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The Pioneers' Arguments for Formulating Economic Problems Mathematically

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Abstract

The nineteenth century pioneers in formulating economic problems mathematically often felt that they needed to explain their reasons for using mathematics. We will look at the arguments of Cournot, Thünen, Gossen, Jevons, Walras, Edgeworth, Marshall, Fisher, Wicksell, and Pareto. Three main arguments can be found: First, mathematics provides greater clarity of presentation, secondly, economics is fundamentally similar to the mathematical natural sciences, especially physics, and third, mathematics can help economists themselves to control the reasoning in their analysis.

Keywords: History of economic thought, economic methodology, mathematics, marginalism

JEL codes: B41, C00

The pioneers in formulating economic problems mathematically worked at a time when few understood this departure from the traditional verbal presentation which, at the very most, could contain statistics. Therefore they often felt that they needed to explain their reasons for using mathematics.

The discussion here will be restricted to early economists who fulfill three requirements: First, they made theoretical contributions which make their work (or at least their names) known to present-day economists with elementary knowledge of the history of economic thought. Second, mathematics in the form of algebra, functions, or derivatives played an essential part in their analyses. Third, they gave thorough arguments for their use of mathematics. We shall focus on those arguments.

These requirements restrict the number of names but do not exclude a certain amount of arbitrariness. We will look closer at Cournot, Thünen, Gossen, Jevons, Walras, Edgeworth, Marshall, Fisher, Wicksell, and Pareto. Table 1 (below) shows these ten economists' use of three chief arguments:

- that using mathematics provides greater *clarity of presentation* including greater ease of checking the logic of the reasoning;
- that using mathematics is appropriate since economics *is like the mathematical natural sciences – especially physics and its subdivisions mechanics and astronomy*; and
- that using mathematics can *help economists themselves to survey and control the reasoning in their analysis,* even if the published result may end up being entirely verbal.

On this last point, Gunnar Myrdal (1972, pp. 144-145) recounts that when he worked on an appendix to his *Asian Drama*, he "actually came up with an illustration in the form of a set of algebraic equations and a huge diagram" which he nevertheless abstained from using in the text as "it would have been pretentious, and conducive to the mistaken view that it would have added anything to the knowledge [he] was trying to convey in the text". Gustav Cassel, who himself had a doctor's degree in mathematics before he became an economist, maintained in a letter to the Danish mathematician Annette Vedel in 1896 that "it's mathematically trained thinking, not mathematics, that is needed to penetrate deeply into the economic questions" (Nycander 2005, p. 17).

Table 1: Pioneers' arguments for mathematics in economics

	Clarity	*Like physics*	*Help the analysis*
Cournot	Yes	Yes	
Thünen	Yes	Yes	
Gossen	Yes	Yes	
Jevons	Yes	Yes	
Walras	Yes	Yes	
Edgeworth	Yes	Yes	
Marshall	(Yes)	(Yes)	Yes
Fisher	Yes	(Yes)	
Wicksell	Yes		Yes
Pareto	Yes	Yes	

Note: Parenthesis means less emphasis on the argument. Blank space does not necessarily mean complete absence of the argument.

The *clarity* and *physics* arguments were used by almost all these economics pioneers, though Marshall emphasized especially that mathematics could *help the economist's analysis* (a point which Wicksell also made). But let's look in some detail at how each of these pioneers argued.

Cournot

Augustin Cournot (1801-1877), a professor of mathematics in Lyon, published also in economics and philosophy of science, but mathematics dominated his thinking. He states in *Researches into the Mathematical Principles of the Theory of Wealth* ([1838] 1963, p. 4) that he had "put aside questions, to which the mathematical analysis cannot apply". Thus he let the method determine his choice of analytical question, rather than the reverse. However, in the opinion of a modern scholar, "No contribution during this period [before Jevons' *Theory of Political Economy*] could approach in quality or in breadth of vision Cournot's *Recherches"* (Theocharis 1993, p. 266).

As algebra or calculus were not common in economic texts in 1838, Cournot expected criticism: "This is a plan likely, I confess, to draw on me at the outset the condemnation of theorists of repute. With one accord they have set themselves against the use of mathematical forms, and it will doubtless be difficult to overcome today a prejudice which thinkers, like Smith and other modern writers, have contributed to strengthen." (p. 1-2)

But the critics had not understood what it was about, Cournot continues. They believe that "the use of symbols and formulas could only lead to numerical calculations", and as "the subject was not suited to such a numerical determination of values by means of theory alone," the mathematical apparatus was meaningless. "But those skilled in mathematical analysis know that its subject is not simply to calculate numbers, but that it is also employed to find relations between magnitudes which cannot be expressed in numbers." If mathematical symbols "are able to facilitate the exposition of problems, to render them more concise, to open the way to more extended development, and to avoid the digression of vague argumentation" (p. 2), they should not be rejected just because they are not equally familiar to all readers. This is the *clarity* argument for mathematics.

Cournot also gave analogies from mathematical astronomy in his analysis of some phenomena in the economy. Concerning variations in the price of wheat, he says, "as in astronomy, it is necessary to recognize *secular* variations which are independent of *periodic variations"* (p. 20). And the concept *"corrected money* would be the equivalent of the mean sun of astronomers" (p. 21). Thus, Cournot also argues by analogy with *physics*.

Thünen

Johann Heinrich von Thünen (1783-1850) – who had studied natural sciences, economics, and mathematics in connection with his studies of agronomy – used an inheritance to buy the estate Tellow in Mecklenburg in 1809. His main work – only parts of which have been translated and published as *Von Thünen's Isolated State* (1966) – is an example of applied economics, mathematics, and agronomy, using data and experience from Tellow. The book – whose first German edition appeared in 1826 – came out in a substantially expanded second edition in 1850 and then in posthumous editions in 1863 and 1875.

Thünen (1966, p. 229) asserts that the results in the first edition ("Part One" in later editions) had been deduced "from a formula on the costs and yield of agriculture, which itself was based on empirical data". Provided that these data was correct and the conclusions were consistent, "this method introduces mathematical certainty into a field where mere reasoning would give rise to wholly contradictory opinions." This again is the *clarity* argument.

Thünen first adopts a verbal marginal analysis while praising the differential calculus, which is its mathematical equivalent. "Our method of determining the maximum net product is thus in accordance with the method which in mathematics has been proved correct for determining the maximum value of a function...." (p. 232)

Like Cournot, Thünen understood that his mathematical approach would be criticized by narrow-minded colleagues:

"I am very much afraid that I have tried the patience of many of my readers with the algebraic calculations, because I am not unaware how burdensome and inconvenient even many learned men find algebraic formulas.

But the use of mathematics must, nevertheless, be allowed where the truth cannot be found without it.

If one in other scientific fields had had such a disinclination to mathematical calculation as in agriculture and political economy, we would still be in complete ignorance of the laws of astronomy; and shipping, which now connects all parts of the world because of the extension of astronomy, would be limited to mere coastwise shipping" (Thünen 1875, 2nd part, 1st section, pp. 177-178)

Here Thünen invokes the *physics* argument.

Gossen

Hermann Heinrich Gossen (1810-1858) – though he was good at mathematics – aimed at a career in public service. However, that did not go well, so, after a short interlude in an insurance company, he retired in 1850 to develop his economic ideas. The result – presented in an 1854 book translated as *The Laws of Human Relations and the Rules of Human Action Derived Therefrom* – attracted attention only after his death. Astronomy was clearly the model for his approach.

"I believe I have accomplished for the explanation of the relation among humans what Copernicus was able to accomplish for the explanation of the relations of heavenly bodies. I believe that I have succeeded in discovering the force, and in its general form also the law of the effect of this force, that makes possible the coexistence of the human race and that governs inexorably the progress of mankind. And just as the discoveries of Copernicus have made it possible to determine the paths of the planets for any future time, I believe that my discoveries enable me to point out to man with unfailing certainty the path that he must follow in order to accomplish the purpose of his life." (p. cxlvii)

Mathematics was important when determining the paths of the planets, and Gossen found an analogy with the economy. But he expected objections.

"As to the exposition, its mathematical foundation will undoubtedly irritate the majority of those who are inclined to concern themselves with economic problems, as it is unfortunately not yet customary to consider mathematical competence as a necessary part of man's education. For the justification of this framework, it suffices to observe that economics concerns itself with the interplay of a variety of forces, and that it is impossible to determine the resultant effect of these forces without calculus. For this reason it is impossible to present the true system of economics without the aid of mathematics – a fact that has long been recognized in the case of pure astronomy, pure physics, mechanics, and so forth. That we have not yet succeeded in finding an appropriate mathematical framework may have contributed substantially to the confusion in which economics finds itself today." (p. cxlviii)

Thus Gossen considered mathematics necessary for economics. But "taking into consideration ... that mathematics is by no means part of general training, [I have] striven constantly to proceed on the assumption that the reader knows only parts of mathematics that are taught in our *Gymnasiums."* (p. cxlviii)

At the end of the book, Gossen interprets the laws which natural scientists have revealed – and his own economic laws – as a sign of God's wisdom:

"Given the mode in which the other laws of nature function, the Creator knew how, through the law of decrease in pleasure, to cause man – the creature to whom He granted the greatest self-determination – to use this freedom only in the most desirable way for the benefit for the whole universe." (p. 299)

To Gossen, mathematics was thus a tool both for deducing and for expressing scientific laws given by God.

Jevons

William Stanley Jevons (1835-1882) – like Cournot and Thünen – had a background in the natural sciences, having studied chemistry and botany as well as mathematics at the University College School in London before moving to Australia in 1854 to work as an assayer at the Mint in Sydney. Here he also became interested in economics, finding the mathematical approach natural. To him, the *essence* of economic questions was mathematical. In 1858 he wrote to a sister who was also reading economics:

"You will perceive that *economy*, scientifically speaking, is a very contracted science; it is in fact a sort of vague mathematics which calculates the causes and effects of man's industry." (quoted by Black 1970, p. 12)

Returning to England and picking up his studies again in 1859, Jevons became so convinced that his understanding of the mathematical nature of economics was correct that, according to a letter to his brother in 1860, he "cannot now read other books on the subject without indignation" (quoted by Black 1970, p. 13). But he struggled with a problem which, today, would probably had been formulated in econometric terms, i.e., the relationship between theoretical principles and data:

"While the theory is entirely mathematical in principle, I show, at the same time, how the data of calculation are so complicated as to be for the present hopeless. Nevertheless, I obtain from the mathematical principles all the chief laws at which political economists have previously arrived, only arranged in a series of definitions, axioms, and theories almost as rigorous and connected as if they were so many geometrical problems." (quoted by Black 1970, p. 13)

In a section in the introductory chapter of his *The Theory of Political Economy* (1871) – under the heading Mathematical Character of the Science – Jevons again maintains that economics is intrinsically mathematical:

"It seems perfectly clear that Economy, if it is to be a science at all, must be a mathematical science. There exists much prejudice against attempts to introduce the methods and language of mathematics into any branch of the moral sciences. Most persons appear to hold that the physical sciences form the proper sphere of mathematical method, and that the moral sciences demand some other method, I know not what. My theory of Economy, however, is purely mathematical in character. Nay, finding that the quantities with which we have to deal are subject to continuous variation, I do not hesitate to use the appropriate branch of mathematical science, involving though it does the fearless consideration of infinitely small quantities. The theory consists in applying the differential calculus to the familiar notions of wealth, utility, value, demand, supply, capital, interest, labor, and all the other notions belonging to the daily operations of industry. As the complete theory of almost every other science involves the use of that calculus, so we cannot have a true theory of Political Economy without its aid.

To me it seems that our science must be mathematical, simply because it deals with quantities." (pp. 3-4)

Jevons gives the example of the "ordinary laws of supply and demand" which "express the mode in which the quantities vary in connection with the price. By this fact the laws *are* mathematical: Economists cannot deprive them of their nature by denying them the name; they might as well try to alter red light by calling it blue." (pp. 4-5)

Jevons believed that the reason why many were skeptical of using mathematics in economics was that they took 'mathematical science' to mean 'exact science'. "They think that we must not pretend to calculate unless we have the precise data." (p. 6) But, besides pure logic, there is no exact science, only more-or-less approximate sciences. Even astronomy is not a fully exact science, because every calculated solution of an astronomic problem "involves hypotheses which are not really true: as, for instance, that the earth is a smooth, homogeneous spheroid" (p. 7).

Consequently, there is no fundamental difference between economics and astronomy, although astronomy is more exact than is economics. “There can be but two classes of science – those which are *simply logical, and those which, besides being logical, are also mathematical.”* (p. 8) In both astronomy and economics, “quantitative notions enter, and the science must be mathematical in nature, by whatever name we call it.” (p. 8)

In his introduction to the second (1879) edition, Jevons comments on a discussion whether economics should be a logical science, *à la* Ricardo, or an inductive science. He maintains that we must accept, and keep apart, different methods for different types of knowledge. He himself works with general principles: “As all the physical sciences have their basis more or less obviously in the general principles of mechanics, so all branches and divisions of economic science must be pervaded by certain general principles.” His book is devoted “to the investigation of such principles – to the tracing out of the mechanics of self-interest and utility” (Jevons 1970, pp. 49-50).

Walras

Léon Walras (1834-1910) has been described as “the most ardent [advocate of “a suitably mathematical economics”] as he saw his relentless and seemingly unyielding advocacy … met with disdain in France” (Turk 2012, p. 150), but this does not mean that he was a brilliant mathematician. As a young man, Walras had twice failed to be admitted to the distinguished *École polytechnique*, and he never got a university position in France, his home country. In 1870, though, he was offered a professorship in Lausanne in Switzerland, which he held until 1892.

The first part of Walras’ best known book, *Elements of Pure Economics*, was published in French in 1874, and the second part in 1877. He never left this work, but revised it thoroughly a few times before the fourth (1900) edition, which he denoted as *“l’édition définitive”*.

Walras was influenced by his father Auguste Walras, who had sought an analogy between the concept of scarcity (*rareté*) in economics and that of velocity in physics. In 1860 the younger Walras tried to introduce a Newtonian model of market relations such that the “price of things is in inverse ratio to the quantity offered and in direct ratio to the quantity demanded” (quoted by Mirowski [1989] 1999, p. 255), which Mirowski regards as an incompetent attempt to link up with Newton’s force law. In 1862 Walras pleads in a letter for financial support to develop “an original creation,” a “new science ... a science of economic forces analogous to the science of astronomical forces ... the analogy is complete and striking” (quoted by Mirowski [1989] 1999, p. 255).

Walras corresponded extensively with the leading French economists of the time – Henri Poincaré, Émile Picard, Émile Borel, etc. – seeking their support for his mathematical economics. But they had reservations and proposals for modifications (which Walras did not follow) about the role of uncertainty; complications with discontinuities; the possibility of using qualitative measures which might require other mathematical approaches; and the role of memory and history in economic systems (Turk 2012, p. 151).

"This whole theory is mathematical," Walras states in the preface of the fourth (1900) edition of *Elements.* Even if it could be described in words, the proof of the theory must be mathematical (Walras 2003, p. 43).

Edgeworth

Francis Ysidro Edgeworth (1845-1926) – who was born in Ireland and educated initially at home – was admitted in 1862 to Dublin's Trinity College, where he specialized in classical languages and literature. In the 1870s he published on ethics, psychology, and utilitarianism, including calculation of utility. In 1888 he became professor of political economy at King's College in London, moving on to Oxford three years later.

As the title suggests, Edgeworth's *Mathematical Psychics* (1881) contains mathematical expressions of psychic phenomena, though quantitatively, ordinary text dominates. He declares in the introduction:

"An analogy is suggested between the *Principles of Greatest Happiness*, Utilitarian or Egoistic, which constitute the first principles of Ethics and Economics, and those *Principles of Maximum Energy* which are among the highest generalizations of Physics, and in virtue of which mathematical reasoning is applicable to physical phenomena as complex as human life." (p. v)

Edgeworth not only draws a parallel with *physics* but also adduces the *clarity* argument:

"He that will not verify his conclusions as far as possible by mathematics ... will hardly realize the full value of what he holds, will want a measure of what it will be worth in however slightly altered circumstances, a means of conveying and making it current." (p. 3)

Marshall (1881, p. 457) published a friendly book review of *Mathematical Psychics*, but Marshall's more reserved attitude towards mathematics is obvious. He thinks it will be interesting to watch the development of Edgeworth's theory "and, in particular, to see how far he succeeds in preventing his mathematics from running away with him, and carrying him out of sight of the actual facts of economics".

Marshall

Alfred Marshall (1842-1924) – the son of a Bank of England official – early showed an aptitude for mathematics, in which he became absorbed. But from the middle of the 1860s he was increasingly interested in philosophy, psychology, and economics. His main work – *Principles of Economics* 1890) – contains elements of mathematics, but placed mainly in an appendix. Marshall had thought carefully about what role mathematics should play in economic analysis. In a 1906 letter to Arthur Bowley he wrote:

"But I had a growing feeling in the later years of my work at the subject that a good mathematical theorem dealing with economic hypotheses was very unlikely to be good economics: and I went more and more on the rules: 1) Use mathematics as a shorthand language, rather than as an engine of inquiry. 2) Keep to them till you have done. 3) Translate into English. 4) Then illustrate by examples that are important to real life. 5) Burn the mathematics. 6) If you can't succeed in 4, burn 3. This last I did often. ... I think you should do all you can to prevent people from using Mathematics in cases in which the English Language is as short as the Mathematical." (quoted by Groenewegen 1995, p. 413)

This restrictive attitude to mathematics is also found in the preface of the various editions of *Principles of Economics.* In the first edition (1890) Marshall takes it for granted that the "notion of continuity with regard to development is common to all modern schools of economic thought, whether the chief influences acting on the them are those of biology ... or of history and philosophy." (Marshall 1961, p. ix) He says that his own view has been influenced from both those directions, "but their form has been most affected by mathematical conceptions of continuity, as represented in Cournot's *Principes Mathématiques de la Théorie des Richesses."* (p. ix)

Our observations of both the moral and physical worlds, he says, relate not so much to aggregate magnitudes as to marginal increments. Particularly, "the demand for a thing is a continuous function, of which the 'marginal' increment is, in stable equilibrium, balanced against the corresponding increment of its cost of production." (p. x) This can be demonstrated with diagrams or mathematical symbols, i.e. derivatives, but diagrams are often more clear. Therefore, Marshall sometimes used diagrams as supplementary illustrations in footnotes.

The conclusion of the preface of the first edition elucidates rules 1-6 above:

"The chief use of pure mathematics in economic questions seems to be in helping a person to write down quickly, shortly and exactly, some of his thoughts for his own use: and to make sure that he has enough, and only enough, premises for his conclusions (i.e., that his equations are neither more nor less in number than his unknowns). But when a great many symbols have to be used, they become very laborious to any one but the writer himself. And though Cournot's genius must give a new mental activity to everyone who passes through his hands, and mathematicians of caliber similar to his may use their favorite weapons in clearing a way for themselves to the centre of some of those difficult problems of economic theory, of which only the outer fringe has yet been touched; yet it seems doubtful whether any one spends his time well in reading lengthy translations of economic doctrines into mathematics, that have not been made by himself.[1] A few specimens of those applications of mathematical language which have proved most useful for my own purpose have, however, been added in an Appendix." (Marshall 1961, pp. x-xi)

In his description of method in the preface of the eighth (1920) edition, Marshall discloses that he had hesitated when working out economic models by analogy with mechanics and physics:

"The Mecca of the economist lies in economic biology rather than in economic dynamics. But biological conceptions are more complex than those of mechanics; a volume of Foundations must therefore give relatively large place to mechanical analogies; and frequent use is made of the term 'equilibrium' which suggests something of statical analogy." (Marshall 1961, p. xiv)

However, it is change that Marshall actually wanted to understand:

"Fragmentary statical hypotheses are used as temporary auxiliaries to dynamical – or rather biological – conceptions; but the central idea of economics, even when its Foundations alone are under discussion, must be that of living force and movement." (p. xv)

Indirectly, Marshall comes to the role of mathematics. Having established that marginal analysis is essential, he concludes:

"The new analysis is endeavouring gradually and tentatively to bring into economics, as far as the widely different nature of material will allow, those methods of the science of small increments (commonly called the differential calculus) to which man owes directly or indirectly the greater part of the control that he has obtained in recent times over physical nature." (p. xvii)

The method was still in its infancy, yet Marshall observed remarkable agreement among those who were working with it, "especially among such of them as have served an apprenticeship in the simpler and more definite, and therefore more advanced, problems of physics" (p. xvii).

Fisher

Irving Fisher (1867-1947) – the son of a Congregational minister – was engaged in many things beyond economics and statistics: the peace movement, eugenics, alcohol prohibition, vegetarianism, calendar reform, map projection….

The contents of his *Mathematical Investigations in the Theory of Value and Prices* ([1892] 1925) was "in substance" the same as his 1891 PhD thesis at Yale. He

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[1] Marshall's critic Paul A. Samuelson ([1947] 1965, p. 6) disapproves of the last statement and thinks it 'should be exactly reversed'.

starts by noting that most persons, including economists, are satisfied with vague notions. Experience may give a practical conception of the working of, say, mechanics, but it “gives no inkling of the complicated dependence on space, time, and mass. Only patient mathematical analysis can do that.” (p. V) The same applies, Fisher says, to the relations between value, utility, and quantity in economics.

The advantage of mathematics varies with the ability of the user and with the complexity of the problem. Advanced physics requires mathematics, and “similarly in economics, mathematical treatment is relatively useful as the relations become relatively complicated. The introduction of mathematical method marks a stage of growth – perhaps it is not too extravagant to say, the entrance of political economy on a scientific era.” (p. 109)

Specific insights have been gained by means of mathematics. Fisher says it “is perhaps fair to credit the idea of marginal utility to mathematical method” (p. 110). About this idea – whose origins he attributes to Dupuit, Gossen, Jevons, Menger, and Walras – he says “All except Menger presented this idea, and presumably attained it, by mathematical method. ... This one achievement is a sufficient vindication of the mathematical method.” (p. 110)

Within physics, he says the chief mission of mathematics had been to correct wrong conclusions. But, also within economics, mathematics “has corrected numerous errors and confusion of thought” (p. 111). This *clarity* argument dominates in Fisher.

Wicksell

Knut Wicksell (1851-1926) – like Fisher – had broad intellectual interests, reflected both in his university studies and in his writings and speeches. Wicksell’s Bachelor studies included such disparate subjects as theoretical philosophy, history, Latin, Scandinavian languages, astronomy, and mathematics. His Licentiate studies of mechanics, physics, and mathematics even included a dissertation *On Proving the Existence of a Root in an Algebraic Equation* (Gårdlund 1996, p. 79).

Wicksell also pursued fine literature: Among his juvenile publications are both a collection of poems and a stage play which was performed some ten times at various locations. But to the general public he was known as an iconoclast and provocateur who loved to challenge traditional moral, religious, and political ideas.

So – despite Wicksell’s Licentiate studies of mathematical sciences – his conception of the world was influenced from many directions, which influenced the role he would give mathematics in political economy.

The first time Wicksell touched on the role of mathematics in economics was in his article “Kapitalzins und Arbeitslohn” (Interest on Capital and Wages, 1892), where he tried to give Böhm-Bawerk’s capital theory a mathematical guise. Among other things, Wicksell points out that the theory is about the length of the period of production, making it difficult to avoid differential calculus. But he expects to meet opposition, and thus begins by defending his approach with the *clarity* argument.

"It is ... difficult to see why the use of mathematical symbols in economics should be prohibited forever, even when, without doubt, it could conduce to an abridgement of expression and to greater clarity of understanding." (p. 858)

Only basic mathematics is involved, Wicksell points out, and, besides, mathematical method is more and more used in the closely related discipline of statistics. But it is important to be aware of the limitations of mathematics: "Of course, one must be careful not to expect more from this method than it can give. From the crucible of the calculation does not an atom more of truth come out than what has been put into it." (p. 858) The results are completely dependent on the assumptions, i.e., "the results can only lay claim to an entirely conditional validity." (p. 858)

Wicksell comes back to the role of mathematics in economics in 1893 in *Value, Capital and Rent*, where he concludes: "Mathematical expression ought to facilitate the argument, clarify the results, and so guard against possible faults of reasoning, that is all." (Wicksell [1893] 1970, p. 53) Both Marshall and Wicksell seem to be concerned about what Romer (2015) recently called the difference between mathematical theory and *mathiness*. Like Marshall, Wicksell emphasizes that mathematics has no intrinsic value, and must not be placed over economic content: "It is, by the way, evident that the *economic* aspects must be the determining ones everywhere: Economic truth must never be sacrificed to the desire for mathematical elegance." (p. 53)

In the second Swedish edition (1911) of *Lectures on Political Economy*, Wicksell again comments on mathematics as a protection – though not absolute protection – against syllogistic errors. But the mathematical method "has a great advantage over the merely descriptive method, in that errors committed cannot long be concealed, and false opinions cannot be defended long after they have been shown to be wrong." (Wicksell 1977, p. xxiii). This is perhaps on overly optimistic judgment.

Pareto

Vilfredo Pareto (1848-1923) – born in Paris but brought up in Italy, where his parents moved in 1852 – began studying mathematical sciences at the University of Turin at age 16, and started a career as an engineer in 1870, first for a railway company. His writings broadened over the years, often arguing for economic liberalism. In 1892 he accepted an invitation to succeed Walras at Lausanne, a position which he held until 1911. During the last decades of his life he also wrote on matters which made his name as a sociologist.

In the first sentence of Pareto's preface to his *Cours d'économie politique* (1896), he says that his object is to "give an outline of economic science considered as a natural science" (p. iii), which hints at the *physics* argument. Concerning mathematics, he also says that he did not find it useful to "refuse the help of this powerful logic" (p. iii).

In the opening paragraph of the main text, Pareto repeats that he is going to explicate a natural science, "like psychology, physiology, chemistry, etc.". This means that the study will not give instructions but will instead solve questions of the type: "Given certain premises, what will the consequences be?" (p. 2)

Conclusion: Mathematical economy established

The economists whom we have scrutinized used quite elementary mathematics. The development of mathematics itself was at quite another level at the time, and advanced mathematics meant anything for economics only later in the 20th century.[2]

Nevertheless, by the beginning of the 20th Century the role of mathematics in economic explication and analysis had been discussed so many times that most authors no longer felt a need to explain it, and if they did, they only reiterated what others had already said.

But that the use of mathematics in economics no longer required explanation or defense did not mean that mathematical presentations had totally displaced verbal texts. It took a long time before most economists mastered even the mathematics of the pioneers. When Wicksell reviewed Pareto's *Manuel d'économique politique* in 1913, he declared that, to the majority of readers, the mathematical appendix "will ... be a closed book ... though from several points of view it is the most important and interesting" part of the work (Wicksell [1913] 1969, p. 160). Landreth and Colander (2002, p. 391) maintain that the mathematical approach was not well received in the United States until the middle of the 20th Century, and that these pioneers were, therefore, "unheeded prophets of the future".

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[2] Weintraub (2002) discusses this from various angles.

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