Mechanism, understanding and silent practice in the teaching of arithmetic. On the intention, critique and defense of Carl Alfred Nyström's *Digit-Arithmetic* 1853-1888

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Contents

Preface

This report was written as part of the project *Conceptions of mathematics in the Swedish world of schooling 1880-1980*, financed by The Swedish Research Council. It contains a presentation of historical material, pertaining to the teaching of elementary mathematics in Sweden in the last third of the 19th century, and a rather far-reaching and critical analysis of this material. Its length, in combination with its somewhat tentative character made it more suitable for publication in a report, rather than in a research article.

My story circles around the civil servant and physicist Carl Alfred Nyström (1831-1891). He worked mainly with telegraphy, a field in which he was a main contributor in Sweden. He was the manager of the educational department of the telegraph office 1873-1879 and then went on to manage the Stockholm telegraphy station a few years in the late 1880's. In 1881 he represented Sweden at the international committee in Paris for the formulation of proposals for international units in the field of electricity and electro-technics.

I am interested in Nyström because of his *Digit-Arithmetic* (*Sifferräkneläran*), which was one of the most popular textbooks for the teaching of arithmetic in Sweden in the second half of the 19th century.[1] As stated in the title of this report, I give an account of the intention, critique and defense (by Nyström himself) of this textbook. My explanation of *why* the *Digit-Arithmetic* was phased out from use in Swedish public education in the 1880's involves the topics of the first part of the title: mechanism, understanding, and a particular way of managing pupils, which at the time was called "silent practice" .

The story about Nyström and the fate of his educational ideas are interesting enough to be made available for a potentially

international readership. The numerous quotations, which I would have had to cut out, had I tried to make the text fit into the article format, will now make it possible for the reader also to interpret the episode in other ways than I have done here.

As regards my own analysis, it is a development and specification of themes that I have presented in my thesis, *The mathematics of schooling* (2008), which is in Swedish, and in two articles: "The missing piece" (2010) and "Hating School, Loving Mathematics" (2012).[2] In the present text I add some historical specificity to the otherwise quite abstract and theoretical points made in the two articles written in English. The argument should be read as work in progress however, and I cordially welcome critique and suggestions for improvement.

I want to use the opportunity here to describe, briefly and informally, how I understand mathematics education on a more general level, and how I conceive of the project to which this report should be seen as a contribution.

For many years now I have been interested in the emergence of modern mathematics education, in Sweden and as a world-wide phenomenon. I understand this process, not as the emergence of a particular way of solving the problem of how to teach children mathematics, but as an aspect of the formation of modernity. What interests me is the modern world-view that makes compulsory mathematics education seem like a worth-while project, despite its great costs in terms of time, frustration, suffering and, of course, money.

I take mathematics education to be intimately connected to a particularly modern conception of the relationship between humanity and nature, namely as partly, but centrally, mediated by mathematical knowledge. There are many images of human nature circulating in the modern world, pertaining to different spheres of society and life. But those images (or metaphors) related to science are among the most powerful. My contention is that mathematics education both derives its significance from, and contributes to the reproduction of, such a scientific imagery.[3]

In *The Theological Origins of Modernity* (2008), Michael Allen Gillespie discusses how the relationship between ”man, God and nature” shifted as we moved into our modern world-view. He suggests that:

> What actually occurs in the course of modernity is not simply the erasure or disappearance of God but the transference of his attributes, essential powers, and capacities to other entities or realms of being. The so called process of disenchantment is thus also a process of reenchantment in and through which both man and nature are infused with a number of attributes or powers previously ascribed to God.[4]

One such attribute of a modern human being is her ability to understand and master the world. Mathematics seems to lend us such divine powers. It plays a double role, of being ”out there”, as a fundamental structure of reality as described by science, and ”in here” as a capacity on our behalf of understanding and mastering this mathematical reality. In this sense it constitutes a link between the human realm and nature.

While the concern of science is to use the power of mathematics to understand and master the world, the concern of mathematics education is the establishment, in the individual human being, of that mathematical power. As I understand it, mathematics education has shouldered the burden of putting the power of mathematics at the disposal of all citizens. If the object

of science is the world, the objects of mathematics education are men and women, as potential masters of the world. The task of mathematics education is to put the human being in her proper place, to bestow on her one of the powers which, according to our modern world view, is properly hers.

The central metaphors of modern mathematics education are of building and growth. The individual is envisioned as actively and purposefully constructing her own knowledge or, alternatively, as the bearer of a growing plant of knowledge. Despite their differences, these metaphors have one important thing in common: they both suggest that mathematical learning takes time. Their image of learning is one of successive increments, each building on the previous ones, finally constituting a functional whole, the ”stability” of which is largely dependent on the ”foundation”.[5]

In this regard they can be fruitfully contrasted with another metaphor of knowledge – that of enlightenment. Contrary to building and growth, this metaphor suggests that knowledge can potentially emerge instantly, as light is ”turned on” at the moment of understanding. The metaphors of light focuses the potential effects of knowledge, what it makes visible, how it makes darkness disappear and can lead the way as a guide.[6] The notions of growth and construction, on the other hand, lend themselves primarily to reflection upon knowledge itself, viewed (as it were) from the outside. They highlight the conditions under which knowledge can develop, interdependence of different parts, the proper sequence of construction, the anatomy and architecture of knowledge, its structural strengths and weaknesses. While light is a *tool for* exploration, knowledge as organism or construction is rather an *object of* exploration.

My studies of the history of mathematics education have made me believe that the emergence of modern mathematics education proper coincided with the metaphors of growth and construction replacing other images of learning and knowing mathematics. I take this shift to be an indication of, firstly, that modern mathematics education is self-referential to a greater extent than earlier forms of mathematics education. By this, I mean that what modern mathematics education *is mainly about* is people taking part in the time-consuming practice of learning mathematics. In this report, Nyström function as an exponent of an earlier and in this regard non-modern form of mathematics education, because of his conception of learning in terms of irreversible insight. Secondly, the shift of metaphoric logic suggests that modern mathematics education is partly defined by its conception of the relationship between learning and time.[7]

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Why did the metaphors of growth and construction become so dominant in mathematics education at the turn of the 19th century? This has to do with the role of schooling in modern society as a place where children (for reasons having nothing to do with learning and knowing) need to stay for a long time and, while being there, need to do something that should preferably *make sense* according to modern standards of what counts as a meaningful activity for children.

The history of science and mathematics is relatively independent of that of education and schooling. And while mathematics (geometry and arithmetic) have always in some way been ”learned”, and geometry in particular has always been associated with more general virtues, I see modern mathematics education as a result of a mixing or merger, of this mathematical tradition with that of schooling. From this perspective, the episode discussed can be seen as a meeting between on the one

hand, a way of thinking about learning and knowing mathematics mostly deriving from the traditions of science, mathematics, engineering and arithmetic, focused on what it means to *have* mathematical knowledge, and on the other, a view of learning and knowing originating in Germany around the turn of the 18th century instead focused on the *formation* of mathematical knowledge.

A rather simple point of the argument presented in the report, is that Nyström's view of learning and knowing did not fit so well with what actually happened in the classroom – and with what had to happen because of the material circumstances of schooling at this particular time and place. His views and his textbooks therefore had to give way for conceptions of learning and knowing that had actually developed in the context of schooling, and that were therefore better suited to *make sense* of that context, i.e. a view of learning drawing on metaphors of formation and growth.

It is, however, important to be clear over what this aptness of the organic metaphors really entailed. It had little to do with success in relation to any particular goals of the learning process. No efforts were made to assess the relevance and usefulness of the result of the various teaching methods in circulation at the time. Not only was knowledge of the effects of schooling generally lacking, this lack was not even acknowledged as a problem. The focal point of the discussion was the activity of the classroom and its interpretation. While this was of course never made explicit, the strength and weaknesses of different conceptions of learning and knowing was decided by their aptness for *making sense* of the activity of the classroom. The criteria of evaluation were implicit in, and internal to, the world of schooling.

In the 1880's, the Swedish *Läroverk* was an elite institution with well-educated teachers and small and relatively homogeneous classes. It was in this setting that Nyström's books were initially put to use and appreciated. In the Swedish *Folkskola* on the other hand, which grew rapidly in the second half of the 19th century, the teachers often lacked education and the classes were big and heterogeneous. On one level, it is unproblematic and true that Nyström's teaching methods ran into difficulties because they were moved from the *Läroverk* to the *Folkskola*.

Following this line of thought one can say that his teaching methods were phased out because the centralized education system that started to emerge around the turn of the 19th century had more in common, on a practical and institutional level, with the previous *Folkskola* than with the previous *Läroverk*.

What this account risks to miss however is that differences in what one could perhaps call *techniques for the management of the classroom* are always reflected in assumptions regarding learning and knowing. The practical problem, viz., that Nyström's teaching method was infeasible on the level of management, administration, governance or technique, thus also made the metaphors of learning and knowing with which it was intertwined problematic. The point is that teaching methods must make sense, and a shift of methodology therefore always entails a shift in ways of sense making. Surprisingly, *a "well functioning" method may very well be one which invariably fails, but for which this failure makes complete sense.*

This is my interpretation of what the metaphors of growth and construction do for mathematics education. For reasons having nothing to do with learning and knowing, children needs to be in school for a long time. Understanding their activity as a slow process of learning, makes sense of they being there. In addition, the organic metaphors constitute the inner world of

children as a rich field of exploration – shifting focus from what the children can do with their knowledge outside school when they eventually "have it", to the process of growth by which this knowledge is understood to be established.

The conceptions of mathematics embedded in the world of schooling can thus be seen as dependent on the particular organization, structure and dynamics of that world. I will now say a few words about what happens when these conceptions are exported to the rest of modern society (where, of course, they have to coexists there with other conceptions, more apt to make sense of other spheres of society and life). My point is that, if I am correct, and now coming back to the thesis of Gillespie mentioned above, mathematics education contributes to the modern world view in a way that is more powerful than is presently acknowledged.

Mathematics education seems to follow from a given relationship between on the one hand the properties and needs of humans, and on the other, the properties of mathematics, society and nature. My suggestion is that our conception of this relationship is to a large extent produced by the materiality, structure and dynamics of mathematics education itself. This means that not only is mathematics education much more self-referential than it first seems.[8] Our modern *common sense* regarding what mathematics is and why it is important – notions which seem to have little to do with mathematics education and if anything is taken to *contrast* sharply with the realities of schooling – nonetheless, behind our backs, to a large extent *derives* from these realities themselves.

Mathematics education fills the crucial function of providing that particularly modern framework for interpretation of our relationship to nature that says that it is mediated by mathematical knowledge, adding that this relationship is not there beforehand when we are born, but that it needs to be *established,* by a process of education. Taking it one step further, this of course also makes it perfectly clear what it means to lack the knowledge that mathematics education is supposed to provide: it means to be in a state of disconnection from the modern world. Processes of *exclusion and stigmatization* can thus be interpreted as failed attempts of *inclusion and empowerment*.

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I would like now to say a few word about the relationship between the story told in this report and the movement of progressive education. The discussion between Nyström and his interlocutors takes place in Sweden in 1888, that is, some time before that movement. It seems to me however that the ”metaphoric logic”, so to speak, of the story I tell here is very much the same as that which came to stand at the center of the much more well known discussions a few decades later.[9] Looking at the history of mathematics education, it is also difficult to recognize any clear break resulting from the progressivist movement. What one can see is rather a strong continuity, at least if one follows the tradition, associated with the Swiss educator Johann Heinrich Pestalozzi, that developed in the German states in the 19th century.[10]

It thus seems to me that, in particular as regards mathematics education, the conception that the progressivist movement entailed a break with earlier ”traditional” and ”mechanical” teaching methods, to a large extent derives from the self-

congratulatory account of history produced by this movement itself. It severely misrepresents the relationship between progressivism and the educational discourse of the 19th century.[11]

My interpretation is instead that that the decades around the turn of the 19th century entailed a sort of loss of sense of continuity. While Nyström and his interlocutors contributed to an ongoing discussion, those taking up this discussion in the early 20th century seem to have felt that they should, and were entitled to, rebuild and rethink everything from scratch. Unsurprisingly they then ended up doing pretty much the same as the generations before them.

It was at this point that mathematics education became secular and scientific. In the 19th century, the organic metaphors were deployed in elementary mathematics education primarily in the then thoroughly Christian setting of the *Volksschule* in the German states. This is clear from the writing of Adolf Diesterweg, August Wilhelm Grube or any other of the prominent educators of this tradition.[12] If you compare them with Jean Piaget, who became the most important educational thinker of the 20th century, it becomes apparent their respective relationships to Christianity are strikingly different.

My point now, is that it is rather easy to recognize, in the mathematics education of the 20th century, under the scientific surface, a framework for interpretation, a set of metaphors, that had developed *as Christian.* While this is not so visible in the Swedish discussion of the 1880s, the organic metaphors were *introduced* into mathematics education (in the late 18th century) as part of a Christian framework of interpretation. Mathematics education thus lend itself to a thesis of ”secularization”,[13] and the Christian origins of progressive education has of course already been acknowledged.[14] What I hope to add to this, is some details pertaining more specifically to the history of mathematics education.

A hypothesis that I would be happy to see tested beyond my own work is that mathematics, as a school subject, functioned as a kind of bridge between the Christian framework of the 19th century *Volksschule* (in Germany, and later, in Sweden), and the scientific framework for the quite similar educational activates of the 20th. Because of the central role of mathematics in modern secular science, it was particularly easy to see mathematics as perhaps the modern secular school subject *par excellence*. It was forgotten how closely intertwined mathematics had been with Christian theology.[15]

My hypothesis is thus, crudely put, that the school subject of mathematics, in the early decades of the 20th century, functioned as a kind of entry point into the scientific world-view for educational doctrines which, just a few decades earlier, had found their significance in a Christian context. The risk is of course great here that I overstate my case because of the specific focus of my own investigations. Taking that risk, my hypothesis is that, for instance, the scientific theories of child development of the 20th century not only draw on an image of the child originating in the context of schooling – something which should be obvious – but that they also have an especially strong relation to mathematics education.[16] For Piaget, the formation of specifically *mathematical* concepts plays a central role for his general theory of development. His theory can thus be seen as a secular variant of the much older theme of mathematics as mediator between humankind and nature. The popularity of Piaget, in the mathematics education of the 20th century, derives then perhaps from the fact that his theories *fit* so well with what was to a large extent already there – both in theory and practice. His scientific account of what was useful and detrimental for

development was perfectly apt to *make sense* of what was already going on in the mathematics education classroom.[17]

As the attentive reader will surely notice, there is a significant gap between the theses I have just presented, and the material and argument of the actual report that now follows. Nonetheless it is perhaps possible to see the relevance of my story about Nyström for the more general project that I have outlined in this preface. However that may be, I hope that this report will invite critical and fruitful reflection upon the place of mathematics education in modern society.

I want to thank Paul Dowling and Marcelo Caruso for useful comments and for lending me confidence that this line of thinking regarding mathematics education is worthwhile. Sven-Eric Liedman was helpful in pointing out that further work would be needed to properly relate this story about Nyström to the contemporary educational context in Sweden. I hope to be able to pursue such work in the future. Until then, this preface will at least make it more clear what kind of questions the story about Nyström intends to answer. I want to thank Aant Elzinga both for correcting my English and for a number of suggestions that made this text significantly more readable. Finally, I want to thank Karin Sjöberg for helping me with layout and typography. I am of course alone responsible not only for remaining mistakes but also for the general contents of this report.

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Introduction

Around the turn of the 19th century a paradigm shift took place in Swedish elementary mathematics education. A new way of teaching, in which the textbook functioned as a guide for both teacher and pupils in a new and quite specific way, were introduced through curricula, teacher seminars and textbooks. This shift amounted to a general loss of agency. Not only teachers and pupils but also textbook authors had to give up their autonomy: Textbook authors had to follow curricula instead of their own ideas; teachers had to follow the textbook instead of using it according to their own principles; the activity of the pupils was organized as progression along a predetermined track.

This is not how the shift was interpreted at the time however. On the contrary, it was claimed that, as mechanical memorization was replaced with the development of concepts, the pupils would finally get access to the full power of mathematical knowledge. The idea, never made explicit, was that proper development of mathematical knowledge requires a period of initial submission; that freedom *in the end* requires submission *on the way*. But this end point of the learning process turned out to be more of a fiction than a reality. The main result of this period in the history of mathematics education, which continued into the 20th century, was a quite problematic combination of a practice, seemingly beyond the control of those participating in it, and a theory of learning and knowing setting an agenda which this practice invariably failed to make good on.

I have introduced the concept of *the standard critique* to describe a central feature of present day discourse on mathematics education.[18] It refers to the claim that, on the one hand, there is a great potential inherent in mathematical

knowledge, but on the other, that current practices of mathematics education generally fail to realize this potential. For example, it might on the one hand be claimed that "[to] think mathematically affords a powerful means to understand and control one's social and physical reality", but on the other that "despite some 12 or so years of compulsory mathematical education, most children in the developed world leave school with only a limited access to mathematical ideas".[19] Thus, mathematics education *could,* potentially, help children understand and control their social and physical reality, but *in fact*, it generally doesn't.

The rhetorical figure of the standard critique performs a temporalization of mathematics education, where the level of ideas, theories and abstract knowledge are conceived of as contemporary, but where teaching practices, on the other hand, are described as belonging to a traditional past, as not yet corresponding to present day thinking. In this way, the gap between the potential and the actual is explained in terms of a lag, resulting from inertia and resistance towards the new.[20]

This report is about is the shift, at the end of the 19th century, into this modern way of conceiving of mathematics education, the shift into seeing it as *always not quite there,* as the bearer of a great mission: To bestow on everybody the power of mathematical knowledge, but at the same time in the unfortunate position of lacking the means necessary to fulfill this mission. This shift thus entails two simultaneous and opposite kinds of emergence: On the one hand of a conception of universal potential in mathematical knowledge, great enough to make compulsory mathematics education seem an obvious necessity. On the other hand, however, an acknowledgement of impotence in the face of the task of realizing this potential.

My argument is based on a quite small material: the reception of an elementary arithmetic textbook – the *Digit-Arithmetic* (*Sifferräkneläran*) – by Carl Alfred Nyström (1831-1891), first published in 1853.[21] It received mostly praise initially, but towards the late 1870's the tone became increasingly more critical. In 1887 his book was finally dismissed as inappropriate for use in public education.

Why Nyström? Firstly, the *Digit-Arithmetic* was among the most used textbooks for the teaching of arithmetic in Sweden in the second half of the 19th century. It was in use well into the 20th century. Second, and more importantly, Nyström's views are interesting in the way they contrast with present day mathematics education. I will associate him with what I have chosen to call an *enlightenment paradigm* for the teaching of mathematics. This will be contrasted with the *paradigm of knowledge development* of his interlocutors, which also corresponds to the view of learning and knowing today associated with "progressive" education. I conceive of my presentation of the reception of Nyström's *Digit-Arithmetic* as an illustration of what was no less than a *clash* between these two paradigms, resulting, as we will see, in mutual incomprehension. Methodologically, I thus see the voices of Nyström and his interlocutors respectively as belonging to mutually different frameworks, characterized by fundamentally different views on what it means to learn and know something.

A contention of this article is that the shift in question, into the paradigm of knowledge development, in Swedish mathematics education principally took place in the decades around the turn of the 19th century. I think it is better to talk about a shift "into", rather than a transition between paradigms, because the paradigm of enlightenment was never hegemonic. What we had before, in Sweden, from about the middle of the century, was a discussion where textbook authors presented and

defended their own views. Thus, Nyström was, until the 1880's only one voice among others, belonging to the same "field" of methodological discussion. It was only from the 1880's onward that only one of these voices, the proponents of the paradigm of knowledge development, rose to hegemonic prominence.

A third reason for focusing on Nyström is that he actually participated in the decisive discussion in the 1880's. He was one of the last to oppose – as a textbook author and teacher, *from within* mathematics education – the consensus among educators, taking form at this time, regarding what it should mean to learn and know mathematics.

This report is based on my dissertation from 2008, *Skolans matematik (The Mathematics of Schooling)* and in particular its eleventh chapter, "The time of mathematics education". The major point of *Skolans matematik* is theoretical. While most of its pages concern history, these pages were not intended primarily as historical description, but as support for a theoretical argument. The same goes for the present report. While I find the material significant in its own right, as part of the under-studied history of mathematics education, the reader will soon notice that more is going on than mere description. Thus, I will not only see the discussion between Nyström and his interlocutors as an illustration of a more general shift in mathematics education. I will also try to explain *why* this particular shift happened at this particular point in time in Sweden, and why it manifested itself in the particular *expression* that appears in the material.

In *Skolans matematik* I was mostly concerned with the "object" of mathematics education. This is the point of the title: *The mathematics of schooling*, that "the mathematics" around which mathematics education circles, is "of schooling"; that it is *its own* in the sense that those properties of mathematics from which corresponding properties of the educational practices *seems to be* *derived*, can be seen to take form, historically, in the same processes as those practices themselves.[22] I thus argue that "mathematics" should be seen more as an *objective counterpart* to "schooling" – in Marxist terms, as a *reification* – than as its objective and natural foundation.

This report further develops this line of thought. I want to say something about the relationship between on the one hand *intentions* and *discourse* and on the other *practical necessities*. [23] My rather simple contention, the support of which is a principal purpose of this report, is that the reason why the paradigm of development became hegemonic at this particular point in time in Sweden, was that its proclaimed goals, as well as the proclaimed means to reach these goals, fitted with the new material circumstances that were taking form as the result of the establishment of a centralized system of compulsory schooling. I will thus try to show that Nyström was not defeated by "rational" arguments, but that it was rather the case that his vision of mathematics education lost its credibility because of its increasing distance to the actualities of mathematics education, its materiality. This is not to say that the paradigm of knowledge development was somehow *caused* by processes of expansion, centralization and standardization. In fact, these "progressivist" ideas were not particularly new at the end of the 19th century, originating as they did in late 18th century Germany. In Sweden they had been around since the beginning of the 19th century and they had had strong defenders in the methodological discussion from the 1850's onward. My point is rather that educational ideas founded on the metaphor of growth, could thrive in the emerging milieu of compulsory, standardized and centralized schooling in a way that ideas of enlightenment could not.

The intention

Carl Alfred Nyström was a civil servant and a physicist. He worked mainly with telegraphy, a field in which he was a main contributor in Sweden. He was the manager of the educational department of the telegraph office 1873-1879 and then went on to manage the Stockholm telegraphy station a few years in the late 1880's. In 1881 he represented Sweden at the international committee in Paris for the formulation of proposals for international units in the field of electricity and electro-technics.[24] These biographical notes indicate that Nyström was no mere school-teacher, but had quite extensive experience, not only of teaching mathematics, but also of activities where mathematics is used.

His *Digit-Arithmetic*, as it was called when it had become a standard reference, was first published in 1853 and became an immediate success.[25] With this book Nyström reacted against two prevalent trends in the teaching of elementary arithmetic: the method of analogies and the algebraic method.

Both the algebraic method and the method based on analogies served to solve problems of *the rule of three*. In its simple form, this rule shows how the fourth number can be found, if three are given and there is an assumption of proportionality. In its composite form, several numbers can be given, and a further one found.

Simplifying somewhat, the method of analogies has three steps. First the exact "kind" of the problem is identified. Arithmetic books explaining this method could contain well over thirty different such "kinds", for instance problems of: Fellowship (single and double), Interest, Commission, Buying and selling stocks, Insurance, Discount, Barter and Exchange.[26] In the second step, the given numbers are matched with a formula specific to this particular kind of problem. Then, finally, the computations are performed according to this formula.

In the preface to his 1853 *Digit-Arithmetic*, Nyström agrees with a then common critique of this method, namely that it more often than not leads the pupils to merely guess how to apply the algorithms, since they do not understand what they are doing. Another apparent problem with the method was that it left the pupils perplexed in situations where there was no formula with which to match the problem at hand. In short, the method could perhaps work very well if it was properly mastered, but it had turned out to work poorly when used as a standardized methodology for teaching.

The algebraic method was actually in itself the result of a strong reaction against the methodology based on analogies. This is clear from the preface of the textbook by which the method was introduced in Sweden: Jacob Otterström's *A tentative textbook in arithmetic* (*Försök till lärobok i aritmetiken*) published in 1849.[27] Otterström (1806-1892) derided the teaching methods of his time:

> […] something more unreasonable and more difficult than arithmetic, as it is generally presented in textbooks and by teachers, has never been constructed if not in astronomy before Copernicus. The results of this method is obvious everywhere from Ystad [in the south of Sweden] to Torneå [in the far north of Sweden], in the fact that very few of those who have learned arithmetic in school, can understand and answer the very simplest of arithmetical questions, beside the practical routine of work.[28]

Otterström sought a way of understanding and answering the questions of the rule of three founded on rational thinking, instead of thoughtless application of formulas. The algebraic method, by which the use of symbols such as ”x” was introduced

from the very beginning, was his solution. The idea was to teach children general techniques for the simplification of algebraic expressions, which could then invariably be used to solve all problems in arithmetic, including of course those of the rule of three.

Nyström did not think this would work. While he agreed that arithmetic may very well be *logically* seen as an application of algebra, from an *educational* perspective he considered this ordering of the subject matter untenable:

> For those, who have not beforehand, through work with numbers, gained some arithmetical insight, the general rules of algebra must appear as highly difficult to understand; and the study of digit-arithmetic must thence be significantly hindered, if not wholly deterred. If it is then further taken into account that the teaching of arithmetic in the schools, usually and for good reasons, starts at an early age, it should be clear that the difficulties with realizing this idea, which is the foundation of the so called *newer* or *algebraic* method, well surpass that significant gain which would doubtless ensue from its full and consequent realization.[29]

It was thus not the method *per se* that Nyström considered inadequate. He clearly saw its advantages! The problem was its infeasibility for the teaching of children. He thought that they needed to work with digits first, to be able to understand the abstract rules of algebra.[30]

Nyström's own method was called ”the method of reduction to the unit” (”enhetsmetoden”).[31] Nyström conceived of it as a *via media* between the mechanism of the method of analogies and the too high level of abstraction of the algebraic method. He wanted to give the ”digit-arithmetic” its own foundation, based on rational thinking rather than on memorization and mechanical computation. His method should be teachable to children and it should in the end lead to an ability to perform computations efficiently. The main point of the method was to reduce any problem of the rule of three to a sequence of divisions and multiplications. The operations were not performed immediately however, but written down to form an expression (somewhat in the style of the algebraic method), because:

> The divisor and multiplier may namely be of such natures, that either one of them is contained in the other or that they have common factors. When, in such cases, the computations are performed immediately, this often gives rise to the large and prolix numbers, something which can be avoided by way of writing down the arithmetical operations as a fraction, and then performing all possible reductions.[32]

Important to note here is that Nyström presents this as a *general* method, by which any problem of the rule of three can be solved. In his book, this is highlighted by that fact that the method is described under the following extensive heading:

> The practical use of the four arithmetic operations for the answering of questions pertaining to simple and composite rule of three, simple and composite interest, discount, company, allegation, payment terms, reduction and chain computations.[33]

In this, Nyström continues the ambition of the 18th century enlightenment to simplify and rationalize arithmetics which in Sweden goes back to the astronomer and textbook author Anders Celsius (1701-1744).[34]

The method of reduction to the unit was closely related to a didactical technique called ”the heuristic method”.[35] It is the method by which the pupil is led, by questions, to discover the solution to a problem ”by himself”. It ultimately derives from

Plato, who in his dialogue *Meno* lets Socrates, by means of clever questions, have a slave boy find, and at the same time supposedly understand, the proof of a theorem in geometry.[36]

Anticipating the later discussion and critique of Nyström's method, it is worth pointing out that Nyström was far from dogmatic in his support for the heuristic method. Its purpose was, for him, strictly limited to the achievement of understanding: it was the proper way to *introduce* the method of reduction to the unit. Then, work on exercises, guided by the textbook, should follow. Already in the first edition of his book, Nyström pointed out that the value mechanical proficiency should not be disregarded as a significant goal in the teaching of arithmetic.[37]

To sum up, there were two dangers that Nyström wanted to avoid: On the one hand a method which was too theoretical, too abstract, and which for this reason reduced the teaching to pointless memorization; on the other hand, a method where there was nothing to understand but only rules to memorize and apply. In his method of reduction to the unit, introduced by heuristic dialogue, he saw a middle ground. The method was based on rational thinking, but avoided the use of algebraic symbols. It could be taught to children by means of the heuristic method, but in the end also lead to effective computations.

It should be clear that, in stark contrast to the view of the ”past” of mathematics education as ”traditional” and unreflectively founded on appreciation of ”mechanism”, Nyström's *Digit-Arithmetic* contributed to a then emerging field of discussion of the proper goals and methods of elementary mathematics education in Sweden.[38]

Early praise and critique

To give an impression of the discussion to which Nyström contributed, I will here present some examples of reactions that his book evoked and show how Nyström responded to these reactions. In an early review, published in *Aftonbladet*, Nyström was praised for having ”investigated and surmounted all meeting problems, instead of proceeding like other theoretical authors, who have omitted everything difficult and, having kept only that which is easy, have made themselves popular”. The reviewer notes that Nyström is the most innovative in his treatment of the rule of three, and acclaims the fact that the method demands that the pupil is ”taught to understand the reasons” for how to go about, that she learns to ”know, that such is the case, instead of just remembering that it is so”.[39] Nyström chose to print this early review in what appears to be a second printing of the first edition of the book.

In 1871 Nyström was again praised for his use of the method of reduction to the unit. His book is described as ”the best example of the use of this method in Sweden” – most probably with implicit reference to how the method had risen to prominence in some of the German states in the previous decade.[40] We read that a most important consequence of the method is a ”consistency and perspicuity [åskådlighet]” of the presentation of arithmetics. Nyström's use of ”examples, the solutions of which are accompanied by explanations” is acclaimed as something which makes explicit rules superfluous.[41]

There was also some critique early on. In *Post och Inrikes Tidningar* a reviewer remarked upon the heuristic dialogue, the use of which is sustained throughout Nyström's book. These questions may very well be a ”useful exercise for the beginner”, the reviewer contends, but they soon become tiring and, in

particular when enhanced proficiency is sought for, unnecessarily time consuming. The book was also claimed to be too ”theoretical”, too ”comprehensive”, having too many exercises and thus being unnecessarily expensive.[42]

Nyström responded that the purpose of the heuristic dialogue is ”not to always constitute an essential part of the computation in itself, but is intended only as a guide for the thinking of the beginner”. It should serve to introduce and explain. When the sought for understanding is achieved, the student should be presented with a concise rule or formula which, since it is understood, can be used to perform the calculations ”as quickly as the hand can write the numbers” without any risk of thoughtless mechanism.[43]

To the complaint that his book was too comprehensive and difficult for public education, Nyström responds sarcastically that it is perhaps in the particular and difficult circumstances of public education that his ”elaborate clarifications” are more needed than elsewhere. As regards the exercises, he explains that he sought to provide ”a sufficient number” not with the intention that *all* of them should necessarily be solved, and he actually goes on to suggest that ”weaker” students could perhaps solve only the first half or third of the problems, since they are ordered ”from the easier to the more difficult”. The ”better” students could instead solve every second or third of the exercises. Regarding the high price, finally, Nyström sees no need to comment since: ”judgment regarding the **price** of goods cannot take place without simultaneously attending to their greater or lesser **value**”.[44]

The rejection

In 1884 Nyström published a version of the *Digit-Arithmetic* specifically designed for use in public education (*folkskolan*). He split the book in two, one containing text, the other containing only exercises. This arrangement would make it possible also for the poorer to acquire that which was most important for participation in the school work, i.e., ”a sufficiently comprehensive collection of exercises”. Concerning these exercises he wrote:

> In general, [the book with exercises] does not contain any directions as to the execution and application of the arithmetical operations, but is the necessary guidance assumed to be provided by the teacher, who can shape her presentation after that method which she finds most suitable.[45]

We should note here the central role that Nyström assigns to the teacher. He writes that he has been careful to ”follow the fundamental rule of teaching of successive progression from the simple and easily applicable, to the more complicated”. In achieving this, he believed the book to be ”in complete agreement with the fundamental tenets of the current curriculum [of 1878]”.[46]

Interestingly, his qualification that he agrees to the ”fundamental tenets” only of the curriculum, left room for a deviation of crucial importance for the ongoing development of mathematics education and for the argument of this article, namely concerning the relation between explanations and exercises. Nyström's formulation on this point seems cryptic at first. Translating literally, he wrote that ”[he] has not found it appropriate, by the breaking-up of the example-complexes and the interspersion of the sections in different parts of the book, to

present [the book] in the form of a kind of mosaic”. (*jag har icke funnit tjenligt att genom de till ett och samma räknesätt (i hela tal) hörande exempel-komplexernas sönderbrytande och styckenas inflikande inom olika delar af boken framställa denna under formen af ett slags mosaikarbete).*

My interpretation is that Nyström wanted his book to be structured as a rational presentation of a *subject matter,* to be read, understood and mastered. As I will shortly return to, the curriculum of 1878 instead demanded that textbooks were structured so as to sustain a specific process of learning, in which the need to keep the pupils from bothering the teacher also had to be taken into account.

The version of the *Digit-Arithmetic* rewritten for use in public education was reviewed in *Svensk Läraretidning* in 1884.[47] The review begins with some recognition of the ambition to contribute to the development of public education in Sweden but then swiftly turns to critique. As regards the text part, the reviewer cannot see any other use for it than as a guide for the teacher. But for this purpose it is clearly not fit, since ”was [the teacher] in need of a guide of this sort, he would certainly be quite incompetent to teach in the subject at hand”. It is not difficult to see the reviewer's point: since Nyström had, to a large extent, simply split his *Digit-Arithmetic* in two, the textbook naturally contained *all* that Nyström thought necessary to do arithmetic. Of course the teacher must be supposed to already know much of this. Thus, if the book for the teacher was supposed to contain *only* that which she actually needed, Nyström's textbook contained *too much*.

As I will soon come back to, Nyström did not see any problem with a book for teachers containing ”too much” however. He thought that teachers should be quite able to sort out *for themselves* what material to use – a selection which he in fact saw as instrumental to the activity of teaching.

The book with exercises, on the other hand, the reviewer calls ”naked” (a term commonly used for collections of this sort). We read that:

> By putting in the hands of the pupil a collection of exercises like this, you force him to search assistance from the teacher in each and every case, and himself generally to become the one merely passive. In this way, the pupil has been deprived of an excellent opportunity for that self-developing activity, which is so important when it comes to the bringing up of independent and energetic individuals. If the teaching was previously often restricted to the memorization of rules and definitions, it seems that an opposite extreme is now at hand. It seems difficult to hit the ”golden” middle which, as regards the task of the teacher [...] actually should be to as far as possible make himself superfluous.[48]

Apparently, the book with exercises contains *too little*. However, three ideas are introduced here which are foreign to the views of Nyström: First, that the learning of arithmetic should be a ”self-developing activity”; second, that receiving assistance from the teacher implies that one is ”merely passive”; third, that the purpose of the teaching of arithmetic is ”the bringing up of independent and energetic individuals”. All these views belong to what I here call the ”paradigm of development”, and to which I will return shortly.

The reviewer goes on to ask what the purpose of the ”theory” contained in the text part can really be. To him, it seems neither possible nor desirable that the pupils learn arithmetics by reading theory, because then ”the role of the teacher would be reduced to nothing”. What the reviewer suggests here is, briefly, that knowledge should be provided *as a process of learning* and not as subject matter presented discursively. The reviewer asks himself how Nyström may have conceived of the actual *use* of the textbook:

> Shall it be made the object of reading-exercises, with thereto adjoined interrogations regarding the contents, or perhaps home-work? – In [the textbook] one can find special directions as to the solution of some of the more complicated exercises in the collection of examples. Precisely these ”guiding directions” should rightly have found their place by the respective example.[49]

What the reviewer suggests is that the book put in the hands of the pupils should have the form of exercises combined, when necessary, with short guiding directions as to how to solve them. This would compel the pupils to ”actively” try to work out the solution by themselves. We remember that this is exactly the form suggested in the 1878 curriculum, which Nyström explicitly rejects in the preface. This indicates that the critique of the review hinges on a more general shift in conceptions of learning and knowing, which had become manifest already in the curriculum of 1878.

The last part of this review concerns the number of exercises and their relative difficulty. The reviewer contends that there are too few exercises and in particular too few exercises with small numbers. Nyström mentions in his preface that he has not kept to the guidelines of the 1878 curriculum in this regard and the reviewer now contends that this reveals that Nyström ”has overlooked the low standpoint [ståndpunkt] of the pupil and the great importance for the success of the teaching, in particular at this stage, of the examples containing small numbers and being numerous”.[50]

We begin to see a pattern: Nyström tends to thinks more highly of both the teachers and the pupils than his reviewer. He thinks the teacher can select material from the book and structure it for teaching *by herself* and that this is central to the very activity of teaching; something similar goes for the pupils, who are invited, by the form of the teaching material, to ask for assistance *when they need it*.

Another critical review was published in 1887 in the report of a national committee, commissioned in 1884 to review all textbooks used in public education and suggest guidelines for their improvement.[51] Here, Nyström's treatment of arithmetic is again and again characterized as "mechanical". For instance, the value of using the method of reduction to the unit is drastically diminished, we read, because of "mechanical directions", such as:

> Move down and write to the right of the just obtained residual the digits of the […] The obtained residue is written after the quote, and under it is drawn a horizontal line, under which […][52]

Furthermore, in the eyes of the committee, the computed examples, which, as we have seen, Nyström intended as means for explanation, contribute to this mechanism, since they are supposedly merely copied by the pupils, making the method one of mechanical application of "formulas" encoded in the solutions provided. The committee further complains about the insufficient number of "introductory examples", as well as their insufficiently systematic ordering.

Characteristic is their claim that such an arrangement would have let the pupils "get used to" the handling of the number system. It is worth pointing out that this "getting used to", in the eyes of Nyström was equivalent to mechanism, because for him it is opposite to *understanding*, in the sense of understanding why something is done in this way and not otherwise. Nyström insists on *explaining*. He *explains* how the number system works, for instance how the value of a digit "is increased by a multiple of ten" when you "move to the left", and how the comma should

be "moved" in multiplication and division. In the eyes of the committee, this amounts to mechanism.

The size of the numbers in arithmetic textbooks was a recurring theme in the Swedish discussion from the 1870's onwards.[53] Many authors thought that numbers should be kept small in the teaching of children and Nyström's book was mentioned as a deterrent example well before 1887.[54] Therefore it is worthwhile to look into Nyström's rationale for using these big numbers (which may seem quite absurd also for a present-day reader).

Those arguing for small numbers conceived of mathematical knowledge in terms of number-concepts. Such concepts needed to be built, or formed. This made it seem reasonable to start by building small numbers, and then to proceed to successively larger ones. Nyström, on the other hand, focused on the algorithms by which the numbers were manipulated. It was these that should be understood, rather than the numbers themselves. While Nyström was by no means unreasonable on this point, conceding to the fact that small children who cannot yet write the numbers properly should not be given tasks with too many digits, his ordering of the exercises was guided by *the difficulty of the operations* rather than the size of the numbers. Hence, when the committee objects that his examples are not ordered according to a "progression of development" giving priority to the size of the numbers involved, they in fact implicitly dismiss Nyström's conception of what it means to learn and know arithmetic.

Nyström's defense

Nyström was not the only author criticized by the reviewing committee of 1887 and during 1888 a heated debate ran through several issues of *Svensk Läraretidning*. The first thing Nyström comments upon in his ”indictment”, was the reproach that his methods would be ”mechanical”.[55] He seems sincerely perplexed that the board had this complaint, at the same time as it admitted that ”the rules are not only followed by explanations but even by 'extensive' explanations” since for Nyström:

> a rule which is founded upon sufficient explanations, cannot lead to [...] a merely mechanical proficiency in arithmetic, since the hallmark for mechanical computations is the inability to present the reason for why one goes about in this or that fashion. The occurrence of rules in the textbook can thus not justify the claim that the treatment is mechanical; and even less can the use of computed examples [mönsterexempel] warrant such a claim. It seems likely that the committee, with their, not only against me but against most other authors, directed accusation concerning ”mechanical treatment” must mean something else than that, which the expression at hand actually means.[56]

It becomes successively clearer that this review, as well as that of 1884, was founded upon a conception of learning which is fundamentally different from Nyström's. A further comment, testifying to this, regards the role of the teacher. Nyström contended that the extent to which rules are needed in an arithmetics textbook ”cannot be decided on objective grounds”, since ”[one] teacher may treat the subject such that both book and rules are largely superfluous, while another teacher may need substantial support from the presentation in the textbook”.[57] Already in the ninth edition of his book (from 1874) he stated that ”he is of the conviction that, as regards the so called mental

arithmetic [involving small numbers], the great importance and value of which is willingly acknowledged, the teacher must do almost everything and that the textbook can thereby in this regards accomplish just about nothing essential”.[58]

The issue at stake here is the relationship between the teacher and the textbook. Nyström repeatedly states that the function of the teacher is to attend to ”the particular capabilities [of each individual] and the particular circumstances [of the teaching situation]”, and that in this, the teacher can only receive very limited support from the textbook. While the book may very well provide examples for the teacher to use ”so to speak as a theme for several variations”, it is strictly impossible for it to provide a learning-track; a row of successively more difficult tasks, anticipating the learning process of the pupils. According to Nyström, there is no such single method that would suit all pupils equally well and even more to the point: There is no method that would equally well suite all teachers. Teachers should instead be given room to teach according to their own preferences.[59]

Nyström tries to understand what the committee may mean with their criticism of the, according to them, prevalent mechanical treatment in almost all contemporary arithmetic textbooks. He arrives at an interpretation that I think is quite to the point: *First* he speculates that they may think it best if the pupils, ”at all times”, had a clear view of the mathematical meaning of the computations they were performing.[60] This would differ from Nyström's stance, since he thinks it necessary to understand, once and for all, how the algorithms work, and then, knowing this, just perform the computations, with as little effort as possible. *Secondly*, he suggests that the reviewers perhaps want the knowledge to ensue even more from ”merely work on exercises”, so that the presentation of rules is avoided.[61] Against this he responds that he is ”an old, warm, but modest friend” of the heuristic method, but that he thinks that its use should be strictly limited to the introduction of new ideas. He notes sardonically that: ”if one has, at a point, come so far as to having solved the problem at hand and [understood] how an operation is to be performed, or how questions of a certain type should be treated, [the pupil] no longer takes the same interest in continued discussions about that same thing”. Thus, as I interpret Nyström, understanding is something that happens at some *point* in time, it is an *event*. What constitutes a meaningful activity is thoroughly different before and after this event, and only the teacher can attend to this difference. Nyström also adds that it is indeed a very good thing if the pupil, after having properly understood an operation, loses interest in its theoretical foundation and instead focuses on its accurate execution.[62]

It seems fitting to add here a comment from another textbook author of the time, Gullbrand Elowson, commenting critically upon the ”method of discovery”, the idea that knowledge should be ”invented” by the pupils by work on exercises. It is, he concludes, utterly impossible for the pupils to invent, by themselves, ”that theory, which is – a work of centuries”. Instead, ”[grateful] for what we have received from past times, we should for all teaching methods lay down as a rule: to make the subject intelligible”.[63] This is something to which Nyström would surely agree.

Summing up, the main disagreement between Nyström and his interlocutors concerned the form of the textbook and its relationship to the activity of the classroom. Nyström conceived of his book as a resource for the teacher and, to some extent, also a resource for the pupils. Its purpose was to provide well structured information as well as material for practice. The classroom activity, on the other hand, was not to be governed by

this book, but by the teacher. Her task was to *use* the textbook, in a way appropriately adjusted to the particularities of the pupils and the circumstances at hand.

The reviewers, of 1884 and 1887, claimed that the burden of governing the classroom activity should to a large extent be lifted from the teacher and instead put on the textbook – which should be designed accordingly to serve this purpose. This managerial idea was supplemented with an educational theory, where knowing was conceived of in terms of development resulting from work on exercises.

How can the shift from Nyström's conception of learning and knowing to that of his interlocutors be explained? As a first step towards an answer to this question I will now turn to a topic that was much discussed at the time, but which was quite forgotten just a few decades later – the use of arithmetic textbooks for *silent practice.*[64]

Silent practice

When public education (*folkskola*) was introduced in Sweden in the first half of the 19th century the actual teaching was often supposed to be arranged according to a method called ”the monitorial system”, or ”the Lancaster method” after one of its protagonists. The characteristic feature of this method was that most of the teaching was conducted by the pupils themselves. This was made possible by a detailed scheme of progression between different ”courses” or ”circles” (each comprising about 10 pupils), strict discipline, special teaching material and not the least a teaching room especially designed for the use of this method – for instance with the places for some of the circles marked out on the floor.[65]

This teaching method was forbidden by law, however, in 1862.[66] It was generally claimed that the pupils only memorized facts, and that the knowledge they gained was useless. Interestingly, the learning was characterized as ”mechanical” in a way quite similar to the judgment of Nyström's methodology just a few decades later. It was decided that only teachers should teach, themselves trained at special teacher seminars. In fact, not only was the full use of the Lancaster method forbidden, teachers were not even allowed to use the more knowledgeable pupils as help. This created problems in the public schools and raised questions as to how the often large groups of pupils, of different ages and abilities, were to be taught by a single teacher.

The solution proposed by the central agencies (connected to the teacher seminars) was called ”silent practice”. The idea was that groups of students were to be given exercises, specially designed for the purpose of keeping them busy – and thus silent. This arrangement would make it possible for the teacher to attend to those in most need of help or introduce new subject matter for a selected group of pupils.[67]

It should come as no surprise that elementary arithmetics was found to provide great material for silent practice. In fact, it was a much discussed topic in teachers magazines in the 1870's and the first half of the 1880's how, more exactly, textbooks in elementary arithmetic should be designed to function optimally for silent practice.[68]

One obvious demand was that the exercises should be sufficiently numerous.[69] Furthermore, the exercises should not be too difficult, as this would force the pupil to seek the assistance of the teacher. On the other hand, the exercises could not be too simple either, as this would make them uninteresting and thus fail to make the pupils stay focused (and hence: silent). It is against this background that we should interpret the

enormous efforts that at this time was put into the systematic arrangement of the exercises, to make them progress steadily from the ”more simple to the more complicated”.[70]

It was for the purpose of silent practice that it became important to intersperse ”guiding directions” between the exercises. Another much discussed means for the smooth running of silent practice was to make the exercises ”realistic” – not in the sense of being problems that actually needed to be solved, but in the sense of being meaningful for the pupils, that is, involving material related to the everyday life or, perhaps, the everyday work life of their parents.

It is against this practical and institutional transformation of public schools that the highly critical reviews of Nyström's books should be interpreted. His books were not designed for silent practice.

The necessity of silent practice has nothing to do with learning. Furthermore, it seems that this technology of "classroom management" had a crucial impact on the activity of the modern mathematics education classroom. Nonetheless, as I pointed out in the introduction, the development to which the introduction of silent practice contributed was not interpreted as externally forced. On the contrary, the changes that took place at this time were celebrated in terms of emancipation from earlier, "traditional", mechanical and deficient, modes of teaching.

This is the paradox to which I will now turn. How it is possible that change, driven by necessities external to the concerns of the involved actors, are nonetheless identified by them as a realization of their own intentions?

Learning as Enlightenment or Development

For Nyström, arithmetic was a set of algorithms and methods founded on theory. To understand arithmetics, in this framework, means to understand how and why the algorithms work. Such understanding would for example make it possible to reconstruct the algorithms, if they were (partly) forgotten. It could function as a support for, if not complete replacement of, the task of remembering them.

As Nyström saw it, understanding could be brought about in many different ways. For instance through heuristic dialog with a knowledgeable teacher, through written explanation or through reflection upon worked out and commented examples.

The primary goal with the teaching of arithmetic, for Nyström, was the competent use of arithmetics outside school. For this, understanding was beneficial and strongly recommended, but not necessary. Nyström did not exclude the possibility of someone becoming and being a competent practitioner of arithmetic, despite lacking insight into the theoretical foundations of this practice.

Nyström regarded his method to be *rational*. This made it easy to understand and master. He conceived of it as a resource for the *rationalization* of the learning and use of arithmetics. In his book he *presented* his method, and provided *resources* for its appropriation. Importantly however, he did not take it to be his task to structure the process by which it was to be appropriated. As he saw it, this task must necessarily be left to the teacher and, to a varying extent, also to the learner.

Nyström had a firm belief in the rational judgment of the teacher and more generally of any reader of his *Digit-Arithmetic*.

He also had a firm belief in his own judgment, as an inventor and presenter of a new methodology. In short, Nyström's goal can be understood as one of *enlightenment*.

I use this label partly because I think there is some continuity between Nyström's conception of learning and knowing in Sweden, in the second half of the 19th century and the educational theories of the enlightenment one century earlier. Another reason for the label is the aptness, for Nyström's views, of the metaphor of turning on a light: Understanding, for Nyström, was an *event,* the bringing about of which the teacher could facilitate by means of explanation. It was a matter of knowing *why* it is right to go about in the way that you do; of the ability to justify it. For Nyström, asking if a person performed actions mechanically or not came down to asking if this person *knew what she was doing,* if she could explain *why* she was doing this and not something else.

Despite these good intentions, Nyström was reproached for representing a *mechanical* methodology for the learning of arithmetics. We can see now that this reproach was the result of his interlocutors being inside a different framework for interpretation, to which I shall now turn.

Nyström's interlocutors saw knowledge and proficiency as inexorably intertwined with the formation of subjectivity. Knowledge, for them, had the form of *concepts* that needed to be *formed*, and these concepts were tools for thinking and apprehending the world. I here see it as comprising a *paradigm of development.* [71] The central tenets of this paradigm were clearly expressed in 1874, in the very first lines of a much used teachers guide:

> The goal of teaching of arithmetics in public schools is: to give the children the ability to understand and solve those computational tasks, which are presented in everyday life to each and every citizen, whichever class or estate he may belong to. If the pupil, through this teaching, is to be led to understand the tasks which are offered in everyday life, this teaching must be developing for the mind.[72]

The point is that usefulness cannot be aimed at directly. Teaching must instead aim to be ”developing for the mind”. Then, as a consequence, useful proficiency will supposedly follow. Many of the differences between Nyström and his interlocutors hinge on this point. Nyström aimed *directly* at understanding and proficiency. In this, he presupposed an already capable, rational, subject. The framework of development is founded on the opposite presupposition, that the subject of teaching needs to be carefully formed (or developed), and that this formation should be the central concern of education. From this perspective, direct explanations of e.g. algorithms seems misdirected as they do not contribute to the ”development” of the pupil.

One of the things that makes the discussion between proponents of this view of learning – with which we are thoroughly familiar today – and Nyström interesting, is that Nyström, when defending himself against the accusation of mechanism, shows how, from *his* perspective, it is actually the framework of development which seems to be the more mechanical. The main reason for this is that development is thought to be a largely unconscious process. It aims to shape the pupils, so to speak, ”behind their backs”. Being conscious of what you know, reflecting upon it, does not play a very important role in this framework of learning and knowing. Even after successful knowledge development, it is not clear that the

pupil in question will be able to present a rational justification of why that knowledge is in fact correct. From Nyström's perspective then, such "knowledge" is not in fact knowledge at all. It amounts to little more than the "mechanical" formation of a habit.

The relationship between Nyström and his interlocutors is apparently symmetric in that they both view *the other's* methodology as mechanical.

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As it turns out, the framework of development became hegemonic in Swedish elementary mathematics education soon after the dust stirred up by the 1887 committee had settled. At the end of the 1880's, Nyström was among the last still living members of the first generation of teachers engaged in the development of ”new” methods for the teaching of elementary mathematics in Sweden. Nyström died in 1891, Otterström one year later, as did Per Adam Siljeström, another prominent reform educator, who had written several influential textbooks and articles on the teaching of elementary arithmetic in the 1860's and 1870's.[73] After 1890, the interest in elementary mathematics education was to recede for some decades.

Testifying to the fact that a paradigm shift had occurred is that the great textbooks of the 19th century were now phased out. Nyström's *Digit-Arithmetic* was kept in use for another few decades, but only to the price of severe modifications of both its form and contents – changes that were facilitated by the death of its author. Per Anton von Zweigbergk's *Textbook in Arithmetics* met a similar fate: its 34th and last edition appeared in 1919.

A number of textbooks that agreed with the tenets of the reviewing committee, many of which were first published in the

1870's, took their place. Among the most successful was Alfred Berg's series of textbooks. One should note that Berg had not written *a* book but *a series* of books. It was a characteristic feature for the new paradigm that each type of school, and increasingly also each age cohort, got its own teaching material, adjusted to its particular curriculum.[74] In the hands of a number of different editors, new editions of these books appeared until 1949, always appropriately adjusted to changes in curricula. These are the textbooks that were used in Swedish elementary mathematics education until the next breaking point in its history: the "new maths" movement of the 1960's. And, some would say, it was to these books that educators turned in the 1980's, after the collapse of the latter movement.[75]

Conclusion

Why was it possible to *settle down* with the framework of development for thinking about the learning and knowing of mathematics? And was this not possible with the framework suggested by Nyström? I don't think so.

The paradigm of enlightenment and the paradigm of development relate very differently to the *practical necessities* of public education as it took shape in Sweden in the last decades of the 19th century. In so far as silent practice became an integral part the classroom activity, this activity became difficult to reconcile with Nyström's vision of what it meant to learn something. For the paradigm of development on the other hand, the situation was quite different:

First, the paradigm of development saw the process of learning as crucially dependent on the *autonomous activity* of the pupil. This requirement fitted very well with silent practice.

Theory and necessity were aligned in the requirement of a diminution of the role of the teacher.

Secondly, the paradigm of development opened up for an understanding of quality of knowledge as positively dependent on the time spent on learning. The metaphor of building suggests that the quality of knowledge depends on thoroughness and the time invested in a similar way as would be the case in, for instance, the building of a house. It also suggests, with this metaphor, that time is better spent on fundamentals than on more complicated matters. In this way, the *slow progression* inevitably following from use of silent practice, was transformed from a vice into a virtue.

Third, since knowledge is conceived of as crucially dependent on the process of learning, it makes sense, in this paradigm, to put efforts into the standardization of the process of learning. The paradigm of development opens up for the conceptualization of learning as a problem of construction, almost as a problem of engineering, and hence opens up for the design of plans, in terms of curricula, and establishment of agencies for their implementation. It thus fits well with processes of centralization on the national level of the institution of public schooling at this time, as well as ambitions among the elite of the emerging teacher profession to establish themselves as experts on this problem of knowledge formation.

Finally, while in the paradigm of enlightenment arithmetic is seen as useful only in the specific circumstances which require computation, in the paradigm of development, the development of knowledge of arithmetics is interpreted as corresponding to the development, on the most general level, of rationality and autonomous adult subjectivity as such. This universalization of the role of the learning of arithmetics fits well with the ambition of the time to make public education compulsory.

My contention is that this complicity between on the one hand the necessities of public education and on the other the paradigm of development to a large extent explains why the paradigm of development became hegemonic in education in general, and in elementary mathematics education in particular, in Sweden at this particular point in time. The power of the vision of development derives from its ability to make sense of the frame set by institutional and practical necessities.

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Let me now come back to the loss of agency that was mentioned in the beginning of this report. In contrast to the situation in which Nyström wrote his *Digit-Arithmetic*, the role of the textbook authors of the last decades of the 19th century had been reduced to the implementation of curricula and the creating of teaching material suited for the management of the class room activity by means of silent practice. The role of the teachers was reduced to being an additional resource in the coaching of pupils to move along a learning track determined beforehand. The pupils, finally, were deprived of the possibility of turning to the textbook in search for further explanation as well as, of course, asking for assistance from the teacher who now, no longer, was supposed to know much about the actual use of arithmetics and, for that matter, did not have sufficient time to give such assistance anyway. Last but not least important is the fact that the activities in which pupils were obliged to participate, now, when they were finally purposefully designed, were not designed exclusively for the purpose of learning, but also for the purpose of keeping them occupied.

The paradigm of knowledge development seems to *require* loss of agency: It aims at the production of mathematical

knowledge as a kind of universal knowledge and therefore requires submission to universal standards of production.

We have seen in this report hat the critique that Nyström's interlocutors directed at Nyström is quite similar to the *standard critique* of today.[76] It claims that any vision of learning other than that of development amounts to mechanism. It claims that the paradigm of development is a force, working for the elimination of mechanism and the propagation of subjective autonomy. My contention is that this is exactly wrong. On the contrary, the paradigm of development emerged together with the practices it aims to overcome, as a response to the necessities resulting from the demand that relatively few teachers keep more children busy for a longer amount of time. It is not a critical response to this new situation, but an identification with it, albeit in a distorted form. Thus, the very conception of what it means to learn and to know mathematics was transformed, so that exactly and only those classroom activities which were necessary anyway, could seem purposeful.

Endnotes

[1] Carl Alfred Nyström, *Försök till lärobok i aritmetiken eller sifferräkneläran, med talrika öfningsexempel och särskildt häftad facitbok* (Stockholm, 1853).

[2] Sverker Lundin, *Skolans matematik. En kritisk analys av den svenska skolmatematikens förhistoria, uppkomst och utveckling. [The mathematics of Schooling. A critical analysis of the prehistory, emergence and development of mathematics education in Sweden]*. (Uppsala: The University of Uppsala, 2008); ”The missing piece. An interpretation of Mathematics Education using some ideas from Žižek” in Christer Bergsten, Eva Jablonka & Tine Wedege, eds., *Mathematics and mathematics education: cultural and social dimensions: proceedings of MADIF 7* (Linköping: Svensk förening för matematikdidaktisk forskning, 2010): 168-178; ”Hating School, Loving Mathematics”, *Educational Studies in Mathematics* 80 (2012): 73-85.

[3] As will soon become clear, one of my central claims is that despite this imagery being ”scientific”, it is nonetheless to a large extent produced in and through schooling.

[4] Michael Allen Gillespie, *The Theological Origins of Modernity* (Chicago and London: The University of Chicago Press, 2008), p. 74.

[5] An rich account of the many metaphors used in educational science can be found in Alexandra Guski, *Metaphern der Pädagogik. Metaphorische Konzepte von Schule, schulischem Lernen und Lehren in pädagogischen Texten von Comenius bis zur Gegenwart.* (Bern: Peter Lang, 2007).

[6] Cf. Hans Blumenberg, *Hans Blumenberg. Ästhetische und metaphorologische Schriften. Auswahl und Nachwort von Anselm Haverkamp* (Frankfurt am Main: Suhrkamp, 2001), p. 140.

[7] Cf. Jacque Rancière, *The ignorant schoolmaster: five lessons in intellectual emancipation* (Stanford, California: Stanford University Press, 1991).

[8] Cf. Paul Dowling, *The Sociology of Mathematics Education* (London: Falmer, 1998), pp. 292-3.

[9] Regarding the origin of this dynamic relationship between the metaphors of machine and organism, see Anthony J. La Vopa, *Fichte: the self and the calling of philosophy, 1762-1799* (Cambridge: Cambridge University Press, 2001), p. 90 et passim.

[10] A rich and illuminating, if somewhat dated, account of this tradition can be found in Berthold Hartmann, *Der Rechenunterricht in der deutschen Volksschule vom Standpunkte des erziehenden Unterrichts* […] (Leipzig: Kesselring, 1904).

[11] See e.g. the concise discussion in Guski, *Metaphern der Pädagogik*, pp. 364-365.

[12] E.g., Edgar Weiss, *Adolph Diesterweg. Politischer Pädagoge zwischen Fortschritt und Reaktion* (Kiel: Peter Götzelmann Verlag, 1996).

[13] A presentation and critique of the secularization thesis can be found in the first part of Hans Blumenberg's *The Legitimacy of the Modern Age* (Cambridge, Mass.: MIT Press, 1983).

[14] Jürgen Oelkers, Fritz Osterwalder and Heinz-Elmar Tenorth, *Das verdrängte Erbe*. (Weinheim und Basel: Beltz, 2003); Jürgen Oelkers, *Pestalozzi - Umfeld und Rezeption.*(Weinheim and Basel: Beltz, 1995).

[15] Amos Funkenstein, *Theology and the scientific imagination from the Middle Ages to the seventeenth century.*(Princeton, N.J.: Princeton University Press, 1986).

[16] Cf. Valerie Walkerdine, "Developmental psychology and the child-centered pedagogy: the insertion of Piaget into early education", in Julian Enriques et al., *Changing the Subject. Psychology, Social Regulation and Subjectivity.* (London: Routledge, 1998), pp. 148-198.

[17] Cf. Hans Blumenberg, *The Genesis of the Copernican World.* Translated by Robert M. Wallace. (Cambridge, Massachusets: MIT University Press, 1987), pp. 124-125.

[18] Sverker Lundin, "Hating School, Loving Mathematics".

[19] Richard Noss & Celia Hoyles, *Windows on Mathematical Meanings: Learning Cultures and Computer* (Dordrecht: Kluwer, 1996), p. 1.

[20] Cf. Jacques Rancière, *The Ignorant Schoolmaster: Five Lessons in Intellectual Emancipation* (Stanford: Stanford University Press, 1991) and Ivan Illich, *Deschooling Society* (New York: Harper and Row, 1972).

[21] Carl Alfred Nyström, *Försök till lärobok i aritmetiken eller sifferräkneläran, med talrika öfningsexempel och särskildt häftad facitbok* (Stockholm, 1853).

[22] I present a similar argument in "The missing piece".

[23] Cf. Lundin, "Hating School, Loving Mathematics".

[24] Bernhard Meijer et al., *Nordisk familjebok: konversationslexikon och realencyklopedi*, vol. 37 (Stockholm: Nordisk familjeboks förlag, 1925), p. 869.

[25] Nyström, *Försök till lärobok i aritmetiken eller Sifferräkneläran* (1853).

[26] E.g., Daniel Staniford, *Staniford's Practical arithmetic ...: adapted principally to federal currency; designed as an assistant to the preceptor in communicating, and to the pupil in acquiring the science of arithmetic; to which is added, A new and concise system of book-keeping, both by single and double entry...* (Cornhill: J. H. A. Frost, for West, Richardson & Lord, no. 75, 1818).

[27] Jakob Otterström, *Utkast till lärobok i aritmetiken (för skolor i allmänhet och folkskolor i synnerhet)* (Stockholm, 1849).

[28] Ibid., preface. All translations in this report are my own.

[29] Carl Alfred Nyström, *Försök till lärobok i aritmetiken eller sifferräkneläran, med talrika öfningsexempel och särskildt häftad facitbok*, 2nd ed. (Stockholm, 1855), preface.

[30] In fact, this critical point closely mirrors that of Axel Theodor Bergius (1817-1897), another teacher and textbook author of the time, who was one of the first to review Otterström's textbook. See Axel Theodor Bergius, "Utkast till Lärobok i Aritmetiken för skolor i allmänhet och folkskolor i synnerhet, af J. Otterström." and "Svar till Hr Otterström med anledning af hans replik", both published in *Ny Tidskrift för lärare och uppfostrare* in 1849. Despite this criticism Otterström's method (if not his textbook) got many followers. The "algebraic method", or "the method of equations" as it was often called, was discussed in Sweden well into the 20th century.

[31] Following the praxis of his time he provided no indications as to how he came up with this method. However, the similarity between Nyström's ideas and those of Ernst Julius Hentschel

suggests that Hentschel was a major source of inspiration. The most influential of Hentschel's textbooks was probably his *Hundert Rechenaufgaben, elementarisch gelöst* (Weißenfels, 1837). The reception of Hentschel's ideas is discussed in Berthold Hartmann, *Die Rechenunterricht in der Deutschen Volksschule: Vom Standpunkte des Erziehenden Unterrichts* (Leipzig and Frankfurt am Main: Kesselring, 1893), p. 85 et passim. The influence of Hentschel on elementary mathematics education in Sweden is mentioned in G. W. Bucht, *Anteckningar i räknemetodik för folkskolan och småskolan* (Stockholm: Norstedts, 1894), p. 3.

[32] Carl Alfred Nyström, *Sifferräkneläran* (1855), p. 80.

[33] Ibid.

[34] Sverker Lundin, *Skolans matematik*, pp. 138–178.

[35] The method was also called "the method of discovery", "the Socratic method" and "the majestic method".

[36] The method was introduced for the teaching of geometry in Sweden by Anders Magnus Kjelldal. Its main protagonist in the second half of the 19th century was Karl Petter Nordlund. See Lundin, *Skolans matematik,* pp. 317-320; "Ur Lektor A. M. Kjelldahls efterlemnade papper" *Pedagogisk Tidskrift* (1872): 131-138 and e.g., Nordlund, *Räkneöfningsexempel för skolor: uppställda med afseende på heuristiska methodens användande* (Gefle, 1867).

[37] Nyström, *Sifferräkneläran* (1853), preface.

[38] For the historical context of the discussion in Sweden, see Lundin, *Skolans matematik*. It was heavily influenced by similar and earlier discussions in the German states. A useful, if somewhat triumphalist introduction to these are provided in

Berthold Hartmann, *Der Rechenunterricht in der deutschen Volksschule*. A similar account can be found in Eduard Jänicke *Geschichte des Unterrichts in den mathematichen Lehrfächern in der Volksschule* (Gotha: E. F. Thinemanns Hofbuchhandlung, 1888). Friedrich Unger, *Die Methodik der praktischen Arithmetik in historischer Entwickelung: vom ausgange des Mittelalters bis auf die Gegenwart* (Leipzig: Teubner, 1888) is much shorter but interesting because of ist many critical remarks.

[39] "Försök till lärobok i aritmetiken eller Siffer-Räkneläran", *Aftonbladet*, 1853-08-03.

[40] Hartmann, *Der Rechenunterricht in der deutschen Volksschule*, pp. 85-6.

[41] Kommissionen för behandling af åtskilliga till undervisningen i matematik och naturvetenskap inom elementarläroverken hörande frågor, *Underdånigt betänkande* (Stockholm, 1872), pp. 86–87. We see here that already in the 1870's there is a tendency to view "rules" as problematic. Nyström was by no means unaffected by these lines of thought.

[42] Nyström quotes this review himself in the 3rd edition of the Digit-Arithmetic: Carl Alfred Nyström, *Försök till lärobok i aritmetiken eller sifferräkneläran, med talrika öfningsexempel och särskildt häftad facitbok*, 3rd ed. (Stockholm 1859), preface.

[43] Ibid.

[44] Ibid. I reproduce Nyström's use of bold letters for emphasis.

[45] Carl Alfred Nyström, *Räknelära för folkskolor* (Stockholm: Kinberg, 1884), preface.

[46] Ibid.

[47] -id-, "Räknelära för folkskolor af Carl Alfred Nyström.", *Svensk Läraretidning* (1884): 321-322.

[48] Ibid.

[49] Ibid.

[50] Ibid.

[51] *Granskning af läroböcker för folkskolan jemte grundsatser för deras uppställning: underdånigt utlåtande* (Stockholm: Kongl. boktryckeriet, 1887).

[52] Ibid., p. 65.

[53] E.g., Johan Peter Velander, "Ämnet räkning i folkskolan" *Svensk Läraretidning* (1884) and by the same author, "Hela tal i folkskolan" *Svensk Läraretidning* (1885).

[54] In e.g., Velander, "Ämnet räkning i folkskolan".

[55] Carl Alfred Nyström, "Vidräkning med kommitterade för granskning af folkskolans läroböcker", *Svensk Läraretidning* (1888): 113-118.

[56] Ibid.

[57] Ibid.

[58] Carl Alfred Nyström, *Siffer-Räknelära*, 9th ed., 1874, preface.

[59] Ibid.

[60] Nyström, "Vidräkning med kommitterade för granskning af folkskolans läroböcker".

[61] Ibid.

[62] Ibid.

[63] Gullbrand Elowsson, "Om den aritmetiska undervisningsmetoden. 1 Diskussion om undervisningen i aritmetik.", *Tidskrift för matematik och fysik* (1868): 294-297 and ibid. (1869): 40-56.

[64] Marcelo Caruso touches upon the history of silent practice in his *Geist oder Mechanik Unterrichtsordnungen als kulturelle Konstruktionen in Preussen, Dänemark (Schleswig-Holstein) und Spanien 1800–1870* (Frankfurt am Main: Lange, 2010), pp. 84–85.

[65] Lundin, *Skolans matematik*, 241–242.

[66] Cf. Thor Nordin, *Växelundervisningens allmänna utveckling och dess utformning i Sverige till omkring 1830* (Stockholm, 1973).

[67] Lundin, *Skolans matematik*, p. 303.

[68] See a number of articles by Carl Kastman: "De tysta öfningarna i skolan", *Tidning för Folkskolan* (1877): 229-234; "De tysta öfningarna i Folkskolan", *Tidning för Folkskolan*: 72-77;"Ytterligare om 'de tysta öfningarna'."; *Tidning för Folkskolan* (1879): 209-215; Sven Kellin, "Ett strå till frågan om 'de tysta öfningarne'.", *Tidning för Folkskolan* (1879): 281-283; Carl Kastman, "Om tysta öfningar i räkning", *Folkskolans vän* (1886): 3-5.

[69] Lundin, *Skolans matematik*, p. 220.

[70] E.g., Velander, "Ämnet räkning i folkskolan".

[71] See e.g. Jürgen Oelkers, *Reformpädagogik: eine kritische Dogmengeschichte* (Weinheim: Juventa-Verlag, 1996).

[72] Christofer Ludvig Anjou m.fl., *Bidrag till pedagogik och metodik för folkskolelärare. Häftet V. Metodik: Räknekonsten i Folkskolan.* (Stockholm: Hjalmar Kinbergs förlagsexpedition, 1876), p. 1.

Accounts of this principle can be found in teacher guides published in the German states at least from the 1840's onwards. Eduard Jänicke put it concisely in 1855: "Durch Übung der geistigen kraft Bildung fürs leben." (quoted in Hartmann, *Der Rechenunterricht in der deutschen Volksschule,* p. 87)

[73] See Lundin, *Skolans matematik,* pp. 274-6.

[74] Ibid., pp. 347-50.

[75] I have noted this myself in *The mathematics of schooling*. It is also mentioned in Leif Hellström, *Undervisningsmetodisk förändring i matematik – villkor och möjligheter* (Malmö: Gleerups, 1985) p. 18.

[76] See the introduction to this report and Lundin, "Hating School, Loving Mathematics".