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Okounkov bodies and geodesic rays in Kähler geometry

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ABSTRACT

This thesis presents three papers dealing with questions in Kähler geometry.

In the first paper we construct a transform, called the Chebyshev transform, which maps continuous hermitian metrics on a big line bundle to convex functions on the associated Okounkov body. We show that this generalizes the classical Legendre transform in convex and toric geometry, and also Chebyshev constants in pluripotential theory. Our main result is that the integral of the difference of two transforms over the Okounkov body is equal to the Monge-Ampère energy of the two metrics. The Monge-Ampère energy, sometimes also called the Aubin-Mabuchi energy or the Aubin-Yau functional, is a well-known functional in Kähler geometry; it is the primitive function to the Monge-Ampère operator. As a special case we get that the weighted transfinite diameter is equal to the mean over the unit simplex of the weighted directional Chebyshev constants. As an application we prove the differentiability of the Monge-Ampère on the ample cone, extending previous work by Berman-Boucksom.

In the second paper we associate to a test configuration for a polarized variety a filtration of the section ring of the line bundle. Using the recent work of Boucksom-Chen we get a concave function on the Okounkov body whose law with respect to Lebesgue measure determines the asymptotic distribution of the weights of the test configuration. We show that this is a generalization of a well-known result in toric geometry.

In the third paper, starting with the data of a curve of singularity types, we use the Legendre transform to construct weak geodesic rays in the space of positive singular metrics on an ample line bundle L . Using this we associate weak geodesics to suitable filtrations of the algebra of sections of L . In particular this works for the natural filtration coming from an algebraic test configuration, and we show how this in the non-trivial case recovers the weak geodesic ray of Phong-Sturm.

Keywords: ample line bundles, Okounkov bodies, Monge-Ampère operator, Legendre transform, Chebyshev constants, test configurations, weak geodesic rays.