# Should We Use Distributional Weights in CBA When Income Taxes Can Deal with Equity?

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**Abstract** - Kaplow (1996) and others argue forcefully in favor of using the standard costbenefit test alone, without any distributional concern, given "standard simplifying assumptions." This paper, on the contrary, demonstrates that distributional weights, equal to the social marginal utility of income, should be applied in cost-benefit analysis, given weak separability in public goods instead of in leisure. This result holds for linear as well as non-linear income taxes, and whether they are optimal or not. A correspondingly modified Samuelson rule is derived and more general policy recommendations discussed.

*Key words*: public goods; distributional weights; equity and efficiency; separability; costbenefit; optimal taxation

JEL-classification: D61; H21; H41

## I. INTRODUCTION

Cost-benefit analysis (CBA) often tends to focus solely on efficiency, by simply comparing aggregate willingness to pay (WTP) figures with costs. Still, from the public policy discussions it is clear that distributional matters are often very important. Distributional weights, when applied, are typically based on the social marginal utility of income; see e.g. Drèze (2000) for recent arguments in favor of weighted CBA. Consider the frequently used assumption of a utilitarian social welfare function (SWF), and utility functions characterized by constant relative risk aversion equal to 2 (in income).<sup>1</sup> The benefit to a person who is 100 times richer than another one should then be given a weight of only  $1/10,000 (1/100^2)$  relative to the poor. Obviously, the outcome of the CBA can then be very different with and without weights for projects that particularly benefit either rich or poor people.

However, given that equity aspects are intrinsically important, it is still not obvious that one should apply distributional weights in CBA. One may instead, following Harberger (1978, 1980), argue that it is more efficient to focus solely on efficiency in CBA and leave distributional considerations to income taxation. Much of the policy discussion, for or against the view of Harberger, has been based on a rather general and intuitive level, and the main issue seems to have been how distortionary the income tax is, that is, how costly it would be to reach distributional goals by income taxes instead; see e.g. Layard (1980), Squire (1980), and Brent (1984, 1996).

The more thorough theoretical analysis, undertaken in a general equilibrium framework, has to a large extent focused on the case when we may use optimal non-linear income taxes. The key-results here are due to Hylland and Zeckhauser (1979), published in

<sup>&</sup>lt;sup>1</sup>According to Dasgupta (1998, p. 145, footnote 11), empirical evidence on choice under uncertainty suggest a value of around 2, or slightly larger. Blanchard and Fischer (1989, p. 44) report that results based on intertemporal choices vary greatly, but are often around or larger than unity.

this journal, soon followed by Christiansen (1981). Both derived conditions for when the basic Samuelson (1954) rule (3 MRS = MRT) can be applied without adjustments for distributional concern and incentive effects. Christiansen (1981) showed the striking result that an optimal non-linear income tax together with weak separability in the utility function (identical for all individuals) between private and public goods on the one hand, and leisure on the other, are sufficient conditions.<sup>2</sup> However, it is somewhat surprising to see how little influence these seminal papers have had on the applied CBA literature, given the obvious policy content. One can only speculate about the reasons for this; possibly, CBA practitioners find the somewhat technical presentations in these papers difficult to follow, or they may simply be skeptical to the applicability of the results outside the assumptions made.

More recently, Kaplow (1996) in an influential paper argued again for the use of the standard cost-benefit test alone, ignoring distributional and excess-burden effects, based on what he denotes "standard simplifying assumptions", which are largely the same as the ones used by Christiansen. Still, from a policy perspective the most relevant issue may not be to find clear results under special assumptions, which one typically knows are not strictly fulfilled anyway, but to get some information about roughly *how* valid the results are in the real world. For example, if the separability assumptions assumed by Christiansen (and Kaplow) do not hold, can we then say anything generally about distributional concern in CBA? Or, in the example above, can we say that, given that we can use income taxation to deal with equity, the theoretically appropriate weights given to rich and poor people,

<sup>&</sup>lt;sup>2</sup>Boadway and Keen (1993) showed that it is sufficient that the utility function for an individual *i* may be written  $u^{i} u^{i}(f(x^{i}, G), l^{i})$ , where *x* and *G* are private and public consumption, respectively, and *l* is leisure. Hence, it is sufficient that all individuals have an identical sub-utility function *f*; the overall utility function need not be identical for the Samuelson rule to be valid.

respectively, would most likely not differ by more than say a factor 5, unless we make very unreasonable or counterintuitive assumptions?

Although no authors have made any explicit claims about the applicability of the results outside the assumptions made, the paper by Kaplow do include recommendations for real policy, and not just special theoretical results. As expressed by Ng (2000a): "...the results of Christiansen and his followers are presented as a special case (of weak separability), while Kaplow argues for the simple benefit/cost ratio as the benchmark, presumably regarding the deviation caused by non-separability (which may go either way) as a secondary complication that should be disregarded in the basic analysis and probably in most real-world applications (where the sign of the required deviation may not be known)." The analysis by Ng (2000a, b), which largely supports Kaplow, is made in the same spirit (as, one may add, is the discussion in this paper), although he insightfully adds some important non-standard qualifications for the Kaplow conclusions. In particular he notes that the existence of relative income effects (Ng, 1987a),<sup>3</sup> or diamond goods (Ng, 1987b), would tend to favor higher public spending, and that there may be other distortionary costs associated with taxation besides those related to labor supply, which would go in the other direction. There is almost no discussion, however, in either Kaplow, or Ng (2000a, b) concerning if, and if so when and how, it would be appropriate to explicitly make distributional concern in public good provision outside the case analyzed. There is some discussion regarding the case when the tax system, for whatever reason, cannot be adjusted in an appropriate way. About this, however, Ng (2000a, 257) colorfully states that

<sup>&</sup>lt;sup>3</sup>See for example Easterlin (1995) and Oswald (1997) for recent surveys concluding that relative income seems to be much more important for individual well-being than absolute income. See Solnick and Hemenway (1998) and Johansson-Stenman et al. (2001) for recent economic experiments suggesting that both absolute and relative income are important.

"while we may try to do good by stealth in the short run, and proceed to use distributional weights, in the long run this will be known and cause disincentive effects." Similarly, Frank (2000, p. 917) recently stated (based on no formal analysis): "We can employ unweighted willingness-to-pay measures without apology, and use the welfare and tax system to compensate low-income families ex ante for the resulting injury. [...] Rich and poor have an interest in making the economic pie as large as possible. Any policy that passes the costbenefit test makes the economic pie larger. And when the pie is larger, anyone can have a larger slice." The same view is repeated in the innovative and potentially best-selling textbook by Frank and Bernanke (2001). But can we really say that the economic pie would typically be smaller if we use distributional weights in CBA?

To analyze distributional concern in CBA outside the special assumption used by Christiansen, Kaplow and others is the main purpose of this paper. One obvious way is to consider other frequently used simplifying assumptions, and to see whether the policy conclusions appears to be fairly similar. Here we will focus on the case where utility is (weakly) separable in the public goods, instead of in leisure. Although empirical evidence on this point seems almost non-existent it is a very often used assumption in the literature, and for example Starret (1988, p. 173) argues that "a general project has no obvious net complementarity."

Diamond (1975) and Atkinson and Stiglitz (1980, pp. 496-7) have derived expressions for the optimal provision of public goods under optimal *linear* taxation, given this type of separability. It is clear from their work that distributional weights should then be applied. One may then wonder whether this result would hold also in the case of non-linear or piecewise linear income taxes? And what about the more realistic case of non-optimal taxes? As demonstrated in Section II, it turns out that in all these cases 'full' distributional weights should be used, so that the relative weights in the initial example would still vary by a factor 10,000!

Another main point in Kaplow's paper is that the practice of adjusting (downwards) the appropriate amount of public goods due to excess burden of taxation can be questioned. Although not the main issue in this paper, in Section 4 we will briefly discuss the optimal amount of public goods provided under optimal (linear or non-linear) taxation relative to the basic Samuelson rule. As we will see, the results imply that for the kind of separable public goods assumed here, the optimal provision may be either below or above the Samuelson rule, depending on the distributional properties of the public good and on the income (*not* the substitution) effects of the labor supply elasticities. Hence, a possible deviation from the Samuelson rule is not based on the conventional efficiency-based marginal cost of funds argument, which supports Kaplow's general criticism in this respect.

We will also discuss some special cases in Section IV, including the one where both types of separability holds simultaneously. In this case it seems that we should both apply weights and not apply weights! However, as will be shown this apparent contradiction is an illusion, since in this case the marginal WTP for all public goods will vary with income in an identical manner. Section V summarizes and discusses conclusions for policy more generally.

## **II DISTRIBUTIONAL WEIGHTS**

#### Separability in public goods

Assume that the government's objective is to maximize a general Bergson-Samuelson SWF  $w(u^1, u^2, ..., u^n)$  satisfying the Pareto criterion, where  $u^i$  is utility for individual *i*, subject to a public budget constraint. Consider now the problem of supplying two different public goods, with different distributional characteristics; one may be preferred mainly by poor people and the other by the rich. Utility is separable in the public goods, as follows:

$$U^{i} = u^{i}(f^{i}(x^{i}, l^{i}), G_{1}, G_{2})$$
(1)

where x is after-tax private income, l is leisure, and  $G_1$  and  $G_2$  are public goods. In other words, the marginal rate of substitution, and hence the choice, between leisure and private consumption is unaffected by provision of  $G_1$  and  $G_2$ . This implies that for any size of the public budget (i.e. optimal or not), and for any taxation system (optimal or not), we should choose the combination of  $G_1$  and  $G_2$  which contributes most to social welfare, and there are no indirect incentive effects through labor supply effects to correct for. Assume for simplicity that the production price of both public goods are normalized to 1. The optimality condition, found from maximizing welfare with respect to the public goods, is then simply given by

$$\sum_{n} \boldsymbol{a}^{i} MRS_{G_{1}x}^{i} = \sum_{n} \boldsymbol{a}^{i} MRS_{G_{2}x}^{i}$$
(2)

where  $MRS_{G_jx}^i = \frac{\Re^i}{\Re^j_j} / \frac{\Re^i}{\Re^i}$  is individual *i*'s marginal rate of substitution (or marginal

willingness to pay in terms of x) between the public good j and x, and  $\mathbf{a}^{i} = \frac{\mathbf{f}_{w}}{\mathbf{f}_{u}^{i}} \frac{\mathbf{f}_{u}^{i}}{\mathbf{f}_{x}^{i}}$  is the

corresponding social marginal utility of income.<sup>4</sup> Hence, rather then comparing the sum of individual marginal willingness to pay for the public goods, we should compare the *weighted* sums, where the weights are given by the social marginal utility of income. Since this holds generally (given the separability assumed), it holds also for optimal non-linear income taxes. Alternatively, we can rewrite (2) to separate out the distributional effects:

<sup>&</sup>lt;sup>4</sup>Following the terminology by Diamond and Mirrlees (1971), and hence not the one by Diamond (1975). For a utilitarian SWF we have of course that  $\mathbf{a}^{i} = \frac{\mathbf{f} \boldsymbol{\mu}^{i}}{\mathbf{f} \boldsymbol{k}^{i}}$ , i.e. the individual marginal utility of income.

$$\frac{\sum_{n} MRS_{G_{1}x}^{i}}{\sum_{n} MRS_{G_{2}x}^{i}} = \frac{1 + \operatorname{cov}\left(\frac{a}{a}, \frac{MRS_{G_{2}x}}{MRS_{G_{2}x}}\right)}{1 + \operatorname{cov}\left(\frac{a}{a}, \frac{MRS_{G_{1}x}}{MRS_{G_{1}x}}\right)} = \frac{1 + d}{1 + d}$$
(3)

where a bar denotes mean value and where d is the distributional characteristics of the public good *i*. Hence, the ratio between the sums of *MRS* should not equal the *MRT* ratio (which is normalized to one here), but rather the *MRT*-ratio times an expression which compares the distributional characteristics, or the normalized covariances between the marginal willingness to pay for the public goods and the social marginal utility of income. Goods which are relatively more preferred by low-income people should then clearly be over-provided compared to other goods, and vice versa.

Consider the following stylized, but fairly realistic, example: There are 2 small public projects to be compared: improving the local road infrastructure, and improving the public transport system. The latter is preferred largely by low-income people, since many rich people will continue not to use public transport irrespective of improvements. One has undertaken surveys to elicit people's marginal willingness to pay (WTP) for small improvements (with equal cost) in these 2 areas, using the contingent valuation (CV) method, and the total WTP is found to be 50% higher for the improved road infrastructure. As is standard in CV analysis, one also asked for people's income (net of taxes), and income elasticities of the WTP for these projects were estimated to 1 for road infrastructure, and 0 for public transport. After-tax income in the economy is Gamma-distributed and given by:  $f(x) = xe^{-x}$ , where  $x \ge 0$ . Assume also that changes in tax revenues (if any), due to adjustments in labor supply, are expected to be the same for these projects. The social welfare function is utilitarian<sup>5</sup> and the social inequality aversion, measured as individual

<sup>&</sup>lt;sup>5</sup>See Kaplow (1995) and Ng (1999, 2000b) for arguments in favor of utilitarian SWFs.

relative risk aversion in income, is (conservatively) assumed to 1. Which project should be preferred? Given constant income elasticities, we can write the distributional characteristics for any of the goods as:

$$\mathbf{d} = \frac{\int_{0}^{\infty} MRS_{G_{i}x} \mathbf{a} f(x) \, \mathrm{d}x}{\int_{0}^{\infty} MRS_{G_{i}x} f(x) \, \mathrm{d}x - \int_{0}^{\infty} \mathbf{a} f(x) \, \mathrm{d}x} - 1 = \frac{\int_{0}^{\infty} x^{1+\mathbf{e}-s} e^{-x} \, \mathrm{d}x}{\int_{0}^{\infty} x^{1-s} e^{-x} \, \mathrm{d}x} - 1$$
(4)

where e is the income elasticity of WTP, and *s* is the social inequality aversion. Imposing the values for e and *s* into (4) we get the distributional characteristics are equal to -0.5 for road infrastructure, and 0 for public transport. Plugging these values into (3), where good 1 is the road improvement, we see that the right-hand-side ratio is equal to 2, which is larger than the total marginal WTP ratio of 1.5. Thus, road improvements are in this case less socially profitable than public transport improvements, despite the fact that they would be more profitable if we only considered efficiency aspects.

#### Separability in leisure

Consider now instead the utility function mainly discussed by Kaplow (1996) and Christiansen (1981), where utility is separable in leisure as follows:

$$U = u(f(x, G^{1}, G^{2}), l)$$
(5)

In this case, as thoroughly demonstrated by both Christiansen and Kaplow, if we adjust the income taxes on the margin so that each individual will pay exactly his marginal WTP for the improvement, then each individual will choose exactly the same amount of leisure (and hence labor) as before this reform. If these payments are larger than the costs of the project the government makes a surplus, which can be distributed back to obtain a Pareto improvement. Similarly, if these payments are not sufficient to cover the costs of the project, a Pareto improvement can be obtained by having the project undone. In an optimal taxation

framework, any combination of marginal tax parameter changes to raise an additional dollar is equally good (or bad) in terms of social welfare (by the envelope theorem), since otherwise the first-order optimality conditions are not fulfilled, and taxes should be redistributed in favor of the better means. Hence, for optimal non-linear income taxes, it is clear that the basic Samuelson rule for provision of public goods still holds in this case.

Now, intuitively, why do the policy conclusions come out so *very* differently for these 2 commonly used types of separability assumptions? In the case of separability in the public goods (eq. 1), an increase of these goods *per se* does not affect labor choices. Hence, there is no 'distortionary cost' of these public goods *per se*. In the case of separability in leisure (eq. 5), on the other hand, a *combined* public good and tax increase (equal to the marginal WTP) leaves the labor choice unaffected. Since tax changes typically affect this choice, so does the public good provision. And the larger marginal WTP, the larger the distorting effect of the public good provision. Hence, possibly large distributional benefits are off-set by large distorting costs, and vice versa.

#### III. THE MODIFIED SAMUELSON RULE

This section will discuss whether pulic goods should be over- or under-provided relative to the Samuelson rule, under (some kind of) optimal income taxation.<sup>6</sup> Let us now apply the same type of reasoning as in the last sub-section, but this type on the case with separability in public goods (eq. 1). Again, given optimal non-linear taxes any combination of marginal tax-parameter changes to finance the public good is equally good in terms of social welfare. But in this case, a payment (through a tax shift) equal to each individual's marginal WTP

<sup>&</sup>lt;sup>6</sup>See Sandmo (1998) for public good provision under optimal and non-optimal linear income taxation. We will not discuss the 'level issue' here, i.e. compare the optimal amount provided in a second-best economy relative to a first-best economy; for this see Wilson (1991) and Mirrlees (1994).

will in general not leave labor unaffected, since there is no off-setting incentive effect from the public good provision here. Whether the overall labor supply effect of the tax increase will increase or decrease labor is then due to whether the income or the substitution effect dominates. This, in turn, is of course largely given by the type of tax increase undertaken. Assume, for example, that the marginal WTP is independent of income, i.e. what Wilson (1991) denotes 'distributionally neutral' public goods, then the payment would imply the same tax increase for all, independent of income. But this means that the tax shift is basically a uniform lump-sum tax, which has no substitution effect. Since the labor supply income effect of a tax increase is typically found to be positive (although theoretically undetermined),<sup>7</sup> we would expect labor to increase in this case, which, in this second-best setting, increases welfare. Hence, it would be positive to increase the provision beyond the Samuelson rule here. This is clearly a somewhat extreme case, an in general we would expect marginal WTP to increase with income. The more it increases with income, the more would the off-setting tax shift increase with income, implying that the marginal tax increases, and the more important becomes the substitution effect, which is theoretically known to be negative. Thus, for a sufficiently 'regressive' public good, we would expect an underprovision relative to the Samuelson rule to be optimal. This gives an alternative, more efficiency-oriented, picture of why it is optimal in this case to take distributional concern also in public good provision.

An optimal *linear* income tax (Sheshinski, 1972) consists of two parts: An optimal uniform lump-sum tax (which is typically negative)<sup>8</sup> and a proportional income tax. An optimal non-linear tax (Mirrlees, 1971), on the contrary, can be seen as consisting of an

<sup>&</sup>lt;sup>7</sup>A sufficient (but not necessary) condition for this is that leisure is a normal (or at least noninferior) good for all individuals.

<sup>&</sup>lt;sup>8</sup>See for example Stern (1976) for simulation results.

infinite number of parameters  $t_0$  to  $t_n$ , where  $t_0$  is the uniform lump-sum tax,  $t_1$  is the marginal tax rate for the first infinitesimal income unit, and so forth. We can also think of intermediate piecewise linear cases (see e.g. Dahlby, 1998) where we can use a finite number of parameters, for example a certain marginal tax rate up to a specific income level, and a higher marginal tax rate beyond that level. Such income-tax systems are (with some modifications) currently used in many countries. We can write the Lagrangean, including all these cases, as:

$$w(\mathbf{n}^{1}(\mathbf{t}_{0},...,\mathbf{t}_{m},G),...,\mathbf{n}^{n}(\mathbf{t}_{0},...,\mathbf{t}_{m},G)) - I(R_{0} - R(\mathbf{t}_{0},...,\mathbf{t}_{m}) + G)$$
(6)

where *m* is 1 for the linear income-tax case, and infinite for the optimal non-linear incometax case; ? is the indirect utility function;  $R_0$  is an exogenous public budget balance requirement; and the function  $R(t_0,...,t_m)$  gives the tax revenues collected.<sup>9</sup> The necessary optimality conditions obviously include the first-order conditions for choosing the public good *G* 

$$\sum_{i} \frac{\mathcal{I}_{M}}{\mathcal{I}_{D}^{i}} \frac{\mathcal{I}_{D}^{i}}{\mathcal{I}_{G}} - \mathbf{I} = \sum_{i} \mathbf{a}^{i} MRS_{Gx}^{i} - \mathbf{I} = 0$$
<sup>(7)</sup>

and the uniform lump-sum tax  $t_0$ 

$$\sum_{i} \frac{f_{i\nu}}{f_{i}n^{i}} \frac{f_{i}n^{i}}{f_{i}t_{0}} + I \frac{f_{i}R}{f_{i}t_{0}} = -\sum_{i} a^{i} + In(1+b) = 0$$
(8)

where **b** can be seen as a labor-supply tax-revenue effect of a uniform lump-sum tax. If we increase the lump-sum tax (or decrease the lump-sum transfer) by 1 USD the additional tax revenues with no adjustments would be *n* USD, and with adjustments n(1 + b) USD.

<sup>&</sup>lt;sup>9</sup>Note that, due to the separability assumption, R is not a function of the public goods. For the purpose here, we need not specifying R further.

Combining (7) and (8) gives

$$\sum_{n} \frac{\mathbf{a}}{\mathbf{a}} MRS_{Gx}^{i} = \left(1 + \cos\left(\frac{\mathbf{a}}{\mathbf{a}}, \frac{MRS_{Gx}}{MRS_{Gx}}\right)\right) \sum_{n} MRS_{Gx}^{i} = \frac{1}{1 + \mathbf{b}}$$

or

$$\sum_{n} MRS_{Gx}^{i} = \frac{1}{\left(1 + \mathbf{b}\right) \left(1 + \operatorname{cov}\left(\frac{\mathbf{a}}{\mathbf{a}}, \frac{MRS_{Gx}}{MRS_{Gx}}\right)\right)} = \frac{1}{(1 + \mathbf{b})(1 + \mathbf{c})}$$
(9)

Thus, whether the public good should be over- or under-provided relative to the Samuelson rule depends on the relative size of  $\boldsymbol{b}$  (typically positive) and the distributional characteristic  $\boldsymbol{d}$  (typically negative). Hence, again, we see that public goods which are sufficiently regressive should be under-provided relative to the Samuelson rule, and vice versa.

#### IV. SPECIAL CASES

#### Additive separability in public goods

Boadway and Keen (1993) derived, using the self-selection approach, a result for the case where the (common for all) utility function is additively separable (and hence also weakly separable) as follows:

$$U(x,G_1,G_2,l) = A(x,l) + B(G_1,G_2)$$
(10)

They showed that, given optimal non-linear income taxes, a public good should be over-

provided relative to the Samuelson rule if  $\frac{\frac{\pi}{2}A}{\frac{\pi}{2}\pi} > 0$ , i.e. if private consumption and leisure

are Edgeworth complements, and vice versa. This condition is clearly independent of the characteristics, including distributional characteristics, of the public goods. Thus, there is no explicit equity consideration here, despite the weakly separable public goods, which seems to contradict the results above (eq. 3 and 9). However, as will be shown, this is an

illusion. The marginal WTP for a public good is given by  $MRS_{G_jx} = \frac{\mathcal{P}}{\mathcal{P}_j} / \frac{\mathcal{P}}{\mathcal{R}}$ , with the

corresponding income elasticity 
$$\frac{\Re RS_{G_{jx}}}{\Re} \frac{x}{MRS_{G_{jx}}} = -x \frac{\Re^2 A}{\Re^2} / \frac{\Re}{\Re}$$
, which is clearly independent

of the public goods. Hence, the marginal WTP varies with income in exactly the same way for all public goods. The distributional characteristic is given by<sup>10</sup>

$$\boldsymbol{d} = \frac{n}{\overline{\boldsymbol{a}}} \frac{\sum_{i} \boldsymbol{a}^{i} \frac{\boldsymbol{f} \boldsymbol{B}}{\boldsymbol{f} \boldsymbol{G}_{j}} / \frac{\boldsymbol{f} \boldsymbol{A}}{\boldsymbol{f} \boldsymbol{k}^{i}}}{\sum_{i} \frac{\boldsymbol{f} \boldsymbol{B}}{\boldsymbol{f} \boldsymbol{G}_{j}} / \frac{\boldsymbol{f} \boldsymbol{A}}{\boldsymbol{f} \boldsymbol{k}^{i}}} - 1 = \frac{n}{\overline{\boldsymbol{a}}} \frac{\sum_{i} \boldsymbol{a}^{i} / \frac{\boldsymbol{f} \boldsymbol{A}}{\boldsymbol{f} \boldsymbol{k}^{i}}}{\sum_{i} \frac{\boldsymbol{f} \boldsymbol{B}}{\boldsymbol{f} \boldsymbol{G}_{j}} / \frac{\boldsymbol{f} \boldsymbol{A}}{\boldsymbol{f} \boldsymbol{k}^{i}}} - 1$$
(11)

which is clearly independent of the public good, implying that the distributional characteristics will be the same for all public goods, and the r.h.s. of eq. (3) will simply be equal to the cost ratio. Thus, given preferences as in (10), comparing 2 (small) public projects in terms of the benefit-cost ratio will give the same result with and without distributional weights.

# Separability in both leisure and public goods

Consider now the special case discussed by Kaplow where people's identical utility function is given by

$$U(x,G_1,G_2,l) = f(x) + g(G_1,G_2) + h(l)$$
(12)

This utility function is clearly separable in both public goods and in leisure. As we have seen,

<sup>&</sup>lt;sup>10</sup>Assuming a utilitarian SWF we get  $d = \frac{1}{\overline{m} 1/m} - 1$ , where **m** is individual marginal utility of

income. Hence, the larger (after-tax) inequality, the larger is the distributional characteristic (in absolute value), reflecting the fact that the public good will then have a more equalizing effect.

according to the former we should apply distributional weights, but according to the latter we should not, which seems again to indicate a contradiction. However, since (12) is a special case of (10), the same arguments are applicable here. Hence, eq. (3), (8) and the Samuelson rule will all hold simultaneously. In appendix it is shown that a sufficient condition for this result is that utility is *weakly* separable in both leisure and the public goods, simultaneously.

## V. DISCUSSION AND CONCLUSIONS

We have seen that distributional weights, equal to the social marginal utility of income, should actually be applied in cost-benefit analysis, also when optimal non-linear income taxes are used, given that an exogenous public good change does not affect tax revenues. A sufficient condition for this, in turn, is that people's utility function (which need not be identical) are weakly separable in the public goods. On the other hand, when utility (the same utility function for all) is weakly separable in leisure, no weights should be used, given that one can make appropriate adjustments of the income taxation. The large difference in policy implications between these two frequently used assumptions may seem surprising. It should be more concerned with equity in the former case compared to the latter, since the optimal income-tax design will generally depend on whether or not distributional weights are used in CBA.

Now, which assumption should be seen as the most natural benchmark case? Sandmo (1998) argues that the assumptions used by Kaplow are very strong, but the same can of course be said about the assumptions used here. Is it reasonable that a public good increase *per se* will keep the individual amount of labor supplied constant (separability in public goods)? Or is it more reasonable that a combined public good and income-tax change, in

order to keep utility constant, will hold labor fixed (separability in leisure)? Neither of these two cases seem to give a representative picture of most public goods, and it seems likely that different assumptions will be more or less suitable for different goods, and which case should serve as the most natural 'bench-mark' appears to be an open question.

Harberger provides several examples of the negative consequences of applying distributional weights in cost-benefit analysis. One of the more amusing deals with the possibility of sending ice-cream on camel-back across the desert, from a richer oasis to a poorer one (Harberger 1978, repeated in Harberger 1984). In an extreme case, when the social inequality-aversion used is large, he asserts (Harberger 1978, p. S113) that "up to 63/64 of the ice-cream could melt away without causing the project to fail the test." He concludes (Harberger 1984, p. 458): "Even ways which by the traditional standards would be scandalously inefficient would have to be explored." But the point is that, given a SWF sufficiently concave in income to be maximized, decreasing inequality by changes in income taxation will have large 'inefficiency losses' as well.<sup>11</sup> Consider again the initial example with a utilitarian SWF, and where the income of a rich person is 100 times that of a poor. Then, everything else equal, a redistribution of one dollar from the rich to the poor, through changed income taxes, would increase social welfare if the poor would receive at least the fraction  $1/100^{e}$ , where e is the inequality aversion parameter, corresponding to the (constant) relative risk aversion in income. Thus, this redistribution would be social welfare improving if the marginal excess burden of this tax change would not be larger then 99%

<sup>&</sup>lt;sup>11</sup>In addition, ice-cream is not a public good and it is not clear why the government should redistribute such a strictly private good in this way. (Although there may be good reasons for public provision of *some* private goods; see e.g. Blomquist and Christiansen, 1995). Second, the distributional weights normally proposed are related to the *benefit*, i.e., a monetary WTP measure, and not to the good *per se*; It is not obvious that the poor would have the same WTP for ice-cream as the rich.

for e = 1, and 99.99% for e = 2! This illustrates clearly the problems of basing policy conclusions on measures of efficiency losses alone. As recently expressed by Sandmo (p. 366): "Presumably the main reason why we have distortionary taxes is precisely the distributional problem; if issues of equity and justice could be disregarded altogether, the design of an optimal tax system would be a much less challenging task,"<sup>12</sup> and by Dahlby (1998, p. 113): "The problem in raising additional tax revenue is not that the marginal cost of raising revenue is high. The problem is that the marginal cost of raising revenue in an equitable fashion may be high." Making non-negligible decreases in inequality between rich and poor people will, at the margin, have large 'inefficiency losses', irrespective of whether the redistribution takes place through tax changes or through public good provision. Sometimes, these losses are shown to be lower when only making tax changes, and sometimes they are lower when also taking distributional concern in CBA.<sup>13</sup> Therefore, the policy conclusion from this paper is not that one should always use distributional weights in CBA. Rather, given the limited information typically available, it is perhaps often not be that important, from an overall welfare point of view, whether the government deals with distribution *solely* through income taxes, or whether it uses both income taxes and some kind of distributional weights in CBA.<sup>14</sup>

<sup>&</sup>lt;sup>12</sup>In the framework used here, an optimal tax would simply be a uniform lump-sum tax, which would be a first-best instrument.

<sup>&</sup>lt;sup>13</sup>This follows from a dual formulation of the welfare maximization problem, in terms of minimizing efficiency losses given a certain inequality. Of course, it is implicitly assumed that it is costly in terms of efficiency to obtain higher equality by adjusting the income tax. If it is not, the income tax cannot be optimal.

<sup>&</sup>lt;sup>14</sup>Simulation results would be a useful contribution for future research to more clearly evaluate this proposition.

In practice, simplicity of analysis is presumably one of the most important arguments against using weights. For example, should the weights be based on individual or household income? Do we have to use equivalent scales and, if so, how should these be calculated? Should we use life-cycle income instead of current per month income (are PhD-students poor or rich?), and if so how should this income be estimated? Which inequality aversion should be assumed? These are clearly difficult questions, but there are also practical arguments in favor of distributional weights. Many poor countries' income-tax system simply works very poorly, and equity-concerns directed through public projects may in such countries have quite a high distributional accuracy compared to the available alternatives. And for global problems, such as the greenhouse effect, there is no government (or other authoritative super-national institution) at all (see Johansson-Stenman, 2000).

Finally, it is important to keep in mind that all results discussed here are derived from very stylized and simplified models, and there are many important problems which one is forced to ignore on the link from fundamental ethics to actual policy recommendations (Hausman and McPherson, 1993; Johansson-Stenman, 1998). So, in concluding it appears very difficult to defend both the proposition that distributional weights should *always* be used in CBA, and that they should *never* be used.

#### APPENDIX:

## Separability in both leisure and public goods

Utility, given these types of separabilities, can be written as the following functions:  $u(f(x,l),G_1,G_2) = U(h(x,G_1,G_2),l)$ . Using the latter function, the ratio between the

marginal WTP for the public goods can then be written  $r = MRS_{G_2x} / MRS_{G_1x} = \frac{fh}{fG_2} / \frac{fh}{fG_1}$ 

which is a function of x,  $G_1$  and  $G_2$ , and clearly independent of l. But, using the former

function, this ratio can also be written  $r = MRS_{G_2x} / MRS_{G_1x} = \frac{f_{\mu}}{fG_2} / \frac{f_{\mu}}{fG_1}$  which is a function of

f,  $G_1$  and  $G_2$ . Hence, we can write  $r = s(x, G_1, G_2) = t(f(x, l), G_1, G_2)$ . But then we have

that 
$$\frac{\mathrm{d}r}{\mathrm{d}l} = \frac{\mathrm{d}s}{\mathrm{d}l} = 0 = \frac{\Re}{\Re} \frac{\Re}{\Re}$$
. But since in general  $\frac{\Re}{\Re} \neq 0$  we must have that  $\frac{\Re}{\Re} = 0$ , so that

*r* is independent of *x* as well. Thus, *r* can be written as a function solely of  $G_1$  and  $G_2$ . This means that all individuals, having the same utility function, will have the same *r*. The distributional characteristics are then the same for good 1 and 2, since

$$\mathbf{d} = \operatorname{cov}\left(\frac{\mathbf{a}}{\overline{\mathbf{a}}}, \frac{MRS_{G_{2}x}}{MRS_{G_{2}x}}\right) = \frac{n}{\overline{\mathbf{a}}} \frac{\sum_{i} \mathbf{a}^{i} \frac{\P}{\PG_{2}} / \frac{\P}{\P_{i}}}{\sum_{i} \frac{\Re}{\PG_{2}} / \frac{\P}{\P_{i}}} - 1 = \frac{n}{\overline{\mathbf{a}}} \frac{\sum_{i} \mathbf{a}^{i} r \frac{\Re}{\PG_{1}} / \frac{\Re}{\P_{i}}}{\sum_{i} r \frac{\Re}{\PG_{1}} / \frac{\P}{\P_{i}}} - 1$$
$$= \frac{n}{\overline{\mathbf{a}}} \frac{\sum_{i} \mathbf{a}^{i} \frac{\Re}{\PG_{1}} / \frac{\P}{\P_{i}}}{\sum_{i} \frac{\Re}{\PG_{1}} / \frac{\P}{\P_{i}}} - 1 = \operatorname{cov}\left(\frac{\mathbf{a}}{\overline{\mathbf{a}}}, \frac{MRS_{G_{1}x}}{MRS_{G_{1}x}}\right) = \mathbf{d}$$

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