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ESSAYS ON INSURANCE ECONOMICS

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To the Memory of My Grandmother

Xiao, Zhi-Hua

Abstract

This thesis investigates two aspects of insurance theory. Essays I, II and III deal with the ownership structure in the insurance industry. Essays IV and V deal with the effects of background risks on an individual's insurance decision against a given risk.

Essay I uses game theory to analyze mutual contracts. Whether or not there are pure risk premiums is assumed to distinguish mutual contracts from insurance contracts. It is found that the mutual game with the absence of pure risk premiums has a nonempty core. Thus, stable mutual sharing is possible. However, the Pareto-efficient allocation may not be in the core, as opposed to the insurance game in which the Pareto can be in the core.

In *Essay II*, a bargaining model is used to study how individuals in a mutual society design mutual contracts in order to share their risks. It is found that, 1) there is a general consistence between the mutual and insurance contracts: The same risk premium is required against the same risk and the high-risks are required to pay higher risk premiums than the low-risks; 2) There are situations where the mutual contract requires only an assessment of the relative value of the probabilities of losses, which shows an advantage of the mutual contract over the insurance contract because the insurance contract generally requires an assessment of the actual value of the probabilities; 3) The way in which an individual's degree of risk aversion affects a contract in the mutual case appears differently from the way in the insurance case.

Essay III uses the transaction cost theory to argue that mutual cooperatives can be formed and developed from a small mutual society, and that they can behave efficiently and similarly to their stock counterparts. The essay also presents some of the important characteristics of mutual cooperatives and gives a few examples from the Swedish insurance industry, which tentatively illustrate the formation and development of mutual cooperatives in Sweden.

Essays IV and V turn to another topic. *Essay IV* uses a general expected-utility approach to examine optimal insurance coverage in presence of both additive and multiplicative risks. It is concluded that there exist cross effects of other risks on insurance decision against a considered risk. And the total effect of both additive and multiplicative risks is not simply the sum of their individual effects, even risks are unrelated to each other. Thus, taking both additive and multiplicative risks into account simultaneously is important.

Essay V studies the effect of derivative securities on an individual's insurance decision. In the framework of a mean-variance utility, it is concluded that derivative securities have an impact on an individual's insurance decision; Because a farmer uses hedging instruments against the price risk, he may not buy the full insurance even if the premium is fair. And there is no monotonic relationship between the farmer's degree of risk aversion and his insurance purchase or his hedging ratio. Thus, the effect of a farmer's degree of risk aversion appears differently when the crop insurance and the derivative securities are concerned separately and when they are concerned simultaneously. The discussion is also connected to the concept of the variable participation contract, which is found to have advantages over a usual insurance contract.

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(*Singapore International Insurance and Actuarial Journal*, 2000, 4(1): 37-49)

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Introduction and Summary

This thesis investigates two aspects of insurance theory. The first part, Essays I, II and III, deals with the ownership structure in the insurance industry. The second part, Essays IV and V, deals with the effects of background risks on an individual's insurance decision against a given risk.

Part I

There are two main types of ownership structure in the insurance industry: stock insurance companies and mutual insurance cooperatives. Mutual cooperatives have a significant position in the industry. In order to see the difference between the mutual cooperatives and the insurance companies, let us first look at the difference between a mutual contract and an insurance contract. In general, the insurance contract includes a pure insurer, the insurance company; Customers come to the company to buy insurance policies. They pay fixed risk premiums, and thus transfer their risks to the insurance company. The fixed risk premium is called a *pure risk premium*. In a mutual sharing society, individuals get together and sign mutual contracts to share their risks. Normally, there are no pure risk premiums involved. To compensate an individual (say individual A) for bearing others' losses, individual A's own loss is born by others. In other words, when some individuals in a mutual society suffer a loss, all the others in the society compensate them according to a share rule signed up before. Thus, in the mutual contract, there is no fixed payment and there is no pure insurer. How much each individual compensates others depends on the actual losses of all individuals in the society. Since losses are random, the individuals' final payments are unfixed. The insurance contract corresponds to the stock insurance companies and the mutual contract corresponds to the mutual cooperatives. Thus, whether or not there is a pure risk premium is assumed to distinguish the mutual cooperatives from the insurance companies.

Part I, which includes three essays, focuses on the mutual cooperatives and investigates the following questions: a) How do the mutual cooperatives form and develop? b) What makes mutual cooperatives different from (or similar to) stock insurance companies?

A number of papers have tried to explain the co-existence of the insurance companies and the mutual cooperatives.[1] In the papers, it is usually argued that mutual cooperatives are

[1] Papers include Born et al. (1995), Cummins, et al. (1997), Doherty (1991), Doherty & Dionne (1993), Hansmann (1985, 1996), Lamm-Tennant & Starks (1993), Mayers & Smith (1981, 1988, and 1994), O'Sullivan

formed by individuals who think that mutuality is a good method of sharing a certain risk (e.g., Hansmann (1985 and 1996), O'Sullivan (1998) and Skogh (1999)). With this argument standing, mutual cooperatives start with some individuals who sign mutually beneficial contracts with each other. Then how do these mutually beneficial contracts develop into efficient enterprises that are able to compete with stock insurance companies? Essay III tries to answer this question. Moreover, in most insurance literature, writers do not distinguish between mutuals and stocks. Similarly, the insurance literature usually focuses on insurance contracts and does not distinguish between an insurance contract and a mutual contract. Why is this so? If it can be argued that the insurance contract and the mutual contract are developed into similar institutions and market performance, then to make the distinction may not be necessary for practical reasons. However, theoretically it is still important to investigate the similarity and the difference between the two ownerships.

Whether does a stable mutual contract exist when there is no pure risk premium? *Essay I* uses cooperative game theory to answer this question. It is found that the mutual game has a nonempty core. Thus, stable mutual sharing is possible. However, the Pareto-efficient allocation may not be in the core. This conclusion is in contrast to the insurance game analyzed by Suijs et al. (1998), which concluded that the Pareto-efficient allocation of the total loss in the insurance game belongs to the core when insurance premiums are calculated according to the zero-utility principle. Here, the Pareto-efficient allocation is the one maximizing a social welfare function. Moreover, this function is the one maximizing the sum of all individuals' expected utilities.

Essay I also finds that, in the mutual game, the core allocation maximizing the social welfare function may require information about who experiences losses, as opposed to the Pareto-efficient allocation, which does not. In other words, to reach the Pareto-efficient allocation, individuals put their entire potential losses into the pool and agree on rules about how to divide the total loss. It does not really matter who experiences the losses. Individual A shares the same amount of individual B's loss as the amount of individual C's loss. However, according to the core allocation the share rule depends on the individual's index. Individual A may share a different amount of individual B's loss from the amount of individual C's loss.

Note that the first essay focuses on allocations, which maximize the whole society's welfare. The essay starts with the Pareto efficient allocation. After proving that the general

& Diacon (1999), Skogh (1999), and Smith and Stutzer (1990 and 1995). O'Sullivan (1998) reviewed most of theoretical and empirical papers.

core exists, and that the Pareto-efficient allocation may not be in the core, the essay looks for a core allocation which maximizes the whole society's welfare. But why should individuals in a mutual society pay attention to having a share rule that maximizes the whole society's welfare? When studying how individuals in a mutual society design contracts to allocate risks, we may think of a more realistic model — a bargaining model with a Nash solution.[2]

In *Essay II*, a bargaining game model focusing on the Nash solution is used to study the mutual contract in a mutual society. Three cases are analyzed: a) all individuals with the same utility function who face the same risk; b) individuals with the same utility function who face different risks; and c) individuals who have different degrees of risk aversion but face the same risk. In this essay, when the mutual contract is compared to the insurance contract, a general consistence between the contracts is found: The same risk premium is required against the same risk; The high-risks are required to pay higher risk premiums than the low-risks. Thus, we do not only see what the mutual contract looks like, but we also see the similarity between the mutual contract and the insurance contract. Furthermore, it is concluded that the mutual contract has an advantage over the insurance contract; There are situations where a mutual contract requires only an assessment of the relative value of the probabilities of losses. The insurance contract, however, requires an assessment of the actual values of probabilities. Relative values are easier to assess than actual values. Finally, we investigate how an individual's degree of risk aversion affects a share rule. The effect in the mutual case appears differently from the effect in the insurance case. This is because the disagreement point in the mutual bargaining game is a risky outcome.

Although the term "mutual cooperative" is used in Essays I and II, it normally refers to a small mutual society in which individuals sign mutual contracts with each other, not to a large mutual cooperative in the insurance industry. In *Essay III*, it is asked why and how a mutual society, which may consist of a few individuals only, develops into a mutual cooperative — an efficient enterprise. The way in which mutual cooperatives can be formed and developed from small mutual societies where individuals sign mutual contracts is explained by the use of transaction cost theory. Indeed, the mutual cooperatives can behave efficiently and be similar to their stock counterparts. The essay describes some of the important characteristics of

[2] In various definitions of solutions in a bargaining model, the Nash solution is the unique one, which maximizes the total combined utility gains due to the players' cooperation, and at the same time considers the equality among the players. See Kalai (1985) and Shapley (1969).

mutual cooperatives. Finally, some examples, which illustrate tentatively the formation and development of mutual cooperatives in the Swedish insurance industry, are given.

Part II

The demand for insurance against loss from a particular risky asset depends on other risks the decision-maker faces. A number of papers[3] have discussed the effect of other risks on the optimal insurance coverage of a given risk. It was pointed out that background risks have significant effects on an individual's hedging decision against any of the risks facing him. Essays IV and V included in this thesis study the effects from different aspects. Both essays take crop insurance as an example.

Essay IV examines optimal insurance coverage in the presence of both additive and multiplicative risks. This distinguishes this essay from most of other studies that consider additive or multiplicative risk separately. In this essay, a farmer's income is taken to include two terms: income from selling a specific crop and other income. The first term, income from selling the specific crop, is equal to the product of the crop's price and its output. A farmer can buy a crop insurance to protect himself against a decrease in the crop output and the crop's price is assumed to be uninsurable. The second term, the other income, is assumed to be insurable or uninsurable. This essay investigates how a farmer's decision on the purchase of the crop insurance is affected by the uninsurable price risk (a multiplicative factor) and an insurable or uninsurable additive risk, included in the other income. By using the general expected utility approach, it is found that there are cross effects of other risks on the insurance decision of the considered risk. The total effect of both additive and multiplicative risks is therefore not simply the sum of their individual effects, even if risks are unrelated to each other. Thus, taking both additive and multiplicative risks into account simultaneously is important.

Essay V studies the effect of derivative securities on an individual's insurance decision. To investigate the effect, a farmer's income is assumed to come from selling a specific crop only, which is equal to the product of the crop's price and its output. In this essay, it is assumed that a farmer can buy a crop insurance to protect himself against a decrease in the crop output, and he can also join the futures market or the futures option market to protect himself against a decrease of the crop's price. This is different from Essay IV that assumes that the price risk is uninsurable. To my knowledge, there has been no paper has assumed an insurable price risk when the effect of the price risk on the farmer's insurance purchasing is discussed.

[3] References can be found in Essays IV and V.

This essay looks at the insurance contract and the derivatives in a symmetric pattern. In the framework of a mean-variance utility, it is concluded that derivative securities have an impact on the individual's insurance decision. There are situations where a farmer will not buy the full insurance even if the premium is fair because he uses hedging instruments against the price risk. And there is no monotonic relationship between the correlation coefficient of price and output and the farmer's hedging amount. When the effect of a farmer's degree of risk aversion on his hedging decision is investigated, there is no monotonic relationship between the farmer's degree of risk aversion and his insurance purchase or his hedging ratio. Therefore, the effect of a farmer's degree of risk aversion appears differently when the crop insurance and the derivative securities are concerned separately and when they are concerned simultaneously.

The discussion in Essay V is also connected to the concept of the variable participation contract, newly initiated by Doherty and Schlesinger (2001). Suppose that an insurance company issues a non-participation contract against a risk C_i facing individual i by requiring a fixed risk premium P_f, and suppose that the company can also issue a full participation contract against the same risk by requiring a random risk premium P_r. Then, *the variable participation contract* allows an insured to choose a degree of him participating in the contract, denoted by α, and to pay for $\alpha P_r + (1-\alpha)P_f$ to have risk C_i insured. An interesting conclusion is that, under certain conditions, the variable participation contract is equivalent to the synthetical use of the hedging instruments in the insurance and the derivative security markets. Thus, the variable participation contract makes it possible to use the advantages of financial instrument, like options and futures. As a result, risk C_i is better hedged, especially when risk C_i is correlated among individuals and the correlation makes the risk uninsurable in a traditional insurance market.

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— Part I —

The Ownership Structure in the Insurance Industry

Essay I

The Mutual Insurance Cooperative as a Game

Essay I

by

Hong Wu

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Essay II

The Mutual Contract: Comparing with the Insurance Contract

by

Hong Wu[*]

1. Introduction

What is the difference between the insurance contract and the mutual contract? The insurance contract is a contract between the insurance company and the insureds (Figure 1.a); Insurance companies are pure insurers. Customers (insureds) come to a company to buy an insurance policy. They pay a fixed risk premium, and thus transfer their risks to the insurance company. The fixed risk premium is called a *pure risk premium*. The mutual contract, on the other hand, is meant for individuals who want to share their risks collectively in a mutual sharing society. Individuals get together and sign mutual contracts. The relationship between these individuals is illustrated in Figure 1.b. Everyone in the society has a direct relationship with others. In this mutual case, there are no pure insurers, and there are no pure risk premiums. To compensate an individual (say individual A) for bearing others losses, individual A's own loss is born by others. In other words, when some of the individuals in the society suffer an actual loss, all the others in the society compensate them according to a share rule signed up before. The individuals in a mutual society are called insureds because they are insured with each other. In this mutual case, there is no fixed payment. The amount that each insured compensates others depends on the actual losses of all individuals. Since losses are random variables, the insureds' final payments are unfixed.

There are two main types of ownership structures in the insurance industry: stock insurance companies and mutual insurance cooperatives. The insurance contract corresponds to the stock insurance companies and the mutual contract corresponds to the mutual cooperatives. Many papers have studied the coexistence of stocks and mutuals and compared their behavior in different ways.[1] This essay distinguishes the mutual contract from the insurance contract according to their contract structures shown in Figure 1. Essay I studied mutuals in this way

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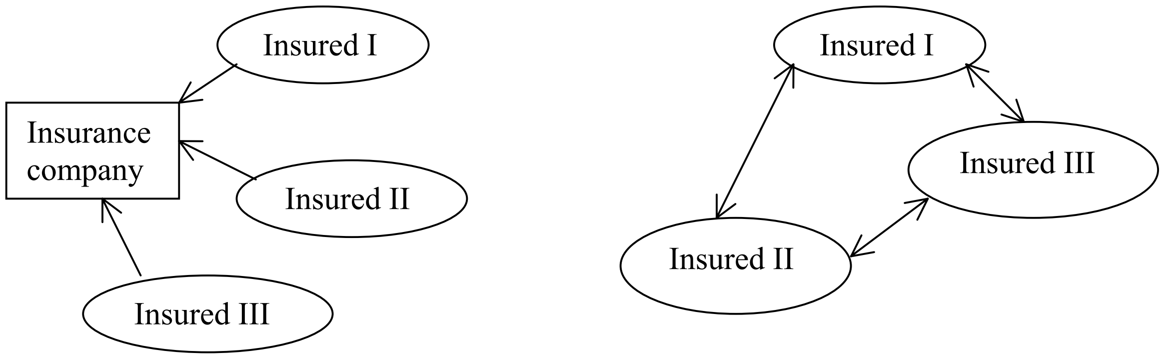
[1] O'Sullivan (1998) reviewed most of theoretical and empirical papers.

and answered yes to a question whether or not there are sharing rules that can stabilize a mutual pool. Now, if a stable mutual sharing is possible, what do the sharing rules, or the mutual contracts, look like? One of the purposes of this essay is to answer this question. In addition, this essay makes comparisons between the mutual contract and the insurance contract.



1.a 1.b

Figure 1: Relationships among parties in the insurance contract and in the mutual contract

In order to compare the mutual contract to the insurance contract, the terminology of "risk premium" is used for the mutual contract, which is distinguished from a *pure* risk premium for the insurance contract. In the mutual contract, the amount that each individual contributes (pays) in order to compensate the losses in the pool is related to both the risk that the individual pours into the pool and the risks that other individuals pour into the pool. The "*risk premium*" is related to *the individual's total contribution to compensate the losses in the pool*. If individual A, who pours risk A into the pool, contributes more than individual B, who pours risk B into the pool, then, by using the terminology of "risk premium", risk A is charged a higher risk premium than risk B. According to this definition, the risk premium does not only depend on the risk itself, but also on risks that other individuals pour into the pool. Since actual losses are unknown ex-ante, the risk premium is unfixed in this mutual contract.

Since the pure risk premium is fixed and the insureds' payments are not related to the insurance company's eventual economic behavior, the insurance company issues non-participation contracts. Since the risk premium in mutual contracts is unfixed and the insureds' eventual payments are related to the pool's eventual economic behavior, the mutuals issue participation contract. Thus, this way of distinguishing the mutual contract from the insurance contract is consistent with that of others, e.g., Smith and Stutzer (1990 and 1995), and Doherty (1991).

To see how the mutual contract is designed, and to compare it with the insurance contract, three cases are analyzed in Sections 2, 3, and 5, respectively. They are a) all individuals with the same utility function who face the same risk; b) individuals with the same utility function who face different risks; and c) individuals facing the same risk with different degrees of risk aversion, which is a special case among different utility functions. It will be shown that, the outcome of the mutual contract is similar to that of the insurance contract when all individuals have the same utility function. This explains why there is a similarity between the mutual and the insurance contracts in the market, although their contract relationships are different. However, the effect of an individual's degree of risk aversion on a share rule in the mutual case appears differently from the effect in the insurance case.

Section 4 investigates a case where the mutual contract requires only an assessment of the pool members' relative probabilities of losses, as opposed to the insurance contract, which generally requires an assessment of the actual value of the probability of loss for each policyholder. Since the relative value is easier to assess than the actual values, the result proposes an advantage of the mutual contract over the insurance contract. That is, the information requirement is less for the mutual contract than for the insurance contract. The conclusion is in favor of Hansmann (1996) and Skogh (1999); Hansmann (1996) reviewed his early paper Hansmann (1985), and suggested a number of reasons for the evolution of mutual insurance cooperatives. Among others, he pointed out that mutual contracts appear when the loss experience is difficult to predict. Skogh (1999) presented a theory on risk-sharing institutions for unpredictable losses where he emphasized that when there is uncertainty on the probability of risk, the mutual contract can be an alternative to the insurance contract. As usual, the final section will draw conclusions and summarize the essay.

It is worth mentioning that after many years, both types of companies have become rather similar in practice, although there maybe a few companies are in their early stage of development. Usually the premiums in mutuals are as fixed as they are in insurance companies and sometimes, variable premiums might also occur in insurance companies. Therefore, the investigation is basically a theoretical one.

The discussion below follows the bargaining game approach and employs the concept of the Nash (bargaining) solution. Therefore, the focus is to find the Nash solution in each case. Some basic concepts about the bargaining game and the Nash solution are presented in Appendix I.

2. A mutual bargaining game — the basic case

Essay I modeled a mutual cooperative as a cooperative game called a mutual game. Suppose that there are n individuals. Let N denote the set $\{1, 2, \ldots, n\}$. Individual i ($i = 1, 2, \ldots, n$) with endowment w_i faces a possible loss, denoted by a random variable L_i. The total loss of all individuals in set N will be $\sum_{i=1}^{n} L_i$. With this assumption, individual i ($i = 1, 2, \ldots, n$) has an initial portfolio $x_i = w_i - L_i$. If individuals in the grand set N share risks based on a mutual contract $R = (r_{ij})_{n\times n}$ with r_{ij} denoting the proportion that individual i bears for individual j's loss: $\sum_{i=1}^{n} r_{ij} = 1$, for any $j = 1, 2, \ldots, n$, and $0 \le r_{ij} \le 1$, for any $i, j = 1, 2, \ldots, n$, then $y_i = w_i - \sum_{j=1}^{n} r_{ij} L_j$ denotes individual i's final portfolio through risk exchanges among individuals in set N. Individual i enters the game with an initial portfolio x_i, exchanges it with other individuals in the mutual pool, and ends with a final portfolio y_i. Obviously, individuals will enter the pool only if they can obtain no less expected utilities than by not joining it, i.e., $EU_i(y_i) \ge EU_i(x_i)$ for all i ($i = 1, 2, \ldots, n$), where $EU_i(\cdot)$ denotes individual i's expected utility function. $EU_i(x_i)$ is individual i's reservation utility. A portfolio y_i satisfying $EU_i(y_i) \ge EU_i(x_i)$ for all i ($i = 1, 2, \ldots, n$) is called a feasible portfolio (solution). It is a solution making no individual worse off.

When individuals get together and discuss a mutual contract, they bargain with each other. This n-person bargaining game can be denoted by a combination (B, d), in which

$$\mathrm{B} = \left\{ (EU_i)_{i\in N} \middle| EU_i = EU_i(w_i - \sum_{j\in N} r_{ij} L_j); 0 \le r_{ij} \le 1, \forall i, j \in N; \sum_{i\in N} r_{ij} = 1, \forall j \in N \right\}$$

and

$$d = \left\{ (EU_i)_{i\in N} \middle| EU_i = EU_i(w_i - L_i) \right\}$$

where $\mathrm{N} = \{1, 2, \ldots, n\}$. Solving this bargaining game entails finding a proper $R = (r_{ij})_{n\times n}$ so that $(EU_i)_{i\in N} \in$ B are accepted by all individuals in the pool.

According to the definition of the Nash solution, the Nash solution solves for $R = (r_{ij})_{n\times n}$,

$$Max \prod_{i\in N} (EU_i(y_i) - EU_i(x_i))$$

$$s.\ t. \quad EU_i(y_i) \ge EU_i(x_i) \qquad (i \in \mathrm{N})$$

The discussion is restricted to a special case with five assumptions.

1. Individuals' utility functions are increasing and strictly concave. This assumption of risk-averse individuals is typical in insurance theory, and it extends the assumption in Essay I, which assumes exponential utility functions.

2. All individuals have an equal endowment w, and w is large enough so that $w - \sum_{j \in N} r_{ij} L_j \geq 0$ for any $R = (r_{ij})_{n \times n}$. This assumption of the same endowment avoids an income effect on a solution. The assumption that an individual's endowments are large enough is for simplifying calculations. Otherwise the constraint of $w - \sum_{j \in N} r_{ij} L_j \geq 0$ has to be added in all maximization problems.[2]

3. All individuals' losses L_i's ($i \in$ N) are distributed independently, but may not be identical. Instead of assuming that losses are distributed exponentially as in Essay I, it is assumed here that individuals' losses satisfy the Bernoulli distribution: the random variable L_i ($i \in$ N) is equal to l_i with probability p_i and equal to 0 with probability $1-p_i$. With this assumption, the difference in the probability of a loss can be separated from the difference in the amount of a loss.

4. There exists a Nash solution so that r_{ij} is independent from j. In other words, for $\forall i = 1, 2, \ldots, n$, there exists a s_i such that $r_{ij} = s_i$ ($j = 1, 2, \ldots, n$) and $\sum_i s_i = 1$, $0 \leq s_i \leq 1$, which is the so-called *equal-proportion-share*,[3] defined in Essay I. $R = (r_{ij})_{n \times n}$ can therefore be rewritten as a vector $S = (s_1, s_2, \ldots, s_n)'$, and, instead of $y_i = w_i - \sum_{j=1}^{n} r_{ij} L_j$, $y_i = w_i - s_i \sum_{j=1}^{n} L_j$. This assumption obviously simplifies the problem. However, whether is this assumption rational? It will be found out that, as long as the constraint set of the Nash problem is nonempty, a Nash solution does not only exist in the type of equal-proportion-share but it is also unique.

[2] Essay I made an assumption of different endowment, but assumed that all individuals have exponential utility functions. This assumption of exponential utility functions also prevents the solutions from an income effect. The essay concluded that both the Pareto efficient allocation and the core allocation considered are unrelated to individuals' endowments, which may be due to the assumption. Essay I also made the assumption that endowments are large enough to simplify calculations.

[3] This equal-proportion-share means that each individual in the pool shares all others' losses in an equal proportion: Individuals agree with a share rule $S = (s_1, s_2, \ldots, s_n)'$ in advance and then put all their losses into the pool. When losses occur, they share the total loss of the pool, based on S, and do not care about who experiences the losses.

Whether can the constraint set of the Nash problem be assumed nonempty? An empty constraint set means that individuals will not get into a mutual pool, which makes the research meaningless. So, it is rational to assume that there is a nonempty constraint set and there is a Nash solution in the type of equal-proportion-share.

5. $|N| = 2$. $|N| = 2$ means that there are two individuals. See Appendix I for comments about this assumption.

In this model, there are two individuals 1 and 2, and their utility functions are $u_i(\cdot)$ ($i = 1, 2$): $u_i'(\cdot) > 0$ and $u_i''(\cdot) < 0$. If both of them do not join any pool, then their expected utility functions (reservation utilities) will respectively be

$$r_1 = p_1 u_1(w-l_1) + (1-p_1)u_1(w)$$

and

$$r_2 = p_2 u_2(w-l_2) + (1-p_2)u_2(w)$$

If the two individuals share the total loss according to $S = (s, 1-s)'$, $0 \le s \le 1$, where s is for individual 1 and $1-s$ is for individual 2, then

$$EU_1 = p_1 p_2 u_1(w-s(l_1+l_2)) + (1-p_1)(1-p_2)u_1(w) + p_1(1-p_2)u_1(w-sl_1) + (1-p_1)p_2 u_1(w-sl_2)$$

and

$$EU_2 = p_1 p_2 u_2(w-(1-s)(l_1+l_2)) + (1-p_1)(1-p_2)u_2(w) + p_1(1-p_2)u_2(w-(1-s)l_1) + (1-p_1)p_2 u_2(w-(1-s)l_2)$$

The Nash maximum problem, called (Na), will be

$$\begin{aligned} &\underset{0\le s\le 1}{Max}(EU_1 - r_1)(EU_2 - r_2) \\ s.t., \quad & EU_1 \ge r_1 \\ & EU_2 \ge r_2 \end{aligned} \tag{Na}$$

Mathematical derivatives give us the following: $\frac{\partial EU_1}{\partial s} < 0$, $\frac{\partial EU_2}{\partial s} > 0$, $\frac{\partial^2 EU_1}{\partial s^2} < 0$, $\frac{\partial^2 EU_2}{\partial s^2} < 0$ and $\frac{\partial^2 (EU_1 - r_1)(EU_2 - r_2)}{\partial s^2} < 0$. Thus, the constraint set $\{s \in [0, 1] \mid EU_1 \ge r_1,\ EU_2 \ge r_2\}$ is a closed convex set. According to mathematical theorems, if the constraint set is nonempty, then maximizing problem (Na) will have a unique solution. Solving the problem (Na) requires the use of the Kuhn-Tucker theorem. The Lagrangian will be

$$L = (EU_1 - r_1)(EU_2 - r_2) + \mu_1(EU_1 - r_1) + \mu_2(EU_2 - r_2)$$

in which μ_i (i = 1, 2) is the Lagrange multipliers related to constraint $EU_i \geq r_i$ (i = 1, 2) and $\mu_i \geq 0$ (i = 1, 2). The first order condition requires the derivative of the Lagrangian with respect to s equal to zero, which produces equation

$$(EU_1 - r_1)\frac{\partial EU_2}{\partial s} + (EU_2 - r_2)\frac{\partial EU_1}{\partial s} + \mu_1 \frac{\partial EU_1}{\partial s} + \mu_2 \frac{\partial EU_2}{\partial s} = 0 \qquad (1)$$

If $EU_i > r_i$ at the optimal solution, then its relative $\mu_i = 0$, for i = 1, 2. Thus, when both $EU_1 > r_1$ and $EU_2 > r_2$, equation (1) becomes

$$(EU_1 - r_1)\frac{\partial EU_2}{\partial s} + (EU_2 - r_2)\frac{\partial EU_1}{\partial s} = 0 \qquad (2)$$

Let us start with the simplest case where both individuals have the same utility function and the same distribution of losses. The Nash solution is risk sensitive. The assumption of the same utility function limits us from the effects of individuals' difference on their utilities on the Nash solution. Section 5 will relax this assumption to specifically investigate the effects of individuals' utilities on the Nash solution. The assumption of the same distribution of losses means that both the amount of losses and the probability of losses for the two individuals are equal: Random variable L_i is equal to L with probability p_i and equal to 0 with probability $1-p_i$. And $p_1 = p_2 = p$.

Theorem 1 According to the Nash solution, when two individuals have the same utility function and face the same distribution of losses, they share the total loss equally. Following the notations in this essay, the Nash solution of (Na) is that $s = ½$.

Proof: Let $u_i(\cdot) =: u(\cdot)$ for i = 1, 2. Obviously, $r_1 = r_2 =: r$. When $s = ½$, $EU_1 = EU_2 =: EU$. Then,

$$\begin{aligned}
&EU - r \\
&= p^2u(w-L) + 2p(1-p)u(w-L/2) + (1-p)^2u(w) - pu(w-L) - (1-p)u(w) \\
&= 2p(1-p)[u(w-L/2) - (u(w-L) + u(w))/2] \\
&> 0
\end{aligned}$$

since $u(\cdot)$ is strictly concave. Hence under the assumptions, the constraint set of (Na) is nonempty and therefore there exists a unique solution. $EU_1 = EU_2 > r$ means that $\mu_1 = \mu_2 = 0$ in Equation (1). Furthermore, $\frac{\partial EU_1}{\partial s} = -\frac{\partial EU_2}{\partial s}$ at $s = ½$ implies that $s = ½$ satisfies equation (2) and therefore becomes the unique Nash solution. #

This conclusion can be simply extended to a case where $|N| > 2$; When several individuals have the same utility function and face the same distribution of losses, they share the total loss equally.

Comparing the mutual contract with the insurance contract: Theorem 1 shows that when two individuals who have the same utility function face the same possible loss, they share their total loss equally. According to the definition of risk premium in Section 1, it means that the two individuals will contribute (pay) the same amount of risk premium in order to share their risks. This is consistent with the insurance contract, in which insurance companies charge the same pure risk premium against the same risk. In other words, if two individuals want to have the same possible loss insured, they have to pay for the same amount of the pure risk premium in order to have their risks insured by the insurance company.

3. A case where two individuals face different distributions of losses

There are two individuals who have the same utility function but different distributions of losses; The amount of possible loss L is the same, but a high-risk individual faces a higher probability of loss than a low-risk individual. p_h and p_l denote their probabilities of losses respectively and $p_h > p_l$. Thus, the high- and the low-risk individuals' reservation utilities are $r_h = p_h u(w-L) + (1-p_h)u(w)$ and $r_l = p_l u(w-L) + (1-p_l)u(w)$ respectively. By joining a pool in which the high-risk individual bears a share of total loss s and the low-risk individual bears the others, i.e. $1-s$, their expected utilities, are

$$EU_h = p_h p_l u(w-2sL) + (1-p_h)(1-p_l)u(w) + p_h(1-p_l)u(w-sL) + (1-p_h)p_l u(w-sL)$$

and

$$EU_l = p_h p_l u(w-2(1-s)L) + (1-p_h)(1-p_l)u(w) + p_h(1-p_l)u(w-(1-s)L) + (1-p_h)p_l u(w-(1-s)L)$$

Substitute EU_h, EU_l, r_h and r_l for EU_1, EU_2, r_1 and r_2 in (Na), respectively, i.e., the high-risk individual corresponds to individual 1 and the low-risk individual to individual 2 in the last section. Then the first order condition will be

$$(EU_h - r_h)\frac{\partial EU_l}{\partial s} + (EU_l - r_l)\frac{\partial EU_h}{\partial s} + \mu_h \frac{\partial EU_h}{\partial s} + \mu_l \frac{\partial EU_l}{\partial s} = 0 \qquad (1')$$

in which $\mu_i \geq 0$ ($i = h, l$). If $EU_i > r_i$, then relative $\mu_i = 0$, for $i = h, l$. Obviously, the Nash solution has to be $0 < s < 1$, because $EU_l < r_l$ at $s = 0$ and $EU_h < r_h$ at $s = 1$.

Theorem 2 According to the Nash solution, when two individuals with the same utility function face different distributions of losses, the high-risk individual bears a greater share of the total loss than the low-risk individual, if the high- and the low-risk individuals join a mutual pool together. It means that under the assumptions, the Nash solution of (Na) satisfies $s > ½$ when the constraint set of (Na) is nonempty. Here, the different distributions of losses mean that the probabilities of losses are different, but the amount of possible losses is the same.

Proof: The high-risk individual has a higher probability of loss than the low-risk individual, which implies that $r_l > r_h$. Obviously, when $s = ½$, $EU_l = EU_h > r_h$. Thus there are three possibilities: 1) when $s = ½$, $EU_h > r_h$ and $EU_l < r_l$ exist; 2) when $s = ½$, $EU_h > r_h$ and $EU_l = r_l$ exist; 3) when $s = ½$, both $EU_h > r_h$ and $EU_l > r_l$ exist.

Case 1) Mathematical derivative gives $\frac{\partial EU_l}{\partial s} > 0$, which implies that $EU_l < r_l$ when $s < ½$. Thus, one of the constraints of (Na), $EU_l \geq r_l$, will not be satisfied if $s \leq ½$. Therefore, $s \leq ½$ will not even be a feasible solution. Thus, if the constraint set of (Na) is nonempty, the Nash solution of (Na) will be $s > ½$.

Case 2) Under this case, the constraint set of (Na) is nonempty because $s = ½$ is a feasible solution of (Na). Therefore, there certainly exists a unique Nash solution. Furthermore, $s < ½$ cannot be the solution, because $\frac{\partial EU_l}{\partial s} > 0$ implies again that $EU_l < r_l$, if $s < ½$. That is, $s < ½$ will not be a feasible solution. If $s = ½$ could be the solution, then it would be $(EU_h - r_h + \mu_l)\frac{\partial EU_l}{\partial s} = 0$ at $s = ½$, where $\mu_l \geq 0$, based on equation (1′) and the Kuhn-Tucker theorem. However, $(EU_h - r_h + \mu_l) > 0$ and $\frac{\partial EU_l}{\partial s} > 0$, at $s = ½$. Therefore, $s = ½$ cannot be the Nash solution, although it is a feasible solution. Thus, the unique Nash solution can only be $s > ½$.

Case 3) Again, there exists a unique Nash solution in this case because $EU_h > r_h$ and $EU_l > r_l$ at $s = ½$ implies that the constraint set of (Na) is nonempty. For the same reason, the Nash solution of (Na) s should be such that $EU_h > r_h$ and $EU_l > r_l$. Thus, instead of equation (1′), the Nash solution should satisfy,

$$(EU_h - r_h)\frac{\partial EU_l}{\partial s} + (EU_l - r_l)\frac{\partial EU_h}{\partial s} = 0 \qquad (2')$$

Let $h(s)$ denote the left side of equation (2′). By differentiating $h(s)$ with respect to s, it is found that $h'(s) < 0$ in the feasible interval, which means that $h(s)$ is a decreasing function of s. Thus, if $h(½) > 0$, then the Nash solution of solving $h(s) = 0$ will be $s > ½$. And $h(½) > 0$ can be easily proved. That is because a) $r_l > r_h$ and $EU_h = EU_l$ at $s = ½$ show us $EU_h - r_h > EU_l - r_l$ at $s = ½$; b) $\frac{\partial EU_l}{\partial s} = -\frac{\partial EU_h}{\partial s}$ at $s = ½$; a) and b) implies that $h(½) = [(EU_h - r_h) - (EU_l - r_l)]\frac{\partial EU_l}{\partial s} > 0$. #

Note that the conclusion is conditional on the assumption that the constraint set of (Na) is nonempty. An empty constraint set may appear if $EU_h > r_h$ and $EU_l < r_l$ at $s = ½$, which appears when $r_l >> r_h$. This is a situation where the low-risk individual is in a much better position than the high-risk individual. If this is the case and the constraint set of (Na) is empty for any s, then the low-risk individual will not join the pool. In order to see why there is a similarity between the mutual and the insurance contracts, let us focus on the way in which both the high- and the low-risk individuals share a risk within a mutual pool. Thus, Theorem 2 shows that according to the Nash solution, the high-risk individual has to share a higher proportion of the total loss than the low-risk individual, if they both join a mutual pool.

The same conclusion as in Theorem 2 can be proved for the case when two individuals have different amount of possible losses, but the same probabilities of losses: the high-risk individual (the one who may suffer a larger amount of loss) bears a greater share of the total loss than the low-risk individual (the one who may suffer a less amount of loss). The proof is omitted as it is rather similar to that of Theorem 2.

Comparing the mutual contract with the insurance contract: That the high-risk individual has to share a higher proportion of the total loss than the low-risk individual ($s > ½$) means that the high-risk individual pays (or contributes) more than the low-risk individual. This is exactly the case in the usual insurance contract, where high-risk individuals pay a higher pure risk premium than low-risk individuals.

What is s exactly equal to? Clearly, s depends on both p_h and p_l, which is similar to the case in the insurance company where a pure risk premium depends on a probability of a loss. In the next section, there is a situation where s depends only on the relative value of the probabilities p_h and p_l.

4. A case where *s* depends on the relative value of the probabilities only

All notations in this section are the same as the ones in Section 3, with the exception of the ones specifically explained.

4.1. If there is no aggregate uncertainty

Suppose that there are N_h high-risk individuals and N_l low-risk individuals. All of them have the same utility functions, and both N_h and N_l are large enough so that there is always $p_hN_h =: M_h$ high-risk individuals and $p_lN_l =: M_l$ low-risk individuals suffering from losses. Thus, when all the individuals get together and form a joint pool, the total loss in the pool will be a certain value, $(p_hN_h + p_lN_l)L$. Since there is always a fixed amount of loss in the pool, this becomes a situation where there is no aggregate uncertainty.

Theorem 3 If there are two types of individuals who have the same utility function but face different probabilities of losses and their number is large enough so that there is no aggregate uncertainty in the pool, then the Nash solution is $s = \dfrac{p_hN_h}{p_hN_h + p_lN_l}$, where s denotes the total proportion of all high-risk individuals bearing the total loss. Furthermore, if there is a t such that $p_h = tp_l$, then the Nash solution s will depend on the relative value of the probabilities, t, only and it will be independent from the actual value of the probabilities, p_h and p_l.

Proof: First, assume that both high-risk individuals and low-risk individuals can have their own separate pool, each for one type of individuals. Since individuals in each pool face *iid* losses, according to Theorem 1, they equally share the total loss in the separate pool. Thus, high-risk individuals end with a utility $r_h' = u(w - p_hL)$ and low-risk individuals end with a utility $r_l' = u(w - p_lL)$.

Then, assume that both high- and low-risk individuals get together and form a joint mutual pool. Let s be the proportion of all high-risk individuals bearing the total loss of the joint pool and $1-s$ be the proportion of all low-risk individuals bearing the total loss. Thus, each high-risk individual will end with a utility $r_h'' = u(w - \dfrac{s(p_hN_h + p_lN_l)}{N_h}L)$ and each low-risk individual will end with a utility $r_l'' = u(w - \dfrac{(1-s)(p_hN_h + p_lN_l)}{N_l}L)$.

In order for both types of individuals to join the joint pool, s has to be such that both $r_l'' \geq r_l'$ and $r_h'' \geq r_h'$. It can be easily proved that the only solution satisfying both $r_l'' \geq r_l'$ and r_h''

$\geq r_h'$ is $s = \dfrac{p_h N_h}{p_h N_h + p_l N_l}$ and therefore $1 - s = \dfrac{p_l N_l}{p_h N_h + p_l N_l}$. Each high-risk individual bears $\dfrac{p_h}{p_h N_h + p_l N_l}$ of the total loss in the joint pool and each low-risk individual bears $\dfrac{p_l}{p_h N_h + p_l N_l}$. As this s is the only feasible solution to the maximum problem, it is the Nash solution as well.

The proof is trivial for the case where $p_h = tp_l$. #

The theorem says that the Nash solution s can only be related to t and unrelated to p_h and p_l. More comments on this point will be presented later on.

With the solution s defined above, $r_l'' = r_l'$ and $r_h'' = r_h'$. Thus, what motivates both high- and low-risk individuals to get together? Let us look at the situation where there is the aggregate uncertainty.

4.2. If the pool is not large enough and therefore there is aggregate uncertainty

Suppose that there are N_h high-risk individuals and N_l low-risk individuals. Each of the high-risk individuals' losses is denoted by L_n (n = 1, 2, …, N_h) and each of the low-risk individuals' losses is denoted by L_m ($m = N_h+1, N_h+2, \ldots, N_h+N_l$). Let us compare two situations in a mean-variance approach: a) both high- and low-risk individuals have their own pool and equally share the total loss in each pool; b) they get together, form a joint pool, and share the total loss according to $s = \dfrac{p_h N_h}{p_h N_h + p_l N_l}$ defined in the above subsection.

In case a), each of the high-risk individuals will have to contribute an amount of $L_h = \dfrac{1}{N_h}\sum_{n=1}^{N_h} L_n$ into the pool. Then

$$EL_h = \frac{1}{N_h}\sum_{n=1}^{N_h} EL_n = \frac{1}{N_h} N_h p_h L = p_h L$$

$$DL_h = \frac{1}{N_h^{\,2}}\sum_{n=1}^{N_h} DL_n = \frac{1}{N_h} p_h (1 - p_h) L^2$$

where $E(\cdot)$ and $D(\cdot)$ denote expected value operator and variance operator respectively.

Similarly, for each of the low-risk individuals, $L_l = \frac{1}{N_l}\sum_{m=N_h+1}^{N_h+N_l} L_m$, $EL_l = p_l L$, and $DL_l = \frac{1}{N_l} p_l (1-p_l) L^2$.

In case b), each of the high-risk individuals will contribute an amount of $L_h{'} = \frac{p_h}{p_h N_h + p_l N_l}(\sum_{n=1}^{N_h} L_n + \sum_{m=N_h+1}^{N_h+N_l} L_m)$ into the joint pool and each of the low-risk individuals will contribute an amount of $L_l{'} = \frac{p_l}{p_h N_h + p_l N_l}(\sum_{n=1}^{N_h} L_n + \sum_{m=N_h+1}^{N_h+N_l} L_m)$ into the joint pool. Then,

$$EL_h{'} = \frac{p_h}{p_h N_h + p_l N_l}(p_h N_h L + p_l N_l L) = p_h L$$

$$DL_h{'} = \frac{p_h^{\,2}}{(p_h N_h + p_l N_l)^2}(N_h p_h (1-p_h) + N_l p_l (1-p_l))L^2 =: \varphi(N_l)$$

$$EL_l{'} = p_l L$$

$$DL_l{'} = \frac{p_l^{\,2}}{(p_h N_h + p_l N_l)^2}(N_h p_h (1-p_h) + N_l p_l (1-p_l))L^2 =: \psi(N_h)$$

Thus, both high- and low-risk individuals have unchanged expected values of losses under cases a) and b). However, the fact that $\varphi(0) = DL_h$, $\psi(0) = DL_l$, $\varphi'(N_l) < 0$ and $\psi'(N_h) < 0$ shows that the more the low-risk individuals join a pool which is initiated with only the high-risk individuals, the less the variance of the high-risk individuals' contribution, and *vice versa*. Similarly, the more the high-risk individuals join a pool which is initiated with only the low-risk individuals, the less the variance of the low-risk individuals' contribution, and *vice versa*. Therefore, in a mean-variance approach, $s = \frac{p_h N_h}{p_h N_h + p_l N_l}$ makes both the high- and low-risk individuals better off under case b) than under case a).

Unfortunately, it cannot be proved that $s = \frac{p_h N_h}{p_h N_h + p_l N_l}$ is the Nash solution. Even so, $s = \frac{p_h N_h}{p_h N_h + p_l N_l}$ is a feasible share rule which can make both types of individuals better off, and therefore individuals can be ended with this share contract. Again, if there is a t such that $p_h = tp_l$, then s will only be related to t and unrelated to the probabilities p_h and p_l.

As mentioned before, the independence of a share contract from the actual values of the probabilities is an important point that deserves attention. Generally, to settle an insurance contract, one needs to assess probabilities of losses in order to define a reasonably pure risk premium. Otherwise, a very high or very low premium will obstruct the prevalence of the insurance contract. Here, it is found that the mutual contract requires only an assessment of t, the relative value of the probabilities, which in some cases is easier to assess than the actual value of the probabilities. For example, if individual A drives twice as long as individual B, then, when all others are equal, we could assume that $t = 2$ without making any assessment on the probabilities of individuals getting involved in any traffic accident. Thus, the advantage of the mutual contract is that mutuals require less information about the distributions of risks than the insurance contracts. As it has been pointed out in the introduction, this conclusion is in favor of Hansmann (1996) and Skogh (1999).

5. A case where two individuals have different utility functions

It has been assumed that all individuals have the same utility to guard the Nash solutions from the effects of bargainers' risk attitudes on the share rule. The effect will be specifically investigated in this section.

Note that "for any model of bargaining that depends in a non-trivial way on the expected utility function of the bargainers, the underlying assumption is that the risk aversion of the bargainers influences the outcome of bargaining. That is, the risk aversion of the bargainers influences the decisions they make in the course of negotiations, which in turn influence the outcome of bargaining" (Kihlstrom and Roth, 1982). Thus, although this section discusses the effect of the individuals' risk attitudes (individuals' risk aversions) on the outcome of bargaining, it is not assumed that the bargainers know one another's risk postures.

Suppose that there are two individuals. Their losses are distributed independently and identically: Both of them face the same amount of possible loss L with the same probability p. However, they have different utility functions: Individual A has an increasing and strictly concave utility function $u(\cdot)$ and individual B has an increasing and strictly concave utility function $v(\cdot)$. Assume that individual A is more risk-averse than individual B, which, according to Pratt (1964), means that, for any x, $R_A(x) > R_B(x)$, where $R_A(x) = -\dfrac{u''(x)}{u'(x)}$ and $R_B(x) = -\dfrac{v''(x)}{v'(x)}$ are individuals A's and B's measures of absolute risk aversions, respectively. Or equivalently, individual A is more risk-averse than individual B if and only if there is an increasing and strictly concave function $G(\cdot)$, such that $u(x) = G(v(x))$.

Thus, individuals A's and B's reservation utilities $r_A = pu(w-L) + (1-p)u(w)$ and $r_B = pv(w-L) + (1-p)v(w)$. By joining a pool in which individual A bears a share of total loss s and individual B bears the others, i.e. $1-s$, their expected utilities are

$$EU_A = p^2u(w-2sL) + (1-p)^2u(w) + 2p(1-p)u(w-sL)$$

and

$$EU_B = p^2v(w-2(1-s)L) + (1-p)^2v(w) + 2p(1-p)v(w-(1-s)L)$$

Substitute EU_A, EU_B, r_A and r_B for EU_1, EU_2, r_1 and r_2 in (Na), respectively, i.e., individual A corresponds to individual 1 and individual B to individual 2 in Section 2. The first order condition will be

$$(EU_A - r_A)\frac{\partial EU_B}{\partial s} + (EU_B - r_B)\frac{\partial EU_A}{\partial s} + \mu_A\frac{\partial EU_A}{\partial s} + \mu_B\frac{\partial EU_B}{\partial s} = 0 \qquad (1'')$$

in which $\mu_i \geq 0$ (i = A, B). And if $EU_i > r_i$, then relative $\mu_i = 0$, for i = A, B.

Obviously, if there is a Nash solution, then it has to be in the open interval (0, 1), because $EU_A < r_A$ at $s = 1$, and $EU_B < r_B$ at $s = 0$. Since both $EU_A > r_A$ and $EU_B > r_B$ exist at $s = ½$ from the proof of Theorem 1, $s = ½$ is a feasible solution and the constraint set of (Na) is nonempty, which means that there exists a unique Nash solution and, at the Nash solution s, $EU_A > r_A$ and $EU_B > r_B$. From the Kuhn-Tucker theorem, the Nash solution s should solve

$$(EU_A - r_A)\frac{\partial EU_B}{\partial s} + (EU_B - r_B)\frac{\partial EU_A}{\partial s} = 0 \qquad (2'')$$

Take into account an extreme case, in which individual B is risk-neutral and individual A is risk-averse.[4] If this is the case, then $EU_A > r_A$ and $EU_B = r_B$, at $s = ½$. Since $EU_A > r_A$ at $s = ½$, and $EU_A < r_A$ at $s = 1$, there exists $s_0 > ½$ such that $EU_A = r_A$ at s_0. Thus, it must be $s \in [0, s_0]$ to satisfy $EU_A \geq r_A$. In addition, because $EU_B < r_B$ at $s = 0$ and $EU_B = r_B$ at $s = ½$, it must be $s \in [½, 1]$ to satisfy $EU_B \geq r_B$. $s \in [0, s_0]$ and $s \in [½, 1]$ gives the Nash solution $s \in [½, s_0]$. Moreover, when $s \in (½, s_0)$, both $EU_A > r_A$ and $EU_B > r_B$. Therefore, the Nash solution will satisfy $s > ½$.

This extreme case suggests that if both individuals are risk-averse, the more risk-averse individual might bear more than the less risk-averse individual. Under the assumptions of the model specified in this section, the Nash solution would satisfy $s > ½$. Unfortunately, this conjecture can only be confirmed in a special case where individuals' utility functions are

[4] When a risk-neutral individual B is assumed, the Nash solution does not necessarily solve (2´´), since, with the assumption, $EU_B = r_B$ at the Nash solution may be the case.

quadratic. In the general expected utility approach, the Nash solution will not be necessarily larger than ½.

Theorem 4 Two risk-averse individuals with different degrees of risk aversion face the same distribution of loss. 1) Although the equal-share (s = ½) is a feasible solution, the Nash solution may be larger than, equal to, or less than ½ if the two risk-averse individuals maximize the general expected utility functions. In other words, according to the Nash solution, the more risk-averse individual may bear more total loss than, or less than, or the same as, the less risk-averse individual. However, 2) in a special case where they both have strictly concave quadratic utility function, the more risk-averse individual bears more total loss than the less risk-averse individual, which means that under the assumptions the Nash solution satisfies that s > ½.

Proof: 1) Let $h(s)$ denote the left side of the equation (2′′). As $h'(s) < 0$ in the feasible interval, $h(s)$ is a decreasing function of s. If $h(½) > 0$, then the Nash solution solving $h(s) = 0$ will be $s > ½$. And if $h(½) < 0$, then the Nash solution will be $s < ½$. To prove that the sign $h(½)$ is not certain, an example where both $h(½) > 0$ and $h(½) < 0$ appear must be given.

From the expression EU_A and EU_B,

$$\frac{\partial EU_A}{\partial s} = -2pL(pu'(w-2sL)+(1-p)u'(w-sL))$$

$$\frac{\partial EU_B}{\partial s} = 2pL(pv'(w-2(1-s)L)+(1-p)v'(w-(1-s)L))$$

Thus,

$$h(½) = 4p^2(1-p)L\{\left(u(w-L/2)-\frac{u(w-L)+u(w)}{2}\right)\left(pv'(w-L)+(1-p)v'(w-L/2)\right) - \left(v(w-L/2)-\frac{v(w-L)+v(w)}{2}\right)\left(pu'(w-L)+(1-p)u'(w-L/2)\right)\}$$

Assume that $v(x) = -1/x$, $G(x) = -e^{-x}$, and $u(x) = G(v(x)) = -e^{1/x}$. One can check that both $v(x)$ and $u(x)$ are increasing and strictly concave functions and $u(x)$ are more concave than $v(x)$. a) If $L = 0$, or if $L << w$ such that $\frac{1}{w-L} \approx \frac{1}{w-L/2} \approx \frac{1}{w}$, then $h(½) = 0$, or $h(½) \approx 0$; b) If $w = 5$, $L = 4$ and $p = ½$, then $h(½) < 0$; c) If $w = 2$, $L = 1$ and $p = ½$, then $h(½) > 0$. The first part of the theorem is thus proved.

2) See Appendix II. #

Comparing the mutual contract with the insurance contract: The second result in Theorem 4 is consistent with Pratt (1964) and with Kihlstrom and Roth (1982). When two individuals have different degrees of risk aversion, the more risk-averse individual will be willing to contribute more to sharing the same risk than the less risk-averse individual. This is consistent with Pratt (1964) who claimed that a more risk-averse individual would be willing to pay more against a risk than a less risk-averse individual. Furthermore, it can also be proved that $\frac{\partial s}{\partial R_A(w)} > 0$.[5] It means that the larger the difference between $R_A(w)$ and $R_B(w)$, the larger the difference in the proportion of individuals sharing the total loss. If individual 1 bargains with either individual 2 or individual 3 for the same risk and individual 2 is more risk-averse than individual 3, then individual 1 shares a smaller proportion by bargaining with individual 2 than by bargaining with individual 3. The conclusion is consistent with Kihlstrom and Roth (1982), who analyzed the negotiation between a risk-neutral insurer and a risk-averse insured. They concluded that a risk-neutral insurer prefers to bargain with a more risk-averse client (insured), since that client will agree to spend more for less insurance, than a less risk-averse client.

However, the consistence exists only in a special case. General results, the first part of Theorem 4, are not consistent with Pratt (1964). This is because in this mutual bargaining game, the disagreement point is a risky outcome. According to Roth and Rothblum (1982), although the Nash solution generally predicts that risk aversion is a disadvantage in bargaining, risk aversion does not always have to be a disadvantage if a bargaining game concerns risky outcomes as well as riskless outcomes. Thus, the sign of $\frac{\partial s}{\partial R_A(w)}$ at the Nash solution might be predicted as generally uncertain.

Do the insureds, who have different levels of risk aversion but still join the same mutual cooperative, contribute differently to the same risk? The answer is yes, if there are not many individuals involved in the bargaining game, but the answer can also be no if there is a large number of individuals in the pool. As in Kihlstrom and Roth (1982), this bargaining model works only if each individual getting into the game has bargaining power. Kihlstrom and Roth (1982) also mentioned that if individuals behave competitively, the risk aversion does not need to be disadvantageous to the insured, because the price of insurance is actuarially fair in

[5] The proof is not included in the essay and, if interested, one can get from the author.

a competitively market equilibrium, regardless of the risk aversion of the insured. This is why we should not bother about the inconsistence found above. If the market is small and not competitive, Theorem 4 works and we may see a different pattern between the mutual and the insurance contracts. But in a competitive situation, the effect of an individual's degree of risk aversion on the (pure) risk premium is not expected to be seen.

6. Concluding remarks

In the field of risk and insurance theory, the focus is mainly on the insurance contract. In this essay the focus is on the mutual contract. As it is pointed out in the introduction, there are two purposes of writing this essay. The first is to find a stable mutual contract. In order to do so, the concept of the Nash solution is used. The second is to compare the mutual contract with the insurance contract. Before we come to the conclusion, it is worth mentioning that the stable sharing rule found may be different from the Pareto efficient one.[6] So it does not mean any contradiction if a different solution is found from Borch's (1960) and Bühlmann's (1980), both of which look for the market equilibrium and the Pareto efficient allocation for the insurance contract, which allows a pure risk premium to exist. The conclusion is summarized as follows.

First, according to the Nash solution, when two individuals with the same utility function face the same distribution of loss, they share the total loss equally. This conclusion is consistent with the one in the insurance contract.

Second, according to the Nash solution, when two individuals with the same utility function face different distributions of losses, the high-risk individual shares a higher proportion of the total loss than the low-risk individual. This is again consistent with the insurance contract.

Third, when two individuals with the same utility function face different distributions of losses, there are situations where the mutual contract requires only an assessment of the relative value of the probabilities. Thus, since the insurance contract generally requires an assessment of the actual value of the probabilities and the relative value is usually easier to assess than the actual value, the conclusion proposes an advantage of the mutual contract over

[6] The Pareto efficient allocation in Essay I focuses on an allocation maximizing the whole society's welfare — the sum of all individuals' expected utility functions, while a Nash solution pays attentions to not only the total combined utility gains due to the players' cooperation, but also to the equality among the players (Kalai, 1985). Besides, Essay I concluded that the Pareto efficient allocation may not be in the core, which means that it may not be stable.

the insurance contract, which is a less information requirement for the mutual contract than for the insurance contract. As explained, this conclusion is in favor of Hansmann (1996) and Skogh (1999).

Finally, when two individuals with different utility functions face the same distribution of loss, the individuals' degree of risk aversion has an impact on a share rule. Furthermore because of the risky outcome involved, the effects in the mutual case may be different from ones in the insurance case. However, the model does not work if the market is competitive.

Appendix I: Some basic concepts of the bargaining game

According to Roth (1979), in a pure bargaining problem, "a group of two or more participants is faced with a set of feasible outcomes, any of which will be the result if it is specified by the unanimous agreement of all the participants. In the event that no unanimous agreement is reached, a given disagreement outcome is the result. If there are feasible outcomes which all the participants prefer to the disagreement outcome, then there is an incentive to reach an agreement; however, as long as at least two of the participants differ over which outcome is most preferable, there is a need for bargaining and negotiation over which outcome should be agreed upon. Each participant has the ability to veto any outcome different than the disagreement outcome, since unanimity is required for any other result." Thus, an n-person bargaining game is a combination (B, d), in which B denotes the set of feasible payoffs (utilities) of the game and point $d \in$ B denotes the disagreement point. A solution of a bargaining game is defined as a point f(B, d) in B, which is a feasible payoff (utility) reached when the bargaining ends. Suppose that there are n individuals. N = $\{1, 2, \ldots, n\}$. Individual i ($i = 1, 2, \ldots, n$) has an initial portfolio x_i. ($x_1, x_2, \ldots, x_n$) could be, although not necessarily, the disagreement point. If individuals join a bargaining game and end with a portfolio y_i for individual i, then ($y_1, y_2, \ldots, y_n$) will be a solution of the game.

Obviously, individuals would agree with ($y_1, y_2, \ldots, y_n$) only if they would obtain higher expected utilities by it, i.e., $EU_i(y_i) \geq EU_i(x_i)$ for all i ($i = 1, 2, \ldots, n$), where $EU_i(\cdot)$ denotes individual i's expected utility function. A portfolio ($y_1, y_2, \ldots, y_n$) satisfying $EU_i(y_i) \geq EU_i(x_i)$ for all i ($i = 1, 2, \ldots, n$) is called a feasible portfolio (solution). It is a solution with which a bargaining game could be ended. Most bargaining games have more than one feasible solution. Then, which one of them will be an actual solution for a bargaining game?

Solving a bargaining game implies finding an actual solution. There are two approaches used to find a solution for bargaining games: the axiomatic approach and the strategic approach. Kalai (1985, p.80) demonstrated that the axiomatic approach "proves to be very useful, since it succeeds in choosing a unique solution through a small number of simple conditions. It saves us from having to get involved in the complicated process of bargaining that the players may be going through. Whatever this process is, the players will end with our solution if our axioms are correct for their behavior." This essay follows the axiomatic approach and uses the concept of the Nash solution, initiated by Nash (1950), which is a classical concept of solutions.

Which conditions (axioms) is the Nash solution concerned with? The Nash solution is concerned with the PAR, the SYM, and the IIA axioms.[7] In other words, a solution satisfying the three axioms is the Nash solution. According to the Nash solution, it is proved that the product of the utility functions of the *n* bargaining parties is maximized in a feasible set. Thus the Nash solution solves

$$\underset{y_i}{Max} \prod_{i \in N} (EU_i(y_i) - EU_i(x_i))$$

$$\text{s. t. } EU_i(y_i) \geq EU_i(x_i) \text{ for all } i \ (i = 1, 2, \ldots, n)$$

Under certain conditions, this maximizing problem will give us a unique Nash bargaining solution.

It is worth making more comments about bargaining solutions. First, the rules of the bargaining problem usually permit the final outcome to be determined only by the coalition of all the participants acting together, or by the individual participants acting alone. Therefore, most of the literature on the bargaining problem has concentrated on the special case where $n = 2$. This case is intended to avoid a situation where intermediate coalitions which contain more than one participant but fewer than n, could reach a better solution by forming their own bargaining problem than by joining grand coalition N. But the assumption of $n = 2$ does not mean that there can be exactly two individuals only. It can also be explained as two types of individuals with the same bargaining power. In this essay, $n = 2$ is assumed.

Second, some bargaining solutions, including the Nash solution, are risk sensitive. In Roth (1979), risk sensitivity is defined as follows: If a two-person (i and j) bargaining game (B, d) is transformed into a game (B′, $d′$) by replacing player i with a more risk averse player, then f_j(B′, $d′$) ≥ f_j(B, d). Thus, the utility, which the Nash solution assigns to a player in a two-person game, increases, as his opponent becomes more risk averse. Roth (1979) maintains

[7] 1) Pareto Efficiency (PAR): There is no agreement which is preferred by all players to the solution, and is strictly prefered by at least one player. 2) Symmetry (SYM): This condition guarantees that the outcome does not depend on the labeling of the players. 3) Independence of Irrelevant Alternatives (IIA): It requires that if a player prefers A to B when C is available, then he should still prefers A to B even when C is not available.

that, since a disagreement may occur, the fear of this eventuality may cause a highly risk-averse player to settle for an unfavorable agreement. To avoid the effect of individual's risk aversion on the solutions, it is assumed that individuals have the same utility function, so that they have the same degree of risk aversion in Sections 2, 3 and 4. To see the effect, individuals are assumed to have different degrees of risk aversions in Section 5.

Third, Nash gives us a solution satisfying PAR, SYM, and IIA axioms. But there are doubts about the axioms. The IIA is recognized as the most problematic one. How should we evaluate the Nash solution? On the one hand, despite the doubts, the axioms are not completely implausible and therefore the results are still interesting. Moreover, the Nash solution is the unique one which maximizes the total combined utility gains due to the players' cooperation and at the same time considers the equality among the players (Kalai, 1985). This turns out to be an advantage of the Nash solution over other solutions. On the other hand, we should not stick to the Nash solution. It is not a unique solution for a bargaining game between rational agents. So it is still interesting to look at other feasible solutions with which we may end in a different bargaining process or in a different set of axioms.

Appendix II: Proving the second part of Theorem 4

The assumption is changed a little in order to prove the second part of the theorem. It is assumed that individuals' losses L_A and L_B are identical and independent as before, but that they are distributed continuously with the same density function of $f(x)$, instead of the discrete distributions which is assumed before. The conclusion should not be affected by this change.

Under the assumptions we have, $EL_A = EL_B =: \mu$ and $DL_A = DL_B =: \sigma^2$, where $E(\cdot)$ and $D(\cdot)$ denote expected value operator and variance operator respectively. The total loss by both individuals A and B will be $L = L_A + L_B$. Obviously, L has its density function $g(x) = \int f(x_1)f(x-x_1)dx_1$. And, $EL = EL_A + EL_B = 2\mu$ and $DL = DL_A + DL_B = 2\sigma^2$.

Individuals A's and B's reservation utilities will be $r_A = Eu(w-L_A)$ and $r_B = Ev(w-L_B)$, respectively. If individual A bears a share of the total loss s and individual B bears the others, their expected utilities become $EU_A = Eu(w-sL)$ and $EU_B = Ev(w-(1-s)L)$. Obviously, $\frac{\partial EU_A}{\partial s} = -Eu'(w-sL)L$ and $\frac{\partial EU_B}{\partial s} = Ev'(w-(1-s)L)L$. Consistent with the discrete case, $\frac{\partial EU_A}{\partial s} < 0$, $\frac{\partial EU_B}{\partial s} > 0$, $\frac{\partial^2 EU_A}{\partial s^2} < 0$, $\frac{\partial^2 EU_B}{\partial s^2} < 0$ and $\frac{\partial^2 (EU_A - r_A)(EU_B - r_B)}{\partial s^2} < 0$ in the constraint set of

(Na). Thus, as long as the constraint set is nonempty, the maximizing problem (Na) has a unique Nash solution.

To prove the conclusion is to prove that the Nash solution satisfies $s > ½$, as individual A is assumed to be more risk-averse than individual B.

Taking a second order Taylor series expansion of $u(w–sL)$ around w, and then taking expectations gives

$$EU_A = Eu(w–sL) = u(w) – su'(w)EL + s^2u''(w)EL^2 \qquad (i)$$

Note that $u'''(w) = 0$ under the assumption of quadratic utility functions. Similarly,

$$EU_B = Ev(w–(1–s)L) = v(w) – (1–s)v'(w)EL + (1–s)^2v''(w)EL^2 \qquad (ii)$$

$$r_A = Eu(w–L_A) = u(w) – u'(w)EL_A + u''(w)EL_A^2 \qquad (iii)$$

$$r_B = Ev(w–L_B) = v(w) – v'(w)EL_B + v''(w)EL_B^2 \qquad (iv)$$

Taking a first order Taylor series expansion of $u'(w–sL)$ around w, multiplying both sides by $–L$, and then taking expectations gives

$$\frac{\partial EU_A}{\partial s} = –Eu'(w–sL)L = –u'(w)EL + su''(w)EL^2 \qquad (v)$$

and similarly,

$$\frac{\partial EU_B}{\partial s} = Ev'(w–(1–s)L)L = v'(w)EL – (1–s)v''(w)EL^2 \qquad (vi)$$

Substitute (i), (ii), (iii), (iv), (v), (vi) and $EL_A = EL_B =: \mu$, $DL_A = DL_B =: \sigma^2$, $EL = 2\mu$ and $DL = 2\sigma^2$ into the left side of equation (2′′), a cubic function of s, $h(s)$, is obtained. The solution of $h(s) = 0$ will be the Nash solution.

To get the obvious form of $h(s)$, note that $EL_A^2 = EL_B^2 = (EL_B)^2 + DL_B = \mu^2 + \sigma^2$ and $EL^2 = (EL)^2 + DL = 2(2\mu^2 + \sigma^2)$. And let $R_A(w)(2\mu^2 + \sigma^2) =: \mathrm{M}$, $R_B(w)(2\mu^2 + \sigma^2) =: \mathrm{N}$, $R_A(w)(\mu^2 + \sigma^2) =: \mathrm{M}_1$, $R_B(w)(\mu^2 + \sigma^2) =: \mathrm{N}_1$. Then,

$$h(s) = –2u'(w)v'(w)\{– 4\mathrm{MN}s^3 – 2(2\mu\mathrm{N}–2\mu\mathrm{M}–3\mathrm{MN})s^2 + (4\mu^2+7\mu\mathrm{N}–\mu\mathrm{M}–2\mathrm{MN}+2\mathrm{N}_1\mathrm{M})s – (2\mu^2+3\mu\mathrm{N}–\mu\mathrm{N}_1+\mu\mathrm{M}_1+\mathrm{N}_1\mathrm{M})\}$$

That $\frac{\partial^2 (EU_A - r_A)(EU_B - r_B)}{\partial s^2} < 0$ in the constraint set of (Na) gives $h'(s) < 0$ in the constraint set. Therefore, $h(s)$ is a decreasing function of s. If $h(½) > 0$, then the Nash solution will be $s > ½$. Substitute ½ into $h(s)$, we will have $h(½) = u'(w)v'(w)(R_A(w)–R_B(w))\mu\sigma^2 > 0$, if $R_A(w) > R_B(w)$. The second part of the theorem is thus proved. #

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Essay III

Mutual Cooperatives: Their Formation and Development

by

Hong Wu*

1. Introduction

How are mutual cooperatives formed and developed in the insurance industry? The general argument is that mutual cooperatives are formed to decrease distortion due to asymmetric information. O'Sullivan (1998) reviewed most of the theoretical and empirical papers related to the formation of mutuals, and concluded that some mutual cooperatives appear to be "due to the coming together of specific professions or industries who perceive themselves as being low risk and who view mutuality as a method of avoiding the diversity of risk types which proprietary companies attract".[1] Two exceptions are Hansmann (1996) and Skogh (1999). Hansmann (1996) reviewed his early paper, Hansmann (1985), and suggested a number of reasons for the evolution of mutual insurance cooperatives. Among others, he pointed out that mutuals appear when the loss experience is difficult to predict. Skogh (1999) highlighted this point by presenting a theory on risk-sharing institutions for unpredictable losses. He argued that when there is uncertainty on the probability of risk, the mutual contract can be an alternative to the insurance contract.[2] While others suggested that mutuals appear on an existing risk market, i.e., on a market where some insurance companies are already operating against the considered risk, Hansmann (1996) and Skogh (1999) suggested that mutuals appear to be against a risk, to which insurance may not be available, or be very expensive.

In a mutual sharing society, individuals get together and share their risks with each other. Essay I proves that the core of the mutual game is nonempty, which means that stable mutual

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[1] An example is Wisconsin Lawyers Mutual Insurance Company (WILMIC). "WILMIC was founded in 1986 by a group of lawyers who wanted to provide an alternative to the commercial insurance market for Wisconsin lawyers in private practice who needed a stable source of professional liability insurance" (See website: http://www.wilmic.com).

[2] Skogh (1999) concluded this by proving that the equal-share allocation of the total risk in the pool makes individuals better off when they presume that their distributions of risks are identical.

sharing is always possible. Essay II discusses how the mutual contract[3] is designed. Although the term "mutual cooperative" is used in Essays I and II, it normally refers to a set of mutual contracts, which individuals sign with each other, not to an organization. So the focus in Essays I and II is in fact on the mutual contracts. The question to be addressed in this essay is why and how a mutual society, which may consist of only a few individuals, develops into the mutual cooperative, an organization that is competitive with insurance companies. This issue is analyzed in Section 2 by using the transaction cost theory. If it can be pointed out that there are transaction costs in a mutual sharing society, then the transaction cost theory tells us why the small mutual society need to be developed into an efficient organization.

Section 3 describes some of the characteristics that make the mutual cooperative an efficient organization. As it will be seen, mutuals are organized in a similar way to the stock insurance companies. This is why the mutuals can exist in the market as efficiently as the stock insurance companies. However, this does not mean that there are no conversions (mutualization or demutualization). The reason why conversions appear is a topic beyond the discussion of this essay, which only looks at the existing mutuals, not the transition.

In Section 4, a few examples that illustrate the formation and development of mutual cooperatives in the Swedish insurance industry will be given. Most of the arguments about the mutual contract and the formation of mutuals will be confirmed by these examples. The final section will summarize the essay and draw conclusions.

2. From mutual contract to mutual cooperative

A large number of studies have investigated the reasons why firms exist. This issue was first raised in Coase (1937) over 60 years ago. Since then, discussions on related topics have continued. There are papers surveying this matter, e.g., Holmstrom and Tirole (1989) and Lin and Nugent (1995). In analyzing the demand for firms, the transaction cost theory has proved to be useful. This theory has also been applied toward explaining alternative institutional arrangements other than firms. Lin and Nugent (1995, p2306) defined an institution as "a set of humanly devised behavioral rules that govern and shape the interactions of human beings, in part by helping them to form expectations of what other people will do." According to this definition, institutions are more close to rules, such as laws, regulations, social norms and so on, while firms, enterprises, families, and states are organizations which have to follow the

[3] The distinctions between the mutual contract and the general insurance contract have been drawn in Essays I and II. Simply speaking, there is no pure insurer and there is no fixed pure risk-premium in the mutual contract, as opposed to the insurance contract, in which they exist.

rules. However, there is no clear line that delimits institutions and organizations. Thus, when the term "institution" is used, it often means organization too. So mutual cooperatives can also be recognized as a type of institution and the transaction cost theory can be used to discuss the formation of mutuals.

Basically, transaction costs include those of organizing, maintaining and enforcing the rules of an institutional arrangement. Lin and Nugent (1995) classified the transaction costs into direct and indirect costs. Direct costs include those of obtaining the information, of negotiating among the parties to reach agreement in the provisions of the contract, and of communicating all such provisions to all the relevant agents. Indirect costs include those of monitoring and enforcing the terms and conditions of the contracts and the output lost due to contractual default. Costs that occur before the transaction takes place are called *ex ante* costs and those that occur after the transaction takes place are called *ex post* costs. It is not the purpose of this essay to show and explain all the transaction costs in general. The main point in the transaction cost theory is that it is the avoidance of the costs of carrying out transactions through the market that explains the existence of the firm in which the allocation of factors comes about as a result of administrative decisions (Coase, 1937). More generally, according to the transaction cost theory, institutions, and therefore mutual cooperatives, arise in order to economize the transaction costs.

Are there transaction costs in a mutual society? Yes, there are. First, they exist because it is generally good to have more individuals in a mutual pool. In other words, a risk averse individual's utility increases as the number of individuals in the same society increases. This conclusion is actually trivial if ones are familiar with a basic result in risk management and insurance that the pooling activity decreases individuals' risk. Let us illustrate the idea by analyzing a simple case in a mean-variance utility approach.

Assume that there are n individuals, each of them facing a possible loss, and all the losses facing them are distributed independently and identically. According to Essay II, the individuals will be better off by joining a mutual pool and sharing the total loss equally, no matter if they have assessed the probabilities of the losses. Thus, if individuals face loss L with probability p and 0 with probability $1-p$, then, when an individual (say individual j) does not join any mutual pool and bears this possible loss by himself, the mean value of his final payment related to this loss will be

$$\begin{aligned} EP_j = EL_j &= pL + (1-p)0 \\ &= pL \end{aligned}$$

and the variance will be

$$DP_j = DL_j = p(1-p)L$$

By signing a mutual contract with other $n-1$ individuals and sharing the total risk equally, individual j will pay for

$$P_j = \frac{1}{n}\sum_{i=1}^{n} L_i$$

in which L_i (i = 1, 2, …, n) is equal to L with probability p and equal to 0 with probability $1-p$. Thus, the mean value of P_j will be

$$EP_j = \frac{1}{n}\sum_{i=1}^{n} EL_i = pL$$

And the variance is

$$DP_j = \frac{1}{n^2}\sum_{i=1}^{n} DL_i = \frac{1}{n}p(1-p)L$$

There is the same mean value when individual j signs the mutual contract with others and when he bears his loss by himself. However, as n increases, DP_j decreases. This proves that the risk-averse individual's utility increases as n increases in a mean-variance utility approach.

As the number of individuals in the pool is important, it is necessary for the individuals to find partners.[4] Thus, the cost of finding partners becomes a direct transaction cost for the mutual pool. However, even if the partners have been found, there are still costs of negotiation on agreements. Essay I found that when individuals' losses are different, stable share rule may depend on an individual's index, which means that individual A may share individuals B's and C's loss in different proportions,[5] which makes the contracting cost very high.

Besides, transaction costs can also appear indirectly. Let us examine Alchian and Demsetz's model (1972). This model considers the incentive problems of team production. The basic idea is that if inputs cannot be verified so that rewards must be based on output alone, team production will lead to a free-rider problem. As a result, one has to introduce a monitoring system into the production, which obviously produces a transaction cost. Although mutual contracts against similar risks can be started on a small scale, which favors mutuals in solving the free-rider problem, this problem will still exist if an individual's potential to deal

[4] Example 1 in Section 4 shows that, when the fire-support was decentralized, people made attempts to extend the recruiting area.

[5] In Examples 1 and 2 in Section 4, it will be seen that distinction was made for different risk-holders to have the mutual cooperative successful.

with the possible loss cannot be observed by others.[6] By using a similar model to the ones in Alchian and Demsetz (1972), and in Holmström (1982), free-rider problems in mutuals can be observed. Thus, according to Lin and Nugent (1995), indirect costs exist.

The existence of the transaction costs proposes a need to develop mutual contracts and to turn them into organizations (enterprises). To initiate an organization, one or more people devotes time and maybe money to the promotional organization. In nearly every case, the operation is guided by a strong promoter, who may eventually become the leader of the mutual. Leadership may come from a small group of potential members of the mutual. However, leadership may also be provided by persons or organizations who are not motivated by the gain as members. For example, some mutual insurance cooperatives have been built by individuals who expected to gain by commissions.[7] However, it is the leadership that launches the efficient organization structure of the mutuals.

Thus, the transaction cost theory gives one of the reasons why the mutual contracts need to be developed into an efficient organization. To organize the mutual as an enterprise can decrease the transaction costs related to the risk sharing management. In addition, the more efficient the management, the more the cooperative can attract and hold members' patronage. The efficiency enables the mutual to achieve its long-term success in the insurance industry.

In the next section, we look at some of the characteristics of existing mutual cooperatives. As we shall see, these cooperatives can function as well as stock insurance companies.

3. The mutual cooperative as an efficient enterprise

Generally, in mutual cooperatives, policyholders as "customers" own the mutual. The objective of mutual cooperatives is not to maximize their own net gain like their stock counterparts. Each mutual cooperative, as an enterprise, strives to spread its members' risks with minimum costs. Of course, mutual cooperatives are, like their stock counterparts, subject to insurance regulations. The regulation system in Sweden is organized on six policy principles: the solvency principle, the fairness principle, the principle about business

[6] Example 3 in Section 4 shows that, when policyholders were discontented about the behavior of mutuals, the mutual company took action to improve the internal monitoring of risks, thus indicating the importance of solving the free-rider problem and of decreasing the related transaction costs in mutual cooperatives.

[7] For instance, James S. Kemper was "an unusually effective entrepreneur". "After a meteoric rise in an Ohio-based retailer-owned mutual he became owner-manager of a sales agency established in Chicago to recruit member-policyholders for a group of fire mutuals of the region. Later he helped form other mutuals, notably what is now the giant Lumbermen's Mutual Casualty Company, and for years was president of some of them. He, not the member-policyholders, provided the entrepreneurship" Heflebower (1980, p 166).

restrictions, the principle of need, the principle of separation, and the principle about the influence of policyholders. See Hägg (1998) for details.

It has been pointed out that in order to initiate a mutual insurance cooperative, promoters are needed. However, a substantial size for an initial mutual cooperative is not really necessary. A mutual pool can be started by only one mutual contract where maybe only two individuals are involved. However, the survival of mutual cooperatives depends on efficiency. The scale of mutual cooperatives helps them in their capacity to spread risks, and it also helps them in raising their investment capital if prepayment is required. To make a mutual cooperative work efficiently and be able to compete with stock insurance companies, it is necessary to expand it by letting more individuals join the pool.

Efficient investment also enables the mutuals to achieve long-term success and to be competitive with stock companies in the market. Mutuals can borrow from commercial banks if their credit standing is good. Debts through bank credits, introduced prepayment, and mutuals' surplus constitute the mutuals' investment capital.

By definition, policyholders are in principle not required to give prepayment. However, to lower the cost of sharing risks, they are required to put some amount of money into the pool in advance. This requirement changes individuals' payment-time, which may affect the decision of those individuals who face the liquidation problem. However, it allows mutuals to achieve long-term efficiency. The prepayment requirement achieves at least three purposes. First, an important precondition for not requiring prepayment is about trust, which implies that *ex ante* committed share rule can be enforced *ex post* in case of accident. Prepayment obviously mitigates possible default. Second, prepayment serves to diversify risks in the time dimension. Thus, risks are diversified in dimensions of both time and space. Third, when prepayment is introduced, collected capital can be used to make investment so that mutuals can earn investment income. Investment income belongs to all policyholders in the pool[8] and therefore, policyholders' final payments will be deducted by the investment income. How does the investment income benefit policyholders in mutuals? If policyholders can make investment somewhere else and obtain the same return as they invest into the mutuals or obtain a higher return, then they will not be better off by investing in the mutuals from the perspective of supplying investment capital. Otherwise, if mutuals can produce the same or a higher return in investment, policyholders will not be worse off.

[8] This is different from stock insurance companies, in which investment income belongs to shareholders.

It is possible that mutuals' revenues exceed the cost of conducting the cooperatively owned activity when prepayment is required. Thus, net savings may arise from the mutuals' operations. Theoretically, these savings belong to the policyholders in proportion to patronage, and therefore they should be paid back to the policyholders as dividends, which adjusts the policyholders' amount of payment. However, part or even all of the savings may be withheld to augment the cooperative's capital accumulation and to protect mutuals against adverse contingencies such as heavy losses. A proportion of the withheld savings can also be used as investment capital of following periods. Investing part or all of these savings rather than distributing the money as dividends can help the mutuals to achieve the efficiency required for long-period success simply due to the economics of scale in investment. The question is how much a mutual cooperative should pay out as dividends to its policyholders and how much it should retain in its capital account. This question of course concerns all firms as well as mutuals and stocks. However, in the case of stock companies, efficiency is reflected in the value of the company, in other words, in the share market. If stockholders do not get full dividends, the benefit of savings is reflected in the share market. When stockholders want to withdraw from the company, they have their shares sold in the market. Therefore, they eventually obtain the value of the savings. In mutual cooperatives, members who withdraw are not paid their shares or that part of the mutual's surplus which their savings have produced.[9] Thus, the problem of how much to pay out and how much to retain becomes more crucial in mutuals than in stocks. A paper by Scordis and Pritchett (1998) considered the matter of policyholder dividend policy in mutual life insurance industry. As shown by the paper, the trade-off between retaining cash as surplus and paying it out as dividends keeps policyholder dividends at a reasonable level.

Another observation about savings is that new members of the mutuals are subsidized by both the current members and the former members because they enjoy benefits from the capital accumulation of the current and the former members. Heflebower (1980) argued that the current members approve because the new members' volume may affect the cooperative's costs per unit favorably and offset the lost volume as old members withdraw.

With regard to the issue of financing, mutual cooperatives cannot turn to the capital market to raise capital. As mentioned above, capital partly comes from a portion of underwriting profits and investment income. In addition, mutuals can borrow from commercial banks if

[9] This does not mean that policyholders in a mutual cooperative will not be paid back when the mutual cooperative is liquidated. Actually, a major concern of the insurance commissioner is the equitable division of

their credit standing is good. Debts do not only supply the mutual cooperative investment capital but they can also help the mutual cooperative with its solvency.[10] Financing is an important issue that differentiates mutual cooperatives from stock insurance companies. Access to capital enables stocks to have an advantage over mutuals, and it is recognized as one of the reasons for the demutualization of mutuals.

Let us turn to look at the contract structure in mutuals and in stocks. Essay II shows that mutuals and stocks enjoy some consistency: The same amount of risk premium is required against the same risk, and the high-risks are required to pay higher risk premiums than the low-risks. Is there an eventual consistency in the contract structure of mutuals and stocks? Since the design of the insurance contract highly depends on the solution of the asymmetric information problem,[11] the way in which mutuals design the mutual contract in order to solve the problem of asymmetric information needs to be discussed.

It is generally believed that mutual cooperatives are in a better position to solve the asymmetric information problem than insurance companies. Smith and Stutzer (1990 and 1995) had a special focus on this. Actually, the success of mutuals is mostly traceable to the reduction of members' losses. But this does not mean that there is no asymmetric information problem in mutuals. First, as it is pointed out in Section 2, although mutual contracts favor mutuals in solving the free-rider problem, the problem still exists. The existence of moral hazards can be attributed to the existence of the free-rider problem. Second, in terms of the existence of adverse selection, Essay II points out that if there are two individuals with different probabilities of losses, then the Nash solution will be that the high-risk individual bears a greater share of the total loss than the low-risk individual. Now, suppose that the high-risk individual can hide his condition. He announces that he is a low-risk individual. By doing this, he shares a proportion which is supposed to be shared by the low-risk individual. Thus, he shares a less amount than he should, and he obtains a higher utility than he should. As a result, the pool will either be running short, or the true low-risk individual has to share more than he should and may therefore ask for split-up from the mutual pool. In either case, the original pool breaks down. Thus, the asymmetric information problem exists in mutuals.

How do the mutuals solve the problem of asymmetric information? Studies about solving the asymmetric information problem usually focus on the typical insurance contract in the

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the mutual's surplus among policyholders, when demutualization is applied for.

[10] Example 2 in Section 4 illustrates this.

[11] For example, solving the problem of moral hazard suggests the use of partial coverage, of coinsurance, of insurance deduction, and of Bonus-Malus system in the insurance contract.

stock insurance company, not on the mutual contract. If it can be confirmed that the way of solving the problem in a mutual society is the same as that of solving it in the stocks, then it is unnecessary to have a detailed discussion about solving the asymmetric information problem in the mutual case and conclusions from the discussion in the stock case can be directly applied to the mutual case.

Asymmetric information can be a problem only if there are more than one individual in the pool. The more individuals in the pool, the more serious the problem will be. Thus, let us assume that the pool is relatively large in order to show that the mutual contracts eventually end with a rather similar structure to the insurance contracts. With this assumption, it is actually assumed that the mutual contracts are signed in a mutual organization and a manager who works with the efficiency of the organization is dealing with the problem of asymmetric information. With the assumption, we will see that the problem in mutual cooperatives can be modeled and solved in the same way as in stock insurance companies.

Let us look at a basic insurance model of discussing the problem of asymmetric information. The most basic model used to analyze this problem considers a single period contract in a competitive market. Thus, it is usually assumed that insurers are risk-neutral, which implies that insurance companies produce zero expected profit at the equilibrium. The zero expected profit means that insurance companies are break-even, which “coincides” with the case of mutual cooperatives; Mutual contracts are by definition designed in such a way that the pools are break-even. Furthermore, in both insurance companies and mutual cooperatives, there are managers who act on behalf of investors, shareholders in insurance companies or policyholders in mutuals, in order to achieve this break-even constraint. Thus, it is obvious that the contracting solution found by managers should be the same, which implies that there is a general consistence in the contract structure of mutuals and stocks.

4. Mutual cooperatives in Sweden

In this section, three examples related to the development of fire-support in the Swedish insurance industry will be given. As mentioned before, the purpose of presenting these examples is to illustrate the formation and development of mutual cooperatives in the Swedish insurance industry.

The arguments about the formation and development of mutuals can be summarized as follows. Mutual cooperatives can be initiated by a small group’s mutual contracts. To survive in the industry and to compete with their stock counterparts, the small mutual society must take the form of an efficient organization. In addition, the mutual cooperative, as an efficient organization, has to be successful in underwriting management and investment management. Finally, there are some similarities between the mutual contract and the insurance contract.

The following examples are taken from Hägg (1998). Although they cover a few cases only, they tentatively illustrate the formation and development of mutual insurance cooperatives in Sweden.

Swedish insurance institutions and insurance organizations were evolved from the late Middle Ages until the implementation of the Insurance Business Act of 1903. During this period, the foundation for a modern Swedish insurance market became established. The New Insurance Business Act in Sweden, with minor changes from the Insurance Business Act of 1948, has been in force since 1982.

Example 1: The mandatory fire-support.

This is the earliest insurance-like institution recorded at the Swedish countryside and it has been a legal institution from the 13th and 14th century. At that time, there were some provincial laws regulating the handling of fire, as well as liabilities, damages, and penalties for fire accidents. Fires were considered to be a result of human activities. The proceedings for stating the cause of and the liability for a fire were undertaken in an open rural court led by a local judge. A person found liable was obligated to pay a fine and/or to compensate victims of the fire. If the person liable was short of resources, the provincial laws offered an alternative or a complement in the form of mandatory fire-support from the people within an administrative county district. Thus, this mandatory fire-support was a complement to the tort and crime rules. All inhabitants that possessed buildings, irrespective of the social class, had to contribute to the approved fire-support. Nevertheless, available information indicates that the fire-support was in fact ignored before the 18th century. It was occasionally replaced by tax exemption and/or by fire begging[12]. The follow-up development in Hägg (1998) was divided into two-steps: *tariff societies* and *parish societies*.

Tariff societies. At the time, taxes were set in relation to the occupation of land, which was mainly owned by the State. When a fire hit a village, the tax income was affected. To secure the stock of buildings in the countryside and maintain the peasantry's ability to pay taxes to the State, the legal fire-support institution underwent reform. The reform of the central government did not alter the main principles of the medieval institution, but encouraged a revision of its organization. As before, the activities were administrated by means of the

[12] Fire begging can be understood as a custom where fire victims within an administrative county district collected support directly, instead of having it enforced by the rural court.

judicial system and the local civil administrative bureaucracy. In collaboration with the county governor, the inhabitants in an administrative county district had to make precise the norms of the local fire-support institution. Hence, the public governance of mandatory fire-support was decentralized and therefore attempts were made to extend the recruiting area in order to diversify the risk. Moreover, some societies started to incorporate crude forms of insurance techniques such as a refined discrimination of contributions, yearly fixed payments, a fund of surpluses, and full compensation for ordinary buildings but limited compensations for larger estates.

Parish societies. The Parliament approved the establishment of new parish societies in 1766. These societies became established among neighbors in local parishes. In some provinces, the county based fire-support institution split up into small mutual, but still mandatory, fire-support societies administrated by the inhabitants in the parish. Despite setbacks in the spreading of the risks, this program was considered a success among the peasantry. In an investigation made in 1813, the peasantry expressed an overall satisfaction with the scheme.

The tariff and parish societies constituted the cradle of the Swedish half-public mutual fire and property insurance organization in the countryside. Despite the fact that the mandatory legal fire-support system tended to degenerate or cease to exist during different periods and different places, and the legal institution of fire-support was finally replaced in 1853-56, many societies became transformed into voluntarily local mutual fire-support companies. As such, they maintained a strong position among the rural population until the 20th century. According to official statistics, there were still 368 local mutual fire-support companies in 1902 that ran a business covering a local area smaller than a county.

<u>*Comments on Example 1*</u>: This earliest widely spread insurance-like fire-support was a mutual-like institution. In this example, the fire-support was only against fire, worked only within local areas, was mandatory and asked for only *ex post* payment at the beginning of its formation. Step by step, the importance of recruiting policyholders to diversify the risk and the importance of using insurance techniques to achieve efficiency are realized, e.g., distinguished contracts for distinguished risk holders and introducing prepayment. All this is consistent with the arguments that similar risk forms mutuals,[13] that mutuals start with a small scale and do not require *ex ante* payment at the beginning of their formation[14] and that as

[13] All papers on mutuals are consistent with regard to the argument of similar risk forming mutuals.

[14] The view that mutuals do not require *ex ante* payment was emphasized in Skogh (1999).

mutuals become mature, advanced insurance techniques should be introduced in order to achieve efficiency.

The fire-support mutual society was first mandatory and then became transformed into voluntary mutual company in some districts. The mandatory fire-support is more like a tax matter rather than an insurance matter. However, the fire-support is intended to protect the peasantry from the fire and thus functions in the same way as the insurance. Actually, the mandatory helps to prevent possible default. In order for mutuals (in a framework of *ex post* payment) to be set up and operate continually and successfully, it is very important that *ex post* are able to enforce *ex ante* committed compensation in case of an accident.

Example 2: The Stockholm fire office.

This fire office (*The Stockholm Stads Brand- och Försäkrings-Contoir*) was set up in 1746 and it constituted the raw prototype and the source of inspiration for later urban mutual fire insurance companies. Membership in the fire office was voluntary. After 12 years, the policyholders were promised insurance protection for all future fire accidents in exchange for a yearly fireguard fee which was less than a permillage of the value of the insured object. By using front-loaded schemes, solvency would be attained relatively quickly. From a long-term perspective, the favorable future terms made the offer look inexpensive. It was assumed that the yield from the reserve would cover the cost for future liabilities. Following the practice at that time, the company code regulated contractual conditions as well as tariffs. A rough distinction was made between fire risks for timber houses, half-timber houses, and stone houses. A board of directors, responsible for 100 representatives of the policyholders, had to be elected to govern the office. Auditors kept the accounts. Executors, partly appointed by the fire victims and partly by the officials of the pool, provided for claim-settlements. Dissatisfaction with the claim settlement could be appealed, and the procedure was redone.

After the set-up of the fire office, the pool faced the challenge of major fires in 1751 and 1759. The claimed compensations were on both occasions larger than the collected fund and far more expensive than what was at disposal. Threatened by bankruptcy, the board managed to get rid of the insolvency crises through bank credits. By having fulfilled promises under severe circumstances twice, the pool attracted attention and built up a favorable reputation. In the year of the first large fire, the pool signed 308 new policies and the recruitment of new policyholders improved in the following years. Despite higher premiums, the growth of the pool was remarkable. In the 1790’s, almost all buildings in Stockholm were insured.

For the rest of the period, Stockholm was spared from large fires. Still, since the fires in the 1750's, the collected fund remained small for decades. The percentage of the total liability covered through funded means was less than 2% in the mid-1760's. Considering the fact that the fire office administered insurance where the yield of the capital aimed at covering all reimbursements, the solvency was extremely low. So Hägg (1998) used the term "sheer luck" to express the survival of the pool.

Comments on Example 2: The fire office was a voluntary system. It introduced prepayment to mitigate insolvency and to produce favorable future return. It also designed different contracts for different risky holders, elected directors from its policyholders to govern the office, and took debts to solve insolvency. All this supports the arguments made before about the development of mutuals.

However, the idea of the fire office's formation was not closely based on the coming together of individuals who want to share their risks with each other. The idea was mostly based on the mutual fund obtained by individuals' prepayment, which could diversify risk in time and also produce yields against the fire. The prepayment was calculated according to experience from fire accidents.

As the additional assessment of the mutual fund was not allowed in this system in the case of shortage of money, large fires challenged the fire office. The office used debts to solve its insolvency crises. As a result, it set up a favorable reputation so that it could recruit many new members and had a larger pool to diversify risks. This supports the view that reputation is actually an intangible and beneficial asset to any firms and it is sometimes called reputation capital. In the theory of the firm, it has been argued that the soul of the firm is its reputation. Here, reputation is of course a beneficial asset for the mutuals. From the next example, we will see that a change in the reputation is an important factor in introducing insurance techniques among mutuals.

Example 3: The reorganization of the mutual fire insurance industry.

In 1888, major fires struck some cities in the northern part of Sweden. These events came to be a watershed for the mutual industry because the stock company[15] could honor the reimbursement that followed thanks to its reserves and reinsurance. Mutuals had not yet made use of reinsurance and thus had insufficient reserves to stand the losses. To avoid bankruptcy,

[15] The first stock insurance company, Skandia, was formed in 1855. Follow-ups include Svea, Sverige, Skåne, and so on.

the mutuals had to take out extra charges. The predicament caused widespread discontent among policyholders, while, at the same time, the credibility of stock companies increased.

The discontent about extra charges triggered a comprehensive modernization of mutual fire insurance companies with the entire country as a recruiting area. In one of the mutuals that was among the worst hit by losses, *Städernas Lösöresförsäkringsbolag*, a far-reaching reorganization took place shortly after the catastrophes. The company substantially increased its reserves, started to make use of risk statistics, set the premiums on business terms, introduced reinsurance, hired agents, improved the internal monitoring of risks, and employed an executive officer who had earlier worked in the stock company, *Svea*. Later, in the second half of the 1890's, the other mutual insurance company that had been severely hit by the fires in 1888, *Städernas Allmänna Brandstodsbolag*, came to effectuate a similar reorganization of its business. In the closing stages of the 19th century, mutual cooperatives followed with a gradual modernization of their organization and operation as well. Eventually, many of the earlier attributes that distinguished mutuals from stock companies were extinguished, as they all offered services on business terms.

Comments on Example 3: This example illustrates how a modern mutual organization is eventually formed and developed. Although mutual cooperatives can originate from a small scale and "primitive" techniques, the development of mutuals depends on the use of comprehensive insurance techniques.

5. Concluding remarks

Typical cooperative companies differ in many ways from investor-owned companies. But the differences have not handicapped successful cooperatives. Mutual insurance cooperatives enjoy a significant position in the insurance industry. As a form of organization, the mutual cooperative is an alternative to investor-owned stock insurance companies. This essay argues that a small mutual society, where mutual contracts are signed, will eventually be developed into an efficient organization and take the form of an enterprise to play its role in the insurance industry. In other words, mutual cooperatives can be initiated from the small mutual societies. In addition, the essay shows that the survival and growth of the mutual depends on its efficiency and on its fighting capacity. When the same type of insurance is available in both mutual cooperatives and stock companies, the success of the new mutual then depends on how long it can grow to a relatively efficient size and on how efficient it works so that it can be competitive with its stock counterparts.

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— Part II —

The Effects of Background Risks on an Individual's Insurance Decision

Essay IV

Optimal Crop Insurance with Multiple Risks

by

Hong Wu

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Essay V

Farmer's Decision Making: Insurance and Derivative Security

by

Hong Wu*

1. Introduction

A number of studies have pointed out that background risks have significant effects on an individual's hedging decision against any one of the risks he faces.[1] This essay focuses on the effect of derivative securities on an individual's insurance decision. To study the effect, crop insurance is taken as an example. A farmer's income from a specific crop is equal to the product of the crop's price and its output. A farmer can buy the crop insurance to protect himself against a decrease in the crop output. He can also join the futures market or the futures option market to protect himself against a decrease in the crop's price. Thus, when both insurance and derivative security markets exist, a farmer's decision making becomes a good example of studying the effect of derivative securities on an individual's insurance decision.

Why does the effect of derivative securities on an individual's insurance decision need to be studied? Most papers studying the effect of price risk on farmer's insurance purchasing (Ramaswami and Roe (1992), Mahul (2000), and Essay IV in the thesis) assume that price risk is uninsurable. However, the price risk can be hedged through the futures market and the futures option market. Hence, to study the effect of derivative securities on an individual's insurance decision becomes an extension of the previous studies. Another reason for analyzing this issue relates to the concept of *variable participation contract*, which is newly defined in Doherty and Schlesinger (2001) and which will be explained in Section 4.

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* Dept. of Economics, Göteborg University, P.O. Box 640, SE 405 30 Gothenburg, Sweden. Email: Hong.Wu@economics.gu.se. This essay was initiated when I was in the Wharton school as a visiting PhD student. I am grateful to Professor Clas Wihlborg and Professor Richard Kihlström for making my visit to Wharton possible. I am also grateful to Professor Neil Doherty, Professor Göran Skogh and Professor Clas Wihlborg for their comments. Finance support from STINT is gratefully acknowledged.

[1] For example, Imai *et al*. (1981), Doherty and Schlesinger (1983a), Doherty and Schlesinger (1983b), Mayers and Smith (1983), Turnbull (1983), Schulenburg (1986), Briys, Kahane, and Kroll (1988), Dionne and Gollier (1992), Eeckhoudt and Kimball (1992), Meyer (1992), Ramaswami and Roe (1992), Gollier and Schlesinger (1995), Guiso and Jappelli (1998), Meyer and Meyer (1998), Mahul (2000) and Essay IV in this thesis.

In this essay, Section 2 presents the basic models, which respectively concern crop insurance only, futures only, and both crop insurance and futures simultaneously. We will see that there are situations where a farmer will not buy the full insurance even if the premium is fair because he uses hedging instruments against the price risk. Thus we see the effect of derivative securities on an individual's insurance decision. In Section 3, two extensions of the basic models are given to first investigate the effect of a farmer's risk aversion on his hedging decision. We will see that the effect appears differently when the crop insurance and the derivative securities are concerned separately and when they are concerned simultaneously. The second extension is to consider the effect of futures option. The aim is to show a symmetric pattern between the effects of the insurance and the derivative security markets. This enables us to explain the model broadly. Section 4 explains the concept of the variable participation contract and investigates the relationship between the concept and the models. The final section summarizes the essay.

2. The basic models

Suppose that all farmers are identical and a representative farmer is risk-averse with a mean-variance utility.[2] Then, when the representative farmer's income is R, his utility function can be defined as $U(R) = ER - \phi VarR$, where $\phi > 0$ gives the degree of the farmer's risk aversion and $E(\cdot)$ and $Var(\cdot)$ denote the expectation operator and the variance operator respectively.

Assume a single period model from a first harvest time, time 0, to a second harvest time, time 1. At time 0, the representative farmer's output of a crop is Q_0 and the crop's market price is p_0. The farmer must make his insurance decision and his hedging decision on the futures at time 0 against a possible income decrease facing him at the second harvest time, time 1. Let Q be the farmer's output and p the market price at time 1. Assume that $Q \geq 0$ and $p \geq 0$. A correlation coefficient $\rho_{p,Q}$ denotes the relationship between the representative farmer's output and the market price at time 1.

2.1. Concern crop insurance only

Crop insurance protects farmers from crop losses caused by natural hazards. Farmers select both yield guarantee and indemnity price at the time of purchasing the insurance. Indemnity

[2] The mean-variance ($\mu-\sigma^2$) and the mean-standard variance ($\mu-\sigma$) models are widely used in studying farmers' decision making (Hazell, 1982).

payments occur when actual yield falls below the yield guarantee and the indemnity payment equals the yield shortfall, times the indemnity price.[3]

In the model, *the yield guarantee* is fixed as *the expected output*, *EQ*, and *the indemnity price*, denoted by p_i, is assumed to be *any price no less than 0*. The constraint of a nonnegative p_i makes the coverage function nonnegative, which is considered as institutional by Arrow (1971). By having the yield guarantee fixed, the representative farmer selects only the indemnity price, instead of both the yield guarantee and the indemnity price. The rationality of having this assumption is that the yield guarantee and the indemnity price appear symmetrically in both the indemnity payment formula and the premium formula[4]. The indemnity payment is equal to the yield guarantee multiplied by the indemnity price. Hyde (1996) showed a negative relationship between the optimal solutions of the yield guarantee and the indemnity price. Therefore, fixing the yield guarantee and at the same time letting the indemnity price remain little restricted does not affect a lump-sum indemnity payment.

In making his decision on the crop insurance at time 0, the representative farmer has to select the indemnity price p_i. With the above assumptions, the *compensation* that the farmer can obtain if he buys the insurance and selects the indemnity price p_i, will be, in currency,

$$I(Q) = \begin{cases} 0 & Q \geq EQ \\ p_i \times (EQ - Q) & Q < EQ \end{cases}$$

$$= p_i \times Max(EQ - Q, 0).$$

Note that $Max(EQ - Q, 0)$ looks like the payment of a PUT option, denoted by PUT_Q, with a "strike price" of EQ and an "asset price" of Q. We therefore rewrite $I(Q)$ as $p_i PUT_Q$.

This insurance contract takes the form of deductible, which is obviously consistent with the general argument on the optimal form of insurance by both Arrow (1971) and Raviv (1979), summarized in Gollier (1992).

[3] According to the Multiple Peril Crop Insurance (MPCI) policy provided in the United States, farmers select both yield-selection and price-selection at the time of purchasing the insurance. The yield-selection and the price-selection are both percentage numbers, with which the insurance's yield guarantee and indemnity price are fixed. Usually, Yield Guarantee = Actual Production History (APH) × Yield-Selection. Indemnity Price = Federal Crop Insurance Corporation (FCIC) Price × Price-Selection. APH and FCIC Price are fixed from the perspective of the farmer's insurance decision. Thus, to select both yield-selection and price-selection are in fact to select yield guarantee and indemnity price.

[4] Premium is related to the total liability, i.e., the total indemnity payment, not to either the yield guarantee or to the indemnity price.

Besides, the *premium*, denoted by P, is defined as *the expected indemnity payment*, i.e., $P = EI(Q) = p_i EPUT_Q$. In other words, a pure premium or a fair premium is defined. The assumption of a pure premium is standard in studying optimal insurance models. Thus, with the consistence in the assumption of the contract structure, and with the standard assumption on the premium, the model can be explained more generally than just within the field of crop insurance, which will be done in Section 3.3.

To put it in another way, the premium P is the price of $p_i PUT_Q$. If there is a linear relationship between the number of commodity and the total payment, and if we let P_{PUT_Q} denote the price of the put PUT_Q, then the premium P is p_i times the price of the put, i.e., $P = p_i P_{PUT_Q}$. Thus, $P = p_i EPUT_Q$ actually means $P_{PUT_Q} = EPUT_Q$.

In order to define the farmer's optimal program by considering the crop insurance only, let us first look at the farmer's income, which will be

$$\begin{aligned} R_i &= pQ + I(Q) - P \\ &= pQ + p_i(PUT_Q - EPUT_Q) \end{aligned}$$

and therefore, $ER_i = E(pQ)$ and $VarR_i = Var(pQ) + p_i^2 Var(PUT_Q) + 2p_i Cov(pQ, PUT_Q)$. Thus, the farmer's optimal program will be

$$\underset{p_i \geq 0}{Max}\, U(R_i) = ER_i - \phi VarR_i$$

Since the objective function $U(R_i) = ER_i - \phi VarR_i$ is a concave function of p_i, the first order condition gives the unique optimal solution. If $\rho_{pQ,PUT_Q} < 0$, the optimal indemnity price will be uniquely

$$p_i^0 = -\frac{Cov(pQ, PUT_Q)}{Cov(PUT_Q, PUT_Q)}$$

or equivalently,

$$p_i^0 = -\frac{\sqrt{Var(pQ)}}{\sqrt{Var(PUT_Q)}}\rho_{pQ,PUT_Q}$$

Otherwise, if $\rho_{pQ,PUT_Q} \geq 0$, the uniquely optimal indemnity price will be $p_i^0 = 0$, which means that the farmer does not buy the insurance at all.

Here, we see that since the purchase of insurance does not change the expected value of the farmer's income, the farmer cares about the variance of his income only. The farmer's income comes from two terms, pQ and $p_i(PUT_Q - EPUT_Q)$. Figure 1 gives an explanation of the optimal solution.

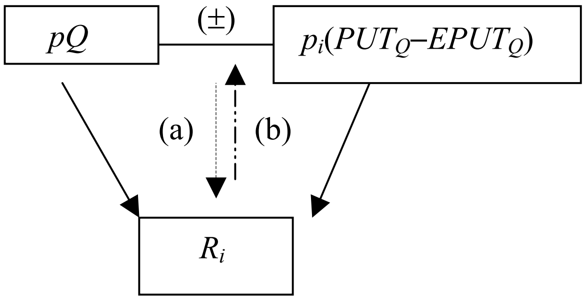

Figure 1: How the individual terms contribute to the variance of the farmer's income

In Figure 1, a term in a rectangle denotes the variance of the term. The sign between terms pQ and $p_i(PUT_Q - EPUT_Q)$ denotes the sign of the covariance between pQ and $p_i(PUT_Q - EPUT_Q)$. The arrows toward R_i show that the considered variance or covariance from which the arrow departs produces a positive amount to the variance of the farmer's income R_i and therefore produces a negative effect for minimizing variances. On the contrary, the arrows departing from R_i show that the considered variance or covariance to which the arrow points produces a negative amount to the variance of R_i and therefore produces a positive effect for minimizing variances.

The variances always produce negative effects on the variance of the farmer's income. The solid lines in Figure 1 denote this. When $\rho_{pQ,PUT_Q} > 0$ (line (a)), all arrows point towards income R_i, which means that not only the variances of pQ and $p_i(PUT_Q - EPUT_Q)$, but also the covariance between pQ and $p_i(PUT_Q - EPUT_Q)$, produce negative effects on the variance. Thus, it is wise to have no insurance at all. However, when $\rho_{pQ,PUT_Q} < 0$ (line (b)), there is one arrow departing from income R_i. The negative effects of the variances of pQ and $p_i(PUT_Q - EPUT_Q)$ are hedged by the positive effect of the covariance through purchasing some amount of insurance, which explains the positive insurance purchase.

The usual argument is that the optimal insurance is the full insurance if the insurance premium is statistically fair. Why is a possible zero insurance purchase regarded as the optimal insurance? The answer is that in usual discussions about the optimal insurance, the market price p is considered as constant, which results in $\rho_{pQ,PUT_Q} = \rho_{Q,PUT_Q} < 0$, according to the definition of PUT_Q. Thus,

$$p_i^0 = -\frac{Cov(pQ, PUT_Q)}{Cov(PUT_Q, PUT_Q)} = -\frac{p \cdot Cov(Q, PUT_Q)}{Cov(PUT_Q, PUT_Q)}$$

$$= -\frac{p\sqrt{Var(Q)}}{\sqrt{Var(PUT_Q)}}\rho_{Q,PUT_Q} > 0$$

Due to the argument that the optimal insurance should be full when the insurance premium is fair, p_i^0 can be defined as the full insurance in this model. To sum up, when the premium of the crop insurance is statistically fair, the representative farmer will buy the full insurance, which is defined as

$$p_i^0 = \begin{cases} -\frac{\sqrt{Var(pQ)}}{\sqrt{Var(PUT_Q)}}\rho_{pQ,PUT_Q} & \rho_{pQ,PUT_Q} < 0 \\ 0 & \rho_{pQ,PUT_Q} \geq 0 \end{cases}$$

In a special case where the price risk p and the output risk Q are independent, $\rho_{p,Q} = 0$ and $\rho_{pQ,PUT_Q} < 0$[5]. The optimal solution will be larger than zero and $p_i^0 = Ep \cdot \frac{(EPUT_Q)^2}{Var(PUT_Q)} > 0$.

The appearance of PUT_Q in the formula shows the role of the yield guarantee (or the yield selection) in the farmer's insurance decision. It will be seen that PUT_Q also appears in the other formulas below. In any situation where PUT_Q appears, the appearance of PUT_Q shows the role of the yield guarantee in the farmer's decision.

2.2. Concern futures only

Now, suppose that the farmer can join the futures market against a possible price decrease. To simplify the discussion, first the perfect hedge in the futures market is assumed to be possible, which means that there is no asset mismatch and no maturity mismatch. Thus, it is actually assumed that there is a futures contract in the futures market, which considers exactly the same type of crop with exactly the same quantity as the one that the representative farmer may produce, and the maturity of the contract is exactly the second harvest time, i.e., time 1. In addition, the time value of money is ignored so that the effect of the clearing margin required in the futures exchange will not be considered. Finally, the crop is considered as a pure asset, and therefore its convenience value is ignored; Convenience assets are assets held for the physical services they offer as well as for their potential investment returns. In this model, it is assumed that the aim of the farmer's hedging in the futures market is to sell his product at a good price at the second harvest time only.

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[5] See Appendix for proof.

With the above assumptions, the *futures price*, denoted by p_f, can be defined as *the expected value of the crop's market price at time 1*, i.e., $p_f = Ep$ in a risk-neutral market. The farmer decides how much amount of crop he would like to hedge in the futures market, i.e., his *hedge ratio*, denoted by Q_f. Assume that Q_f is *no less than 0*. Thus, the farmer's income with only the effect of the futures market considered will be

$$\begin{aligned} R_f &= p(Q - Q_f) + p_f Q_f \\ &= pQ + (Ep - p)Q_f \end{aligned}$$

Therefore, $ER_f = E(pQ)$ and $VarR_f = Var(pQ) + Q_f^2 Var(p) - 2Q_f Cov(pQ, p)$. Thus, the farmer's optimal program will be

$$\underset{Q_f \geq 0}{Max} U(R_f) = ER_f - \phi VarR_f$$

Again, since the objective function $U(R_f) = ER_f - \phi VarR_f$ is a concave function of Q_f, the first order condition gives the unique optimal solution. If $\rho_{pQ,p} > 0$, the optimal hedge ratio will uniquely be

$$Q_f^0 = \frac{Cov(pQ, p)}{Cov(p, p)}$$

or equivalently,

$$Q_f^0 = \frac{\sqrt{Var(pQ)}}{\sqrt{Var(p)}} \rho_{pQ,p}$$

If $\rho_{pQ,p} \leq 0$, the uniquely optimal hedge ratio will be $Q_f^0 = 0$. The farmer does not join the futures market because the risk of a price decrease facing him is naturally hedged by the negative relationship between the market price p and pQ. Although a figure similar to Figure 1 can be drawn to clarify, it is unnecessary to repeat the same explanation. To sum up,

$$Q_f^0 = \begin{cases} \dfrac{\sqrt{Var(pQ)}}{\sqrt{Var(p)}} \rho_{pQ,p} & \rho_{pQ,p} > 0 \\ 0 & \rho_{pQ,p} \leq 0 \end{cases}$$

Under the special case where the price risk p and the output risk Q are independent, $Cov(pQ, p) = E(pQ \cdot p) - E(pQ)E(p) = E(Q)[E(p^2) - (E(p))^2] = E(Q)Var(p) > 0$ which results in $\rho_{pQ,p} > 0$. Therefore the optimal solution Q_f^0 will be positive and $Q_f^0 = E(Q)$.

2.3. Concern both crop insurance and futures

Following the above two sections, when the effect of both insurance and futures markets is considered, the farmer's income will be

$$R_{if} = pQ + (p_f - p)Q_f + I(Q) - P$$
$$= pQ + (Ep - p)Q_f + p_i(PUT_Q - EPUT_Q)$$

And therefore, $ER_{if} = E(pQ)$ and $VarR_{if} = Var(pQ) + Q_f^2 Var(p) + p_i^2 Var(PUT_Q) - 2Q_f Cov(pQ, p) + 2p_i Cov(pQ, PUT_Q) - 2p_i Q_f Cov(p, PUT_Q)$. Thus, the farmer's optimal program will be

$$\underset{Q_f \geq 0, p_i \geq 0}{Max} U(R_{if}) = ER_{if} - \phi VarR_{if}$$

Once more, the objective function $U(R_{if}) = ER_{if} - \phi VarR_{if}$ is a concave function of Q_f and p_i. The first order condition gives the unique optimal solution. The optimal solutions Q_f and p_i should satisfy

$$\begin{pmatrix} -Cov(p,p) & Cov(p,PUT_Q) \\ Cov(p,PUT_Q) & -Cov(PUT_Q,PUT_Q) \end{pmatrix} \cdot \begin{pmatrix} Q_f \\ p_i \end{pmatrix} = \begin{pmatrix} -Cov(pQ,p) - \dfrac{\lambda_{Q_f}}{2\phi} \\ Cov(pQ,PUT_Q) - \dfrac{\lambda_{p_i}}{2\phi} \end{pmatrix} \qquad \text{(Eq)}$$

where both λ_{Q_f} and λ_{p_i} are no less than zero, and $\lambda_{Q_f} = 0$, if the optimal $Q_f > 0$, and $\lambda_{p_i} = 0$, if the optimal $p_i > 0$.

In this model when the farmer's insurance and futures decisions are put together, the relationship between the considered crop's price and its output becomes more important than in the models where the decisions are made separately. Under the special case where the price risk p and the output risk Q are independent, $\rho_{p,Q} = 0$, $\rho_{p,PUT_Q} = 0$, $\rho_{pQ,PUT_Q} < 0$, and $\rho_{pQ,p} > 0$. Equation (Eq) gives the optimal solution $Q_f^* = Q_f^0$ and $p_i^* = p_i^0$.

If the change in the output is related to a whole geographical area, then there may be a dependent relationship between the price risk p and the output risk Q. Assume that the price risk p and the output risk Q are correlated and the correlation coefficient is negative, i.e., $\rho_{p,Q} < 0$.[6] When this is the case, the solution of the equation (Eq) becomes more complicated. Let us first look at the relationships of the correlation coefficients.

According to the definition of the PUT_Q, $\rho_{Q,PUT_Q} < 0$. Thus, under the assumption of $\rho_{p,Q} < 0$, $\rho_{PUT_Q,p} > 0$, and $\rho_{PUT_Q,pQ} \cdot \rho_{pQ,p} > 0$ to be consistent with a positive $\rho_{PUT_Q,p}$, which tells

[6] Assuming inputs are given, then the covariance between p and Q will depend entirely on the shape of the demand curve. Therefore the correlation coefficient $\rho_{p,Q}$ is related to elasticity of demand curve for the product.

us that the signs of ρ_{pQ,PUT_Q} and $\rho_{pQ,p}$ are the same. We assume that any of the correlation coefficients will not be zero. Figure 2 summarizes the signs of the correlation coefficients.

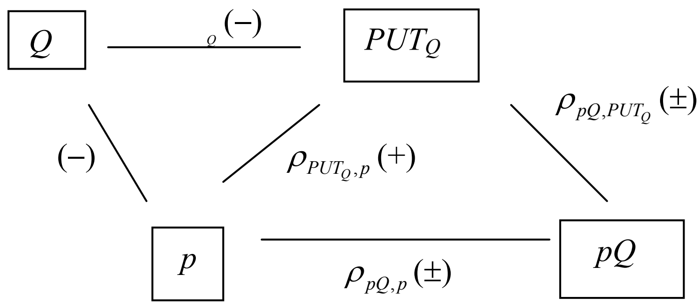

Figure 2: The signs of the correlation coefficients

Define Set I, if $\rho_{pQ,p} > 0$ (i.e., $\rho_{pQ,PUT_Q} > 0$), and Set J, if $\rho_{pQ,PUT_Q} < 0$ (i.e., $\rho_{pQ,p} < 0$). Then the universal set, U = I∪J and I∩J = ∅.

Furthermore, define Set A, if $\rho_{pQ,PUT_Q} - \rho_{pQ,p} \cdot \rho_{p,PUT_Q} \geq 0$ and $\rho_{pQ,p} > 0$, Set B, if $\rho_{pQ,p} - \rho_{pQ,PUT_Q} \cdot \rho_{PUT_Q,p} \leq 0$ and $\rho_{pQ,PUT_Q} < 0$, Set C_A, if $\rho_{pQ,PUT_Q} - \rho_{pQ,p} \cdot \rho_{p,PUT_Q} < 0$ and $\rho_{pQ,p} > 0$, and Set C_B, if $\rho_{pQ,p} - \rho_{pQ,PUT_Q} \cdot \rho_{PUT_Q,p} > 0$ and $\rho_{pQ,PUT_Q} < 0$. Obviously, $A \cap C_A = \emptyset$, $A \cup C_A = I$, $B \cap C_B = \emptyset$, and $B \cup C_B = J$. Therefore the universal set, U, can be separated into four un-joint subsets: $U = (A \cup C_A) \cup (B \cup C_B) = A \cup B \cup (C_A \cup C_B)$.

$C_A \cap C_B = \emptyset$. Define Set C, if $\rho_{pQ,PUT_Q} - \rho_{pQ,p} \cdot \rho_{p,PUT_Q} < 0$ and $\rho_{pQ,p} - \rho_{pQ,PUT_Q} \cdot \rho_{PUT_Q,p} > 0$. It can be proved that $C = C_A \cup C_B$. First, since $A \cap C = \emptyset$ and $B \cap C = \emptyset$, then $C \subseteq C_A \cup C_B$. Secondly, if $\rho_{pQ,p} > 0$, and $\rho_{pQ,PUT_Q} - \rho_{pQ,p} \cdot \rho_{p,PUT_Q} < 0$, i.e., C_A is the case, then $\rho_{pQ,PUT_Q} - \rho_{pQ,p} \cdot \rho_{p,PUT_Q} < 0$ gives $\rho_{pQ,p} > \dfrac{\rho_{pQ,PUT_Q}}{\rho_{p,PUT_Q}}$, which deduces that

$$\rho_{pQ,p} - \rho_{pQ,PUT_Q} \cdot \rho_{PUT_Q,p} > \frac{\rho_{pQ,PUT_Q}}{\rho_{p,PUT_Q}}(1 - \rho_{p,PUT_Q}{}^2) > 0.$$

Hence, $C_A \subseteq C$. Similarly, we can prove $C_B \subseteq C$. Therefore, $C_A \cup C_B \subseteq C$.

Thus, the universal set, U, is separated into three un-joint subsets: $U = A \cup B \cup C$, and Set C is separated into two un-joint subsets: $C = C_A \cup C_B$. Table 1 summarizes the separations.

Table 1: The separations of the universal set

<table>
<tr><td>A: $\rho_{pQ,PUT_Q} - \rho_{pQ,p} \cdot \rho_{p,PUT_Q} \geq 0$,
$\rho_{pQ,p} > 0$ (i.e., $\rho_{pQ,PUT_Q} > 0$)</td><td>B: $\rho_{pQ,p} - \rho_{pQ,PUT_Q} \cdot \rho_{PUT_Q,p} \leq 0$,
$\rho_{pQ,PUT_Q} < 0$ (i.e., $\rho_{pQ,p} < 0$)</td></tr>
<tr><td colspan="2">C: $\rho_{pQ,PUT_Q} - \rho_{pQ,p} \cdot \rho_{p,PUT_Q} < 0$, $\rho_{pQ,p} - \rho_{pQ,PUT_Q} \cdot \rho_{PUT_Q,p} > 0$</td></tr>
<tr><td>C_A: $\rho_{pQ,PUT_Q} - \rho_{pQ,p} \cdot \rho_{p,PUT_Q} < 0$,
$\rho_{pQ,p} > 0$</td><td>C_B: $\rho_{pQ,p} - \rho_{pQ,PUT_Q} \cdot \rho_{PUT_Q,p} > 0$,
$\rho_{pQ,PUT_Q} < 0$</td></tr>
</table>

Now, let us solve equation (Eq). If $\rho_{p,PUT_Q} \neq 1$, then the determinant $\begin{vmatrix} -Cov(p,p) & Cov(p,PUT_Q) \\ Cov(p,PUT_Q) & -Cov(PUT_Q,PUT_Q) \end{vmatrix} = Var(p) \cdot Var(PUT_Q) \cdot (1 - \rho_{p,PUT_Q}{}^2) \neq 0$. The equation has a unique solution.

1) If both the optimal Q_f and p_i were equal to zero ($Q_f = 0$ and $p_i = 0$), then we would have to find two nonnegative λ_{Q_f} and λ_{p_i} so that $\begin{pmatrix} -Cov(pQ,p) - \dfrac{\lambda_{Q_f}}{2\phi} \\ Cov(pQ,PUT_Q) - \dfrac{\lambda_{p_i}}{2\phi} \end{pmatrix} = 0$, which is impossible because it is argued that in general $\rho_{PUT_Q,pQ} \cdot \rho_{pQ,p} > 0$. Thus, the optimal indemnity price and hedge ratio cannot be zero simultaneously, which means that the optimal solution excludes the possibility of the farmer neither buying the insurance nor joining the futures market.

2) If both the optimal Q_f and p_i are not equal to zero ($Q_f > 0$ and $p_i > 0$), then both λ_{Q_f} and λ_{p_i} will be zero. Solving the equation with both $\lambda_{Q_f} = 0$ and $\lambda_{p_i} = 0$, we get, if $\rho_{pQ,PUT_Q} - \rho_{pQ,p} \cdot \rho_{p,PUT_Q} < 0$ and $\rho_{pQ,p} - \rho_{pQ,PUT_Q} \cdot \rho_{PUT_Q,p} > 0$, i.e., if C is the case, the optimal indemnity price and hedge ratio will be

$$p_i^* = -\frac{\sqrt{Var(pQ)}}{\sqrt{Var(PUT_Q)}} \frac{\rho_{pQ,PUT_Q} - \rho_{pQ,p} \cdot \rho_{p,PUT_Q}}{1 - \rho_{p,PUT_Q}{}^2} > 0$$

and

$$Q_f^* = \frac{\sqrt{Var(pQ)}}{\sqrt{Var(p)}} \frac{\rho_{pQ,p} - \rho_{pQ,PUT_Q} \cdot \rho_{PUT_Q,p}}{1 - \rho_{p,PUT_Q}{}^2} > 0$$

Otherwise, when C is not the case, there will be no optimal solution satisfying both Q_f and p_i positive. Furthermore, when $\rho_{pQ,PUT_Q} - \rho_{pQ,p} \cdot \rho_{p,PUT_Q} < 0$ and $\rho_{pQ,p} - \rho_{pQ,PUT_Q} \cdot \rho_{PUT_Q,p} > 0$, it can be proved that $p_i^* > p_i^0$, and $Q_f^* > Q_f^0$.

3) If the optimal Q_f is positive and the optimal p_i is equal to zero ($Q_f > 0$ and $p_i = 0$), then, if $\lambda_{p_i} = \rho_{pQ,PUT_Q} - \rho_{pQ,p} \cdot \rho_{p,PUT_Q} \geq 0$ and $\rho_{pQ,p} > 0$, i.e., if A is the case, then the optimal solution will be $p_i^* = 0$ and

$$Q_f^* = \frac{\sqrt{Var(pQ)}}{\sqrt{Var(p)}} \rho_{pQ,p} = Q_f^0$$

Otherwise, when A is not the case, there will be no optimal solution with a positive Q_f and a zero p_i.

4) If the optimal Q_f is equal to zero and the optimal p_i is positive ($Q_f = 0$ and $p_i > 0$), then, if $-\lambda_{Q_f} = \rho_{pQ,p} - \rho_{pQ,PUT_Q} \cdot \rho_{PUT_Q,p} \leq 0$ and $\rho_{pQ,PUT_Q} < 0$, i.e., if B is the case, then the optimal solution will be $Q_f^* = 0$ and

$$p_i^* = -\frac{\sqrt{Var(pQ)}}{\sqrt{Var(PUT_Q)}} \rho_{pQ,PUT_Q} = p_i^0$$

Otherwise, when B is not the case, there will be no optimal solution with a zero Q_f and a positive p_i.

Table 2: Summary of the results conditional on $\rho_{p,PUT_Q} \neq 1$.

		The farmer thinks about insurance and futures *separately*.		The farmer thinks about insurance and futures *simultaneously*.	
		Insurance decision (p_i)	Futures decision (Q_f)	Insurance decision (p_i)	Futures decision (Q_f)
$\rho_{p,Q} < 0$ and therefore the price risk p and the output risk Q are dependent.					
A		$p_i^0 = 0$	$Q_f^0 > 0$	$p_i^* = p_i^0$	$Q_f^* = Q_f^0$
B		$p_i^0 > 0$	$Q_f^0 = 0$	$p_i^* = p_i^0$	$Q_f^* = Q_f^0$
C	C_A	$p_i^0 = 0$	$Q_f^0 > 0$	$p_i^* > p_i^0$	$Q_f^* > Q_f^0$
	C_B	$p_i^0 > 0$	$Q_f^0 = 0$	$p_i^* > p_i^0$	$Q_f^* > Q_f^0$
The price risk p and the output risk Q are independent and therefore $\rho_{p,Q} = 0$.					
All		$p_i^0 > 0$	$Q_f^0 > 0$	$p_i^* = p_i^0$	$Q_f^* = Q_f^0$

Table 2 summarizes the results, and like Figure 1, Figure 3 helps to explain the results. The analyses for the cases $\rho_{pQ,PUT_Q} < 0$ (B and C_B in Tables 1 and 2) and $\rho_{pQ,p} > 0$ (A and C_A in Tables 1 and 2) are similar. So Figure 3 considers cases A and C_A only.

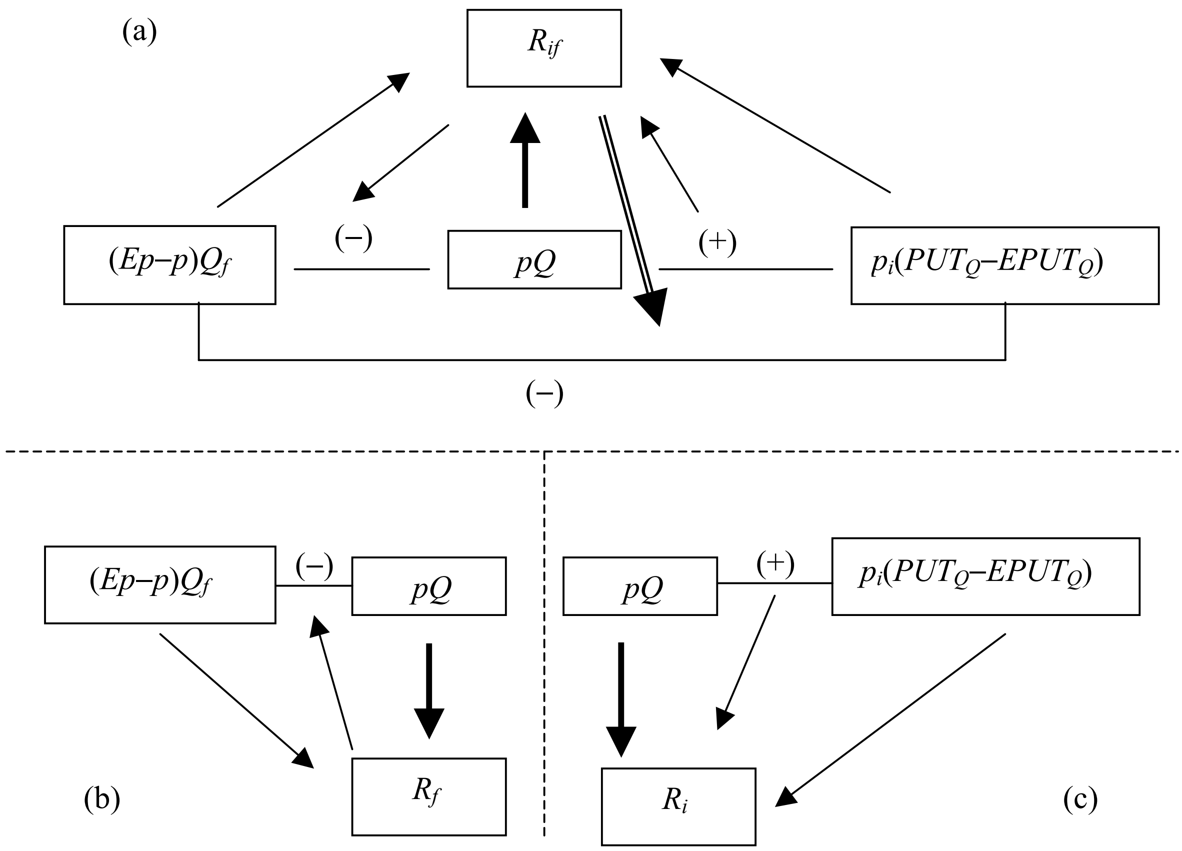

Figure 3: The individual terms' contribution to the variance of the farmer's income

When the farmer fails to protect himself against any price or output change by using insurance and futures instruments, his income is pQ. The effect of this term on the variance of the farmer's income is out of his control from the perspective of him making insurance and futures decisions, and therefore remains unchanged no matter how much he buys the insurance or how much he hedges in the futures market. The dark arrows related to this term denote the unchanged effect. The $p_i(PUT_Q-EPUT_Q)$ denotes the change in the farmer's income caused by his purchase of the insurance. The $(Ep-p)Q_f$ denotes the change in the farmer's income caused by his decision on the futures market.

Panel (c) is the same as in Figure 1. When he makes the insurance decision alone, the farmer will buy zero insurance because all terms related to his insurance purchase increase the variance of his income. However, when he considers his decision of joining the futures market shown in Panel (b), there are both positive and negative effects on the variance of his income. A zero Q_f will not be optimal because at $Q_f = 0$ the positive effect of the negative relationship between $(Ep-p)Q_f$ and pQ is not used. The farmer will make a tradeoff between the positive and negative effects. The positive Q_f such that its positive effect optimally dominates the negative effect will be the optimal solution for the farmer.

Panel (a) illustrates a case where the farmer thinks of his insurance decision and futures decision simultaneously. From this panel, we can see a symmetric pattern between the effects of insurance and the derivative security markets. Once again, there are both positive and negative effects on the variance of the farmer's income. Due to the negative relationship between $p_i(PUT_Q-EPUT_Q)$ and $(Ep-p)Q_f$, i.e., the positive ρ_{p,PUT_Q}, which does not show up in Panels (b) and (c), a positive insurance purchasing may have a positive effect on the variance of the farmer's income. Thus, the farmer's insurance purchasing will not necessarily be zero. The double line in the figure shows the effect of the positive ρ_{p,PUT_Q}.

The optimal solution depends on the correlation coefficients. Among others, the correlation coefficient ρ_{p,PUT_Q} plays an important role. The term $(Ep-p)$ relates to the realized value of the futures contract and the term PUT_Q-EPUT_Q relates to the realized value of the insurance contract. The higher the market price p, the lower the realized value of the futures contract. The higher the farmer's output Q, the lower the realized value of the insurance contract. Thus, in the case of $\rho_{p,Q} < 0$ discussed here, the higher the realized value of the futures contract, the lower the realized value of the insurance contract, and *vice versa*. This relationship is denoted by $\rho_{p,PUT_Q} > 0$.

When $\rho_{p,PUT_Q} \le \dfrac{\rho_{pQ,PUT_Q}}{\rho_{pQ,p}}$, i.e., A is the case, the added positive effect, caused by the positive ρ_{p,PUT_Q}, of a positive insurance purchase on the variance of the farmer's income is not so large that it dominates the negative effect. The optimal insurance decision is still a zero insurance purchase. Thus, the optimal hedge ratio on the futures will be the same as that in Panel (b) because the zero p_i makes the relationships in Panel (a) exactly the same as that in Panel (b). When $\rho_{p,PUT_Q} > \dfrac{\rho_{pQ,PUT_Q}}{\rho_{pQ,p}}$, i.e., C_A is the case, the added positive effect of a positive insurance purchase dominates its negative effect, so that a positive insurance purchase is optimal. At this time, the optimal hedge ratio on the futures will still be positive. Otherwise, zero Q_f should produce the same solution as Panel (c), which means a zero p_i, which in turn contradicts the pre-conclusion of a positive insurance purchase. Moreover, the optimal hedge ratio will be higher than the one from Panel (b), i.e., $Q_f^* > Q_f^0$; A positive insurance purchase cannot fulfil the purpose of having the total positive effect dominate the negative effect. It is because a positive insurance purchase increases the negative effect of the terms $p_i^2 Var(PUT_Q)$ and $2p_i Cov(pQ, PUT_Q)$ as well. Therefore, a higher hedging ratio than the one with zero insurance purchase is required in the futures market.

Thus, when, the farmer is thinking about different kinds of hedging strategy simultaneously his strategy may be different from the one when he thinks about his hedging strategy separately, depending on the correlation coefficients ρ_{p,PUT_Q}, ρ_{pQ,PUT_Q}, and $\rho_{pQ,p}$. If, as in Section 2.1, p_i^0 is defined as the full insurance, then, when the price risk p and the output risk Q are independent, introducing the farmer's futures decision into the insurance model does not affect his insurance decision. The farmer will still buy the full insurance if the premium is fair. When $\rho_{p,Q} < 0$ and therefore the price risk p and the output risk Q are dependent, introducing the farmer's futures decision into the insurance model affects his insurance decision in some cases. However the farmer will at least buy the full insurance if the premium is fair. In some cases, he will buy more than the full insurance. It is therefore argued that the derivative markets have an effect on the insurance market and *vice versa*. However, because of the cross dependence among the correlation coefficients, there is no monotonic relationship between any of the optimal solutions and any of the correlation coefficients.

The effect of the derivative securities on individuals' insurance purchase will also be found in Section 3.

3. Two extensions from the basic models

In this section, two extensions from the basic models will be given. One of the extensions will relax the assumptions of both $P_{PUT_Q} = EPUT_Q$ and $p_f = Ep$ and the other will introduce the effect of the futures option market. The purpose of doing this will be presented in related sections.

3.1. A model where β's are introduced into the pricing procedure

In the above section, the farmer's degree of risk aversion, denoted by ϕ, does not play any role on his hedging decision. This is because a risk-neutral market in which risk premiums do not play any role has been assumed. The assumption of $p_f = Ep$ does not let the holders of futures position be compensated by the risk they bear for holding the position. This also applies to the assumption of $P_{PUT_Q} = EPUT_Q$. When these assumptions are relaxed by introducing β's, systematic risks defined in the Capital Asset Pricing Model (CAPM), into the model, the effect of the farmer's degree of risk aversion, i.e., the effect of ϕ, will be seen. To see the effect of ϕ is the purposes of this extension.

The assumption that $p_f = Ep$ only functions in a risk-neutral market. The equilibrium futures price is usually not equal to the expected value of the output price. Farmers can hedge their output price risk by taking a short futures position, and producers who use the crop as an input can hedge their input price risk by taking a long futures position. When the short and long positions turn to the equilibrium, the equilibrium futures price is decided. Holders of the futures positions bear a risk and they must be compensated for it. The risk is abstractly denoted by β.

As it has been mentioned, the crop is considered as a pure asset. Thus, according to Siegel and Siegel (1990), the fundamental no-arbitrage futures price will be, by using their notations,

$$F_{t,T} = P_t(1+r_{t,T})$$

where $F_{t,T}$ is the futures price at time t for a considered asset to be delivered at time T, P_t is the considered asset's spot price at time t, and $r_{t,T}$ is the "T-bill" interest rate from time t to time T. However, the expected value of P_T at time t, denoted by $E_t(P_T)$, will be

$$E_t(P_T) = P_t(1+E_t(r_{t,T}^*))$$

where $r_{t,T}^* = \dfrac{P_T - P_t}{P_t}$ is the rate of return on the underline asset at the time interval t to T, and $E_t(\cdot)$ is the expectation operator at time t. When a crop is considered as the underline asset, $r_{t,T}^*$ can be the rate of return on an asset which traces the same systematic price risk as the considered futures contract. Thus,

$$\begin{aligned} F_{t,T} &= P_t(1+r_{t,T}) \\ &= E_t(P_T) + P_t(r_{t,T} - E_t(r_{t,T}^*)) \end{aligned}$$

If there is a difference between $r_{t,T}$ and $E_t(r_{t,T}^*)$, then $F_{t,T} \neq E_t(P_T)$.

In this essay, $t = 0$ and $T = 1$. Following the notations defined in this essay and having the subscripts 0 and 1 omitted, the above formula can be rewritten as $r^* = \dfrac{p - p_0}{p_0}$ and $p_f = Ep + p_0(r - E(r^*))$. According to the CAPM,

$$Er^* = r + \beta(Er_m - r)$$

where r_m = the rate of market return at the time interval 0 to 1, $\beta = Cov(r^*, r_m)/Var(r_m)$. Then,

$$p_f = Ep - p_0\beta(Er_m - r)$$

Obviously, $p_f \neq Ep$ if $r \neq Er^*$, i.e., $Er_m \neq r$. $r^* = \dfrac{p - p_0}{p_0}$ results in

$$\beta = \frac{Cov(r^*, r_m)}{Var(r_m)} = \frac{Cov(p, r_m)}{p_0 Var(r_m)}$$

Then, let $\beta_p = \dfrac{Cov(p, r_m)}{Var(r_m)}$,

$$p_f = Ep - \beta_p(Er_m - r) \tag{1}$$

in which the β-risk is playing its role on the futures market.

The β-risk also plays a role on the insurance market. According to the so-called insurance CAPM, the insurance premium (Fairley, 1979 and Hill, 1979)

$$P = \frac{EI(Q) - \lambda Cov(I(Q), r_m)}{1 + kr}$$

where $\lambda = \dfrac{Er_m - r}{Var(r_m)}$ denotes the market price of the risk and k denotes the so-called funds generating coefficient, which will be zero if investment income is not considered in the model. Let $k = 0$, $\beta_{PUT_Q} = \dfrac{Cov(PUT_Q, r_m)}{Var(r_m)}$ and note that $I(Q) = p_i PUT_Q$. Then

$$P = p_i(EPUT_Q - \beta_{PUT_Q}(Er_m - r)) \tag{2}$$

Now, let us look at the farmer's decision when both β_p and β_{PUT_Q} are introduced into the model. The farmer's income,

$$R_{if} = pQ + (p_f - p)Q_f + p_i PUT_Q - P$$

Substitute formulas (1) and (2) into R_{if}, we get

$$ER_{if} = E(pQ) - \beta_p(Er_m - r)Q_f + p_i\beta_{PUT_Q}(Er_m - r)$$

and

$$VarR_{if} = Var(pQ) + Q_f^2 Var(p) + p_i^2 Var(PUT_Q) - 2Q_f Cov(pQ, p) + 2p_i Cov(pQ, PUT_Q) - 2p_i Q_f Cov(p, PUT_Q)$$

The farmer's optimal program will be

$$\underset{Q_f \geq 0, p_i \geq 0}{Max} U(R_{if}) = ER_{if} - \phi VarR_{if}$$

The fact that the objective function $U(R_{if}) = ER_{if} - \phi VarR_{if}$ is a concave function of Q_f and p_i tells us that the first order condition gives the unique optimal solution. Thus, the optimal Q_f and p_i satisfy

$$\begin{pmatrix} -Cov(p,p) & Cov(p,PUT_Q) \\ Cov(p,PUT_Q) & -Cov(PUT_Q,PUT_Q) \end{pmatrix} \cdot \begin{pmatrix} Q_f \\ p_i \end{pmatrix} = \begin{pmatrix} -Cov(pQ,p) - \dfrac{\lambda_{Q_f} - \beta_p(Er_m - r)}{2\phi} \\ Cov(pQ,PUT_Q) - \dfrac{\lambda_{p_i} + \beta_{PUT_Q}(Er_m - r)}{2\phi} \end{pmatrix}$$

where both λ_{Q_f} and λ_{p_i} are no less than zero, and $\lambda_{Q_f} = 0$, if the optimal $Q_f > 0$, and $\lambda_{p_i} = 0$, if the optimal $p_i > 0$.

Solving the equation, we get the optimal solution

$$p_i^{**} = -\frac{\sqrt{Var(pQ)}}{\sqrt{Var(PUT_Q)}} \frac{\rho_{pQ,PUT_Q} - \rho_{pQ,p} \cdot \rho_{p,PUT_Q}}{1 - \rho_{p,PUT_Q}{}^2} + \frac{\sqrt{Var(p)}\beta_{PUT_Q} - \rho_{p,PUT_Q}\sqrt{Var(PUT_Q)}\beta_p}{\sqrt{Var(p)Var(PUT_Q)}(1 - \rho_{p,PUT_Q}{}^2)} \frac{Er_m - r}{2\phi}$$

and

$$Q_f^{**} = \frac{\sqrt{Var(pQ)}}{\sqrt{Var(p)}} \frac{\rho_{pQ,p} - \rho_{pQ,PUT_Q} \cdot \rho_{PUT_Q,p}}{1 - \rho_{p,PUT_Q}{}^2} + \frac{\rho_{p,PUT_Q}\sqrt{Var(p)}\beta_{PUT_Q} - \sqrt{Var(PUT_Q)}\beta_p}{\sqrt{Var(PUT_Q)Var(p)}(1 - \rho_{p,PUT_Q}{}^2)} \frac{Er_m - r}{2\phi}$$

if the right sides of both p_i^{**} and Q_f^{**} are strictly larger than zero. Discussions on other situations where the right sides are not strictly larger than zero are omitted.

Obviously, the farmer's degree of risk aversion (ϕ) gets into the formulas now. Note that because the signs of $\sqrt{Var(p)}\beta_{PUT_Q} - \rho_{p,PUT_Q}\sqrt{Var(PUT_Q)}\beta_p$ and of $\rho_{p,PUT_Q}\sqrt{Var(p)}\beta_{PUT_Q} - \sqrt{Var(PUT_Q)}\beta_p$ are undetermined, there is no obvious monotonic relationship between the farmer's degree of risk aversion and his insurance purchase (p_i) or his hedging ratio (Q_f). However, if the farmer considers his insurance decision and hedging decision separately, and therefore maximizes $U(R_i)$ and $U(R_f)$ separately as in the maximization problems in Sections 2.1 and 2.2, then there will be a clear monotonic relationship between his degree of risk aversion ϕ and the optimal solutions. Thus, we see that the effect of the farmer's degree of risk aversion appears differently when the crop insurance and the derivative securities are concerned separately and when they are concerned simultaneously.

Furthermore, since $0 < \rho_{p,PUT_Q} < 1$, then, 1) when $\sqrt{Var(p)}\beta_{PUT_Q} - \rho_{p,PUT_Q}\sqrt{Var(PUT_Q)}\beta_p < 0$, $\rho_{p,PUT_Q}\sqrt{Var(p)}\beta_{PUT_Q} - \sqrt{Var(PUT_Q)}\beta_p < 0$, and 2) when $\rho_{p,PUT_Q}\sqrt{Var(p)}\beta_{PUT_Q} - \sqrt{Var(PUT_Q)}\beta_p > 0$, $\sqrt{Var(p)}\beta_{PUT_Q} - \rho_{p,PUT_Q}\sqrt{Var(PUT_Q)}\beta_p > 0$; If the farmer's degree of risk aversion has a positive relationship with his insurance purchase, then it has a positive relationship with his hedging ratio too. If the farmer's degree of risk aversion has a negative relationship with his hedging ratio, then it has a negative relationship with his insurance purchase too.

3.2. A model concerning the effect of the futures option market

In this section, let us look at the effect of the futures option market. The reason for this extension is mainly because this model will give us a more symmetric pattern between the effects of the insurance and the derivative security markets than the one in Panel (a) of Figure 3. This will enable us to explain the model broadly.

Suppose that the representative farmer can hedge the risk of the price decrease by buying options on futures. A put option on a futures contract gives the holder the right to enter into a short futures position at a predetermined exercise price. The exercise price is the fixed price at which the clearinghouse enters the option holder into the futures position. In other words, after exercising a put, the original holder of the put takes a short position on the futures contract, in which the futures price is the predetermined exercise price.

To simplify the discussion, it is assumed that the farmer can only buy a put option at time 0 and exercise it at time 1 in order to be consistent with the time interval assumed before. The exercise price of the put is assumed as the expected value of the market price at time 1, Ep. Let PUT_p denote this put, and P_{PUT_p} denote the price of this put PUT_p. The payment of this put will be $Max(Ep - p, 0)$. Recall the notations in Section 2 where another put, $PUT_Q = Max(EQ - Q, 0)$, is defined. The price of this PUT_Q is defined as the expected value of the put, if the insurance premium is statistically fair. That is, $P_{PUT_Q} = EPUT_Q$. When the farmer's insurance decision and his option decision are combined together without using the assumption $P_{PUT_Q} = EPUT_Q$, the farmer's income will be

$$R_{io} = pQ + p_i(PUT_Q - P_{PUT_Q}) + (PUT_p - P_{PUT_p})Q_o$$

where Q_o denotes the farmer's hedge ratio on the futures option.

Figure 4 is similar to Figures 1 and 3. We see a "dual" relationship between $(PUT_Q - P_{PUT_Q})$ and $(PUT_p - P_{PUT_p})$. If $\rho_{p,Q} < 0$ and therefore $\rho_{PUT_Q,PUT_p} < 0$, then $\rho_{pQ,PUT_Q} \cdot \rho_{pQ,PUT_p} < 0$.

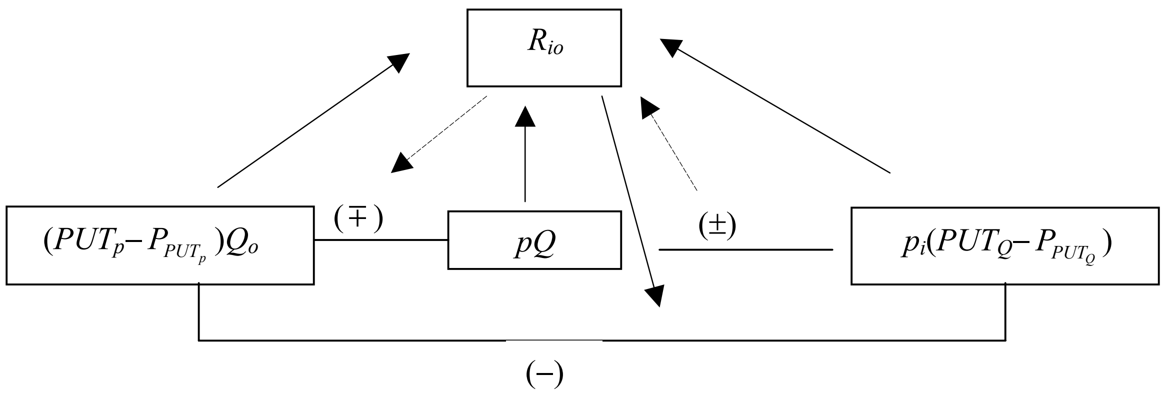

Figure 4: The individual terms' contribution to the variance of the farmer's income

Furthermore, if $P_{PUT_p} = EPUT_p$, to be consistent with the assumption $P_{PUT_Q} = EPUT_Q$, then $ER_{io} = E(pQ)$ and $VarR_{io} = Var(pQ) + p_i^2 Var(PUT_Q) + Q_o^2 Var(PUT_p) + 2p_i Cov(pQ, PUT_Q) + 2Q_o Cov(pQ, PUT_p) + 2p_i Q_o Cov(PUT_p, PUT_Q)$. The farmer's optimal program will be

$$\underset{Q_o \geq 0, p_i \geq 0}{Max} U(R_{io}) = ER_{io} - \phi VarR_{io}$$

Again, since the objective function $U(R_{io}) = ER_{io} - \phi VarR_{io}$ is a concave function of Q_o and p_i, the first order condition gives the unique optimal solution Q_o and p_i.

$$\begin{pmatrix} Cov(PUT_p, PUT_p) & Cov(PUT_p, PUT_Q) \\ Cov(PUT_p, PUT_Q) & Cov(PUT_Q, PUT_Q) \end{pmatrix} \cdot \begin{pmatrix} Q_o \\ p_i \end{pmatrix} = \begin{pmatrix} -Cov(pQ, PUT_p) + \dfrac{\lambda_{Q_o}}{2\phi} \\ -Cov(pQ, PUT_Q) + \dfrac{\lambda_{p_i}}{2\phi} \end{pmatrix}$$

where both λ_{Q_o} and λ_{p_i} are no less than zero, and $\lambda_{Q_o} = 0$, if the optimal $Q_o > 0$, and $\lambda_{p_i} = 0$, if the optimal $p_i > 0$. If we only look at the case where both the optimal Q_o and p_i are strictly larger than zero, then,

$$p_i^{***} = -\frac{\sqrt{Var(pQ)}}{\sqrt{Var(PUT_Q)}} \frac{\rho_{pQ,PUT_Q} - \rho_{pQ,PUT_p} \cdot \rho_{PUT_p,PUT_Q}}{1 - \rho_{PUT_p,PUT_Q}{}^2}$$

and

$$Q_o^{***} = -\frac{\sqrt{Var(pQ)}}{\sqrt{Var(PUT_p)}} \frac{\rho_{pQ,PUT_p} - \rho_{pQ,PUT_Q} \cdot \rho_{PUT_Q,PUT_p}}{1 - \rho_{PUT_p,PUT_Q}{}^2}$$

if $\rho_{PUT_Q,PUT_p} \neq 1$, $\rho_{pQ,PUT_Q} - \rho_{pQ,PUT_p} \cdot \rho_{PUT_p,PUT_Q} < 0$ and $\rho_{pQ,PUT_p} - \rho_{pQ,PUT_Q} \cdot \rho_{PUT_Q,PUT_p} < 0$.

If P_{PUT_p} and P_{PUT_Q} are defined as the Black formula from Black (1975), then we can achieve another symmetric formula for the optimal p_i and Q_o. In one word, the insurance and the derivative contracts can be treated symmetrically.

3.3. Explaining the models broadly

In section 2, it was mentioned that the crop insurance contract set is consistent with general arguments on a optimal insurance. Therefore, the models can be explained broadly. Suppose that there are two risks, p-risk and Q-risk, which are not necessary price and output risks. One of them, for example, the p-risk, can be hedged in a derivative market and another one, the Q-risk, can be hedged in the insurance market. The relationship between the p-risk and the Q-risk can be any; If simply denoted by the correlation coefficient $\rho_{p,Q}$, then the relationship can be $\rho_{p,Q} > 0$, $\rho_{p,Q} = 0$, or $\rho_{p,Q} < 0$. A representative individual does not care about p, or Q, alone, but about the product of p and Q, i.e., pQ.[7] To hedge the p-risk in the derivative market, the individual has to choose an optimal hedging ratio, denoted by Q^* and to hedge the Q-risk in the insurance market, the individual has to choose an optimal insurance amount, denoted by p^*.

With these hedging instruments, the individual's final wealth is basically specified by three terms: 1) $Q^*(DERIV_p - P_{DERIV_p})$, where $DERIV_p$ denotes a hedge instrument of the individual hedging the p-risk and P_{DERIV_p} denotes the price of $DERIV_p$. So this first term denotes the contribution of the individual hedging the p-risk by Q^*. 2) $p^*(DERIV_Q - P_{DERIV_Q})$, where $DERIV_Q$ denotes the compensation of the individual hedging the Q-risk in the insurance market and P_{DERIV_Q} denotes the price of the insurance contract for getting the compensation $DERIV_Q$. Since an insurance contract can be treated as a derivative security as we did, it can be denoted by $DERIV_Q$. Thus, this second term denotes the contribution of the individual hedging the Q-risk by p^*. And 3) pQ. The definitions of the derivatives and the relationship between the p-risk and the Q-risk, i.e., the sign of $\rho_{p,Q}$, determine the relationships among the three terms: $\rho_{pQ,DERIV_Q}$, $\rho_{pQ,DERIV_p}$, and $\rho_{DERIV_p,DERIV_Q}$, which in turn affect the optimal solution Q^* and p^*. Thus, the above analysis of the crop insurance is only a special example of this general model.

4. The concept of the variable participation contract

The concept of the variable participation contract was initiated by Doherty and Schlesinger (2001). Its purpose is to allow an endogenous mixture of participation contract and non-participation contract where insureds can choose the degree of participation.[8] The basic ideas are presented below.

Let N be the set of individuals. Suppose individual i ($i \in$ N) faces a risk C_i. The C_i's are correlated among individuals. The correlation may make the risk "uninsurable" in a traditional insurance market. So we could, for example, think of risk C_i as some kind of catastrophic risk.

Suppose that an insurance company issues a non-participation contract against risk C_i by requiring a fixed risk premium P_f, and suppose that the company also issues a full participation contract against the same risk by requiring a random risk premium P_r. Then, *the variable participation contract* allows an insured to choose a degree of him participating in the contract, denoted by α, and to pay for $\alpha P_r + (1-\alpha)P_f$ to have risk C_i insured. When the variable participation contract is supplied, an insured can choose both the amount of insurance he is going to purchase and the degree of his participation. Since a new selected variable, the degree of participation, is introduced, the variable participation contract will improve the welfare of the insureds, in comparison to either a non-participation contract or a full participation contract.

Now, let us look at another possible hedging strategy against risk C_i, if this risk can be separated into two factors — an identical factor $(1+\varepsilon)$ for all individuals, where $E(\varepsilon) = 0$, and a mutually independent factor L_i among individuals — so that risk C_i is equal to the product of $(1+\varepsilon)$ and L_i, i.e., $C_i = (1+\varepsilon)L_i$. A general argument is that since $C_i = (1+\varepsilon)L_i = L_i + \varepsilon L_i$, risk C_i can be well hedged in two separate markets; The first term L_i, can obviously be insured via the insurance market, because L_i's are mutually independent for all individuals and therefore the insurance company can diversify risk L_i's and supply a insurance contract against it. With regard to the second term εL_i, notice that risk ε includes all the correlated factors in risk C_i and

[7] The crop insurance is one of the examples. In the property and liability insurance related to import and export, not only the amount of liability but also the exchange rate, affect the total loss and this loss is the product of the amount of liability and the exchange rate. In the domestic trade, the inflation rate may be of influence, too.

[8] For further reading, see Doherty and Schlesinger (2001). See Schlesinger (1999) and Louberge and Schlesinger (1999) for extensions of Doherty and Schlesinger (2001).

the correlation helps to mitigate the basis risk[9], so risk ε can be well hedged through a derivative security market. Thus, if εL_i can be somehow swapped for $\varepsilon E(L_i)$, then the second term εL_i, i.e., $\varepsilon E(L_i)$ after the swap, can be hedged by using the hedging instrument against risk ε since $E(L_i)$ is now a real number, instead of a random variable L_i. The crucial point of using the synthetical hedging strategy successfully is to achieve the swap from εL_i to $\varepsilon E(L_i)$. Individuals cannot achieve it by themselves. Due to the pooling effect of the insurance company, it is argued that, in theory, the insurer should be able to achieve this swap (Doherty and Schlesinger (2001)).

The most interesting and important conclusion is that, under certain conditions, if the individuals are supplied a variable participation contract against risk $(1+\varepsilon)L_i$ as a whole, the optimal solution is the same as that when they are supplied an insurance contract against L_i with a fair premium $E(L_i)$ and at the same time hedge risk ε in a derivative market. Thus, from the point of the same hedging solution, the variable participation contract can be interpreted as the synthetical use of the hedging instruments in the insurance and the derivative security markets. To put it in another way, it is the synthetical use of the hedging instruments in the insurance and the derivative security markets that constructs the concept of the variable participation contract. By combining the hedging instruments available in both the insurance and the derivative security markets, the variable participation contract makes use of the advantages of the financial instrument, like options and futures, and as a result risk C_i is better hedged. It is important to note that the insurance intermediation, which in theory could achieve the swap of εL_i and $\varepsilon E(L_i)$, is essential in this conclusion.

The variable participation contract is in fact the synthetical use of the hedging instruments in the insurance and the derivative security markets. We therefore see the connection between the concept of the variable participation contract and the models discussed in the above sections.

In the above sections, the individual's optimal decision against the p-risk and the Q-risk is investigated when the former is hedged in a derivative market and the latter is hedged in the

[9] Basis risk (also called spread risk) is the market risk relating to differences in the market performance of two similar positions. For example, a portfolio manager who wants to temporarily eliminate the market exposure of a diversified stock portfolio might short S&P 500 futures. If the composition of the portfolio does not exactly mirror the S&P 500, the hedge will not be perfect, and the portfolio manager will be taking basis risk.

insurance market. Thus, the optimal solutions, like p_i^* and Q_f^* and p_i^{***} and Q_o^{***},[10] which are obtained by assuming that the individual makes the hedging decisions in the insurance and the derivative markets *simultaneously*, can be equally explained as the optimal solutions of a variable participation contract which an insurance company issues against risk pQ as a whole.

Note that Doherty and Schlesinger (2001) assumed the independence between $(1+\varepsilon)$ and L_i by assuming that $(1+\varepsilon)$ is identical among all individuals and that L_i is mutually independent. We relate $(1+\varepsilon)$ to the p-risk and L_i to the Q-risk.[11] The result in Table 2 shows that, when the p-risk and the Q-risk are independent, $p_i^* = p_i^0$ and $Q_f^* = Q_f^0$. It tells us that, if the individual can separate the p-risk and the Q-risk and hedge them by themselves, the variable participation contract does not produce anything different from the separate insurance and derivative contracts. This is conditional on the assumptions of a fair premium and the lack of transaction cost. Thus the advantage of the variable participation contract may appear, 1) if the individual cannot separate and hedge the p-risk and the Q-risk by themselves and therefore has to ask an intermediation, e.g., an insurance firm, to do it which is consistent with Doherty and Schlesinger (2001) that an insurance intermediation is essential in the concept; and 2) if the transaction cost is considered. When the transaction cost is considered, the variable participation contract is expected to have an advantage over the separate contracts. Thus, the conclusions in Doherty and Schlesinger (2001) are further confirmed.

Let us go one-step further. From Table 2, we see that when the p-risk and the Q-risk are correlated and therefore are not independent, there are cases in which $p_i^* \neq p_i^0$ and $Q_f^* \neq Q_f^0$. If the variable participation contract is defined as the synthetical use of the insurance and derivative contracts no matter whether the two risks are independent or not, the variable

[10] To assert the equivalency between the optimal solutions in a unique insurance market supplying the variable participation contract and ones in separate insurance and derivative markets, Doherty and Schlesinger (2001) assumed that risk L_i can be well diversified by insurance company so that the premium is actually fair, $E(L_i)$. Although it is not clear if this is a necessary condition for a general equivalency, it is better to be careful with the condition. Thus, the model resulting in p_i^{**} and Q_f^{**} in Section 3.1 will not be related to a solution of the variable participation contract.

[11] If the p-risk denotes the market price and the Q-risk denotes an individual farmer's output risk net of the effects of some common factors like climate, which would affect a whole area's output, then the p-risk is identical for all individuals and the Q-risk is *iid* for individuals. Furthermore, if the crop insurance is based on an individual farmer, then it is reasonable to relate $(1+\varepsilon)$ to the p-risk and L_i to the Q-risk.

participation contract may produce different hedging strategies from the separate insurance and derivative contracts.

5. Concluding remarks

As it has been mentioned, this essay serves two purposes. The first is to study the effect of the derivative securities on individuals' insurance purchase as an extension of the previous studies. It is found that the insurance and the futures contracts can be treated symmetrically and the derivative markets have an impact on the insurance market and *vice versa*. A farmer may not buy the full insurance even if the premium is fair because he uses hedging instruments against the price risk. However, there are no monotonic relationships between any of the optimal solutions and any of the correlation coefficients, when the farmer's concern on derivative securities is introduced into his insurance model. Table 2 summarizes the main results from the models. When the effect of a farmer's degree of risk aversion on his hedging decision is discussed, there is no monotonic relationship between the farmer's degree of risk aversion and his insurance purchase or his hedging ratio. Thus, the effect appears differently when the crop insurance and the derivative securities are concerned separately and when they are concerned simultaneously.

The second purpose is to draw some conclusions related to the concept of the variable participation contract. This contract, which is actually a synthetical contract combining the insurance and the derivative contracts, has been presented in Section 4. The optimal solution in the model on the crop insurance, which allows the individual to make his hedging decisions on both the insurance and the derivative markets simultaneously, is equivalent to the one produced by the variable participation contract. Thus, from the perspective of the optimal hedging solution, it is concluded that the advantage of the variable participation contract may appear: 1) if the individual cannot separate and hedge the p-risk and the Q-risk by themselves and therefore has to ask an intermediary, e.g., an insurance firm, to do it; 2) if the transaction cost is considered. This conclusion is consistent with Doherty and Schlesinger (2001).

The concept of the variable participation contract is quite new, which is expected to be useful in solving the problems facing the catastrophic insurance market. One of the most important advantages of this contract is that it allows the insurers to make use of the capital advantages of the financial market, so that the financial dilemma facing the insurers may be solved more efficiently. Another advantage is that it allows the insureds to choose the degree of participation, thus allowing more flexible risk sharing between the insurers and the insureds. Making the participation degree endogenous will improve the welfare of the insureds. It is interesting to note that, when the variable participation contract allows more flexible risk sharing between the insurers and the insureds, the extension by Louberge and Schlesinger (1999) allows risk sharing between different pools of insureds, i.e., the insureds insuring risk A at an insurance company are allowed to take advantage of risk pools B or C, handled by the same insurance company.

Appendix: Proving that ρ_{pQ,PUT_Q} < 0 when the price risk *p* and the output risk *Q* are independent.

Suppose that the market price p has a density function of $p(y)$ $(0 \le y < +\infty)$ and the farmer's output Q has a density function of $q(x)$ $(0 \le x < +\infty)$. Since p and Q are independent, the joint density function of p and Q will be $p(y)\cdot q(x) =: f(x, y)$ $(0 \le x < +\infty,\ 0 \le y < +\infty)$. $PUT_Q = Max(EQ - Q, 0)$ gives $E(PUT_Q) = \int_0^{EQ}(EQ-x)q(x)dx$ and $E(PUT_Q)^2 = \int_0^{EQ}(EQ-x)^2 q(x)dx$.

$\rho_{pQ,PUT_Q} < 0$ if and only if $Cov(pQ, PUT_Q) < 0$. As $Cov(pQ, PUT_Q) = E(pQ\cdot PUT_Q) - E(pQ)\cdot E(PUT_Q)$, let us calculate $E(pQ\cdot PUT_Q)$ first. Since $pQ\cdot PUT_Q = pQ\cdot Max(EQ - Q, 0)$, we have

$$E(pQ\cdot PUT_Q) = \iint\limits_{\substack{0\le x<EQ \\ 0\le y<+\infty}} xy(EQ-x)f(x,y)dxdy$$

$$= \int_0^{+\infty} yp(y)dy \cdot \int_0^{EQ} x(EQ-x)q(x)dx$$

$$= Ep\cdot\left(\int_0^{EQ} EQ(EQ-x)q(x)dx - \int_0^{EQ}(EQ-x)^2 q(x)dx\right)$$

$$= Ep\cdot(EQ\cdot EPUT_Q - E(PUT_Q)^2)$$

$$Cov(pQ, PUT_Q) = -Ep\cdot E(PUT_Q)^2 < 0.$$

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