The Dirac Equation: Numerical and Asymptotic Analysis

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Thesis for the degree of Doctor of Philosophy to be defended in public on
Thursday, December 20, 2012, at 10.15 in room Pascal,
Department of Mathematical Sciences, Chalmers Tvärgata 3, Gothenburg.

The defence will be conducted in Einglish.
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Abstract

The thesis consists of three parts, although each part belongs to a specific subject area in mathematics, they are considered as subfields of the perturbation theory. The main objective of the presented work is the study of the Dirac operator; the first part concerns the treatment of the spurious eigenvalues in the computation of the discrete spectrum. The second part considers G-convergence theory for positive definite parts of a family of Dirac operators and general positive definite self-adjoint operators. The third part discusses the convergence of wave operators for some families of Dirac operators and for general self-adjoint operators.

In the first and main part, a stable numerical scheme, using finite element and Galerkin-based *hp*-cloud methods, is developed to remove the spurious eigenvalues from the computational solution of the Dirac eigenvalue problem. The scheme is based on applying a Petrov-Galerkin formulation to introduce artificial diffusivity to stabilize the solution. The added diffusion terms are controlled by a stability parameter which is derived for the particular problem. The derivation of the stability parameter is the main part of the scheme, it is obtained for specific basis functions in the finite element method and then generalized for any set of admissible basis functions in the *hp*-cloud method.

In the second part, G-convergence theory is applied to positive definite parts of the Dirac operator perturbed by h-dependent abstract potentials, where h is a parameter allowed to grow to infinity. After shifting the perturbed Dirac operator so that the point spectrum is positive definite, the spectral measure is used to obtain projected positive definite parts of the operator, in particular the part that is restricted to the point spectrum. Using the general definition of G-convergence, G-limits, as h approaches infinity, are proved for these projected parts under suitable conditions on the perturbations. Moreover, G-convergence theory is also discussed for some positive definite self-adjoint h-dependent operators. The purpose of applying G-convergence is to study the asymptotic behavior of the corresponding eigenvalue problems. In this regard, the eigenvalue problems for the considered operators are shown to converge, as h approaches infinity, to the eigenvalue problems of their associated G-limits.

In the third part, scattering theory is studied for the Dirac operator and general self-adjoint operators with classes of h-dependent perturbations. For the Dirac operator with different power-like decay h-dependent potentials, the wave operators exist and are complete. In our study, strong convergence, as h approaches infinity, of these wave operators is proved and their strong limits are characterized for specific potentials. For general self-adjoint operators, the stationary approach of scattering theory is employed to study the existence and convergence of the stationary and time-dependent h-dependent wave operators.

Keywords: Dirac operator, eigenvalue problem, finite element method, spurious eigenvalues, Petrov-Galerkin, cubic Hermite basis functions, stability parameter, meshfree method, *hp*-cloud, intrinsic enrichment, G-convergence, Γ-convergence, scattering theory, identification, wave operator, stationary approach.