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Individual Technologies for Health

- the implications of distinguishing between the ability to produce health investments and the capacity to benefit from those investments

Kristian Bolin, Bengt Liljas, Björn Lindgren

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INDIVIDUAL TECHNOLOGIES FOR HEALTH

– the implications of distinguishing between the ability to produce health investments and the capacity to benefit from those investments

KRISTIAN BOLIN[*],[1-2] BENGT LILJAS,[3] AND BJÖRN LINDGREN[2,4,5]

[1] Department of Economics, University of Gothenburg, Gothenburg, Sweden

[2] Centre for Health Economics at the University of Gothenburg

[3] AstraZeneca, R&D Mölndal, Sweden

[4] Lund University, Lund, Sweden

[5] National Bureau of Economic Research, Cambridge MA, United States

ABSTRACT

People differ in their ability to produce health investments and in their capacity to benefit from such efforts. In this paper, we assume (1) that the individual's health-investment production function exhibits diminishing returns to scale and (2) that the individual's capacity to benefit from the investments is diminishing in the stock of health. Previous research has only shown the importance of the first assumption for the health-capital adjustment process. The simultaneous effects go well beyond those results, however. Thus, this paper provides an extended demand-for-health framework that distinguishes between individuals both by their capacities to benefit and by their abilities to produce, when transforming health efforts into health increments. The potential usefulness of this framework for health-policy purposes is demonstrated by solving a numerically specified version of the model, and computing individual welfare effects of medical-care goods changes.

Keywords: Investments in health; Diminishing returns; Capacity to benefit; Human capital; Grossman model.

JEL classification: I10, I12, J24.

* Address for correspondence: Kristian Bolin, Department of Economics, Gothenburg University, Box 640, 405 30 Gothenburg, Sweden. E-mail: Kristian.Bolin@economics.gu.se.

1. Introduction

An individual's efforts to transform time and other resources into health may conceptually be divided into two parts: (1) his or her ability to transform time and other resources into gross health investments and (2) his or her capacity to benefit from the investment, i.e., to transform the investment into changes in health status. Since the two processes may have quite different properties, it is an important conceptual distinction to make. So far, it does not seem to have been recognised or explored in the health-economics literature in general or within the demand-for-health (or individual-as-producer-of-health) theoretical framework in particular.

Moreover, in previous treatments, including Grossman's original model (Grossman, 1972a, b) and several later extensions (for instance, Dardanoni and Wagstaff, 1987, 1990; Selden, 1993; Chang, 1996; Liljas, 2000; Jacobson, 2000; Bolin et al., 2001a, 2002b; 2002c; Galama and Kapteyn, 2011; and Bolin and Lindgren, 2012), it was further assumed that the marginal cost of producing health capital from own time and other resources is constant. Exceptions are Liljas (1998), introducing a decreasing marginal cost function by assuming that the depreciation of health is negatively dependent on the stock of health capital, and Galama (2011), assuming decreasing returns to scale.

Ehrlich and Chuma (1990) claimed that the assumption of a linear homogenous health production process generally implies that no interior equilibrium would exist in the demand-for-health model and that a "bang-bang" equilibrium would follow instead.[1] In a continuous time model and under certain conditions, the "bang-bang" equilibrium means that the initial investment would be undertaken at an infinite rate in order to reach the

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[1] Technically, a "bang-bang" solution means that there are discontinuities in the control path. See, for instance, Chiang (1992), p.164.

demanded level of health immediately and that no investments would be conducted after that (in a discrete-time model the interpretation is analogous).[2] Thus, decreasing returns to scale would instead lead to gradual adjustment of actual to desired level of health capital. Grossman (2000) argued that the assumption of constant returns to scale was not too restrictive (a) since the focus of his model was not on the adjustment process and (b) since the assumption of constant returns to scale in the *production* of gross health investment did not impede the analysis of optimal amounts of health capital and health investment over the lifecycle – a unique optimum is assured by the marginal-benefit-of-health curve sloping downward.

However, it is not possible to reject the assumption of gradual adjustment based on existing empirical studies; see van Doorslaer (1987), Wagstaff (1993), Bolin et al. (2001b, 2002a), and Galama et al. (2012) for population-based studies; Bolin and Lindgren (2002) for a study on individuals, who suffer from asthma or chronic obstructive pulmonary disease; and Bolin et al (2005) for a study on overweight and obese individuals. So, the empirical evidence seems to be in favour of allowing decreasing returns to scale.

In our exploration of the importance of distinguishing between an individual's ability to produce gross health investments and the individual's capacity to benefit from those investments, we will allow for decreasing returns to scale for the production technology. In addition, introducing the concept of capacity to benefit, we will allow for the rate at which that unit of investment is transformed into health capital to differ between healthier and less healthy individuals.[3] The rationale for this is the following: even though

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[2] Ried (1998) noted that an infinite rate of investment is not consistent with equilibrium.

[3] Naturally, this is a simplifying abstraction, since some production of gross health investments depends on whether or not the individual is able to perform the actions required. For instance, jogging is not possible if the stock of health is too small. However, several measures that can be undertaken in order to enhance

those who suffer from a chronic illness may be unable to replicate their production process of gross health investments, their ability of transforming gross health investments into health capital may be greater due to their low levels of actual health. Sick individuals may obviously benefit more than healthy individuals from medication and healthcare, and further health improvements may be more difficult to obtain for individuals with already high levels of health.

Our model will be presented in the next section. Then, some general implications regarding steady state and dynamics of health and health investments will be drawn. After that, we will perform more detailed analyses of a health policy that targets health goods along specific solution curves, using numerical methods. The models by Grossman (1972b) and Ehrlich and Chuma (1990) serve as benchmarks. Conclusions and discussion end the paper.

2. The model

Our model extends the original demand-for-health model (Grossman, 1972a, b) both by allowing the health-investment production technology to exhibit decreasing returns to scale and by assuming that the capacity to benefit, i.e., the rate at which gross health investments will be assimilated into the stock of health depends negatively on the health stock.

Preferences

We consider a version of the demand-for-health model in which time is continuous and the individual is the ultimate producer of his or her health. The individual derives utility

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current health and make future good health more likely can be performed independently of the current health state; for instance, choices regarding smoking, drinking, and eating behaviour.

from health, H, and consumption, G. Preferences are represented by a quasi-linear and additive separable utility function, which is strictly concave in H ($u_{HH} < 0$):[4]

$$\boldsymbol{U}(H(t), G(t)) = U(H_t) + \gamma_G \cdot G(t), \tag{1}$$

where $\gamma_G > 0$ is a constant.

Ability to produce health investments

The individual's investments in the stock of health capital - *gross* investments, at time t, are denoted $I(t)$. For convenience, the technology is formally represented by a production function that is homogenous in the quantity of investment, of degree $\frac{1}{2} \leq \alpha \leq 1$. Thus, the dual cost-of-gross-investment function is homogenous of degree $1 \leq \alpha \leq 2$ with respect to the quantity of gross health investment, i.e., the cost of producing an additional unit of gross health investments is increasing in the size of the investment. The rate at which the marginal cost increases is reflected by the parameter α, which is determined by the individual's ability to transform resources into gross investments in health.[5] The production technology, and other conditions for the production of gross health investments, implies a cost-of-gross-investment function, which is increasing and convex in the quantity of investment. Formally, the cost of gross investment is:

$$\varphi(I(t)) = \pi(p, w) \cdot I(t)^\alpha. \tag{2}$$

where $\pi(p, w)$ is the one-unit cost of gross health investment, p is the composite price of market goods and services used in the production of gross health investments (health

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goods), and w is the wage rate. The one-unit cost function fulfils the usual cost-function properties, in particular, concavity in input prices. Technically, the parameter α reflects the efficiency of the production technology with respect to the instantaneous quantity of gross health investment.

Capacity to benefit from health investments

The individual is assumed to have a diminishing capacity to benefit from health investments – the larger the actual stock of health capital the smaller the rate at which gross health investments are transformed into health capital. The rate at which a certain amount of gross health investment is transformed into health capital – the capacity to benefit – is given by the function $\theta(H(t); \beta)$, which is strictly decreasing in $(H(t))$. The parameter β reflects the rate at which the capacity to benefit diminishes as the stock of health increases. Strict convexity of $\theta(H(t); \beta)$ would imply that there is no upper limit on the health stock, whereas concavity would imply the existence of such a limit (perfect health). In both cases, the size of the gross investment needed in order to add further to the health stock increases as the health stock grows. When $\theta(H(t); \beta)$ is strictly convex, the required investment grows at a decreasing rate. More specifically, in the convex case we have: $\lim_{H(t) \to 0} \theta(H(t); \beta) = 1$, and $\lim_{H(t) \to \infty} \theta(H(t); \beta) = 0$. Similarly, in the concave case, in which the investment required in order to add a unit to the health stock grows at an increasing rate, we have: $\theta(0; \beta) = 1$, and $\theta(\widehat{H}; \beta) = 0$ ($\widehat{H}$ is perfect health). In both cases, the capacity to benefit is $0 < \theta \leq 1$, and the impact on health from the gross investment, I_t, is $\theta(H(t)) \cdot I_t$. Figure 1 shows the graph of the function

θ in the two cases. θ has been specified so that the domain $H(t) \in [0,5]$ yields the functions range in both cases.[6]

Figure 1 about here

The motion of health capital over time

Gross investments in health are partially offset by natural depreciation of the stock. In order to facilitate our analysis, we assume that depreciation of the stock of health capital occurs at each point in time at a time-independent rate, δ $(0 < \delta < 1)$.[7] The equation for the motion of the stock of health capital over time then becomes:

$$\frac{dH(t)}{dt} = \theta(H(t)) \cdot I(t) - \delta \cdot H(t). \tag{3}$$

Budget constraint

For simplicity, we assume that there are no financial markets, and that the price of the consumption good is 1.[8] Thus, income equals consumption and investment spending at each point in time:

$$G(t) + \varphi(I(t)) = y(H(t)), \tag{4}$$

where y is full market income, which is a function of wage and healthy time, which, in turn, is a function of health capital, $\tau(H(t))$, which is increasing and strictly concave in $H(t)$. Thus, potential market income is:

$$y(t) = w \cdot \tau(H(t)), \tag{5}$$

where w denotes the wage rate.

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[6] The convex case is specified as $\theta = e^{-\beta \cdot H(t)}$ and the concave case as $\theta = 1 - 0.04 \cdot \beta \cdot H(t)^2$. $\beta = 1$ in both cases.

[7] In the original formulation of the model (Grossman, 1972a,b) and in several later extensions and applications, for instance, Muurinen (1982), Wagstaff (1986), Liljas (1998), Jacobson (2000), and Bolin et al. (2001a), the depreciation rate is assumed to increase over the individual's lifetime. Since we are not interested in analysing the effects of ageing in this paper, we follow Forster (2001) in adopting the assumption of a time-independent depreciation of health.

[8] This follows in the tradition of Forster (2001), who also assumed that there are no financial markets.

The individual's control problem and optimality conditions

The intertemporal problem that faces the individual in our model is to choose the time path of the health stock so that his or her lifecycle utility is maximised, given that the (exogenous) planning horizon is T.[9] Future utility is discounted at the rate ρ and, hence, the individual acts as if solving the following optimization problem (omitting function arguments):

$$W = \max_{I(t),H(t)} \int_0^T e^{-\rho \cdot t} \cdot U\big(H(t), y(H(t)) - \varphi(I(t))\big)dt,$$

subject to:
$$\frac{dH(t)}{dt} = \theta(H(t)) \cdot I(t) - \delta \cdot H(t),$$

and the start value $H(0) = H_0$. We will consider the vertical-terminal line version of the problem, that is, the transversality condition is $(H(T) - H_{min}) \cdot \lambda(T) = 0$; $H_{min} = 0$.[10] The current value Hamilton function for the maximisation problem is:

$$\mathcal{H}(t) = U(H(t), G(t)) + \lambda(t) \cdot (\theta(H(t)) \cdot I(t) - \delta \cdot H(t)) \qquad (6)$$

The maximum principle provides sufficient conditions for the optimal control of $I(t)$, provided that the Hamiltonian is concave in $(I(t), H(t))$.[11] Concavity of the Hamiltonian cannot be guaranteed for all (permitted) specifications of the model, since $\theta(H(t)) \cdot I(t)$ is convex in $(I(t), H(t))$. Thus, for a specification of the model that involves a concave Hamiltonian, the following equation of motion describes the optimal time path of the shadow price of health capital (again, arguments are left out for brevity):

$$\frac{d\lambda}{dt} = -\mathcal{H}_H + \lambda \cdot \rho = -u_H - \gamma_G \cdot y_H - \lambda \cdot (\theta_H \cdot I - \rho - \delta) \qquad (7)$$

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[9] We formulate the individual's optimisation problem as a truncated vertical terminal line problem. This means that the terminal time, *T*, is fixed but the terminal state is free to vary above a certain permissible level. This is to be distinguished from a horizontal terminal line problem (see, for instance, Ehrlich and Chuma, 1990) in which the terminal state, *T*, is free and the terminal state is restricted (Chiang, 1992, p.182). In the latter case, optimal length of life is, implicitly, determined by the transversality conditions.

[10] See, for instance, Chiang (1992), p183.

[11] This follows from the Mangasarian sufficiency theorem; see, for instance, Seierstad and Sydsaeter (1987). We show, in the appendix, that the Hamiltonian will be concave for some parametrizations.

The first-order condition for the control variable, I, is:

$$\mathcal{H}_I = \lambda \cdot \theta + \gamma_G \cdot (-\varphi_I) = 0. \tag{8}$$

It is straightforward to derive the health-stock equilibrium condition, and the condition for optimal investments in health (flow condition). Rearranging (7) yields the health-stock equilibrium condition:

$$u_H + \gamma_c \cdot y_H = \left(\rho + \delta - \theta_H \cdot I - \frac{\frac{d\lambda}{dt}}{\lambda}\right) \cdot \lambda. \tag{9}$$

This corresponds to the equilibrium condition in Grossman's original demand-for-health model. The differences are (1) the term $\theta_H \cdot I$, which reflects the impact on the marginal cost of health capital induced by the diminishing capability of benefitting from gross health investments, and (2) the shadow price of health capital (λ). The absence of capital markets means that the inter-temporal allocation of resources is completely achieved through the health stock and, hence, that a time derivative of either the shadow price or the control variable ($I(t)$) appears in the stock condition. The equation system (3) and (7), where (8) has been used to substitute for the control variable, cannot in general be solved explicitly.

3. General implications

Diminishing productivity in the production of gross health investments, and in the capacity to benefit from those investments, influence both optimal time paths and steady-state levels of $I(t)$ and $H(t)$. The transversality condition, however, will be violated in many cases. Thus, steady-state levels may not be relevant in the sense of being part of a solution to the dynamic optimization problem. Nevertheless, the location of any steady state(-s) of the system is an essential part of determining the dynamics of the system, since an equilibrium is also a demarcation (or reference) point between different behaviours of the system dynamics.

We summarize our findings regarding the relationship between technology and capacity to benefit from gross health investments, respectively, and steady state, H^*, I^*, in the following claim:

Claim 1: (*i*) the health good price effect on steady state is: $\frac{dH^*}{dp} < 0; \frac{dI^*}{dp} < 0$

(*ii*) the technology effect on steady state is: $\frac{dH^*}{d\alpha} < 0; \frac{dI^*}{d\alpha} < 0;$

(*iii*) the capacity-to-benefit effect on steady state is: $\frac{dH^*}{d\beta} < 0; \frac{dI^*}{d\beta} <> 0.$

Proof: The proof is a straightforward comparative static computation. See the appendix.

A comment is called for: an increase in β will decrease the demand for health but also decrease the supply of health capital. Thus, a smaller capacity to benefit would mean a smaller steady-state health stock but not necessarily a smaller quantity of steady-state gross health investments.

Dynamics

The dynamics of the model outlined above is illustrated in the phase diagram (the *I-H* domain) below.[12] The model dynamics is given by the following equation system (see the appendix):

$$\frac{dH(t)}{dt} = \theta \cdot I(t) - \delta \cdot H(t) \equiv h(H(t), I(t)), \text{ and} \tag{10}$$

$$\frac{dI(t)}{dt} = \frac{1}{\gamma_c \cdot \varphi_{II}} \cdot \left[-(u_H + \gamma_G \cdot y_H) \cdot \theta + \gamma_G \cdot \varphi_I \cdot \left(\rho + \delta - \frac{\theta_H}{\theta} \cdot \delta \cdot H(t) \right) \right] \equiv g(H(t)). \tag{11}$$

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[12] State-phase analysis has been widely applied within economics, in order to illustrate the properties of dynamic theoretical models; see, for instance, Caputo (2005) for several examples. Within health economics this technique has been applied less often, though. Forster (2001) illustrated the qualitative properties of a theoretical model, taking adverse and beneficial consumption-induced health effects into account, using state-phase analysis.

The shape of the null clines for the state and control variable are given by equations (10) and (11), above. Let, $f(H_t) \equiv -(u_H + \gamma_G \cdot y_H) \cdot \theta + \gamma_G \cdot \varphi_I \cdot \left(\rho + \delta - \frac{\theta_H}{\theta} \cdot \delta \cdot H(t) \right)$.

Using $h(H(t), I(t))$ and $f(H(t))$, we show that the health locus has a negative slope and the investment loci have positive slopes, if $\theta_{HH} > 0$. Adding the assumption that all third-order derivatives are 0 assures that both loci are convex. A steady state, H^*, I^*, is defined by the equation system $[h(H^*, I^*) = 0, f(H^*) = 0]$, and is saddle-point stable (see the appendix). It follows from (11), that in steady state, it is necessarily true that $\left(\rho + \delta - \frac{\theta_H}{\theta} \cdot \delta \cdot H^* \right) > 0$, which implies (assuming $\rho = 0$) that any health stock for which the capacity-to-benefit elasticity with respect to health is larger than -1, i.e., $\theta_H \cdot \frac{H}{\theta} > -1$, cannot be a steady state. For purpose of illustration, we will assume that this condition is fulfilled, that a steady state exists for permissible combinations of control and state variables, and that the IH-plane is divided into four similarly sized quadrants.

The direction of the system in each of these quadrants can be found by computing the derivatives h_H and g_H (see the appendix), and using the fact that they are 0 at their respective null cline.

Figure 2 about here

The qualitative properties of the model are illustrated in Figure 2, in which individuals start at $t = 0$ with the initial health stock, H_0, or H_0', in one of four possible regions. Each region is characterized by the direction of the system in IH space. The transversality condition $(H(T) - H_{min}) \cdot \lambda(T) = 0$ is satisfied either for $H(T) = H_{min}$

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or for $\lambda(T) = 0$. We will proceed by assuming $\lambda(T) = 0$ and checking whether or not the resulting $H(T)$ satisfies $H(T) \geq H_{min}$. If it does, we have found a solution; if not, we have to solve the fixed terminal point problem given by the condition $H(T) = H_{min}$. We have marked four particular streamlines, **A − D**, which are candidates for an optimal solution. All four are consistent with the maximum principle, but only two of them are solutions to our problem − **A** and **B** violates the transversality condition which dictates that $I(T) = 0$.

The welfare impact of a change (at $t = 0$) in the price of health goods and services, brought about by altered lifecycle choices, is given by dynamic envelope results (Caputo, 2005; Seierstad and Sydsaeter, 1987). Here, we will calculate (the present value of) the change more directly as:

$$W = W(p_1) − W(p_0) = \int_0^T e^{-\rho t} \left[U\left(\hat{H}(t; p_1), \hat{C}(t; p_1) \right) − U(\hat{H}(t; p_0), \hat{C}(t; p_0)) \right] dt,$$

where $\hat{H}(t)$ and $\hat{G}(t)$ are the optimal time paths of health and consumption. Obviously, not only the direction but also the magnitude of the changes induced by a policy matter for the prospect for constructing efficient health policies as well as for beneficial individual welfare effects. Individual health-production technologies and capacities to benefit from health investments (partly) determine individual responses to and welfare outcomes of exogenous changes.

4. Numerical computation

In order to quantify the effects, we need to employ a specific specification and parameterization of our model. We proceed by (1) specifying the model, (2) solving the model and illustrating the solutions for two specific parameterizations, (3) solving for

steady states corresponding to a range of parameterizations, and (4) performing a policy experiment. [13]

Specification

The following functional specifications and parameter choices were made:

(1) Preferences: $U(H(t), C(t)) = \left(1 - e^{-\gamma_u \cdot H(t)}\right) + \gamma_G \cdot G(t)$, where γ_u and γ_G are (strictly) positive constants;

(2) Ability to produce health investment (technology): $\varphi(I(t)) = \pi(p, w) \cdot I(t)^\alpha$;

(3) Capacity to benefit from health investment: $\theta(H(t)) = e^{-\beta \cdot H(t)}$;

(4) Budget constraint: $\tau(H(t)) = \Omega - \Omega \cdot e^{-\gamma_\tau \cdot H(t)}$ (Ω is total available time), which means that total market income is: $y(H(t)) = w \cdot \left(\Omega - \Omega \cdot e^{-\gamma_\tau \cdot H(t)}\right)$.

The individual's control problem and optimality conditions revisited

The current value Hamilton function for the maximisation problem of the specific model is (arguments are left out):

$$\mathcal{H} = (1 - e^{-\gamma_u \cdot H}) + \gamma_G \cdot (w \cdot (\Omega - \Omega \cdot e^{-\gamma_\tau \cdot H}) - \pi \cdot I^\alpha) + \lambda \cdot \left(e^{-\beta \cdot H} \cdot I - \delta \cdot H\right)$$

Concavity of the Hamiltonian depends on the relative size of the parameters entering the function. In particular, β determines the size of the cross-derivative term in the Hessian matrix of the Hamilton function and, hence, a small enough β guarantees that $\mathcal{H}$ is concave in (I, H). Concavity of $\mathcal{H}$ was checked for all the parameterizations that were used for numerical analysis.

The following equation of motion of the shadow price of health capital results from the specified Hamiltonian (again, arguments are left out for brevity):

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[13] All calculations were performed using Maple17.

$$\frac{d\lambda_t}{dt} = -\gamma_u \cdot e^{-\gamma_u \cdot H_t} - \gamma_G \cdot w \cdot \Omega \cdot \gamma_\tau \cdot e^{-\gamma_\tau \cdot H_t} - \lambda_t \cdot \left(\beta \cdot e^{-\beta \cdot H_t} \cdot I_t - \rho - \delta\right).$$

The first-order condition for the control variable, I_t, is:

$$\mathcal{H}_I = \lambda_t \cdot e^{-\beta \cdot H_t} - \gamma_G \cdot \alpha \cdot \pi \cdot I_t^{\alpha-1} = 0.$$

Estimation method

All estimations were carried out using the software Maple 17. The model is solved using the Trapezoid method, which is a finite difference method (Judd, 1998). Essentially, this means that the time interval is partitioned into a finite number of subintervals, and that the differential equations are used to approximate the change in the values of the unknown variables associated with each such subinterval. In more detail, the method comprises three steps:

(1) partitioning of the planning interval, $[0, T]$, into a finite sequence of equally sized subintervals, $[0, l, 2l, \dots, nl]$, by defining the mesh as $l = \frac{T}{n}$;

(2) defining the difference equations for the movement of the state and co-state variables (in line with previous denotation, $\frac{dH}{dt}$ and $\mathcal{H}_{H_t}$ denote the equation of motion for the stock of health and the partial derivative of the Hamiltonian with respect to the health stock at time t, respectively): $H_t - H_{t-1} = \frac{l}{2} \cdot \left(\frac{dH}{dt}_t - \frac{dH}{dt}_{t-1}\right)$, and $\lambda_t - \lambda_{t-1} = -\frac{l}{2}\left(\mathcal{H}_{H_t} - \mathcal{H}_{H_{t-1}}\right)$; and

(3) choosing the production of gross health investments so as to maximize the Hamiltonian, i.e., choosing I_t so that $I_t = \text{argmax}_I \, \mathcal{H}$. Adding our knowledge about the initial health stock, $H_0 = H^0$, and the requirement that the co-state variable shall be 0 at $t = T$, i.e., $\lambda_T = 0$, sufficient information for the problem to be solvable is available. In all calculations we set $l = 0.005882$ and $n = 8500$.

The following base-case values were assigned to the parameters of the specified model: $\gamma u = 2$; $\gamma_H = 2$; $\gamma_G = 0.001$; $\gamma_\tau = 2.5$; $w = 1$; $\Omega = 365$; $\pi = 10$; $\delta = 0.05$; $\rho = 0.01$. For these values, the Hamiltonian is concave in (H, I) at all $t \in [0,50]$. The analysis focused different technologies for the production of health investments and capacities to benefit from those investments, parameterized by different combinations of $\alpha \in [1,2]$ and $\beta \in [0,2]$.

Figure 3 about here

Figure 4 about here

Illustration of solutions for two different individuals

In Figures 3 and 4, we illustrate the solutions paths for two different individuals – both characterized by the base-case values (above), the starting value of health capital, $H_0 = 5$, and individual-specific technology and capacity to benefit given by (1) $\alpha = 2$; $\beta = 2$ and (2) $\alpha = 1.1$; $\beta = 0$. The figures illustrate optimal time paths of health capital and gross health investments for these cases. The combination of an inefficient technology and diminishing capacity to benefit from health investments induce the individual to postpone his or her investments until health has fallen below a certain level. Without the diminishing capacity to benefit there would have been a more pronounced smoothing-out of investments over the lifecycle (Erlich and Chuma, 1990).

Figure 5 about here

Steady state

Figure 5 illustrates possible steady states as a function of the capacity to benefit from health investment for two different health technologies ($\alpha = 1.1$ and $\alpha = 2$). The more efficient technology implies larger health stocks and more health investments. The specification that we used predicts that a weaker capacity to benefit from health investments leads to less health capital and more gross health investments. Moreover, the model predicts that the increase in the demand for gross health investments grow more rapidly for the relatively efficient technology as the capacity to benefit decline.

Figure 6 about here

Figure 7 about here

Policy experiments

We calculated the p-elasticities of W, $\frac{\Delta W}{\Delta p} \cdot \frac{p}{W}$, associated with different technologies and capacities to benefit (α, β). The results of the computations are reported in Figures 6 and 7. In Figure 6, we report the calculated elasticities as a function of α, for three different β values; in Figure 7, we report the calculated elasticities as a function of β, for three different α values. The results in Figure 6 and 7 show that the welfare impact of a price change is considerably larger in the case of a poor capacity to benefit from health investments (high β) compared to when the capacity is stronger. Similarly, when investment are produced at a fast-increasing marginal cost (high α), the welfare effect of a price change is larger than in the case of less fast increasing marginal cost. The impact of the capacity to benefit is larger than the marginal-cost effect.

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4. Discussion

In this paper we extended the demand-for-health model by making a conceptual distinction between (1) an individual's ability to transform time and other resources into gross health investments and (2) his or her capacity to benefit from the investment, i.e., to transform the investment into an increase in health. We examined the case, in which the first part depends on the size of the investment (decreasing returns to scale) and the second part depends on the actual stock of health (diminishing capacity to benefit). Analytical results concerning the effect of these two features on steady-state levels of health and health investments were derived. We showed: (1) that people will hold less health capital, ceteris paribus, compared to what would be the case (a) in a pure Grossman setting (constant returns to scale and capability to benefit) or (b) in a setting in which the individual produces gross health investments using a decreasing returns-to-scale technology but is not subject to diminishing capacity to benefit (Ehrlich and Chuma setting) and (2) that a steady state is saddle-point stable for certain technologies and capabilities to benefit from health investments (a sufficient condition for this to be the case is that $1 \leq \alpha \leq 2$ and that $\theta(H)$ is concave or has the form $\theta(H) = e^{-\beta \cdot H}$, $\beta > 0$).

Moreover, the introduction of a capability to benefit from gross health investments that diminish with the size of the health stock has implications for the adjustment of the health stock from the actual to the demanded amount. Since the effective price of health capital increases in the health stock, individuals who are not in steady state will face a price of health capital that varies over time due to the changes in the stock itself. More specifically, an individual who is on a health-increasing path will face a higher future price

of health capital due to the capability-to-benefit effect; the opposite is true if that individual is on a health-decreasing time path. These incentives are akin to the incentives created by market-price inflation and deflation, which also create incentives for reallocation of economic activities over time. The incentives created by the diminishing capability to benefit are weakened by decreasing returns to scale in the production of gross health investments: when health is on a declining path, the incentive created by the diminishing capability to benefit to postpone investments is balanced by the increase in production cost of doing so (due to decreasing returns to scale). An analogous argument can be made in the case of an increasing health stock.

The analysis of how technology and capacity influences individual responses to a health policy that lowers the price of health-care goods was performed using numerical analysis. We found that individuals that faces a capacity to benefit from health investments that diminishes rapidly are more sensitive to changes in exogenous prices than individuals with a capacity that diminishes more slowly, ceteris paribus. Moreover, the effect of increasing marginal cost of gross health investments was found to be similar: more rapidly increasing marginal costs corresponds to larger responses to a change in exogenous prices. The technology effect was found less important than the diminishing capacity effect, however. This suggests that the largest total willingness to pay for health policies that target prices of health goods and services is found, when an individual is characterised by rapidly decreasing returns in the production of gross health investments and a rapidly diminishing capacity to benefit from these investments.

APPENDIX

In the appendix, the time argument (t) is omitted where convenient.

Concavity of the Hamiltonian

The components of the Hessian matrix of the Hamiltonian are:

$\mathcal{H}_H = u_H + \gamma_G \cdot y_H + \lambda(t) \cdot (\theta_H \cdot I(t) - \delta);$

$\mathcal{H}_{HH} = u_{HH} + \gamma_G \cdot y_{HH} + \lambda(t) \cdot (\theta_{HH} \cdot I(t)) < 0,$ if $\theta_{HH} < 0;$

$\mathcal{H}_I = -\gamma_G \cdot \varphi_I + \lambda(t) \cdot \theta;$

$\mathcal{H}_{II} = -\gamma_G \cdot \varphi_{II} < 0;$ and

$\mathcal{H}_{IH} = \mathcal{H}_{HI} = \lambda(t) \cdot \theta_H < 0.$

The determinant of the Hessian matrix is: $\mathcal{H}_{HH} \cdot \mathcal{H}_{II} - \mathcal{H}_{IH} \cdot \mathcal{H}_{HI}$, which is > 0, if $\theta_{HH} < 0$, and $\gamma_G \cdot \varphi_I < 1$ (use $\mathcal{H}_I = 0$) which proves that the Hamilton function is strictly and jointly concave in the control and state variables in this case.

Steady state

The qualitative properties of the steady-state loci resulting are depicted by implicit differentiation of equations (10) and (11). The slope of both loci are determined by the assumptions made so far – the health loci is upward sloping (in the *I-H* domain) and the investment loci is downward sloping. This means that there can be only one steady state – a saddle point (this is shown below). The curvature of each loci is determined by the properties of the capacity-to-benefit function, $\theta(H(t))$. The health loci is convex if $\theta(H(t))$ is either concave or has the exponential form $\theta(H_t) = e^{-\beta \cdot H}$. The investment locus has no definite curvature (given the model specification).

The dynamics of health is expressed by equation (3): $\frac{dH(t)}{dt} = \theta \cdot I(t) - \delta \cdot H(t) = h(H(t), I(t))$. As regards the dynamics of gross health investment, begin by taking the time derivative of (8), which yields: $g(H(t)) = -\frac{\frac{d\lambda}{dt} \cdot \theta + (\lambda(\cdot \theta_H) \cdot \frac{dH}{dt}}{\gamma_G \cdot \varphi_{II}} \equiv -\frac{f(H(t))}{\gamma_G \cdot \varphi_{II}} = \frac{dI}{dt}$. Substitute for $\frac{d\lambda}{dt}, \lambda(t)$, and $\frac{dH}{dt}$, using (7), (8) and (3), respectively, gives the following expression for $f(H(t))$:

$f(H(t)) = -(u_H + \gamma_G \cdot y_H) \cdot \theta + \gamma_G \cdot \varphi_I \cdot \left(\rho + \delta - \frac{\theta_H}{\theta} \cdot \delta \cdot H\right).$

Setting $h(H(t), I(t)) = 0$ and $f(H(t), I(t)) = 0$ yields the steady-state loci.

Partial derivatives of $f(H(t), I(t))$

The partial-derivate components of f, which will be used below, are:

$f_I = v_c \cdot \varphi_{II} \cdot \left(\rho + \delta - \frac{\theta_H}{\theta} \cdot \delta \cdot H\right) > 0$, since $\left(\rho + \delta - \frac{\theta_H}{\theta} \cdot \delta \cdot H\right) > 0$ when $f = 0$.

$f_{II} = v_c \cdot \varphi_{III} \cdot \left(\rho + \delta - \frac{\theta_H}{\theta} \cdot \delta \cdot H\right) < 0$.

$f_{IH} = v_c \cdot \varphi_I \cdot \left(\frac{\theta_H \cdot \theta_H}{\theta^2} \cdot \delta \cdot H - \left(\frac{\theta_{HH}}{\theta} \cdot H + \frac{\theta_H}{\theta}\right) \cdot \delta\right) > 0$ if $\theta_{HH} < 0$, or if $\theta(H)$ has the

form: $\theta(H) = e^{-\beta \cdot H}$.

$f_H = -(u_H + \gamma_G \cdot y_H) \cdot \theta_H - (u_{HH} + \gamma_G \cdot y_{HH}) \cdot \theta + \gamma_G \cdot \varphi_I \cdot \left(\frac{\theta_H \cdot \theta_H}{\theta^2} \cdot \delta \cdot H - \left(\frac{\theta_{HH}}{\theta} \cdot H + \frac{\theta_H}{\theta}\right) \cdot \delta\right)$,

which is > 0 if $\theta_{HH} < 0$, or if $\theta(H)$ has the form: $\theta(H) = e^{-\beta \cdot H}$.

$f_{HH} = -2 \cdot (u_{HH} + \gamma_G \cdot y_{HH}) \cdot \theta_H - (u_H + \gamma_G \cdot y_H) \cdot \theta_{HH} - (u_{HHH} + \gamma_G \cdot y_{HHH}) \cdot \theta$

$+ \left(-2\frac{\theta_H \cdot \theta_H \cdot \theta_H}{\theta^3} \cdot \delta \cdot H + \frac{\theta_H \cdot \theta_H}{\theta^2} \cdot \delta + \frac{2(\theta_H) \cdot \theta_{HH}}{\theta^2} \cdot \delta \cdot H - (\frac{\theta_{HHH}}{\theta} \cdot H + \frac{\theta_{HH}}{\theta} - \frac{\theta_{HH} \cdot \theta_H}{\theta^2} \cdot\right.$

$\left. H + \frac{\theta_{HH}}{\theta} - \frac{\theta_H \cdot \theta_H}{\theta^2}) \cdot \delta\right)$, which is < 0 if $\theta(H)$ has the form: $\theta(H) = e^{-\beta \cdot H}$, and all third

order derivatives are 0.

Slope and curvature of the steady-state loci

For the $\frac{dH}{dt} = 0$ −loci we have the following expression:

$I = \frac{\delta \cdot H}{\theta}$, which implies $\frac{dI}{dH} = \frac{\delta \cdot \theta - \delta \cdot H \cdot \theta_H}{\theta^2} > 0$. The curvature is given by: $\frac{d^2 I}{dH^2} =$

$\frac{(\delta \cdot \theta_H - \delta \cdot \theta_H - \delta \cdot H \cdot \theta_{HH}) \cdot \theta^2 - (\delta \cdot \theta - \delta \cdot H \cdot \theta_H) \cdot 2 \cdot \theta \cdot \theta_H}{\theta^4}$, which is > 0 if $\theta_{HH} < 0$, or if $\theta(H)$ has the

form: $\theta(H) = e^{-\beta \cdot H}$. In order to infer the shape of the $\frac{dI_t}{dt} = 0 -$ loci we make use of

the implicit function theorem (and the second equation in system 10): $\frac{dI}{dH} = -\frac{f_H}{f_I} < 0$ if

$\theta_{HH} < 0$, or if $\theta(H)$ has the form: $\theta(H) = e^{-\beta \cdot H}$. The curvature is given by: $\frac{d^2 I}{dH^2} =$

$-\frac{\left(f_{HH} + f_{HI} \cdot \frac{dI}{dH}\right) \cdot f_I - f_H \cdot \left(f_{IH} + f_{II} \cdot \frac{dI}{dH}\right)}{(f_I)^2}$, which is > 0 if $f_{HH} < 0$. In our formulation of the

model this cannot be assured without further assumptions. Thus, the $\frac{dI}{dt} = 0 -$ loci is

downward sloping and either convex or concave. In the EC formulation, the loci will be

convex, since $f_{HH} < 0$ in this case.

The determinant of the Jacobian matrix

It is possible to infer the type of equilibrium by analysing the determinant of the Jacobian matrix, and evaluating it at the steady state point. The Jacobian matrix equals: $J = \begin{pmatrix} h_H & h_I \\ g_H & g_I \end{pmatrix}$. The two components not already calculated are: $h_I = \theta > 0$, and $g_I = \frac{f_I \cdot (\gamma_G \cdot \varphi_{II}) - f \cdot (\gamma_G \cdot \varphi_{III})}{(\gamma_G \cdot \varphi_{II})^2} = \frac{f_I \cdot (\gamma_G \cdot \varphi_{II})}{(\gamma_G \cdot \varphi_{II})^2} > 0$. Thus, the sign of the determinant of the Jacobian matrix is negative ($\begin{vmatrix} - & + \\ + & + \end{vmatrix}$), which proves that a steady state is a saddle-point stable.

General model – Steady-state comparative statics w.r.t α, β, and p^x.

Differentiating the system (10) gives:

$\begin{pmatrix} h_H & h_I \\ f_H & f_I \end{pmatrix} \cdot \begin{pmatrix} \frac{dH}{d\beta} \\ \frac{dI}{d\beta} \end{pmatrix} = \begin{pmatrix} -h_\beta \\ -f_\beta \end{pmatrix}$; for the components of the Jacobian matrix, see above.

The Jacobian determinant is negative, i.e., $|J| = \begin{vmatrix} h_H & h_I \\ f_H & f_I \end{vmatrix} < 0$, if $\theta_{HH} < 0$, or if $\theta(H)$ has the form: $\theta(H) = e^{-\beta \cdot H}$.

Cramers rule gives: $\begin{pmatrix} \frac{dH}{d\beta} \\ \frac{dI}{d\beta} \end{pmatrix} = \frac{\begin{pmatrix} \begin{vmatrix} -h_\beta & h_I \\ -f_\beta & f_I \end{vmatrix} \\ \begin{vmatrix} h_H & -h_\beta \\ f_H & -f_\beta \end{vmatrix} \end{pmatrix}}{|J|}$, where

$f_\beta = -(u_H + \gamma_G \cdot y_H) \cdot \theta_\beta + \gamma_G \cdot \varphi_I \cdot \left(\frac{\theta_H \cdot \theta_\beta}{\theta^2} \cdot \delta \cdot H - \frac{\theta_{H\beta}}{\theta} \cdot \delta \cdot H \right) > 0$,

if $\theta(H)$ has the form: $\theta(H) = e^{-\beta \cdot H}$;

$h_H = \theta_H \cdot I - \delta < 0$,

$h_\beta = \theta_\beta \cdot I < 0$.

$f_H = -(u_H + \gamma_G \cdot y_H) \cdot \theta_H - (u_{HH} + \gamma_G \cdot y_{HH}) \cdot \theta + \gamma_G \cdot \varphi_I \cdot \left(\frac{\theta_H \cdot \theta_H}{\theta^2} \cdot \delta \cdot H - \left(\frac{\theta_{HH}}{\theta} \cdot H + \frac{\theta_H}{\theta} \right) \cdot \delta \right)$,

which is > 0 if $\theta_{HH} < 0$, or if $\theta(H)$ has the form: $\theta(H) = e^{-\beta \cdot H}$. This shows that:

$\frac{dH}{d\beta} < 0$ and $\frac{dI}{d\beta} > < 0$.

The corresponding calculations w.r.t α are straightforward:

$h_\alpha = 0$;

$f_\alpha = \left(\rho + \delta - \frac{\theta_H}{\theta} \cdot \delta \cdot H_t \right) \cdot (\gamma_c \cdot \pi \cdot I^{\alpha-1} + \gamma_c \cdot \alpha \cdot (\alpha - 1) \cdot \pi \cdot I^{\alpha-2}) > 0$.

This shows that that $\frac{dH}{d\alpha} < 0$ and $\frac{dI}{d\alpha} < 0$.

A change in p^x involves the following derivatives:

$$f_{p^x} = \gamma_c \cdot \alpha \cdot \left(\rho + \delta - \frac{\theta_H}{\theta} \cdot \delta \cdot H_t\right) \cdot I^{\alpha-1} \cdot \frac{d\pi}{d_{p^x}} > 0;$$

$$h_{p^x} = 0.$$

Thus, $\frac{dH}{d_{p^x}} < 0$ and $\frac{dI}{d_{p^x}} < 0$.

Dynamics

The direction of the integral curves are given by (arguments omitted):

$$h_H = \theta_H \cdot I - \delta < 0, \text{ and } g_H = \frac{f_H \cdot \gamma_G \cdot \varphi_{II} - f \cdot 0}{(\gamma_G \cdot \varphi_{II})^2} = \frac{f_H}{(\gamma_G \cdot \varphi_{II})^2} > 0 \text{ if } \theta_{HH} < 0, \text{ or if } \theta(H)$$

has the form: $\theta(H) = e^{-\beta \cdot H}$.

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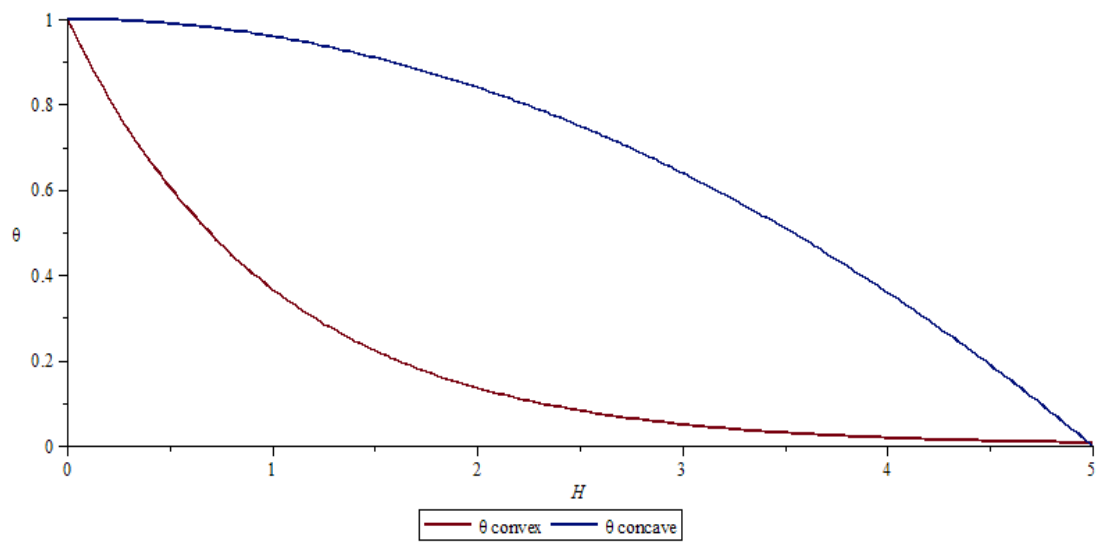Figure 1. The value of the θ function as a function of the health stock

Figure 2. State-phase diagram illustrating the direction of the system for each point in the H-I-plane, when the isoclines are convex, and the steady state is a saddle-point equilibrium. The vertical terminal line problem implies that $I_T = 0$. The two dashed-dotted curves illustrate possible solutions, while the two dashed curves illustrate paths that are not solutions to the problem at hand. The dashed-dotted-dotted line and curve are the isocline in the Ehrlich-Chuma case ($\beta = 0$).
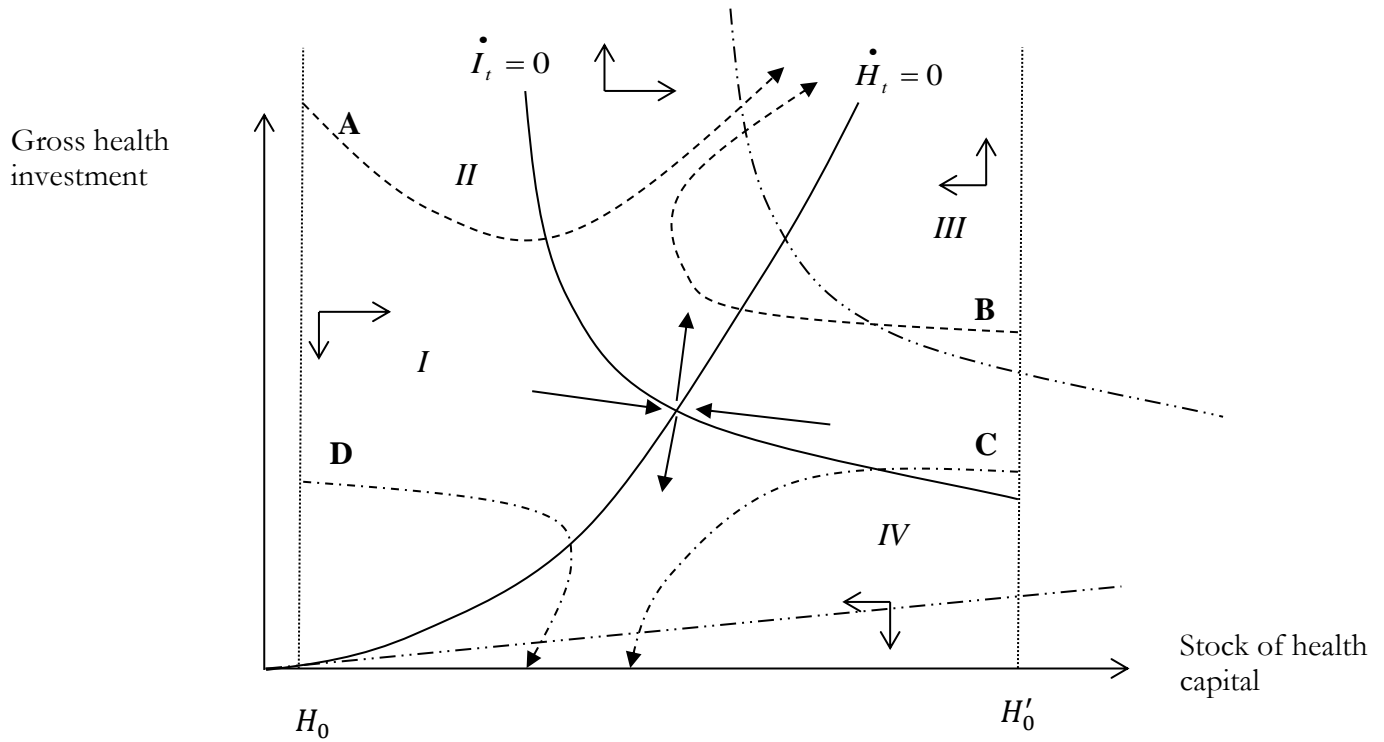
Figure 3. Time paths for health capital and health investments. The $\alpha = 2$ and $\beta = 2$ case
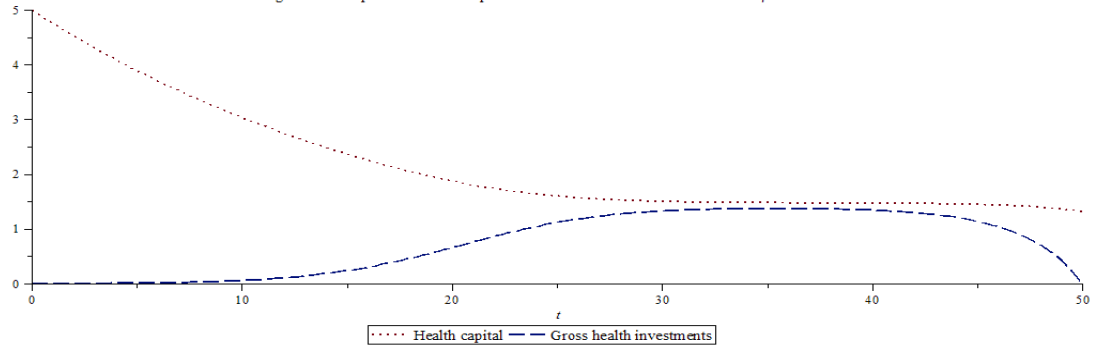
.... Health capital   — — Gross health investments

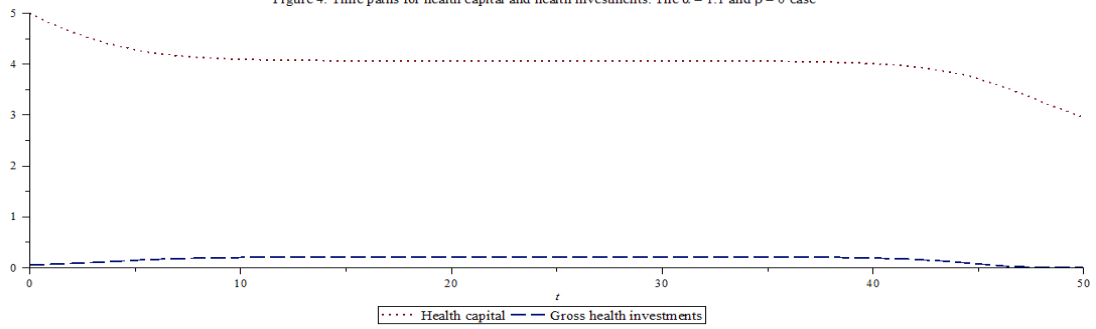Figure 4. Time paths for health capital and health investments. The $\alpha = 1.1$ and $\beta = 0$ case

.... Health capital   — — Gross health investments

Figure 5. Steady state amounts of health capital and health investments as a function of capacity to benefit
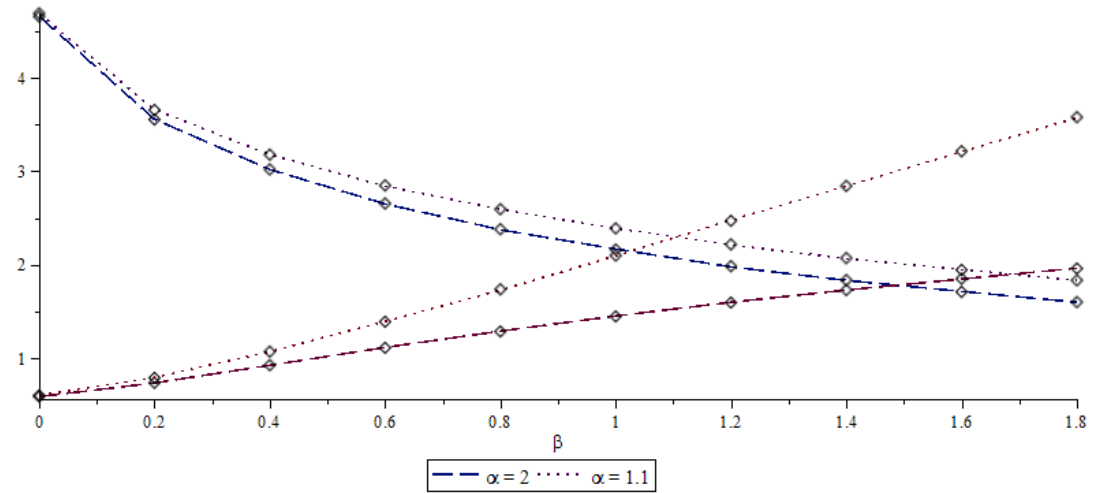

— — $\alpha = 2$ .... $\alpha = 1.1$

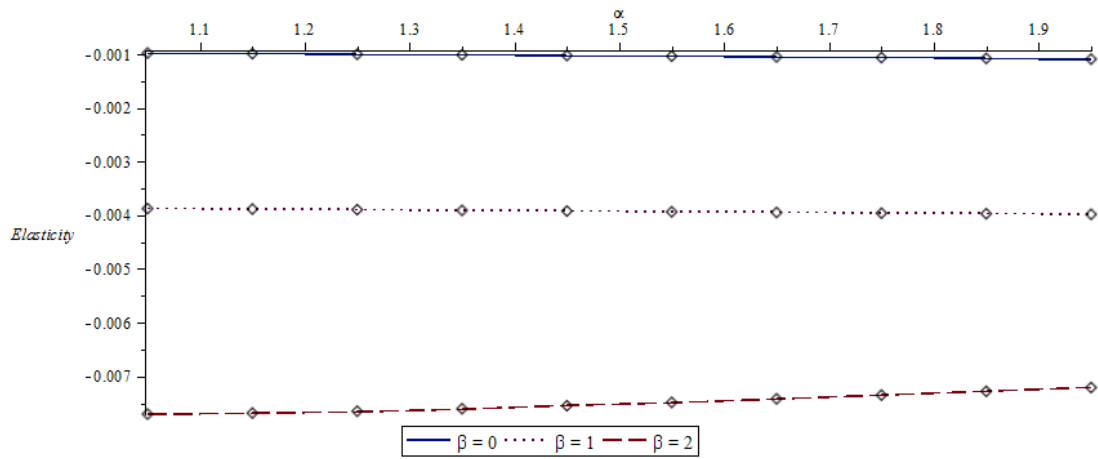Figure 6. Welfare elasticity as a function of technology, for 3 different capacities to benefit

Figure 7. Welfare elasticity as afunction of the capacity to benefit from health investments, for 3 different technologies