental quality (instead of just assuming a natural rate of depreciation). Yet, we refrain from this extension here as it is not important for the qualitative results derived below.

$$\lambda_t^p \rho = \mu_t^p \tag{23a}$$

$$\dot{\mu}_t^p = -\frac{\partial H_t^p}{\partial g_t} = -\upsilon_g(c_t, z_t, g_t, \overline{c}_t)e^{-\theta t} + \mu_t^p \gamma \tag{23b}$$

$$\lim_{t \to \infty} \mu_t^p \geq 0 \ (=0 \text{ if } \lim_{t \to \infty} g_t > 0). \tag{23c}$$

Equations (6a)-(6e) and (23) characterize the social optimum. For further use, let us also solve the differential equation (23b) forward subject to the transversality condition (23c),[17] which gives

$$\mu_t^p = \int_t^\infty \upsilon_g(c_s, z_s, g_s, \overline{c}_s)e^{-\theta s}e^{-\gamma(s-t)}ds . \tag{24}$$

As in Subsection 3.2, if the positional externality has not become internalized, equation (6a) should be replaced with equation (7) (again modified to reflect the instantaneous utility function [19] that contains the public good). This *partly regulated equilibrium* can be implemented in a decentralized setting (where the consumers choose their consumption and work hours at each point in time) by assuming that the planner (or government) raises revenue to finance the public good through lump-sum taxation, although it does not use the tax system to correct the individual first-order conditions for externalities.

We are now ready to present the main results of this section. As before, to separate the social optimum from an allocation with uninternalized positional externalities, we use superscript * to denote the socially optimal resource allocation and superscript 0 to denote the economy with uninternalized externalities. The value function (social welfare function) at any time t will be defined in the same way as above, i.e.,

$$V_t = \int_t^\infty \upsilon(c_s, z_s, g_s, \overline{c}_s)e^{-\theta(s-t)}ds .$$

Then, by recognizing that genuine saving now reads $\lambda_t \dot{k}_t + \mu_t \dot{g}_t$ (since the capital concept is two-dimensional here), we can use the same procedure as in Observation 1 and Proposition 1 to derive an analogue to equation (10) as follows:

$$\dot{V}_t^* = \lambda_t^* \left[\dot{k}_t^* + \rho \dot{g}_t^* \right], \tag{25a}$$

and a corresponding analogue to equation (13):

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[17] We assume that $\lim_{t \to \infty} g_t > 0$.

$$\dot{V}_t^0 = \lambda_t^0 \left[\dot{k}_t^0 + \rho \dot{g}_t^0 - \int_t^\infty \alpha_s^0 \exp(-R_{s-t}) \dot{\bar{c}}_s^0 ds \right]. \tag{25b}$$

Except for the public investment component, equations (25a) and (25b) are interpretable in exactly the same way as their counterparts in the simpler benchmark model, i.e., equations (10) and (13).[18]

Equations (25a) and (25b) together imply the following treatment of public investment:

Proposition 3. *Irrespective of whether the resource allocation is first best or characterized by uninternalized positional externalities, the accounting price of public investment is given by the marginal rate of transformation between the public good and the private consumption good, ρ.*

Proposition 3 has a strong implication, as it means that the valuation of public investment might be based on observables and that the same valuation procedure applies in a social optimum and a distorted market economy. Although convenient, this result may seem surprising at first sight. Yet, note that the information content in ρ differs between the two regimes due to differences in the underlying policy rules for public provision. To see this more clearly, let

$$MRS_{gc} = \frac{\upsilon_g}{\upsilon_c} = \frac{u_g}{u_c + u_\Delta}$$

denote the marginal rate of substitution between the public good and private consumption measured with the reference consumption, $\bar{c}$, held constant, and, following Aronsson and Johansson-Stenman (2008),

$$CMRS_{gc} = \frac{\upsilon_g}{\upsilon_c + \upsilon_{\bar{c}}} = \frac{u_g}{u_c}$$

denote the corresponding marginal rate of substitution measured with the relative consumption, Δ, held constant. Therefore, MRS_{gc} refers to a conventional marginal willingness to pay measure with other people's consumption held constant. This means that an increase in the public good will not only imply reduced (absolute) consumption, the

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[18] If we extend the model by allowing ρ to vary over time, e.g., due to disembodied technological change, an additional (forward-looking) term representing the marginal value of this technological change must be added to equations (25a) and (25b).

individual will also take into account the fact that his/her relative consumption decreases. $CMRS_{gc}$, on the other hand, reflects each respondent's marginal willingness to pay for the public good conditional on that other people have to pay the same amount, implying that relative consumption is held fixed. Thus, in this case there is no additional cost in terms of reduced relative consumption. Indeed, it is straightforward to show that $CMRS_{gc} = MRS_{gc} / (1-\alpha)$.

The policy rule for public provision in the first-best optimum can now be written as

$$\frac{\mu_t^*}{\lambda_t^*} = \int_t^\infty CMRS_{s,gc}^* \, e^{-(R_{s-t} + \gamma(s-t))} ds = \int_t^\infty \frac{MRS_{s,gc}^*}{1-\alpha_s^*} e^{-(R_{s-t} + \gamma(s-t))} ds = \rho , \qquad (26a)$$

while the corresponding policy rule implicit in the economy with with uninternalized externalities becomes

$$\frac{\mu_t^0}{\lambda_t^0} = \int_t^\infty MRS_{s,gc}^0 \, e^{-(R_{s-t} + \gamma(s-t))} ds = \rho . \qquad (26b)$$

Equations (26a) and (26b) are different variants of the Samuelson condition for a state variable public good. In a social optimum where all positional externalities are internalized, the marginal rate of substitution between the public good and private consumption is given by $CMRS_{gc}$, which recognizes that relative consumption is pure waste. In the economy with uninternalized externalities, on the other hand, agents behave as if others' consumption, $\bar{c}$, is exogenous, meaning that the marginal rate of substitution implicit in the first-order conditions is given by MRS_{gc}. In either case, however, it is optimal to equate the weighted sum of marginal rates of substitution with the marginal rate of transformation, which explains Proposition 3. There is an important practical implication here: regardless of whether we are in a first-best economy or market equilibrium with positional externalities, public investment can be valued by the instantaneous marginal cost, such that there is no need to estimate future generations' marginal willingness to pay for current additions to the public good.

6. Heterogeneity, Redistribution Policy and Genuine Saving

In the preceding sections we have consistently considered a representative-agent economy, which is the typical framework used in earlier literature on genuine saving. In this section, we extend the analysis to a model where consumers are heterogeneous in terms of productivity and productivity is private information. As explained in the introduction, this enables us to generalize the study of genuine saving to a second-best economy where the social planner

redistributes and internalizes externalities subject to an incentive constraint.[19] Such an extension is of course not without costs in terms of greater complexity and less transparency. However, the benefits are also considerable in terms of greater realism. Our analysis recognizes that an indicator of social welfare may reflect a desired distribution of welfare among individuals, and that this objective is costly to reach. In turn, the incentive constraint directly affects the marginal social value of positional externalities, which further motives this extension.

We make two simplifying assumptions. First, we do not consider public investments. Since Proposition 3 can be shown to apply also in the model set out below, little additional insight would be gained from studying such investments here as well. Second, to avoid unnecessary technical complications with many different consumer types, we use the two-type setting originally developed by Stern (1982) and Stiglitz (1982). The consumers differ in productivity, and the high-ability type (type 2) is more productive (earns a higher before-tax wage rate) than the low-ability type (type 1).[20] The population is constant and normalized to one for notational convenience, and there is a constant share, n^i, of individuals of type i, such that $\sum_i n^i = 1$.

6.1 Preferences and Social Decision-Problem

We allow for type-specific differences in preferences. The instantaneous utility function facing each individual of type i can then be written as

$$U_t^i = u^i(c_t^i, z_t^i, \Delta_t^i) = \upsilon^i(c_t^i, z_t^i, \overline{c}_t), \tag{27}$$

where $\Delta_t^i = c_t^i - \overline{c}_t$ denotes the relative consumption of type i. The functions $u^i(\cdot)$ and $\upsilon^i(\cdot)$ in equation (27) have the same general properties as their counterparts in equation (1). Also, and similarly to the benchmark model presented in Section 2, the relative consumption concerns in equation (27) solely reflect comparisons with other people's current consumption, i.e., we

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[19] To our knowledge, the paper by Aronsson, Cialani, and Löfgren (2012) is the only earlier study dealing with genuine saving in a second-best economy with distortionary taxation. Their study is based on a representative-agent model (and as such does not consider redistribution policy) and assumes that individuals only care about their own absolute consumption and use of leisure (meaning that relative consumption is not dealt with at all).

[20] Aronsson (2010) uses a similar two-type model to derive a second-best analogue to the comprehensive net national product. However, his study neither addresses the implications of relative consumption concerns nor examines genuine saving, which are the main issues here.

abstract from the catching-up mechanism briefly discussed above. The reference level is assumed to be the average consumption in the economy as a whole, $\bar{c}_t = n^1 c_t^1 + n^2 c_t^2$.[21] As before, each consumer is small relative to the overall economy and hence treats $\bar{c}_t$ as exogenous.

In a way similar to Sections 2 and 3, it is useful to be able to measure the extent to which relative consumption matters for individual utility. By using the function $u^i(\cdot)$ in equation (27), we can define the degree of positionality of an individual of type i at time t such that

$$\alpha_t^i = \frac{u_\Delta^i(c_t^i, z_t^i, \Delta_t^i)}{u_c^i(c_t^i, z_t^i, \Delta_t^i) + u_\Delta^i(c_t^i, z_t^i, \Delta_t^i)} \in (0,1) \text{ for } i=1,2, \text{ all } t. \tag{28}$$

Therefore, except that α_t^i is type specific, it has the same interpretation as in the benchmark model. Since we assume that all individuals compare their own consumption with the average consumption in the economy as a whole, we can define the positional externality in terms of the average degree of positionality. The average degree of positionality measured over all individuals at time t can be written as (recall that the total population size is normalized to unity)

$$\bar{\alpha}_t = n^1 \alpha_t^1 + n^2 \alpha_t^2 \in (0,1), \tag{29}$$

which is interpretable as the sum of the marginal willingness to pay to avoid the positional consumption externality, measured per unit of consumption. Estimates of $\bar{\alpha}_t$ can be found in empirical literature on relative consumption concerns, as discussed above (see the introduction).

The social objective is given by a general social welfare function

$$W_0 = \int_0^\infty \omega\left(U_t^1, U_t^2\right) e^{-\theta t} dt, \tag{30}$$

and the instantaneous social welfare function, $\omega(\cdot)$, is assumed to be differentiable and increasing in the instantaneous individual utilities. The resource constraint can now be written as

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[21] This is the most common assumption in the literature dealing with optimal policy responses to relative consumption concerns. Although the definition of reference consumption at the individual level (i.e., whether it is based on the economy-wide mean value or reflects more narrow social reference groups) matters for public policy, it is not equally important here, where the main purpose is to characterize an aggregate measure of social savings.

$$\dot{k}_t = f(\ell_t^1, \ell_t^2, k_t) - \sum_i n^i c_t^i, \tag{31}$$

where $\ell_t^i = n^i l_t^i$ is interpretable as the aggregate input of type i labor at time t. As before, the production function, $f(\ell_t^1, \ell_t^2, k_t)$, is characterized by constant returns to scale.

Following Aronsson and Johansson-Stenman (2010), we assume that the social planner (or government) can observe labor income and saving at the individual level, while ability is private information. We also assume that the social planner wants to redistribute from the high-ability to the low-ability type. To eliminate the incentive for the high-ability type to mimic the low-ability type (in order to gain from this redistribution profile), we impose a self-selection constraint such that each individual of the high-ability type weakly prefers the allocation intended for his/her type over the allocation intended for the low-ability type. Also, to simplify the analysis, we assume that the social planner commits to the resource allocation decided at time zero.[22] One way to rationalize this assumption in our framework is to interpret the model in terms of a continuum of perfectly altruistic generations, i.e., dynasties (instead of single consumers with infinite lives),[23] implying that the ability of agents living at any time t is not necessarily observable beforehand by the social planner. Therefore, the self-selection constraint is assumed to take the form

$$U_t^2 = \upsilon^2(c_t^2, z_t^2, \overline{c}_t) \ge \upsilon^2(c_t^1, \overline{l} - \phi_t l_t^1, \overline{c}_t) = \hat{U}_t^2 \tag{32}$$

for all t. The left-hand side represents the utility of the high-ability type at time t and the right-hand side the utility of the mimicker (a high-ability type choosing the same income and saving as the low-ability type). The variable

$$\phi_t = w_t^1 / w_t^2 = f_{\ell^1}(\ell_t^1, \ell_t^2, k_t) / f_{\ell^2}(\ell_t^1, \ell_t^2, k_t) = \phi(\ell_t^1, \ell_t^2, k_t) < 1$$

denotes the relative wage rate at time t, and $\phi_t l_t^1 < l_t^1$ is the hours of work the mimicking high-ability type needs to supply to reach the same labor income as the (mimicked) low-ability type.

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[22] See Brett and Weymark (2008) for an analysis of optimal taxation without commitment based on the self-selection approach to optimal taxation.

[23] We realize that this interpretation is not unproblematic, since each individual in a succession of generations faces his/her own objective and constraints; see Michel, Thibault, and Vidal (2006) for a discussion. However, our main concern here is to examine the implications of the self-selection constraint for how positional externalities ought to be treated in savings-based measures of welfare change. Since the externality is atemporal in our model (i.e., a keeping-up-with-the-Joneses externality), the exact way in which successive generations interact is of no major importance for the qualitative results.

6.2 Second-Best Optimum and Genuine Saving

The social optimum can be derived by choosing c_t^1, l_t^1, c_t^2, and l_t^2 for all t to maximize the social welfare function given in equation (30), subject to the resource constraint and self-selection constraint in equation (31) and (32), respectively, and to initial and terminal conditions for the capital stock (as before). Also, the social planner recognizes that the reference consumption is endogenous and given by $\bar{c}_t = n^1 c_t^1 + n^2 c_t^2$.[24] The present value Hamiltonian at any time t is written as

$$H_t^p = \omega\left(U_t^1, U_t^2\right)e^{-\theta t} + \lambda_t^p \dot{k}_t. \tag{33}$$

Adding the self-selection constraint gives the present value Lagrangean

$$L_t^p = H_t^p + \eta_t^p[U_t^2 - \hat{U}_t^2], \tag{34}$$

where η_t^p denotes the present value Lagrange multiplier. If written in terms of the reduced form utility formulation $\upsilon^i(\cdot)$ in equation (27), and by using $\hat{\upsilon}_t^2 = \upsilon^2(c_t^1, \bar{l} - \phi_t l_t^1, \bar{c}_t)$ as a short notation for the instantaneous utility facing the mimicker at time t, the first-order conditions can be written as (in addition to equations [31] and [32] and the initial and terminal conditions for the capital stock)

$$\frac{\partial L^p}{\partial c^1} = \omega_{U^1}\upsilon_c^1 e^{-\theta t} - \eta^p \hat{\upsilon}_c^2 - \lambda^p n^1 + \frac{\partial L^p}{\partial \bar{c}}n^1 = 0 \tag{35a}$$

$$\frac{\partial L^p}{\partial l^1} = -\omega_{U^1}\upsilon_z^1 e^{-\theta t} + \eta^p \hat{\upsilon}_z^2[\phi + \frac{\partial \phi}{\partial l^1}l^1] + \lambda^p n^1 w^1 = 0 \tag{35b}$$

$$\frac{\partial L^p}{\partial c^2} = \omega_{U^2}\upsilon_c^2 e^{-\theta t} + \eta^p \upsilon_c^2 - \lambda^p n^2 + \frac{\partial L^p}{\partial \bar{c}}n^2 = 0 \tag{35c}$$

$$\frac{\partial L^p}{\partial l^2} = -\omega_{U^2}\upsilon_z^2 e^{-\theta t} + \eta^p[-\upsilon_z^2 + \hat{\upsilon}_z^2 \frac{\partial \phi}{\partial l^2}l^1] + \lambda^p n^2 w^2 = 0 \tag{35d}$$

$$\dot{\lambda}^p = -\frac{\partial L^p}{\partial k} = -\lambda_p r - \eta^p \hat{\upsilon}_z^2 \frac{\partial \phi}{\partial k}l^1. \tag{35e}$$

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[24] We will not go into how such an allocation can be implemented by optimal nonlinear taxation. Here, the reader is referred to Aronsson and Johansson-Stenman (2010). Other literature on optimal taxation under relative consumption concerns includes Boskin and Sheshinski (1978), Oswald (1983), Corneo and Jeanne (1997, 2001), Ireland (2001), Dupor and Liu (2003), Wendner and Goulder (2008), and Aronsson and Johansson-Stenman (2014a).

The optimal resource allocation also fulfills transversality conditions analogous to equations (6d) and (6e). In equations (35), we have suppressed the time indicator to avoid notational clutter (since all components refer to the same point in time), and subscripts attached to the instantaneous utility functions and social welfare function refer to partial derivatives (as before). The factor prices are determined by marginal products, i.e., $w^1 = f_{\ell^1}(\ell^1, \ell^2, k)$, $w^2 = f_{\ell^2}(\ell^1, \ell^2, k)$, and $r = f_k(\ell^1, \ell^2, k)$.

The final term on the right-hand side of equations (35a) and (35c) reflects the positional consumption externality and measures the partial instantaneous welfare effect of increased reference consumption times the effect of each type's consumption on the reference measure. This partial welfare effect can be either positive or negative and is given as follows:

$$\frac{\partial L_t^P}{\partial \overline{c}_t} = \omega_{U_t^1} \upsilon_{t,\overline{c}_t}^1 e^{-\theta t} + \omega_{U_t^2} \upsilon_{t,\overline{c}_t}^2 e^{-\theta t} + \eta_t^p \left[\upsilon_{t,\overline{c}_t}^2 - \hat{\upsilon}_{t,\overline{c}_t}^2 \right]. \tag{36}$$

Although the first two terms on the right-hand side are negative, since $\upsilon_{t,\overline{c}}^i < 0$ for $i=1,2$ by the assumptions made earlier, the third term (which is proportional to the Lagrange multiplier of the self-selection constraint) can be either positive or negative. We will return to equation (36) in greater detail below.

We are now ready to derive a measure of welfare change for the second-best economy set out above. The main differences compared with the model analyzed in Sections 2 and 3 are that (i) the individuals differ in productivity and preferences here, meaning also that they may differ in terms of relative consumption concerns (see equation [28] above), (ii) the social planner has an objective that may necessitate redistribution policy, and (iii) the social planner is unable to implement the first-best resource allocation (provided that the self-selection constraint binds). Despite these differences, we will also in this case use superscript * to denote social optimum (let be a second-best optimum) such that

$$\left\{ c_t^{1,*}, l_t^{1,*}, c_t^{2,*}, l_t^{2,*}, k_t^*, \lambda_t^{p,*} \right\} \forall t$$

satisfy equations (6d), (6e), (31), (32), and (35a)-(35e) as well as the initial and terminal conditions. The optimal value function at time t (based on the social welfare function) then becomes

$$W_t^* = \int_t^{\infty} \omega \left(\upsilon(c_s^{1,*}, z_s^{1,*}, \overline{c}_s^*), \upsilon(c_s^{2,*}, z_s^{2,*}, \overline{c}_s^*) \right) e^{-\theta(s-t)} ds. \tag{37}$$

We can then derive the following strong result regarding the change in social welfare over a short time interval, $(t, t+dt)$:

Proposition 4. *If the resource allocation satisfies equations (6d), (6e), (31), and (35a)-(35e), and irrespective of whether the self-selection constraint binds, genuine saving constitutes an exact measure of social welfare change such that*

$$\dot{W}_t^* = \lambda_t^* \dot{k}_t^*, \tag{38}$$

where $\lambda_t^* = \lambda_t^{p,*} e^{\theta t}$.

Proof: See Appendix.

Proposition 4 constitutes a remarkable result. It means that the procedure for measuring welfare change in a first-best representative-agent economy (as discussed in Section 3) carries over to the second-best economy analyzed here, where the agents differ in productivity, and where productivity is private information. The only difference is that the shadow price of capital is measured in a slightly different way (which is seen by comparing equations [6c] and [35e]). The intuition behind equation (38) is that the redistribution is optimal given the social welfare function and constraints, and the positional externality is internalized. As a consequence, there is no market failure or failure to reach the distributional objectives that may cause a discrepancy between the welfare change and the conventional measure of genuine saving.[25] This holds true irrespective of whether the resource allocation is first best or second best.

To take this discussion a bit further, note that the social welfare, as defined in equation (30), is non-declining (declining) over the time interval $(t, t+dt)$ if, and only if, $\lambda_t^* \dot{k}_t^* \geq 0$ (< 0). Consequently, non-negative genuine saving can be interpreted as an indicator of local sustainable development for the same reason as in Subsection 3.1, with the only modification that the social welfare function takes a different form here. Similarly, and again by analogy to the first-best model examined in Subsection 3.1, it is straight forward to show that negative genuine saving means that the instantaneous social welfare, $\omega(U^1, U^2)$, must eventually decline. Therefore, the qualitative results and interpretations are analogous to those derived

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[25] Aronsson and Löfgren (1999) show that if the distribution of resources among consumers is not optimal during a time interval, the comprehensive net national product may for this reason fail as a welfare-level measure. In their study, a first-best resource allocation constitutes the reference case.

and discussed under first-best conditions in the representative-agent model. The strong implication is, of course, that the results derived from the representative-agent model in Subsection 3.1 are more general than might be apparent at first thought.

6.3 Welfare Change and Positional Externalities

To be able to focus on the welfare contributions of uninternalized positional externalities, without introducing any other discrepancy from the second-best optimum, we take the same type of shortcut to the imperfect economy as we did in Sections 2 and 3. More specifically, we assume that the social planner does not internalize the positional externality, although the redistribution among consumer types continues to be optimal (in this case conditional on the equilibrium path for $\bar{c}$) in terms of the social welfare function described above. Thus, instead of examining the decision problems faced by consumers and firms in an economy without externality correction and then characterizing the general equilibrium, we just note that this equilibrium would be equivalent to the special case of the model set out in subsection 6.2 where the planner (erroneously) treats $\bar{c}_t$ as exogenous for all *t*. As a consequence, the allocation that this planner implements means that the private consumption, hours of work, and the capital stock are chosen *as if* the consumer preference for relative consumption does not give rise to externalities.[26]

In this case, the social planner's first-order conditions for l_t^1 and l_t^2 remain as in equation (35b) and (35d), respectively, and the equation of motion for the shadow price remains as in equation (35e), whereas the first-order conditions for c_t^1 and c_t^2 reduce to read

$$\frac{\partial L^p}{\partial c^1} = \omega_{U^1} \upsilon_c^1 e^{-\theta t} - \eta^p \hat{\upsilon}_c^2 - \lambda^p n^1 = 0 \tag{39a}$$

$$\frac{\partial L^p}{\partial c^2} = \omega_{U^2} \upsilon_c^2 e^{-\theta t} + \eta^p \upsilon_c^2 - \lambda^p n^2 = 0. \tag{39b}$$

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[26] It is straight forward to implement this resource allocation in a decentralized setting with nonlinear labor and capital income taxation. The "optimal" marginal labor income tax rates would then take the same general form as those derived by Stiglitz (1982) for an economy without relative concerns. An alternative approach to the decentralized economy is to assume away government intervention completely, in which case none of the social first-order conditions would be fulfilled. With the social welfare function in equation (30), it would then be difficult to separate the welfare consequences of uninternalized externalities from those of a suboptimal distribution among individuals.

Now, let us return to the partial instantaneous welfare effect of increased reference consumption in equation (36), which is no longer internalized by the social planner. By using $v_c^i = u_c^i + u_\Delta^i$ and $v_{\bar{c}}^i = -u_\Delta^i$ together with equation (28), we have $v_{\bar{c}}^i = -\alpha^i v_c^i$, which means that equation (36) can be written as

$$\frac{\partial L_t^P}{\partial \bar{c}_t} = -\omega_{U_t^1} v_{t,c_t}^1 e^{-\theta t} \alpha_t^1 - \omega_{U_t^2} v_{t,c_t}^2 e^{-\theta t} \alpha_t^2 - \eta_t^p \left[v_{t,c_t}^2 \alpha_t^2 - \hat{v}_{t,c_t}^2 \hat{\alpha}_t^2 \right],$$

where $\hat{\alpha}_t^2$ denotes the degree of positionality of the mimicker at time t. Solving equation (39a) for $\omega_{U^1} v_c^1 e^{-\theta t}$ and equation (39b) for $\omega_{U^2} v_c^2 e^{-\theta t}$, and substituting into the above expression gives

$$\frac{\partial L_t^P}{\partial \bar{c}_t} = -\lambda_t^p \bar{\alpha}_t + \eta_t^p \hat{v}_c^2 \left[\hat{\alpha}_t^2 - \alpha_t^1 \right]. \tag{40}$$

Equation (40) is analogous to a result derived by Aronsson and Johansson-Stenman (2008) and implies that the partial welfare effect of an increase in $\bar{c}_t$ can be decomposed in two parts. The first term on the right-hand side depends on the average degree of positionality and is interpretable as the pure efficiency cost of increased reference consumption at time t, ceteris paribus. This is so because $\bar{\alpha}_t$ reflects the sum of marginal willingness to pay to avoid the externality, measured over all consumers, and the multiplication by λ_t^p converts this number into utility units. The second component in equation (40) depends on the difference in the degree of positionality between the mimicker and the low-ability type. Note that this component is proportional to the Lagrange multiplier of the selection constraint, which means that it vanishes in a first-best setting where the differences in productivity are observed by the social planner (in which case the self-selection constraint becomes redundant). If the mimicker is more positional than the low-ability type at time t such that $\hat{\alpha}_t^2 > \alpha_t^1$, an increase in $\bar{c}_t$ contributes to higher welfare by relaxing the self-selection constraint, in which case the sign of the right-hand side of equation (40) is ambiguous. In other words, the pure efficiency cost of a higher $\bar{c}_t$ is accompanied by a welfare gain through this relaxation of the self-selection constraint. On the other hand, if $\hat{\alpha}_t^2 < \alpha_t^1$, i.e., if the low-ability type is more positional than the mimicker, the right-hand side of equation (40) is unambiguously negative since an increase in $\bar{c}_t$ in that case contributes to tighten the self-selection constraint.

To distinguish the resource allocation analyzed here from the second-best optimum in Subsection 6.2, we shall once again use the superscript 0 to denote the economy with uninternalized positional externalities, i.e.,

$$\left\{c_t^{1,0}, l_t^{1,0}, c_t^{2,0}, l_t^{2,0}, k_t^0, \lambda_t^{p,0}\right\} \forall t .$$

The corresponding optimal value function becomes

$$W_t^0 = \int_t^\infty \omega\left(\upsilon(c_s^{1,0}, z_s^{1,0}, \bar{c}_s^0), \upsilon(c_s^{2,0}, z_s^{2,0}, \bar{c}_s^0)\right)e^{-\theta(s-t)}ds , \tag{41}$$

which measures the social welfare at time t.

For notational convenience, define the instantaneous welfare effect of an increase in $\bar{c}_t$ measured in consumption units (by dividing equation [40] by the shadow price of capital) such that

$$\Lambda_t = -\bar{\alpha}_t + \frac{\eta_t^p \hat{\upsilon}_c^2}{\lambda_t^p}\left[\hat{\alpha}_t^2 - \alpha_t^1\right]. \tag{42}$$

We can then derive the following result:

Proposition 5. *If the economy satisfies (6d), (6e), (31), (32), (35b), (35d), (35e), (39a), and (39b), the measure of welfare change can be written as*

$$\dot{W}_t^0 = \lambda_t^0 \dot{k}_t^0 + \int_t^\infty \lambda_s^0 \Lambda_s^0 \dot{\bar{c}}_s^0 e^{-\theta(s-t)}ds . \tag{43}$$

Proof: See Appendix.

Proposition 5 shows how the conventional measure of genuine saving should be modified to reflect the change in welfare over the short time interval $(t, t+dt)$, if the otherwise second-best optimal resource allocation contains uninternalized positional externalities. As in the analogous measure of welfare change derived in the simpler representative-agent model in Section 3, the second term on the right-hand side of equation (43) – which adjusts the conventional genuine saving measure for these externalities – is forward looking. To be able to write equation (43) in exactly the same format as its counterpart in equation (13), i.e., as the product of the welfare change in consumption units and the marginal utility of consumption, we would need an additional, and simplifying, assumption; namely, that the relative wage rate, $\phi_t = w_t^1 / w_t^2 = f_{\ell^1}(\ell_t^1, \ell_t^2, k_t) / f_{\ell^2}(\ell_t^1, \ell_t^2, k_t)$, is independent of k_t for all t (which it is for

standard constant returns to scale production functions such as the Cobb-Douglas or CES). In this case, the shadow price of capital takes the simple form

$$\lambda_t = \lambda_0 \exp(-\int_0^t r_\tau d\tau + \theta t)$$

and equation (43) can be rewritten as

$$\dot{W}_t^0 = \lambda_t^0 \left[\dot{k}_t^0 + \int_t^\infty \Lambda_s^0 \exp(-R_{s-t}) \dot{\bar{c}}_s^0 ds \right]. \tag{44}$$

The only important difference between equations (13) and (44) is that the partial instantaneous welfare effect of increased reference consumption is no longer necessarily negative here, i.e., Λ_t can be either positive or negative, while it would be unambiguously negative in a full information setting where the self-selection constraint does not bind (in which case $\eta_t^p = 0$ and $\Lambda_t = -\bar{\alpha}_t < 0$ for all t). As such, the second term on the right-hand side of equation (44) reflects not only the pure efficiency cost of the positional externality (as represented by the average degree of positionality), but also that an increase in this externality may either facilitate or hinder redistribution.

To be more specific, if the low-ability type is always more positional than the mimicker such that $\alpha_t^1 > \hat{\alpha}_t^2$ for all t, then $\Lambda_t < 0$ for all t and equation (44) is interpretable in the same general way as the measure of welfare change in the representative-agent model in equation (13). However, if the mimicker is more positional than the low-ability type such that $\hat{\alpha}_t^2 > \alpha_t^1$ during a certain time interval $t \in (t_1, t_2)$, and if this difference in degrees of positionality is sufficiently large, we cannot rule out that Λ_t is positive during this time interval. As a consequence, an increase of the reference measure $\bar{c}_t$ through increased consumption along the equilibrium path may actually be associated with a net social benefit (instead of cost), meaning that the conventional measure of genuine saving underestimates (instead of overestimates) the change in social welfare.

What determines differences in the degree of positionality between the mimicker and the low-ability type? In our (quite general) model where the utility functions are allowed to differ between types, these positionality differences may arise for a variety of reasons. To simplify the interpretation, consider the special case where all individuals share a common utility function such that the functions $u^i(\cdot)$ and $\upsilon^i(\cdot)$ are the same for both types. In that case, the only difference between the mimicker and the low-ability type is that the mimicker spends

more time on leisure. Therefore, $\hat{\alpha}_t^2 > (<) \alpha_t^1$ if the degree of positionality increases (decreases) with the use of leisure. Although this information is not directly observable, it is (at least in principle) recoverable via econometric methods.

An implication of the above is also that the first term on the right-hand side of equations (43) and (44), $\lambda_t \dot{k}_t$, is less informative here than in the context of equation (13). More specifically, even if we were to assume that the economy is growing such that $\overline{c}_t$ increases along the equilibrium path, we can no longer use $\lambda_t \dot{k}_t < 0$ for a one-sided test of welfare decline and local unsustainable development (as we did in Subsection 3.2) without making additional assumptions. This is so because $\lambda_t \dot{k}_t < 0$ does not imply that the right-hand side of equation (44) is negative. Since the instantaneous welfare effect of increased reference consumption can be either positive or negative here, $\lambda_t \dot{k}_t < 0$ neither implies declining intertemporal welfare nor that the current instantaneous social welfare in unsustainable. The applicability of the one sided test of welfare decline presupposes that the second term on the right hand side of equation (44) is negative. If $\overline{c}_t$ increases along the equilibrium path, a sufficient condition for this term to be negative is that

$$\Lambda_t = -\overline{\alpha}_t + \frac{\eta_t^p \hat{\upsilon}_c^2}{\lambda_t^p} \left[\hat{\alpha}_t^2 - \alpha_t^1 \right] < 0 \text{ for all } t.$$

This inequality is satisfied if the low-ability type is more positional than the mimicker (as explained above), or if the difference in the degree of positionality between the mimicker and the low-ability type is small enough in absolute value relative to the average degree of positionality. In either case, the information requirements are more demanding here.

7. Numerical Illustration

In previous sections, we have shown that conventional measures of genuine saving do not generally constitute valid measures of welfare change (unless the positional externality has become fully internalized), and we have also derived exact welfare change measures for economies with uninternalized positional externalities. While this is important *per se*, it is also important to be able to say something about likely orders of magnitude and whether or not the modifications are likely to be similar between countries in percentage terms. In this section we illustrate first that the discrepancies between measures of genuine saving and welfare

change are indeed likely to be substantial and second that these modifications may also differ greatly between countries.

Our numerical simulation example is based on data for Sweden and the U.S., and serves to provide a rough estimate of the right-hand side of equation (44), i.e., the genuine saving adjusted for uninternalized positional externalities. To be able to manage this task without a lengthy numerical extension of the paper, we make four additional, simplifying assumptions: (i) all consumers share a common utility function where leisure is weakly separable from the other goods, (ii) the positionality degree is constant over time for all consumers, (iii) the interest rate is constant over time, and (iv) the average consumption in the economy can be approximated by a consumption function with a constant growth rate.

Assumption (i) means that the mimicker and the mimicked agent are equally positional, i.e., $\hat{\alpha}_t^2 = \alpha_t^1$, implying, in turn, that the second term on the right-hand side of equation (42) vanishes.[27] Taken together, assumptions (i) and (ii) imply $\Lambda_t = -\bar{\alpha}_t = -\bar{\alpha}$ for all t,[28] while assumption (iii) implies $r_t = r$ for all t. Finally, assumption (iv) is consistent with models used in literature on endogenous growth and implies that we can write the average consumption at any time $s > t$ as

$$\bar{c}_s = \bar{c}_t e^{\kappa(s-t)}, \tag{45}$$

where κ is the growth rate.

Our aim is now to operationalize the welfare change measure given in equation (44). As such, the calculations below presuppose that the positional externality is uninternalized, while all other aspects of public policy are optimally chosen conditional on the level of reference consumption. By using assumptions (i)-(iv), equation (44) can be written in the following convenient way:[29]

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[27] To our knowledge, there is no empirical evidence available here, which suggests that the relationship between the degree of consumption positionality and the use of leisure is a relevant issue for future research. Therefore, lack of empirical evidence on this relationship motivates us to assume leisure separability.

[28] Assumption (ii) can alternatively be replaced with a functional form assumption on the instantaneous utility function, such that the degree of positionality is constant over time; see, e.g., Ljunqvist and Uhlig (2000).

[29] Since assumption (i) eliminates any difference in the degree of positionality between the mimicker and the low type, the second line of equation (46) would also follow from the welfare change measure of the representative-agent model given in equation (13).

$$\dot{W}_t \frac{1}{\lambda_t} = \dot{k}_t + \int_t^\infty \Lambda_s \exp(-R_{s-t})\dot{\bar{c}}_s ds$$

$$= \underbrace{\dot{k}_t}_{GS} - \underbrace{\frac{\bar{\alpha}\kappa}{r-\kappa}\bar{c}_t}_{ME} \qquad . \tag{46}$$

The left-hand side of equation (46) is a money-metrics version of the welfare change (through division by the marginal utility of consumption), while the right-hand side decomposes this into the conventional genuine saving (*GS*) and the value of the change in the marginal positional externality (*ME*). In particular, note that the higher the average degree of positionality, $\bar{\alpha}$, or the higher the consumption growth rate, κ, ceteris paribus, the larger the marginal externality will be, and the more the conventional measure of genuine saving will overestimate the welfare change.

To estimate equation (46), we use data from Sweden and the U.S. for the period 1990-2012. Our point of departure is the World Bank measure of adjusted net saving, which we interpret as a conventional indicator of genuine saving that does not reflect any positional externalities (i.e., our *GS* above).[30] We shall then examine how this measure ought to be modified in response to uninternalized positional externalities through the second term on the right-hand side of equation (46). The raw data contains information on the adjusted net saving, aggregate private consumption, total population, and the GDP deflator, allowing us to calculate both the conventional genuine saving and the value of the marginal externality in real per capita terms.[31] We use per capita variables to avoid possible welfare consequences of population size and growth (which we have abstracted from above). The average degree of positionality is assumed to be in the interval 0.2-0.4, which is thus a conservative assumption given the empirical research discussed in the introduction. The calculations are presented in Table 1.

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[30] The adjusted net saving is defined as the net national savings plus education expenditure minus an estimated value of energy depletion, mineral depletion, forest depletion, carbon dioxide emissions, and particulate emissions damage. We realize, of course, that this definition captures several aspects not addressed above (mainly referring to valuation of human capital and environmental capital). Yet, our more narrow definition of net investment is not important here, since adding investments in human and environmental capital does not affect any of the qualitative results derived above. In fact, we showed in Section 5 that a straightforward extension of the model would allow us to reinterpret $\dot{k}$ to reflect a broader capital concept.

[31] These data are from the database World Development Indicators of the World Bank: http://data.worldbank.org/data-catalog/world-development-indicators.

Table 1 HERE

The numbers in Table 1 should be seen for what they are, i.e., the outcome of a computational example and not the result of empirical research. With this reservation in mind, we would nevertheless like to emphasize three important insights from the table. First, the value of the change in the marginal positional externality is substantial. Note that this is so despite our quite conservative estimates of the average degree of positionality, $\bar{\alpha}$. Indeed, even based on the lower-bound estimate of 0.2, we obtain quite large values of *ME*. This suggests that the conventional measure of genuine saving may largely overestimate the true welfare change. Second, the conventional measure of genuine saving may give incorrect qualitative information as to whether the development is locally sustainable. For several combinations of growth rate, interest rate, and average degree of positionality, *GS-ME* is less than zero despite that *GS* is positive. Third, this discrepancy will generally not be similar between countries. In our case, the discrepancy is considerably larger for the U.S. than for Sweden, as the conventional genuine saving is smaller and the per capita consumption larger in the U.S. than in Sweden.

8. Discussion and Conclusion

This paper deals with the measurement of welfare change by developing an analogue to genuine saving in an economy where the consumers are positional. Recent empirical evidence shows that people derive utility from their own consumption relative to that of referent others, and also that the corresponding negative externalities might be substantial, implying that measures of social savings ought to be modified accordingly. This is further emphasized by the now widely accepted role that genuine saving plays as an indicator of local sustainable development.

In the context of first-best representative-agent economies, earlier research shows that the genuine saving is an exact indicator of welfare change and also interpretable as an exact indicator of local sustainable development. We show that this result carries over to a second-best economy based on asymmetric information about individual productivity and a binding self-selection constraint, in the sense that the change in social welfare (measured by a general social welfare function) over a short time interval is exactly equal to the genuine saving. We also show how the value of public investment affects genuine saving and derive a simple

accounting price that applies irrespective of whether or not the positional externalities have become internalized.

However, in the arguably more interesting case where the positional externality has not become fully internalized, the equality between welfare change and genuine saving no longer applies, and we show how this discrepancy depends on the degrees of positionality (i.e., measures used in empirical literature to indicate the extent to which relative consumption matters for individual well-being). In a representative-agent model, and if the consumption increases along the general equilibrium path (which the ideas of a positional arms race may suggest), uninternalized positional externalities typically imply that the genuine saving overestimates the change in social welfare. As such, negative genuine saving can still be used as an indicator of local unsustainable development, i.e., negative genuine saving implies that the social welfare declines.

In an economy with productivity differences and asymmetric information, the conventional measure of genuine saving is even less informative. Indeed, if the positional externalities are not internalized we can in this case not any longer use negative genuine saving as an indicator of local unsustainable development. We also demonstrate in a numerical exercise that the modifications are in general likely to be substantial and that they are likely to differ between countries to a large extent.

Future research may take several possible directions. First, recent empirical evidence shows that individuals do not only compare their own consumption with that of other domestic residents, they also engage in between-country comparisons (Becchetti et al., 2013). This suggests to us that a multi-country framework might be useful in future research on genuine saving. We conjecture that genuine saving is less useful as an indicator of welfare change at the national level in such a setting. Second, although empirical evidence shows that people are concerned with their relative consumption, people are also found to have an aversion against too much inequality. This suggests that it might be relevant to extend the analysis of genuine saving by allowing also for other types of social comparisons.

Appendix

The Benchmark Model

The value function is given by

$$V_t = \int_t^\infty \upsilon(c_s, z_s, \bar{c}_s) e^{-\theta(s-t)} ds. \tag{A1}$$

Differentiating equation (A1) with respect to t gives

$$\dot{V}_t = \theta V_t - \upsilon(c_t, z_t, \bar{c}_t). \tag{A2}$$

To write equation (A2) in the format of equation (10) or (13), we must find an expression for θV_t.

Proof of Observation 1

Differentiating the present value Hamiltonian in equation (5) with respect to time and using $\bar{c}_t = c_t$ gives

$$\frac{dH_t^P}{dt} = -\theta \upsilon(c_t, z_t, \bar{c}_t) e^{-\theta t} + \frac{\partial H_t^P}{\partial c_t} \dot{c}_t + \frac{\partial H_t^P}{\partial l_t} \dot{l}_t + \frac{\partial H_t^P}{\partial k_t} \dot{k}_t + \frac{\partial H_t^P}{\partial \lambda_t} \dot{\lambda}_t. \tag{A3}$$

The first term on the right-hand side is the direct effect of time through the utility discount factor, while the remaining three terms represent indirect effects of time through the control, state, and costate variables. Equations (5), (6a), (6b), and (6c) imply $\partial H_t^P / \partial c_t = \partial H_t^P / \partial l_t = 0$, $\dot{\lambda}_t = -\partial H_t^P / \partial k_t$, and $\partial H_t^P / \partial \lambda_t = \dot{k}_t$, and equation (A3) reduces to

$$\frac{dH_t^P}{dt} = -\theta \upsilon(c_t, z_t, \bar{c}_t) e^{-\theta t}. \tag{A4}$$

Solving equation (A4) forward subject to the transversality condition $\lim_{t \to \infty} H_t^P = 0$ implies

$$\theta \int_t^\infty \upsilon(c_s, z_s, \bar{c}_s) e^{-\theta s} ds = H_t^P. \tag{A5}$$

Multiplying both sides of equation (A5) by $e^{\theta t}$ gives Weitzman's (1976) welfare measure $\theta V_t = H_t$. Substituting into equation (A2) and using $H_t = \upsilon(c_t, z_t, \bar{c}_t) + \lambda_t \dot{k}_t$ gives equation (10). ∎

Proof of Proposition 1

Differentiating equation (5) totally with respect to time, while treating $\bar{c}_t$ as a separate variable, we can rewrite equation (A3) as

$$\frac{dH_t^P}{dt} = -\theta \upsilon(c_t, z_t, \bar{c}_t) e^{-\theta t} + \frac{\partial H_t^P}{\partial c_t} \dot{c}_t + \frac{\partial H_t^P}{\partial l_t} \dot{l}_t + \frac{\partial H_t^P}{\partial k_t} \dot{k}_t + \frac{\partial H_t^P}{\partial \lambda_t} \dot{\lambda}_t + \frac{\partial H_t^P}{\partial \bar{c}_t} \dot{\bar{c}}_t. \tag{A6}$$

Then, by using equations (6b), (6c), and (7), equation (A6) simplifies to

$$\frac{dH_t^P}{dt} = -\theta \upsilon(c_t, z_t, \bar{c}_t) e^{-\theta t} - \lambda_t e^{-\theta t} \alpha_t \dot{\bar{c}}_t, \tag{A7}$$

where we have used $\partial H_t^p / \partial \bar{c}_t = \upsilon_{\bar{c}} e^{\theta t} = -\alpha_t \upsilon_c e^{-\theta t} = -\alpha_t \lambda_t^p = -\alpha_t \lambda_t e^{-\theta t}$. Solving equation (A7) forward subject to $\lim_{t \to \infty} H_t^P = 0$, and multiplying by $e^{\theta t}$ to convert into current value, gives

$$\theta V_t = H_t - \int_t^\infty \lambda_s \alpha_s \dot{\bar{c}}_s e^{-\theta(s-t)} ds . \tag{A8}$$

Substitute equation (A8) into equation (A2) and use $H_t = \upsilon(c_t, z_t, \bar{c}_t) + \lambda_t \dot{k}_t$

$$\dot{V}_t = \lambda_t \dot{k}_t - \int_t^\infty \lambda_s \alpha_s \dot{\bar{c}}_s e^{-\theta(s-t)} ds . \tag{A9}$$

Using $\lambda_s / \lambda_t = \exp(-\int_t^s r_\tau d\tau + \theta(s-t))$, and substituting into equation (A9), gives equation (13). ∎

The Model with Heterogeneous Consumers

The social welfare function is given by

$$W_t = \int_t^\infty \omega\left(\upsilon(c_s^1, z_s^1, \bar{c}_s), \upsilon(c_s^2, z_s^2, \bar{c}_s)\right) e^{-\theta(s-t)} ds . \tag{A10}$$

The welfare change measure can then be written as

$$\dot{W}_t = \theta W_t - \omega\left(\upsilon(c_s^1, z_s^1, \bar{c}_s), \upsilon(c_s^2, z_s^2, \bar{c}_s)\right). \tag{A11}$$

Proof of Proposition 4

Differentiating the present value Lagrangean in equation (34) with respect to time, and using that the first-order conditions in equations (35a)-(35e) are fulfilled, gives

$$\frac{dL_t^p}{dt} = -\theta \omega\left(\upsilon(c_t^1, z_t^1, \bar{c}_t), \upsilon(c_t^2, z_t^2, \bar{c}_t)\right) e^{-\theta t} . \tag{A12}$$

By solving equation (A12) in the same way as we solved equation (A4) above, and using that $L_t^p = H_t^p$, the proof follows by analogy to the proof of Proposition 1. ∎

Proof of Proposition 5

Differentiating the present value Lagrangean in equation (34) with respect to time, while using the first-order conditions in equations (35c), (35d), (35e), (39a), and (39b), gives

$$\frac{dL_t^p}{dt} = -\theta \omega\left(\upsilon(c_t^1, z_t^1, \bar{c}_t), \upsilon(c_t^2, z_t^2, \bar{c}_t)\right) e^{-\theta t} + \lambda_t e^{-\theta t} \Lambda_t \dot{\bar{c}}_t , \tag{A13}$$

where we have used $\partial L_t^p / \partial \overline{c}_t = \lambda_t^p \Lambda_t = \lambda_t e^{-\theta t} \Lambda_t$. Solving equation (A13) forward subject to the transversality condition $\lim_{t \to \infty} H_t^p = 0$, using $L_t^p = H_t^p$, and multiplying by $e^{-\theta t}$ to convert into current value terms, gives

$$\theta \int_t^\infty \omega\left(\upsilon(c_s^1, z_s^1, \overline{c}_s), \upsilon(c_s^2, z_s^2, \overline{c}_s)\right) e^{-\theta(s-t)} ds = H_t + \int_t^\infty \lambda_s \Lambda_s \dot{\overline{c}}_s e^{-\theta(s-t)} ds. \qquad (A14)$$

Substituting into equation (A11) and using $H_t = \omega\left(U_t^1, U_t^2\right) + \lambda_t \dot{k}_t$ gives equation (43). ■

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Table 1   A computation example

	Sweden			U.S.		
(i) $\kappa = 0.01$ and $r = 0.03$						
	GS	*ME*	*GS-ME*	*GS*	*ME*	*GS-ME*
$\bar{\alpha} = 0.2$	6.26	1.81	4.45	3.84	2.64	1,20
$\bar{\alpha} = 0.4$	6.26	3.61	2.65	3.84	5.27	-1.43
(ii) $\kappa = 0.02$ and $r = 0.04$						
	GS	*ME*	*GS-ME*	*GS*	*ME*	*GS-ME*
$\bar{\alpha} = 0.2$	6.26	3.61	2.65	3.84	5.27	-1.43
$\bar{\alpha} = 0.4$	6.26	7.23	-0.97	3.84	10.55	-6.71

Note: All numbers are given in thousands of U.S. dollars at the 2005 price level. To avoid yearly fluctuations, the calculations are based on mean values for the genuine saving per capita and aggregate consumption per capita over the period 1990-2012.