

# Technological Opportunities and Growth in the Natural Resource Sector

By

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***Abstract:***

Both technological and natural resource possibilities seem to evolve in cycles. The “Resource Opportunity Model” in this paper introduces the technological opportunity thinking into natural resource modeling. The natural resource industries’ choice between incremental, complementary innovations, and drastic, breakthrough innovations, will give rise to long-run cycles in the so-called familiar resource stock, which is the amount of natural resources determined by the prevailing paradigm. Incremental innovations will increase the exhaustion of the stock, and drastic innovations will create a new paradigm and, thereby, new technological opportunities and a new stock of familiar resources. Drastic innovations are endogenously affected by the knowledge level and induced either by scarcity of technological opportunities or by scarcity of resources. Generally, increased innovation ability increases the knowledge stock and cumulative income over time, but does not affect the sustainability of the resource stock even though the intensity of the resource cycles increases. However, too low innovation ability might drive the sector into technological stagnation, and resource exhaustion in the long run; and too high innovation ability might drive the sector into extraction stagnation, and resource exhaustion in the short run.

***Keywords:*** Cycles, Economic growth, Induced innovations, Natural resources, Paradigm shifts, Technological opportunities.

***JEL classification:*** O11, O13, O31, Q30, Q43, N50

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# 1 INTRODUCTION

The typical dynamics of the abundance of many natural resources are characterized by periods of pessimism with restricted resource opportunities, which are finally replaced by new eras of optimism (see e.g. Simon, 1997), even though there are examples of stagnation (see e.g. Ponting, 1994). The periodic pattern of innovations, and hence economic growth, has also been accepted as a stylized fact. Drastic innovations are followed by periods of less revolutionary innovations. Hence, both technological and natural resource opportunities seem to evolve in cycles, which give rise to several interesting questions about their possible interrelations. Are the effects of innovation on resources different depending on the type of innovation? Can technological change be the source of both prosperity and stagnation in natural resource industries? Are limited natural resources or technological opportunities the driving force of technological shifts?

Several authors highlight the importance of analyzing the underlying mechanisms of changes in resource abundance. David and Wright (1997) argue that resource abundance is not exogenously given by geological conditions, since it is to a large extent an endogenous social construction. They give several examples of how the combined effects of legal, institutional, technological and organizational responses to resource scarcity created a highly elastic supply for American mineral products between 1850 and 1950. In a survey of technological change and the environment, Jaffe et al. (2000) conclude that the “modeling of how the various stages of technological change are interrelated, how they unfold over time, and the differential impact that various policies may have on each phase of technological change” is of great importance to be able to understand the interaction between innovations and the environment. It is the purpose of this paper to model the innovation decisions of the natural resource sector using the technological opportunity approach, which is one way to create a long-run cyclic pattern of natural resources, and thereby to identify the crucial variables at different stages.<sup>1</sup>

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<sup>1</sup> It is not the purpose of this paper to model the effects of resource-saving technologies on the demand side, i.e. end-use technologies. These are, of course, of importance, but to keep the dynamics of supply responses tractable, this effect will only be discussed in the section where price changes are analyzed.

Previous models of innovations and natural resources have usually modeled jumps in the extractable resource stock by assuming a Poisson process with a constant probability of discovery (see Krautkraemer, 1998, for an overview). In some models the frequency of discovery or innovation activity is exogenous, but in others it is a function of research expenditures. As the known stock decreases, the cost of extraction increases and investments in research become profitable. Once the discoveries of new sources or new technologies are made, costs decrease and there is a new period of extraction without any innovation activity.

In this paper, however, the innovation activity is not a discrete but a continuous process, just like extraction, even though the type of innovations might differ from period to period. The focus is therefore on the choice of the type of innovation: incremental or drastic. Research could be of different characters: revolutionary or non-revolutionary, resource consuming or resource creating. However, few studies make this distinction.<sup>2</sup> Moreover, the uncertain outcome of the innovation process does not have to be modeled as completely random, but preferably as endogenously influenced by the level of technical knowledge. Another shortcoming of previous models is the inducing mechanism. Many innovations in the natural resource sector do seem to be induced by the scarcity of resources. However, one should not overlook the fact that many drastic innovations occurred without any physical resource restrictions (Jaffe et al., 2000). In the model developed in this paper this is explained by restrictions on technological opportunities.

The cyclic pattern of innovations is in Olsson (2000, 2001) explained by a theory of technological opportunities, i.e. the abundance or scarcity of technological opportunities. Incremental innovations are developed from a stock of technological opportunities. The more this stock is exhausted, the lower the returns to this activity. Consequently, at some point innovators turn to drastic innovations, which introduces new technological opportunities. This approach is especially suitable for the natural

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<sup>2</sup> One exception is Smulders and Bretschger (2002) who present a model where one type of innovation is undertaken at a certain moment in time, either a revolutionary general purpose technology, or a diffusion process of this new technology. However, it is rather cycles in pollution, not resource stocks, that is modeled and the inducing mechanism is increasing costs (as in the traditional models) because of environmental taxes, and not innovation constraints.

resource sector in which the scarcity thinking is crucial. Therefore, this paper adds a stock of natural resources to this model and studies simultaneously the interaction between technology and natural resources.

Drastic innovations in the natural resource sector can either be connected to the introduction of a new, unexpected technical solution or to the finding of a new type of resource. Some clear-cut examples of major breakthroughs of importance for the natural resource industry are the energy system shifts between horse power, wind power, coal, oil and nuclear power. The common feature of these drastic innovations is that they gave rise to sequences of “follow-up” or complementary innovations. These are non-revolutionary, or incremental innovations in the sense that they are only combinations of already existing ideas. By introducing oil as an energy resource, the mechanical revolution became possible; the steam engine revolutionized the mining industry, etc. It is through these incremental progresses, the combination of a new idea and old knowledge, that the drastic innovation becomes fruitful.

In the Resource Opportunity Model (ROM) presented in this paper, the choice of the natural resource producer is, as mentioned, not between extraction and innovations, but between the types of innovations, even though extraction affects this choice.<sup>3</sup> The alternation between incremental innovations and drastic innovations will give rise to long-run cycles in the so-called familiar resource stock, which is the stock of natural resources determined by the prevailing paradigm. Incremental innovations will increase the exhaustion of the stock, while drastic innovations will create a new paradigm and thereby a new stock of familiar resources. Drastic innovations are not only induced by resource constraints, but also by incremental innovation constraints, as in the technological opportunity model. That is, they are now created either by scarcity of technological opportunities or by scarcity of natural resources. The expected success of these drastic innovations, in introducing new technological and resource opportunities, is not constant but endogenously determined by the increasing stock of knowledge, and the society’s ability to innovate.

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<sup>3</sup> The focus of this paper is on the structural parts of the model, which to some extent results in strong simplifications on the behavioral part with the purpose of clarifying the main points.

The inclusion of restricted resources opens up the analysis for stagnation outcomes. The drastic innovation jumps in resource availability can be more or less successful, which either increases or decreases the probability of economic stagnation caused by technological constraints. Moreover, the rate of incremental innovations might differ, increasing or decreasing the probability of stagnation caused by a too intensive extraction.

The cyclic behavior of the resource stock will also be connected to economic growth in the resource sector. The incremental phase of technological development follows the pattern of exogenous growth models with decreasing returns to scale, both in technological opportunities and natural resources. On the other hand, the sharp increase in marginal returns is dependent on the endogenously determined knowledge level and not really on a “manna from heaven” change in technology. The drastic innovation is therefore characterized by endogenous technological change. This combination of both exogenous and endogenous growth periods may give us new insights about natural resource scarcity.

The main message of this paper is that technological opportunities affect resource dynamics and that sustained growth is only possible if research keeps increasing technological and resource opportunities enough. The general result is that an increased level of ability to turn technological opportunities into innovations does not affect the sustainability of the resource stock (even though the fluctuations increases), but increases the knowledge stock and the total extraction, and hence the stock of income. However, an innovation ability level that is too low might lead the sector to technological stagnation and resource exhaustion in the long run, and a level that is too high might lead the sector to extraction stagnation and resource exhaustion in the short-run.

Section 2 gives the background of the ROM by presenting the definitions of the resource stocks and innovations, plus the idea of innovation cycles. Section 3 introduces the ROM, first by presenting the modeling of technological opportunities and the resource stock dynamics during different types of innovation periods, then by modeling the profits that determine the type of innovation period, and finally by connecting the dynamics to economic growth. The results are presented by simulations in Section 4.

Alternative assumptions and stagnation outcomes are analyzed in Section 5. Section 6 concludes the paper.

## 2 BACKGROUND

### 2.1 Resource Stocks

First of all, it is important to make clear the distinction between familiar and potential resources. *Familiar resources* are the physical quantity of resources, discovered or undiscovered, under the prevailing paradigm, i.e. resources that in some way are seen as useful given the normal science at that time. *Potential resources* are the physical quantity of resources that might be seen as useful resources under other paradigms.<sup>4</sup>

The *familiar resource stock*,  $S_t$ , includes all familiar resources and it is cycles in this stock that are the focus of this paper. The stock includes both discovered and undiscovered resources. The “discovered familiar resource stock”, is the stock often referred to in previous studies of natural resources and growth, i.e. the stock of familiar resources available for extraction. The “undiscovered familiar resource stock” includes familiar resources, i.e. they are known according to the prevailing paradigm, but they have to be physically discovered before they can be extracted. Incremental innovations increase the extraction rate and hence speed up the decrease in the stock of familiar resources, while a paradigm shift increases the quantity of familiar resources, either by introducing an unexpected technology that improves the availability of already familiar resources or by adding to the number of types of familiar resources. The effects of innovations will be further explained in the next section.

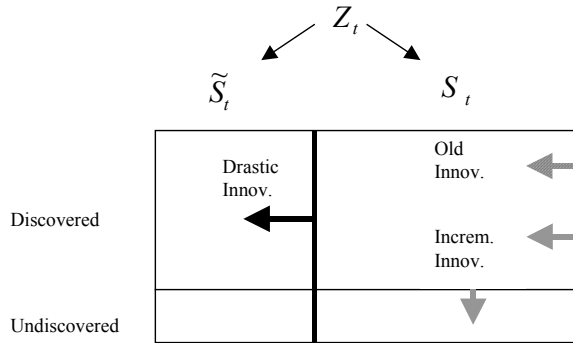
Figure 1 helps clarify the definition of  $S_t$ .  $\tilde{S}_t$  is defined as the *potential resource stock*, including the physical quantity of resources available under all possible future paradigms.  $Z_t$  is then the *total resource stock*, i.e.  $Z_t = S_t + \tilde{S}_t$ , and the only actual restriction on resources by this definition would be the thermodynamic laws.

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<sup>4</sup> The concepts “normal science” and “paradigm shifts” are borrowed from Kuhn (1962). Normal science is conducted until enough anomalies are discovered. The anomalies can no longer be ignored which induces a paradigm shift. The new normal science then created includes the earlier ignored anomalies. Normal science in the ROM is incremental innovations and a paradigm shift is a drastic innovation.

However, in this paper we will, as a simplification, assume that  $\tilde{S}_t$  is unlimited.<sup>5</sup> Note that  $\tilde{S}_t$  also includes discovered and undiscovered resources.

**Figure 1:** Resource Stocks



Assume that the total resource stock we are interested in is the stock connected to the use of energy. Then examples of discovered familiar resource stocks are oil sources whose physical locations are known. They are sources ready to be extracted using the technological knowledge at that time, or sources that you expect to be able to extract using non-revolutionary incremental extraction innovation. Examples of undiscovered familiar resource stock are oil sources not yet physically discovered but are expected to be identified using incremental discovery innovation. They differ from the potential resource stocks, which are not expected to be available at all. They are added to the familiar resources by a completely unexpected new paradigm. An example of a discovered potential resource stock in the energy context is uranium. The finding of nuclear power made uranium become a resource. Uranium was discovered long before but not seen as a resource. An undiscovered potential resource might be an oil source not even conceivable under the current paradigm. With a new revolutionary technology such as oil drilling at sea, large sources became possible to discover.

<sup>5</sup> In the “very-long-run” the long-run waves in  $S_t$  would also be negligible and the availability of familiar resources would be more or less constant. If, however, we had included the thermodynamic restrictions on  $Z_t$ , there would probably be a downward sloping trend and not a constant.

## 2.2 Innovations

The view of the innovation process as consisting of both small non-revolutionary and large revolutionary innovations, is shared by many researchers (see e.g. Schumpeter 1934, 1942; Kuhn, 1962; Dosi, 1988; Jovanovic and Rob, 1990; Mokyr, 1990; and Helpman and Trajtenberg, 1998). Olsson (2001) presents three kinds of technological innovations related to knowledge in general: incremental innovations, drastic innovations and potential innovations.<sup>6,7</sup> *Incremental innovations* are non-revolutionary changes in technology generated by combining various elements of old knowledge. The costs and risks are low, and the innovations are carried out by profit-seeking entrepreneurs. Incremental innovations are the “normal activity” in the technology field and are only bounded by the prevailing technological paradigm. *Drastic innovations* are revolutionary new ideas that combine new knowledge, potential innovations, with the old knowledge. The costs and risks are high, but the financial rewards can be substantial. Most importantly, the drastic innovations open up for new technological possibilities due to the new knowledge, creating a new technological paradigm. However, the returns and the success of the innovations are uncertain and the risks of free-riding are high. The *potential innovations* are the pieces of new knowledge that drastic innovations can connect to the prevailing paradigm. These are considered to be anomalies at first, since they do not fit into the normal science in the old knowledge. They are not a result of systematic entrepreneurship but of random findings, often while conducting normal science.

In this study we look at the technological innovations on the supply side affecting the natural resource sector. Potential innovations are “islands” outside the natural resource knowledge. A potential innovation might have been used in another sector but may still be irrelevant to the science of natural resources. This is actually the typical situation for the natural resource industry which is not a research intensive

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<sup>6</sup> These are similar to other concepts such as micro and macro inventions (Mokyr, 1990), or secondary and fundamental innovations (Aghion and Howitt, 1998). The concepts are also related to the so-called technology “lock-in”, where a particular technology might create path dependence for the follow-up innovations (Dosi, 1988; Jaffe et al., 2000).

<sup>7</sup> Olsson (2001) defines potential innovations as discoveries but because of the possible confusion with resource discoveries we will use potential innovations.



sector, but instead innovative when it comes to implying technological solutions from other parts of the economy (Simpson, 1999). Note that a potential innovation can be either a completely new technology or a completely new type of resource.

As we will see, the drastic innovations are induced by the low returns to incremental innovations, which in the ROM is either due to a low level of technological opportunities or a low level of physical resource availability.<sup>8</sup> To put it another way, the low productivity of extraction cannot be improved enough by the few technological opportunities left. Since drastic innovations are assumed to be induced by low returns in the natural resource industry, we assume they have positive effects on the stock of resources. First, if the potential innovation was a technology, the new knowledge may have made the already familiar resources last longer by a more efficient technology than was available, or even conceivable, under the previous paradigm.<sup>9</sup> Second, if the potential innovation was a resource, the new paradigm may have made materials previously unknown or judged as non-valuable “become” familiar resources.<sup>10</sup> The drastic innovations can in some sense be interpreted as general purpose technologies since they have the potential to influence large parts of the economy. A drastic innovation in the ROM could be seen as a general purpose technology, but only in the sense that it affects large parts of the natural resource sector.

Incremental innovations are connected to the already familiar resources known under the current paradigm.<sup>11</sup> They can be divided into two categories: incremental extraction technology and incremental discovery technology. Incremental extraction innovations increase the efficiency, and hence the rate of extraction, of the discovered

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<sup>8</sup> This assumption is of course only valid for the drastic innovation connecting the potential innovations to knowledge in the natural resource sector and not to drastic innovations in a more general sense. Note that the possibility of natural resource scarcity inducing a completely new technology (i.e. a potential innovation not connected to knowledge in any sector) is possible but it could, as mentioned, also be a technology already used in other sectors but induced to be used in the natural resource sector.

<sup>9</sup> An example from the petroleum industry is the introduction of the computer making new imaging technologies possible, which made it possible to map oil sources previously hidden (Bohi, 1999).

<sup>10</sup> A straightforward example is, as mentioned, the discovery of uranium as a source of energy by the drastic innovation of nuclear power.

<sup>11</sup> Of course even incremental innovations may have a drastic innovation character, i.e. combining old ideas may have revolutionary impacts. In reality it might be difficult to separate the two innovations. However, we define drastic innovations as innovations introducing completely new knowledge to the natural resource sector.

resources.<sup>12</sup> Incremental discovery innovations increase the efficiency in finding undiscovered sources of the already familiar resources, which also affects the rate of extraction.<sup>13</sup> Notice that while a drastic innovation introduces completely unpredicted sources, a source discovered from an incremental innovation is not surprising in the same sense. In the latter case there is much less doubt about the existence of the source, knowing that the non-revolutionary technology of identifying the source was simply lacking.

Hence, under the prevailing paradigm there is a certain set of familiar resources, of which some sources are discovered and some are not, and the exhaustion of these are increased by incremental innovations. However, drastic innovations can introduce a new stock of familiar resources by establishing a new paradigm.

### **2.3 Innovation Cycles**

There is a large body of literature on growth cycles connected to innovation (see Stiglitz, 1993, and Aghion and Howitt, 1998, Chapter 8, for an overview). Some studies analyze the effect of growth cycles on the innovation pattern (see e.g. Stadler, 1990), while others study the impacts of changes in innovation on growth (see e.g. David, 1990; Juhn et al., 1993; Bresnahan and Trajtenberg, 1995; and Helpman and Trajtenberg, 1998). However, for this study it is important to find a model that formalizes the distinctions between drastic and incremental innovations and their different impacts on growth, and that endogenizes the frequency and the success of the drastic innovations instead of just letting them occur in a stochastic process. I will therefore follow the tradition of studies like Jovanovic and Rob (1990), Boldrin and Levine (2001) and Olsson (2001) where the driving force of the growth cycles is the trade-off between new major innovations and refinements of old ones.

Olsson (2001) presents a model to explain the cyclic behavior of technology and economic growth that puts technological opportunities in the center of the analysis

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<sup>12</sup> An example is when the traditional vertical oil drilling technique was replaced by horizontal drilling, making it possible to approach a reservoir from any angle and hence drain it more thoroughly (Bohi, 1999).

<sup>13</sup> An example from the coal industry is the development of the longwall mining, which made it possible to more efficiently exploit deeper and thinner seams of coal (Darmstadter, 1999).

rather than changes in firm and consumer behaviors. Unlike other work in the area, technological opportunities are modeled explicitly and determined endogenously. Rational innovators choose between two basic strategies: to carry out incremental or drastic innovations. The choice depends on which innovation gives the highest expected profit. During periods of normal activities, rational entrepreneurs use the existing technological opportunity to make non-revolutionary, incremental innovations. The technological opportunities are limited by the prevailing paradigm, so as the opportunities becomes exhausted, profits and economic growth decrease. Eventually, profits from incremental activities fall below the expected profits from the revolutionary, drastic innovations. This shifts the interest of the entrepreneurs, and the cluster of drastic innovation activities introduces a new technological paradigm with new technological opportunities. Once again incremental innovation becomes profitable. It is hence through the incremental innovations that the drastic innovation diffuses into the economy.

### **3 THE RESOURCE OPPORTUNITY MODEL**

An important difference between the dynamics of technology as presented in Olson's general technological opportunity model and the ROM presented here is, as mentioned, the driving force of technological development and the effect of technology on resources. In the previous case it was the scarcity of technological opportunities that created incremental innovation constraints and drove the economy into a shift, while it is the scarcity of resources *or* technological opportunities that create incremental innovation constraints in this model. The resource stock is a rival good needed for production and consumption outside the resource sector, and therefore always tends to decline. Because of this complementarity between resources and production, the resource stock determines the size of the market in which incremental innovations can be applied. Hence, the market for incremental innovations continuously shrinks until a drastic innovation creates new resources and technological opportunities. Note that both these scarcities are only indirectly driving the technological changes by their effects on the entrepreneurs' expected profits from incremental versus drastic innovations.

We begin by presenting the dynamics of technological opportunities. We then describe the resource stock dynamics and its connections to innovations depending on the type of innovation in that period. After that, we look at the changes in innovation profits, which determine whether innovations are incremental or drastic in the following period. Finally, a simple growth function of the natural resource sector is presented. Since an analytical solution of the model would be intractable, we will present the result using simulations in Section 4.

### 3.1 Technological Opportunities

There are three fundamental variables of technology:  $A_t$ ,  $B_t$  and  $D_t$ .<sup>14</sup>  $A_t$  is the technology stock, or the set of all known technological ideas at  $t$ .  $B_t$  is the technological opportunities, and  $D_t$  is the success of the drastic innovation in terms of increase in the amount of technological opportunities. The knowledge stock evolves in the following way. A technological opportunity exists if it is possible to connect two technologically close ideas. By connecting two ideas you create a new idea that in turn can be used for new combinations. These unions of old ideas are the incremental innovations and they systematically add new knowledge and thereby increase  $A_t$ ; but at the same time they decrease the technological opportunities left to explore,  $B_t$ . Hence, at each point in time there is a stock  $B_t$ , the technological opportunity, which is the stock of potential ideas left to exploit until  $A_t$  is maximized under the current paradigm.

As  $B_t$  becomes close to exhaustion, entrepreneurs realize that the profits from incremental innovations are coming to an end, and when these profits drop to the level of expected profits from the more uncertain drastic innovations, the entrepreneurs switch over to these activities instead. This phenomenon can be described as follows. Apart from incremental and drastic innovations there is the third component in the technological advancement - potential innovations. These ideas outside  $A_t$ , regarded as irrelevant, do not directly contribute to new knowledge since they do not have any

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<sup>14</sup> See Olsson (2000, 2001), on which this section is based, for a more extended discussion and a set theory approach of the innovation dynamics.

immediate commercial value. For this new knowledge to be used as normal science it has to be connected to the old knowledge,  $A_t$ , by a drastic innovation,  $D_t$ . As mentioned above, entrepreneurs turn to drastic innovation activities when there is a small  $B_t$  left to explore by incremental innovations. A successful drastic innovation that connects a potential innovation with  $A_t$ , introduces new technological opportunities and a new  $B_t$  can be explored. This is called a technological paradigm shift and some of the old anomalies, the potential innovations, are now included in the normal science stock  $A_t$ . Definition 1 gives a formal definition of a technological paradigm shift.

**Definition 1** If  $B_t > B_{t-1}$  then a *technological paradigm shift* has occurred at  $t$ .

After a technological paradigm shift a new period of systematic incremental innovations begins.

Let us assume that  $\phi_t = 1$  in a period of incremental innovations, and  $\phi_t = 0$  in a period of drastic innovations.<sup>15</sup> Note that a period could be seen as a period longer than a year.<sup>16</sup> As mentioned, entrepreneurs have a myopic behavior and form their decision only on the basis of the expected profits in the next period. If the profits from drastic innovations are higher than the profits from incremental innovations, all entrepreneurs shift their efforts to drastic innovation activities that period. The determinants of  $\phi_t$  will be further discussed in Section 3.3. The two sources of change in  $B_t$ , namely (i) incremental innovations that decrease  $B_t$  and (ii) drastic innovations that increase  $B_t$ , can formally be described as in Equation 1.<sup>17</sup>

$$B_t = B_{t-1} - \phi_t \delta B_{t-1} + (1 - \phi_t) D_t \quad (1)$$

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<sup>15</sup> The assumption that only one type of technological innovation takes place at the same time is a simplification to reduce the complexity of the model.

<sup>16</sup> Hence, a period of drastic innovations may also include the possible downturn in the economy before the new technological opportunities are adopted. This paper will however not model this explicitly.

<sup>17</sup>  $X_t$  refers to the stock of  $X$  in the end of period  $t$ . Therefore,  $X_{t-1}$  is the stock available for use in the beginning of period  $t$ .

Thus, during periods of incremental technological progress, the stock of technological opportunities declines according to  $B_t - B_{t-1} = -\delta B_{t-1}$ , where  $\delta$  is a measure of the capacity of society to exploit intellectual opportunities, i.e. the ability to innovate.  $\delta$  is mainly a function of the number of innovators and the human capital level, but also of underlying institutions such as the educational system, corporate laws and the general attitude towards rationalism and scientific curiosity.  $\delta$  is modeled as a constant, and since  $B_t$  decreases every period of normal science the entrepreneurs get less and less output from incremental activity.

During periods of drastic innovations  $B_t - B_{t-1} = D_t$ , i.e. the paradigm shift increases the technological opportunities with the random variable  $D_t$ , which can be used for incremental innovations in the next period.  $E_{t-1}(D_t) = f(\delta, A_{t-1})$  describes the expected technological “success” of the drastic innovation and increases in both  $\delta$  and  $A_{t-1}$ . Hence, the periods of incremental innovations are highly predictable while the outcome of a paradigm shift is not.

Equation 2 describes the dynamics of the knowledge stock.

$$A_t = A_{t-1} + \phi_t \delta B_{t-1} \quad (2)$$

During periods of incremental innovation the knowledge stock increases with the amount of technological opportunities that are turned into new innovations ( $A_t - A_{t-1} = \delta B_{t-1}$ ). During periods of drastic innovations the knowledge stock is constant ( $A_t - A_{t-1} = 0$ ). Even though there are new technological opportunities created by a drastic innovation, they can only be turned into new knowledge during a period of incremental innovations.

We will now turn to the resource stock and see how its dynamics are connected to the waves of technology, and how this in turn affects economic growth. We are interested in the knowledge and technological opportunities related to the natural resource sector, so in the rest of this paper  $A_t$  and  $B_t$  will refer to these more specific

stocks. As we will see,  $\delta$  and  $D_t$  are crucial determinants for long-term resource availability and economic growth.

### 3.2 Resource Stock Dynamics

In the ROM, a paradigm shift is induced either by a small  $B_t$  or by a small familiar resource stock,  $S_t$ . We know about the dynamics of  $B_t$ , but what determines changes in  $S_t$ ? During both incremental and drastic innovation periods there is extraction determined by old knowledge, and hence  $S_t$  decreases independent of technological changes in the natural resource sector in that specific period. The effects of technological changes on  $S_t$  are dependent on the type of innovation period, i.e. on  $\phi_t$ . The dynamics of  $S_t$  are presented in Equation 3.

$$S_t = S_{t-1} - \mu A_t S_{t-1} + (1 - \phi_t) \lambda D_t, \quad (3)$$

where  $\mu$  is a parameter representing the effect of the technological level on the physical resource quantity and  $\lambda$  is a parameter representing the effect of drastic innovation on the physical resource quantity.<sup>18</sup> The extraction rate is a function of the stock of innovation at the end of period  $t$ ,  $A_t$ . Using the expression for knowledge in Equation 2 we get:

$$S_t = S_{t-1} - \mu A_{t-1} S_{t-1} - \phi_t \mu \delta B_{t-1} S_{t-1} + (1 - \phi_t) \lambda D_t. \quad (4)$$

Let us call the second term on the right hand side the knowledge stock effect, the third the incremental innovation effect and fourth the drastic innovation effect. During incremental innovation periods ( $\phi_t = 1$ ) we have  $S_t = S_{t-1} - \mu A_{t-1} S_{t-1} - \mu \delta B_{t-1} S_{t-1}$ , i.e.

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<sup>18</sup> A more general model would take into account that old vintages of technology are of limited use when it comes to extraction of new familiar resources. This would imply that the effect of the aggregate technology on the resource stock is reduced as the technological level increases, i.e.  $(\partial \mu_t / \partial A_t) > 0$ . However, the assumption would still be that the final effect of aggregated technology on the extraction rate is positive.

the resource stock is continuously decreasing by the knowledge stock effect and the incremental innovation effect. As long as  $\mu(A_{t-1} + \delta B_{t-1}) < 1$ , the resource stock is not depleted during the period, i.e.  $S_t > 0$ .<sup>19</sup> During drastic innovation periods ( $\phi_t = 0$ ) we have  $S_t = S_{t-1} - \mu A_{t-1} S_{t-1} + \lambda D_t$ . The resource stock still tends to decrease because of the extraction possible due to the knowledge stock from previous periods, but the stock may now show a net increase by the drastic innovation effect. This gives us a second definition:

**Definition 2** If  $S_t > (1 - \mu A_{t-1}) S_{t-1}$  then a *resource paradigm shift* has occurred at  $t$ .

A resource paradigm shift always follows a technological paradigm shift. However, because of the continued extraction through the knowledge stock effect, the resource stock does not have to increase (it decreases if  $\mu A_{t-1} S_{t-1} > \lambda D_t$ ) as a consequence of a paradigm shift, even though technological opportunities always increase (see Definition 1).

The *knowledge stock effect*,  $\mu A_{t-1} S_{t-1}$ , affects the stock during both periods since there is extraction taking place regardless of the innovation activities. Since all incremental innovations add to the knowledge, the effect on the extraction rate is due to all previous innovations, i.e. the sum of innovations during  $t \in [0, t-1]$ . The knowledge stock,  $A_{t-1}$ , is non-decreasing over time but the knowledge stock effect may decrease if the resource stock decreases, since the marginal resource effect of knowledge is  $\mu S_{t-1}$ .

During periods of incremental innovations, extraction of  $S_t$  increases with the *incremental innovation effect*,  $\mu \delta B_{t-1} S_{t-1}$ . This effect on the extraction rate is due to the

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<sup>19</sup> This would imply that, since  $A_{t-1}$  is non-decreasing as  $t$  increases, all societies would end up with depleted resources at some  $t$ . Pessimists would maybe argue that this is the case: if technology that is powerful enough is available, the myopic behavior of individuals will lead to resource depletion. However, in this study we will, by choosing a small enough  $\mu$ , only analyze the time interval where a society's innovation ability must be close to its maximum ( $\delta$  close to 1) to reach such critical levels of technology. A country with a lower ability to innovate will reach these extraction rates after a longer time interval than included in this study, and then other precautionary principles may have arisen.



amount of incremental innovations during  $t$ , i.e.  $\delta B_{t-1}$ . First, improved extraction technology decreases the extraction costs per unit of the discovered resources, and thereby increases the rate of extraction. Second, discovery technology may improve with incremental innovation, lowering the costs of discovery per unit, which increases the transformation rate of undiscovered resources to discovered, and hence extractable, resources.<sup>20</sup> This negative effect of incremental innovation on  $S_t$  decreases during the period for two reasons. First, the rate of technological improvements decreases since the amount of technological opportunities decreases (less idea combinations possible). Second, the resource stock decreases and the remaining technological opportunities can only be applied to a smaller amount of resources. The marginal resource effect of technological opportunities is  $\mu\delta S_{t-1}$ , i.e. it decreases as  $S_{t-1}$  decreases.

During periods of paradigm shifts, there is a possible positive effect on  $S_t$  through the *drastic innovation effect*,  $\lambda D_t$ . This is the result of the same entrepreneurial effort that simultaneously leads to an increase in the technological opportunity set of a size  $D_t$ , as described in Equation 1.  $S_t$  might increase for two reasons: (i) discoveries of more efficient technology make the already familiar resources last longer, and (ii) earlier potential resources become familiar resources.  $\lambda$  is a parameter representing the effect of the drastic innovations on the physical resource quantity.<sup>21</sup> The expected value of  $D_t$  is non-decreasing in time since it is a function of  $\delta$  and  $A_{t-1}$ , and the knowledge stock is, as mentioned, non-decreasing in time.

To summarize, during the process of incremental innovation the familiar resource stock continuously shrinks. The familiar resource stock or the technological opportunities tend to get exhausted. At a certain point (determined by the relative profits from incremental and drastic innovation shown in the next section) the critical level of resources or technological opportunities is reached. Drastic innovations then become more profitable and increase not only the physical amount of familiar resources, but also

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<sup>20</sup> The type of technological change that occurs during the incremental innovation period (extraction technology which decreases  $S_t$  or discovery technology which keeps  $S_t$  constant) depends on the expected profits from the two technological improvements. This creates short-run waves in the stock of discovered familiar resources not modeled in this paper.

<sup>21</sup>  $\lambda \geq 0$  since the drastic innovation is induced to relax resource scarcity.

the technological potential to extract the familiar resources. With a successful drastic innovation, these effects take the natural resource sector away from the critical level and create new possibilities for incremental innovations.

Looking at a time period  $t = 0$  to  $t = T$ , what determines if the resource stock has increased, decreased or remained constant is the relationship between the amounts added from drastic innovations and total extraction. Hence,

$$\text{if } \lambda \sum_0^T (1 - \phi_t) D_t(\delta, A_{t-1}) \begin{cases} > \\ < \\ = \end{cases} \mu \sum_0^T (A_{t-1} + \phi_t \delta B_{t-1}) S_{t-1} \text{ then } S_t \text{ is } \begin{cases} \text{increased} \\ \text{decreased} \\ \text{unchanged} \end{cases} \text{ at } t = T. \quad (5)$$

The main reasons to analyze the interactions between technology and natural resources as dependent on the type of innovation, are the following: the types of innovation are induced by different kinds of scarcity, their success is dependent on different institutional arrangements and they result in different resource availability effects. Incremental technology is induced by straightforward “profit scarcity”, i.e. the continuous thrive for lower costs in a competitive market. Profit maximization is the indirect reason for drastic innovations as well, but the directly inducing mechanism is a low  $S_{t-1}$  or a low  $B_{t-1}$ . The success of incremental extraction or discovery technology depends mainly on non-revolutionary, entrepreneurial incentives. Drastic technology, however, is a public good with free-riding problems and high risks involved. When it comes to the resource availability effects, incremental technology decreases  $S_t$  while drastic innovations increases  $S_t$ .

### 3.3 Determinants of the Innovation Period

In the previous sections we have identified the three state variables  $B_t$ ,  $A_t$  and  $S_t$ , whose equations of motion are shown in Equations 1, 2 and 4. We will now look closer at the profitability during the two innovation periods, depending on these variables. They are important since the expected profits determine the innovation direction during the next period, i.e.  $\phi_t$ . Innovators are assumed to be risk neutral and

their planning horizon is only one period ahead. They form their innovation decision on the information available at the beginning of the period and do not revise this decision until the next period.<sup>22</sup>

The total profit ( $\Pi_t$ ) of the natural resource industry is  $\Pi_t = p(S_{t-1} - S_t) + (1 - \phi_t)\Pi_t^{ID}$ , i.e. the profit from extraction where the costs are assumed to be zero, plus the direct profits from the drastic innovation in the case of a paradigm shift. This can also be expressed as profits from the knowledge stock effect ( $\Pi_t^A$ ) and profits from innovations ( $\Pi_t^I$ ):  $\Pi_t = \Pi_t^A + \Pi_t^I$ , where  $\Pi_t^I$  is either profits from incremental innovations ( $\Pi_t^{II}$ ) or drastic innovations ( $\Pi_t^{DI}$ ).  $\Pi_t^A = p\mu A_{t-1} S_{t-1}$  where  $p$  is the price index of the resource that we for now assume is constant (see Section 5.2 for an extended price effect analysis). The extraction costs are, as mentioned, assumed to be zero since they are small compared to the costs of drastic innovations.

Since the profits from the knowledge stock effect are present independent of the type of innovation in that period, this effect is not of interest when it comes to determining the type of innovation activity. The determinant of the innovation activity looks as follows:

$$\Pi_t^I = \phi_t \Pi_t^{II} + (1 - \phi_t) \Pi_t^{DI} \quad \text{where} \quad \phi_t = \begin{cases} 1 & \text{if } \Pi_t^{II} > \Pi_t^{DI} \\ 0 & \text{if } \Pi_t^{II} \leq \Pi_t^{DI} \end{cases}, \quad (6)$$

which means that  $\Pi_t^I$  is maximized with regard to  $\phi_t$ , given the dynamics of the three stock variables  $B_t$ ,  $A_t$  and  $S_t$ .<sup>23</sup> The profit maximization hence determines where the

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<sup>22</sup> Hence, the decision is more of a “rule of thumb” based on what gives the highest profits at that moment, than a continuous profit maximization problem. In the long run these principles give the same result, but the discontinuous decision opens up the possibility of stagnation during a running period. A rationale for this is the confidence that, since new revolutionary discoveries have solved the depletion problems previously, the depletion possibility may be ignored. Moreover, decisions are path-dependent, and livelihood may therefore be dependent on a continuing unsustainable extraction rate. Finally, open access conditions may pertain in the natural resource sector making it optimal to deplete the resource completely.

<sup>23</sup> Note that it should have been the expected profits that were maximized, but we will simplify the analysis by assuming non-stochastic profits.

ability to innovate,  $\delta$ , should be used, which is the same as determining where the innovators and their human capital should be allocated.

The profits from incremental innovations are determined by variables already known at  $t - 1$ , so the expected profits equal the actual profits. Profits from incremental innovation evolve according to Equation 7.

$$\Pi_t^I = p\mu\delta B_{t-1}S_{t-1} \quad (7)$$

The incremental profits are simply the product of the extraction based on incremental innovations and the price level.  $\Pi_t^I$  will always be lower after periods of incremental innovation because of decreasing technological opportunities and resources, but is usually higher after a period of drastic innovations. These dynamics are more thoroughly explained in Appendix 1.

The profits from drastic innovations are highly simplified. In reality the actual profits are uncertain, and might even be negative, even though the expected profits might be constant.<sup>24</sup> However, in this model the expected profits equal actual profits as a simplification. This does not change the results except for leaving out the possibility of very high or strongly negative growth during the temporary drastic innovation period. The profits from drastic innovations can therefore be expressed as in Equation 8.

$$\Pi_t^{DI} = \bar{\Pi} \quad (8)$$

where  $\bar{\Pi}$  is a constant. Note that the profit from a drastic innovation is the direct profit to the entrepreneurs, i.e. the profit from the patent. However, the increase in technological opportunities and natural resources from this drastic innovation, i.e. the success of the innovation, will produce extraction profits in future periods.

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<sup>24</sup> Olsson (2001) models the profits from drastic innovations as  $\Pi_t^{DI} = R - c$ , where  $R$  is random revenue, with zero and maximum profit equally likely, and  $c$  is a substantial cost. Hence, even though the expected profits from drastic innovations are constant, as in this paper, the actual profit might vary a lot and even become negative. These stochastic assumptions are, however, not needed for the purpose of this paper.

We can now determine the breakeven point between the different innovation periods by equating their profits, i.e.  $\Pi_t'' = \Pi_t^{D'}$ . The product of the stock of familiar resources and the technological opportunities left at this breakeven point is a constant  $(SB)^*$  and is described in Equation 9.

$$(SB)^* = \frac{\bar{\Pi}}{p\mu\delta} \quad (9)$$

The breakeven point for the familiar resource stock increases with profits from drastic innovations, but decreases with the price of the resource, the effects on the quantity of resources from incremental innovations and the capability of turning technological opportunities into innovations, since these decrease the profits from incremental innovations. Interestingly, the shift can be induced, and thus generate more familiar resources, even in a situation with abundant resources, if there is a lack of technological opportunities. This is the case of a *technological opportunity induced shift*. This shift can however be delayed because of a large resource stock, since even small progresses in incremental technology give high pay-offs with abundant resources. On the other hand, if there is a small stock of resources a shift may occur even if there are a lot of technological opportunities. In this case we have a *resource induced shift*. This is derived logically from the assumption that profits from incremental innovations in the natural resource sector are dependent on how much resources are left on which to apply the new technology.

### 3.4 Economic Growth

Income growth,  $g_t$ , for the natural resource sector is presented in Equation 10 and is simply defined as the proportional rate of change in profits in this sector. As we will see, the growth rate is mainly determined by the changes in the extracted amount of resources, but also by the direct profit in the case of a drastic innovation.

$$g_t = \frac{\Pi_t - \Pi_{t-1}}{\Pi_{t-1}} \quad (10)$$

Assuming that we had drastic innovations in period  $t-1$  ( $\phi_{t-1} = 0$ ), the growth rates in period  $t$  can be described as in Equation 11 and 12. Assuming instead that we had incremental innovations in period  $t-1$  ( $\phi_{t-1} = 1$ ), the growth rate in period  $t$  can be described as in Equation 13 and 14.  $g_t^I$  is the growth rate if there are incremental innovations at  $t$  ( $\phi_t = 1$ ), and  $g_t^{DI}$  is the growth rate if there are drastic innovations at  $t$  ( $\phi_t = 0$ ).<sup>25</sup>

$$g_t^I(\phi_{t-1} = 0) = \frac{p\mu[A_{t-2}(S_{t-1} - S_{t-2}) + \delta B_{t-1}S_{t-1}] - \bar{\Pi}}{p\mu A_{t-2}S_{t-2} + \bar{\Pi}} \quad (11)$$

$$g_t^{DI}(\phi_{t-1} = 0) = \frac{p\mu A_{t-2}(S_{t-1} - S_{t-2})}{p\mu A_{t-2}S_{t-2} + \bar{\Pi}} \quad (12)$$

$$g_t^I(\phi_{t-1} = 1) = \frac{A_t}{A_{t-1}} \frac{S_{t-1}}{S_{t-2}} - 1 \quad (13)$$

$$g_t^{DI}(\phi_{t-1} = 1) = \frac{S_{t-1}}{S_{t-2}} - 1 + \frac{\bar{\Pi}}{p\mu A_{t-1}S_{t-2}} \quad (14)$$

The growth rate from a drastic innovation period to an incremental innovation period,  $g_t^I(\phi_{t-1} = 0)$ , is expected to be large since we know the drastic innovation was successful.<sup>26</sup>  $\phi_{t-1} = 0$  means that Equation 4 gives  $S_{t-1} - S_{t-2} = -\mu A_{t-2}S_{t-2} + \lambda D_{t-1}$  and Equation 1 gives  $B_{t-1} = B_{t-2} + D_{t-1}$ . The successful drastic innovation in the preceding period, i.e. the large  $D_{t-1}$ , therefore induces substantial increases in  $S_{t-1} - S_{t-2}$  and  $B_{t-1}$ . Hence, as long as the profit from the preceding drastic innovation is not extremely large, the growth potential will be large for the incremental innovation period.

<sup>25</sup> See Appendix 2 for more detailed calculations on the growth rates and the conditions for positive or negative growth.

<sup>26</sup> An unsuccessful drastic innovation would be followed by another drastic innovation period.

The growth rate from one drastic innovation period to another drastic innovation period,  $g_t^{DI}(\phi_{t-1} = 0)$ , is small. As mentioned,  $S_{t-1} - S_{t-2} = -\mu A_{t-2} S_{t-2} + \lambda D_{t-1}$  if  $\phi_{t-1} = 0$ . With an unsuccessful drastic innovation at  $t-1$ , which is the case when period  $t$  is also a drastic innovation period, the increase in the resource stock is very small. The knowledge stock effect might even outweigh the innovation effect on the resource stock. Hence, growth might be both positive and negative, but in both cases the rate is small.

The growth rate from one incremental innovation period to another incremental innovation period,  $g_t^{II}(\phi_{t-1} = 1)$ , is positive if the percentage increase in knowledge is larger than the percentage decrease in the resource stock during the incremental innovation period, and vice versa. This depends to a large extent on the choice of parameters, since  $(A_t/A_{t-1}) = 1 + \delta B_{t-1}/A_{t-1}$  and  $(S_{t-1}/S_{t-2}) = 1 - \mu A_{t-1}$ , if  $\phi_t = 1$  (see Equations 2 and 3).<sup>27</sup> However, we do know that during a time interval of incremental innovation periods the growth rate will decrease, since the positive effect on the knowledge stock decreases with decreases in  $B_{t-1}$ , and the negative effect on the resource stock increases with increases in  $A_{t-1}$ . However, this decrease in the growth rate becomes smaller and smaller every incremental innovation period since there will be less and less technological opportunities and resources.

The growth rate from an incremental innovation period to a drastic innovation period,  $g_t^{II}(\phi_{t-1} = 1)$ , is expected to be small, especially if the extraction rate was large in the preceding incremental period.<sup>28</sup> Since  $(S_{t-1}/S_{t-2}) = 1 - \mu A_{t-1}$  if  $\phi_t = 1$ , we know that the probability of a drastic innovation to increase the growth rate decreases over time, since the knowledge stock (and hence the extraction rate) increases over time.

The cumulative income in the natural resource sector during a period from  $t = 0$  to  $t = T$ ,  $Y_T$ , is the sum of profits as is shown in Equation 15.

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<sup>27</sup> Remember that  $A_{t-1} = A_{t-2} + \delta B_{t-2}$ , i.e. the resource stock decreases both by a knowledge effect and an innovation effect.

<sup>28</sup> In that case the profits level during the preceding period might have been substantial even though the expected profit for a new incremental innovation period is very low.

$$Y_T = \sum_{t=0}^T \Pi_t = \sum_{t=0}^T \{p\mu(A_{t-1} + \phi_t \delta B_{t-1})S_{t-1} + (1 - \phi_t)\bar{\Pi}\} \quad (15)$$

This income stock is highly correlated with the total extraction of  $S_t$ .  $Y_T$  is of interest since it indicates the potential value of the resource sector during a certain time interval.

## 4 SIMULATION RESULTS

In this section we will analyze the results from the dynamics presented in the previous sections by simulations, and discuss the possibilities of stagnation. The effects depend, to a large extent, on the uncertain outcome of the paradigm shift, i.e. on the success ( $D_t$ ) of the drastic innovation period.<sup>29</sup> Figure 2 gives an example of how the dynamics of  $S_t$  might look depending on the outcome of  $D_t$  (see Equation 4), and Figure 3 illustrates the cycles of  $g_t$  (see Equations 11, 12, 13, and 14) during the same period.<sup>30</sup>

Notice first that the drastic innovation occurs at different levels of the resource stock, i.e. the value of  $S^*$  changes depending on the amount of technological opportunities left at that moment. This reflects the fact that a drastic innovation is either technological opportunity induced or resource induced. We will first analyze what happens during a period of drastic innovations, and then the implication of this on the following period.

If the drastic innovation was successful, in the sense that it contributed enough to the technological opportunities,  $B_t$ , and to the resource stock,  $S_t$ , by a large  $D_t$ , the economy would be saved from the critically low levels of  $S_t$  and a new era of economic growth would be starting (see Periods 0, 2, 5, 10, 13, and 16 in Figure 2). What happens is that  $D_t$  increases  $S_t$  directly by  $\lambda D_t$ , and the higher the  $B_t$ , the lower the critical level  $S^*$ , since  $S^* = \bar{\Pi}/p\mu\delta B_{t-1}$ . Both of these effects increase the possibilities for

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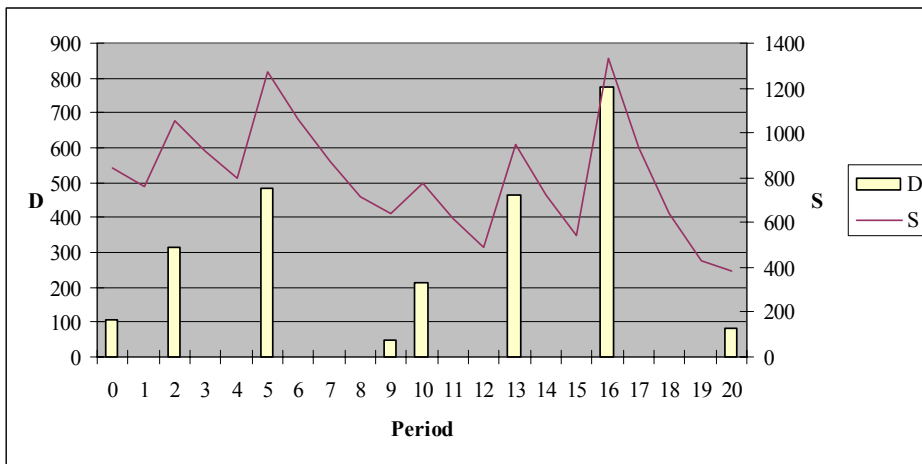
<sup>29</sup> Note that the cycles would prevail if  $D_t$  was assumed to be deterministic. The cycles would be more uniform, only increasing over time because of the increase in  $A_{t-1}$ .

<sup>30</sup> For all simulations we have used  $\delta = 0.5$ ,  $\mu = 0.02$ ,  $\lambda = 500$ ,  $p = 10$ ,  $\bar{\Pi} = 100$ ,  $B_0 = 2$ ,  $S_0 = 1000$ ,  $A_0 = 10$  and  $Y_0 = 10000$ .  $D_t = RAND(1 + A_t/1000)(1 + 10\delta)$  where  $RAND$  is a random number between 0 and 1. Alternative assumptions will be discussed in Section 5.

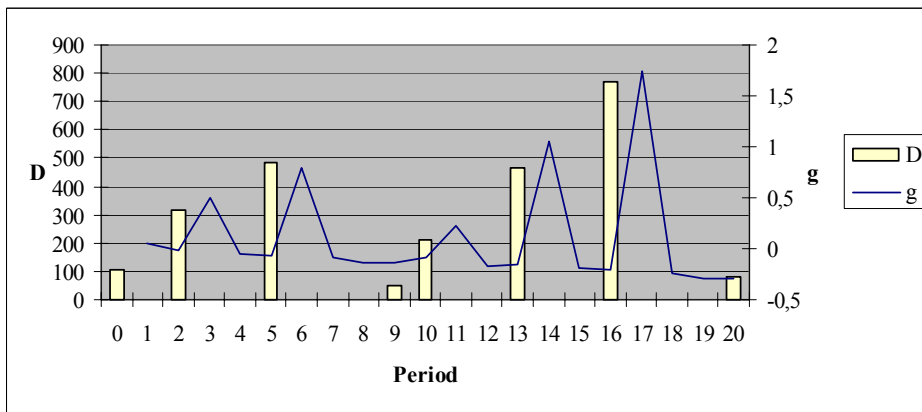


incremental innovations.<sup>31</sup> If, however, the drastic innovation only led to a small paradigm shift, then  $S_t$  would increase only slightly and maybe not even exceed the new lower critical level  $S^*$  (see Period 9). In that case the paradigm shift was not large enough to compensate for the decrease in  $S_t$  due to the knowledge stock effect (which continues independent of the type of innovation period).

**Figure 2:** The dynamics of the familiar resource stock and drastic innovations.  $\delta = 0.5$  and  $(BS)^* = 100000$ .



**Figure 3:** Economic growth in the natural resource sector and drastic innovations.



<sup>31</sup> During the period of incremental innovations,  $S_t$  decreases and  $S^*$  increases, “closing the gap” for these kinds of innovations.

The profit from the drastic innovation is independent of the success of the innovation, since this profit is assumed to be constant. Hence, the growth rate depends on the preceding period's profits in relation to this constant (see Equation 12 and 14). As mentioned in the previous section, both these growth rates are most often small.

So, what happens in the period following a drastic innovation period? The economy continues with a period of incremental innovations if  $B_{t-1}S_{t-1} > (BS)^*$ , since incremental innovations then have higher expected profits.<sup>32</sup> This incremental innovation period leads to a new drastic innovation once the critical level  $(BS)^*$  is reached again (see Period 1, 3-4, 6-8, 11-12 and 17-19). The endogenously induced growth rate following a successful drastic innovation is high, since the new  $B_t$  and  $S_t$  speed up extraction (see Equation 11). The growth rate then decreases for every new incremental innovation period, since there are decreasing returns both with respect to  $B_t$  and  $S_t$  (see Equation 13).

If, however,  $B_{t-1}S_{t-1} < (BS)^*$ , there would be a new period of drastic innovations immediately after the preceding one, since expected profits from drastic innovations still are higher than profits from incremental innovations. Hopefully this new drastic innovation is more successful so that a period of incremental innovations is profitable again. However, since there is always extraction in terms of the knowledge stock effect,  $B_{t-1}S_{t-1}$  continues to decrease during the drastic innovation periods and the gap between the actual level of  $B_{t-1}S_{t-1}$  and the critical level  $(BS)^*$  increases.<sup>33</sup>

The evolution of  $A_t$  and  $Y_t$  (see Equations 2 and 15) during the period illustrated above is presented in Figure 4.  $A_t$  increases with  $\delta B_{t-1}$  during periods of

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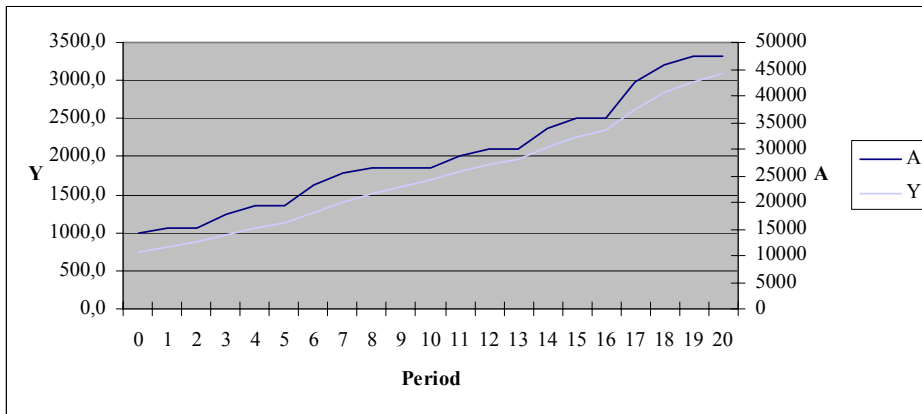
<sup>32</sup> However if the extraction rate is very high, there might be a case where the resource stock is depleted and economic growth in the natural resource sector ceases. This is called the *extraction stagnation case* and is discussed further in Section 5.1.

<sup>33</sup> Low expected success of drastic innovation therefore increases the possibilities of getting trapped in a situation where the needed size of the drastic innovation increases, making it harder and harder to exceed  $S^*$  again. This process may continue until  $S_t$  is exhausted and the growth rate in the natural resource industry drops to zero. This is called the *technological stagnation case* and is discussed further in Section 5.1.

incremental innovation and is constant during drastic innovation periods.  $Y_t$  increases during both types of periods.<sup>34</sup>

Remember that a higher  $A_t$  affects both the expected success of the drastic innovation and the knowledge stock effect. We therefore have a non-decreasing effect on the probability of drastic innovation success and the stock effect over time (see the increasing trend of  $D_t$  in, for example, Figure 2). The size of these intertemporal effects depends to a large extent on the ability to innovate,  $\delta$ , as we will see in the next section.

**Figure 4:** The dynamics of the knowledge stock and the income stock.



## 5 ANALYSIS

### 5.1 Effects of Changes in the Innovation Ability

A crucial variable is  $\delta$ , the ability to turn technological opportunities into innovations. Assume that  $\delta$  differs in societies, for example, because of different educational systems. What would happen, during a longer period, to a given resource, knowledge and income stock, depending on the societies'  $\delta$ ? A direct effect of a higher  $\delta$  is a

<sup>34</sup> Remember that  $Y$  is cumulative income, or profits in the natural resource sector. Hence, even if the total profits,  $\Pi$ , decreases from one period to another, i.e.  $g < 0$ ,  $Y$  will always increase.

higher rate of incremental innovations, given the technological opportunities. This also means that the cumulative effect on  $A_t$  increases. Both of these effects increase the depletion rate of the resource stock,  $S_t$ . Technological opportunities are exploited at a faster rate, which increases the rate of extraction in each period, and the higher  $A_t$  intensifies the knowledge stock effect over time. Hence, an increased  $\delta$  is in this sense negative for the familiar resource stock.

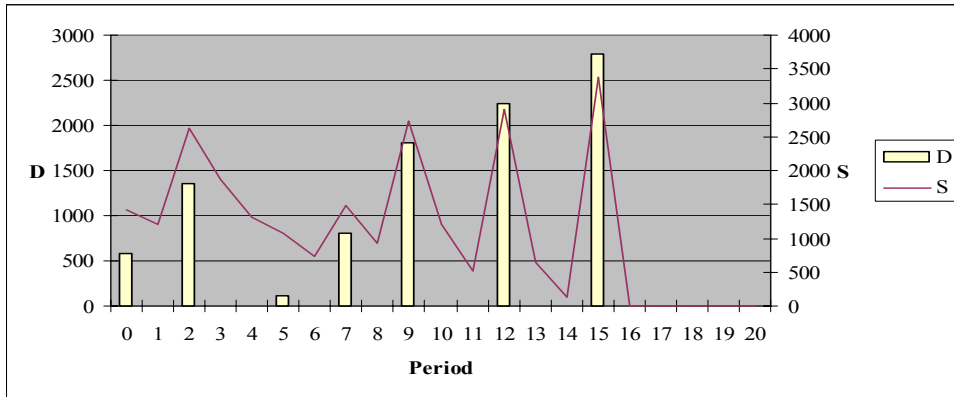
There are however positive effects as well. A higher  $\delta$  increases the probability of a drastic innovation success ( $D_t$ ), increasing the amount of technological opportunities each period. The probability of success increases also over time since  $\delta$  also affects  $A_t$ , which is non-decreasing.

Hence, regardless of a society having a low or a high  $\delta$ , we could expect a sustainable resource stock, as long as the drastic innovations are fruitful enough to compensate for the increased extraction rate (see Equation 5). The only difference is that the frequency and amplitude of the cycles with a high  $\delta$  are larger than with a low  $\delta$ . There are however other important differences in the two cases. As mentioned, since the technological opportunities add to the knowledge stock while being used up, an increased  $\delta$  also increases  $A_t$ . Moreover, even though the sustainability of the resource stock is probable in both cases, the total amount extracted and hence the cumulative income  $Y_t$ , are larger with a high  $\delta$ . Therefore, in a society with a high  $\delta$  we could expect a sustainable resource stock with high fluctuations, a large knowledge stock and a high level of cumulative income (because of a large total extraction). In a society with a low  $\delta$  there could also be a sustainable resource stock but with low fluctuations, a small knowledge stock and a low cumulative income (because of a small total extraction).

The analysis above referred to the increases or decreases of  $\delta$  in a certain interval. Let us instead turn to the extreme cases. A  $\delta$  that is too high drives the resource sector to the *extraction stagnation* case, and a  $\delta$  that is too low drives the sector into the *technological stagnation* case. With a very high  $\delta$ , the possibility of unsuccessful drastic innovations becomes negligible, especially over time, since  $A_t$

increases dramatically. However, the speed of depletion of  $S_t$  also increases drastically, both because of the direct effect on incremental innovations and the indirect effect on the knowledge stock effect, and hence the probability of extraction stagnation increases. These effects are functions of the amount of resources left from the previous period. Hence, even though the resources decrease drastically during the prevailing period, the rate of extraction is not adjusted, which makes the depletion outcome possible. Figure 5 gives an example of resource exhaustion in the short run because of a high  $\delta$ .<sup>35</sup> The amount of  $S_t$  and  $B_t$  are large in Period 15, because of a successful drastic innovation, and through a myopic decision of a large extraction rate, stagnation is a fact in Period 16. Hence,  $\mu(A_{t-1} + \delta B_{t-1}) > 1$  for  $t = 16$ .

**Figure 5:** The dynamics of the familiar resource stock in the extraction stagnation case.  $\delta = 0.95$  and  $(BS)^* = 52632$ .



With a very low  $\delta$  the probability of a successful drastic innovation is also very low, and hence the probability of technological stagnation increases. This leads to a large number of drastic innovations, since the probability for a paradigm shift to compensate for the decrease in  $S_t$  (due to the knowledge stock effect) is very small, i.e. the probability of a new drastic innovation period is high. Since no technological opportunities are used up during drastic innovation periods, there is no increase in  $A_t$

<sup>35</sup> Notice the different scales of  $D_t$  and  $S_t$  compared to Figure 2.

which otherwise would have increased the probability of a larger  $D_t$ . This in turn may have compensated for the increased gap between the higher  $(BS)^*$  and the lower  $B_{t-1}S_{t-1}$ . Figure 6 gives an example of a declining resource stock in the long run

because of a low  $\delta$ , i.e. the case of  $\lambda \sum_0^T (1 - \phi_t) D_t(\delta, A_{t-1}) < \mu \sum_0^T (A_{t-1} + \phi_t \delta B_{t-1}) S_{t-1}$ .<sup>36</sup>

**Figure 6:** The dynamics of  $S_t$  in the technological stagnation case.  $\delta = 0.1$  and  $(BS)^* = 500000$ .

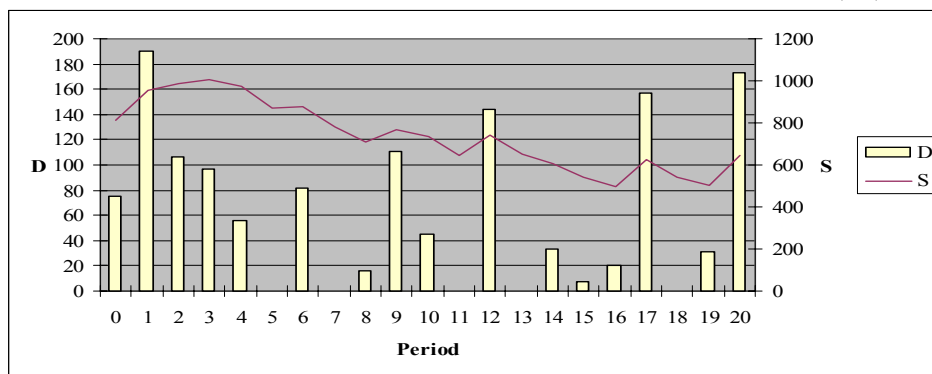


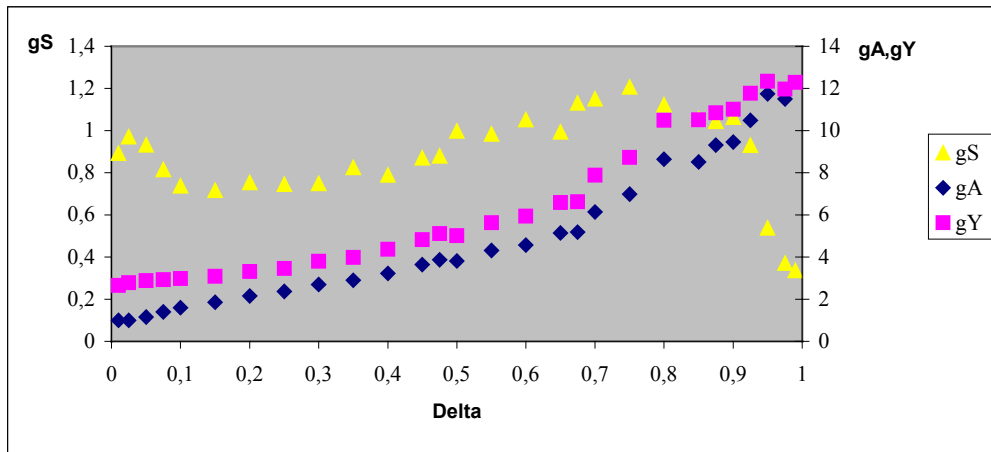
Figure 7 illustrates the effects of different  $\delta$  on the change in the stock of familiar resources, knowledge and cumulative income over 20 periods. Note that it is the change in the stock over the whole period that is examined. Hence, as long as the value is larger (smaller) than one, the stock has grown (declined). The initial stocks are the same in all cases. At a low  $\delta$  the change in  $S_t$  is below 1, i.e. the resource stock has decreased significantly because of the high probability of technological stagnation.<sup>37</sup> The outcome of an unsuccessful drastic innovation is probable throughout the period, and the resource stock is driven towards depletion in the long run by the knowledge stock effect and the incremental innovation effect during the few periods of incremental innovation, even though these effects decrease as  $S_t$  decreases. Note that since  $\delta$  is low, the depletion rate is also low, which means that the stock might not be completely

<sup>36</sup> Again notice the different scales of  $D_t$  and  $S_t$  compared to Figure 2 and Figure 5.

<sup>37</sup> At extremely low levels of  $\delta$  there are only drastic innovations, since  $(BS)^*$  is so much higher than the initial stock of  $B_{t-1}S_{t-1}$ .

exhausted after the 20 periods. Neither  $A_t$  nor  $Y_t$ , which is mainly determined by the total extraction, increases much because of restricted amounts of technological opportunities. Then there is the intermediate interval where the resource stock is unchanged or increased at the end of the period. An increased  $\delta$  means a sustainable (or even increasing)  $S_t$ , although with intensified cycles, and larger stocks of both  $A_t$  and  $Y_t$ . At very high levels of  $\delta$ ,  $S_t$  approaches zero, reflecting the high probability of resource exhaustion in the short run because of too intensive extraction.

**Figure 7:** Effects of the innovation ability on the growth of the familiar resource stock, the knowledge stock and the cumulative income.



For each value of  $\delta$  we run 20 simulations, and the points in the figure represent the average value from these.  $gX = X_T/X_0$  represents the change in the stock during the whole period, where  $X = A, Y, S$ , i.e. the stock of knowledge, income or familiar resources.  $X_0$  is the stock at Period 0, which is the same in all simulations, and  $X_T$  is the average of the last three periods.

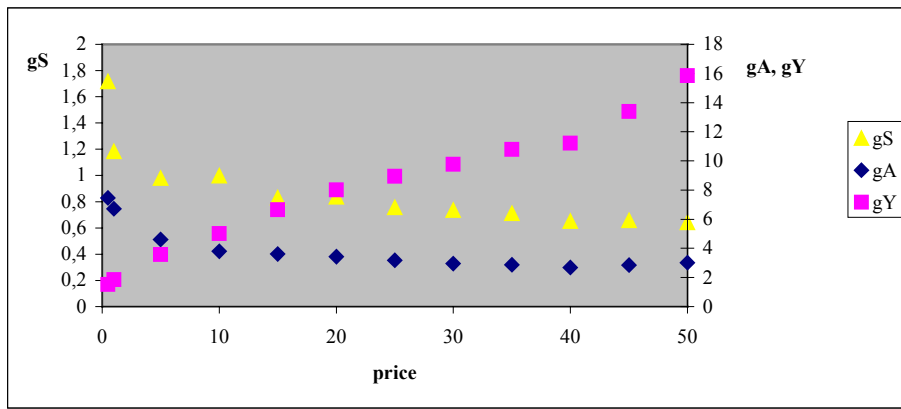
## 5.2 Effects of Changes in the Resource Price

In the basic analysis we treated the resource price as constant. However, the price may be higher because of a higher demand that may be a result of, for example, a large population or a high general technological level, which gives a high resource demand per capita. The price may also be lower because of a low demand caused by structural changes decreasing the importance of the resource sector, or because of the development of more resource efficient end-use technologies. In this section we will

first discuss how the price level that is still assumed to be constant throughout the 20 periods, affects the stocks and the total extraction. Then we will discuss how the resource cycles would be affected if we assumed that the price of a natural resource left in the ground increased as the resource becomes exhausted, i.e.  $\partial p_t / \partial S_t < 0$ .

Figure 8 shows the effect on the resource and knowledge stock and the cumulative income over 20 periods, depending on the price of the familiar resources.

**Figure 8:** Effects of price changes on the growth of the familiar resource stock, the knowledge stock and the cumulative income.



For each value of  $p_t$  we run 20 simulations, and the points in the figure represent the average value from these.  $gX = X_T / X_0$  represents the change in the stock during the whole period, where  $X = A, Y, S$ , i.e. the stock of knowledge, income or familiar resources.  $X_0$  is the stock at Period 0, which is the same in all simulations, and  $X_T$  is the average of the three last periods.

The familiar resource stock decreases with the price. The critical resource level at which it is worth switching to the insecure drastic innovations, is lower since even small extracted amounts may pay off with the high price. The extraction rate is not affected by a higher price, but the periods of incremental innovations are longer. Total extraction during the whole period may therefore decrease with a higher price level. This may help explain why the development of new resources or of new resource technologies is sometimes hard to induce by an increased price of the remaining resources. The continued extraction of these becomes more profitable. It is important to keep in mind that turning to new solutions in new paradigms is not in the option set of the innovators as long as the profits from innovations are not critically low.



Also when it comes to the knowledge stock and cumulative income, the price matters. A high price level decreases the search for a new paradigm, and the lack of an increase in technological opportunities dampens the increase in the knowledge stock. However, a high price increases the profits from extraction, even though the total extraction may decrease, and hence enforce the increase in the income stock.

But what happen if there are price changes between the 20 periods analyzed? According to Hotelling's rule the price of a resource increases as the resource decreases (Hotelling, 1931). This conclusion has been criticized not the least because of the induced resource efficiency technology in the rest of the society, and the entrepreneur's faith in incremental discovery technology, which dampens the increase in the price. However, accepting the Hotelling's rule, what would happen to the ROM? First of all, the critical level  $(BS)^*$  would no longer be constant throughout the periods analyzed. The declining extraction as  $S_t$  and  $B_t$  decline during incremental innovations would increase the price, and the critical level would therefore decline. This means that the number of incremental innovation periods between paradigm shifts would increase. Since the lower  $S_t$  is compensated by a higher  $p_t$ , there are incremental profits to be made even though the amount extracted is low. Moreover, the possibility of a drastic innovation being unsuccessful increases, since even though it could cause  $S_t$  and  $B_t$  to increase, the critical level would also increase due to the lower price.

Hence, even though there might be a price change after each period, we would still have the cyclic pattern of natural resources. Also, even though the probability of an unsuccessful drastic innovation would increase because of increasing critical levels during drastic innovation periods, the incremental innovation opportunity created by a successful innovation would increase, because of declining critical levels during incremental innovation periods.

## 6 CONCLUSIONS

Cycles in the resource stocks have in previous models usually been explained by exogenous and random arrivals of new sources or innovations, or by the choice between extraction and innovation. The model in this paper introduces the technological

opportunity thinking into natural resource modeling by the so-called Resource Opportunity Model, which provides a new explanation for the cyclic pattern of resource availability. The cycles are created by the natural resource sector's profit maximizing choice between the types of innovations: incremental or drastic. Incremental innovations are non-revolutionary, or complementary, innovations that make the drastic innovations diffuse into the production under decreasing returns. Drastic innovations are major breakthroughs that give new possibilities for incremental innovations.

Incremental innovations increase the efficiency of extraction and discovery of already familiar resources under the prevailing paradigm, which increase the rate of exhaustion. When the incremental innovation constraints, and hence profits from this kind of innovation, reach a critical level, drastic innovations become profitable. This shift to drastic innovations is induced either by scarcity of technological opportunities or scarcity of resources, and not only by resource scarcity as is often assumed in previous models.

A drastic innovation, a paradigm shift, increases the quantity of familiar resources, either by introducing an unexpected technology that improves the availability of already familiar resources, or by adding to the number of types of familiar resources. These two forces create a new familiar resource stock, offsetting the decreasing returns from incremental innovations, and enable continued extraction and economic growth. The expected success of this resource-creating innovation is not a constant as is often assumed in previous studies, but endogenously determined by the level of knowledge and innovation ability in the natural resource sector.

This way of modeling innovations in the natural resource sector results in a cyclic behavior of technological opportunities, resource abundance and economic growth, as long as the success of the drastic innovations is large enough compared to the levels of extraction. However, if there are too many unsuccessful paradigm shifts, the resource sector will collapse because of technological stagnation and drive the sector toward long-run resource exhaustion. Stagnation also becomes the case when the speed of extraction during an incremental innovation period is too high, leading to short-run resource exhaustion. Generally, however, an increased level of ability to turn technological opportunities into innovations does not affect the sustainability of the

resource stock, even though the fluctuations increase. The knowledge stock increases with the innovation ability and so does the cumulative income, since the total amount of extraction increases. However, an innovation ability level that is too low might drive the sector into technological stagnation, and resource exhaustion in the long run, and a level that is too high might drive the sector into extraction stagnation and resource exhaustion in the short run.

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## APPENDIX 1

To make the dynamics clearer we will look at how the level of  $\Pi_t^H$  is determined by the nature of innovation in the previous period,  $t-1$ . First we need to determine the level of  $B$  and  $S$  at  $t-1$ , depending on the nature of innovation at  $t-2$ . By substituting  $B_{t-1}$  and  $S_{t-1}$  into the expressions in Equation 7 we get the profits from incremental innovation at  $t$  as follows:

$$\Pi_t^H = \begin{cases} p\mu\delta[(1-\delta)B_{t-2}][S_{t-2} - \mu A_{t-2}S_{t-2} - \mu\delta B_{t-2}S_{t-2}] & \text{if increm. innov. at } t-1 \\ p\mu\delta[B_{t-2} + D_{t-1}][S_{t-2} - \mu A_{t-2}S_{t-2} + \lambda D_{t-1}] & \text{if drastic innov. at } t-1 \end{cases}$$

The profits from extraction at  $t$  are for sure lower than at  $t-1$ , if  $t-1$  was an incremental innovation period. A period of drastic innovations at  $t-1$  can give positive effects on the profits if the drastic innovation was successful enough, i.e. if  $D_t$  was large enough to outweigh the knowledge stock effect. In this last case, we see that a paradigm shift both increases the technological opportunities,  $B$ , and the physical quantity of resources,  $S$ .

## APPENDIX 2

Equation 10 and 6 gives:

$$g_t = \frac{\Pi_t - \Pi_{t-1}}{\Pi_{t-1}} = \frac{(\Pi_t^A + \phi_t \Pi_t^H + (1-\phi_t)\Pi_t^D) - (\Pi_{t-1}^A + \phi_{t-1}\Pi_{t-1}^H + (1-\phi_{t-1})\Pi_{t-1}^D)}{(\Pi_{t-1}^A + \phi_{t-1}\Pi_{t-1}^H + (1-\phi_{t-1})\Pi_{t-1}^D)}.$$

**Drastic innovation period followed by an incremental innovation period.**

Assume  $\phi_{t-1} = 0$ , and  $\phi_t = 1$  then

$$g_t^H(\phi_{t-1} = 0) = \frac{(\Pi_t^A + \Pi_t^H) - (\Pi_{t-1}^A + \Pi_{t-1}^D)}{(\Pi_{t-1}^A + \Pi_{t-1}^D)} = \frac{(p\mu A_{t-1}S_{t-1} + p\mu\delta B_{t-1}S_{t-1}) - (p\mu A_{t-2}S_{t-2} + \bar{\Pi})}{(p\mu A_{t-2}S_{t-2} + \bar{\Pi})}.$$

Following Equation 2 we know that  $A_{t-1} = A_{t-2}$  if  $\phi_{t-1} = 0$ . Hence,

$$g_t''(\phi_{t-1} = 0) = \frac{p\mu[A_{t-2}(S_{t-1} - S_{t-2}) + \delta B_{t-1}S_{t-1}] - \bar{\Pi}}{p\mu A_{t-2}S_{t-2} + \bar{\Pi}} \quad (\text{Equation 11})$$

$$g_t''(\phi_{t-1} = 0) \begin{cases} > 0 \\ < 0 \end{cases} \text{ if } p\mu[A_{t-2}(S_{t-1} - S_{t-2}) + \delta B_{t-1}S_{t-1}] \begin{cases} > \\ < \end{cases} \bar{\Pi}.$$

### **Drastic innovation period followed by a drastic innovation period.**

Assume again that  $\phi_{t-1} = 0$  but  $\phi_t = 0$ , then

$$g_t^{DI}(\phi_{t-1} = 0) = \frac{(\Pi_t^A + \Pi_t^D) - (\Pi_{t-1}^A + \Pi_{t-1}^D)}{(\Pi_{t-1}^A + \Pi_{t-1}^D)} = \frac{(\Pi_t^A + \bar{\Pi}) - (\Pi_{t-1}^A + \bar{\Pi})}{(\Pi_{t-1}^A + \bar{\Pi})} = \frac{p\mu A_{t-1}S_{t-1} - p\mu A_{t-2}S_{t-2}}{p\mu A_{t-2}S_{t-2} + \bar{\Pi}}.$$

Again we know that  $A_{t-1} = A_{t-2}$ . Hence,

$$g_t^{DI}(\phi_{t-1} = 0) = \frac{p\mu A_{t-2}(S_{t-1} - S_{t-2})}{p\mu A_{t-2}S_{t-2} + \bar{\Pi}} \quad (\text{Equation 12})$$

$$g_t^{DI}(\phi_{t-1} = 0) \begin{cases} > 0 \\ < 0 \end{cases} \text{ if } S_{t-1} \begin{cases} > \\ < \end{cases} S_{t-2}.$$

### **Incremental innovation period followed by an incremental innovation period.**

Assume now that  $\phi_{t-1} = 1$  and  $\phi_t = 1$ , then

$$\begin{aligned} g_t''(\phi_{t-1} = 1) &= \frac{(\Pi_t^A + \Pi_t^I) - (\Pi_{t-1}^A + \Pi_{t-1}^I)}{(\Pi_{t-1}^A + \Pi_{t-1}^I)} = \frac{p\mu(A_{t-1} + \delta B_{t-1})S_{t-1} - p\mu(A_{t-2} + \delta B_{t-2})S_{t-2}}{p\mu(A_{t-2} + \delta B_{t-2})S_{t-2}} \\ &= \frac{(A_{t-1} + \delta B_{t-1})S_{t-1}}{(A_{t-2} + \delta B_{t-2})S_{t-2}} - 1. \end{aligned}$$

Following Equation (2) we know that  $A_{t-1} = A_{t-2} + \delta B_{t-2}$  if  $\phi_{t-1} = 1$ , and

$A_t = A_{t-1} + \delta B_{t-1}$  if  $\phi_t = 1$ . Hence,

$$g_t''(\phi_{t-1} = 1) = \frac{A_t S_{t-1}}{A_{t-1} S_{t-2}} - 1. \quad (\text{Equation 13})$$

Moreover, Equation (3) gives us  $(S_{t-1}/S_{t-2}) = 1 - \mu A_{t-1}$  and Equation (2)  $(A_t/A_{t-1}) = 1 + \delta B_{t-1}/A_{t-1}$ , if  $\phi_t = 1$ . We therefore have

$$g_t^H(\phi_{t-1} = 1) = (1 + \delta B_{t-1}/A_{t-1})(1 - \mu A_t) - 1.$$

Hence,

$$g_t^H(\phi_{t-1} = 1) \begin{cases} > 0 \\ < 0 \end{cases} \text{ if } (1 + \delta B_{t-1}/A_{t-1})(1 - \mu A_t) \begin{cases} > \\ < \end{cases} 1.$$

### **Incremental innovation period followed by a drastic innovation period.**

Finally, assume again that  $\phi_{t-1} = 1$  but  $\phi_t = 0$ , then

$$\begin{aligned} g_t^{DI}(\phi_{t-1} = 1) &= \frac{(\Pi_t^A + \Pi_t^D) - (\Pi_{t-1}^A + \Pi_{t-1}^H)}{(\Pi_{t-1}^A + \Pi_{t-1}^H)} = \frac{(p\mu A_{t-1} S_{t-1} + \bar{\Pi}) - p\mu(A_{t-2} + \delta B_{t-2}) S_{t-2}}{p\mu(A_{t-2} + \delta B_{t-2}) S_{t-2}} = \\ &= \frac{A_{t-1} S_{t-1}}{(A_{t-2} + \delta B_{t-2}) S_{t-2}} + \frac{\bar{\Pi}}{p\mu(A_{t-2} + \delta B_{t-2}) S_{t-2}} - 1. \end{aligned}$$

Again we know that  $A_{t-1} = A_{t-2} + \delta B_{t-2}$  if  $\phi_{t-1} = 1$ , which gives,

$$g_t^{DI}(\phi_{t-1} = 1) = \frac{S_{t-1}}{S_{t-2}} + \frac{\bar{\Pi}}{p\mu A_{t-1} S_{t-2}} - 1. \quad (\text{Equation 14})$$

Finally, since  $(S_{t-1}/S_{t-2}) = 1 - \mu A_{t-1}$  if  $\phi_t = 1$ , we have

$$g_t^{DI}(\phi_{t-1} = 1) = \frac{\bar{\Pi}}{p\mu A_{t-1} S_{t-2}} - \mu A_{t-1}.$$

Hence,

$$g_t^{DI}(\phi_{t-1} = 1) \begin{cases} > 0 \\ < 0 \end{cases} \text{ if } \frac{\bar{\Pi}}{p\mu A_{t-1} S_{t-2}} \begin{cases} > \\ < \end{cases} \mu A_{t-1}.$$