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Abstract

In this paper we analyze the effects of environmental policies on the size distribution of firms. We model a stationary industry where the observed size distribution is a solution to the profit maximization problem of heterogeneous firms that differ in terms of their energy efficiency. We compare the equilibrium size distribution under *emission taxes*, *uniform emission standards*, and *performance standards*. Our results indicate that, unlike emission taxes and performance standards, emission standards introduce regulatory asymmetries favoring small firms. These asymmetries cause significant detrimental effects on total output and total welfare, yet lead to reduced emissions and help preserve small businesses.

Keywords: Environmental regulations, energy efficiency, size distribution, emission taxes, emission standards, performance standards.

Jel Codes: Q58, L25, Q55

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1 Introduction

In recent years, several environmental regulations have been introduced to control emissions of several pollutants. These policies have a clear objective: to induce firms to reduce emissions by investing in cleaner/energy-saving technologies and promoting industrial turnover by modifying, among other things, the possibility of entry of new firms, exit of incumbent firms, and the relative competitive advantage of active firms. Environmental regulations may also affect the distribution of market shares and the related size distribution of firms if compliance changes the optimal plant size. As pointed out by Evans (1986), the differential effect of regulation across firm size is important since society may have an interest in preserving small businesses because of antitrust or other noneconomic reasons. When there are scale economies in regulatory compliance, it might be optimal to exempt or impose lighter regulatory burden on smaller firms, or design regulations that are neutral across firm size to minimize the disproportionate impact of environmental regulatory requirements on small businesses (see also Brock and Evans 1985). The incidence of regulatory costs across firm size may also tell us something about the interest of certain groups of businesses in supporting alternative regulatory policies.

The size distribution of firms has been extensively studied in the industrial organization literature. Most of the literature deals with the distributional properties of firm size (see, e.g., Cabral and Mata 2003 and Angelini and Generale 2008). However, more recent research has integrated the size distribution of firms into standard economic theory. Attempts to explain the size dynamics have investigated the effects of bad productivity shocks (Hopenhayn 1992 and Ericson and Pakes 1995), learning (Jovanovic 1982), inefficiencies in financial markets (Clementi and Hopenhayn 2006), the exogenous distribution of managerial ability in the population (Lucas 1978 and Garicano and Rossi-Hansberg 2004), and the efficient accumulation and allocation of factors of production (Rossi-Hansberg and Wright 2007).

In the environmental economics literature, some studies have identified two counteracting effects through which environmental policies affect the distribution of size. First, the studies by Pashigian

(1984), Dean et al. (2000), and Sengupta (2010) indicate that due to economies of scale, environmental regulation modifies the optimal scale of firms and puts small firms at a unit cost disadvantage. Second, Becker et al. (2013) argue that there are statutory and/or enforcement asymmetries that favor smaller establishments. Hence, the final incidence of environmental regulations depends on whether these regulatory asymmetries outweigh any scale economies in regulatory compliance. In this paper, we study the effects of the choice of policy instruments on the size distribution of firms. We find that the relative magnitude of these two effects is quite dependent on the type of environmental policies in place since different environmental policies redistribute intra-industry rents differently. Moreover, over time, different regulations might lead to a different distribution of the share of polluting inputs in the production process as they foster investments in different advanced technologies to different extents.

To study the effects of the choice of policy instruments on the size distribution of firms, we follow the seminal model by Lucas (1978), where the underlying size distribution of firms in the industry is the result of the existence of a productive factor of heterogeneous productivity. In Lucas' model, such a factor is the managerial technology, while in ours it is the energy efficiency of firms.¹ In such a setting, we introduce different environmental policies and analyze the resulting size distributions, as well as the variations in size distribution that arise as a result of investments that reduce the cost of compliance with environmental regulations. The heterogeneity of the available physical capital with respect to energy intensity is well established in the literature. Small firms typically spend more of their operating costs on energy than do large firms due to the lack of knowledge about and expected profitability of available energy-efficient technologies (de Groot et al. 2001 and Ruth et al. 2004).

We compare three environmental policies, namely *emission taxes*, *emission standards*, and *performance standards*. As shown in the paper, under emission taxes and performance standards, the intensity of emissions is determined by the stringency of the regulation and it is the same across

¹Our model also resembles that of Melitz (2003), who derives a simple model of industry equilibrium in an open economy with heterogeneous firms. Firms differ in terms of their marginal productivity of labor (the only factor of production). The productivity of each firm is randomly drawn from some distribution, but unlike our model, firms do not know their productivity prior to starting production. One of the predictions of the Melitz model is that opening up to trade will increase aggregate productivity.

firms. In contrast, under emission standards, the regulatory goal is expressed as an absolute emission limit, which favors smaller firms as the limit might not bind their emissions. Our results indicate that emission taxes and performance standards do not introduce regulatory asymmetries, but do modify the optimal scale of the firms. Moreover, the existence of economies of scale implies that these policies reduce to a lower extent profits for larger firms than for smaller firms. Further, emission taxes reduce the profits of larger firms to a larger extent than performance standards. However, when it comes to emission standards, the incidence of the regulatory costs across firm size depends on the two counteracting effects described above. In line with Becker et al. (2013), our results indicate that emission standards create regulatory asymmetries as they distort the emission intensity of large firms the most. This effect is likely to exceed the economies of scale effect, implying that emission standards reduce the profits of large firms to a larger extent. Finally, unlike previous studies suggesting that market-based instruments create more effective technology adoption incentives than conventional regulatory standards,² our results indicate that when the regulatory asymmetries created by emissions standards are taken into account, the profitability of emission saving biased technological change is higher under emission standards than under market-based instruments.

The paper is organized in six sections. The next section presents the model and the underlying size distribution of firms in the absence of environmental policies. The third section analyzes the incidence of regulatory costs across size and how the choice of a policy instrument modifies the size distribution of firms. The fourth section analyzes the effects of the choice of policy instruments on the share of the polluting input and technological choice. The fifth section presents some numerical simulations and analyzes welfare implications. The final section concludes.

2 The Model

We assume a perfectly competitive stationary industry consisting of a continuum of risk-neutral single-plant polluting firms of mass 1. Firms produce a homogeneous good using two inputs: energy (e)

²There is a large body of research analyzing the incentives provided by different environmental policy instruments for adoption of advanced abatement technology. See Requate (2005) for a review.

and labor (l). Moreover, each unit of energy e used as an input generates γ units of emissions ξ , i.e., $\xi_i = \gamma e_i$. Firms differ in terms of the parameter ϕ , which reflects energy efficiency and is assumed to be uniformly distributed on the interval $[\underline{\phi}, \bar{\phi}]$.

Assuming a Cobb–Douglas technology, the production function of firm i is then characterized as:

$$q(\phi_i, e, l) = \theta [\phi_i e_i]^\alpha l_i^\beta \quad \forall \alpha, \beta > 0, \alpha + \beta < 1, \quad (1)$$

where q is the amount of output produced by a firm using e units of energy and l units of labor, ϕ_i is the energy efficiency of firm i , and θ is a technology index. In the absence of environmental regulation, firm i maximizes net profits π_i^{NR} through the choice of inputs:

$$\max_{e_i, l_i} \pi_i^{NR} = p\theta [\phi_i e_i]^\alpha l_i^\beta - wl_i - ze_i - F, \quad (2)$$

where w and z are the equilibrium wage rate and energy price, respectively. p represents the output price, and F corresponds to a fixed cost. The first order conditions (FOCs) are given by:

$$p\alpha\theta\phi_i^\alpha e_i^{\alpha-1} l_i^\beta = z, \quad (3)$$

$$p\beta\theta\phi_i^\alpha e_i^\alpha l_i^{\beta-1} = w. \quad (4)$$

Dividing by parts, we obtain:

$$\frac{e_i^{NR}}{l_i^{NR}} = \frac{\alpha w}{\beta z}. \quad (5)$$

Substituting equation (5) in the FOCs, we can solve for e_i^{NR} and l_i^{NR} as:

$$e_i^{NR} = \left[p\theta\beta^\beta w^{-\beta} \alpha^{1-\beta} z^{-[1-\beta]} \phi_i^\alpha \right]^{\frac{1}{1-\alpha-\beta}}, \quad (6)$$

$$l_i^{NR} = \left[p\theta\beta^{1-\alpha} w^{-(1-\alpha)} \alpha^\alpha z^{-\alpha} \phi_i^\alpha \right]^{\frac{1}{1-\alpha-\beta}}. \quad (7)$$

We assume that $2\alpha + \beta < 1$, which means that the demand of energy is a concave function of the energy efficiency.

Replacing equations (6) and (7) in equations (1) and (2), we can solve for individual output and profits as:

$$q_i^{NR} = \left[p^{\alpha+\beta} \theta \beta^\beta w^{-\beta} \alpha^\alpha z^{-\alpha} \right]^{\frac{1}{1-\alpha-\beta}} \phi_i^{\frac{\alpha}{1-\alpha-\beta}}, \quad (8)$$

$$\pi_i^{NR} = [1 - \alpha - \beta] \left[p \theta \phi_i^\alpha \beta^\beta w^{-\beta} \alpha^\alpha z^{-\alpha} \right]^{\frac{1}{1-\alpha-\beta}} - F. \quad (9)$$

From equations (8) and (9), it is possible to see that output and profits increase as energy efficiency ϕ_i increases. Firm i would operate in this market as long as its profits are larger than F . Consistent with this, in the continuum of firms the minimum energy efficiency ϕ_0^{NR} satisfies the condition $\pi^{NR}(\phi_0) = F$, or:

$$\phi_0^{NR} = \left[\frac{F^{1-\alpha-\beta} z^\alpha w^\beta}{p \theta \beta^\beta \alpha^\alpha [1 - \alpha - \beta]^{1-\alpha-\beta}} \right]^{\frac{1}{\alpha}}. \quad (10)$$

Thus, the energy efficiency of the firms operating in the market is uniformly distributed on the interval $[\phi_0^{NR}, \bar{\phi}]$, where $\phi_0^{NR} \geq \underline{\phi}$. Note that ϕ_0^{NR} is an increasing function of the inputs prices z and w and a decreasing function of the output price p and the technology index θ . Moreover, the existence of the cost F implies economies of scale since large firms can spread the fixed cost across more output units than small firms.

We can compute aggregate emissions in the absence of environmental regulation Σ^{NR} by integrating individual emissions ξ_i over the range $[\phi_0^{NR}, \bar{\phi}]$, which leads to:

$$\Sigma^{NR} = \gamma \int_{\phi_0^{NR}}^{\bar{\phi}} \left[p \theta \beta^\beta w^{-\beta} \alpha^{1-\beta} z^{-[1-\beta]} \right]^{\frac{1}{1-\alpha-\beta}} \phi_i^{\frac{\alpha}{1-\alpha-\beta}} d\phi. \quad (11)$$

Let $h = \frac{1-\beta}{1-\beta-\alpha} > 1$ and $k_1 = \left[p \theta \beta^\beta w^{-\beta} \alpha^{1-\beta} z^{-[1-\beta]} \right]^{\frac{1}{1-\alpha-\beta}}$. The solution to equation (11) can be represented as:

$$\Sigma^{NR} = \frac{\gamma k_1}{h} \left[\bar{\phi}^h - \left[\phi_0^{NR} \right]^h \right]. \quad (12)$$

To compute total output with no regulation Q^{NR} , we integrate (8) over the interval $[\phi_0^{NR}, \bar{\phi}]$, which leads to:

$$Q^{NR} = \frac{z}{p\alpha\gamma} \Sigma^{NR}.$$

Dividing individual emissions γe_i^{NR} by individual output q_i^{NR} , we can see individual emission intensity in the absence of environmental regulations correspond to $\frac{\alpha p \gamma}{z}$. Note that it coincides with the average emission intensity Σ^{NR} / Q^{NR} . Moreover, (individual and average) emission intensity is a decreasing function of the price of energy z . It is also an increasing function of the share of energy in the production process α and of the output price p . Furthermore, the lower the coefficient γ , the lower the emission intensity.

3 Environmental Regulation

Let us now analyze the effects of environmental policies on the size distribution in equilibrium. We assume that given the initial size distribution of firms, the regulatory goal is to limit aggregate emissions at some exogenously given level \bar{E} by means of one of the following three regulatory instruments: a per-unit emission tax τ , a uniform emission standard $\bar{\xi}$, and a uniform performance standard that defines the maximum intensity of emissions κ . Finally, we assume that the stringency of each policy remains unchanged regardless of the effects of the instruments on the initial size distribution of firms.

Emission Taxes

In the case of emission taxes, firm i maximizes its profits π_i^T :

$$\max_{e_i, l_i} \pi_i^T = p\theta [\phi_i e_i]^\alpha l_i^\beta - w l_i - [z + \hat{\tau}] e_i - F, \quad (13)$$

where $\hat{\tau} = \tau\gamma$ is the tax per unit of energy.

The FOCs are:

$$p\alpha\theta\phi_i^\alpha e_i^{\alpha-1} l_i^\beta = z + \hat{\tau}, \quad (14)$$

$$p\beta\theta\phi_i^\alpha e_i^\alpha l_i^{\beta-1} = w. \quad (15)$$

Dividing by parts, we obtain:³

$$\frac{e_i^T}{l_i^T} = \frac{\alpha w}{\beta [z + \hat{\tau}]}. \quad (16)$$

Substituting equation (16) in the FOCs, we can solve for e_i^T and l_i^T as:⁴

$$e_i^T = \left[p\theta\beta^\beta w^{-\beta} \alpha^{1-\beta} [z + \hat{\tau}]^{-[1-\beta]} \phi_i^\alpha \right]^{\frac{1}{1-\alpha-\beta}}. \quad (17)$$

$$l_i^T = \left[p\theta\beta^{1-\alpha} w^{-(1-\alpha)} \alpha^\alpha [z + \hat{\tau}]^{-\alpha} \phi_i^\alpha \right]^{\frac{1}{1-\alpha-\beta}}. \quad (18)$$

Replacing equations (17) and (18) in equations (1) and (13), we can solve for individual output and profits as:

$$q_i^T = \left[p^{\alpha+\beta} \theta \beta^\beta w^{-\beta} \alpha^\alpha [z + \hat{\tau}]^{-\alpha} \right]^{\frac{1}{1-\alpha-\beta}} \phi_i^{\frac{\alpha}{1-\alpha-\beta}}, \quad (19)$$

$$\pi_i^T = [1 - \alpha - \beta] \left[p\theta\phi_i^\alpha \beta^\beta w^{-\beta} \alpha^\alpha [z + \hat{\tau}]^{-\alpha} \right]^{\frac{1}{1-\alpha-\beta}} - F. \quad (20)$$

The cutoff value of the energy efficiency in the case of taxes ϕ_0^T satisfies the condition $\pi_0^T(\phi_0) = F$, which yields:

$$\phi_0^T = \left[\frac{F^{1-\alpha-\beta} [z + \hat{\tau}]^\alpha w^\beta}{p\theta\beta^\beta \alpha^\alpha [1 - \alpha - \beta]^{1-\alpha-\beta}} \right]^{\frac{1}{\alpha}}. \quad (21)$$

By simple inspection of equations (10) and (21), it is easy to see that $\forall \hat{\tau} > 0$, $\phi_0^T > \phi_0^{NR}$. Moreover, as in the previous case, we can compute aggregate emissions and output under taxes (Σ^T, Q^T) by integrating individual emissions and output over the range $[\phi_0^T, \bar{\phi}]$, which leads to:

$$\Sigma^T = \gamma k_2 \int_{\phi_0^T}^{\bar{\phi}} \phi_i^{\frac{\alpha}{1-\alpha-\beta}} d\phi = \frac{\gamma k_2}{h} \left[\bar{\phi}^h - \left[\phi_0^T \right]^h \right]. \quad (22)$$

³Compared with equation (5), we can see that the use of energy per unit of labor is lower $\forall \hat{\tau} \neq 0$.

⁴As in the previous case, we analyze the case where the optimal use of energy is a concave function of the energy efficiency.

$$Q^T = \frac{[z + \hat{\tau}] \Sigma^T}{p\alpha\gamma}, \quad (23)$$

$$\text{where } k_2 = \left[p\theta\beta^\beta w^{-\beta} \alpha^{1-\beta} [z + \hat{\tau}]^{-[1-\beta]} \right]^{\frac{1}{1-\alpha-\beta}}.$$

Thus, the average emission intensity in the industry correspond to $\frac{\Sigma^T}{Q^T} = \frac{p\alpha\gamma}{z+\hat{\tau}}$. Dividing individual emissions γe_i^T by individual output q_i^T , we can see that individual emission intensity also corresponds to $\frac{p\alpha\gamma}{z+\hat{\tau}}$. Note that with regard to the situation with no regulation, the average emission intensity of the industry is decreased under taxes. Moreover, like in the case with no regulation, the emission intensity of each firm in the industry is the same at the margin and given by the price ratio of emissions to output. Before the imposition of the regulation, firms whose energy efficiency was lower than ϕ_0^T earned positive profits, but they did not take the social externality cost into consideration. The tax on emissions corrects the divergence between private and social incentives by forcing firms whose energy efficiency is in the range $[\phi_0^{NR}, \phi_0^T]$ out of business.

Let $\delta_i^T = \pi_i^{NR} - \pi_i^T > 0$ represent the gap in profits under emissions taxation vis-a-vis no regulation. Subtracting equation (20) from equation (9), it is easy to show that δ_i^T is given by:

$$\delta_i^T = [1 - \alpha - \beta] \left[p\theta\beta^\beta w^{-\beta} \alpha^\alpha \right]^{\frac{1}{1-\alpha-\beta}} \phi_i^{\frac{\alpha}{1-\alpha-\beta}} \left[z^{\frac{-\alpha}{1-\alpha-\beta}} - [z + \hat{\tau}]^{\frac{-\alpha}{1-\alpha-\beta}} \right]. \quad (24)$$

Moreover, let $\Delta\pi_i^T = \frac{\delta_i^T}{\pi_i^{NR}} > 0$ represent the percentage reduction in firm i 's profits under emissions taxation vis-a-vis no regulation. To study the incidence of the regulatory costs of environmental taxation across firm size, we compute the first and second order derivative of $\Delta\pi_i^T$ with regard to ϕ_i , which leads to the following proposition:

Proposition 1 *Emission taxes reduce by a larger percentage profits for smaller firms than for larger firms.*

Proof. Substituting equation (24) in $\Delta\pi_i^T$ and differentiating with respect to ϕ_i yields:

$$\begin{aligned}\frac{\partial\Delta\pi_i^T}{\partial\phi_i} &= -\underbrace{\left[\frac{\alpha}{1-\alpha-\beta}\right]}_{\text{Economies of Scale}} \frac{F}{\phi_i} \frac{\Delta\pi_i^T}{\pi_i^{NR}} < 0 \\ \frac{\partial^2\Delta\pi_i^T}{(\partial\phi_i)^2} &= \left[\frac{\alpha}{1-\alpha-\beta}\right] \left[\frac{F\Delta\pi_i^T}{\phi_i^2\pi_i^{NR}} + \frac{F\Delta\pi_i^T}{\phi_i[\pi_i^{NR}]^2} \frac{\partial\pi_i^{NR}}{\partial\phi_i} - \frac{F}{\phi_i\pi_i^{NR}} \frac{\partial\Delta\pi_i^T}{\partial\phi_i}\right] > 0.\end{aligned}$$

Hence, $\Delta\pi_i^T$ decreases at decreasing rate as ϕ_i increases, implying that in relative terms, emission taxes increase the cost of compliance (and thus reduce the profits) of the smaller firms more than they reduce the profits of larger firms. The intuition behind this result is the existence of economies of scale. As mentioned before, under emission taxes the energy and emission intensity of each firm in the industry is the same at the margin. However, in absolute terms, large firms produce more output and release more emissions. The fixed cost F puts the smaller firms at a unit cost disadvantage; the normalized fixed cost $\frac{F}{\phi_i}$ reflects the fact that the percentage reduction in profits of the larger firms is smaller since they can spread the fixed cost across a larger output. The percentage reduction in profits decreases at a decreasing rate since the use of energy (and emission tax payments) is a concave function of the energy efficiency. ■

Emission Standard

Under a uniform emission standard, the government restricts the individual emissions generated during the production process to the level $\bar{\xi}$. In our setting, this restriction is equivalent to a restriction on the use of the energy input. Thus, firm i maximizes profits given by the constraint $\xi_i \leq \bar{\xi}$, or:

$$\max_{e_i, l_i} \pi_i^S = p\theta [\phi_i e_i]^\alpha l_i^\beta - wl_i - ze_i - F \quad \text{s.t.} \quad e_i \leq \bar{\xi}\gamma^{-1}. \quad (25)$$

Since the standard is uniform and firms are heterogenous, we should expect it to be binding only for some firms. Taking the case without regulation as the baseline, we should expect the standard to be binding if $\xi_i^{NR} \geq \bar{\xi}$. If the standard is *not* binding, the choice of inputs proceeds as in the case

without regulation. If the standard *is* binding, the FOC wrt l_i is:

$$l_i^S = \left[\frac{p\theta\beta\phi_i^{\alpha}\bar{\xi}^{-\alpha}\gamma^{-\alpha}}{w} \right]^{\frac{1}{1-\beta}}, \quad (26)$$

while the energy to labor ratio is equal to:

$$\frac{e_i^S}{l_i^S} = \left[\frac{[\bar{\xi}\gamma^{-1}]^{1-\alpha-\beta} w}{p\theta\beta\phi_i^{\alpha}} \right]^{\frac{1}{1-\beta}}. \quad (27)$$

Substituting l_i^S and e_i^S in equations (1) and (25) yields to output and profits for those firms for which the standard is binding:

$$q_i^S = \left[p^{\beta}\beta^{\beta}w^{-\beta}\theta \right]^{\frac{1}{1-\beta}} [\bar{\xi}\gamma^{-1}]^{\frac{\alpha}{1-\beta}} \phi_i^{\frac{\alpha}{1-\beta}}. \quad (28)$$

$$\pi_i^S = [1 - \beta] \left[p\theta\beta^{\beta}\phi_i^{\alpha} [\bar{\xi}\gamma^{-1}]^{\alpha} w^{-\beta} \right]^{\frac{1}{1-\beta}} - z [\bar{\xi}\gamma^{-1}] - F. \quad (29)$$

In order to compare environmental policies, we assume that aggregate emissions under the emission standard and emission tax are equivalent ex-ante. Therefore, we can solve for $\bar{\xi}$ by integrating emissions over the range $[\phi_0^{NR}, \bar{\phi}]$ and equalizing this to Σ^T in equation (22), which leads to the following condition:

$$\int_{\phi_0^{NR}}^{\bar{\phi}} \bar{\xi} d\phi = \frac{\gamma k_2}{h} \left[\bar{\phi}^h - [\phi_0^T]^h \right].$$

Therefore, the standard $\bar{\xi}$ can be represented as:

$$\bar{\xi} = \frac{\gamma k_2}{h} \left[\frac{\bar{\phi}^h - [\phi_0^T]^h}{\bar{\phi} - \phi_0^{NR}} \right]. \quad (30)$$

By comparing equations (6) and (30), it is possible to show that there is a critical value $\widehat{\phi}_1$ that defines whether the standard $\bar{\xi}$ is binding. The critical value $\widehat{\phi}_1$ corresponds to:

$$\widehat{\phi}_1 = \left[h \left[\frac{z}{z + \widehat{\tau}} \right] \left[\frac{\bar{\phi}^h - [\phi_0^T]^h}{\bar{\phi} - \phi_0^{NR}} \right] \right]^{\frac{1}{h-1}}.$$

Note that $\widehat{\phi}_1$ is positively related to the length of the interval of the energy efficiency distribution $[\phi_0^{NR}, \bar{\phi}]$, meaning that the more heterogeneous the firms are in terms of the energy efficiency, the larger the critical value defining whether emission standards are binding.

Regarding the emission intensity, if $\phi_i \in [\phi_0^{NR}, \widehat{\phi}_1]$, the standard $\bar{\xi}$ is not binding and energy used and output are given by equations (6) and (8), respectively. The average (and individual) emission intensity in this interval is the same as in the case without regulation and equal to $\frac{p\alpha\gamma}{z}$. If $\phi_i \in [\widehat{\phi}_1, \bar{\phi}]$, the standard is binding and individual emissions are equal to $\bar{\xi}$. Dividing $\bar{\xi}$ by (28) leads to an individual emission intensity equal to $\gamma^{\frac{\alpha}{1-\beta}} \bar{\xi}^{\frac{1-\alpha-\beta}{1-\beta}} \left[p^\beta \beta^\beta w^{-\beta} \theta \phi_i^\alpha \right]^{\frac{-1}{1-\beta}}$. Note that unlike emission taxes, under emission standards the individual emission intensity depends on the energy efficiency parameter ϕ_i . It decreases as energy efficiency increases at a decreasing rate, implying that large firms use the input that generates emissions less intensively.

Let $\delta_i^S = \pi_i^{NR} - \pi_i^S > 0$ represent the gap in firm i 's profits under emission standards vis-a-vis no regulation. Subtracting equation (29) from equation (9), it is easy to show that if $\phi_i > \widehat{\phi}_1$, δ_i^S is given by:

$$\delta_i^S = [1 - \alpha - \beta] \left[p\theta\beta^\beta w^{-\beta} \alpha^\alpha z^{-\alpha} \right]^{\frac{1}{1-\alpha-\beta}} \phi_i^{\frac{-\alpha}{1-\alpha-\beta}} - [1 - \beta] \left[p\theta\beta^\beta \phi_i^\alpha [\bar{\xi}\gamma^{-1}]^\alpha w^{-\beta} \right]^{\frac{1}{1-\beta}} + z [\bar{\xi}\gamma^{-1}]. \quad (31)$$

Moreover, let $\Delta\pi_i^S = \frac{\delta_i^S}{\pi_i^{NR}} > 0$ represent the percentage reduction in firm i 's profits under emission standards vis-a-vis no regulation. To study the incidence of the regulatory costs of emission standards across firm size, we compute the first and second order derivative of $\Delta\pi_i^S$ with regard to ϕ_i , which leads to the following proposition:

Proposition 2 *Emission standards reduce by a larger percentage profits for larger firms than for smaller firms.*

Proof. Substituting equation (31) in $\Delta\pi_i^S$ and differentiating with respect to ϕ_i yields

$$\frac{\partial\Delta\pi_i^S}{\partial\phi_i} = \underbrace{\left[\frac{\alpha}{1-\alpha-\beta} \right] \left[\frac{\alpha \left[p\theta\beta^\beta \phi_i^\alpha [\bar{\xi}\gamma^{-1}]^\alpha w^{-\beta} \right]^{\frac{1}{1-\beta}} - z [\bar{\xi}\gamma^{-1}]}{\phi_i \pi_i^{NR}} \right]}_{\text{Regulatory Asymmetry}} - \underbrace{\left[\frac{\alpha}{1-\alpha-\beta} \right] \frac{F}{\phi_i} \frac{\Delta\pi_i^S}{\pi_i^{NR}}}_{\text{Economies of Scale}} > 0. \quad (32)$$

Hence, the incidence of the regulatory costs of emission standards across firm size depends on two counteracting effects. The first effect - regulatory asymmetry (RA)- is positive and captures the fact that emission standards distort the emission intensity of larger firms the most. Compared with non regulation (where emission intensity is the same across firms), under emission standards the larger firms are forced to use the energy input less intensively. Thus, their profits are reduced by a larger percentage than those of smaller firms. ■

Like in the case of taxation, the second effect, the scale effect (SE), is negative and captures the fact that the fixed cost F puts the smaller firms at a unit cost disadvantage, and hence, vis-a-vis no regulation, the profits of the smaller firms are reduced to a larger extent than those of larger firms.

We can show that the regulatory asymmetry effect is larger than the scale effect implying that profits under emission standards are reduced by a larger percentage for larger firms than for smaller firms (see appendix A). Moreover, differentiating $\frac{\partial\Delta\pi_i^S}{\partial\phi_i}$ with respect to ϕ_i yields:

$$\frac{\partial^2\Delta\pi_i^S}{(\partial\phi_i)^2} = \left[\frac{\partial RA}{\partial\phi_i} + \frac{\partial SE}{\partial\phi_i} \right].$$

As in the case of taxes we can show that $\frac{\partial SE}{\partial\phi_i} > 0$. Hence, the scale effect decreases at a decreasing rate since the use of energy is a concave function of the energy efficiency. The derivative $\frac{\partial RA}{\partial\phi_i}$ is given by:

$$\frac{\partial RA}{\partial\phi_i} = - \left[\frac{1-\alpha-\beta}{1-\beta} \right] \frac{RA}{\phi_i} - \frac{RA}{\pi_i^{NR}} \frac{\partial\pi_i^{NR}}{\partial\phi_i} + \left[\frac{\alpha^2}{[1-\alpha-\beta][1-\beta]} \right] \left[\frac{z [\bar{\xi}\gamma^{-1}]}{\phi_i^2 \pi_i^{NR}} \right] < 0.$$

That is, if $\phi_i > \widehat{\phi}_1$, the regulatory asymmetry effect increases at an increasing rate as ϕ_i increases. Since the first part of the $\frac{\partial^2 \Delta \pi_i^S}{(\partial \phi_i)^2}$ is negative and the second part is positive, the sign of the $\frac{\partial^2 \Delta \pi_i^S}{(\partial \phi_i)^2}$ is ambiguous.

Performance Standard

Under a performance standard, emission intensity is fixed by policy at $\frac{e_i}{q_i} \leq \kappa$. Firms can meet partly this restriction by reducing emissions in the numerator and partly by increasing output in the denominator. In our setting, this restriction is equivalent to a restriction on the use of input energy equal to $e_i \leq \kappa \gamma^{-1} q_i$. Thus, firm i maximizes

$$\max_{e_i, l_i} \pi_i^{PS} = p\theta(\phi_i e_i)^\alpha l_i^\beta - w l_i - z e_i - F \quad \text{s.t.} \quad e_i \leq \kappa \gamma^{-1} q_i.$$

If the constraint is binding, the choice of the energy input is given by:

$$e_i = \kappa \gamma^{-1} \theta (\phi_i e_i)^\alpha l_i^\beta$$

or:

$$e_i^{PS} = \left[\kappa \gamma^{-1} \theta \phi_i^\alpha l_i^\beta \right]^{\frac{1}{1-\alpha}}, \quad (33)$$

and the profit maximization problem becomes:

$$\max_{l_i} \pi_i^{PS} = p\theta(\phi_i e_i^{PS})^\alpha l_i^\beta - w l_i - z e_i^{PS}. \quad (34)$$

Substituting equation (33) into equation (34) and solving the FOC wrt l_i yields:

$$l_i^{PS} = \left[\frac{\kappa \gamma^{-1} \theta \beta^{1-\alpha} [p \kappa^{-1} \gamma - z]^{1-\alpha}}{[1-\alpha]^{1-\alpha} w^{1-\alpha}} \phi_i^\alpha \right]^{\frac{1}{1-\alpha-\beta}}. \quad (35)$$

Substituting equation (35) into equation (33), we solve for e_i^{PS} as:

$$e_i^{PS} = \left[\frac{\kappa \gamma^{-1} \theta \beta^\beta [p \kappa^{-1} \gamma - z]^\beta \phi_i^\alpha}{[1 - \alpha]^\beta w^\beta} \right]^{\frac{1}{1 - \alpha - \beta}}, \quad (36)$$

where

$$\frac{e_i^{PS}}{l_i^{PS}} = \frac{[1 - \alpha] w}{\beta [p \kappa^{-1} \gamma - z]}. \quad (37)$$

Finally, substituting l_i^{PS} and e_i^{PS} in (34) yields:

$$\pi_i^{PS} = [1 - \alpha - \beta] \left[\kappa \gamma^{-1} \theta \left[\frac{p \kappa^{-1} \gamma - z}{1 - \alpha} \right]^{1 - \alpha} \phi_i^\alpha \beta^\beta w^{-\beta} \right]^{\frac{1}{1 - \alpha - \beta}} - F. \quad (38)$$

As in the case of the emission standard, we assume that aggregate emissions under the performance standard and the emission tax are equivalent ex-ante. Therefore, the performance standard is equal to the average emission intensity under taxes and corresponds to $\kappa = \frac{p \alpha \gamma}{z + \hat{\tau}}$. Substituting κ in equation (38) and solving for the cutoff value ϕ_0^{PS} that satisfies the condition $\pi_0^{PS}(\phi_0^{PS}) = F$ yields:

$$\phi_0^{PS} = \left[\frac{F^{1 - \alpha - \beta} [z + \hat{\tau}] [1 - \alpha]^{1 - \alpha} w^\beta}{p \theta \beta^\beta \alpha^\alpha [z [1 - \alpha] + \hat{\tau}]^{1 - \alpha} [1 - \alpha - \beta]^{1 - \alpha - \beta}} \right]^{\frac{1}{\alpha}}. \quad (39)$$

As usual, aggregate emissions under performance standard Σ^{PS} are calculated by integrating individual emissions ξ_i^{PS} over the range $[\phi_0^{PS}, \bar{\phi}]$, which leads to:

$$\Sigma^{PS} = \frac{\gamma k_2}{h} \left[\frac{z [1 - \alpha] + \hat{\tau}}{[z + \hat{\tau}] [1 - \alpha]} \right]^{\frac{\beta}{1 - \alpha - \beta}} \left[\bar{\phi}^h - [\phi_0^{PS}]^h \right]. \quad (40)$$

Let $\delta_i^{PS} = \pi_i^{NR} - \pi_i^{PS} > 0$ represent the gap in profits under performance standards vis-a-vis no regulation. Subtracting equation (38) from equation (9), it is easy to show that δ_i^{PS} is given by:

$$\delta_i^{PS} = [1 - \alpha - \beta] \left[\theta \beta^\beta w^{-\beta} \right]^{\frac{1}{1-\alpha-\beta}} \phi_i^{\frac{\alpha}{1-\alpha-\beta}} \left[\left[p \alpha^\alpha z^{-\alpha} \right]^{\frac{1}{1-\alpha-\beta}} - \left[\left[\kappa \gamma^{-1} \left[\frac{p \kappa^{-1} \gamma - z}{1 - \alpha} \right]^{1-\alpha} \right] \right]^{\frac{1}{1-\alpha-\beta}} \right]. \quad (41)$$

Moreover, let $\Delta \pi_i^{PS} = \frac{\delta_i^{PS}}{\pi_i^{NR}} > 0$ represent the percentage reduction in firm i 's profits under performance standards vis-a-vis no regulation. To study the incidence of the regulatory costs of emission standards across firm size, we compute the first and second order derivative of $\Delta \pi_i^{PS}$ with regard to ϕ_i , which leads to the following proposition:

Proposition 3 *Performance standards reduce by a larger percentage profits for smaller firms than for larger firms.*

Proof. Substituting equation (41) in $\Delta \pi_i^{PS}$ and differentiating with respect to ϕ_i yields:

$$\begin{aligned} \frac{\partial \Delta \pi_i^{PS}}{\partial \phi_i} &= - \underbrace{\left[\frac{\alpha}{1 - \alpha - \beta} \right] \frac{F \Delta \pi_i^{PS}}{\phi_i \pi_i^{NR}}}_{\text{Economies of Scale}} < 0, \\ \frac{\partial^2 \Delta \pi_i^{PS}}{(\partial \phi_i)^2} &= \left[\frac{\alpha}{1 - \alpha - \beta} \right] \left[\frac{F \Delta \pi_i^{PS}}{\phi_i^2 \pi_i^{NR}} + \frac{F \Delta \pi_i^{PS}}{\phi_i [\pi_i^{NR}]^2} \frac{\partial \pi_i^{NR}}{\partial \phi_i} - \frac{F}{\phi_i \pi_i^{NR}} \frac{\partial \Delta \pi_i^{PS}}{\partial \phi_i} \right] > 0. \end{aligned}$$

Like in the case of emission taxes, $\Delta \pi_i^{PS}$ decreases at a decreasing rate as ϕ_i increases, implying that in relative terms, performance standards increase the cost of compliance (and thus reduce the profits) of the larger firms to a lower extent. As in the case of emission taxes, the emission intensity of each firm in the industry is the same at the margin and given by the regulation. The regressive incidence of performance standards is explained by the existence of economies of scale. This effect decreases at a decreasing rate since the use of energy is a concave function of the energy efficiency. ■

Proposition 4 *Emission taxes reduce the profits of larger firms by a larger percentage than do performance standards.*

Proof. The condition $\left| \frac{\partial \Delta \pi_i^T}{\partial \phi_i} \right| > \left| \frac{\partial \Delta \pi_i^{PS}}{\partial \phi_i} \right|$ holds if $\pi_i^T - \pi_i^{PS} < 0$. Subtracting equation (38) from (20) yields:

$$\pi_i^T - \pi_i^{PS} = [1 - \alpha - \beta] \left[\theta \phi_i^\alpha \beta^\beta w^{-\beta} \alpha^\alpha \right]^{\frac{1}{1-\alpha-\beta}} \left[\left[p \alpha^\alpha [z + \hat{\tau}]^{-\alpha} \right]^{\frac{1}{1-\alpha-\beta}} - \left[\kappa \gamma^{-1} \left[\frac{p \kappa^{-1} \gamma - z}{1 - \alpha} \right]^{1-\alpha} \right]^{\frac{1}{1-\alpha-\beta}} \right].$$

We can see that $\pi_i^T - \pi_i^{PS} < 0$ if $p \alpha^\alpha [z + \hat{\tau}]^{-\alpha} < \kappa \gamma^{-1} \left[\frac{p \kappa^{-1} \gamma - z}{1 - \alpha} \right]^{1-\alpha}$. Replacing κ we have that this condition simplifies to:

$$\hat{\tau} [1 - \alpha] < \hat{\tau},$$

which is true since $0 < \alpha < 1$. Hence, and not surprisingly, emission taxes reduce firm profits by larger percentage. As pointed out by Fullerton and Heutel (2010) a restriction on emissions per unit of output is equivalent to a combination of a tax on emissions and subsidy to output. The actual cost of the regulation is larger under emission taxes since firms must pay the tax for each unit of emissions they release. Instead, under performance standards, firms are granted κq_i^{PS} units of emission free of charge. The higher the level of output q_i^{PS} , the larger the amount of emissions granted free of charge. Hence, as ϕ_i increases, and so does output, the actual cost of the regulation under performance standard decreases, implying that vis-a-vis taxation performance standards reduce the profits of larger firms by a lower percentage than do emission taxes. ■

Proposition 5 *We have the following ranking regarding how environmental policies modify the optimal scale of firms.*

(a) The minimum optimal firm size is larger under emission taxes and performance standards than under no regulation. Further, the minimum optimal firm size is larger under emission taxes than under performance standards, i.e., $\phi_0^{NR} < \phi_0^{PS} < \phi_0^T$.

(b) There is a critical threshold $\hat{\phi}_1$ that defines whether emission standards are binding. Since $\hat{\phi}_1 > \phi_0^{NR}$, it follows that emission standards do not affect the minimum optimal firm size.

Proof. (a) Comparing equations [10] and [21], it is easy to show that for any $\hat{\tau} > 0$, $\phi_0^{NR} < \phi_0^T$.

Comparing equations [21] and [39], it follows that $\phi_0^T > \phi_0^{PS}$ if:

$$[z[1-\alpha] + \hat{\tau}]^{1-\alpha} > [z + \tau]^{1-\alpha} [1-\alpha]^{1-\alpha},$$

which holds since $0 < \alpha < 1$. Hence, $\phi_0^{NR} < \phi_0^{PS} < \phi_0^T$. ■

As shown above, since emission taxes reduce firm profits by a larger percentage, the marginal firm in the case of taxation should be more energy efficient than the corresponding one in the case of performance standards.

(b) We show in page 11 that emission standards are binding only for larger firms. A more formal proof is provided in the Appendix B.

Finally, since $\phi_0^{PS} < \phi_0^T$, it can be shown that aggregate emissions are higher under performance standards than under emission taxes, i.e., $\Sigma^{PS} > \Sigma^T$. Indeed, $\Sigma^{PS} > \Sigma^T$ if:

$$\frac{\gamma k_2}{h} \left[\frac{z[1-\alpha] + \hat{\tau}}{[z + \hat{\tau}][1-\alpha]} \right]^{1-\frac{\beta}{\alpha-\beta}} \left[\bar{\phi}^h - [\phi_0^{PS}]^h \right] > \frac{\gamma k_2}{h} \left[\bar{\phi}^h - [\phi_0^T]^h \right],$$

which simplifies to:

$$\left[\frac{z[1-\alpha] + \hat{\tau}}{[z + \hat{\tau}][1-\alpha]} \right]^{1-\frac{\beta}{\alpha-\beta}} \left[\bar{\phi}^h - [\phi_0^{PS}]^h \right] > \left[\bar{\phi}^h - [\phi_0^T]^h \right].$$

This inequality holds since $\bar{\phi}^h - [\phi_0^{PS}]^h > \bar{\phi}^h - [\phi_0^T]^h$ and $\frac{z[1-\alpha] + \hat{\tau}}{[z + \hat{\tau}][1-\alpha]} > 1$.

4 The Choice of Policy Instruments and the Distribution of Factors

To analyze the effects of the choice of policy instruments on the distribution of factors, we model the choice between two technologies. In particular, technology 1 (T_1) increases the technology index from θ to $\hat{\theta}$, while technology 2 (T_2) reduces the generation of emissions per unit of energy from γ to $\tilde{\gamma}$.

From the analysis above, recall that if the regulations are binding, individual emissions, the energy to labor ratio and individual emission intensity under emission taxes (T), emission standards (S) and performance standard (PS) correspond to:

Table 1: Individual Emissions, Energy/Labor Ratio and Emission Intensity			
	Individual Emissions	Energy/Labor	Emission Intensity
T	$\gamma \left[p\theta\beta^\beta w^{-\beta} \alpha^{1-\beta} [z + \tau\gamma]^{-[1-\beta]} \phi_i^\alpha \right]^{\frac{1}{1-\alpha-\beta}}$	$\frac{\alpha w}{\beta[z+\tau\gamma]}$	$\frac{p\alpha\gamma}{z+\tau\gamma}$
S	$\bar{\xi}$	$[\gamma^{-1}\bar{\xi}]^{\frac{1-\alpha-\beta}{1-\beta}} [p\theta\beta w^{-1}\phi_i^\alpha]^{\frac{-1}{1-\beta}}$	$\gamma^{\frac{\alpha}{1-\beta}} \bar{\xi}^{\frac{1-\alpha-\beta}{1-\beta}} \left[p^\beta \theta \beta^\beta w^{-\beta} \phi_i^\alpha \right]^{\frac{-1}{1-\beta}}$
PS	$\gamma^{-\frac{[\alpha+\beta]}{1-\alpha-\beta}} \left[\frac{\kappa\theta\beta^\beta [p\kappa^{-1}\gamma-z]^\beta \phi_i^\alpha}{[1-\alpha]^\beta w^\beta} \right]^{\frac{1}{1-\alpha-\beta}}$	$\frac{[1-\alpha]w}{\beta[p\kappa^{-1}\gamma-z]}$	κ

Hence, under taxes and performance standards, individual emissions increase when θ increases. However, the firms' relative use of inputs and emission intensity do not depend on θ . In contrast, under emission standards the firms' relative use of energy and emission intensity are reduced if θ increases. θ has no effect on individual emissions if the standard is binding.

Under taxes and emission standards, T_2 increases the firms' relative use of energy while reducing the emission intensity (as well as individual emissions in the case of taxes). Finally, under performance standards, T_2 increases individual emissions and the firms' relative use of energy but has no effect on the emission intensity, which is fixed by the regulation. However, if the adoption of T_2 makes the standard κ no-binding, the emission intensity is reduced to $\frac{p\alpha\tilde{\gamma}}{z}$ and the energy to labor ratio is increased to $\frac{\alpha w}{\beta z}$.

All in all, the technologies affect individual emissions, the relative use of inputs, and emission intensity differently depending on the policy instrument in place. However, for simplicity, let us refer to T_1 as a neutral technical change (which holds for all cases but the emission standards) and T_2 as an emission-saving technological change. Without loss of generality, we assume that both options have the same investment cost G , and normalize γ to 1, meaning that $\hat{\tau} = \tau$ and $\tilde{\gamma} < 1$. We also assume that both technologies are profitable and focus instead on the choice of technology to understand how the distribution of factors is affected by the choice of different environmental policies.

Emission Taxes

Let $\Gamma_i^T(\hat{\theta}, \theta)$ represent firm i 's profits from adoption of T_1 . Using equation (20), $\Gamma_i^T(\hat{\theta}, \theta)$ can be represented as:

$$\Gamma_i^T(\hat{\theta}, \theta) = [1 - \alpha - \beta] \left[p\phi_i^\alpha \beta^\beta w^{-\beta} \alpha^\alpha [z + \tau]^{-\alpha} \right]^{\frac{1}{1-\alpha-\beta}} \left[\hat{\theta}^{\frac{1}{1-\alpha-\beta}} - \theta^{\frac{1}{1-\alpha-\beta}} \right] - \Lambda.$$

Profits from adoption of T_2 can be represented as $\Gamma_i^T(\tilde{\gamma}, 1)$:

$$\Gamma_i^T(\tilde{\gamma}, 1) = [1 - \alpha - \beta] \left[p\theta\phi_i^\alpha \beta^\beta w^{-\beta} \alpha^\alpha \right]^{\frac{1}{1-\alpha-\beta}} \left[[z + \tilde{\tau}]^{\frac{-\alpha}{1-\alpha-\beta}} - [z + \tau]^{\frac{-\alpha}{1-\alpha-\beta}} \right] - \Lambda,$$

where $\tilde{\tau} = \tau\tilde{\gamma}$, and $\Lambda = G - F > 0$.

From these equations we can show that T_1 is most profitable when the technology index $\hat{\theta}$ exceeds a critical threshold given by $\theta_T^* = \left[\frac{z+\tau}{z+\tilde{\tau}} \right]^\alpha \theta$. For $\hat{\theta} = \theta_T^*$, both technologies are equally profitable, while T_2 is more profitable if $\hat{\theta} < \theta_T^*$.

Emission Standards

From equation (29) we can see that if the standard is binding, the profits from adopting T_1 can be represented as:

$$\Gamma_i^S(\hat{\theta}, \theta) = \left[1 - \beta^{\frac{1}{1-\beta}} \right] \left[p\phi_i^\alpha [\bar{\xi}\gamma^{-1}]^\alpha w^{-\beta} \right]^{\frac{1}{1-\beta}} \left[\hat{\theta}^{\frac{1}{1-\beta}} - \theta^{\frac{1}{1-\beta}} \right] - \Lambda.$$

Profits from adopting T_2 can be represented as:

$$\Gamma_i^S(\tilde{\gamma}, 1) = \left[1 - \beta^{\frac{1}{1-\beta}} \right] \left[p\theta\phi_i^\alpha [\bar{\xi}\gamma^{-1}]^\alpha w^{-\beta} \right]^{\frac{1}{1-\beta}} \left[\tilde{\gamma}^{\frac{-\alpha}{1-\beta}} - 1 \right] - \Lambda.$$

Like in the previous case, there is critical threshold $\theta_S^* = \tilde{\gamma}^{-\alpha}\theta$ that defines which technology is the most profitable. T_1 is more profitable than T_2 when $\hat{\theta} > \theta_S^*$, while the reverse holds for $\hat{\theta} < \theta_S^*$. For $\hat{\theta} = \theta_S^*$, both technologies are equally profitable.

Performance Standards

Finally, from equation (38) we can see that the profits from adoption of T_1 can be represented as:

$$\Gamma_i^{PS}(\hat{\theta}, \theta) = [1 - \alpha - \beta] \left[\kappa \left[\frac{p\kappa^{-1} - z}{1 - \alpha} \right]^{1-\alpha} \phi_i^\alpha \beta^\beta w^{-\beta} \right]^{\frac{1}{1-\alpha-\beta}} \left[\hat{\theta}^{\frac{1}{1-\alpha-\beta}} - \theta^{\frac{1}{1-\alpha-\beta}} \right] - \Lambda.$$

Profits from adoption of T_2 can be represented as:

$$\Gamma_i^{PS}(\tilde{\gamma}, 1) = [1 - \alpha - \beta] \left[\frac{\theta \phi_i^\alpha \beta^\beta w^{-\beta} \kappa}{[1 - \alpha]^{1-\alpha}} \right]^{\frac{1}{1-\alpha-\beta}} \left[[\tilde{\gamma}^{-1} [p\kappa^{-1}\tilde{\gamma} - z]^{1-\alpha}]^{\frac{1}{1-\alpha-\beta}} - [p\kappa^{-1} - z]^{1-\alpha} \right]^{\frac{1}{1-\alpha-\beta}} - \Lambda.$$

Again, there is a critical threshold $\theta_{PS}^* = \theta \tilde{\gamma}^{-1} \left[\frac{z[\tilde{\gamma}-\alpha] + \tau\tilde{\gamma}}{z[1-\alpha] + \tau} \right]^{1-\alpha}$ that defines which technology is the most profitable. Investment in T_1 is more profitable than T_2 when $\hat{\theta} > \theta_{PS}^*$, while the reverse holds for $\hat{\theta} < \theta_{PS}^*$. For $\hat{\theta} = \theta_{PS}^*$, both technologies are equally profitable.

Proposition 6 *Compared with neutral technological change, the profitability of emission-saving technological change is the highest under emission standards, followed by emission taxes and performance standards.*

Proof. Comparing the thresholds we can show that:

$$\theta_S^* > \theta_T^* \text{ if } [z + \tau\tilde{\gamma}] > [z + \tau]\tilde{\gamma}, \text{ which holds since } \tilde{\gamma} < 1.$$

$$\theta_S^* > \theta_{PS}^* \text{ if } [z + \tau][1 + \tilde{\gamma}] > z\alpha, \text{ which holds since } \alpha < 1.$$

$$\theta_T^* > \theta_{PS}^* \text{ if } \tilde{\gamma}^{\frac{1}{1-\alpha}} \left[\left[\frac{z+\tau}{z+\tilde{\tau}} \right]^{\frac{\alpha}{1-\alpha}} - \tilde{\gamma}^{\frac{\alpha}{1-\alpha}} \right] > -\frac{z\alpha[1-\gamma]}{z[1-\alpha]+\tau}, \text{ which holds since } \frac{z+\tau}{z+\tilde{\tau}} > 1 > \tilde{\gamma}.$$

Hence, it follows that $\hat{\theta}^S > \hat{\theta}^T > \hat{\theta}^{PS}$. ■

This result is interesting. As discussed above, emission standards distort the choice of inputs the most, affecting quite significantly the profits of those firms for which the standard is binding. T_2 allows firms to increase the use of the energy input, reducing the shadow cost of the regulation. The finding that T_2 is most likely to be adopted under emission standards goes against previous studies suggesting that market-based instruments create more effective technology adoption incentives than conventional regulatory standards (see Requate 2005 for a survey). This result relies on the logic that

under emission standards, the incentive for adoption is given by the increased profits resulting from using new technology when firms are restricted to emit no more than $\bar{\xi}$. In comparison, under market-based instruments, firms would instead increase their emission reductions even further to reduce tax payments. Our analysis shows, however, that when the regulatory asymmetries created by emission standards are taken into account, the profitability of emission-saving-biased technological change is higher under emission standards than under market-based instruments. The most productive firms are more likely to invest in new technology. Under emission standards they are the firms that face the larger percentage reduction in profits due to the regulation, and hence benefit the most from investing in T_2 .

Finally, the finding that adoption of T_2 is more likely under emission taxes than under performance standard is in line with Proposition 4. For equivalent stringencies of these policy instruments, firms face a larger percentage reduction in profits under emission taxes, which creates incentives to invest in technologies that reduce the cost of the regulation.

5 Numerical Example

In this section we present a numerical example of the size distribution induced by the different policies under analysis. We provide values for some of our key parameters and calculate the resulting choice of inputs, profits, and aggregate emissions and output.

Table 2: Parameter Values												
α	β	θ	p	w	z	τ	F	γ	$\tilde{\gamma}$	N	$\bar{\phi}$	$\underline{\phi}$
0.2	0.5	2	5	1	1.6	0.2	21	1	0.8	50	1	0

The production elasticity of emissions and labor is set at $\alpha = 0.2$ and $\beta = 0.5$, respectively. The general productivity parameter, θ , is equal to 2. The price of the output is set at $p = 5$, while the wages and the price of energy are set at $w = 1$ and $z = 1.6$, respectively. We set the emission tax at $\tau = 0.2$ and the fixed entry cost of firms at $F = 21$. Finally, we assume that the initial number of

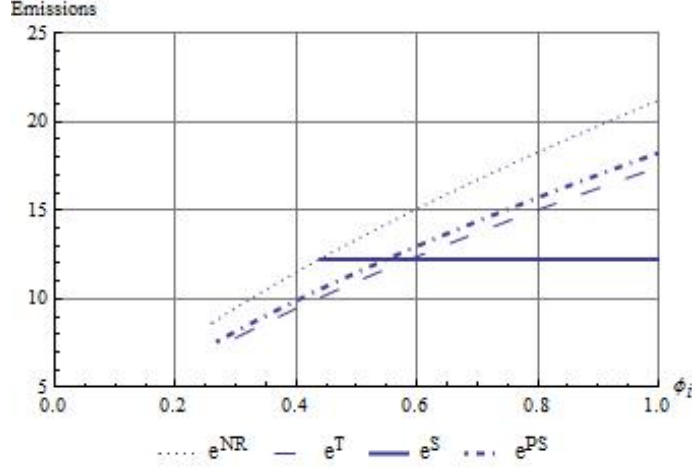


Figure 1: Distribution of Emissions across the Different Types of Firms (NR: Non-regulation, T: taxes, S:emission standards, PS: performance standards)

firms is $N = 50$, and these firms are uniformly distributed in the interval $]0, 1]$, which means that the upper bound of the distribution is given by $\bar{\phi} = 1$ and the lower bound $\underline{\phi} \simeq 0$.

Using the parameter values presented in Table 2, we study the size distribution of firms under different environmental regulations. Table 3 summarizes the main results. In the case without regulation, 12 out of 50 firms cannot operate since they are not profitable enough. Firms with energy efficiency lower than $\phi = 0.26$ are not profitable even in the case of no regulation.

Firms need to be more energy efficient in order to stay in the market if environmental taxes are imposed. The cutoff value in this specific numerical example is 0.3. Hence, the internalization of the cost of emissions made firms in the interval $[0.26, 0.3)$ exit the market. The case of standards is different. For the firms with energy efficiency in the range $[0.26, 0.44)$, the emissions standard is not binding. Those firms for which the standard is binding, i.e., $\phi_i \in [0.44, 1]$, produce less than before since they are restricted in the use of the energy (or equivalently in the generation of emissions), as illustrated in Figure 1. Finally, the cutoff value in the case of performance standards is equal to 0.28, which implies that firms in the interval $[0.28, 0.3)$ will still find it profitable to operate under performance standards but not under taxes.

When it comes to output, it is clear from Table 3 that output is higher under performance standards

than under taxation, which is easily explained if we take into account that firms pay taxes for the total emissions they generate, while in the case of performance standards firms are granted a certain number of emissions free of charge. The lowest level of total output is observed in the case of emission standards, since the larger firms that used to produce a lot are now restricted by the regulation. Given the specific parameter values assumed here, we can rank the total output under the four regimes as $Q^{NR} > Q^{PS} > Q^T > Q^S$.

Table 3: Numerical Results							
	ϕ_0	$\hat{\phi}_1$	Σ	Q	Σ/Q	WI	$\hat{\theta}$
Non-regulation	0.26		584	934	0.63	603	
Emission Tax	0.3		465	837	0.56	592	2.0091
Emission Standard	0.26	0.44	450	838	0.53	577	2.0912
Performance Standard	0.28		494	890	0.56	596	2.0003

Table 3 also shows the total emission level in each case, while Figure 1 presents the emissions generated by each type of firm. To start with the aggregate amount, we have that $\Sigma^{NR} > \Sigma^{PS} > \Sigma^T > \Sigma^S$. So, emission standards result in lower levels of emissions than taxes and performance standards. Emission standards lead to the same emission level as with non-regulation when the standard is not binding, but there is a significant decrease in the emissions generated by the firms for which the standard is binding. Compared with taxes, emissions are higher under performance standards (as expected). Table 3 also shows the average emissions-output ratio. We can see that the average emission intensity is lowest under emission standards, followed by taxes and performance standards, i.e., $(\Sigma/Q)^S < (\Sigma/Q)^T = (\Sigma/Q)^{PS} < (\Sigma/Q)^{NR}$. Again, this ranking is explained by the significant effect of emission standards on the emission intensity of large firms.

When it comes to welfare effects, Table 3 provides the values of a welfare indicator equal to aggregate profits of active firms under the different policy instruments. In calculating our indicator in the case of taxes, we sum back aggregate tax payments as they only represent a transfer between

firms and the government. This is to say, they should not be considered as a reduction in aggregate profits. Further, the aggregate profits in the case of emission standards correspond to the weighted average of the profits of those firms for which the standard is not binding and those for which the standard is binding. Two factors determine the observed differences in aggregate profits: (i) the cost of compliance of the different environmental policies and (ii) the different number of firms exiting the market after the implementation of each environmental policy. Our numerical example provides the following ranking: $WI^{NR} > WI^{PS} > WI^T > WI^{ES}$. This means that performance standards lead to higher aggregate profits, followed by emission taxes and emission standards. The fact that emission standards lead to lower profits is interesting since firms are not required to pay for their emissions in this case. However, the fact that the regulation significantly affects the choice of inputs and restricts firms with the highest energy efficiency implies that this policy has the most negative effects on aggregate profits, though it impacts small firms to a lower extent. The extent to which our welfare indicator corresponds to actual welfare depends on the social cost of emissions. If we would assume, for example, that the cost of emissions is given by the tax (and that the marginal damage is constant), the welfare under taxes would be the largest, followed by performance standards and emission standards.

In order to illustrate Propositions 1, 2, and 3, we calculate the percentage gap in profits under each policy instrument vis-a-vis no regulation, i.e., $\Delta\pi_i^j = \frac{\pi_i^{NR} - \pi_i^j}{\pi_i^{NR}}$, $\forall j = T, S, PS$. As we can see in Figure 2, in relative terms, emission standards are much more stringent for larger firms than for smaller firms. As expected, taxes and performance standards impose a higher cost to smaller firms. Moreover, under performance standards large firms lose a smaller part of their profits (vis-a-vis no regulation) than under emission taxes. As discussed before, this is explained by the fact under taxes firms have to pay for all the emissions they release, while in the case of performance standards emissions below the level imposed by the standard are free of charge.

Finally, in Table 3 we also present some numerical results for the two technology options. In particular, we compute the thresholds $\hat{\theta}^S$, $\hat{\theta}^T$, and $\hat{\theta}^{PS}$. Since the adoption of these technologies implies an investment cost for the firms, only those firms whose surplus exceeds the investment cost

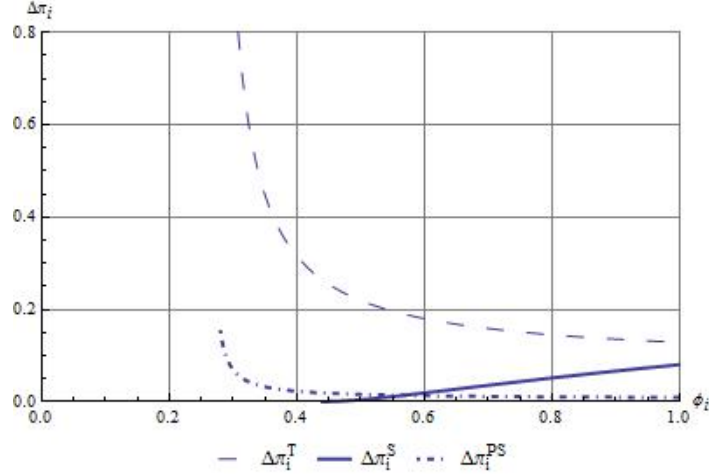


Figure 2: Percentage reduction in firm i 's profits under environmental policy vis-a-vis no regulations.

will be able to invest. However, as expected, our simulations indicate that $\hat{\theta}^S > \hat{\theta}^T > \hat{\theta}^{PS}$ implying that firms are most likely to invest in the emissions-saving technology under emission standards.

6 Conclusions

In this paper we study the effects of the choice of policy instruments on the size distribution of firms. We have shown that each regulation affects firms of heterogeneous size differently, favoring either small or large firms. For instance, compared with taxes or performance standards, uniform emission standards are much more stringent for larger firms which despite using the output that generates emissions less intensively emit more than small firms in absolute terms. Moreover, we have shown that a different number of firms go out of business under different policy instruments.

To sum up, the internalization of the social cost coming from the polluting activity of firms leads to lower production levels for each "type" of firm. Emission taxes affect small firms with significantly low surpluses (needed to cover the fixed costs) the most, as the use of energy now becomes more expensive. Emission standards affect the most those large firms for which the standard is binding. These firms would have to distort their choice of inputs significantly as well as reduce their production and profits in order to comply with the standard. Finally, performance standards favor large firms

that produce high levels of output and do not find the regulation so restrictive. Moreover, compared with the other two policy instruments, they lead to a higher output (though at the expense of higher emissions). Last but not least, assuming that firms can invest in two different technologies, a neutral technology and an emission-saving-biased technology, we show that emission standards favor the use of emissions-saving technologies the most.

The fact that each regulation affects the size distribution differently has important welfare consequences. In our setting, the underlying size distribution of firms in the industry is the result of the existence of heterogeneity in available physical capital with respect to energy intensity. Any environmental policy introducing regulatory asymmetries favoring small firms might have significant detrimental effects on total output and total welfare, yet it might also lead to reduced emissions and help preserve small businesses, which might be desirable because of antitrust or other non-economic reasons. Alternatively, one could exempt smaller firms from the regulation, though this might create additional distortions and discontinuities on the size distribution - an interesting issue for future research.

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Appendix A

Proposition 1

Note that $\Delta\pi_i^T = \frac{\pi_i^{NR} - \pi_i^T}{\pi_i^{NR}} = 1 - \frac{\pi_i^T}{\pi_i^{NR}}$. Differentiating $\Delta\pi_i^T$ with respect to ϕ_i leads to:

$$\frac{\partial\Delta\pi_i^T}{\partial\phi_i} = \frac{\pi_i^T \frac{\partial\pi_i^{NR}}{\partial\phi_i} - \pi_i^{NR} \frac{\partial\pi_i^T}{\partial\phi_i}}{[\pi_i^{NR}]^2}. \quad (\text{A1})$$

Differentiating equations (9) and (13) with respect to ϕ_i and replacing in (A1) leads to:

$$\frac{\partial\Delta\pi_i^T}{\partial\phi_i} = \left[\frac{\alpha}{1 - \alpha - \beta} \right] \left[\frac{\pi_i^T [\pi_i^{NR} + F] - \pi_i^{NR} [\pi_i^T + F]}{\phi_i [\pi_i^{NR}]^2} \right], \quad (\text{A2})$$

which simplifies to:

$$\frac{\partial\Delta\pi_i^T}{\partial\phi_i} = - \left[\frac{\alpha}{1 - \alpha - \beta} \right] \frac{F}{\phi_i} \frac{\Delta\pi_i^T}{\pi_i^{NR}} < 0. \quad (\text{A3})$$

Proposition 2

Note that $\Delta\pi_i^S = \frac{\pi_i^{NR} - \pi_i^S}{\pi_i^{NR}} = 1 - \frac{\pi_i^S}{\pi_i^{NR}}$. Differentiating $\Delta\pi_i^S$ with respect to ϕ_i leads to:

$$\frac{\partial\Delta\pi_i^S}{\partial\phi_i} = \frac{\left[\pi_i^S \frac{\partial\pi_i^{NR}}{\partial\phi_i} - \pi_i^{NR} \frac{\partial\pi_i^S}{\partial\phi_i} \right]}{[\pi_i^{NR}]^2}. \quad (\text{A4})$$

Differentiating equations (9) and (29) with respect to ϕ_i and replacing in (A4) leads to:

$$\frac{\partial\Delta\pi_i^S}{\partial\phi_i} = \left[\frac{\left[\frac{\alpha}{1 - \alpha - \beta} \right] \frac{\pi_i^S [\pi_i^{NR} + F]}{\phi_i} - \left[\frac{\alpha}{1 - \beta} \right] \frac{\pi_i^{NR} [\pi_i^S + z[\bar{\xi}\gamma^{-1}] + F]}{\phi_i}}{[\pi_i^{NR}]^2} \right], \quad (\text{A5})$$

which simplifies to:

$$\frac{\partial\Delta\pi_i^S}{\partial\phi_i} = \left[\frac{\alpha}{1 - \alpha - \beta} \right] \left[\frac{\alpha \left[p\theta\beta^\beta \phi_i^\alpha [\bar{\xi}\gamma^{-1}]^\alpha w^{-\beta} \right]^{\frac{1}{1-\beta}} - z [\bar{\xi}\gamma^{-1}]}{\phi_i \pi_i^{NR}} \right] - \left[\frac{\alpha}{1 - \alpha - \beta} \right] \frac{F}{\phi_i} \frac{\Delta\pi_i^S}{\pi_i^{NR}}. \quad (\text{A6})$$

The first term in brackets on the RHS of equation (A6) corresponds to the regulatory asymmetry effect (*RA*), which captures the fact that emission standards distort the emission intensity of larger firms the most.

Adding and subtracting $\left[\frac{1}{1-\alpha-\beta}\right] \pi_i^{NR} [\pi_i^S + z [\bar{\xi}\gamma^{-1}] + F]$ to equation (A6) yields:

$$\frac{\partial \Delta \pi_i^S}{\partial \phi_i} = \frac{\alpha}{\phi_i [\pi_i^{NR}]^2} \left[\begin{aligned} & \left[\frac{1}{1-\alpha-\beta}\right] [\pi_i^S [\pi_i^{NR} + F] - \pi_i^{NR} [\pi_i^S + z [\bar{\xi}\gamma^{-1}] + F]] \\ & + \left[\frac{1}{1-\alpha-\beta}\right] \pi_i^{NR} [\pi_i^S + z [\bar{\xi}\gamma^{-1}] + F] - \left[\frac{1}{1-\beta}\right] \pi_i^{NR} [\pi_i^S + z [\bar{\xi}\gamma^{-1}] + F] \end{aligned} \right].$$

Or:

$$\frac{\partial \Delta \pi_i^S}{\partial \phi_i} = \frac{1}{\phi_i [\pi_i^{NR}]^2} \left[\frac{\alpha}{1-\alpha-\beta} \right] \left[- [F [\pi_i^{NR} - \pi_i^S] + \pi_i^{NR} z [\bar{\xi}\gamma^{-1}]] + \left[\frac{\alpha}{1-\beta} \right] \pi_i^{NR} [\pi_i^S + z [\bar{\xi}\gamma^{-1}] + F] \right].$$

$\frac{\partial \Delta \pi_i^S}{\partial \phi_i}$ is positive if:

$$\alpha > \frac{F [\pi_i^{NR} - \pi_i^S] + \pi_i^{NR} z [\bar{\xi}\gamma^{-1}]}{\pi_i^{NR} [\pi_i^S + z [\bar{\xi}\gamma^{-1}] + F]} [1 - \beta]. \quad (\text{A7})$$

Note that $\frac{F [\pi_i^{NR} - \pi_i^S] + \pi_i^{NR} z [\bar{\xi}\gamma^{-1}]}{\pi_i^{NR} [\pi_i^S + z [\bar{\xi}\gamma^{-1}] + F]} < 1$. Hence, the constraint in (A7) should be consistent with the condition $\alpha < 1 - \beta$ since $\frac{F [\pi_i^{NR} - \pi_i^S] + \pi_i^{NR} z [\bar{\xi}\gamma^{-1}]}{\pi_i^{NR} [\pi_i^S + z [\bar{\xi}\gamma^{-1}] + F]}$ is something smaller than 1.

For the concavity condition to hold we need:

$$\alpha < \frac{1 - \beta}{2}. \quad (\text{A8})$$

Combining conditions (A7) and (A8) yields:

$$\frac{F [\pi_i^{NR} - \pi_i^S] + \pi_i^{NR} z [\bar{\xi}\gamma^{-1}]}{\pi_i^{NR} [\pi_i^S + z [\bar{\xi}\gamma^{-1}] + F]} [1 - \beta] < \alpha < \frac{1 - \beta}{2}.$$

Then the two conditions will hold simultaneously if and only if:

$$\frac{F [\pi_i^{NR} - \pi_i^S] + \pi_i^{NR} z [\bar{\xi}\gamma^{-1}]}{\pi_i^{NR} [\pi_i^S + z [\bar{\xi}\gamma^{-1}] + F]} < \frac{1}{2}. \quad (\text{A9})$$

Equation (A9) can be represented as:

$$2F [\pi_i^{NR} - \pi_i^S] + 2\pi_i^{NR} z [\bar{\xi}\gamma^{-1}] < \pi_i^{NR} [\pi_i^S + z [\bar{\xi}\gamma^{-1}] + F].$$

Or:

$$0 < \pi_i^{NR} [\pi_i^S - z [\bar{\xi}\gamma^{-1}] - F] + 2F\pi_i^S,$$

which always holds since $\pi_i^S \geq z [\bar{\xi}\gamma^{-1}] + F$.

Hence, $\frac{F[\pi_i^{NR} - \pi_i^S] + \pi_i^{NR} z [\bar{\xi}\gamma^{-1}]}{\pi_i^{NR} [\pi_i^S + z [\bar{\xi}\gamma^{-1}] + F]} < \frac{1}{2}$ and $\frac{\partial \Delta \pi_i^S}{\partial \phi_i} > 0$. This is to say, the RA is larger than the SE effect implying that emission standards reduce the profits of larger firms by a larger percentage than those for smaller firms.

Proposition 3

Differentiating $\Delta \pi_i^{PS}$ with respect to ϕ_i leads to:

$$\frac{\partial \Delta \pi_i^{PS}}{\partial \phi_i} = \frac{\pi_i^{PS} \frac{\partial \pi_i^{NR}}{\partial \phi_i} - \pi_i^{NR} \frac{\partial \pi_i^{PS}}{\partial \phi_i}}{[\pi_i^{NR}]^2}. \quad (\text{A11})$$

Differentiating equations (9) and (38) with respect to ϕ_i and replacing in (A11) leads to:

$$\frac{\partial \Delta \pi_i^{PS}}{\partial \phi_i} = \left[\frac{\alpha}{1 - \alpha - \beta} \right] \left[\frac{\pi_i^{PS} [\pi_i^{NR} + F] - \pi_i^{NR} [\pi_i^{PS} + F]}{\phi_i [\pi_i^{NR}]^2} \right], \quad (\text{A12})$$

which simplifies to:

$$\frac{\partial \Delta \pi_i^{PS}}{\partial \phi_i} = - \left[\frac{\alpha}{1 - \alpha - \beta} \right] \frac{F}{\phi_i} \frac{\Delta \pi_i^{PS}}{\pi_i^{NR}} < 0. \quad (\text{A13})$$

Appendix B

The emission standard is binding for some of the firms belonging to the upper part of the distribution $[\phi_0^{NR}, \bar{\phi}]$. In particular, $\hat{\phi}_1 > \phi_0^{NR}$ if:

$$\left[\frac{h \left[\bar{\phi}^h - [\phi_0^T]^h \right]}{\left[\bar{\phi} - \phi_0^{NR} \right] \left[\phi_0^{NR} \right]^{\frac{\alpha}{1-\alpha-\beta}}} - 1 \right] > \frac{\hat{\tau}}{z}.$$

Since $h > 1$, $\left[\bar{\phi}^h - [\phi_0^T]^h \right] > \left[\bar{\phi} - \phi_0^{NR} \right]$, and $\left[\phi_0^{NR} \right]^{\frac{\alpha}{1-\alpha-\beta}} < 1$, it is easy to show that for reasonable values of $\hat{\tau}$, this inequality holds. In our numerical example: $\left[\frac{1.67[1-0.13]}{[1-0.26][0.26]^{0.66}} - 1 \right] > 0.125 \Rightarrow 3.8 > 0.125$. It is clear that even if the tax would double the price of energy, the inequality would still hold.