

Exercising Mathematical Competence Practising Representation Theory and Representing Mathematical Practice

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ABSTRACT

This thesis assembles two papers in mathematics and two papers in mathematics education. In the mathematics part, representation theory is practised. Two Clebsch-Gordan type problems are addressed. The original problem concerns the decomposition of the tensor product of two finite dimensional, irreducible highest weight representations of $GL_{\mathbb{C}}(n)$. This problem is known to be equivalent with the characterisation of the eigenvalues of the sum of two Hermitian matrices. In this thesis, the method of moment maps and coadjoint orbits are used to find equivalence between the eigenvalue problem for skew-symmetric matrices and the tensor product decomposition in the case of $SO_{\mathbb{C}}(2k)$. In addition, some irreducible, infinite dimensional, unitary highest weight representations of $\mathfrak{gl}_{\mathbb{C}}(n+1)$ are determined.

In the mathematics education part a framework is developed, offering a language and graphical tool for representing the exercising of competence in mathematical practices. The development sets out from another framework, where competence is defined in terms of mastery. Adjustments are made in order to increase the coherence of the framework, to relate the constructs to contemporary research and to enable analysis of the exercising of competence. These modifications result in two orthogonal sets of essential aspects of mathematical competence: five competencies and two aspects. The five competencies reflect different constituents of mathematical practice: representations, procedures, connections, reasoning and communication. The two aspects evince two different modes of the competencies: the productive and the analytic. The operationalisation of the framework gives rise to an analysis guide and a competency graph.

The framework is applied to two sets of empirical data. In the first study, young children's exercising of competencies in handling whole numbers is analysed. The results show that the analytical tools are able to explain this mathematical practice from several angles: in relation to a specific concept, in a certain activity and how different representations may pervade procedures and interaction. The second study describes university students' exercising of competencies in a proving activity. The findings reveal that, while reasoning and the analytic aspect are significant in proving, the other competencies and the productive aspect play important roles as well. Combined, the two studies show that the framework have explanatory power for various mathematical practices. In light of this framework, this thesis exercises both aspects of mathematical competence: the productive aspect in representation theory and the analytic aspect in the development of the framework.

Keywords: Mathematical competence, exercising competencies, young children, whole number arithmetic, tertiary level, proving, highest weight representation, tensor product decomposition, skew-symmetric matrix, moment map, infinite dimensional unitary representation.