Master Degree Project in Finance

A Regime Switching model

Applied to the OMXS30 and Nikkei 225 indices

Ludvig Hjalmarsson

UNIVERSITY OF GOTHENBURG
SCHOOL OF BUSINESS, ECONOMICS AND LAW

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Author:

Ludvig Hjalmarsson

Supervisor:

Mattias Sundén

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Abstract

This Master of Science thesis investigates the performance of a Simple Regime Switching Model compared to the GARCH(1,1) model and rolling window approach. We also investigate how these models estimate the Value at Risk and the modified Value at Risk. The underlying distributions that we use are normal distribution and Student's t-distribution. The models are fitted to the Nasdaq OMXS30 and the Nikkei 225 indices for 2013. This thesis shows that the Simple Regime Switching Model with normal distribution performs superior to the other models adjusting for skewness and kurtosis in the residuals. The best model for estimating risk is the Simple Regime Switching Model with normal distribution in combination with the classic Value at Risk. In addition, we show that financial institutions using the Simple Regime Switching Model will possibly lower their cost of risk, compared to using the GARCH(1,1) model.

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1 Introduction

During the last years there have been two major financial crises, the Global financial crisis and the European sovereign crisis [25], both of which have once again raised the awareness of the importance for financial institutions to manage risk. Many European financial institutions are now implementing the Basel III framework to handle risk [22]. However, it is also of great importance for financial institutions to apply internal risk models in order to managing risk on a daily basis [21].

In this thesis we compare the risk measure Value at Risk (VaR) and a version that adjusts for skewness and kurtosis, called modified VaR (mVaR). In order to estimate the parameters needed to calculate VaR and mVaR, we use the classic rolling window approach, the GARCH(1,1) model, and additionally the Simple Regime Switching Model (SRSM).

In portfolio management, it is essential to be aware of the risk in the portfolio. In 1996, J.P. Morgan together with RiskMetrics™ developed the Value at Risk (VaR). This risk measure provides the actual number of the maximum loss in the portfolio over a predefined time horizon, given the chosen probability [19].

In 1989, James Hamilton published his first paper discussing the Simple Regime Switching Model (SRSM), which is used to estimate the parameters' mean and variance of financial time series [9]. The SRSM was further developed in two subsequent articles by Hamilton.

The SRSM assumes two states, either one with a high return of an asset with low volatility or one with a low return of an asset with high volatility. Today these states are respectively known as "bull" and "bear" market among financial professionals and in the academia. The bull market is a market with increasing asset prices, a typical market where the investors are interested in a long position. A bear market is a market where the prices of assets are declining, therefore, a short position is preferred [24].

The purpose of this thesis is to analyze the quality of the SRSM, and how well this model adjusts for skewness and kurtosis in the residuals of the returns. We also test the quality of VaR, estimated by the parameters from the SRSM. Furthermore, there has been no research on the Swedish stock market using the SRSM and the quality of the model is tested in more extreme environment using the volatile Nikkei 225 index for 2013. This implies that this thesis can contribute to the existing work within the field of risk management, as well as provide new findings for the Swedish stock market

and the quality of the SRSM.

There are many interesting research questions that can be analyzed within the field of regime switching models. The four major questions that are addressed in this thesis are:

- Which of the models will best adjust for kurtosis and skewness in the residuals?

- Which of the models for parameter estimation in combination with the risk measure produces the best model for estimating risk?

- Is there a clear difference between the risk measures produced by the GARCH(1,1) model compared to the SRSM?

- How good are the models at adjusting for violations arriving in clusters?

In order to define a framework for the thesis, we introduce some limitations. The limitations and their motivations are as follows:

- We are limiting the data to the OMXS30 and Nikkei 225 indices. We could of course consider more assets to improve the quality of the work, but some trade-off has to be done since running backtesting is time consuming.

- The backtesting is done for 255 observations. A longer time period may improve the results, but some trade off has to be made since running backtesting is time consuming.

- A one-day forecast is chosen, since it is an established method among researchers in the area and greatly simplifies the calculation of VaR for the SRSM.

The SRSM was introduced by James Hamilton in 1989 [9], in order to explain discrete shifts among the parameters. Hamilton developed the Markov switching regression by Goldfeld and Quandt [7], where Hamilton presented a nonlinear filter and smoother used to estimate the probability of the states based on observations of the output. The GARCH model was introduced by Bollerslev [3] and is one of the most used models in volatility estimation.

VaR is covered in the majority of the books in risk management, and the topic is probably one of the most discussed in articles covering the risk of financial assets. The VaR model was first presented in the original paper by J.P. Morgan and Reuters called RiskMetrics[TM][19]. The modified VaR model was introduced by Favre and Galeano [17] in 2002 and works as a complement when distribution of residuals are not normally distributed.

2 Theory

2.1 Returns

In this thesis we assume that the prices take either a lognormal distribution or a logged Student's t-distribution, hence we use logarithmic returns, which are defined as

$$r_t = \ln \left(\frac{P_t}{P_{t-1}} \right) = \ln P_t - \ln P_{t-1}, \tag{1}$$

where P_t is the price of a security at time t.

2.2 Value at Risk (VaR)

When managing a portfolio of equity or other financial assets it is important to know the risk. Important questions for financial institutions are *what is the potential loss tomorrow?* and *how does the portfolio react to the market movements?*, in order to be able to manage the portfolio and reallocate the weights of the assets efficiently. The natural response to the question of *what is the potential loss tomorrow?*, would be *everything!*, but this is quite a vague answer and probably not an acceptable answer for the risk- or portfolio managers. The Value at Risk (VaR) model is a way for risk managers to get stimulating answers to the aforementioned questions [18].

As mentioned earlier, the VaR model is a risk measure that provides the actual number of what the maximum loss will be over a predefined time horizon given the chosen probability [19].

The VaR can also be expressed as the probability for the return being less than $\text{VaR}_\alpha(r_t)$ during the time period h is α, namely $\mathbf{P}[r_{t+h} \leq \text{VaR}_\alpha(r_t)$ [14], in this thesis the one day VaR is being used and, therefore, we assume that $h = 1$. In addition, r_{t+1} is the return over the period $]t, t+1]$, from now on we will express this return as r_t and replace (α, h) as mentioned above. $\text{VaR}_\alpha(r_t)$ is thus given by the smallest number y, for which r_t exceeds y with probability $1 - \alpha$ at time t [12].

We start to define $F_{r_t}(x) = \mathbf{P}[r_t \leq x]$ for any x, we have that $F_{r_t}(x)$ is the distribution function for the return variable r_t. Thus

$$\text{VaR}_\alpha(r_t) = \inf\{y \in \mathbb{R} : \mathbf{P}[r_t \le y] \le 1 - \alpha\}$$
$$= \inf\{y \in \mathbb{R} : F_{r_t}(y) \ge \alpha\}. \tag{2}$$

$F(x)_{r_t}$ is a nondecreasing function on $\mathbb{R}$, $F(x)_{r_t} : \mathbb{R} \to \mathbb{R}$. Then the generalized inverse to F_{r_t} is $F_{r_t}^{\leftarrow}$, this is thus defined as

$$F_{r_t}^{\leftarrow}(y) = \inf\{x \in \mathbb{R} : F_{r_t}(x) \ge y\}. \tag{3}$$

If $F(x)_{r_t}$ is a continuous and strictly increasing function then $F_{r_t}^{\leftarrow} = F_{r_t}^{-1}$, so the generalized inverse $F_{r_t}^{\leftarrow}(y)$ will then be $F_{r_t}^{-1}$, hence VaR_α is

$$\text{VaR}_\alpha(r_t) = F_{r_t}^{\leftarrow}(\alpha) = F_{r_t}^{-1}(\alpha). \tag{4}$$

Assuming r_t is normally distributed random variable with mean μ_t and variance σ_t^2, we have $r_t \sim N(\mu_t, \sigma_t^2)$, then $F_{r_t}(x)$ is

$$F_{r_t}(x) = \mathbf{P}[r_t \ge x] = \mathbf{P}\left[\frac{r_t - \mu_t}{\sigma_t} \ge \frac{x - \mu_t}{\sigma_t}\right] = \Phi\left(\frac{x - \mu_t}{\sigma_t}\right), \tag{5}$$

where $\Phi(x)$ describes the cumulative distribution function.

The cumulative distribution function of a standard normal random variable with $\Phi(x)$, is described as

$$\Phi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} exp\left\{-\frac{z^2}{2}\right\} dz. \tag{6}$$

In order to find $F_{r_t}^{-1}(y)$, we solve for x in the equation $y = F_{r_t}(x)$, and get

$$\Phi\left(\frac{x - \mu_t}{\sigma_t}\right) = y \Leftrightarrow \frac{x - \mu_t}{\sigma_t} = \Phi^{-1}(y) \Leftrightarrow x = \mu_t + \sigma_t \Phi^{-1}(y). \tag{7}$$

Then we can see that $F_{r_t}^{-1}(y)$ is

$$F_{r_t}^{-1}(y) = \mu_t + \sigma_t \Phi^{-1}(y). \tag{8}$$

We know that $\text{VaR}_\alpha(r_t) = F_{r_t}^{-1}(\alpha)$ from equation 4, hence we can express $\text{VaR}_\alpha(r_t)$ as

$$\text{VaR}_\alpha(r_t) = \mu_t + \sigma_t \Phi^{-1}(\alpha), \tag{9}$$

where Φ^{-1} can not be explicitly expressed.

If f is the density function of the return series, then VaR can also be expressed as

$$1 - \alpha = \int_{-\infty}^{VaR_\alpha(r_t)} f(x)\mathrm{d}x. \tag{10}$$

2.3 Modified Value at Risk (mVaR)

VaR measures the risk in a portfolio with returns that are normally distributed. This implies that if a time series is not normally distributed, VaR may give misleading results. Therefore, we introduce a model that does not assume a normal distribution among the returns; instead the model uses the skewness and kurtosis of the time series to estimate VaR. This model is called modified VaR (mVaR) [17].

The mVaR measure is due to this more adaptable and dynamic. For instance, we will overestimate risk if we try to estimate VaR at low confidence levels, using the normal distribution when the distribution is in fact leptorkurtic. At high confidence levels, we will instead underestimate the risk. The mVaR therefore adjusts for the non-normal distribution and gives a more correct estimate of the risk, even if the returns are non-normally distributed.

The mVaR is expressed as

$$\mathrm{mVaR}_\alpha(r_t) = \mu_t + \left(\Phi^{-1}(\alpha) - \frac{1}{6}(z_\alpha^2 - 1)\mathrm{S}_t - \frac{1}{24}(z_\alpha^3 - z_\alpha)\mathrm{K}_t + \frac{1}{36}(2z_\alpha^3 - 5z_\alpha)\mathrm{S}_t^2 \right) \sigma_t, \tag{11}$$

where we have that

- $\Phi^{-1}(\alpha)$: standard normal quantile for α

- S_t: skewness

- K_t: excess kurtosis which is defined as kurtosis-3

- μ_t: mean

- σ_t: standard deviation

In equation 11, we can see that when skewness and excess kurtosis are zero, then mVaR is equal to VaR. If excess kurtosis or skewness are deviating from zero, the mVaR will

not be equal to VaR, and the risk will be adjusted for the different distribution of the returns.

2.4   The Simple Regime Switching Model (SRSM)

The Simple Regime Switching Model (SRSM) (also known as the Markov state switching model) is a model that allows for the parameters to switch states. This implies that if the mean and variance are Markov switching then they will change depending on the state of the market. A classic example of this is the stock market where we can have either a bull or a bear market. A bull market has a positive trend and low volatility while a bear market has a negative trend and higher volatility. In the SRSM model, we know, that in a bull market, we have positive mean and low variance compared to a bear market were the mean is lower or even negative, and the variance is considerably higher. The volatility is represented by the variance. The SRSM model gives us the mean, the variance and the probability for the two different states [11].

We assume the returns for the SRSM to be

$$r_t = \mu_{S_t} + \sigma_{S_t}\epsilon_t, \tag{12}$$

where r_t is a time series of returns, S_t is a Markov chain with k possible states and the innovation ϵ_t is an i.i.d process. We have that $t = 1, \ldots, T$. From now on we define the SRSM when we have $k = 2$, which means having two different states or regimes. S_t is defined as

$$S_t = \begin{cases} 1 & \text{with probability } \pi, \\ 2 & \text{with probability } 1 - \pi. \end{cases} \tag{13}$$

The Markov chain, S_t, transition matrix is

$$\mathbf{P}^* = \begin{pmatrix} p_{11} & p_{21} \\ p_{12} & p_{22} \end{pmatrix}. \tag{14}$$

In the diagonal we have p_{11} and p_{22} that represents the probability of staying in regime 1 and 2, respectively. Then $p_{12} = 1 - p_{11}$ and $p_{21} = 1 - p_{22}$, which represent the probabilities of switching from regime 1 to 2 and from regime 2 to 1.

We have the following model for r_t

$$r_t = \begin{cases} \mu_1 + \sigma_1 \epsilon_t & \text{if } S_t = 1, \\ \mu_2 + \sigma_2 \epsilon_t & \text{if } S_t = 2, \end{cases} \tag{15}$$

for our two states. Hence the innovations ϵ_t are i.i.d $N(0,1)$ and

$$\epsilon_t \sim \begin{cases} N(\mu_1, \sigma_1^2) & \text{if } S_t = 1, \\ N(\mu_2, \sigma_2^2) & \text{if } S_t = 2. \end{cases} \tag{16}$$

In equation 15, there are two different equations for r_t, depending on which state we are in.

The unconditional probabilities for the states are given by the following vector,

$$\begin{pmatrix} \frac{(1-p_{11})}{(1-p_{11}-p_{22})} \\ \frac{(1-p_{22})}{(1-p_{11}-p_{22})} \end{pmatrix}, \tag{17}$$

these are used and explained in the Hamilton filter section. This is also the long run equilibrium of the weights for our two states. When using the Hamilton filter we assume the starting values to be $\{0.5, 0.5\}$ since we do not know the unconditional probabilities [11].

2.4.1 VaR for the SRSM

When we estimate VaR using the SRSM, we use the standard VaR for each state with the given parameters and then weight our different VaR calculations depending on the probability for each state. Hence, the one day VaR at time t for SRSM is the weighted VaR for the states, as can be seen in

$$\text{VaR}_\alpha(r_t) = \sum_{S_{t+1}=1}^{k} \mathbf{P}(S_t|\psi_t)(\mu_{S_{t+1}} + \sigma_{S_t}^2 \Phi^{-1}(\alpha)). \tag{18}$$

Here we have that $\mathbf{P}(S_{t+1}|\psi_t)$ is the probability for the different states given all the information up to time t [14].

2.4.2 Hamilton Filter with Maximum Likelihood Estimation

When estimating the parameters of the SRSM using the Hamilton filter, we may use either maximum likelihood estimation or Bayesian inference (Gibbs-Sampling) [20]. In this thesis maximum likelihood estimation is used, since it is the method recommended and used by Hamilton in his papers about regime switching models [10], the Hamilton filter will be described in this section [11].

We start by considering a standard regime switching model

$$r_t = \mu_{S_t} + \sigma_t \epsilon_t, \tag{19}$$

where the innovations, ϵ_t, are i.i.d $N(0,1)$ and the states are $S_t = 1, 2$.

The log likelihood of the aforementioned model is

$$\ln L = \sum_{t=1}^{T} \ln \left(\frac{1}{\sqrt{2\pi\sigma_{S_t}^2}} \exp\left\{ -\frac{(r_t - \mu_{S_t})^2}{2\sigma_{S_t}^2} \right\} \right) = \sum_{t=1}^{T} \left(-\frac{1}{2}\ln(2\pi\sigma_{S_t}^2) - \frac{(r_t - \mu_{S_t})^2}{2\sigma_{S_t}^2} \right). \tag{20}$$

We want to maximize $\ln L$ (20), which is equivalent to maximizing

$$-\frac{1}{2}\sum_{t=1}^{T} \left(\ln(\sigma_{S_t}^2) + \frac{(r_t - \mu_{S_t})^2}{\sigma_{S_t}^2} \right). \tag{21}$$

Using maximum likelihood for the above specified model, everything is relatively easy if we know the states of the world, S_t. Then we only have to maximize equation (20) with respect to the parameters μ_1, μ_2, σ_1 and σ_2.

However, in the Markov switching case the states of the world are not known. Therefore, the log likelihood equation for the case when the states are unknown is calculated.

We have that $p_{ij} = \mathbf{P}[S_{t+1} = j | S_t = i]$ $i = 1, 2$, $j = 1, 2$ which is our transition probabilities from our transition matrix. Our six parameters are thus $\Theta = \{\mu_1, \mu_2, \sigma_1, \sigma_2, p_{12}, p_{21}\}$.

The likelihood for our observations is defined as

$$L(\Theta) = f(r_1|\Theta)f(r_2|\psi_1, \Theta)f(r_3|\psi_2, \Theta)\ldots f(r_t|\psi_{t-1}, \Theta), \tag{22}$$

where $\psi_t = \{r_t, r_{t-1}, \ldots, r_1\}$ is the information available at time t and f is the probability density function for r_t.

We start the maximum likelihood estimation for the case when $t = 1$.

In order to start with the first recursion we need a value (given Θ) for $\mathbf{P}(S_0)$ and we want to find $f(y_1|\Theta)$.

Then we start the recursion by calculating for the parameters Θ

$$f(S_1 = 1, r_1|\Theta) = \pi_1 \varphi \left(\frac{r_1 - \mu_1}{\sigma_1} \right), \tag{23}$$

and

$$f(S_1 = 2, r_1|\Theta) = \pi_2 \varphi \left(\frac{r_1 - \mu_2}{\sigma_2} \right), \tag{24}$$

where φ is the standard normal probability density function and the total is

$$f(r_1|\Theta) = f(S_1 = 1, r_1|\Theta) + f(S_1 = 2, r_1|\Theta). \tag{25}$$

Calculate the probabilities for each state, that is $S_1 = 1, 2$:

$$\mathbf{P}(S_1|r_1, \Theta) = \frac{f(S_1, r_1|\Theta)}{f(r_1|\Theta)}. \tag{26}$$

We now advance to when $t = 2$.

$f(r_2|r_1, \Theta)$ is the sum over $S_t = 1, 2$ and $S_{t-1} = 1, 2$ for

$$f(S_2, S_1, r_2|r_1, \Theta) = \mathbf{P}(S_1|r_1, \Theta)\mathbf{P}(S_2|S_1, \Theta)f(r_2|S_2, \Theta), \tag{27}$$

where the first factor of the right hand side is the probability function from the previous recursion, in this case when t=1. The second factor on the right hand side is the transition probabilities between the regimes (p_{ij}). The last factor is the probability density function

$$f(r_2|S_2, \Theta) = \varphi \left(\frac{r_2 - \mu_{S_2}}{\sigma_{S_2}} \right). \tag{28}$$

To find the $\mathbf{P}(S_2|r_2, \Theta)$, which is the probabilities for the different states, we use the following equation where $S_2 = 1, 2$

$$\mathbf{P}(S_2|r_2, \Theta) = \frac{f(S_2, S_1 = 1, r_2|r_1, \Theta) + f(S_2, S_1 = 2, r_2|r_1, \Theta)}{f(r_2|r_1, \Theta)}. \tag{29}$$

Now consider an arbitrary t, then the log-likelihood for t'th observation is

$$\ln f(r_t|\psi_{t-1}, \Theta). \tag{30}$$

We calculate this recursively by calculating for each t

$$f(S_t, S_{t-1}, r_t|\psi_{t-1}, \Theta) = \mathbf{P}(S_{t-1}|\psi_{t-1}, \Theta) \ \mathbf{P}(S_t|S_{t-1}, \Theta) \ f(r_t|S_t, \Theta), \tag{31}$$

where $\mathbf{P}(S_t|S_{t-1}, \Theta)$ is the transition probability for the regimes

$$f(r_t|S_t, \Theta) = \varphi\left(\frac{r_t - \mu_{S_t}}{\sigma_{S_t}}\right). \tag{32}$$

The probability function $\mathbf{P}(S_{t-1}|\psi_{t-1}, \Theta)$ is found from the previous recursion (29), and is

$$\mathbf{P}(S_{t-1}|\psi_{t-1}, \Theta) = \frac{f(S_{t-1}, S_{t-2} = 1, r_{t-1}|\psi_{t-2}, \Theta) + f(S_{t-1}, S_{t-2} = 2, r_{t-1}|\psi_{t-2}, \Theta)}{f(r_{t-1}|\psi_{t-2}, \Theta)}. \tag{33}$$

We can now calculate $f(r_t|\psi_{t-1}, \Theta)$ as the sum over the possible values of $S_t = 1, 2$ and $S_{t-1} = 1, 2$ in formula 31.

This can now be recursively done for $t = 1, 2, \ldots, T$ by maximizing the likelihood function over our parameters $\Theta = \{\mu_1, \mu_2, \sigma_1, \sigma_2, p_{12}, p_{21}\}$ by using the function fminsearch in Matlab.

2.5 Rolling window

A common approach when testing statistical models is to use a rolling window (moving average, rolling analysis). It is a simple alternative to capture the changing mean and variance over time. The approach is used in this thesis, and it works as follows; first we divide the data into an estimation sample and a prediction sample. Then we estimate the parameters from the estimation sample and compare how well they fit the prediction sample. Once this is completed we roll one time period ahead, and the estimation sample now becomes the old estimation sample, but with one observation added from

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the prediction sample and the oldest observation taken away in the estimation sample [27]. The prediction sample is now one observation less than what was earlier the case.

In our analysis we are not using all data in our sample, we only use the recent m observations, therefore we have the following mean

$$\mu_{t,m} = \frac{1}{m} \sum_{i=0}^{m-1} r_{t-i}, \tag{34}$$

and the variance is given by

$$\sigma_{t,m}^2 = \frac{1}{m-1} \sum_{i=0}^{m-1} (r_{t-i} - \mu_{t,m})^2. \tag{35}$$

The mean and variance is updated in each time period by replacing the oldest observation with a new observation [27].

The m is chosen by testing for different lengths and observing the results, then choosing the length of m that produces the best result of a skewness and kurtosis.

2.5.1 Value at Risk for rolling window

We plug in the mean and variance from the rolling window and then get the VaR by

$$\text{VaR}_\alpha(r_t) = \mu_{t,m} + \sigma_{t,m}^2 \Phi^{-1}(\alpha). \tag{36}$$

2.6 The GARCH(1,1) model

In the GARCH(1,1) model the returns are conditionally normally distributed with conditional mean μ_t and conditional variance σ_t^2 , where ψ_t is information available at time t [3],

$$r_t | \psi_{t-1} \sim N(0, \sigma_t). \tag{37}$$

Then the expected mean, $\hat{\mu}_t$, can be expressed as

$$\hat{\mu}_t = \text{E}[r_t | \psi_{t-1}], \tag{38}$$

or alternatively as an AR(1) model

$$\hat{\mu}_t = \alpha_0 + \alpha_1 r_{t-1}, \tag{39}$$

or an ARMA(p, q) model.

σ_t^2 can be expressed as

$$\sigma_t^2 = Var[r_t|\psi_{t-1}] = \mathrm{E}[(r_t - \mu_t)^2|\psi_{t-1}]. \tag{40}$$

In order to adjust for non zero mean, we subtract the estimated mean at time period t from r_t. We therefore introduce the variable a_t

$$a_t = r_t - \hat{\mu}_t, \tag{41}$$

where $\hat{\mu}_t$ is the estimated mean at time t [26].

The general GARCH(p, q) model by Bollerslev[3] is defined as

$$\hat{\sigma}_t^2 = \alpha_0 + \sum_{i=1}^{p} \alpha_i a_{t-i}^2 + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^2, \tag{42}$$

where a_t is a weighted (with α_i) random variable (in this paper the demeaned daily return of the portfolio at time t), expressed as

$$a_t = \sigma_t \epsilon_t, \tag{43}$$

where the innovation $\epsilon_t \sim$ i.i.d $N(0, 1)$. We have that σ_{t-j}^2 is the weighted (with β_j) conditional variance at the time period t.

The GARCH(1,1) model for the conditional variance is

$$\hat{\sigma}_{t+1}^2 = \alpha_0 + \alpha_1 a_t^2 + \beta_1 \sigma_t^2. \tag{44}$$

In addition we have the restriction

$$\alpha_0, \alpha_1, \beta_1 > 0, \tag{45}$$

and

$$\alpha_1 + \beta_1 < 1, \tag{46}$$

in order for the GARCH(1,1) to be considered a stationary process.

A log-likelihood function or least squares regression can be used to estimate the parameters of the GARCH(1,1) model. The log likelihood function for a conditionally normally distributed series $\{a_t\}$ with parameters $\Theta = \{0, \sigma_t^2\}$ is

$$\ln L = \sum_{t=1}^{T} \ln \left(\frac{1}{\sqrt{2\pi\sigma_t^2}} \exp \left\{ -\frac{a_t^2}{2\sigma_t^2} \right\} \right) = -\frac{1}{2} \sum_{t=1}^{T} \left(\ln(2\pi\sigma_t^2) + \frac{a_t^2}{\sigma_t^2} \right), \tag{47}$$

When the parameters are estimated, the conditional mean, $\hat{\mu}_{t+1}$ and conditional variance $\hat{\sigma}_{t+1}^2$ can be forcasted. It is also possible to use a Student's t-distribution instead of assuming a normal distribution.

2.6.1   Value at Risk for GARCH(1,1)

From the results of the GARCH(1,1) model, we plug in the estimated conditional mean, $\hat{\mu}_{t+1}$, and estimated conditional variance, $\hat{\sigma}_{t+1}^2$, into the VaR$_\alpha$

$$\text{VaR}_\alpha(r_t) = \hat{\mu}_{t+1} + \hat{\sigma}_{t+1}^2 \Phi^{-1}(\alpha). \tag{48}$$

3 Methodology

3.1 Software

There are several softwares that can be used for this type of time series analysis. We choose to work in MatLab from MathWorks since this is a software for which our knowledge is good. In addition, MatLab is widely used among professionals and academics, and it offers many toolboxes with relevant functions.

3.1.1 Toolboxes

The toolbox "MS Regress - The MATLAB Package for Markov Regime Switching Models" by Marcelo Perlin [20] is used to run the SRSM. The toolbox "MFE MATLAB Function Reference Financial Econometrics" by Kevin Sheppard [23] is used for the other econometrical calculations and estimations. For the BDS test, the toolbox by Ludwig Kanzler [15] is used.

3.2 Value at Risk method

When calculating VaR and mVar the results will be a positive number since it is denoting the value of the negative return.When the calculations are performed, a minus sign is used in front in order to denote that the value of the VaR is negative and that we are comparing the results with the actual negative returns from the time series.

3.3 Normality test for residuals

Here we describe three tests that controls for a normal distribution, skewness and kurtosis in the residuals of the estimated parameters.

Introducing the variable e_t

$$e_t = \frac{r_t - \hat{\mu}_t}{\hat{\sigma}^2}. \tag{49}$$

Then e is a vector with residuals gathered from backtesting, $e = \{e_t, e_{t-1}, ..., e_1\}$.

3.3.1 Anderson-Darling test (AD test)

For the Anderson-Darling test we have that H_0 : data follows a normal distribution, and the test statistic is [1]

$$A^2 = -T - S_{AD}, \tag{50}$$

where

$$S_{AD} = \sum_{t=1}^{T} \frac{2t-1}{T} [\ln\Phi(e_t) + \ln(1 - \Phi(e_{T+1-t}))]. \tag{51}$$

the non-rejection region for 5% significance level is ± 1.96.

3.3.2 Jarque-Bera test (JB test)

For the JB-test we have the H_0 : skewness and excess kurtosis is zero, test statistic is

$$JB = \frac{S(e_t)}{6/T} + \frac{(K(e_t) - 3)^2}{24/T} \tag{52}$$

which is asymptotically $\chi^2(2)$ under the assumption of normal distribution. Thus, under the H_0, of skewness and excess kurtosis being zero, will be rejected at the 5% significance level when $|JB| > 3.84$.

If we want to test only for skewness the test statistic is

$$JB_{\text{skewness}} = \frac{S(e_t)}{6/T}. \tag{53}$$

If we want to test only for kurtosis the test statistic is

$$JB_{\text{kurtosis}} = \frac{(K(e_t) - 3)^2}{24/T}. \tag{54}$$

For both skewness and kurtosis the non-rejection region for 5% significance level is ± 1.96.

3.3.3 BDS test

The BDS test is a test by Brock, Dechert and Scheinkman. We will use the notations from [27].

The focus of the BDS test is the correlation dimension, in order to test for the distribution of impermanent patterns in time series.

The time series of residuals is defined as e_t for $t = 1, 2, ..., T$, and its m-history is defined as $e_t^m = (e_t, e_{t-1}, ..., e_{t-m+1})$.

Start of by estimating the correlation integral at the embedded dimension m

$$C_{m,\epsilon} = \frac{2}{T_m(T_m - 1)} \sum_{m \leq s} \sum_{<t \leq T} I(e_t^m, e_s^m; \epsilon), \tag{55}$$

where $T_m = T - m + 1$ and $I(e_t^m, e_s^m; \epsilon)$ is an indicator function that is taking the value one if $|e_{t-i} - e_{s-i}| < \epsilon$ for $i = 0, 1, ..., m - 1$ and it is equal to zero otherwise.

The joint probability is estimating the probability of two m-dimensional points are being located within a distance of ϵ from each other, which is the correlation integral, by the following formula $\mathbf{P}(|e_t - e_s| < \epsilon, |e_{t-1} - e_{s-1}| < \epsilon, ..., |e_{t-m+1} - e_{s-m+1}| < \epsilon)$

If e_t is i.i.d, then the probability will be

$$C_{1,\epsilon}^m = \mathbf{P}(|e_t - e_s| < \epsilon)^m \tag{56}$$

The DBS statistic is defined by

$$V_{m,\epsilon} = \sqrt{T} \frac{C_{m,\epsilon} - C_{1,\epsilon}^m}{\sigma_{m,\epsilon}}, \tag{57}$$

where $\sigma_{m,\epsilon}$ represents the standard deviation for $\sqrt{T}(C_{m,\epsilon} - C_{1,\epsilon})$.

Hence the BDS statistic will converge to standard normal distribution. Thus, under the H_0 of i.i.d residuals will be rejected at the 5% significance level when $|V_{m,\epsilon}| > 1.96$.

3.4   Kupiec test - Probability of Failure

In order to evaluate if the number of violations is in line with the given confidence level, we use one of the most widely known tests, the Kupiec test, also known as Probability of Failure test(PoF) [16].

This is a Bernoulli trial, which is a sequence of observations that either succeeds or fails, whom follow a binomial distribution. The probability of observing x observations of return below our given level of VaR_α out of a total of T observations, where $x \sim$

Bin(T, α), the binomial probability mass function is

$$\mathbf{P}(x|\alpha, T) = \binom{T}{x} (1 - \alpha)^x \alpha^{T-x}. \tag{58}$$

The null hypothesis is $H_0 : \hat{\alpha} = 1 - \alpha$ where $\hat{\alpha}$ is

$$\hat{\alpha} = \frac{1}{T} I(\alpha) \tag{59}$$

and $I(\alpha)$ is the number of violations and $I_t(\alpha)$ takes the value 0 if no violation at time t and 1 if there is a violation at time t, which can be described by

$$I(\alpha) = \sum_{t=1}^{T} I_t(\alpha). \tag{60}$$

The test statistic of the Kupiec test is

$$LR_{\text{POF}} = 2\ln \left(\left(\frac{1 - \hat{\alpha}}{\alpha} \right)^{T - I(\alpha)} \left(\frac{\hat{\alpha}}{1 - \alpha} \right)^{I(\alpha)} \right) \sim \chi^2(1). \tag{61}$$

In order to evaluate this we use a $\chi^2(1)$ distribution, e.g. for the 95% percentile the $\chi^2(1)$ is 3.84.

VaR Confidence Level	Non-rejection T=255 days
99%	$x < 7$
97.5%	$2 < x < 12$
95%	$6 < x < 21$
92.5%	$11 < x < 28$
90%	$16 < x < 36$

Table 1: *Non-rejection region for Kupiec test for different confidence levels.*

3.4.1 Criticism of Kupiec

Kupiec test has been criticized for the fact that it only takes into account the number of failures [4] and not that failures may come in clusters.

3.5 Christoffersen's Independence test

As discussed in 3.4.1, it is important to be able to make sure that the violations do not come in clusters, for this purpose we can use the Christoffersen's Independence test.

In the test, we have an indicator variable taking the value 1 if the $\text{VaR}_\alpha(r_t)$ value is larger than the actual return and taking the value 0 if the value of $\text{VaR}_\alpha(r_t)$ is lower than the actual return [5].

$$I_t = \begin{cases} 1 & \text{if violation occurs,} \\ 0 & \text{if no violation occurs.} \end{cases} \tag{62}$$

n_{ij} illustrates the value at day i given j. The four different outcomes are displayed in the matrix below.

	$I_{t-1} = 0$	$I_{t-1} = 1$	
$I_t = 0$	n_{00}	n_{10}	$n_{00} + n_{10}$
$I_t = 0$	n_{01}	n_{11}	$n_{01} + n_{11}$
	$n_{00} + n_{01}$	$n_{10} + n_{11}$	$n_{00} + n_{01} + n_{10} + n_{11}$

Table 2: *Outcomes of violations clustering for Christoffersen's Independence test.*

In addition, the variable π_i represents the probability of observing a violation conditional on state i.

$$\pi_0 = \frac{n_{01}}{n_{00} + n_{01}}, \tag{63}$$

$$\pi_1 = \frac{n_{11}}{n_{10} + n_{11}}, \tag{64}$$

and

$$\pi = \frac{n_{01} + n_{11}}{n_{00} + n_{01} + n_{10}n_{11}}. \tag{65}$$

Our test statistic for independence is thus given by

$$LR_{ind} = -2\ln\left(\frac{(1-\pi)^{n_{00}+n_{10}}\pi^{n_{01}+n_{11}}}{(1-\pi_0)^{n_{00}}\pi_0^{n_{01}}(1-\pi_1)^{n_{10}}\pi_1^{n_{11}}}\right), \tag{66}$$

which is evaluated based on a $\chi^2(1)$ distribution. With the Christoffersen's Independence test we test if the violations are arriving in clusters or not, the null hypothesis is

thus

$$H_0 : \pi_{01} = \pi_{11}. \tag{67}$$

3.6 Violation ratio

An additional way to compare the relative performance of VaR and mVaR is to use the violation ratio. The violation ratio is simply the number of violations divided by the expected number of violations [6].

We find the number of violations with

$$I_t = \begin{cases} 1 & \text{if violation occurs,} \\ 0 & \text{if no violation occurs.} \end{cases} \tag{68}$$

Then we divide I_t with the expected number of violations

$$\frac{\sum_{t=1}^{T} I_t}{(1 - \alpha)T}. \tag{69}$$

4 Data

4.1 Data background

In this thesis data is used from the Nasdaq OMX30 index and the Nikkei 225 index from 2012 to 2013[1]. In total, there are 255 observations over the years. The data has been retrieved from Bloomberg terminals and are displayed in figure 1 and figure 2.

OMXS30 is the index of the Stockholm Stock Exchange's 30 most actively traded stocks. By limiting the index to the 30 most traded stocks, we know for sure that they have good liquidity, which means that the market is effective, and investors can enter and exit their positions when they feel that the asset has reached the target price, in this way the prices are the actual market prices. Also with good liquidity in the underlying assets the index is suitable for derivative products. The OMXS30 index is a weighted basket on a market weighted price index [8].

The Nikkei 225 is the index for the First Section of the Tokyo Stock Exchange and consists of the 225 most traded companies that are listed. The index is price-weighted average of the companies [2].

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[1]more specifically 2012-12-13 to 2013-12-30
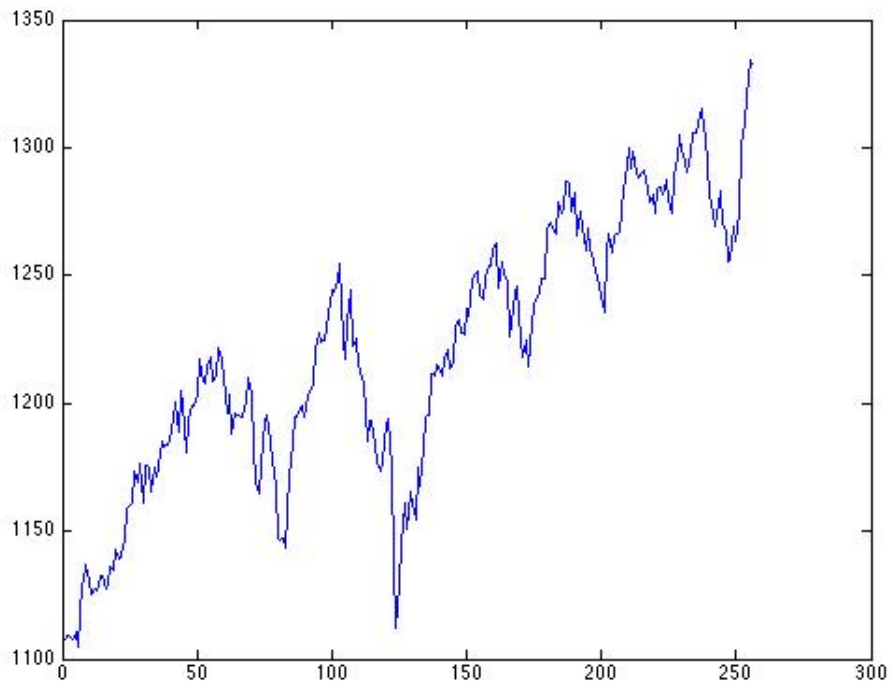
Figure 1: *Value of OMXS30 from 2012-12-13 to 2013-12-30.*
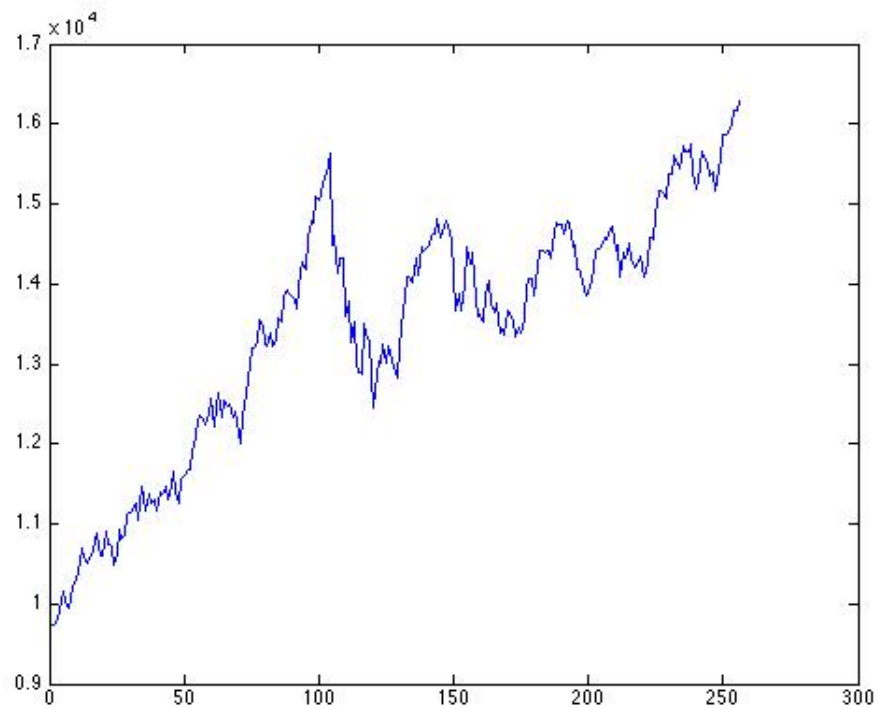
Figure 2: *Value of Nikkei 225 from 2012-12-13 to 2013-12-30.*

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The daily log returns for the indices is calculated, see chapter 2.1.

For all the indices, we estimate the risk models for 255 consecutive days, for each day of estimation we are using a rolling sample of the 64 observations, that we estimate our parameters based on. One day VaR forecasts for 95% and 99% confidence levels are generated. Model parameters are re-estimated every trading day and all tests are performed using the information for the last 64 days.

4.2   Descriptive statistics of the daily log returns

	Nasdaq OMXS30	Nikkei 225
Mean	0.0727%	0.2016%
Median	0.0784%	0.1999%
Max	2.55%	4.83%
Min	-3.13%	-7.60%
Std	0.0082	0.0169
Skewness	-0.3497	-0.7659
Kurtosis	4.1172	5.2634
Excess Kurtosis	1.1172	2.2634
Observations	255	255

Table 3: *Descriptive statistics of the daily log returns.*

For the Nikkei 225, we have excess kurtosis and fat tails which indicates a leptokurtic distribution. For OMXS30, we have a lower kurtosis value, but still the excess kurtosis is almost equal to one.

The mean and median are quite the same and very low, almost zero for OMXS30. For Nikkei 225 the mean and median are not deviating a lot, but they are not close to zero.

Leptokurtosis may affect the VaR analysis since at low levels of significance when estimating VaR, using the normal distribution instead of leptokurtic distribution, will overestimate the risk. At high levels of significance, it will instead underestimate the risk; this is since leptokurtic distribution has fatter tails than the normal distribution; the risk comes from outliers and extreme observations will be more likely to occur [13].

By observing the sample autocorrelation plot for each iteration, there seem to be no problem with first-order autocorrelation for any of the indices, therefore there is no need to use the AR(1) model for estimating the residuals.
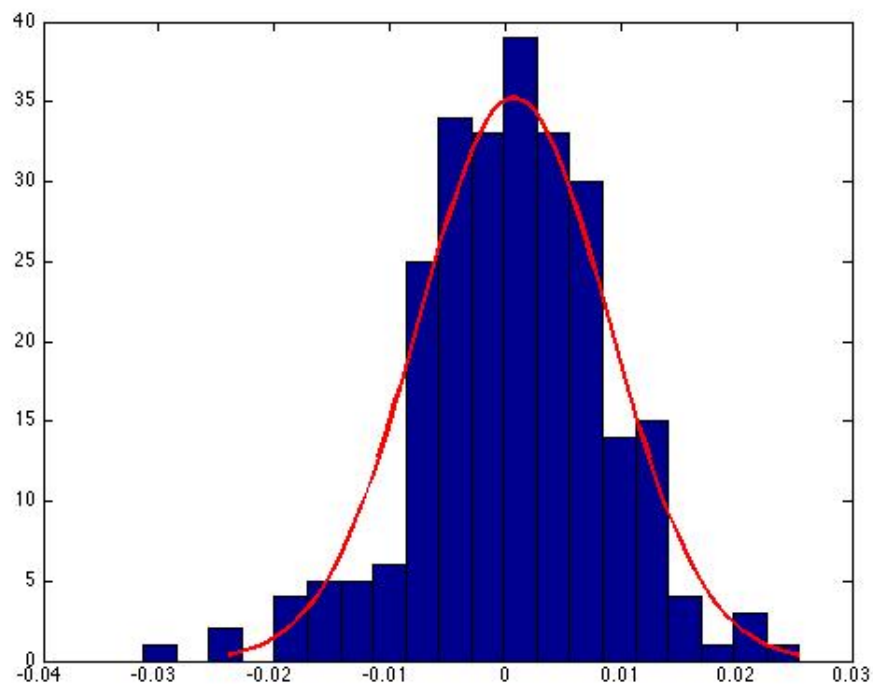
Figure 3: *Histogram of returns for OMXS30.*
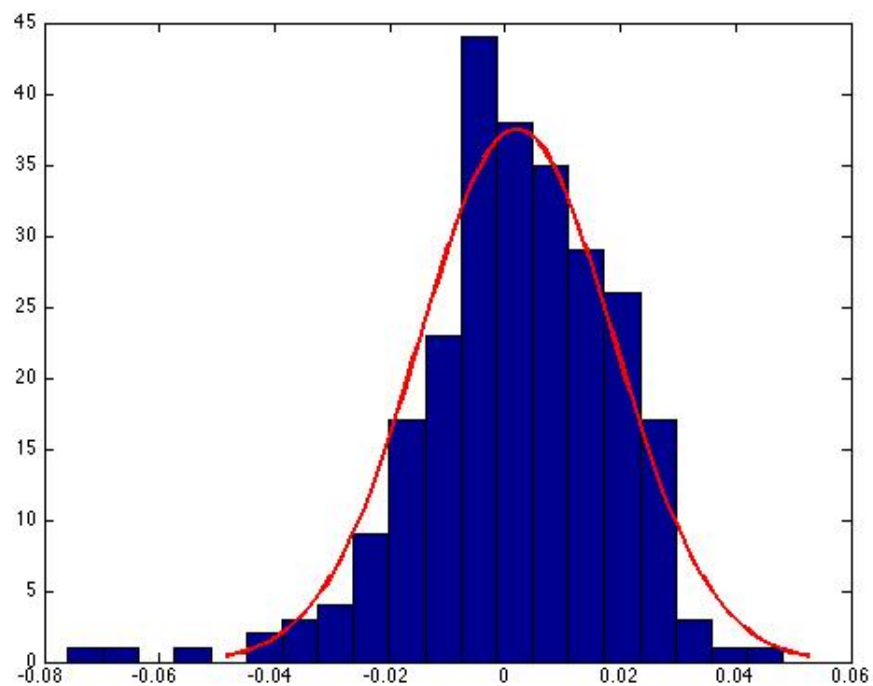
Figure 4: *Histogram of returns for Nikkei 225.*

5 Analysis

5.1 Residuals

5.1.1 Distribution of residuals

As concluded in the data section, the log returns of OMXS30 and Nikkei 225 are leptokurtically distributed. In this section, we further analyze the distribution of the residuals for our three different models with OMXS30 and Nikkei 225.

Model	Anderson-Darling		Jarque–Bera		BDS test	
Distribution	OMXS30	Nikkei 225	OMXS30	Nikkei 225	OMXS30	Nikkei 225
GARCH	0.1525*	0.4317*	0.1610*	0.0228	0.2617*	0.0407
Normal						
GARCH	0.1487*	0.3063*	0.0993*	0.001	0.3384*	0.0361
Student's t						
SRSM	0.6736*	0.7368*	0.5000*	0.5000*	0.8133*	0.3206*
Normal						
SRSM	0.6229*	0.9226*	0.5000*	0.5000*	0.5932*	0.5453*
Student's t						
Rolling	0.3542*	0.4040*	0.2420*	0.0926*	0.7632*	0.6053*
Window						

Table 4: *The test statistic for normal distribution of residuals, an asterisk (*) means that we can not reject the null hypothesis at 5% significance level.*

In table 4, we present the results of the three different tests that are performed in order to test whether the residuals of our indices are normally distributed or not.

We first consider at the distribution among the residuals for OMXS30. The GARCH(1,1) model with normal distribution gives normally distributed residuals, as expected. The GARCH(1,1) model with Student's t-distribution gives normally distributed residuals, from table 7 we can note that it does not have high degrees of freedom, but it has high enough for the residuals to be normally distributed.

The SRSM with normal distribution produces residuals that are normally distributed, which is what we expected. When we using the SRSM with Student's t-distribution we also get normally distributed residuals. In table 7 we can see that the degrees of freedom, for the number of observations are high, therefore we can assume the residuals from the SRSM with Student's t-distribution to follow a normal distribution.

According to our test, the rolling window approach produces normally distributed residuals, as we want when performing the risk models, since those are based on normal

distribution among the residuals. Hence all our models perform well with the OMXS30 index.

The outcome for the distribution of the residuals is slightly different for Nikkei 225 compared to OMXS30. The GARCH(1,1) model with normal distribution should have normally distributed residuals to estimate our risk models good, but this is not the case as shown by Jarque-Bera and the BDS test. However, even if we can not reject the null hypothesis for the Anderson-Darling test, we can still assume the residuals to be normally distributed. When breaking down the distribution of the residuals by Jarque-Bera, the main problem lies within the skewness, as can be seen in table 5. Since the GARCH model does not adjust for skewness, it is not a surprise that the skewness has test statistic above 1.96. Further, the kurtosis is relatively high with a value of 3.5114, but we should take into account that it originally was 5.2634 for the log return distribution of Nikkei 225, and hence this is an improvement.

Analyzing the GARCH(1,1) model with Student's t-distribution, we can see that it is normally distributed according to the Anderson-Darling test, but not according to the Jarque-Bera or the BDS test. Considering the degrees of freedom, we can see that they are high, therefore the residuals should be approaching the normal distribution. Since 194 out of 255 observations have degrees of freedom higher than 100, we would assume the residuals of the GARCH(1,1) model to be normally distributed, but still the residuals have not taken a normal distribution.

When estimating the risk models we assume the residuals to be normally distributed, therefore we want the residuals of the SRSM with normal distribution to be normally distributed. The test results for all models imply that we can not reject the null hypothesis at 5% significance level, we can therefore assume the residuals to be normally distributed.

Applying the SRSM with Student's t-distribution, the residuals seem to be normally distributed, since when the degrees of freedom increase, the Student's t-distribution becomes normally distributed (in table 7 the degrees of freedom are presented).

Using the rolling window approach for Nikkei 225 the residuals becomes normally distributed, this is what we want when we estimate the risk models as they assume normal distribution among the residuals.

We can clearly see that SRSM with both normal and Student's t-distribution and rolling window perform better than the GARCH(1,1) model with normal and Student's t-distribution.

The result of the Anderson-Darling test implies normal distribution among the residuals, but both the Jarque-Bera and the BDS test rejects the null hypothesis for the GARCH(1,1) model with normal and Student's t-distribution. We want to analyze further what the problem with the residuals may be, due to the rejected null hypothesis in the aforementioned test. We break down Jarque-Bera into one test for skewness and one test for kurtosis. The skewness and kurtosis, for the indices and the volatility models, can be seen in table 5 below.

Model	Skewness		Kurtosis	
Distribution	OMXS30	Nikkei 225	OMXS30	Nikkei 225
GARCH Normal	-0.2237	-0.3570	3.3203	3.5114
GARCH Student's t	-0.1802	-0.5047	3.5101	4.2009
SRSM Normal	-0.0206	-0.0361	2.9924	2.8117
SRSM Student's t	0.0531	-0.0986	3.0613	2.8821
Rolling Window	-0.1525	-0.3139	3.3767	3.0999

Table 5: *Skewness and kurtosis with Jarque-Bera for the models.*

From table 5, we can note that the model that best adjusts for the skewness is the SRSM, both with normal and Student's t-distribution. From table 6 we see that these models have lower test statistic than the other models. It is hard to tell which one of the models that is better, but SRSM with normal distribution has a slightly lower test statistic, see table 6. Further, the SRSM with normal distribution has a skewness closer to zero as can be seen in table 5.

Model	Jarque–Bera			
Distribution	Skewness		Kurtosis	
	OMXS30	Nikkei 225	OMXS30	Nikkei 225
GARCH Normal	2.1260	5.4154	1.0899*	2.7791
GARCH Student's t	1.3802*	10.8270	2.7651	15.3241
SRSM Normal	0.0181*	0.0554*	0.0006*	0.3766*
SRSM Student's t	0.1197*	0.4133*	0.0399*	0.1477*
Rolling Window	0.9881*	4.1865	1.5078*	0.1060*

Table 6: *Test statistics for Jarque-Bera Skewness and Kurtosis test. Market with asterisk (*) means that we can not reject the null hypothesis at 5% significance level.*

Furthermore, all the models do better adjustments the skewness better for the OMXS30 index comparing with the Nikkei 225 index. Three of the models do not adjust the skewness properly. The skewness for OMXS30 was originally -0.3497 and for Nikkei 225 it was -0.7659, all models improve the skewness, hence the skewness approaches zero. The GARCH(1,1) does not adjust as much as the other models, since it is a volatility model and should not adjust that much for skewness compared with the other models.

We want to have a kurtosis close to 3 for our models with normal distribution. Considering the degrees of freedom for all our models, we can see that they are sufficiently high for assuming normal distribution. The model that performs best is once again SRSM with normal and Student's t-distribution. Comparing the test statistics, the SRSM with normal distribution performs better with the OMXS30 index, while the SRSM Student's t-distribution performs better with the Nikkei 225 index. Furthermore, we can see that rolling window performs well for OMXS30 but for Nikkei 225 the residuals are too skewed and we also reject the model at 95% confidence level as the test statistic is 4.1865 and the critical value is 1.96.

In table 7 we can see the degrees of freedom for the models where we use the Student's t-distribution. The degrees of freedom have high values and when the degrees of freedom approaches infinity, the Student's t-distribution curve approaches the normal distribution curve.

Model	Median		Observations larger than 100	
Distribution	OMXS30	Nikkei 225	OMXS30	Nikkei 225
GARCH Student's t	11.2594	3442.7	39	194
SRSM Student's t	342.2479	342.2480	177	239

Table 7: *The degrees of freedom for our models with Student's t-distribution.*

5.1.2 Correlation of residuals for OMXS30

In order to detect if we have a problem with autocorrelation, we analyze the results of the sample autocorrelation plot for our models.

The GARCH(1,1) model with normal distribution has one lag outside the boundary, as can be expected at 95% confidence level. The GARCH(1,1) model with Student's t-distribution, SRSM with normal distribution and SRSM with Student's t-distribution all have two lags outside, which we can assume to not be any problem at 95% confidence

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level. Using rolling window we obtain three lags that are outside the boundary and two of them are close to each other, which can be seen in the figure 5 below.
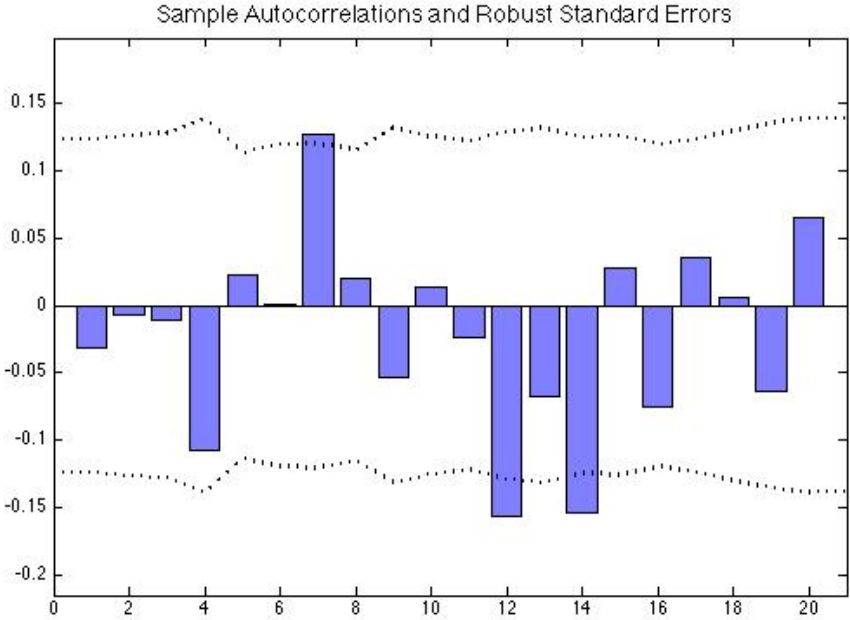
Figure 5: *Autocorrelation plot for OMXS30 using rolling window.*

Since we are unsure about the independence of the residuals, we decide to perform a Ljung-Box test. Under the null hypothesis, the residuals are uncorrelated. We perform the Ljung-Box test for the residuals of our models since we have between one and three lags outside the boundary in our sample autocorrelation plot. Therefore want to be sure that the residuals are uncorrelated. The test is performed for 20 lags, the same properties as for the test of the autocorrelation. The result is that no p-value is below 0.05, and we can therefore not reject the null hypothesis that the residuals are uncorrelated for all our models.

5.1.3 Correlation of residuals for Nikkei 225

From the autocorrelation plot, the GARCH(1,1) model with normal and Student's *t*-distribution and rolling window have no lag outside the boundary. The SRSM with normal distribution has two lags outside the boundary (figure 6), and SRSM with Student's *t*-distribution has one lag that is outside the boundary.
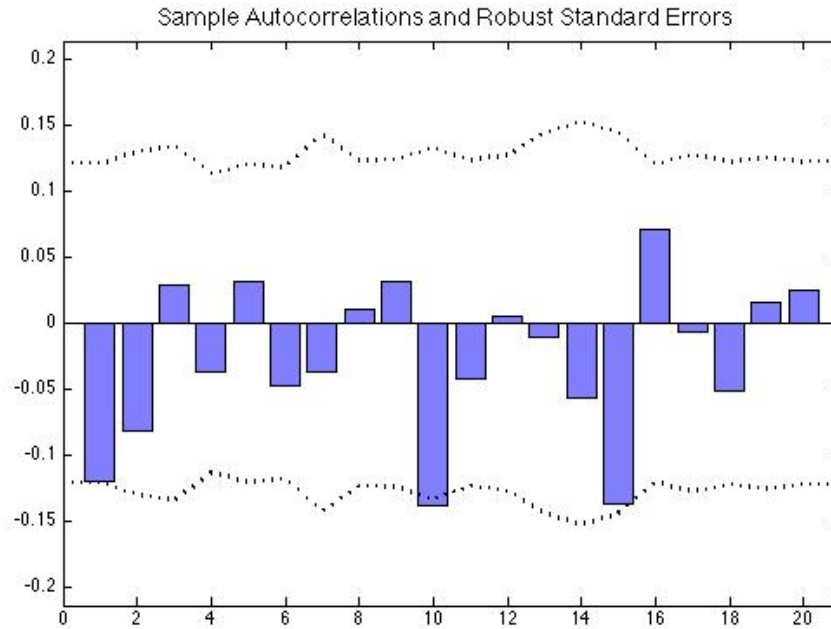
Figure 6: *Autocorrelation plot for Nikkei 225 using SRSM with normal distribution.*

The Ljung-Box test is used for all models in order to be sure that the residuals are uncorrelated, even though we can assume to not have any problems with autocorrelation from the sample autocorrelation plot. The result is that no p-value is below 0.05, and we can therefore not reject the null hypothesis that the residuals are uncorrelated for all our models.

5.1.4 Summary residual analysis

From table 4 at 95% confidence level, the SRSM with normal and Student's t-distribution and rolling window does not reject the null hypothesis about normal distribution. However, the GARCH(1,1) model has some minor problems with the Nikkei 225 index.

We can conclude that the superior model for skewness is the SRSM with normal distribution as it performs best for both the OMXS30 and Nikkei 225 index, this because the test statistic is close to zero and significant and the 5% significance level. It is also the SRSM with normal distribution that performs better for kurtosis as the test statistic of the Jarque-Bera test is almost zero.

We can also see that for our models with Student's t-distribution the degrees of freedom are high and we can therefore assume the residuals to be normally distributed.

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Finally, we want to make sure that the residuals are uncorrelated and considering the sample autocorrelation plots we can conclude that there is no problem with the majority of the models. Some models have two lags outside the boundary and one model has three lags outside the boundary, so in order to check if this is any problem we perform a Ljung-Box test. For all models with both the indices, we have no p-value that is lower than 0.05, therefore we can conclude that we have no problem with correlated residuals.

As can be seen in the data section, the period for Nikkei 225 has been volatile, and this is a property that of course makes it more difficult for our models to estimate the parameters. Even though our models are challenged, the SRSM model performs good and give us exactly the results that we would like to have, as can be seen above.

5.2 Backtesting of risk models

5.2.1 Frequency test

Risk model Distribution	Confidence level	Kupiec test Test statistic		Critical Value $\chi^2(1)$	Test Outcome	
		OMXS30	Nikkei 225		OMXS30	Nikkei 225
Garch						
Normal distribution						
Value at risk	95%	0.1252	2.1351	3.84	Not Rejected	Not Rejected
Value at risk	99%	0.7100	0.0759	3.84	Not Rejected	Not Rejected
mVaR	95%	0.0473	4.6411	3.84	Not Rejected	Rejected
mVaR	99%	0.7100	1.2373	3.84	Not Rejected	Not Rejected
Garch						
Student's t-distribution						
Value at risk	95%	0.3965	2.1351	3.84	Not Rejected	Not Rejected
Value at risk	99%	0.0759	0.0759	3.84	Not Rejected	Not Rejected
mVaR	95%	0.0473	4.6411	3.84	Not Rejected	Rejected
mVaR	99%	0.0759	0.1294	3.84	Not Rejected	Not Rejected
SRSM						
Normal distribution						
Value at risk	95%	0.0473	0.6722	3.84	Not Rejected	Not Rejected
Value at risk	99%	0.7100	0.7100	3.84	Not Rejected	Not Rejected
mVaR	95%	0.1252	3.2407	3.84	Not Rejected	Not Rejected
mVaR	99%	0.1294	0.1294	3.84	Not Rejected	Not Rejected
SRSM						
Student's t-distribution						
Value at risk	95%	0.0473	0.3965	3.84	Not Rejected	Not Rejected
Value at risk	99%	0.7100	3.4154	3.84	Not Rejected	Not Rejected
mVaR	95%	0.1252	0.6722	3.84	Not Rejected	Not Rejected
mVaR	99%	0.0759	3.4154	3.84	Not Rejected	Not Rejected
rolling window						
Value at risk	95%	0.3965	1.2882	3.84	Not Rejected	Not Rejected
Value at risk	99%	0.7100	0.0759	3.84	Not Rejected	Not Rejected
mVaR	95%	0.3965	3.2407	3.84	Not Rejected	Not Rejected
mVaR	99%	0.1294	1.2373	3.84	Not Rejected	Not Rejected

Table 8: *Kupiec test for OMXS30*

With the OMXS30 index no model is rejected, and all models produce low test statistic for the Kupiec test, as can be seen in table 8. No test statistic is higher than 0.7100 in the Kupiec test for OMXS30, and the critical value is 3.84. The GARCH(1,1) model in combination with mVaR produces the best result when searching for the lowest test statistics, where the SRSM Student's t-distribution with mVaR and SRSM normal distribution with mVaR are the second best performing models.

With the Nikkei 225, most of the models are not rejected, as can be seen in table 8. For the SRSM with normal distribution, the SRSM with Student's t-distribution and rolling window no risk model is rejected. This is in line with the result about the distribution of residuals. SRSM with normal and Student's t-distribution are the models that have the best results in the test statistics for distribution of residuals, while the GARCH(1,1) model both with normal and Student's t-distribution have some problems with the residuals not being normally distributed. In table 8 we can see that the models have considerably higher test statistics compared to the OMXS30 index and all models have at least one test statistic larger than three. The GARCH(1,1) model with normal and Student's t-distribution produces test statistic above 3.84, which is higher than the critical value.

When considering the test statistic of the Kupiec test, the model that performs best for the Nikkei 225 is the SRSM with normal distribution and VaR as risk measure, as this is the only model producing both of the test statistics below one.

In total, the model that produces the best result in the Kupiec test is the SRSM with normal distribution and VaR. Only considering the OMXS30, it is hard to tell which one is superior as all the models produce good results. Even though several models produce slightly better results than the SRSM with normal distribution and VaR, it produces results that are good enough. The results for the Nikkei 225 imply that this model performs better and have lower test statistics than the other models, it is the only model that has no test statistic above one.

5.2.2 Violations ratio

Risk model Distribution	Confidence level	Number of Violations		Violations ratio Risk model		Standard deviation	
		OMXS30	Nikkei 225	OMXS30	Nikkei 225	OMXS30	Nikkei 225
GARCH							
Normal							
VaR	95%	14	8	1.0980	0.6275	0.0026	0.0081
VaR	99%	4	3	1.5686	1.1765	0.0037	0.0114
mVaR	95%	12	6	0.9412	0.4706	0.0031	0.0116
mVaR	99%	4	1	1.5686	0.3922	0.0056	0.0183
GARCH							
Student's t							
VaR	95%	15	8	1.1765	0.6275	0.0025	0.0082
VaR	99%	3	3	1.1765	1.1765	0.0037	0.0116
mVaR	95%	12	6	0.9412	0.4706	0.0030	0.0103
mVaR	99%	3	2	1.1765	0.7843	0.0051	0.0196
SRSM							
Normal							
VaR	95%	12	10	0.9412	0.7843	0.0025	0.0088
VaR	99%	4	4	1.5686	1.5686	0.0037	0.0116
mVaR	95%	14	7	1.0980	0.5490	0.0029	0.0101
mVaR	99%	2	2	0.7843	0.7843	0.0048	0.0181
SRSM							
Student's t							
VaR	95%	12	15	0.9412	1.1765	0.0025	0.0088
VaR	99%	4	6	1.5686	2.3529	0.0037	0.0116
mVaR	95%	14	10	1.0980	0.7843	0.0029	0.0101
mVaR	99%	3	6	1.1765	2.3529	0.0048	0.0181
rolling window							
VaR	95%	15	9	1.1765	0.7059	0.0029	0.0070
VaR	99%	4	3	1.5686	1.1765	0.0042	0.0103
mVaR	95%	15	7	1.1765	0.5490	0.0034	0.0090
mVaR	99%	2	1	0.7843	0.3922	0.0056	0.0187

Table 9: *Violations and violation ratios for the risk models and confidence levels.*

It is now clear that we can not reject the majority of the risk models from the Kupiec test, but we further want to compare how good the estimates are compared to the expected level. We do this by observing the violations ratio, as presented in table 9.

The violations ratio, for our four different risk measures, is as described in the theory chapter 3.6, the number of violations divided by the expected violations. As we have 255 observations, the expected value for 95% confidence level is 12.75 and for 99% confidence level 2.55. We want a violation ratio as close to one as possible, since this indicates that the actual violations are the same as the expected violations.

For OMXS30, we can see that all models give quite good results for the violations ratio. The worst performing model seems to be the GARCH(1,1) model with normal distribution, while any of the other models can be the performing good.

For Nikkei 225, it is harder to tell which one is superior and which one that performs worse, SRSM with normal distribution seems to be the best performing, but is still inferior when comparing the violations ratio that we obtain from OMSX30.

In total, we can conclude that the SRSM with normal distribution and VaR seems to produce the best results, just as in the Kupiec test. We should also notice that, for the 99% confidence level, a very small change in the number of violations highly affect the outcome and can, therefore, be misleading.

As can be expected; higher confidence levels generate more volatility in the risk measure, this because larger changes need to be made in the risk measure in order to capture the violations. We know from table 3 that the log returns of the Nikkei 225 are more volatile than OMXS30, and we find in table 9 that the VaR and mVaR of Nikkei 225 are considerably more volatile than OMXS30.

One of the reasons to why OMXS30 gives better results than Nikkei 225 may be that it is less volatile, as can be seen in table 3. In addition, we can in table 9 see that the standard deviation of VaR and mVaR are considerably higher for Nikkei 225 compared to OMXS30. In table 9, we can also observe that the mVaR is more volatile than the classic VaR.

One conclusion from the results is that, as the standard deviation increases for the risk models, the models becomes less reliable. Which is in accordance with the models, since increased volatility in the underlying assets forces the VaR to adjust more and thus it becomes harder to predict when movements are large from day to day.

5.2.3 Comparing risk measures

In appendix B, the output from the models is plotted for parameter estimation and the risk models. From the plots it is clear that SRSM with both normal distribution and Student's t-distribution produces a VaR and mVaR value that is considerably closer to zero than the equivalence of GARCH(1,1); hence they are underestimating the risk more than the GARCH(1,1) models. In figure 7 the results for VaR 95% confidence level with GARCH(1,1) and SRSM with normal distribution is plotted. From this it is clear that SRSM is closer to zero than the GARCH(1,1), and this is also common for all our

models, but still the SRSM only produces 12 violations while the GARCH(1,1) model produces 14 violations. So considering the number of violations we can see that SRSM produces fewer violations; hence SRSM must be better in adjusting for the actual risk in this case.
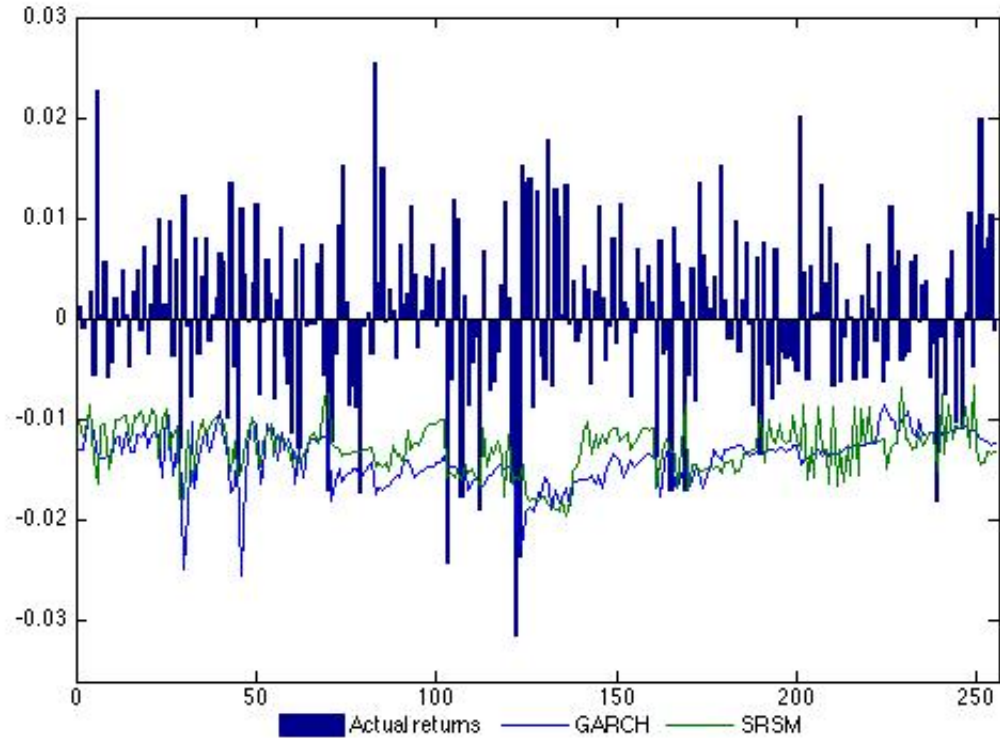

Figure 7: *VaR 95% for GARCH(1,1) and SRSM with normal distribution from 2012-12-13 to 2013-12-30.*

It is only mVaR at 99% confidence level for the GARCH(1,1) model and SRSM with normal distribution for the OMXS30 index that has a longer time period, about 70 observations, where GARCH(1,1) has a risk measure that is closer to zero than the SRSM, displayed in figure 8.
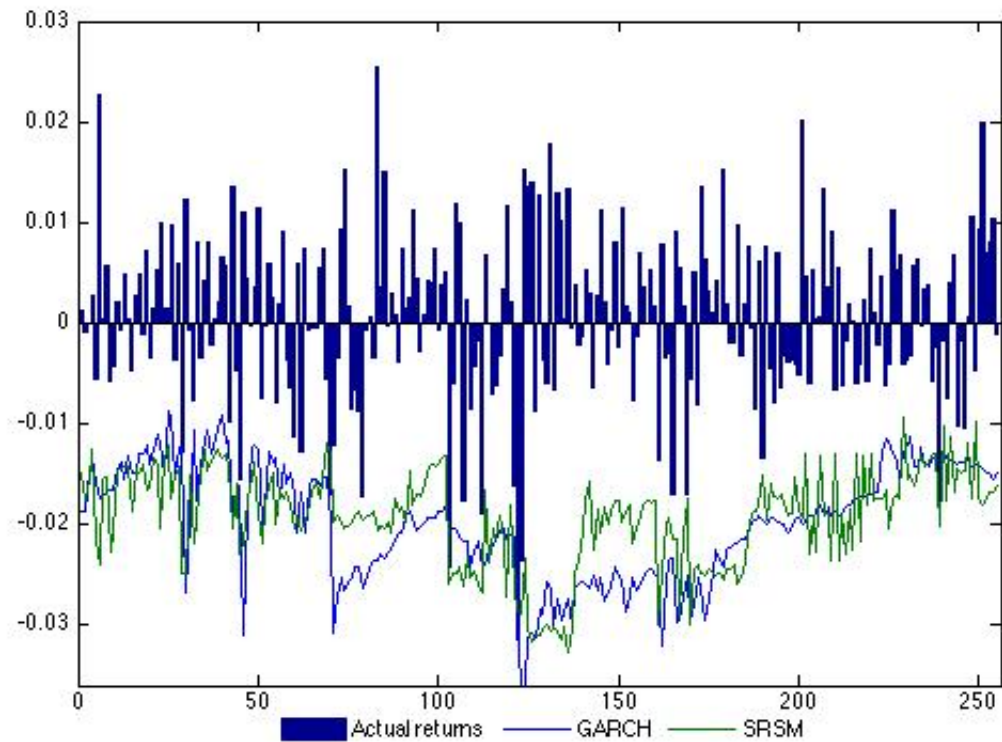
Figure 8: *OMXS30 mVaR 99% for GARCH(1,1) and SRSM with normal distribution from 2012-12-13 to 2013-12-30.*

This indicate that, since there is no major difference in the number of violations, a financial institution using SRSM instead of GARCH(1,1) would be able to reduce the amount of regulatory capital to hold, hence lower the costs of risk.

5.2.4 Independence test

Risk model Distribution	Confidence level	Independence test Christoffersen	
		OMXS30	Nikkei 225
GARCH			
Normal			
VaR	95%	Not rejected	Not rejected
VaR	99%	Not rejected	Not rejected
mVaR	95%	Not rejected	Not rejected
mVaR	99%	Not rejected	Not rejected
GARCH			
Student's t			
VaR	95%	Not rejected	Not rejected
VaR	99%	Not rejected	Not rejected
mVaR	95%	Not rejected	Not rejected
mVaR	99%	Not rejected	Not rejected
SRSM			
Normal			
VaR	95%	Not rejected	Not rejected
VaR	99%	Not rejected	Not rejected
mVaR	95%	Not rejected	Not rejected
mVaR	99%	Not rejected	Not rejected
SRSM			
Student's t			
VaR	95%	Rejected	Not rejected
VaR	99%	Not rejected	Not rejected
mVaR	95%	Rejected	Not rejected
mVaR	99%	Not rejected	Not rejected
rolling window			
VaR	95%	Not rejected	Not rejected
VaR	99%	Rejected	Not rejected
mVaR	95%	Not rejected	Not rejected
mVaR	99%	Not rejected	Not rejected

Table 10: *Tests of independence among residuals for VaR and mVaR.*

With Nikkei 225, there is no problem with independence among the violations, as can be seen in table 10. Table 15 (appendix) confirms this as there is only one occasion, using the rolling window with VaR, when violations are arriving consecutively. For all other models and confidence levels there are no consecutive violations.

OMXS30 has no problem with violations arriving in clusters for the GARCH(1,1) models and for the SRSM with normal distribution. There is, however, problem with SRSM Student's t-distribution and rolling window, both for Var and mVaR at 95% confidence level. For SRSM with Student's t-distribution we reject that there is no independence in

the violations for both VaR and mVaR at 95% confidence level. For the rolling window, we reject the independence for VaR at 95% confidence level.

Table 13 (appendix) confirms the aforementioned, for all the rejected models there are three times when violations are occurring consecutively. In addition, we find that, for all models with OMXS30 at 95% confidence level, there is at least one time that violations occur consecutively.

Analyzing how the violations occur and if they do so consecutively we can conclude that the SRSM with normal distribution gives the best result, both for VaR and mVaR. As aforementioned, we can not distinguish with Nikkei 225 which one that performs best or worst as all produce enough satisfying results. With OMXS30, it is clear that the SRSM with normal distribution definitely produces the best results and, therefore, we can conclude that this is the model that in general gives the best results.

6 Conclusion

In this thesis, we have shown that the Simple Regime Switching Model (SRSM) with normal distribution generates superior results, both for the distribution of the residuals and the estimation of risk models. This result is expected as SRSM is more adapted to non-normally distributed time series than the GARCH(1,1) model. The result becomes obvious when applying the models to the Japanese Nikkei 225 index as it is considerably more volatile and have stronger kurtosis than the Swedish OMXS30 index.

The SRSM with normal distribution is the model that best adjusts for both skewness and kurtosis, while the SRSM with Student's t-distribution gives almost as good results. Those results are considerably better than the results for the GARCH(1,1) model and rolling window.

When backtesting Value at Risk (VaR) and modified Value at Risk (mVaR), the model producing the best result is the SRSM with normal distribution and VaR. When only considering the OMXS30 it is hard to tell which model that is superior, as all the models produce good results. From the results for the Nikkei 225, we can see that the SRSM with normal distribution and VaR performs better.

In addition, the SRSM produces VaR and mVaR results that are considerably closer to zero compared to the GARCH(1,1) model, this result is the same for all confidence levels and distributions.

Regarding consecutive occurrence of violations, we have showed that the SRSM with normal distribution generates the best result, both for VaR and mVaR. For Nikkei 225, we can not distinguish which model performs best, as all models produce satisfying results. With OMXS30, it is clear that the SRSM with normal distribution performs better; hence the model generates the best results in this thesis.

6.1 Further studies

In this field, it would be interesting to apply several other distributions and investigate how these affect the result, as the returns are not normally distributed.

There are further extensions of the Simple Regime Switching Model available, and it would be interesting to use a Regime Switching Market Model for the OMXS30 index together with the Swedish industrial indices, alternatively several Swedish stocks, and see how these estimate the residuals and the risk models.

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Appendix A Tables

A.1 Kupiec test

Model Distribution	Risk measure	Confidence level	Test statistic Kupiec test	Critical Value $\chi^2(1)$	Test Outcome
Garch					
Normal distribution					
	Value at risk	95%	0.1252	3.84	Not Rejected
	Value at risk	99%	0.7100	3.84	Not Rejected
	Modified VaR	95%	0.0473	3.84	Not Rejected
	Modified VaR	99%	0.7100	3.84	Not Rejected
Garch					
Student's t-distribution					
	Value at risk	95%	0.3965	3.84	Not Rejected
	Value at risk	99%	0.0759	3.84	Not Rejected
	Modified VaR	95%	0.0473	3.84	Not Rejected
	Modified VaR	99%	0.0759	3.84	Not Rejected
SRSM					
Normal distribution					
	Value at risk	95%	0.0473	3.84	Not Rejected
	Value at risk	99%	0.7100	3.84	Not Rejected
	Modified VaR	95%	0.1252	3.84	Not Rejected
	Modified VaR	99%	0.1294	3.84	Not Rejected
SRSM					
Student's t-distribution					
	Value at risk	95%	0.0473	3.84	Not Rejected
	Value at risk	99%	0.7100	3.84	Not Rejected
	Modified VaR	95%	0.1252	3.84	Not Rejected
	Modified VaR	99%	0.0759	3.84	Not Rejected
rolling window					
	Value at risk	95%	0.3965	3.84	Not Rejected
	Value at risk	99%	0.7100	3.84	Not Rejected
	Modified VaR	95%	0.3965	3.84	Not Rejected
	Modified VaR	99%	0.1294	3.84	Not Rejected

Table 11: *Kupiec test for OMXS30*

Model Distribution	Risk measure	Confidence level	Test statistic Kupiec test	Critical Value $\chi^2(1)$	Test Outcome
Garch					
Normal distribution					
	Value at risk	95%	2.1351	3.84	Not Rejected
	Value at risk	99%	0.0759	3.84	Not Rejected
	Modified VaR	95%	4.6411	3.84	Rejected
	Modified VaR	99%	1.2373	3.84	Not Rejected
Garch					
Student's t-distribution					
	Value at risk	95%	2.1351	3.84	Not Rejected
	Value at risk	99%	0.0759	3.84	Not Rejected
	Modified VaR	95%	4.6411	3.84	Rejected
	Modified VaR	99%	0.1294	3.84	Not Rejected
SRSM					
Normal distribution					
	Value at risk	95%	0.6722	3.84	Not Rejected
	Value at risk	99%	0.7100	3.84	Not Rejected
	Modified VaR	95%	3.2407	3.84	Not Rejected
	Modified VaR	99%	0.1294	3.84	Not Rejected
SRSM					
Student's t-distribution					
	Value at risk	95%	0.3965	3.84	Not Rejected
	Value at risk	99%	3.4154	3.84	Not Rejected
	Modified VaR	95%	0.6722	3.84	Not Rejected
	Modified VaR	99%	3.4154	3.84	Not Rejected
rolling window					
	Value at risk	95%	1.2882	3.84	Not Rejected
	Value at risk	99%	0.0759	3.84	Not Rejected
	Modified VaR	95%	3.2407	3.84	Not Rejected
	Modified VaR	99%	1.2373	3.84	Not Rejected

Table 12: *Kupiec test for Nikkei 225*

A.2 Christoffersen's Independence test

Model Distribution	Risk measure	Confidence level	Violations	n_{00}	n_{01}	n_{10}	n_{11}	π_0	π_1	π
Garch										
Normal distribution										
	Value at risk	95%	15	229	12	12	2	0.0498	0.1429	0.0549
	Value at risk	99%	4	247	4	4	0	0.0159	0	0.0157
	Modified VaR	95%	12	232	11	11	1	0.0453	0.0833	0.0471
	Modified VaR	99%	4	247	4	4	0	0.0159	0	0.0157
Garch										
Student's t-distribution										
	Value at risk	95%	15	227	13	13	2	0.0542	0.1333	0.0588
	Value at risk	99%	3	249	3	3	0	0.0119	0	0.0118
	Modified VaR	95%	12	232	11	11	1	0.0453	0.0833	0.0471
	Modified VaR	99%	3	249	3	3	0	0.0119	0	0.0118
SRSM										
Normal distribution										
	Value at risk	95%	12	232	11	11	1	0.0453	0.0833	0.0471
	Value at risk	99%	4	247	4	4	0	0.0159	0	0.0157
	Modified VaR	95%	14	228	13	13	1	0.0539	0.0714	0.0549
	Modified VaR	99%	2	251	2	2	0	0.0079	0	0.0078
SRSM										
Student's t-distribution										
	Value at risk	95%	12	234	9	9	3	0.0370	0.2500	0.0471
	Value at risk	99%	4	247	4	4	0	0.0159	0	0.0157
	Modified VaR	95%	14	230	11	11	3	0.0456	0.2143	0.0549
	Modified VaR	99%	3	249	3	3	0	0.0119	0	0.0118
rolling window										
	Value at risk	95%	15	228	12	12	3	0.0500	0.2000	0.0588
	Value at risk	99%	4	248	3	3	1	0.0120	0.2500	0.0157
	Modified VaR	95%	15	227	13	13	2	0.0542	0.1333	0.0588
	Modified VaR	99%	2	251	2	2	0	0.0079	0	0.0078

Table 13: *Christoffersen's Independence test for OMXS30*

Model Distribution	Risk measure	Confidence level	Test statistic Chris ind	Critical Value $\chi^2(1)$	Test Outcome
Garch					
Normal distribution					
	Value at risk	95%	1.6057	3.84	Not Rejected
	Value at risk	99%	0.1275	3.84	Not Rejected
	Modified VaR	95%	0.3067	3.84	Not Rejected
	Modified VaR	99%	0.1275	3.84	Not Rejected
Garch					
Student's t-distribution					
	Value at risk	95%	1.2253	3.84	Not Rejected
	Value at risk	99%	0.0714	3.84	Not Rejected
	Modified VaR	95%	0.3067	3.84	Not Rejected
	Modified VaR	99%	0.0714	3.84	Not Rejected
SRSM					
Normal distribution					
	Value at risk	95%	0.3067	3.84	Not Rejected
	Value at risk	99%	0.1275	3.84	Not Rejected
	Modified VaR	95%	0.0719	3.84	Not Rejected
	Modified VaR	99%	0.0316	3.84	Not Rejected
SRSM					
Student's t-distribution					
	Value at risk	95%	6.2952	3.84	Rejected
	Value at risk	99%	0.1275	3.84	Not Rejected
	Modified VaR	95%	4.5285	3.84	Rejected
	Modified VaR	99%	0.0714	3.84	Not Rejected
rolling window					
	Value at risk	95%	3.7968	3.84	Not Rejected
	Value at risk	99%	4.1530	3.84	Rejected
	Modified VaR	95%	1.2253	3.84	Not Rejected
	Modified VaR	99%	0.0316	3.84	Not Rejected

Table 14: *Results for Christoffersen's Independence test for OMXS30*

Model Distribution	Risk measure	Confidence level	Violations	n_{00}	n_{01}	n_{10}	n_{11}	π_0	π_1	π
Garch										
Normal distribution										
	Value at risk	95%	8	239	8	8	0	0.0324	0	0.0314
	Value at risk	99%	3	249	3	3	0	0.0119	0	0.0118
	Modified VaR	95%	6	243	6	6	0	0.0241	0	0.0235
	Modified VaR	99%	1	253	1	1	0	0.0039	0	0.0039
Garch										
Student's t-distribution										
	Value at risk	95%	8	239	8	8	0	0.0324	0	0.0314
	Value at risk	99%	3	249	3	3	0	0.0119	0	0.0118
	Modified VaR	95%	6	243	6	6	0	0.0241	0	0.0235
	Modified VaR	99%	2	251	2	2	0	0.0079	0	0.0078
SRSM										
Normal distribution										
	Value at risk	95%	10	235	10	10	0	0.0408	0	0.0392
	Value at risk	99%	4	247	4	4	0	0.0159	0	0.0157
	Modified VaR	95%	7	241	7	7	0	0.0282	0	0.0275
	Modified VaR	99%	2	251	2	2	0	0.0079	0	0.0078
SRSM										
Student's t-distribution										
	Value at risk	95%	15	225	15	15	0	0.0625	0	0.0588
	Value at risk	99%	6	243	6	6	0	0.0241	0	0.0235
	Modified VaR	95%	10	235	10	10	0	0.0408	0	0.0392
	Modified VaR	99%	6	243	6	6	0	0.0241	0	0.0235
rolling window										
	Value at risk	95%	9	238	8	8	1	0.0325	0.1111	0.0353
	Value at risk	99%	3	249	3	3	0	0.0119	0	0.0118
	Modified VaR	95%	7	241	7	7	0	0.0282	0	0.0275
	Modified VaR	99%	1	253	1	1	0	0.0039	0	0.0039

Table 15: *Christoffersen's Independence test for Nikkei 225*

Model Distribution	Risk measure	Confidence level	Test statistic Chris ind	Critical Value $\chi^2(1)$	Test Outcome
Garch					
Normal distribution					
	Value at risk	95%	0.5183	3.84	Not Rejected
	Value at risk	99%	0.0714	3.84	Not Rejected
	Modified VaR	95%	0.2892	3.84	Not Rejected
	Modified VaR	99%	0.0079	3.84	Not Rejected
Garch					
Student's t-distribution					
	Value at risk	95%	0.5183	3.84	Not Rejected
	Value at risk	99%	0.0714	3.84	Not Rejected
	Modified VaR	95%	0.2892	3.84	Not Rejected
	Modified VaR	99%	0.0316	3.84	Not Rejected
SRSM					
Normal distribution					
	Value at risk	95%	0.8166	3.84	Not Rejected
	Value at risk	99%	0.1275	3.84	Not Rejected
	Modified VaR	95%	0.3952	3.84	Not Rejected
	Modified VaR	99%	0.0316	3.84	Not Rejected
SRSM					
Student's t-distribution					
	Value at risk	95%	1.8762	3.84	Not Rejected
	Value at risk	99%	0.2892	3.84	Not Rejected
	Modified VaR	95%	0.8166	3.84	Not Rejected
	Modified VaR	99%	0.2892	3.84	Not Rejected
rolling window					
	Value at risk	95%	1.0411	3.84	Not Rejected
	Value at risk	99%	0.0714	3.84	Not Rejected
	Modified VaR	95%	0.3952	3.84	Not Rejected
	Modified VaR	99%	0.0079	3.84	Not Rejected

Table 16: *Results for Christoffersen's Independence test for Nikkei 225*

Appendix B   Graphs

B.1   Comparing results OMXS30
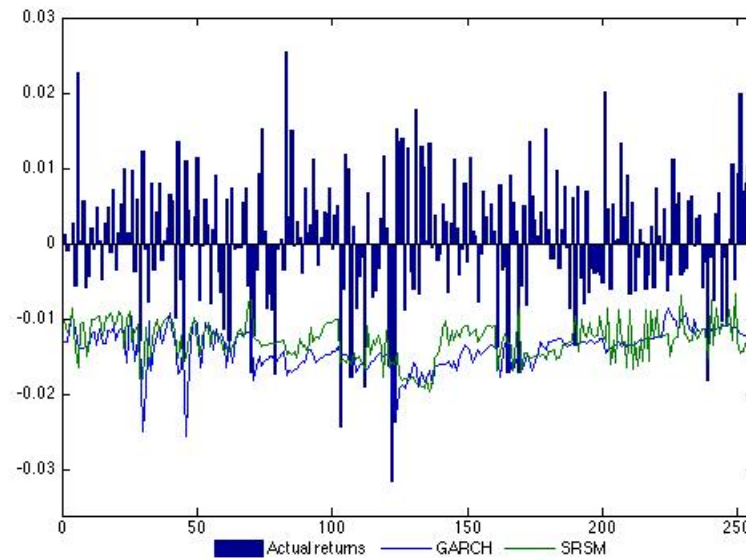
Figure 9: *VaR 95% for GARCH and SRSM with normal distribution from 2012-12-13 to 2013-12-30.*
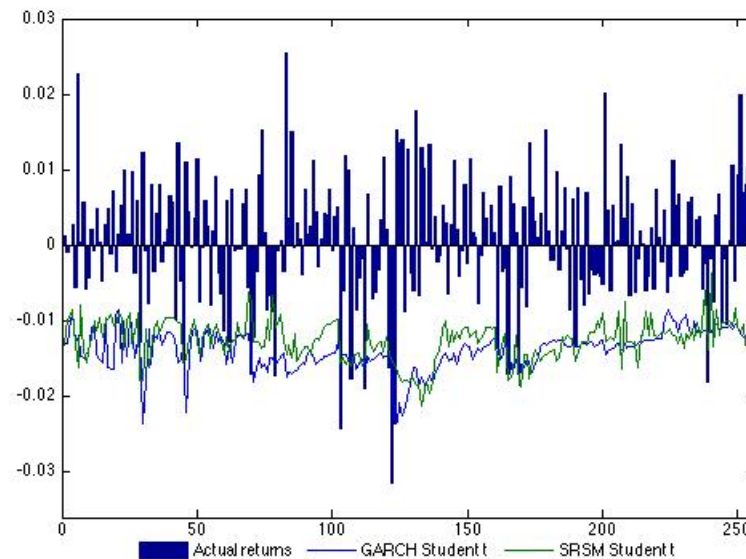
Figure 10: *VaR 95% for GARCH and SRSM with Student's t-distribution from 2012-12-13 to 2013-12-30.*
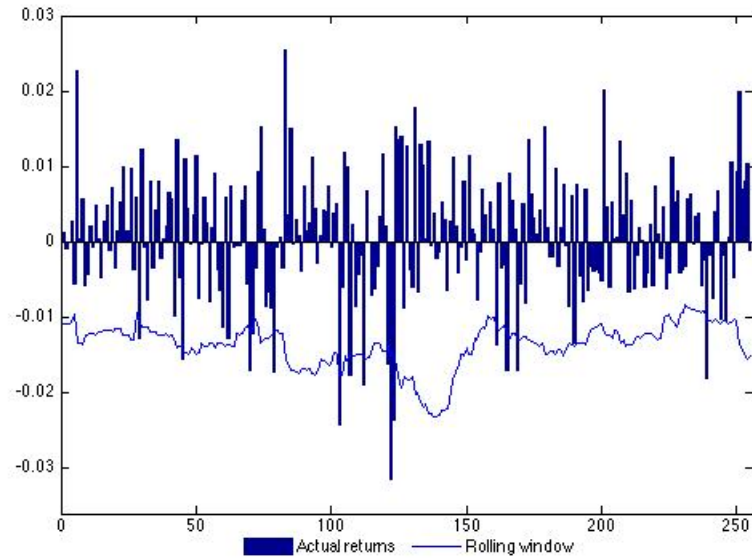
Figure 11: *VaR 95% for rolling window from 2012-12-13 to 2013-12-30.*
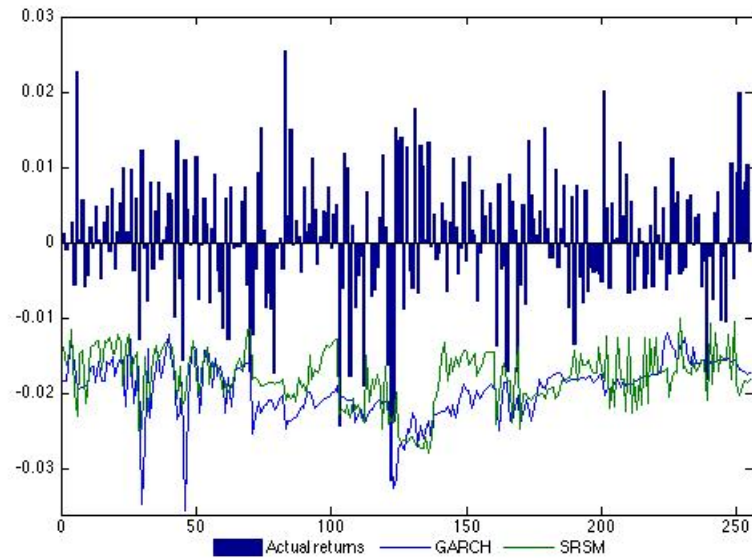
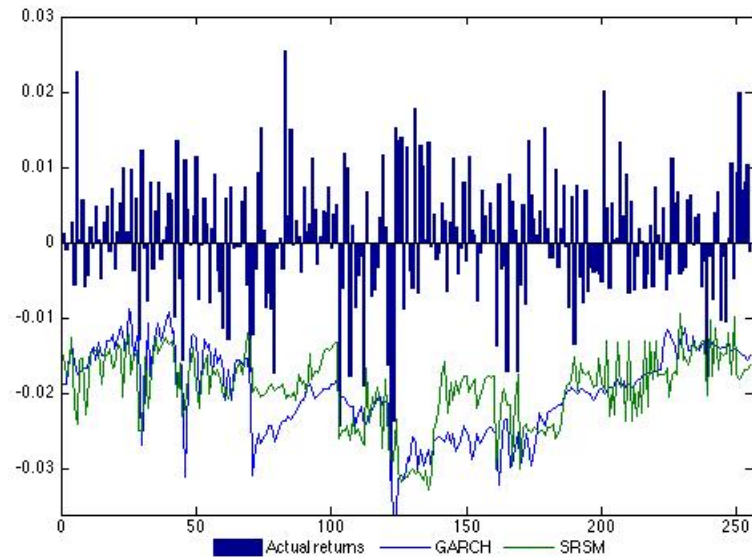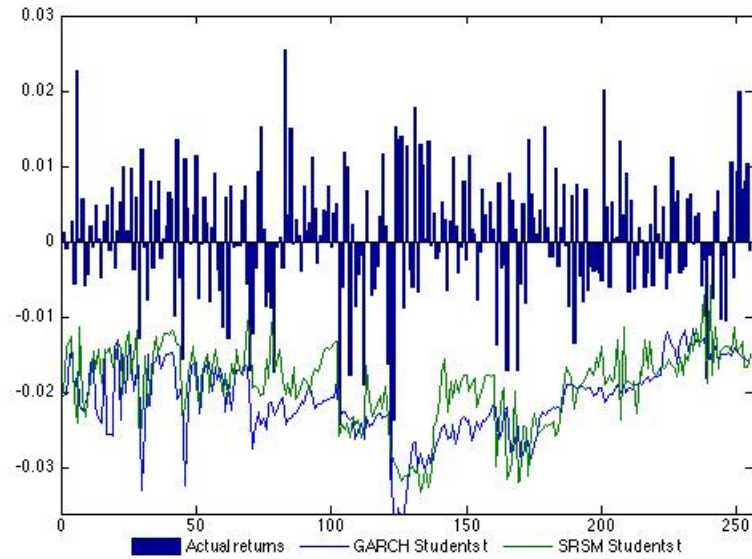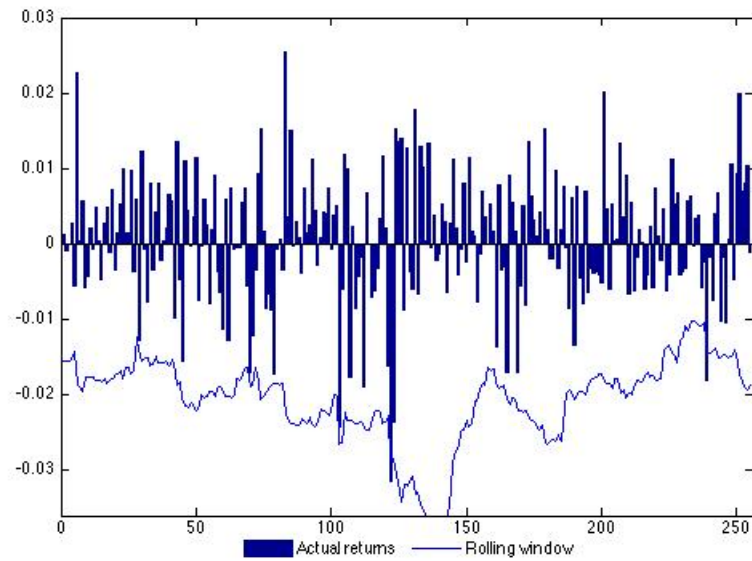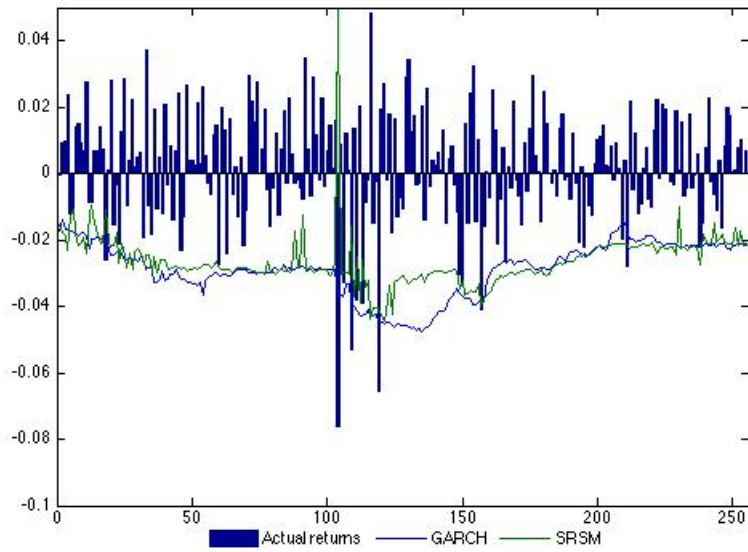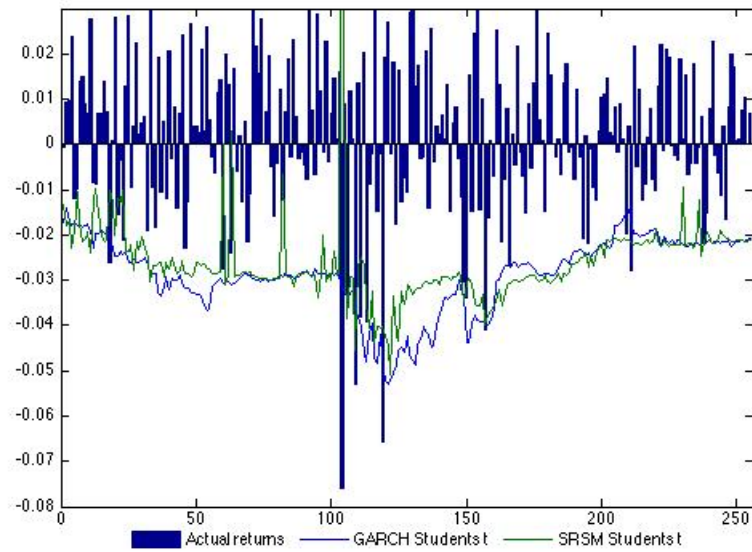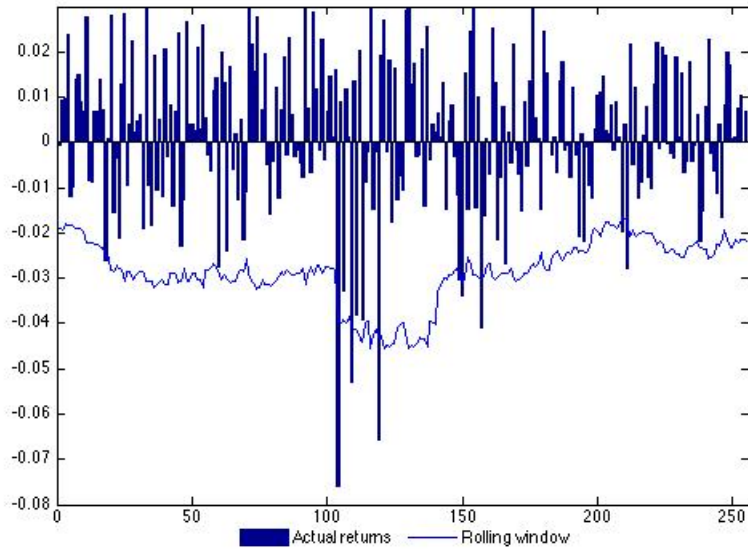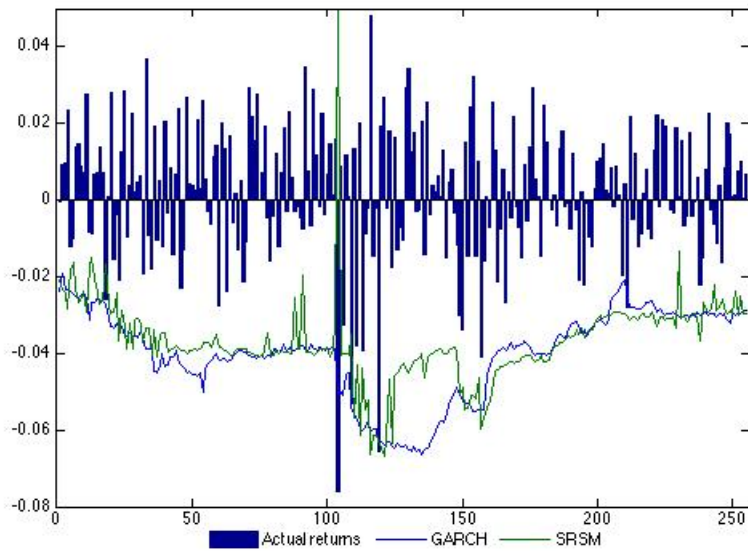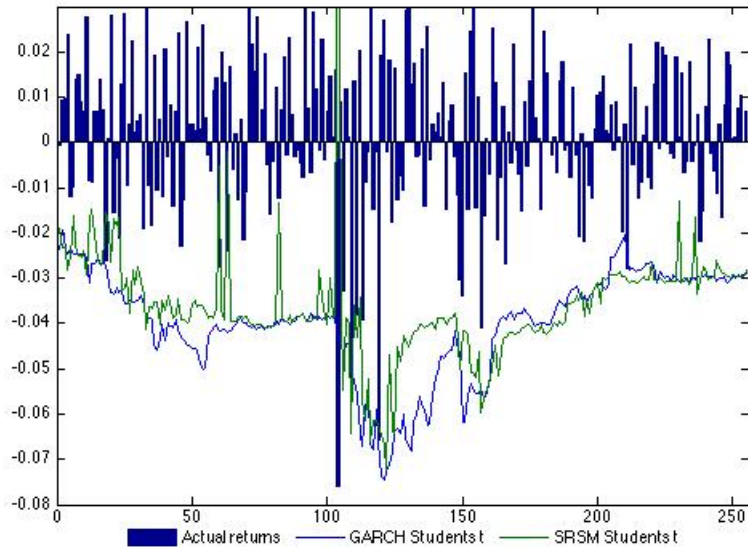Figure 12: *VaR 99% for GARCH and SRSM with normal distribution from 2012-12-13 to 2013-12-30.*

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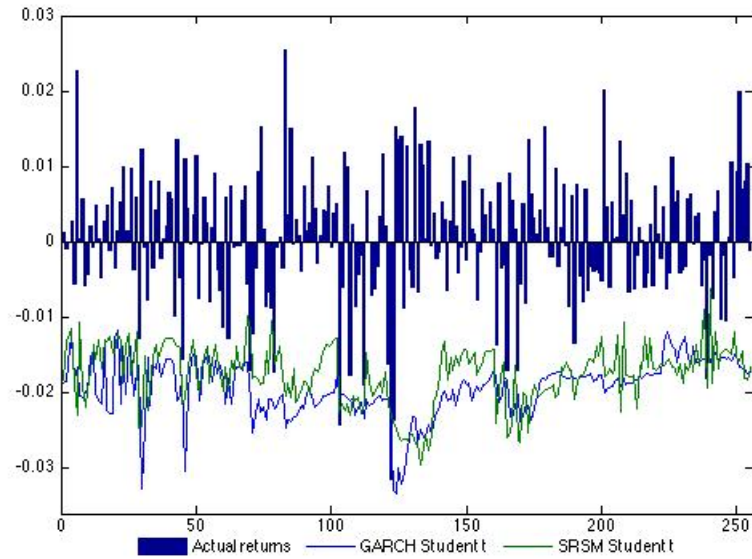
Figure 13: *VaR 99% for GARCH and SRSM with Student's t-distribution from 2012-12-13 to 2013-12-30.*
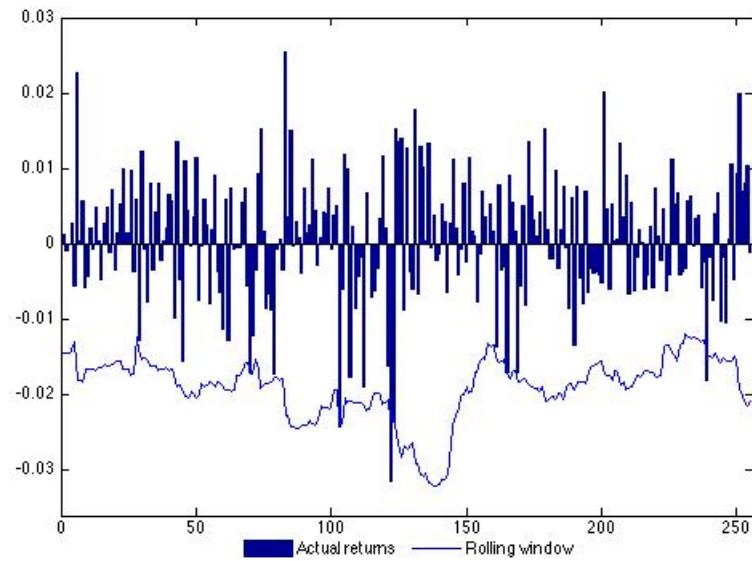
Figure 14: *VaR 99% for rolling window from 2012-12-13 to 2013-12-30.*
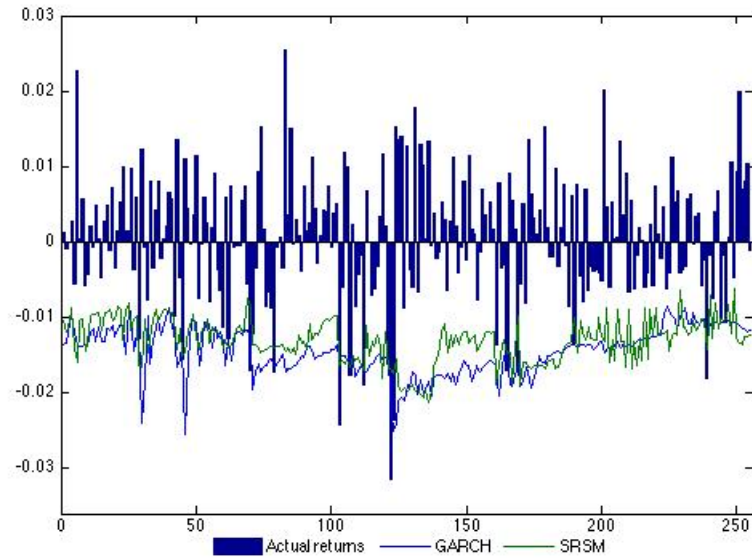
Figure 15: *mVaR 95% for GARCH and SRSM with normal distribution from 2012-12-13 to 2013-12-30.*
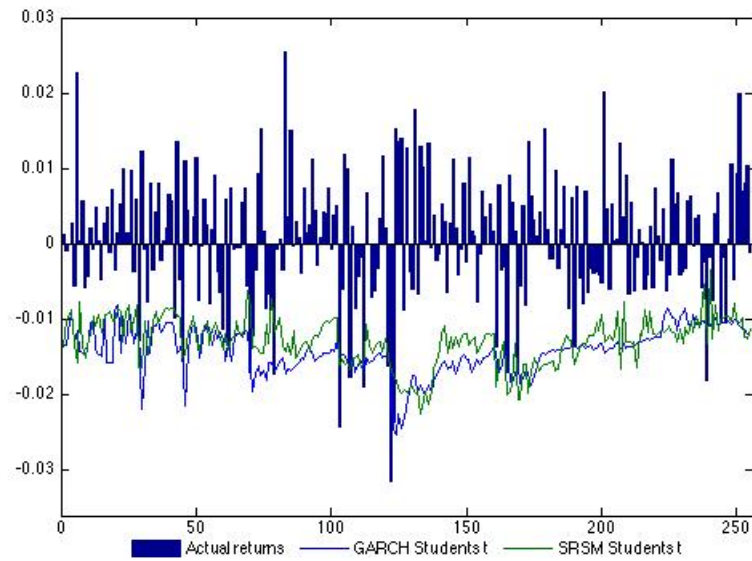
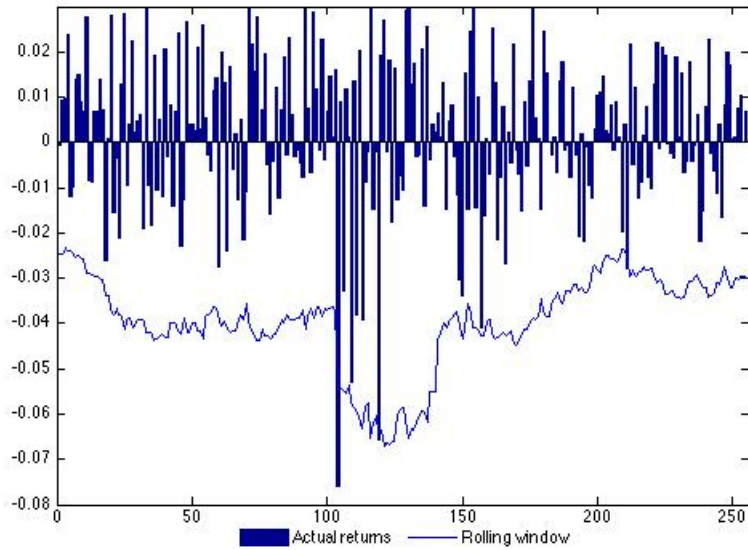Figure 16: *mVaR 95% for GARCH and SRSM with Student's t-distribution from 2012-12-13 to 2013-12-30.*

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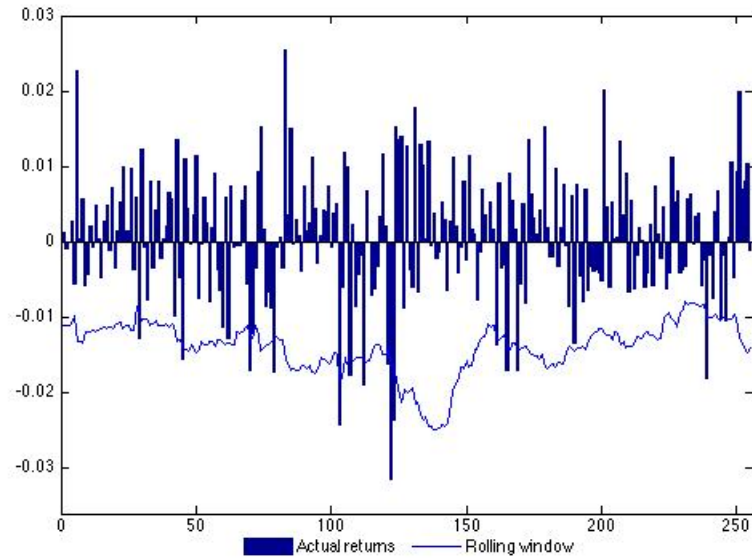
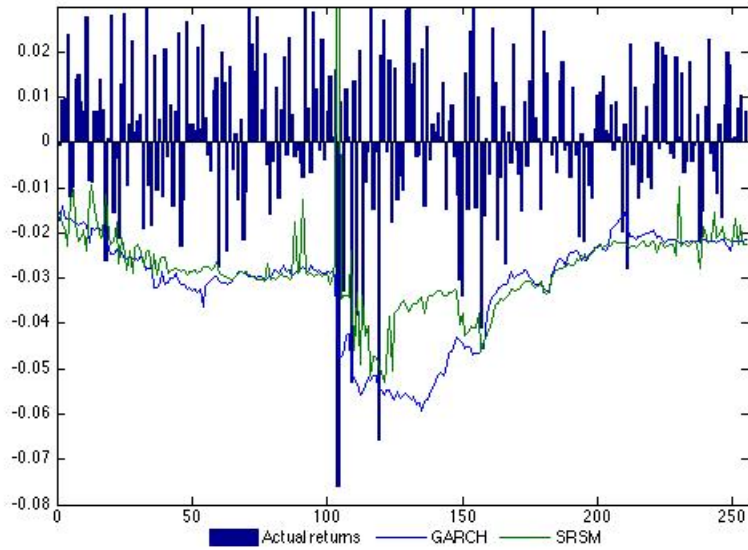Figure 17: *mVaR 95% for rolling window from 2012-12-13 to 2013-12-30.*

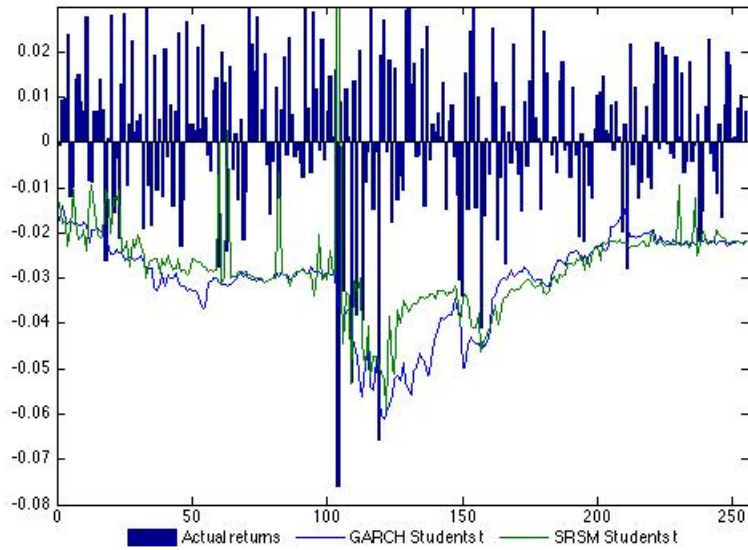Figure 18: *mVaR 99% for GARCH and SRSM with normal distribution from 2012-12-13 to 2013-12-30.*

Figure 19: *mVaR 99% for GARCH and SRSM with Student's t-distribution from 2012-12-13 to 2013-12-30.*

Figure 20: *mVaR 99% for rolling window from 2012-12-13 to 2013-12-30.*

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Figure 21: *VaR 95% for GARCH and SRSM with normal distribution from 2012-12-13 to 2013-12-30.*

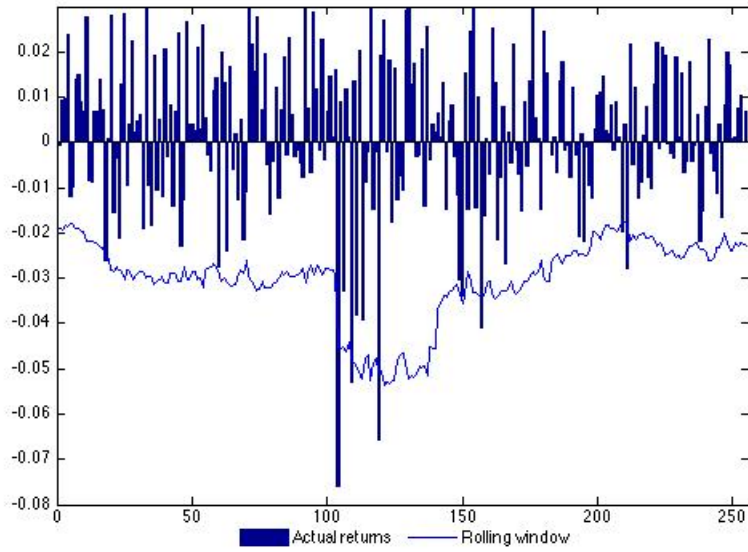Figure 22: *VaR 95% for GARCH and SRSM with Student's t-distribution from 2012-12-13 to 2013-12-30.*

Figure 23: *VaR 95% for rolling window from 2012-12-13 to 2013-12-30.*

Figure 24: *VaR 99% for GARCH and SRSM with normal distribution from 2012-12-13 to 2013-12-30.*

Figure 25: *VaR 99% for GARCH and SRSM with Student's t-distribution from 2012-12-13 to 2013-12-30.*

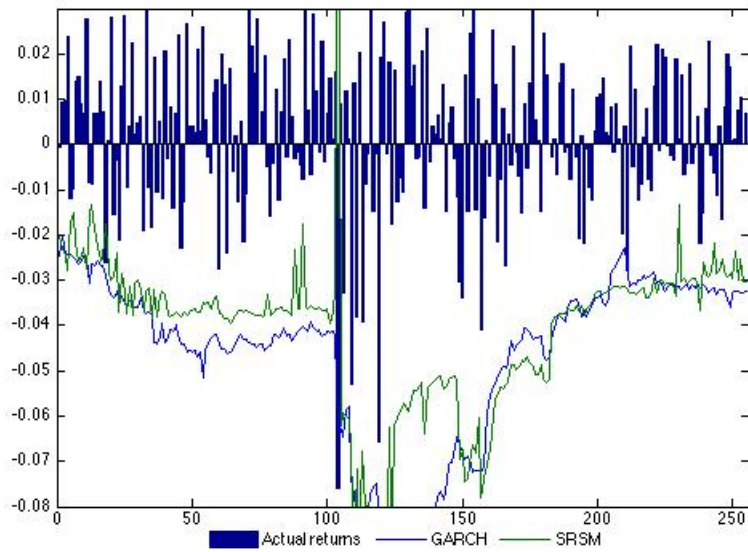Figure 26: *VaR 99% for rolling window from 2012-12-13 to 2013-12-30.*

Figure 27: *mVaR 95% for GARCH and SRSM with normal distribution from 2012-12-13 to 2013-12-30.*

Figure 28: *mVaR 95% for GARCH and SRSM with Student's t-distribution from 2012-12-13 to 2013-12-30.*

Figure 29: *mVaR 95% for rolling window from 2012-12-13 to 2013-12-30.*

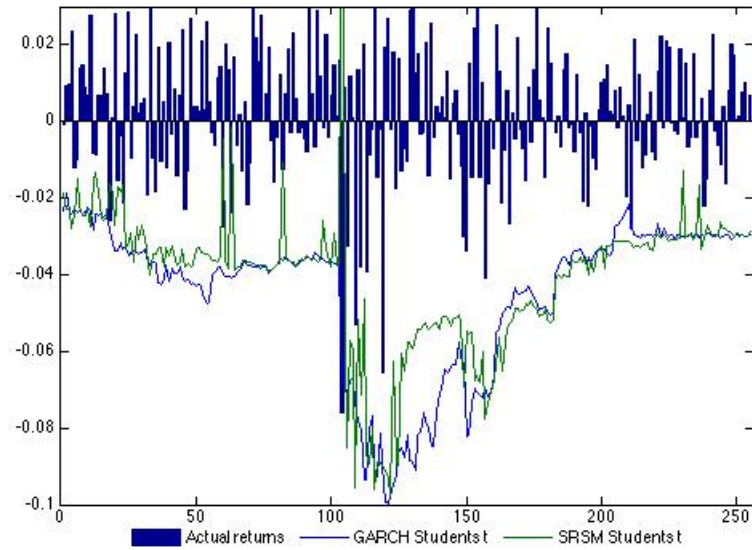Figure 30: *mVaR 99% for GARCH and SRSM with normal distribution from 2012-12-13 to 2013-12-30.*

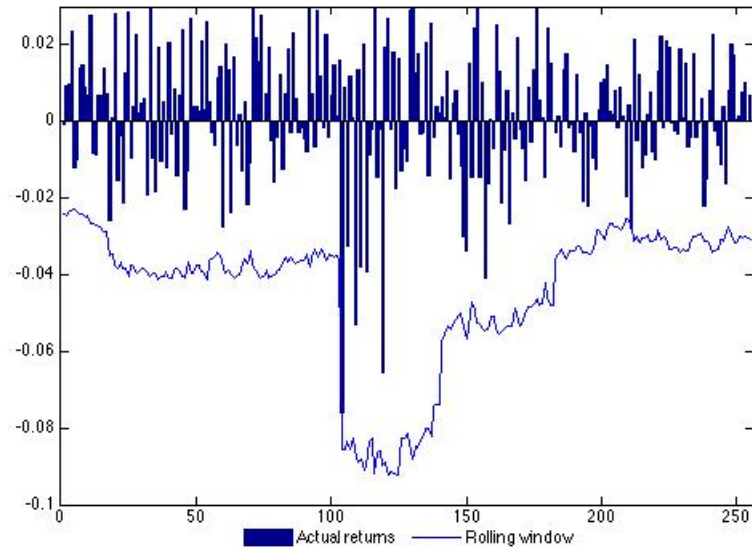Figure 31: *mVaR 99% for GARCH and SRSM with Student's t-distribution from 2012-12-13 to 2013-12-30.*

Figure 32: *mVaR 99% for rolling window from 2012-12-13 to 2013-12-30.*