**Industrial and Financial Economics**
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# THE VALUE OF FLEXIBILITY

## A Real Option Approach to Capital Budgeting

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Chance favours the prepared mind.

*Louis Pasteur (1822 - 1895)*

## Executive Summary

In this paper we present a simple and intuitive real option based framework for analyzing and valuing capital investment opportunities. The framework is applied in a case study for Gothenburg Energy AB which to this day has not used Real Option based valuation frameworks in their capital budgeting. Our analysis showed that its usefulness varied depending on the project characteristics.

We evaluated a project involving district cooling, assuming three different scenarios. In the case when there was no possibility of postponing the investment decision and the project had very limited strategic value, our results showed that the real option framework did not add any value to the capital budgeting decision. However, in the case when the investment decision could be postponed over a period of time the real option based valuation framework gave a result superior to a simple NPV analysis. The expanded valuation framework captured the extra value that postponing the investment added to the total project value. This was also true in the case when the project was assumed to have strategic value in the sense the investment could be expanded considerably 5 years after the initial investment was made.

In spite of the limitations of the ROV, presented in this paper, in some cases it is still able to compensate for many of the major shortcomings DCF valuation methods face.The framework is able to incorporate the value inherent in strategic opportunities imbedded in many capital projects. It is also able to value the flexibility given by the opportunity to defer an investment over a period of time in which valuable information may become available as uncertainty unfolds.

A major advantage of the approach used in the case study is that it is simple and easily implementable as most of the information needed for the valuation is already present in the traditional DCF spreadsheet used by most corporations. At the same timeas simplicity is an advantage, it is also a drawback, as it requires some liberties being taken which lead to an outcome that is more of an approximation than an exact answer.

Using option-pricing models to analyze capital projects presents some practical problems. Comparatively few of these have completely satisfactory solutions; on the other hand, some insight is gained just from formulating and articulating the problems. Still more, perhaps, is available from approximations. When interpreting an analysis, it helps to remain aware of whether it represents an exact answer to an approximated problem, or an approximate answer to an exact problem. Either may be useful.

We believe that the results form the case study show sufficient evidence to support a recommendation to Gothenburg Energy AB to implement a real option based valuation framework to their capital budgeting process in addition to their existing valuation methods.

## Abstract

This paper discusses different approaches to the capital investment process. The main focus is on investigating the practical aspects when applying real option theory to capital budgeting. We apply an extended framework introduced by Timothy A. Luehrman (1997) to a case study for Gothenburg Energy AB. The project is evaluated based on different assumptions in three separate scenarios. The results form each scenario are discussed in context of the applicability of the real options approach to the investment decision. Further, the robustness of the results to different assumptions about the evolation of the project value is examined.



**Key words:** Capital budgeting, Real option theory, Financial option theory, Flexibility, Discounted cash flow analysis.

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# Part I – Theory

# 1 Background

Most managers today use some kind of cash flow analysis, in one form or another, to value capital investments[1]. This typically involves a simple rule to apply to the decision process. The first step is usually to calculate the present value of the expected cash flow from the potential investment. The next step is to calculate the present value of the expected expenditures from the potential project. The final step involves determining the difference between the expected cash inflow and the expected expenditures from the investment opportunity. This procedure gives the so-called net present value (NPV) of the potential investment. The decision rule is then based on the simple logic that if the NPV is greater than zero the manager gives his approval to go ahead with the project. If, however, the NPV is close to or below zero the project will most likely not be undertaken.

Certainly there is more to the net present value calculation. The issue about how to estimate the expected cash flows generated by the project has to be resolved, as well as accounting for taxes and inflation. But maybe the most important variable in the calculation, the discount rate, can be very hard to assess correctly. These issues complicate the method somewhat but in fact the general methodology is quite simple and easy to understand. Simply determine if the difference between the future cash inflows and cash outflows involved in the project is negative or positive.

Despite the popularity and simplicity of this approach, it often results in inaccurate or outright incorrect estimation of the potential profitability of the project under valuation. This unfortunate side effect, on an otherwise

[1] Segelod (1998)

attractive method, arrives from the fact that it is built on unrealistic assumptions.

NPV usually assumes one of two scenarios. First, the investment is reversible or in other words that it can somehow be undone and the expenditures recovered should market conditions turn out to be unfavourable. The other scenario is that if the investment is irreversible, it is a now or never proposition. This means that if the company does not make the investment now, it will lose the opportunity forever.

It is certainly the case that some investment decisions fall into either of those categories but most do not. The reality is that in many cases, investments are more or less irreversible and can in one way or another be delayed for shorter or longer periods of time.

## 1.1 Problem discussion

A growing body of research shows that the ability to delay irreversible investment expenditures can profoundly affect the decision to invest. Ability to delay also undermines the validity of the net present value rule. Thus, for analysing investment decisions, we need to establish a richer framework, one that enables managers to address the issues of irreversibility, uncertainty, and timing more directly (Dixit and Pindyck, 1995).

Most of the recent research on capital investment stresses that some of the most important aspects of most investments are in fact the timing of the investment and the flexibility involved. Not only is the investment opportunity itself important, but more so, how can managers decide how to exploit those opportunities most effectively to increase shareholder value.

The research is based on an important analogy with financial options. A company with an opportunity to invest is holding much like a financial call option: it has the right but not the obligation to buy an asset (namely, the entitlement to the stream of profits from the project) at a future time of its choosing. When a company makes irreversible investment expenditure, it "exercises," in effect, its call option. So the problem of how to exploit an investment opportunity boils down to this: how does the company exercise that option optimally? Academics and financial professionals have been studying the valuation and optimal exercising of financial options for the past two decades. Thus we can draw from a large body of knowledge about financial options (Dixit and Pindyck, 1995).

The recent research on investment offers a number of valuable insights into how managers can evaluate opportunities, and it highlights a basic weakness of the NPV rule. When a company exercises its option by making an irreversible investment, it effectively "kills" the option. In other words, by deciding to go ahead with a project, the company gives up the possibility of waiting for new information that might affect the desirability or timing of the investment; it cannot disinvest should market conditions change adversely. The lost option value is an opportunity cost that must be included as part of the cost of the investment. Thus the simple NPV rule needs to be modified. Instead of just being positive, the present value of the expected stream of cash from a project must exceed the cost of the project by an amount equal to the value of keeping the investment option alive (Dixit and Pindyck, 1995).

Flexibility is the ability to defer, abandon, expand, or contract an investment. Because the NPV rule does not factor in the value of uncertainty, it is inherently less robust than an options approach in valuing flexibility. For example, a company may choose to defer an investment for some period of time until it has more information on the market. The NPV rule would value that investment at zero, while the real options approach would correctly allocate some value to that investment's potential.

Conventional capital budgeting techniques, such as DCF models, ignore the operating flexibility that gives management the option to revise decisions while a project is underway. Real options analysis recognises the flexibility inherent in many capital projects and the value of that flexibility. A real option captures the value of a company's opportunity to start, expand, constrain, defer, or scrap a capital investment, depending on the investment's prospects (Trigeorgis, 1996).

Despite the popularity of the real option approach among academics, only a few corporations[2], in very selective industries have begun to employ this framework. Vast majority of corporations use valuations methods based on discounted cash-flow evaluations. According to Segelod (1998) the proportion of "Fortune 500 corporations" using discounted DCF for investment appraisal has risen from 38% in 1962, to 64% in 1977, to over 90% in 1990-1993.

The energy industry, including electricity distribution as well as district heating and cooling production, is a very capital intensive industry with huge investments in infrastructure that are often expected to pay off over several decades. Long construction lead times and operating lives imply the need for *capacity planning* to determine the *type*s, *size*s, and *timing* of new plants to be built as older plants are retired. These decisions are made in the face of great uncertainty, and the often-irreversible commitments are translated into future costs. In the presence of rapidly changing technology, economics, and shifting social attitudes, new commitments may quickly become obsolete and inadequate. These attributes provide the ideal conditions to apply a real option base valuation framework to the capital budgeting process. Hence, we decided to apply the option framework discussed in this paper to a case study for Gothenburg Energy AB involving an investment in a district-cooling project.

## 1.2 Purpose

The main purpose of this paper is to introduce and apply a real option based valuation framework on a real project that is able to incorporate the value of flexibility in the capital budgeting process. The approach is intended to supplement, not replace capital budgeting analysis and investment criteria based on standard discounted cash flow methodologies. Further, it is aimed at bridging the gap between the practical problems of applying real option theory on real projects, and the complicated mathematics associated with formal option pricing theory.

In order to arrive at the main purpose, four sub purposes have been formulated.

- First, what are the major shortcomings of the traditional capital budgeting methods and in what way can real option theory compensate for these shortcomings?

- Second, how can financial valuation techniques be used to value real assets in a relatively simple and easily implementable manner?

- Third, under what circumstances is the real option valuation approach appropriate for capital budgeting?

- Finally, what are the most common options embedded in a project and how can these options be identified?

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2 Corman (1997) reports on companies that have adopted explicit option valuation methods.

## 1.3 Methodology

The thesis is divided into two main parts. The first part contains the theory on which the application framework used in the case study in part two is based on. The approach used in the paper is in large part descriptive in the sense that it draws on extensive existing knowledge about the problem, which it is fairly well structured in the theory. The following research methods are employed: literature review, model replication, conceptual development, interviews and comparative theoretical evaluations.

The application framework is based on a methodology introduced by Luehrman (1994, 1998), which has also been applied by Trigeorgis (1996) on hypothetical examples. However, as far as we know this framework has never been applied on a real project before. Details of the methodology applied in the case study are presented in part II of the paper.

### 1.3.1 Literature

The literature review is based on secondary data, i.e. books and articles. Most of the information has been gathered through an extensive search in various databases and Internet search engines. The search was focussed on titles and key words as well as key authors on the topic. The databases include Altavista, Harvard Business Review, Gunda, Libris and others. Going through references in key articles and books on the topic extended the data collection process further.

Gothenburg Energy provided us with some of the data directly connected with district cooling, both their own publications as well as other key articles on the topic.

### 1.3.2 Framework construction for case study

A single case study is used to explore the applicability of the framework presented in the paper. When constructing the valuation framework, our goal was that the study would be able to demonstrate how relatively easily the real option based valuation framework can be applied to a wide range of different capital projects.

The data for the case study is mostly primary data collected from Gothenburg Energy consisting of project valuation spreadsheets. Data for the risk-free interest rate was gathered from Riksbanken.

Part of the data from Gothenburg Energy AB was collected through interviews with project managers. Interviews were conducted with Anders Eriksson, project manager, and Stefan Hellberg, manager of district cooling. Most of these interviews were informal and customarily aimed at getting general information about the projects' characteristics.

We also had informal discussions, about various topics concerning the paper, with professors at the Integrated Masters program at Gothenburg University. These included for example professor Clas Wihlborg and professor Ted Lindblom.

### 1.3.3 Critique of the Sources

In the last few years the available data about the real option approach has increased significantly. There is an enormous amount of academic and business articles and books on the topic from various authors available. These sources are of various quality so we made an effort to ensure that we used only sources that are widely considered reliable, for example sources that have been referred to by leading researchers in the field of real option theory as well as suggestions from professor Clas Wihlborg.

Data for the case study in part II is provided by Gothenburg Energy (GE). We do not make any judgments about the reliability of that data as first of all we do not have an insight into the industry to dispute it, nor is it an important factor in the analysis[3]. The main purpose of the analysis is to illustrate how the framework can be applied to areal project, but not necessarily to provide a detailed insight to this particular industry.

## 1.4 Delimitations

All formal option pricing models, including Black-Scholes, assume that the riskiness of an asset can be expressed as a probability distribution for returns, prices or payouts for the asset. Some of the assumed distributions are elegantly simple, such as the lognormal distribution assumed by Black-Scholes. But corporate data for most real projects is usually not that elegant and may be inconsistent with, for example, a lognormal distribution. However, we were unable to tackle this problem directly for lack of data but instead we apply a sensitivity analysis to interpret the results. One approach to this problem might be to figure out in which direction a simplified distribution biases the analysis and then interpret the output accordingly, as an upper or lower bound for the actual project's value (Luehrman, 1994).

More fundamental than the particular distribution assumed by a given model is the type of world being modelled. The Black-Scholes world, for example, is one in which underlying assets are securities that are traded continuously.

[3] Some of the data and details behind the calculations are not provided in the general version of the paper and will only be available to Gothenburg Energy AB, for reasons of confidentiality.

Many real options involve underlying assets that are not traded continuously or, in some cases, not traded at all. For such assets, the five variables (six if dividends are allowed) of the Black-Scholes model may not be sufficient to characterise and price a call option. Whether one model or another remains useful as a way to price a simplified version of the project is a judgement the analyst must make. One alternative to such modelling is brute force, in the form of computing power. High-speed computers and advanced spreadsheet software make it possible to simulate some projects as a complicated decision tree. Decision-tree analysis is not, formally speaking, option pricing, but if well executed, it provides a better treatment of uncertainty and of manager's scope for decision making than conventional discounted cash flow analysis alone (Luehrman, 1998).

When it is assumed to give a better understanding to the reader, we also mention the delimitation in the text in more detail.

### 1.5 Organisation of the Thesis

The basic structure of the thesis is divided into two main parts. The first part presents the building blocks for theoretical framework that the case study in part two of the paper is basedon. Part I is based on a literature review and focuses on traditional capital valuation methods, financial option theory and the fundamentals of real options theory. In order to get a comparatively deep understanding of real options theory we discuss each of these topics in some detail and try to show how they are connected. Further we discuss the development and recent advances in the field of real option theory.

The second part starts by presenting a valuation framework that combines traditional cash flow analysis and real option theory. We then apply this framework in a case study for GE, where we value a real project concerning District Cooling.

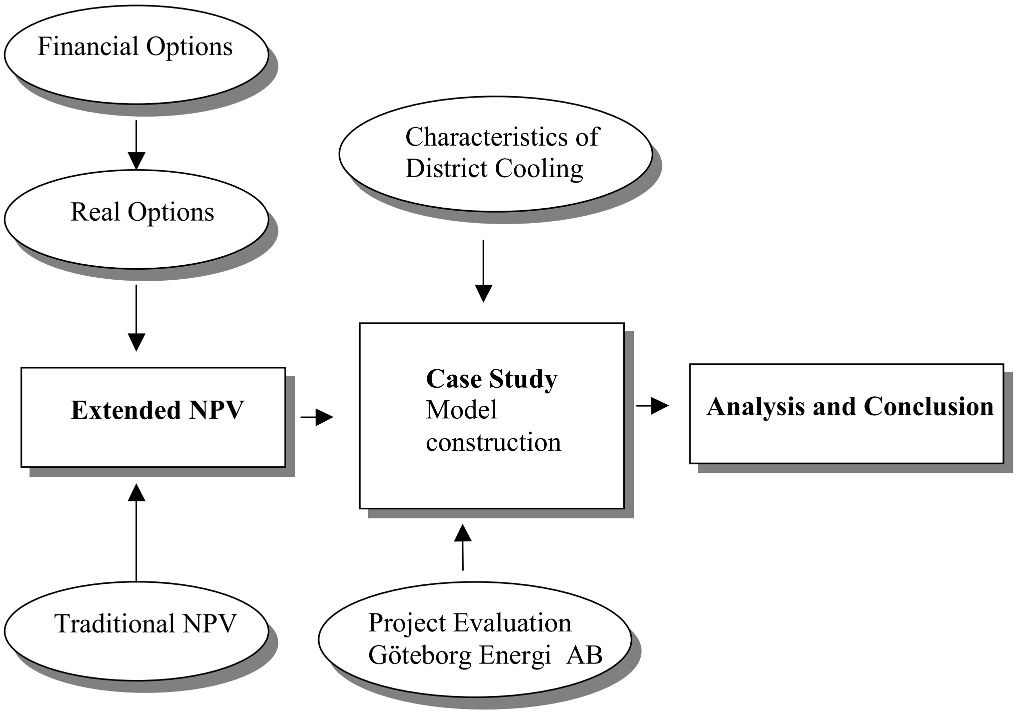


**Figure 1-1.** Structure of the thesis

The paper is structured as presented in figure 1.1. Before turning to real option theory we discuss the underlying theory of traditional capital budgeting methods and financial option. Chapter two explains the most popular traditional evaluation methods and addresses some aspects of the first sub purpose proposed in section 1.2. Chapter three is directly linked to the second sub purpose and presents an introduction into financial option theory, which provides the direct link into real option theory. In chapter four we extend the discussion on financial option theory to real option theory and address sub purposes three and four. Part II continues to address sub purposes three and four, as well as the main purpose, through the framework construction and the case study for GE. A more detailed overview of the structure for part two is presented in the beginning of part II.

## 2 Traditional capital budgeting methods

This section presents a brief overview of the most widely known "traditional" capital budgeting methods. According to Brigham and Gapenski (1996), seven primary methods have proved to be most popular to rank projects and to decide whether or not they should be accepted: payback, accounting rate of return (ARR), net present value (NPV), internal rate of return (IRR), modified IRR (MIRR), profitability index (PI) and decision tree analyses. We first explain how each ranking criterion is calculated and then discuss briefly how well each performs in terms of identifying those projects that will maximize the firm's value.

The term capital refers to fixed assets used in production, while a budget is a plan which details projected inflows and outflows during some future period. Therefore, the capital budget is an outline of planned expenditures on fixed assets, and capital budgeting is the entire process of analysing projects and deciding which one to include in the capital budget (Brigham and Gapenski, 1996).

### 2.1 Payback period

The payback period, defined as the expected number of years required to recover the original investment, was the first formal method used to evaluate capital budgeting projects. The easiest way to calculate the payback period is to accumulate the project's net cash flows and see when they sum to zero. Some firms use a variant of the regular payback, the discounted payback, which is similar to the regular payback except that the expected cash flows are discounted by the project's cost of capital[4].

Note that the payback is a type of "breakeven" calculation. If cash flows come in at the expected rate until the payback year, then the project will break even in the sense that the initial cash investment will be recovered. However, the regular payback does not take account of the cost of capital, no cost for the debt or equity used to undertake the project is reflected in the cash flows or the calculation. Even though the discounted payback method takes account of the cost of capital, both methods have serious deficiencies, especially the fact that they ignore all cash flows after the payback period.

Although the payback method has some serious faults as a project ranking criterion, it does provide information on how long funds will be tied up in a project, giving an idea about the projects liquidity (Brigham and Gapenski, 1996).

## 2.2 Accounting rate of return (ARR)

The second oldest evaluation technique is the accounting rate of return. It essentially focuses on a project's net income rather than its cash flow and is measured as the ratio of the project's average annual expected net income to its average investment.

Although this method may (or may not) be useful for measuring performance, it is not a good capital budgeting decision method as it completely ignores the time value of money.

[4] A project's cost of capital reflects the corporate cost of capital to the firm and the differential risk between the firm's existing projects and the project being evaluated. This is discussed further in section 2.8

## 2.3 Net present value (NPV)

In the NPV method the expected future cash flows for each period are discounted using the company's discount rate to account for the time value of money. The basic formula[5] for calculating the NPV of a project is shown below.

$$NPV = \sum_{t=1}^{T} \frac{E(c_t)}{(1+k)} - I_0$$

$E(c_t)$ = Expected future cash inflow at time t
$k$ = required rate of return
$I_0$ = Investment outlay in period 0

The expected future cash inflows $E(c_t)$ for each period are discounted back to present time using the required rate of return $(1+k)$. The investment outlay in period 0 is subtracted from the present value of the cash inflows. An NPV greater than zero will mean that the project should be undertaken.

The intuition behind discounted cash flow analysis is that a project must generate a higher rate of return than the one that can be earned in the capital markets. Only if this is true will a project's NPV be positive (Ross, Westerfield and Jaffe 1999). If a firm takes on a zero-NPV project, the position of the stockholders remains constant, the firm becomes larger, but the price of its stock remains unchanged.

## 2.4 Internal rate of return (IRR)

Internal rate of return is defined as the discount rate that makes the present value of the expected cash outflows equal to the present value of the cash inflows. In effect, the IRR on a project is its expected rate of return. If the IRR exceeds the cost of the funds used to finance the project, a surplus remains

[5] See for example Ross, Westerfield and Jaffe, 1999

after paying for the capital, and this surplus accrues to the firm's stockholders. Hence, taking on a project whose IRR exceeds its cost of capital increases shareholders wealth. On the other hand, if the IRR is less than the cost of capital, then taking on the project imposes a cost on current stockholders. It is this "break even" characteristic that makes the IRR useful in evaluating capital projects.

The same basic equation is used for both the NPV and IRR. However, in the NPV method the discount rate, k, is specified and the NPV is found, whereas in the IRR method the NPV is specified to equal zero, and the value of IRR that forces this equality is determined (Brigham and Gapenski, 1996).

In spite of a strong academic preference for NPV, surveys indicate that business executives prefer IRR to NPV by a margin of 3 to 1. Apparently, managers find it intuitively more appealing to analyse investments in terms of percentage rates of return than dollars of NPV (Brigham and Gapenski, 1996).

## 2.5 Modified internal rate of return (MIRR)

The IRR can be modified to make it a better indicator of relative profitability, hence better for use in capital budgeting. This measure is called the modified IRR or MIRR, and is defined as that discount rate which forces the PV of the investment outlays to equal the PV of the project's terminal value.

The modified IRR has a significant advantage over the regular IRR. MIRR assumes that cash flows from all projects are reinvested at the cost of capital, while the regular IRR assumes that the cash flows from each project are reinvested at the project's own IRR.

If two projects are of equal size and have the same life, then NPV and MIRR will always lead to the same project selection decision. If, however, the projects differ in scale (or size), then conflicts can occur (Brigham and Gapenski, 1996).

## 2.6 Profitability Index (PI)

Another method used to evaluate projects is the profitability index (PI), sometimes called the benefit/cost ratio:

$$\text{PI} = \frac{PV(benefits)}{PV(\cos ts)} = \sum_{t=0}^{n} \frac{CIF_t}{(1+k)^t} \Bigg/ \sum_{t=0}^{n} \frac{COF_t}{(1+k)^t}$$

Here $CIF_t$ represents the expected cash inflows, or benefits, and $COF_t$ represents the expected cash outflows, or costs. The PI shows the relative profitability of a project, or the present value of benefits per present "dollar value" of costs. A project is acceptable if its PI is greater than 1.0 and the higher the PI, the higher the project's ranking.

Mathematically, the NPV, the IRR and the PI methods will always lead to the same accept/reject decisions for independent projects. If a project's NPV is positive, its IRR will exceed k and its PI will be greater than 1.0. However, NPV, IRR and PI can give conflicting rankings for mutually exclusive projects (Brigham and Gapenski, 1996).

## 2.7 Decision tree analysis (DTA)

Decision tree analysis (DTA) takes the NPV method a little further. Instead of presuming a single scenario of future cash flows, many different scenarios are being considered. By solving the problem in this way, several possibilities of futures states of the world and also the set of decisions made each time in each state will be incorporated into the analysis. The future cash flows and probabilities used in the analysis reflect the information available to the company at the present time. The values are derived from the basis of past information (Brigham and Gapenski, 1996)[6].

## 2.8 The reqiured rate of return

A central question of capital budgeting concerns the specification of an appropriate required rate of return. The required rate of return represents the time value of money and the relative risk of the project in the discounted cash flow models. If the cash flows from the project under consideration were known for certain the required rate of return would be the risk free interest rate. However, the future cash flows for projects are usually associated with uncertainty. The uncertainty is then incorporated into the analysis by using a risk adjusted required rate of return (Buckley, 1998). The Capital Asset Pricing Model provides a very helpful tool in calculating the risk adjusted required rate of return and has wide applicability in the real world. Another popular method is the Arbitrage Pricing Theory APT, which can also be used for calculating the required rate of return.

[6] See appendix I for an example of a decision tree analysis.

## 2.9 Pitfalls of traditional methods

According to Dixit and, Pindyck (1994) the most important mistake in the basic NPV method is that it assumes the investment to be either reversible or irreversible. In other words the method assumes management's passive commitment to a certain "operating strategy", which is usually not the case.

The basic NPV method also ignores the synergy effects that the investment project can create. A project of a certain kind might allow the company to expand into a second project, which would not have been possible without the first project (e.g., many research and development projects). It is the value of this second project that NPV ignores.

Using sensitivity analysis in combination with NPV is an attempt to deal with uncertainty of future cash flows by making different scenarios using the NPV approach. The sensitivity analysis begins with the creation of a base case scenario the most likely value of the relevant variables in a NPV calculation. Then, some key primary variables, which have an impact on NPV (IRR), are identified. Each key variable will be changed to a best and worst value while holding the others constant at their base case value. The resulting NPV values can then give a picture of the possible variation in, or sensitivity of the NPV to each of the key variables. In turn, the impact of misestimating each key variable can then be observed. The sensitivity analysis could also be used to see when the project's return is zero, i.e. a break-even analysis (Buckley, 1998).

However this method has its limitations as well. It considers the effect on NPV of only one error in a variable at a time, thus ignoring combinations of errors in many variables at the same time. This is a major shortcoming since in many cases a change in one variable will affect another. The variables may also be serially dependent over time i.e. the variables could effect themselves through time. By using Monte Carlo simulation these shortcomings are considered. However, this is a very complex and time consuming procedure and the results can be hard to interpret which means that management often has to delegate this task to experts (Trigeorgis, 1996).

As opposed to basic NPV analysis, the DTA incorporates the issues concerning flexibility mentioned above into the analysis. This makes DTA analysis a better tool than basic NPV to evaluate projects. However, to find the appropriate required return of return is a problem in both basic NPV and DTA. A seemingly small difference in the discount rate can have a huge impact on the overall result.

Trigeorgis (1996) argues that the most serious problem in DTA analysis is to find the appropriate discount rate. This is because the presence of flexibility would alter the project's risk, hence altering the discount rate that would prevail without the flexibility. For example the possibility to abandon the project would clearly reduce the project's risk and lower the discount rate. Then, using the same discount rate as in a basic NPV would undervalue the project. Cortazar (1999) supports this argument by emphasising that whichever pricing model is used (CAPM or APT) most investment projects will find their risk structure change over time. This means that the risk-adjusted discount rate also will change over time, which in turn will lead to errors in the result.

Moreover, the example (presented in appendix I) of a DTA analysis only considers high low and middle values of the cost. In reality the market consists of a range of values in between. Also, the events do not simply occur at some discrete points in time, rather, the resolution of uncertainty may be continuous.

Another critique of the DTA analysis refers to its complexity in the sense that when it is applied in most realistic investment settings, it will easily turn out to be a unmanageable "decision-bush analysis", as the number of paths through the tree expands geometrically with the number of decisions, outcome variables, or states considered for each variable (Trigeorgis, 1996).

Buckley (1998) argues that in reality, managers frequently pursue policies that maintain flexibility on as many fronts as possible and thereby maintain options that promise upside potential. However a tool is needed to account for the flexibility in projects in a more correct and simple way than DTA analysis does and it is here that the real options approach comes in.

# 3 Financial Options

In order to appreciate and understand real options a strong knowledge of financial options and option-pricing models is required. In this section we will introduce the basic concepts of financial options as well as different methods for their valuation.

## 3.1 Options characteristics

Options are special contractual arrangements giving the owner the right to buy or sell an asset at a fixed price on/or anytime before a given day. Options are a unique type of financial contract because they give the buyer the right, but not the obligation, to buy or sell an underlying asset. The holder exercises the option only if it ends up "in the money"[7] otherwise the option can be left to expire unexercised (Ross, Westerfield, and Jaffe 1999).

There are two basic types of options. A *call option* is a contract giving the owner the right to *buy* a specified asset at a fixed price at any time on or before a given date. A *put option* is a contract giving the owner the right to *sell* a specified asset at a fixed price at any time on or before a given date (Cox and Rubinstein, 1985).

The basic features of option contracts are the following:

- *Exercise or strike price* is the price by which the owner buys or sells the underlying asset.
- *Expiration (exercise date) or maturity* is the date on which the owner of the option can exercise his or her right.

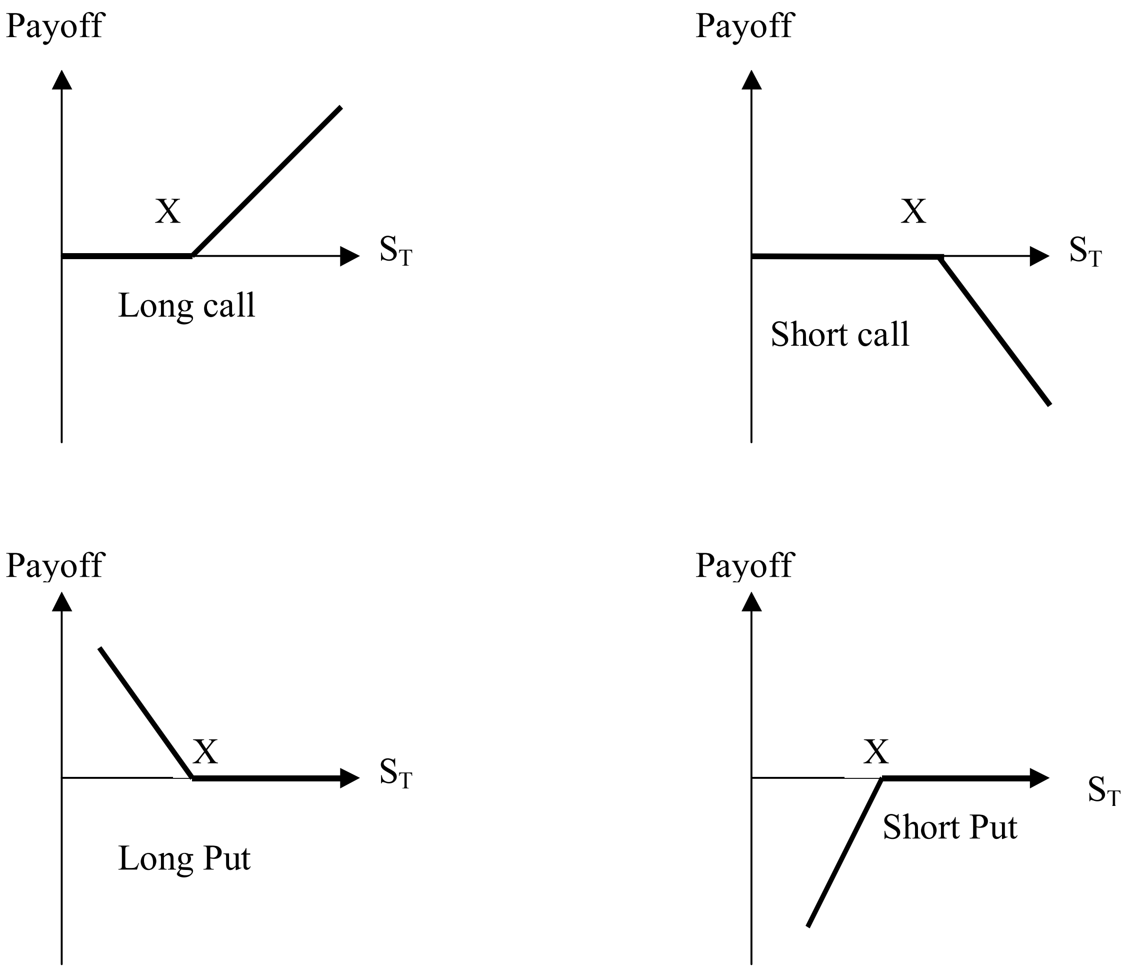


**Figure 3-1.** Call and put option values. Source: Hull (1997)

Options can be distinct between *American* and *European*. American options can be exercised at any time prior to maturity, whereas European options can only be exercised at the expiration date. Most of the options that are traded on exchanges are American.

[7] "In the money options" are those that have a positive intrinsic value. "At the money options" are those that have an intrinsic value of zero. "Out of the money options" are those that have a negative intrinsic value.

Options belong to the derivative family, because their underlying asset determines their value. Financial options have the same purposes as other financial instruments, that is, to reallocate resources and risks, to hedge financial positions and to be used to take speculative positions.

Option values are determined by the difference between the exercise price and the current price of the underlying asset (also called intrinsic value).

Often European option positions are characterised in terms of payoff to the investors at maturity. If X is the strike price and $S_T$ is the final price of the underlying asset, the payoff from a long position in a European call is

$$\max(S_T - X, 0)$$

This reflects the fact that the option will be exercised if $S_T > X$ and it will not be exercised if $X \geq S_T$. The payoff to the holder of a short position in European call is

$$\min(X - S_T, 0).$$

The payoff to the holder of a long position in a European put option is

$$\max(X - S_T, 0)$$

and the payoff from a short position in a European put option is

$$\min(S_T - X, 0).$$

All these payoff positions are represented graphically in figure 3.1.

## 3.2 Factors affecting option value

The factors affecting an option's value can be divided in to two groups. The first contains the option contract contractual features and the second one concerns the characteristics of the underlying asset and the market. These factors are the following (Cox and Rubinstein, 1985):

- The exercise price (X)
- The underlying asset price (S)

- The time to expiration (t)
- The volatility of the underlying asset (σ)
- The interest rate (r)
- Dividend/Yield

In this section we will examine the effects that these factors have on the value of put and call options.

### 3.2.1 Exercise price and asset price

The difference between these two values determines the intrinsic value[8] that an option already has. Although the final intrinsic value is used to value an option, the current intrinsic value is important because the probability of a higher intrinsic value at maturity is greater if it is already high today.

In particular, the higher the *exercis*e *price* the lower the value of a call option. However, the value of an option cannot be negative no matter how high the exercise price is set and as long as there is some possibility that the exercise price will exceed the price of the underlying asset then the call option will still have some value. Since for put options the payoff on exercise is the difference between the strike price and the underlying asset price, then it is obvious that their value increases when the exercise price increases.

The effect of the *asset's price* is exactly the opposite, since the payoff for a call option is the amount by which the asset price exceeds the exercise price. Therefore, call options become more valuable as the asset price increases. The opposite is true for put options, where as stock price increases the difference

[8] The intrinsic value of an option is defined as the maximum of zero and the value it would have if it were exercised immediately (Hull 1997).

between strike price and stock price gets smaller and so does the value of the option.

### 3.2.2 Expiration date and interest rate

These two variables together determine the time value of money on the exercise price or the discounting of the expected intrinsic value on maturity back to the present.

A longer time to expiration always has a positive effect on a call option value. First of all it reduces the present value of the exercise price on maturity, if the option ends up in the money. Second, a longer time horizon gives potentially higher intrinsic values on maturity, since the volatility of the underlying assets grows with the square root of time. For European style options the effects of longer time to maturity cannot definitely be determined but for short-term puts it is usually positive. Concerning long-term options, there are influences that may have contradicting effects. The positive effect of the increased volatility, mentioned above, may be overcompensated by the fact that one receives the exercise price only at maturity. Especially for American style put options, a longer time period has a positive effect on the value of the option, because one has the right to receive the exercise price at any time prior to maturity (Hull, 1997).

A higher risk-free interest rate will have a positive effect on a call option because the exercise price will only have to be paid at the maturity date, if paid at all. Thus, making it possible to invest the money somewhere else gaining at least the risk-free rate for the time period left to maturity. For put options, a higher interest rate has a negative impact, since it decreases the present value of the money received by the sale of the underlying asset in the future.

### 3.2.3 Volatility of the underlying asset

The greater the volatility of the underlying asset the more valuable a call option will be. The same is true for the value of a put option. The owner of a call option benefits from stock price increases but has limited downside risk in the case of price decreases, since the most he can lose is the price of the option. In a similar way, the owner of a put benefits from price decreases and has limited downside risk in the case of price increases. Hence, the value of puts and calls increase, as volatility gets higher (Beer, 1994).

### 3.2.4 Dividends

Dividends have the effect of reducing the stock price on the ex-dividend day. This means that the value of a call is negatively correlated to the size of any dividends because options do not participate on the dividends. The value of a put option is positively correlated to size of dividends.

| **Option type** | Call Option | Put Option |
|---|---|---|
| Exercise price (X) | - | + |
| Stock price (S) | + | - |
| Interest rate (r) | + | - |
| Time to maturity (t) | + | +(-) |
| Volatility (σ) | + | + |
| Dividend/Yield | - | + |

**Table 3-1.** Determinants of valuation. Source: Beer (1994)

Table 3.1 summarises the effects the different factors may have on the on the value of put and call options. The plus sign indicates that the value of the option changes in the same direction as the value of the relevant factor does.

The minus sign indicates exactly the opposite, that is, the value of the option moves in the opposite direction than the value of relevant factor does. For example, an increase in the exercise price would result in a decrease of a call option value and an increase in put option value.

## 3.3 Valuation methods

This section presents the two basic methods for valuing options and other derivatives. We start with the Binomial Approach to valuation and its relationship to the principle of risk-neutral valuation. We then continue with the Black-Scholes model.

### 3.3.1 The Binomial Model

The binomial approach is both a simple and intuitive method for valuing complicated real and financial options that may arise in practice, especially in cases where the Black-Scholes formula is not a perfect fit. Using the binomial model we can price options other than European, like American options, that can be exercised at any time prior to maturity. This is because in the binomial model the time to maturity is divided into $n$ discrete intervals rather than constituting a continuous time framework, as in the Black-Scholes model. Therefore, the model can take option values into consideration before the maturity date of the option (Beer, 1994).

At each of these intervals, the stock price is assumed to have only two possible movements that are either up or down compared to the initial price. In order to present the binomial valuation method we will consider an example about a non-dividend paying underlying asset and a derivative on this asset whose current price is $f$. The only additional assumption we need to make is that there are no arbitrage opportunities (Cox and Rubinstein, 1985).

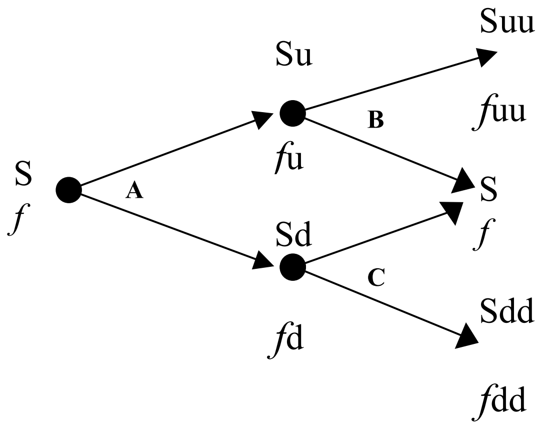


**Figure 3-2.** Two step binomial tree

If at time *t* the asset's value is *S*, at time $t_1$ it will be either Su or *Sd*. The proportional increase when the price moves upwards is u-1 and when it moves downwards the proportional decrease is 1-d. In this case u and d correspond to the upward or downward movement of the asset's value respectively. If the asset's price moves to *Su* the payoff from the option will be $f_u$ and if it moves to *Sd* then it will be $f_d$. The two step binomial tree in figure 3.2 illustrates this assumption of price movements made in the binomial model.

Lets now take a portfolio that consists of a long position in *Δ* shares of equity and a short position in a call option and calculate the value of *Δ* that makes the portfolio risk-less. The value of the portfolio will be either $Su\Delta - f_u$ or $Sd\Delta - f_d$ in the case of an upward and downward movement of the asset's price respectively. Since we want *Δ* to be such that the portfolio is risk- less then the two outcomes should be equal. That means that:

$$Su\Delta - f_u = Sd\Delta - f_d \text{ or,}$$

$$\Delta = \frac{f_u - f_d}{Su - Sd} \tag{1}$$

In this case the portfolio is risk-less and earns only the risk-free interest rate. *Δ* is the ratio of change in the option price to the change in the stock price as we move between different time points.

The present value of the portfolio should then be: $(Su\Delta - f_u)e^{-rT}$, where r is the risk-free interest rate.

The cost of setting up the portfolio is: $S\Delta - f$. It follows that $S\Delta - f = (Su\Delta - f_u)e^{-rT}$. Substituting Δ from equation (1) we get:

$$f = e^{-rT}\left[pf_u + (1-p)f_d\right] \tag{2}$$

where,

$$p = \frac{e^{rT} - d}{u - d} \tag{3}$$

Equations (2) and (3) enable a derivative to be priced using a one-step binomial model (Hull 1997).

The analysis can be extended to a two-step binomial tree such as the one in figure 1. The stock price is initially *S* and during each time period, it either moves up to u times its initial value or down to d times its initial value. We suppose that the risk-free rate is *r* and the length of the time period is *Δt* years. Applying equation (2) repeatedly gives:

$$f_u = e^{-r\Delta t}\left[pf_{uu} + (1-p)f_{ud}\right] \tag{5}$$

$$f_d = e^{-r\Delta t}\left[pf_{ud} + (1-p)f_{dd}\right] \tag{6}$$

$$f = e^{-r\Delta t}\left[pf_u + (1-p)f_d\right] \tag{7}$$

Substituting equations (5) and (6) in equation (7) we get:

$$f = e^{-r\Delta t}\left[p^2 f_{uu} 2p(1-p) f_{ud} + (1-p)^2 f_{dd}\right] \quad (8)$$

These results are consistent with the principle of risk-neutral valuation, which we discuss in more detail in the following section. The variables $p^2$, $2p(1-p)$ and $(1-p)^2$ are the probabilities that the upper, middle, and lower final nodes price values will be reached. The option price is equal to its expected value in a risk-neutral world discounted at the risk-free rate. The use of binomial trees can be generalised even further by adding more time steps. The risk-neutral valuation principle will continue to hold and the price of the option will be its expected payoff in a risk-neutral world discounted at the risk-free interest rate (Hull, 1997).

### 3.3.2 Risk neutral valuation

The variable p in the above equations is the probability that the price of the asset will have an upward movement. In the same manner the probability that the price will have a downward movement is $1-p$. The expected payoff from the option is then $pf_u + (1-p)f_d$. This means that equation (2), in the previous section, states that the value of the option today is its expected future value discounted at the risk-free interest rate. Since $p$ is the probability that the price will move up, then the expected price of the stock at time *t*, $E(S_T)$, is given by the following equation:

$$E(S_T) = pSu + (1-p)Sd$$

$$or$$

$$E(S_T) = pS(u-d) + Sd$$

If we then substitute $p$ from equation (3) we get:

$$E(S_T) = Se^{rT} \qquad (4)$$

This equation states that the price of the asset grows at the risk-free rate, meaning that setting the probability equal to *p* is the same as assuming that the return on the stock equals the risk-free rate. In other words, when we set the probability of an upward movement equal to *p* we assume the investors are risk-neutral. This is an important principle in the pricing of options and other derivatives known as the risk-neutral valuation. This principle simply states that since the risk preferences of the investors do not enter the solution, then any set of risk preferences may be used when evaluating *f*, including risk neutrality (Cox and Rubinstein, 1985). Hence, the very simple assumption that all investors are risk neutral can be made. When moving from a risk-neutral world to a risk-averse world the principle still holds and two things happen. The expected return on the underlying asset will be higher to compensate for the risk taking, while at the same time, the required interest rate to discount all future payoffs of the option increases. Those two effects compensate each other (Hull, 1997).

### 3.3.3 Black-Scholes option pricing formulas

The Black-Scholes formula is the most common method to value financial options and one of the most important ones in finance. The formula gives the price of a call or a put option on the basis of five variables: underlying stock price, exercise price, time to expiration, the risk-free interest rate and volatility of the stock. The idea behind their approach was the construction of a portfolio comprising of options and the underlying asset, which would be risk-less for a very short time period and therefore should earn, in order to avoid arbitrage opportunities, the return on a risk-free short-term security. Such a risk-less portfolio can be constructed because the option and the stock have the same source of uncertainty, which are the share price changes. If the portfolio has the right proportion of short options and a long position in the underlying asset shares, for very small changes in the stock price the gains on one side will be offset by the losses on the other side. The value of the portfolio at the end of the short time period is known in advance. Using this argument in the Black-Scholes model, it is possible to derive the valuation of options regardless of risk preferences. Therefore, any attitude towards risk may be used.

The Black-Scholes valuation formula relies on the following assumptions:

- The underlying asset price follows a generalised Brownian motion (see Appendix II).
- The prices of the underlying asset are log-normally distributed. Since prices cannot fall bellow zero, it is not reasonable to assume that they follow a normal distribution. Instead, it is more reasonable to assume that they are log-normally distributed.
- The short-term interest rate is assumed to be constant or known over time.
- The underlying asset has no form of dividend payoffs. This is an assumption that is not always true and it is dealt with as an appropriate extension of the Black-Scholes model.
- The option is European. This is another restriction to the formula and it should be taken into account when dealing with American style options. Several researchers have tried to extend the formula so it can deal with American options as well.
- The markets are considered to be frictionless. That means four things: (a) there are no transactions costs or taxes; (b) there are no restrictions on short sales, such as margin requirements, and full use of proceeds is allowed; (c) all shares of all securities are infinitely divisible; and (d) borrowing and lending are unrestricted. These assumptions allow continuous trading.

Again, the fundamental assumption is a risk-neutral world, in which the expected value of a European call option is:

$$\hat{E}[\max(S_T - X, 0)] \ .$$

From the risk-neutral valuation argument we know that the call's price, *c*, is the expected value of the call option discounted at the risk-free interest rate. That is:

$$C = e^{-r(T-t)} \hat{E}[\max(S_T - X, 0)]$$

Solving for the right hand side of the equation we obtain the Black-Scholes pricing formula for a European call option.

$$C = SN(d_1) - Xe^{-r(T-t)} N(d_2) \Rightarrow$$
$$C = e^{-r(T-t)} \left[ SN(d_1) e^{r(T-t)} - XN(d_2) \right]$$

where,

$$d_1 = \frac{\ln(S/X) + (r + \frac{1}{2}\sigma^2)T}{\sigma\sqrt{T}}$$

$$d_2 = \frac{\ln(S/X) + (r - \frac{1}{2}\sigma^2)T}{\sigma\sqrt{T}} = d_1 - \sigma\sqrt{T} \ .$$

N (.) is the cumulative probability distribution for a variable that is normally distributed with a mean of zero and a standard deviation of one. In particular, $N\ (d_2)$ is the probability that the option will be exercised in a risk-neutral world so that $XN(d_2)$ is the strike price times the probability that the strike price will be paid. The expression $SN(d_1)e^{r(T-t)}$ is the expected value equal to $S_T$ if $S_T > X$ and zero otherwise in a risk-neutral world. This interpretation of the terms shows that the formula is consistent with risk-neutral valuation (Hull 1997). The above equation, also, gives the value of an American call option that doesn't pay any dividends.

The value of a European put can be obtained in a manner similar to the above or by using the put-call parity[9]. The result is:

$$P = X[1 - N(d_2)]e^{-rT} - S[1 - N(d_1)].$$

[9] $C+Xe^{-r(T-t)}=P+S$. This relationship, known as the put-call parity, shows that the value of a European put with a certain exercise price and exercise date can be deducted by from the value of a European call with the same exercise price and date (Hull, 1997).

# 4 Real Options Theory

Real options theory can be described as a new valuation, project management and strategic decision making paradigm that makes up for many of the traditional methods by allowing for making flexible or staged decisions under uncertainty (Trigeorgis, 1996). Analogous to financial options a company that owns a real option has the right, but not the obligation to make a potentially value creating investment. The main difference between a financial option and real options is that the real options are applicable to “real” assets. The real assets are usually tangible such as a factory or a special production device while a financial asset typically consists of stocks, bonds or currency.

The Economist (1999) compared the real options approach to playing poker. If players had to place their final bets right as the first hand was dealt (as the CAPM requires them to), most would (reasonably) opt out quickly. Instead, they merely put down a small initial stake to stay in the game. Depending on the next card, they then pass, match or raise, and so on. This corresponds to an option to wait until more information becomes available. As in poker, the higher the volatility (higher stake) and the longer the option lasts, the more valuable it becomes. This is in sharp contrast to the CAPM, which deals harshly with both long time horizons and uncertainty.

If competitors share the right to exercise and may be able to take part (or all) of the project’s value away from the firm (option holder), then the option is shared. Shared real options can be seen as jointly held opportunities of a number of competing firms or of a whole industry, and can be exercised by any one of their collective owners (Trigeorgis, 1996). Examples of shared real options are the opportunity to introduce a new product unprotected from possible introduction of close substitutes and the opportunity to penetrate a new geographic market without barriers to competitive entry. The loss in value suffered by a firm as a result of competitive interaction when a competitive firm exercises its shared rights will be subsequently called competitive loss.

## 4.1 Key concepts

In this section we discuss a few key concepts related to real options and introduce the real options paradigm.

### 4.1.1 Investment

Economics define investment as the act of incurring an immediate cost in the expectation for future rewards (Dixit and Pindyck, 1994). Companies make capital investments in order to exploit profit opportunities. Investments in research and development, for example, can lead to patents and new technologies that open up those opportunities. Somewhat less obviously, companies that shut down money losing operations are also investing. The payments they make to extract themselves from contractual agreements, such as severance pay for employees, are the initial expenditure. The payoff is the reduction of future losses (Dixit and Pindyck, 1995)

### 4.1.2 Sunk cost

Brigham and Capenski (1996) define sunk cost as an outlay that has already occurred (or been committed). Since it has already occurred, it is an outlay that is not affected by the accept/reject decision under consideration.

Investment expenditures are more likely to be irreversible when they are specific to a company or to an industry. For example, most investments in marketing and advertising are company specific and cannot be recovered. They are sunk costs. Irreversibility can also arise because of government regulations, institutional arrangements, or differences in corporate culture. For example, capital controls may make it impossible for foreign (or domestic) investors to sell their assets and reallocate their funds. In the same way, investments in new workers may be partly irreversible because of the high costs of hiring, training and firing (Dixit and Pindyck, 1995).

Irreversible investments require good up-front analyses because, once the assets are in place, the investment cannot be reversed without losing much of its value. Irreversible investments are often managed by delaying a project until a significant amount of the uncertainty is resolved or by breaking the investment into stages (Amram and Kulatilaka, 1999)

### 4.1.3 Uncertainty

"Uncertainty" is a generic term used to describe something that is not known either because it occurs in the future or has an impact that is unknown. Uncertainty relates to the unknown at a given point in time, although it is not necessarily the "unknow-able." The term "uncertainty" has been used to mean an "unknown" that cannot be solved deterministically or an "unknown" that can only be resolved through time. Schweppe (1989) defines uncertainties as quantities or events that are beyond the decision maker's foreknowledge or control. Paraskevopolous (1991) attributes the origins of uncertainties to errors in specification, statistical estimation of relationships, and assumptions of exogenous variables. Uncertainty arises because of incomplete information such as disagreement between information sources, linguistic imprecision, ambiguity, impreciseness, or simply missing information. Such incomplete information may also come from simplifications and approximations that are necessary to make models tractable. Uncertainty sometimes refers to randomness in nature or variability in data.

Certainty refers to situations when the investor knows with probability 1 what the return on his investment is going to be in the future. Uncertainty then is when a collection of values (associated with individual uncertain "states of nature") can happen, with strictly positive probabilities for, at least, two different possible values (Levy and Sarnat, 1984). Uncertainty means for example that the future price of electricity will be up or down, in relation to the forecasted price. So there are two sides to uncertainty, a "good" one and a "bad" one.

For our purposes uncertainty can be classified into two main categories, economic uncertainty and technical uncertainty. These different types of uncertainty have opposite effect on an investment decision.

*Economic uncertainty*

Economic uncertainty is correlated with the general movements of the economy and the industry involved. The interest rate and price of oil or gas are examples of variables with economic uncertainty. This uncertainty is exogenous to the decision process, as the variables do not change depending on the investment decision of a particular project (Levy and Sarnat, 1984). Economic uncertainty gives an incentive to wait to invest. This can lead to the postponement of investments, even those with a considerable positive net present value.

*Technical uncertainty*

Technical uncertainty is not correlated with the general movements of the economy or a particular industry. This uncertainty is influenced by the investment decision and therefore endogenous to the decision process. An example is a new gold mine, the amount of gold and its quality are variables with technical uncertainty. Waiting does not influence the value of these variables and does therefore not decrease the uncertainty involved. Only by making an initial investment in exploration can this uncertainty be resolved or reduced. A step by step investment strategy can provide valuable information and reduces the variance of expected future cash flow from the project. This additional value has sometimes been called shadow value (Dixit and Pindyck, 1994), because it is not a directly measurable cash flow with traditional DCF methods. Technical uncertainty, unlike economic uncertainty, encourages an initial investment, although it is necessary that the investment be done in stages. It may be economically optimal to start a staged investment in a project with a negative NPV if substantial technical uncertainty is present. As new information becomes available management must revise the investment decision, that is if to abandon, proceed or even speed up the investment process depending on the information gained.

*Uncertainty and Hedging*

According to Dixit and Pindyck (1994), the option value for an investment opportunity is not affected if the firm is able to hedge the risk by trading in forward or futures market. In efficient markets such risk is fairly priced, so any decrease in risk is offset by the decrease in return and the financial operation has no effect on firm's real decisions.

### 4.1.4 Flexibility

In order to be able to identify and value flexibility it is useful to fully understand the meaning of the concept.

The idea of flexibility appears in many disciplines. In banking and finance, investors' preference for flexibility translates into the notion of liquidity, or the ease in which assets can be transformed. In operations management, flexible manufacturing systems replace the function and product-specific machines of the past. In the labour markets, employers allow flexible hours to attract better skilled workers. In turn, a multi-skilled worker can entertain more job opportunities. Flexible information systems offer users more functionality. In all of these areas, flexibility represents a desirable property or goal (Ku, 1995).

For our purposes, flexibility can be defined as the ability to defer, abandon, expand, or contract an investment.

As new information arrives and uncertainty about future cash flows is resolved management may find that that various projects allow it varying degrees of flexibility to depart from and revise the operating strategy it originally anticipated.

Management's flexibility to adapt its future actions depending on the future environment introduces an asymmetry or skewness in the probability distribution of NPV that expands the investment opportunitys true value by improving the upside potential and at the same time limiting downside losses. If managerial flexibility were absent the probability distribution of NPV would be reasonably symmetric. When managerial flexibility is significant, however, by providing a better adaptation to future events turning out differently than from what management expected at the outset, it introduces a transition with enhanced upside potential so that the resulting actual distribution is skewed to the right (Trigeorgis, 1996).

### 4.1.5 Contingency

Trigeorgis (1996) defines contingency as a situation when future investments are contingent on the success of today's investment. Managers may make investments today, even those deemed to be NPV negative, to access future investment opportunities. Traditional budgeting models inadequately value these options-creating investments. Pharmaceutical company investments are a good example. Future spending on drug development is often contingent on the product clearing certain efficacy hurdles. This is valuable because investments can be made in stages, rather than all up-front.

Amram and Kulatilaka (1998) support the idea that managers intuitively use options, such as when they delay completing an investment program until the results of a pilot project are known. The decision about whether to complete the investment program is a contingent investment decision, one that depends on an uncertain outcome. Valuing investment opportunities that contain future contingent decisions is hard, but it can be done with the options approach to valuation.

### 4.1.6 Volatility

Under uncertainty, a future variable is characterized not by a single value but by a probability distribution of its possible outcomes. The amount of dispersion or volatility of possible outcomes is a measure of how risky that uncertain variable is.

Somewhat counter intuitively, investments with greater uncertainty have higher option value. In standard finance, higher volatility means higher discount rates and lower net present values. In option theory, higher volatility, because of asymmetric payoff schemes, leads to higher option value. In a sense, real option theory allows us to value the unimaginable. This means that industries with high uncertainty, like the Internet, actually have the most valuable options.

Micalizzi and Trigeorgis (1999) point out that even though a project has a positive NPV if undertaken immediately, an appropriate delay may result in even more value, due to uncertainty being resolved.

## 4.2 From financial options to real options

The analogy between financial options and corporate investments that creates future opportunities is both intuitively appealing and increasingly well accepted (Luehrman, 1998)

Leslie and Michaels (1997) describe some aspects of flexibility that are common to financial and real options. In each case, an option holder can decide whether to make the investment and realize the payoff, and if so, when to invest which is important since the payoff will be optimal at a particular moment. These aspects are essentially reactive flexibility: flexibility an option holder exploits to respond to environmental conditions and maximize his or her payoff.

When it comes to proactive flexibility, that is to influence the value of the option once it has been acquired, the characteristics of real and financial options differ. A financial option is acquired in a deep and transparent market while options on real assets are usually influenced by a limited number of players interacting with one another. Each of these players has the opportunity to influence the real option levers and hence the option values (Amram and Kulatilaka,1999).

## 4.3 The Real Options Theory Potential

The real option valuation method is most valuable when there are situations of high

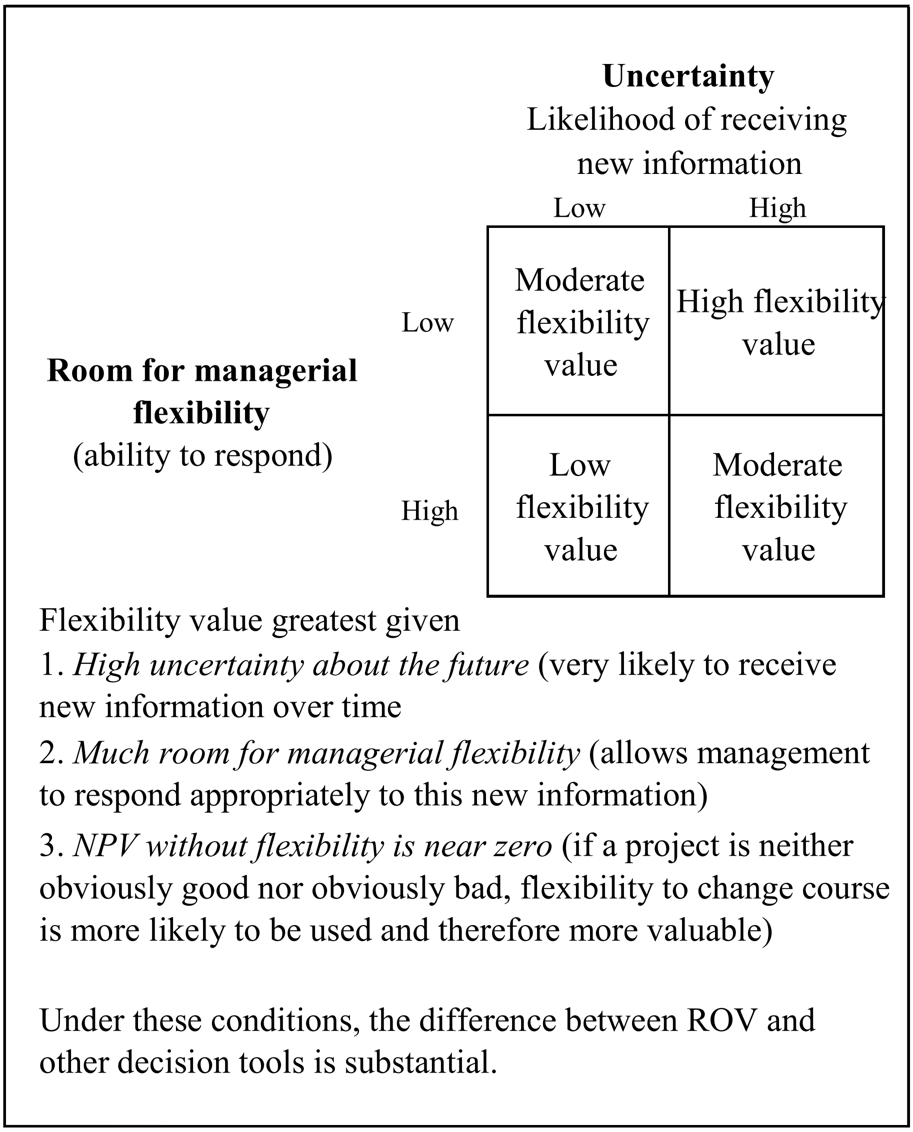


**Figure 4-1** When managerial flexibility is valuable. Source: Copeland and Keenan (1998)

uncertainty and management has flexibility to respond to new information. The value of the method is further enhanced when traditional methods like NPV are near the break-even point without the value of flexibility, Copeland and Keenan (1998). However, if the project already has a high NPV, real options adds little to the go-ahead decision because it is likely that the project will be undertaken and the value of flexibility will not be exercised. The same situation is true if the project has a strong negative NPV value so it is unlikely that any value of flexibility will be able to make up for projections of negative returns.

Mauboussin (1999) argues that real option theory adds most value to the investment decision when one or more of the following factors are present. Flexibility is valuable, future investment is contingent on the success of today's investment, the expected payoff from the investment is very volatile and the investment is largely irreversible.

Under conditions of high uncertainty managerial flexibility has great value, as new information is likely to influence the investment decision. Figure 4.1 summarizes the relationship between managerial flexibility and uncertainty.

### 4.4 Pitfalls of the real option approach

The major millstone for the real option approach so far has been that it has been considered relatively complex compared to the more traditional valuation methods. The criticism is aimed at the complexity of the mathematical tools needed for the real option evaluation. Although problems to the technical aspect of ROV can be overcome it might still be too complex to be worthwhile for minor decisions.

Another aspect is that it is not very useful for projects that require a full commitment right away, since much of the value of an option lies in the ability to spend a little now and decide later whether to continue.

ROV has also received criticism aimed at the difficulties of identifying all options involved in the project and framing the model to fit a particular problem. When the problem under consideration consists of only one single option the valuation does not pose any major complications. However for most applications the project usually involves several options, which complicates matters considerably. The complication arises when several options are present which at the same time might interact and change not only the value of the project but also the critical boundaries at which exercise of each option becomes optimal. In some cases, the presence of one real option may complement the value of another; while in other cases the option values may be substitutes (Kulatilaka, 1999)

Amram and Kulatilaka (1999) mention some common errors made when identifying all options. These are not understanding the exposure; using quick-fixed solutions too value complex options and paying to much attention to private risk and too little attention to bundles of market price risks.

However, even thought there may be problems with arriving at a precise number when valuing flexibility with the real options approach the analysis itself can provide valuable information. A real option analysis, frames the different risk and opportunities of the project and it is the insights from that which might be valuable.

## 4.5 When should either method be used?

The real option approach may not always be the optimal method when evaluating projects. There are circumstances when the traditional methods are a more practical choice though ROV in combination with NPV should produce basically the same results but the extra effort might not be worthwhile. In some situations the choice of method may be clear and in some cases not.

Option valuation is a good choice if the analysis is intended to estimate the market value of the project or decision and if the underlying asset value and foregone earnings can be estimated correctly. Dynamic DCF method is a good alternative if foregone earnings cannot be estimated in a meaningful way but market valuation remains the goal and risk remains relatively constant. Clearly, traditional tools work better when there are no options present at all, or when there is very little uncertainty in the project. However, ROV is needed when uncertainty is high. More specifically, when uncertainty is so high that it might be sensible to wait for more information before the investment is made or when uncertainty is high enough to make flexibility (switch use, scale up or down etc.) a consideration. Further, an option approach is helpful in valuing contingent investment decisions and future growth options (Amram and Kulatilaka, 1999).

## 4.6 Types of Real options

Copeland and Keenan (1998) classify individual real options into three main categories: growth options, deferral/learning options, and abandonment options. We describe these three main groups and the basic individual options each group contains below[10].

4.6.1 Invest/growth options

Growth options are options on investments that create additional growth in standard business situations, such as investments in advertising and improved customer service. Investments in R&D also contain growth options because they cerate a platform of knowledge for future products (Amram and Kulatilaka, 1999).

*Scale up*
Early entrants have an opportunity to scale up their operations later through cost-effective sequential investments as market grows.

This is where the initial investments scale up to future value-creating opportunities. Scale-up options require some prerequisite investments. For example, a distribution company may have valuable scale-up options if the served market grows (Mauboussin, 1999).

*Switch up*
Speedy commitment to first generation of product or technology can give the company preferential position to switch to next generation.

The option to switch (up or down) refers to the feasibility of choosing among alternative operating modes – for example, switching among alternative energy sources in the case of a chemical plant, or switching production among various locations internationally for a multinational (Micalizzi and Trigeorgis 1999).

[10] Appendix III presents an summary of the different types of options and lists examples of industries where they proved to be important.

The ability to choose between families of different sized aircraft, manufactured on the same production line, at option exercise, can be termed a switching option (Stonier 1999).

A switch, or flexibility, option values an opportunity to switch products, process, or plants given a shift in the underlying price or demand of inputs or outputs. One example is a utility company that has the choice between three boilers: natural gas, fuel oil, and dual-fuel. Although the dual fuel boiler may cost the most, it may be the most valuable, as it allows the company to always use the cheapest fuel (Mauboussin 1999).

*Scope up*
An investment in proprietary assets in one industry creates an option that can enable companies to enter another industry cost effectively.

This option values the opportunity to leverage an investment made in one industry into another, related industry. A company that dominates one sector of e-commerce and leverages that success into a neighbouring sector is exercising a scope-up option (Mauboussin 1999).

An example of a company that has exercised its scope-up options is Amazon. They have used their position in key markets to expand into similar businesses. Amazon used its market leading bookselling platform to move from selling books to selling music and from there on into films and video games. This is an example of a contingency option.

### 4.6.2 Options to defer/learn

The timing decision is much discussed in the option-pricing literature, that is, "when is the optimal time to invest and to exercise your option?" The timing decision is relevant if uncertainty can be resolved by waiting for or acquiring more information before deciding (thus deferring the decision).

*Study/start*

This option captures the value of waiting to invest until more information or skill is acquired. By delaying an investment, valuable information may be gained as uncertainty due to economic conditions unfolds and more knowledge becomes available. The traditional valuation methods treat most investments as "now or never" opportunities and therefore do not capture this value of waiting. NPV implicitly assumes that all information needed in order to maximize the allocation of capital is available at time zero.

Under uncertainty, it is important to consider the issue of investment timing. This now involves the cost (or value) of renouncing the option to defer a project's implementation until an optimal future moment, and of conditioning the investment decision with a favorable evolution of the state (or reference) variables. (Micalizzi and Trigeorgis, 1999)

The option to defer is important when evaluating investment decisions in industries where there is high uncertainty about output prices and market development. They have been used widely in for example natural-resource extraction industries, real-estate development, farming and the launch of new products.

This is a case where management has an opportunity to invest in a particular project, but can wait some period before investing. The ability to wait allows for a reduction in uncertainty, and can hence be valuable. For example, a real estate investor may acquire an option on a parcel of land and exercise it only if the contiguous area is developed (Mauboussin 1998).

Trigeorgis (1996) points out that in general, under exogenous competitive entry, management may find it justifiable to exercise relatively early under the following circumstances.

a) When its real option is shared with competitors and the anticipated loss in project value due to competitive entry is large and can be pre-empted,
b) When competitive pressure is intense,
c) When project uncertainty and interest rates are low
d) When the "competitive loss" pre-empted or the strategic benefit gained exceeds the "deferability value" sacrificed by early exercise.

### 4.6.3 Options to disinvest/shrink

*Scale down*
Shrink or shut down a project partway through if new information changes the expected payoffs.

Here, a company can shrink or downsize a project in midstream as new information changes the payoff scheme. An example would be an airline's option to abandon a non-profitable route (Mauboussin 1998).

*Switch down*
As with the option to switch up this option refers to the feasibility of choosing among alternative operating modes, that is to switch to more cost effective and flexible assets as new information is obtained.

*Scope down*

Limit the scope of (or abandon) operations when there is no further potential in a business opportunity.

A scope-down option is valuable when operations in a related industry can be limited or abandoned based on poor market conditions and some value salvaged. A conglomerate exiting a sector is an example (Mauboussin 1998).

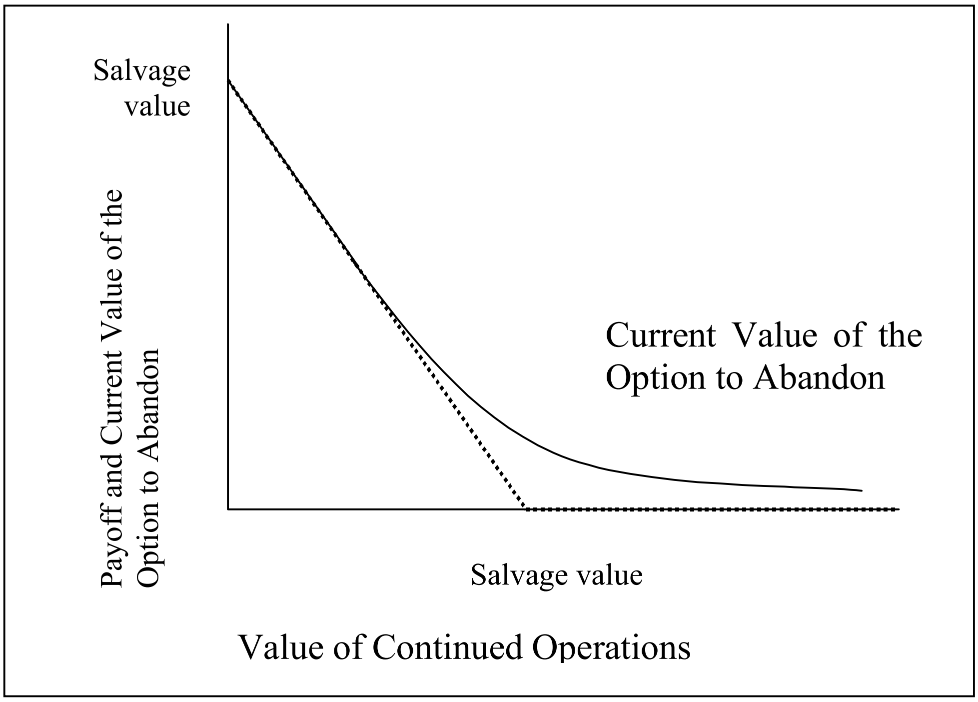


**Figure 4-2.** Option to Abandon. Source: Amram and Kulatilaka (1999)

The payoff from abandonment has its greatest value when the value of continued operations is zero. Uncertainty about the value of continued operations keeps the value of the option above its payoff in the area to the right of the salvage value.

## 4.7 Compound Options

Compound options are basically options on options. There are four main types of compound options: a call on a call, a call on a put, a put on a put and a put on a call. These options have two strike prices and two exercise dates. For example the owner of a compound call option has, at date $T_1$, the right to pay the first strike price, $X_1$, and receive a call option. This call option gives the owner the right to buy the underlying asset at the second exercise date $T_2$ paying the second strike price $X_2$. The compound option will be exercised at the first exercise date only if the value of the option at that date is greater than the first strike price (Hull, 1997).

Smit and Trigeorgis (1999) suggest that compound, or multistage, real options involve more pure growth-option value. Furthermore they (compound options) are better seen as a first link in a sequence of interrelated investment opportunities, the earlier of which are options to proceed to the next stage, but only if this appears to be beneficial. According to their examples of such options they are very often observed in "strategic investments", such as R&D projects, exploration drilling for oil and pilot projects. Such projects derive most of their value from the creation of follow on investments opportunities.

Trigeorgis (1996), when discussing real option compoundness distinguishes between multistage projects and project interdependence. In the first case, to which he refers to as "intra-project compoundness", the investment outlay is not viewed as a single one-time expenditure at the beginning of the project, but rather as a sequence of investment-cost "instalments" starting immediately and continuing throughout much of the project lifetime. In such a case an investment can be viewed as a compound option, where the initial investment-cost instalment represents the exercise price required to acquire an option to continue operating the project until the next instalment comes due and so on.

The second case which he calls "inter-project compoundness", concerns contingent or interdependent projects, where undertaking the first project is a prerequisite for the next or where the first project provides the opportunity to acquire, at maturity, the benefits of a new investment by making a new outlay. Compoundness between projects is an interaction of considerable strategic importance, since it may justify the undertaking of projects with negative NPV on the basis of opening up subsequent future investment opportunities or growth options.

## 4.8 Applying the Real Options Approach to capital investments

One of the first industries to apply the ROA to capital investments was the natural resources extraction industry, more specifically the oil and gas industry. The idea of investments as real options is clearly illustrated in the context of decisions to acquire and exploit deposits of natural resources.

Oil companies first applied the ROA when valuing license blocks (rights to explore and produce oil and gas). Consider an oil company that wants to value license blocks[11]. The company has the opportunity to acquire a five-year license on a block. When developed, the block is expected to yield 50 million barrels of oil. The current price of a barrel of oil from this field is $10 and the present value of the development cost is $600 million. Thus the NPV of the opportunity is simply:

*$500 million - $600 million = -$100 million.*

Faced with this valuation, the company would obviously pass up the opportunity. This is an example of what would happen if the manager of the oil company tried to value the undeveloped oil reserve using only the standard

[11] This example follows an example from Leslie and Michaels (1997)

NPV approach. Depending on the current price of oil, the expected rate of change of the price (ignored in the example) and the cost of developing the reserve, the company might construct a scenario for the timing of development and hence the timing and size of future cash flows from production. Because oil price uncertainty is not completely diversifiable, the greater the perceived volatility of oil prices the higher the discount rate used in DCF analysis. The higher the discount rates the lower the estimated value of the undeveloped reserve (Dixit and Pindyck, 1995)

This is a classic example of a real option, in which paying the license fee (acquiring the option to explore and produce) gives the owner the right to invest (at the exercise price) after uncertainty over the value of the developed reserves (stock price) is resolved. So what would option valuation make of the same case? To begin with, such a valuation would recognize the importance of uncertainty, which the NPV analysis effectively assumes away. There are two major sources of uncertainty affecting the value of the block: the quantity and the price of the oil. One can make a reasonable estimate of the quantity of the oil by analyzing historical exploration data in geologically similar areas. Similarly, historical data on the variability of oil prices is readily available.

Assume for the sake of argument that these two sources of uncertainty jointly result in a 30 percent standard deviation ($\sigma$) around the growth rate of the value of operating cash inflows. Holding the option also obliges one to incur the annual fixed costs of keeping the reserve active, for example, \$15 million. This represents a dividend-like payoff of 3 percent (ie, 15/500) of the value of the asset. We already know that the duration of the option, *t,* is five years and the risk-free rate, *r,* is 5 percent. To get the option value we plug these variables directly into the Black – Scholes formula, presented in section 3.3.3:

$$C = e^{-r(T-t)}\left[SN(d_1)e^{r(T-t)} - XN(d_2)\right]$$

$$d_1 = \frac{\ln(S/X) + (r + \frac{1}{2}\sigma^2)T}{\sigma\sqrt{T}}$$

$$d_2 = \frac{\ln(S/X) + (r - \frac{1}{2}\sigma^2)T}{\sigma\sqrt{T}} = d_1 - \sigma\sqrt{T}$$

This gives the option value of:

*ROV*[12] $= [(500e^{-0.03*5}) * (0.58)] - [(600e^{0.05*5}) * (0.32)]$

*= $251 million - $151 million = +$100 million.*

The difference between the two outcomes, $200 million represent the value of the flexibility inherent in not having to decide on full investment today, but instead being able to wait and invest when uncertainty is resolved.

[12] The ROV is derived by plugging the relevant variables directly into the Black-Scholes formula.

## 4.9 Comparing Real Options Theory and Traditional Methods

The decision approach for investments using the traditional discounted cash flow (DCF), relies in the net present value (NPV) rule: invest if NPV > 0; reject projects with NPV < 0; and for mutually exclusive projects, choose the higher NPV one.

These rules can result in wrong decisions, investing in projects where waiting is better, or not investing in good R&D (or others with high technical uncertainty or growth options) because the static NPV is negative, or even choosing large projects to the detriment of small ones, because higher NPV (high absolute NPV doesn't mean "deep in the money").

| Traditional Discounted Cash Flow (DCF) Rules | Real - Options |
|---|---|
| *Invest in all projects with NPV > 0* | *Invest when the project is "deep in the money"* |
| *Reject all projects with NPV < 0* | *Can recommend to start "Strategic Projects" (projects with technical uncertainty or growth options)* |
| *Among mutually exclusive projects, choose the one with higher NPV* | *Frequently chooses smaller projects, which are sufficiently "deep in the money" for their size* |

**Figure 4-3.** NPV Rules versus ROV. Source: Muralidhar (1992)

The real options approach rules: the investment opportunity needs to be "deep in the money", so NPV positive is not sufficient because there are probabilities that the prices will fall and the project would turn unprofitable. Waiting for better information is valuable and can prevent decisions mistakes. At a sufficiently high price ("critical price") it will be optimal to invest. At this critical value point the project value might need to be two or three times the investment value (not equal, as the traditional DCF rules).

### 4.10 A new way of thinking

In a way uncertainty can be seen as a factor that creates opportunities rather than decreasing their value. Managers should welcome, not fear uncertainty. In rethinking strategic investments, managers must try to view their markets in terms of the source, trend, and evolution of uncertainty; determine the degree of exposure for their investments (how external events translate into profits and losses); and then respond by positioning the investments to best take advantage of uncertainty (Amram and Kulatilaka, 1999).

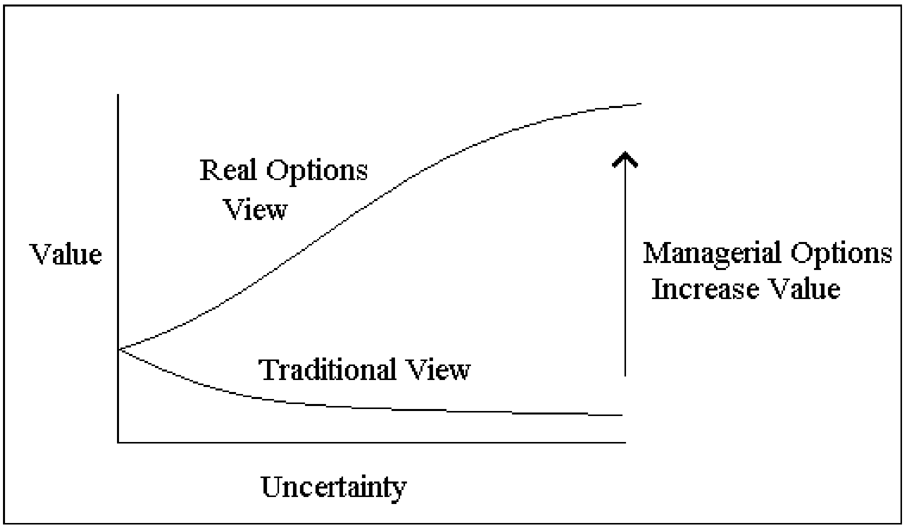


**Figure 4-4.** Project value and Uncertainty. Source: Amram and Kulatilaka (1999)

When applying traditional methods in asset valuation, a higher level of uncertainty leads to a lower asset value. The real options approach shows that increased uncertainty can lead to a higher asset value if managers identify and use their options to flexibly respond to unfolding events.

## 5 Real Options Evaluation

This section presents ways to frame and implement capital budgeting using the Real Options approach to Valuation, as well as the necessary variables used in the implementation of the Binomial and the Black and Scholes valuation techniques.

### 5.1 Fundamental Assumptions of the Real Options Valuation

Standard option valuation relies on four basic assumptions. First of all, the markets are considered to be frictionless. That means four things: (a) there are no transactions costs or taxes; (b) there are no restrictions on short sales, such as margin requirements, and full use of proceeds is allowed; (c) all shares of all securities are infinitely divisible; and (d) borrowing and lending are unrestricted. These assumptions allow continuous trading (Trigeorgis, 1996).

The second assumption made concerns the risk-free rate, which is presumed to be constant over the life of the option or known over time.

The third assumption concerns dividends. It is assumed that the underlying asset pays no dividends. This assumption can be relaxed with appropriate dividend adjustments.

The final assumption states that asset prices follow a stochastic diffusion Wiener process[13] of the form: $\frac{dA}{A} = adt + \sigma dz$.

In the discrete time case this diffusion process is replaced by a multiplicative binomial process or random walk which in the limit, as the trading interval

[13] Wiener process is presented in appendix II

gets smaller, becomes equivalent to the log-normal distribution underlying the process in the above equation (Trigeorgis 1998)

## 5.2 Input variables

In order to use the Black-Scholes equation or the binomial model to find the value of an option the relevant variables must be collected. These variables are the following:

S: present value of the underlying asset.
This is equal to the current value of the cash flows that the asset is expected to generate. These expected cash flows can be estimated by prognosis or by using a simulation model.

t: time to maturity
As in the case of financial options this is the time left to exercise the option before the right to do so disappears. In some cases this can be a fixed time period deriving, for example, from the ownership of a patent. After the expiration of the patent the firm loses the opportunity to gain a competitive advantage over the other firms (Dixit and Pindyck 1994). However, in other cases management has to make a subjective estimation of the life time of the option. For instance management may have to estimate the time it will take the competitors to exploit the same opportunity.

$\sigma$: volatility
By volatility we mean the variability of the return of the underlying asset. Volatility is a function of market-priced risk as well as private risk. There are several different approaches one can use for creating or judging estimates of volatility. First of all, one could take an educated guess. Assets to which a higher hurdle rate would be assigned (because of a higher than average

systematic risk[14]) are also likely to have a higher volatility. A good starting point would be to look at the returns on broad-based stock indexes. Building up from there, we can adjust for the higher σ that individual companies usually have from the market and for an even higher σ that individual projects have from the company as a whole (Luehrman 1998).

Another way to estimate volatility would be to gather some data. For some businesses we can estimate volatility using historical data on investment returns in the same or related industries. Alternatively, where this is not possible, another approach would be to use the prices of option contracts on the same underlying asset. The prices of these contracts are observed and can be used along with other option pricing inputs to solve for volatility. This estimate is known as the implied volatility (it is implied from the price of the option and the other inputs) and is viewed as the financial market's forecast of the volatility expected to prevail until the maturity date of the contracts (Amram and Kulatilaka 1999).

Luerhman (1998) suggests that volatility can also be estimated using Monte Carlo simulation techniques. These techniques together with a project's future cash flow and spreadsheet-based projections can be used to synthesise a probability distribution for the project's returns. Once we have the probability-synthesised distribution, the computer can quickly calculate the corresponding standard deviation. Another factor that might influence the estimates of uncertainty is private risk. The current level of private risk and the estimate of the range of uncertainty about that value are based on historical data, actuarial information, engineering estimates, and so on. The nature of data available and data desired about private risk varies tremendously across applications (Amram and Kulatilaka 1999).

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[14] Systematic risk is a part of total risk that affects a large number of assets and can be diversified away.

X: the cost of the investment to be made
The value of the investment is equivalent to the exercise price of a financial option. In reality this value might not be constant or known in the beginning of a project, however, in practice it not considered unreasonable to assume it to be certain (Trigeorgis, 1996).

r: the risk-free rate of return
This is the return on risk-free treasury instruments. The difference of real options approach with traditional valuation tools is that the short-term rate is used even for long-lived projects. In the real options approach, the risk-free rate is the return to the hedge position over a short time interval (Amram and Kulatilaka 1999).

## 5.3 Valuation using the Black-Scholes option pricing model

The Black-Scholes formulas for the valuation of financial options can also be used when valuing a real option.
The formula the model uses for the valuation of a call option is[15]: $C = N(d_1)S - N(d_2)Xe^{-rT}$ , where

- C is the current value of the call optio.,
- S is the current price of the underlying asset, or the present value of future cash inflows.
- σ is the volatility of future cash inflows .
- T is time to expiration of the option or the time until the investment opportunity disappears.
- r is the risk free interest rate.
- N (.) is the cumulative standard normal distribution function.

[15] Amram and Kulatilaka, 1996

In the Black-Scholes equation the values of $d_1$ and $d_2$ are as follows:

$d_1 = \left[\ln(S/X) + (r + 0.5\sigma^2)T\right] / \sigma\sqrt{T}$ and $d_2 = d_1 - \sigma\sqrt{T}$ .

In order to get a better understanding of the Black-Scholes breakthrough in Real Options Evaluation, it is helpful to go through an example. Trigeorgis (1996) presents a good example about valuing a pioneer venture where the growth option makes the difference. In this example he considers a type of high-tech project, which involves high initial costs and insufficient projected cash flows. The project's cash flows are represented in figure 5.1. The initial investment outlay is $I_0$ = \$500 million and the expected cash inflows over the 4 years are $C_1$ = \$100 million, $C_2$ = 200\$ million, $C_3$ = 300\$ million, and $C_4$ = 400\$ million. The management feels the need to prove the new technology in order to enhance the company's market position if that market should develop. Even if the pioneer venture itself does not appear profitable, valuable expertise and opportunity to enter a potential growth market may be lost to competitors if the investment is not made.

Investing in the initial project derives strategic value from the generation of growth opportunities to invest in future commercial projects. If the technology is proven, commercial production can be many times the size of the pioneer project. The follow up project (see figure 5.1) would become operational in year 4 and is assumed to be 3 times the size of the pioneer venture. The present value expected from the pioneer venture, discounted by 20% (k = 0.2) discount rate is V[16]$_0$ = \$444 million. Thus, the NPV is $V_0$-I = 444-500 = -\$56 million.

[16] Note in this example $V_0$ (present value of the operating assets) corresponds to S in the Black-Scholes equation.

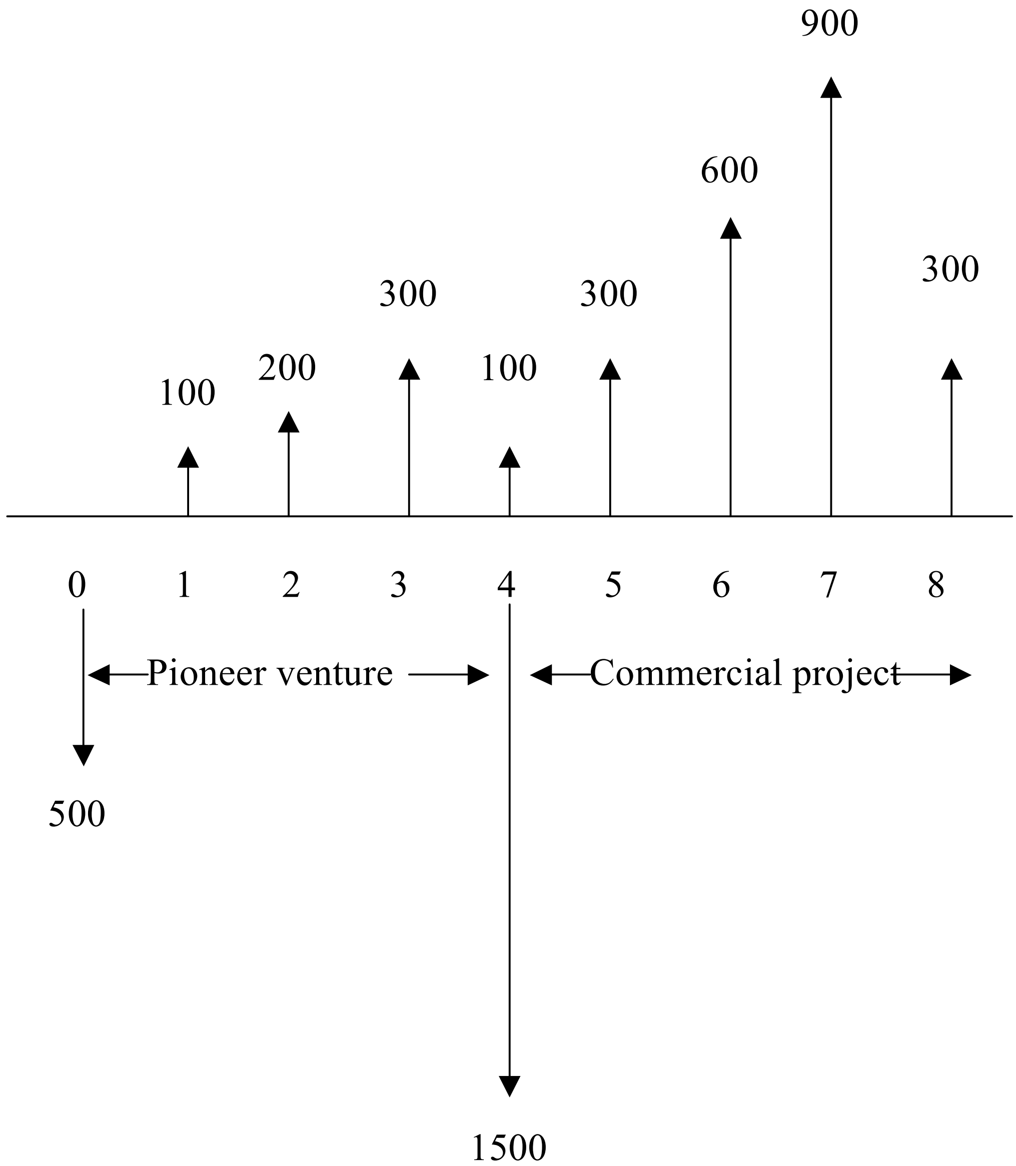


**Figure 5-1.** Capital outlays and expected inflows.

Looking only at the expected value of the pioneer project, investment opportunity does not look very attractive. The expected value of the follow up project does not look much better. It requires an outlay of $I_4 = \$1.5$ billion as of year 4 and it is only expected to generate a discounted value of subsequent cash inflows at that time $V_4$ = \$1.332 billion. The NPV at year 4 is -\$168 million, which amounts to an NPV of -\$81 million at time-0, after discounting for 3 years with 20%. The total expected loss in net value would amount to \$56 million + \$86 million = \$127 million.

However, the commercial project investment will be realised in year 4 only if the market is proven by that time and the project then appears profitable. Thus, investing in the negative-NPV pioneer venture is like incurring a cost to buy the option, giving the firm the right (with no obligation) to acquire the follow-up commercial venture. That option will be exercised at year 4 (an exercise cost of $X = I_4$ = \$1.5 will be incurred) only if the estimated value of the subsequent cash inflows at that time is sufficiently high. The -\$56 million NPV of the pioneer venture is the price that must be paid to acquire the growth option in the commercial project.

The more uncertain the potential of the technology or the future market demand, the higher the value of this option will be. Is the value of that option worth that cost?

| **Pioneer Project** | | **Follow on Project** | |
|---|---|---|---|
| Initial investment | \$500 million | Initial investment | \$1500 million |
| PV of inflows | \$444 million | PV of inflows | \$1332 million |
| NPV | -\$56 million | NPV | -\$81 million |

**Table 5-1**.NPV of initial and follow on projects.

The growth option represented by the right to invest in the commercial venture is like a European call option with time to maturity t = 4 years and exercise price X = \$1.5 billion. The underlying asset value is the current (time 0) value of a claim on the commercial project's expected future cash inflows. This can be obtained by discounting the time-4 value of the cash inflows (\$1332 billion) back to the present at the 20% discount rate, that is,

$$V_0=V_4e^{-kr}=1332e^{-0.20*4}=\$598.5 \text{ million.}$$

If we assume that the technology to be tested is quite uncertain, represented by a standard deviation $\sigma$ = 0.35, while the risk-free rate, r, is 10%. Given this information, we can now follow the short cut, practical procedure to obtain the Black-Scholes option values, which is discussed in detail in chapter 7.

$$\sigma\sqrt{\tau}=0.35*\sqrt{4}=0.7,$$

$$\frac{V_0}{Xe^{-r\tau}}=\frac{5985}{1500 \quad e^{-0.10\times 4}}=0.6$$

From appendix IV we can find that the value at the intersection 0.7 and 0.6 is 0.1185, or 11.85% of $V_0$. Thus, the value of the growth option to acquire the commercial project in year 4 if the market is proven by that time is currently worth 0.1185 * 598.5 = \$71 million. Therefore, the total strategic (or expanded) NPV is –56 + 71 = \$15 million. Management's intuition that it must invest in a pioneer venture for the strategic value of proving the new technology and positioning itself to take advantage of a future growth option is justified in this case, despite the negative NPV of its own direct cash flows.

## 5.4 Valuation using the Binomial model

As we have already seen in the Financial Options section of the paper, the binomial model is based on a simple representation of the evolution of the value of the underlying asset. At discrete points in time the present value of the asset can evolve to only one of two possible prices. These up or down movements lay out the possible paths. The asset has an initial value, *S* and within a short time period either moves up to *Su* or down to *Sd*. In the next period, the possible asset values are $Su^2$, $Sud$ or $Sd^2$. A step-by-step binomial option pricing formula makes it possible to value the project at every point in time. When the risk-neutral approach is applied to the binomial model, the expected return to the underlying asset is the risk-free rate of interest, r, but its volatility, $\sigma$, will be the same as that observed in the real economy. Using continuous compounding[17], the expected return during each period is: $\frac{pSu + (1-p)Sd}{S} = e^r$. The probability p weights the outcomes to obtain the risk-free rate of return and called risk-neutral probability (Amram and Kulatilaka 1999). In the same way equating the variance of the return from the binomial model to that of the observed normal distribution we get: $pu^2 + (1-p)d^2 - [pu + (1-p)d]^2 = \sigma^2$. Assuming that the underlying asset has symmetric up and down movements, one solution to the above equations would be:

$$u = e^{\sigma}; d = e^{-\sigma}$$

$$p = (e^r - d)/(u - d).$$

[17] Continuous compounding is the most general form of the binomial model (Amram and Kultilaka 1999).

The following example[18] is a simple application of vacant urban land valuation seen as an option to choose at a future date among different types for building construction. We will consider the choice between constructing a six-unit apartment building or a nine-unit apartment building. The optimal type of a building to be constructed in the next period is currently unknown and it will be determined by future real estate prices that are currently unknown. Committing to either type of building today might be suboptimal compared to waiting one more period and making the decision after additional information about market conditions has been revealed.

Assume the price, P, per unit is currently $100,000 and in the next period it will either rise to $P^+$ = $150,000 or, with an equal probability, it will decrease to $P^-$ = $90,000, in case the market moves favourably or unfavourably respectively. The construction cost now as well as next year is assumed to be $80,000 per unit for a six-unit building and $90,000 per unit for a nine-unit building. This gives an exercise cost of $480,000 and $810,000 in each case. The current risk-free rate is assumed to be $r = 0.10$.

The value of the vacant land is viewed as an option on the maximum of the values from the alternative building types. First we will consider the case where the land is developed immediately. The NPV at time 0 from the future cash flows would be $NPV_0 = nP - C$, where n is the number of units and C the total cost of construction is C.  For a six-unit building the net present value would be $NPV_{n=6}$ = 6 * $100,000 - $480,000 = $120,000, whereas for a nine-unit building it would be $NPV_{n=9}$= 9 * $100,000 - $810,000 = $90,000. The land, if the construction begins immediately, will be worth $120,000, which is the maximum value given the two different types of buildings considered.

[18] The example is based on "Real Options. Managerial flexibility and strategy in recourse allocation" pp.347-348 by L. Trigeorgis.

In the case where the construction is delayed for one year, the NPV for a six-unit building will be $NPV_{(1,\ n=6)}^{+}$= 6 * $100,000 - $480,000 = $420,000, in the case that market conditions move favourably and $NPV_{(1,\ n=6)}^{-}$ = $6 * $90,000 - $810,000 = $60,000 if the conditions are unfavourable. Similarly, for a nine-unit building the NPV will be $NPV_{(1,\ n=9)}^{+}$ = 9 * $150,000 -$810,000 = $540,000, in an upward price movement and $NPV_{(1,\ n=9)}$- = 9 * $90,000 - $810,000 = 0 if the market moves down. Since the vacant land provides the option to choose the building type next year after we have learnt which type is more appropriate; we will select the one with the highest value at that time. That means we select to build the nine-unit building worth $V^{+}$ = $540,000 if the market moves up and the six-unit worth $V^{-}$ = $60,000 if the market moves down. $p = \frac{(1+r)P - P^{-}}{P^{+} - P^{-}} = \frac{1.10 * 100 - 90}{150 - 90} = \frac{1}{3}$, if the market moves up and 1-p = 2/3 if the market moves down. The current value of the vacant land seen as an option must then be:

$$V = \frac{pV^{+} + (1-p)V^{-}}{1+r} = \frac{\frac{1}{3}(540) + \frac{2}{3}60}{1.10} = \$200{,}000 .$$

# Part II – Case Study

# 6 Case Introduction

The purpose of this case study is to present a simple framework for applying the real options approach to capital budgeting. We will value a project for Gothenburg Energy, which is considering investing in a district cooling system.

Part II of the paper is organized as follows. Chapter 6 gives a general introduction to the project and explains some of the properties of the technology involved. Chapter 7 presents the framework applied in the analysis. Chapter 8 contains the numerical analyses and presents some of the results. Chapter 9 examines robustness of the results to different assumptions about the evolution of the project value. We use sensitivity analysis to study the impact of changes in input variables such as time and volatility. Finally, chapter 10 contains our conclusions.

## 6.1 District Cooling

The principle of district cooling is similar to that of district heating: cold water is produced in a large central plant and distributed through pipes to customers. District cooling is used primarily by offices and shops, although also for the cooling of various industrial processes.

The market for district cooling in Sweden has expanded rapidly since it was introduced in 1992. This expansion has been fuelled by such factors as new building regulations, the greater use of computers, more awareness of the importance of good working conditions, a relatively extensive expansion of the district-heating system and the entry of new suppliers to the market. Demand is expected to continue to increase, in response to greater pressure for comfort cooling and the replacement of existing individual refrigeration/air conditioning plants by more environmentally sound alternatives (Energy in Sweden, 1999).

The most common method is to use either a so-called compressor machine or an absorption machine; the third alternative would be to use a combination of these two. The machine is usually placed centrally in an area and a closed network is then built to connect the customers to the cooling generator.

The compressor machine, which is powered by electricity, is more common today. It has some advantages over the other alternative, as it is relatively small and flexible. However, it has some major disadvantages. The most serious one is that it uses cooling substances, which damage the ozone layer. It is also quite noisy and causes vibrations.

The absorption machine, which is powered by hot water from district heating, has the advantage that it does not make use of materials that are believed to damage the ozone layer. Further, it is less noisy and does not cause vibrations.

A new environmental regulation will take affect in the beginning of the year 2002. This regulation prohibits the refilling of cooling substances that contain the chemical HCFC. The reason for this is that this chemical contains Chloride, which has a negative effect on the ozone layer. The HCFC is the dominating substance used in compressor machines to day. When the new regulation takes effect these machines will have to go through costly reconstructions to be able to use more environmental friendly cooling substances.

## 6.2 Objective and policy

GE is considering building a district-cooling network centrally located in Gothenburg. At present, GE already has 10 customers, to which they provide cooling using individual production units. This is locally produced cooling for each costumer, which means that the district-cooling network they are considering building will be the first one of its kind for GE. One advantage ofbuilding a district-cooling network, opposed to building individual facilities at each location, is that it may result in economies of scale. GE expects the cost of building one large cooling plant and a network to be lower than building several smaller plants locally.

Today most of the buildings in the district have compressor machines that have to be adjusted or go through a costly reconstruction due to the new regulation. This puts some time pressure on the project, as these potential customers will have to come up with an alternative solution. In order for GE to be able to provide this solution through a district-cooling network, a decision has to be made in the near future, about whether to go ahead with the project or not. If the project will be approved construction is expected to start early year 2001.

GE's policy is to provide the most environmental friendly and long-term sustainable energy solutions. District cooling is believed to be the most environmental friendly cooling system available today. This is because the energy that is used comes mainly from waste heat instead of electricity. GE believes this is a more effective use of energy recourses and therefore aligns with their policy.

## 6.3 Structure of the project

The cooling generation will consist of a combination of a compressor machine, absorption machine and free cooling produced by use of outside air. The purpose of this combination is to take advantage of the ability to switch between inputs and thereby always be using the lowest cost input at each time (heat, electricity or cold air). The optimal time to switch between inputs depends on the outside temperature. Based on statistics of historical temperatures GE has estimated that the absorption machine and free cooling will each be used approximately 45% of total usage time and compressor machine will be used 10% of total usage time.

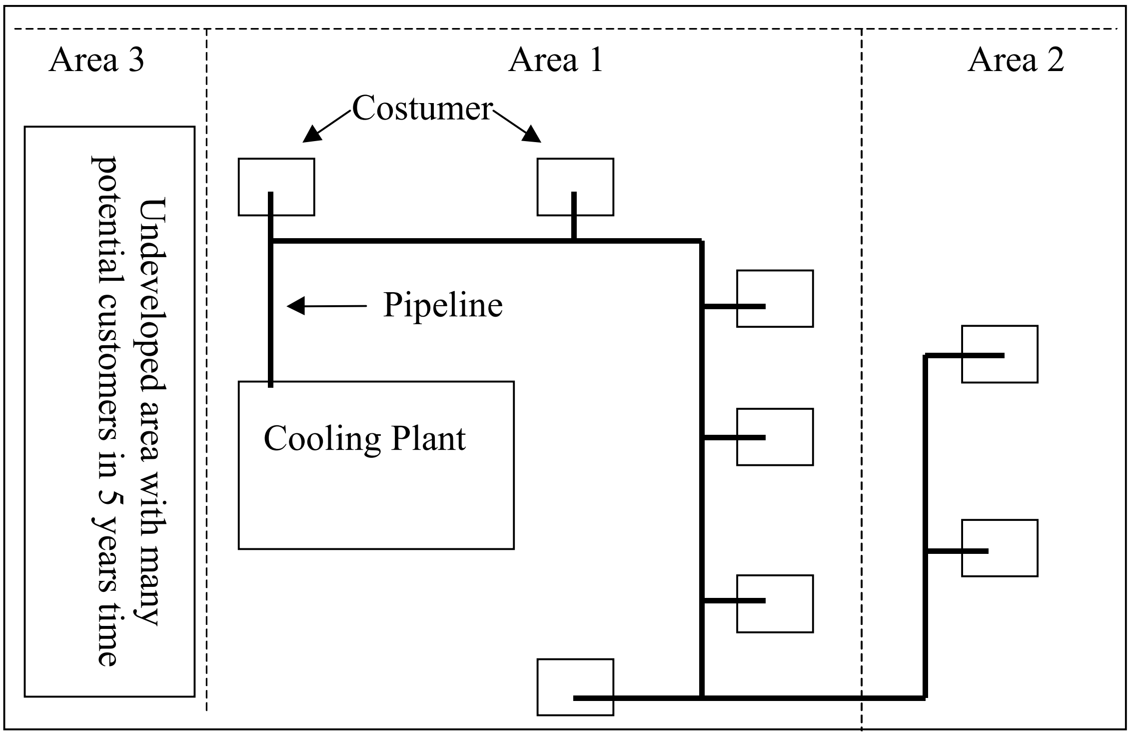


**Figure 6-1**. Possible layouts of the district cooling project.

Figure 6.1 is a simplified layout of the district-cooling project. The map is divided into three areas. Currently GE is considering three investment scenarios, depending on the scope of the project. Each of these areas represents different scenarios and possible layouts for the project. Area 1 is the main area on which the basic project evaluation is based. The first scenario would only include production capacity for area 1 with very limited possibilities for expansion. Option two assumes a larger network, which would be able to serve more customers, and hence has higher growth potential. This option is represented by area 1 and area 2.

Area 2 includes more potential custumers and the network could be built to be able to cover these custumers as well. However, to expand the network to cover these custumers, higher capacity pipes have to be used for the whole network in order to cope with the increased water pressure. These pipes are more expensive and require more effort to be put in place compared to the lower capacity pipes. The lower capacity pipes can be put in place by drilling but the higher capacity pipes will have to be dug down with the resulting extra costs.

Scenario 3 would be the best-case scenario, and would include area, one and three which is an undeveloped area with many potential customers. Area 3 is rather close to the location where the cooling plant is to be built. The plant can be expanded to be able to cover area 3 as well as area 1. Higher capacity pipes for the whole network are not required to cover area 3, because new pipes would be installed as an extension directly from the cooling plant rather than as an extension of the network itself.

## 6.4 What options are embedded in the project

It takes practice to recognize the options that may be buried in conventional projects. However, there are at least two points of departure that are useful when locating real options in projects. The first is simply to look beyond the numbers and examine the project's description for large discretionary expenditures. The other is to examine the pattern of the project cash flows over time and determine if the company can choose not to make the investment involved depending on how things look when the time comes.

It is important to identify the most valuable options embedded in the project since the majority of the options that theoretically could exist in a project, most likely have limited or no effect on the valuation. It is therefore of great interest for the option analysis that the most important options are identified at an early stage.

### Option to defer (option to learn)

As the situation is today there is little or no ability to defer the investment due to the environmental regulations discussed above. However, if these regulations did not exist there could potentially be some value in deferring the investment in order to resolve uncertainty. In the analysis that follows we will evaluate both scenarios, first with the regulation and second we will add the assumption that the investment can be delayed for a period of time, in order to demonstrate the value of flexibility when a deferral option is present.

## Invest/Growth options

*Scope Up options*

A growth option that is embedded in the project is that it offers GE the opportunity to sell their new clients other products they have to offer. GE aims at providing specially adapted, comprehensive solutions where they offer to take responsibility for lighting, indoor climate control, production security, ventilation and broadband solutions. Unfortunately we were not able to gather enough data in time to be able to value this option.

*Switch Up options*

As mentioned in section 4.4.1, speedy commitment into first generations of product or technology gives the company a preferential position to switch to the next generation technology. For instance, GE follows closely the advances in development of cooling generators and an extensive investment in district cooling at any time will give a preferential position to take advantage of any major advances or breakthroughs in cooling generation technology. However, at this point in time, taking GE market position into consideration it is unlikely that this option will affect the investment decision in any great way.

*Scale Up options*

If this project is successful it could open up opportunities to launch a follow on project in the future, which could be to build similar district cooling centers in other parts of the city. This growth option is evaluated in Scenario III which includes area 1  and the option to expand into area 3 in a five years time.

By just looking at the project description is quite obvious that the opportunity to expand the initial investment into area 2 is an obvious growth option embedded in the project. However, to realize this option the investment in the distribution network in area 1 has to be able to handle considerably more pressure, which is much more expensive. This extra cost could be seen as an option premium or cost of the option to expand the initial investment into area 2. We believe this to be one of the most important aspects the management has to consider in this particular project. Unfortunately we were not able to gather the relevant information and data to be able to value this option reasonably.

**Options to disinvest/shrink**

As mentioned in section 4.2.3 this category includes option to scale down, scale switch and scope down. Due to the fact that GE makes long term contracts with their customers they have limited flexibility to reduce their services during the contract period. However, disinvestments or even selling off the whole division can be done in stages at the end of the contract period, if economic conditions so demand. This option is, however, not included in the analysis that follows.

# 7 The solution (valuation) framework

This section presents the basic investment decision problem and explains the procedure used for valuing the project.

## 7.1 Assumptions

The framework builds up on a solution framework introduced by Timothy A. Luehrman (1994, 1998). The basic idea behind the method is to reduce the relevant variables that need to be evaluated from five to two basic variables. This is done in order to simplify the application and emphasize that RO can be applied to support and improve the basic cash flow analysis, not replace it.

### 7.1.1 Volatility

As there is limited public information about the volatility of returns from companies in the district-cooling sector, the volatility used in the analysis is based on estimates from the project managers involved. They base their assessments on the predicted demand for district cooling which depends on factors such as the price of alternative cooling sources and changes in outside temperature among other things. Another important factor is the relative risk of the project compared to other projects the company is involved with. The district cooling business is a new field for GE and at this time, predicted demand is very uncertain. Based on these factors this project is considered relatively riskier than other projects in the company and therefore has higher volatility. All things considered, 50% was considered a fair estimate of the volatility of the project returns.

As described in section 5.2 about volatility, there are several other methods to estimate the volatility of returns. As the main purpose of the paper is to present an option valuation framework and illustrate how it can be implemented, rather than to derive complicated measures of volatility, we settle for the estimation described above. Further, the sensitivity analysis in chapter 9, demonstrates the sensitivity of the results to changes in volatility.

### 7.1.2 Risk neutrality

Risk neutral valuation will be applied, as it was introduced in the financial options chapter, when calculating the values of the options embedded in the project for each scenario. In effect, this means that the assets are considered to be frictionless and the markets complete. These assumptions are needed since they are of fundamental importance when applying the Black-Scholes model, which we are using when calculating the option values.

### 7.1.3 Data from Gothenburg Energy AB

Most of the data used in the case study was received from GE and it is assumed to be correct. However, in certain parts of the scenario analysis we had to rely on assumptions made after discussing the issue first with the company's managers that are in charge of the project. The assumptions concern the growth option scenario we analyze and the cash flow related to it. The specific cash flows for the second stage of the scenario were derived from the first phase after taking into consideration relevant factors. Furthermore, the discount rate we use is the discount rate used by the company today.

### 7.1.4 Economic uncertainty movement

The economic uncertainty is assumed to influence the present value of the project and thus make it follow a geometric Brownian motion, as it was introduced in the section about the ROV fundamental assumptions. The underlying movement of the project's cash flow value is described by the following formula

$$\frac{dA}{A} = \alpha dt + \sigma dz \text{ , } dA = \alpha A dt + \sigma A dz$$

where A is the gross present value of the project, $\alpha$ is the instantaneous expected return on the asset, $\sigma$ is the constant instantaneous standard deviation of asset returns and $dz$ is the differential of a standard Wiener process. The reason that the Brownian motion has been chosen is that it is a prerequisite in the Black-Scholes valuation model and it is a very widely used model.

### 7.1.5 Production limits

We assume that there are no production limits in the scenarios we examine. This means that the company is able to satisfy any demand that may occur in the future. This assumption is made in order for the distribution of the present value of the project to be able to follow the geometric Brownian motion.

### 7.1.6 Deferral option and Growth option

The underlying asset of both options evaluated is assumed to follow the geometric Brownian motion as described above. Again, this is a prerequisite when we use the Black-Scholes valuation model to calculate the option values.

For the expansion project the economic uncertainty is assumed to be the same as for the initial project. This is a realistic assumption since the expansion project is of the exact same nature to the initial project and hence, has approximately the same underlying movement as the first phase

#### 7.1.7 Investments costs

The investment costs are assumed to be certain. If this were not the case, it would the same as assuming that the options embedded in the project have an uncertain exercise price making the calculation of their value very complicated.

## 7.2 Linking NPV and Option Value

As described previously in the paper, NPV is the difference between how much the operating assets are worth and how much it costs to acquire them.

NPV = (present value of assets to be acquired) – (required capital expenditures)

The decision rule was to reject all projects that have a negative NPV, because they do not add any value to the firm but they actually reduce it. If the NPV is positive and sufficiently large the decision is usually to go ahead with the project.

When the project has no strategic growth options or can no longer be deferred (the options embedded in the project have reached their expiration date) the real option valuation (ROV) and NPV yield the same result. At that time the option value is: max (S-X, 0), or in other words, either **ROV = S – X** or **ROV = 0** whichever is greater. But note that **NPV = S – X** as well, because S corresponds to the present value of the project assets and X to the required capital expenditure.

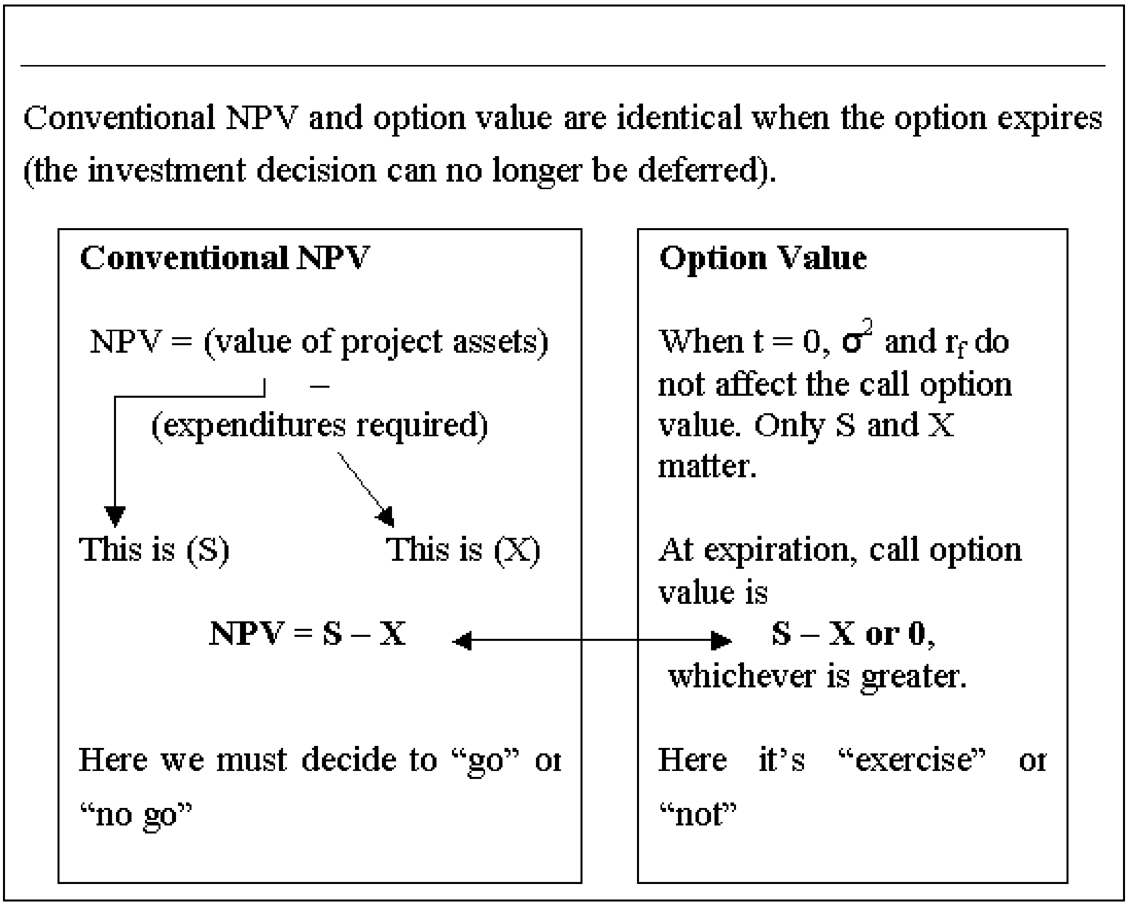


**Figure 7-1**. When Are Conventional NPV and Real Option Value Identical? Source: Luehrman 1997

Figure 7.1 explains how to reconcile the two methods. When the NPV of the project is negative, the management will, in most cases, decide not to invest, so the project value is effectively zero rather than negative. Equally, the call option value can never be less than zero, so ultimately both approaches arrive at the same conclusion.

It is this common ground between the two methods that is the main building block in the framework used in the case study. Spreadsheet programs set up to compute conventional NPV already contain the information necessary to compute S and X, which are two of the five option-pricing variables.

The two measures, NPV and ROV, diverge when there is an opportunity to defer the investment decision, whether it is the project as a whole or parts of the project when the investment can be done in stages. The possibility of deferral gives rise to two additional sources of value[19]. The first source is the time value of money, by delaying the investment it is always possible to earn at least the risk free interest rate on the deferred expenditure. Second, during the additional time new information might become available which, reduces uncertainty concerning the value of the operating assets.

Traditional cash flow analyses do not capture this added value derived from deferring and/or staging the investment. Real option analyses, however, provide a way to quantify this value and include it in the project evaluation. The framework used to value these factors is discussed in detail in the coming sections.

## 7.3 $NPV_q$

The simplest way to account for the time value of the required capital is to discount the necessary capital expenditures to the present time. In option notation, it's the present value of the exercise price, or

$$\mathbf{PV\ (X) = X / (1 + r_f)^t}$$

[19] This topic is discussed in more detail in section 4.4.2 of the paper.

where (t) is the number of time periods until the investment is to be made and ($r_f$) is the risk free rate of return. This approach is supported by both Luehrman (1994, 1998) and Trigeorgis (1996). The extra value is then the difference between (X) and PV (X).

We can now include this time value element into the NPV analyses by constructing a modified NPV measure; this is done by substituting PV (X) for (X). Thus:

**"Modified" NPV = S – PV (X)**

By definition, the modified NPV is greater than or equal to the regular NPV as it explicitly includes the interest rate element that can be earned in the respective periods (t).

This measure can be positive, negative or zero. To simplify the calculations it is convenient to express the relationship in such a way that the number can neither be negative nor zero. Instead of expressing the modified NPV as the difference between S and PV (X), it is advantageous to create a new metric: S divided by PV (X). By converting the difference to a ratio, all we are doing, essentially, is converting negative values to decimals between zero and one. This metric is called $NPV_q$ (Luehrman, 1997), where "q" indicates that the relationship between cost and value is expressed as a quotient;

$$\mathbf{NPV_q = S / PV (X)}$$

Note that the modified NPV and $NPV_q$ are not equivalent, that is, they do not yield the same numeric answer. However, we have not lost any information about the project by substituting one metric for another. When modified NPV is positive, $NPV_q$ will be greater than one; when NPV is negative, $NPV_q$ will be less than one and anytime modified NPV is zero, $NPV_q$ equals one. There is a perfect correspondence between the two measures as shown in the figure "Substituting NPVq for NPV".

| **Substituting $NPV_q$ for NPV** | | |
|---|---|---|
| $NPV < 0$ | → | $NPV_q < 1$ |
| $NPV = 0$ | → | $NPV_q = 1$ |
| $NPV > 0$ | → | $NPV_q > 1$ |

The difference between NPV and NPVq contains a useful managerial insight. As time runs out, these two must converge to some agreement: at expiration they will be either greater than 0 and 1, respectively, or less than these values. But prior to expiration, NPVq may be positive even when NPV is negative.

## 7.4 Uncertainty as a source of value

The second source of value mentioned in the beginning of the chapter was that while the project can be postponed new information that may affect the investment decision might become available. This factor is very important, but at the same time more difficult to value. First of all, it is uncertain that the asset value will change at all and more important, if it changes, will it increase or decrease.

One way to measure uncertainty is to assess the probability of different outcomes. As discussed previously, the most common probability weighted measure of dispersion is variance ($\sigma^2$). Another factor that has to be accounted for is the time element involved as these variables are closely connected together. In option terminology it is common to speak in terms of variance per period. That way the total amount of uncertainty is; variance per period times the number of periods or, $\sigma^2 t$.

This is sometimes called cumulative variance. An option expiring in two years has twice the cumulative variance as an otherwise identical option expiring in one year, given the same variance per period (Luehrman, 1997).

Like Luehrman (1994, 1997) and Trigeorgis (1996), we make two modifications on the measure for variance for mathematical convenience, without loosing any information. First, instead of using the variance of project values, we will use the variance of the project returns. That is instead of working with the actual currency value of the project, we convert it to percentage gained (or lost) per year. There is no loss of content because a project's return is completely determined by the project's value:

**return = (future value – present value) / present value**

The probability distribution of possible values is usually quite asymmetric; value can increase greatly but cannot drop below zero. Return, in contrast, can be positive or negative, sometimes symmetrically positive or negative, which makes their probability distribution easier to work with (Luehrman, 1997).

Instead of working with the variance it is more convenient to work with the standard deviation, which is simply the square root of the variance. This measure has the advantage of being denominated in the same units as the object being measured.

To summarize, the refinements to our measure of total uncertainty are the following. First, stipulate that $\sigma^2$ denotes the variance of returns per unit of time on the project. Second, multiply variance per period by the number of periods (t) to get cumulative variance ($\sigma^2 t$). Finally take the square root of cumulative variance to change units, expressing the metric as standard deviation rater than variance. We call this last quantity cumulative volatility ($\sigma\sqrt{t}$) to distinguish it from cumulative variance (Luehrman, 1997).

## 7.5 Valuing the option

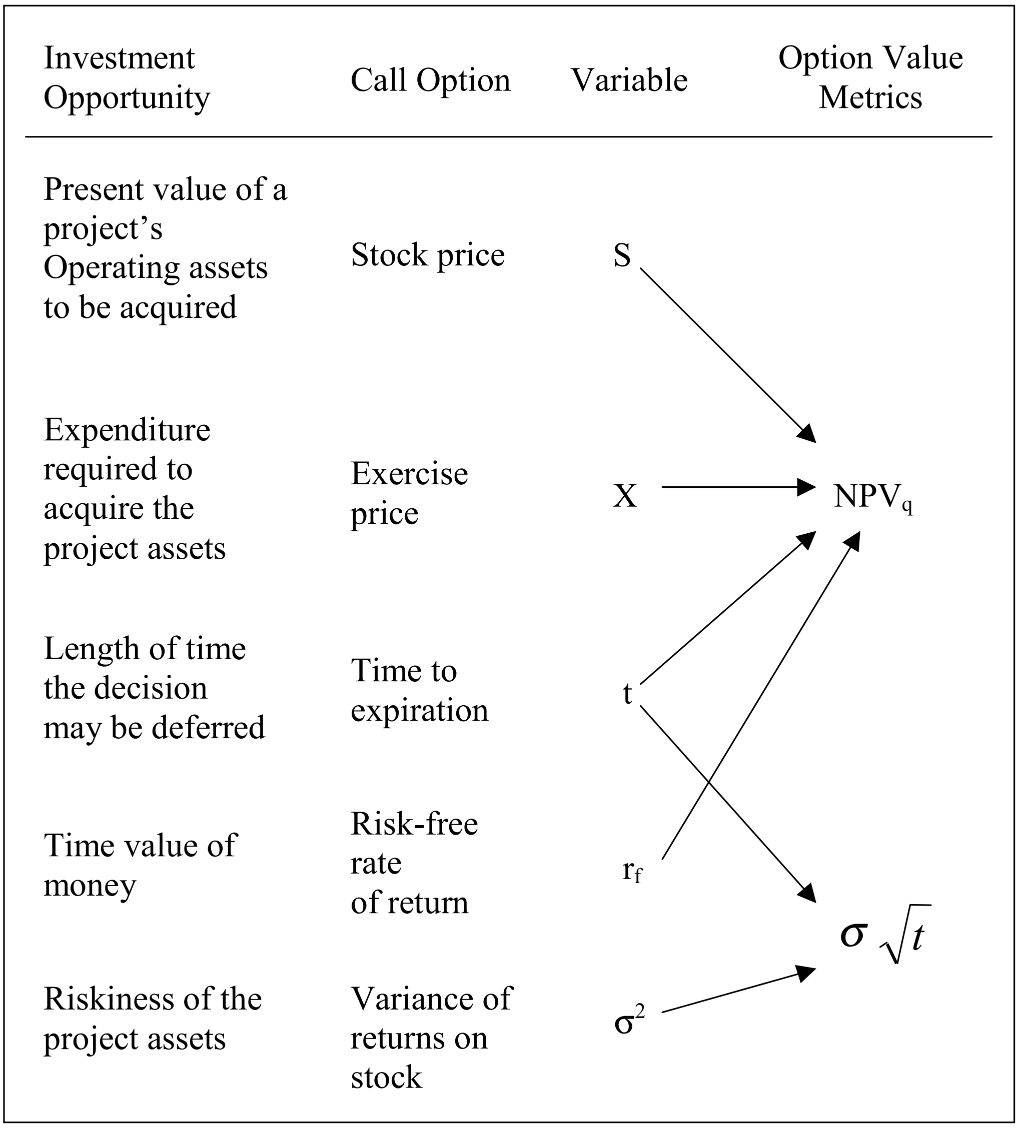


**Figure 7-2.** Combining the Black-Scholes variables to form the two option value metrics. Source: Luehrman 1997

All the five basic variables in the Black – Scholes model are accounted for and contained in the two measures ($\sigma\sqrt{t}$) and $NPV_q$, defined in the previous sections. The connection between the variables is depicted in figure 7.2.

The latter is actually a combination of four of the five option variables; S, X, $r_f$ and t. Cumulative volatility combines the fifth, σ, with t

Combining the variables this way has some major advantages. To begin with, it simplifies the whole process and makes it easier to grasp. The second advantage is that it enables us to draw up the solution in two-dimensional diagrams, which makes the interpretation of the results more intuitive.

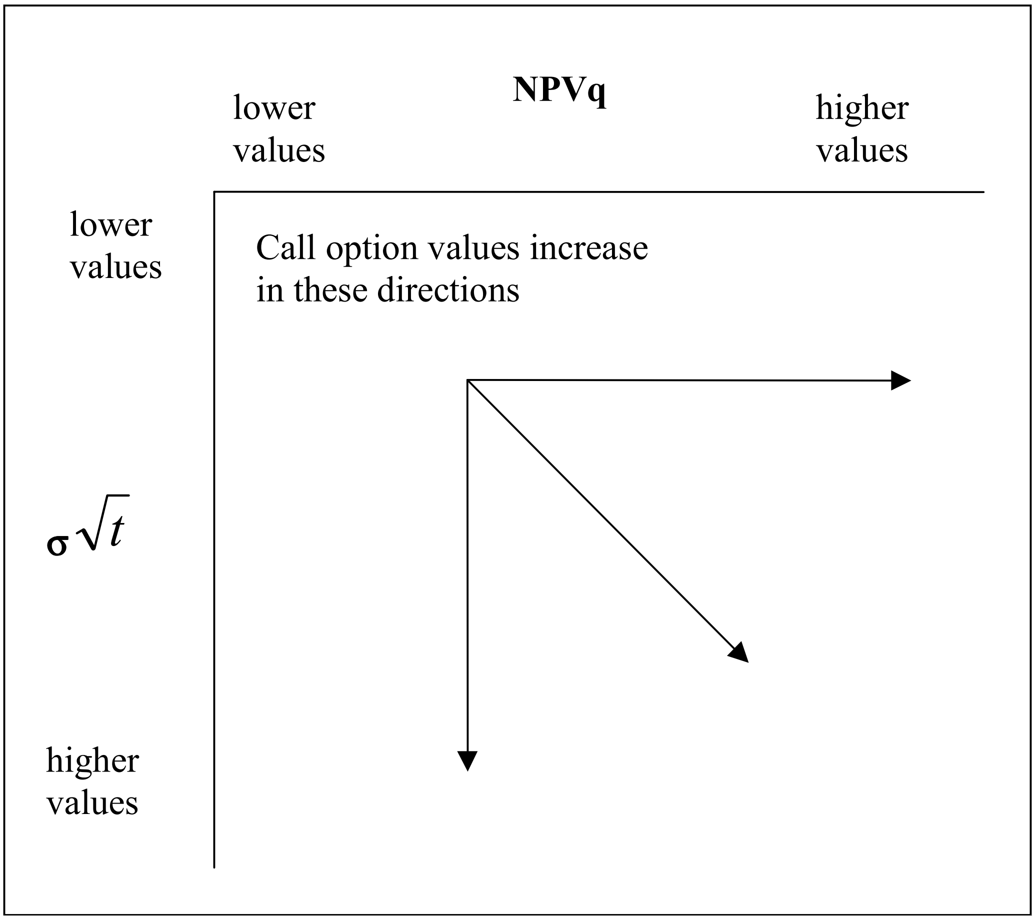


**Figure 7-3.** Locating the Option Value in Two-Dimensional Space.
Source: Luehrman (1994)

Figure 7-3 shows how to use $NPV_q$ and ($\sigma\sqrt{t}$) to obtain a value for the option. $NPV_q$ is on the horizontal axis, increasing from left to right. As $NPV_q$

increases, so does the value of the call option. Cumulative volatility is on the vertical axis of the graph, increasing from top to bottom. As ($\sigma\sqrt{t}$) increases, so does the call value.

To get an actual number for the option value, we fill in a table with Black-Scholes call values that correspond to every pair of NPVq and ($\sigma\sqrt{t}$) coordinates (this table is presented in appendix IV).

## 8 Numerical Solution

In this section we evaluate 3 different scenarios. The first scenario is based on the assumption that there are limited possibilities to expand in the future and further, the investment is a now or never decision, meaning that a decision has to made to go ahead now or not undertake the project at all. The purpose of evaluating this scenario is to demonstrate that when there are limited growth opportunities and the investment decision cannot be deferred, real option theory and the traditional NPV method yield basically the same result.

Scenario 2 is mostly based on the same project characteristics as in scenario 1, except we assume that the project can be delayed for at least two years. This is done to demonstrate that an option to defer the project for a period of time adds to the total project value.

In scenario 3 we assume the same characteristics as in scenario 1, except that we now add the assumption that the project can be expanded considerably in year 5. By adding this assumption we tend to demonstrate the growth option value embedded in the project.

### 8.1 Scenario 1

This is the base case scenario and it is based on calculations and assumptions received from GE (see section 6.4 for the project description). The other scenarios are extensions of this scenario and build on the assumptions made here.

In this scenario there is limited opportunity to expand the project to include new customers in the future. However, in year 5 the company has a moderate opportunity to expand its capacity about 10%. The additional investments required for this expansion are very small compared to the initial investments and therefore we expect this opportunity to have a relatively low option value compared to the initial investment, and hence yield a similar result as NPV calculations.

We divide the project into two phases, phase 1 and phase 2.

**Phase 1:** refers to the initial investment and the associated cash flows. It includes the investments in buildings, equipment and distribution network. This scenario refers to area 1 in figure 6.1.

We value phase 1 with NPV as usual, based on an estimated lifetime of 20 years. Note however, that the time period here is based on the estimated lifetime of the least durable capital investment (the actual distribution pipes) though most of the other capital has a longer estimated lifetime.

**Phase 2:** refers to the limited opportunity to expand, which may or may not be exploited in year 5. The extra investment is only expected to amount to 3 million SEK while it is expected to yield a one-time connection fee of 3,5 million SEK immediately after the additional investment. Additionally, the price, of that extra capacity output, to the end customer is higher than for the customers that have joined in the first stage of the project[20].

[20] The reason for the higher rate is that the new customers are not expected to require any adjustments to their internal systems in order to connect to the distribution network. Therefore they do not qualify for a subsidised rate the initial customers get for having to adjust their existing systems.

Viewed in option terminology we already have a fair idea what the outcome is going to be as this option to expand is considerably “deep in the money” and therefore very likely to be exercised[21].

As the option embedded in the project cannot be evaluated based on similar options traded on an exchange we need to create a synthetic option. To value phase 2, we will use the framework outlined above to synthesize a call option and value it.

The value of the underlying assets (S) will be the present value of the assets acquired when and if the company exercises the option to expand in phase 2. The exercise price (X) is expenditures required to acquire the phase 2 assets. The time to expiration (t) is five years according to Gothenburg Energy’s projections.

The five years risk free rate ($r_f$) is 4,77% (which is the interest on a five-year Swedish government bond).

We assume that volatility is 50% per year (see the section on estimating the volatility).

The cumulative volatility is therefore: $\sigma\sqrt{t} = 0,5 * \sqrt{5} = 1,12$

We begin by rearranging the DCF projections for two purposes: first to separate phase 1 from phase 2 and second, to isolate values for S and X. This procedure requires identifying what expenditures belong to each phase and what spending is considered discretionary versus non-discretionary.

[21] According to basic option pricing theory, an option that has a high intrinsic value today, is very likely to have a high intrinsic value at maturity (Hull, 1997).

To get the NPV value for phase 2 separately, we first calculate the total value of the project as if both phases will be carried out. We then value the project without the additional investment, that is, as phase 2 will not be executed. By deducting the value of phase one from the value of the entire project we arrive at the value of phase 2. Note that when we discount the two phases separately, we obtain the same NPV as before.

The next step is to establish a benchmark for phase 2's option value based on the rearranged DCF analysis. Phase 1 alone has a positive NPV of 14.378.000 SEK while phase 2's NPV is 2.237.000 SEK. NPV of the project as a whole is then 16.615.000 SEK[22].

Having reformulated the DCF spreadsheet, it is now possible to attach values to the option pricing variables S and X.  X is the amount the company will have to invest in net working capital and fixed assets (capital expenditures) in year 5, if it wants to proceed with the expansion, that is 3 million SEK. We then discount this number for five periods, using the risk-free rate.

We do not use the risk-adjusted corporate discount rate of 7% because it is almost certainly too high. Discretionary expenditures in phase II are rarely subject to the same operating and product market forces that make the project's cash flows risky[23].

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[22] The details behind these calculations are presented in an appendix, which is only available for Gothenburg Energy. Note however, that even thought this seems to be a quite high NPV it does not comply with the specified company policy of requiring at least 7% internal rate of return over a 10 year period. Based on the same cash flow analysis the 10-year IRR is only 3,9%.

[23] Construction costs, for example, may be uncertain but they are usually much more dependent on engineering factors, weather conditions, and contractors performance than on customers taste, competitive conditions, industry capacity utilisation, and such. Over-discounting future discretionary spending leads to an optimistically biased estimate of NPV. (Luehrman 1998)

$$PV(X) = \frac{X}{(1+r_f)^t} \Rightarrow PV(X) = \frac{3.000.000}{(1+0,0477)^5} = 2.376.000$$

S is the present value of the new phase 2 operating assets (discounted with the corporate discount rate of 7%) and it amounts to 4,376, million SEK. This is the DCF value now (at time zero) of the cash flows phase 2 assets are expected to generate from the fifth year to end of year 20.

The next step is to combine the five option pricing variable into our two option value metrics: $NPV_q$ and $\sigma\sqrt{t}$ . In this case:

$$NPV_q = \frac{S}{PV(X)} \quad \rightarrow \quad NPV_q = \frac{4.376}{2.376} = 1,841$$

Finally we look up the call value as a percentage of asset value in our Black-Scholes option-pricing table in appendix IV. According to the table the option value is approximately 0,597, or 59,7% of the estimated present value of phase II assets. To get an actual number, we multiply this number by S:

**0,597 * 4.376.000 SEK = 2.612.253 SEK**

The value of the entire project is then the sum of phase 1 and the value of the option.

NPV (entire proposal) = NPV (phase I assets) + call value (phase II assets)

NPV (entire proposal) = 14.378.000 SEK + 2.612.253 SEK = 16.990.253

This is approximately the same figure (16.615.000 SEK) we get by using the traditional NPV method. This result is not surprising as the additional investment in phase II is quite small compared to the initial investment, meaning that the additional investment has limited strategic value.

This result demonstrates that when there are limited strategic growth opportunities embedded in the project, ROV and NPV method yield approximately the same results.

## 8.2 Scenario 2

This scenario is based on the assumption that the investment decision can be deferred for two years. The assumptions are the same as in scenario 1 in all other aspects. We use the same framework as outlined above to value the deferral option in this scenario[24].

The underlying assumption here is that the company has secured the right to the building site and has to decide in two years time whether to go ahead with the project or not.

The value of the underlying assets (S) is now the present value of the assets from the total project, which was divided into two phases in scenario I.
According to the cash-flow analysis this amounts to S = 55.866.420, SEK.

The spending required in year 2 to obtain the assets associated with the project is X = 44.949.700 SEK. We then discount X for two years using the risk free rate. The two year risk free rate ($r_f$) is 4,15% (which is the interest on a two-year Swedish government bond).

$$PV(X) = \frac{X}{(1+r_f)^t} \Rightarrow PV(X) = \frac{44.949.700}{(1+0,0415)^2} = 43.158.664 \text{ SEK}$$

The next step is to derive the value for $NPV_q$:

$$NPV_q = \frac{S}{PV(X)} \quad \rightarrow \quad NPV_q = \frac{55.866.420}{43.158.664} = 1,294$$

As before we need to incorporate the estimated volatility into the calculations. We assume the same volatility as in scenario I, that is $\sigma$ = 50% per year. As stated in the introduction the time to maturity is two years, t = 2. Hence, the cumulative volatility is

$$\sigma\sqrt{t} = 0,5 * \sqrt{2} = 0,707$$

We have now derived both metrics necessary to find the option value from the table in Appendix IV. The value of the option, as percentage of the value of the required assets is approximatelly 39,26%.

The final step is to multiply the option value with S to get the numerical value of the investment opportunity:

ROV = 0,3926 * 55.866.420 SEK = 21.410.805 SEK

This number is considerably higher than the value the traditional NPV yielded (16.615.000 SEK), assuming that the company did not have the ability to defer the project for two years.

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[24] Note that we value the whole project as a European call option

The difference (between the ROV and NPV) of 4,796,805 SEK reflects the value of the flexibility the option to defer the investment adds to the project. By delaying the investment valuable information may be gained as uncertainty due to economic conditions unfolds and more knowledge becomes available. The basic NPV method treats the investment as “now or never” opportunity and therefore does not capture this value of waiting.

This result emphasizes that when investments are to a large part irreversible and economic environment is stochastic, the option value of maintaining flexibility is important.

## 8.3 Scenario 3

In this scenario we assume GE has an opportunity to expand into area 3 in year five (see figure 6.1). This is a comparatively large expansion, approximately a 50% increase of the initial capacity. As stated in the project description, area 3 is currently an undeveloped area but there are plans to build both large commercial and residential buildings in the zone in five years time.

The expansion requires considerable additional investments and expenditures. The opportunity to expand represents a classical growth option to the company; they have the opportunity, but not the obligation to expand their production and distribution capacity. As all projections assume that at least five years will pass until the area will be developed, this opportunity resembles a European call option rather than an American option, as there is no possibility of early exercise.

The expansion is based on the following assumptions:

- Production capacity of cooling is increased by 50%.

- The distribution network is expanded by 50% compared to the initial capacity.

- Due to the lack of better information, we assume that the associated costs are proportional to the expansion, that is costs related to the building, distribution network, cooling generators and customer connection pipes are estimated to be 50% of the initial costs.

- Additionally we assume a constant additional expansion cost of 3.000 SEK per kW of extra capacity, based on estimated costs from GE for the small expansion in scenario I.

- All other costs associated with the extra capacity are assumed to be proportional to corresponding initial costs and are included in the cash flow analysis.

- Because of the additional investments in year five, we extend the lifetime of the project by an additional five years[25]. The NPV analysis is therefore extended by five years compared to the other two scenarios.

We follow the same procedure as in scenario I, that is divide the project into phase I and phase II, based on the additional assumption. We rearrange the cash flows for two purposes, to separate phase I from phase II and isolate the values for X and S.

[25] The additional investments in the fifth year are quite substantial (50% of the initial investment). We therefore assume that the lifetime of the operating assets is extended by five years.

As before phase one refers to the initial investment and the associated cash flows, that is the initial investments in buildings, equipment and distribution network.  We value it with NPV over a 25-year period.

Now phase 2 refers to the opportunity to expand into area III, which may or may not be exploited in year 5. We will use the same framework as before to synthesise a comparable European style call option and value it.

The time to expiration is five years as stated above; it is five years until the area will be developed.

Again the five year risk free rate ($r_f$) is 4,77%.

Having separated phase I and II, we calculate the conventional cash flow NPV for each phase.

| NPV phase 1 | NPV phase 2 | NPV phase 1+2 |
|---|---|---|
| 20.768.686 SEK | -1.198.613 SEK | 19.570.073 SEK |

The table above shows that phase I has an NPV of 20,769 million SEK while phase II has a negative NPV of 1,199 million SEK. The NPV of the whole project is 19,57 million SEK.

The sum of the NPV of each phase separately equals the NPV of the entire project. The value of the whole proposal must be at least 20,769 million SEK because the option value of phase II, what ever it turns out to be, cannot be less than zero. In fact if the option value of the second phase turns out to be substantial, the value of the project will be considerably higher than 20,769 million SEK. The only way to realise this is by separating the project into two phases and conceiv that the company has an choice of whether to undertake the second phase of the project, or not.

The estimation of the standard deviation is 50% per year .

The cumulative volatility is therefore: $\sigma\sqrt{t}$ = 0,5 * $\sqrt{5}$ = **1,12**

After having reformulated the DCF analysis, we attach values to the option pricing variables X and S. As before these variables represent the required capital expenditures for phase 2 and the present value of phase II assets, respectively. This procedure follows the steps described in section 9.5.

$$\text{PV(X)} = \frac{X}{(1+r_f)^t} \Rightarrow \text{PV(X)} = \frac{37.474.874}{(1+4{,}77)^5} = 29.686.259$$

According to the rearanged cash flow analysis ,S (the present value of the new phase 2 operating assets) is 25.682.832 SEK. This is the DCF value now (at time zero) from the fifth year until the end of year 25.

We now combine the five option pricing variables into the two option value metrics: $\text{NPV}_q$ and $\sigma\sqrt{t}$ .

$$\text{NPV}_q = \frac{S}{PV(X)} \quad \rightarrow \quad \text{NPV}_q = \frac{25.682.832}{29.686.259} = \mathbf{0{,}865}$$

The corresponding option value from the table in appendix IV is 0,38[26]. In other words, this means that the option value is 38% of the present value of phase II assets (S). Accordingly the value of the option is

0,38 * 25.682.832 = 9.759.476 SEK.

Recall that the value of the entire project is given by:

NPV (entire proposal) = NPV (phase I assets) + call value (phase II assets)

or

**NPV (entire proposal) = 20.768.686 + 9.759.476 = 30.528.162 SEK**

This figure is considerably higher than the initial NPV value of the project of 19.570.073 SEK. Even though the option-pricing analysis relies on the same input variables as the NPV analysis the total project value is approximately 35% higher when calculated with ROV.

When this result is compared to the outcome of scenario I, where the ROV added little to the investment decision, it is clear that real option theory adds value to the investment decision when investment can be staged and the future investment is contingent on the success of today's investment. The fact that the expected payoff from the investment is relatively volatile, adds further to the option value of phase II.

[26] The table does not show values that correspond exactly to the computed values for the two metrics, but the value of the option can be reasonably approximated by the use of interpolation.

## 9 Sensitivity Analysis

In this section we examine robustness of the results to different assumptions about the evalution of the project value. We perform experiments (scenario analysis) in which we fix the investment rule and vary the project characteristics such as the project volatility and the time frame of the project.

As the analysis of scenario I showed that ROV and NPV yielded approximately the same results, sensitivity analysis is not expected to alter the results significantly. Hence, this section concentrates on the results from scenarios II and III.

### 9.1 Deferral option

The option to defer, or option to wait refers to the time the investment decision can be delayed without losing the investment opportunity. The result from the analysis of scenario II indicated that postponing the investment decision for two years had considerable real option value. The sensitivity analysis in this section will show the sensitivity of the option value to time and volatility.

In the numerical analysis the option to defer is basically valued as a European call option on the project, with an exercise price equal to the necessary investment outlays. More accurately this option should be valued as an American style call option using a binomial discrete time model. Valuing it as an American call gives the opportunity of an early exercise opposed to the European call option that can only be exercised at maturity. However, the framework applied above to value the option is useful to get a minimum value for the option, that is the option value is at least equal to the European call but probably worth even more. If the option were valued as an American call, the value would be equal to or greater than the European call option.

Figure 9.1[27] shows the sensitivity of the option value to time, when the variance is fixed at 50% and all other factors are kept constant. The analysis is made with a timeframe of half a year up to 4 years showing the value of the option depending on how long the investment decision can be deferred. The result shows that the value of the option increases with time, that is, the longer the investment can be delayed, the more valuable the option becomes.

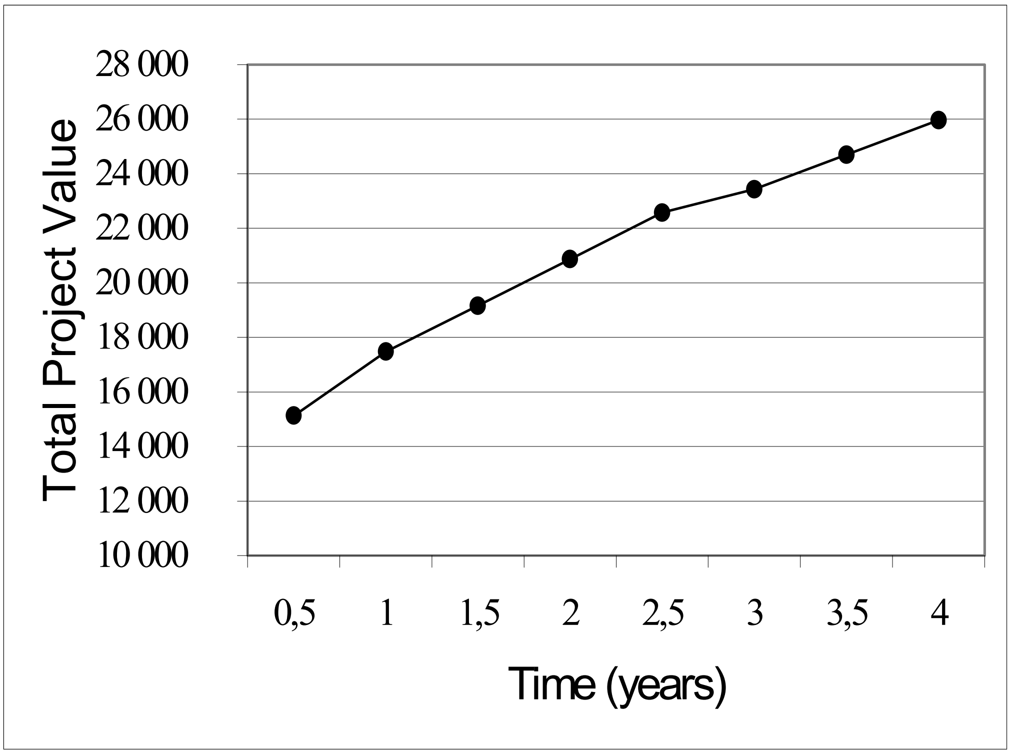


**Figure 9-1.** Sensitivity of total project value (with the option to wait) to time

[27] Note that in this, and the following graphs in this section, the values on the Y-axis are in thousands of SEK.

As discussed in section 3.2. a longer time to expiration always has a positive effect on a call option value. First of all it reduces the present value of the exercise price on maturity, if the option ends up in the money. Second, a longer time horizon gives potentially higher intrinsic values on maturity, since the volatility of the underlying assets grows with the square root of time.

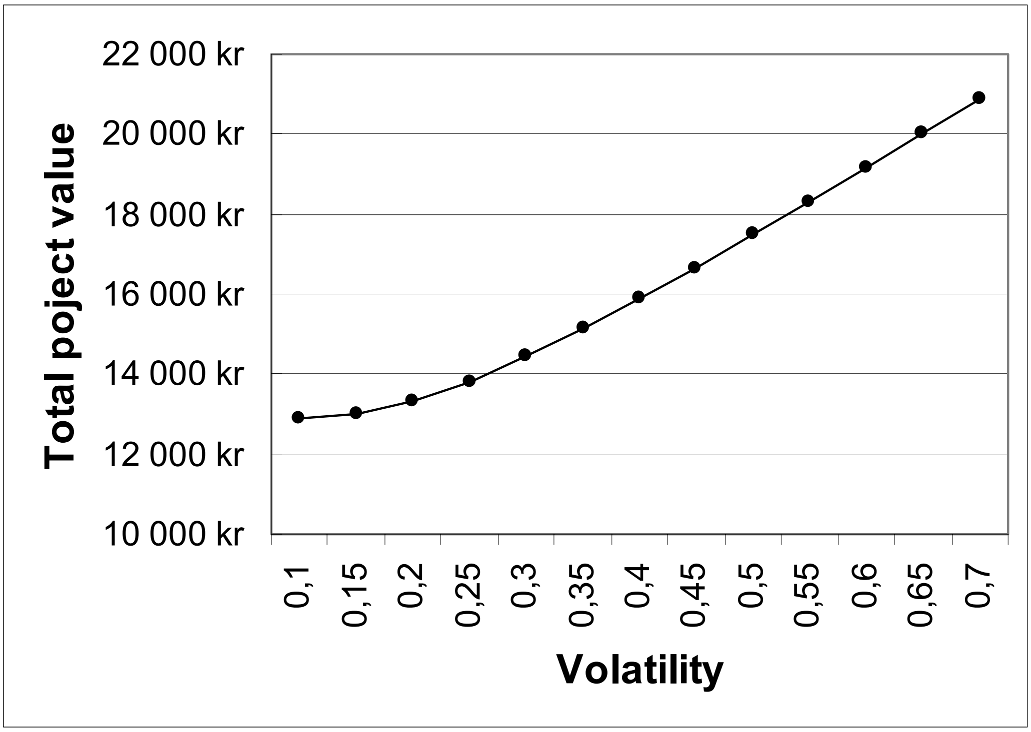


**Figure 9-2.** Sensitivity of the total project value to volatility

We also examined the sensitivity of the option value to changes in volatility of the expected returns of the investment. The time to expiration is fixed to two years and all other factors are constant. The results show, as depicted in figure 9.1, that the option value is quite sensitive to the volatility of returns. As the volatility increases above 20% the option value increases more rapidly, that is to say the slope of the line increases.

This result is aligned with basic option pricing theory, the greater the volatility of the underlying asset the more valuable a call option will be. The owner of a call option benefits from price increases in the underlying asset but has limited downside risk in the case of price decreases, since the most he can lose is the price of the option.

## 9.2 Option to expand

Scenario III assumes that GE has an option to expand its cooling production and distribution capacity in year 5. The underlying assumption was that the volatility of the expected returns was 50%. The analysis showed that opportunity embedded a real option value of approximately 9,8 million SEK. The static NPV analysis of the project, ignoring the option value, yielded a value of about 21 million SEK. Combining the NPV value and the option value gave a total project value of 30,5 million SEK. This section studies the sensitivity of the real option value to changes in volatility of expected returns when all other factors are kept constant.

Graph 9.3 shows the sensitivity of both the real option value and total project value to changes in volatility.

When the volatility increases, the value of the option to expand increases as well. As the total project value is just the sum of the projects static NPV and the option value, the total project value increases as well with increased volatility. The reason why the option value increases with higher volatility is that the company has the option, but not the obligation, to expand their operations. This results in a higher option value because higher volatility means higher upside potential while the downside risk is limited and constant. If high volatility results in favorable economic and market conditions the company exercises their option to expand, while if the volatility results in a

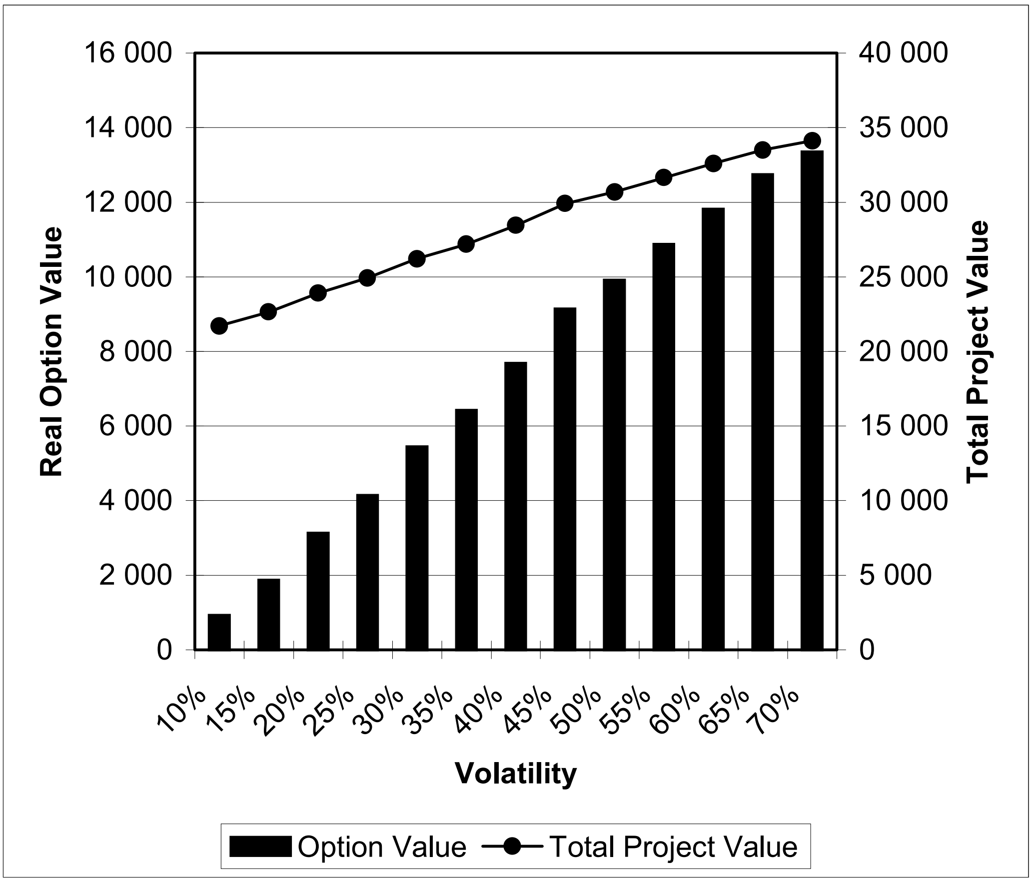


**Figure 9-3.** Sensitivity of Option and Project value to changes in volatility

unfavorable conditions, the company will decide not to expand and will therefore not experience any additional expenditures.

## 9.3 Summary

In the first scenario there is a limited opportunity to expand the project in order to include more customers in the same service area. Our calculations yielded approximately the same results when using the traditional NPV method and the modified (extended) NPV, which included value of the option to expand. This is because of the small size of the additional investment (compared to the initial investment) made in phase II. In effect that means that there is limited strategic value in the second phase investment and this value can be captured by the traditional NPV framework.

The second scenario is based on the assumption that the investment expenditures can be deferred for a period of two years. All other assumptions are the same as in scenario I. After valuing the investment as a call option, we found that there is an extra value of approximately 4.8 million SEK that is was not included in the project value when the calculations excluded the deferral option. This difference reflects the value of flexibility the option to wait adds to the project. By delaying the investment valuable information may be gained, as uncertainty due to economic conditions is resolved and more knowledge becomes available. The NPV method treats the investment as "now or never" opportunity and therefore do not capture this value of waiting.

In scenario III we value the opportunity that the company may have to expand into area 3 in the fifth year. We valued this opportunity as a classical growth option and we found that the conventional NPV grossly undervalued the project as the extended NPV we used to calculate the value of the project yielded considerably different results. The extended NPV yielded approximately 35% higher value than the basic NPV produced, even though the analysis in both cases relies on the same input variables. After comparing this result to the outcome from scenario I, it becomes clear that real option valuation is a more suitable method (which captures the additional value embedded in a project) when an investment can be staged and the future investment is contingent on the success of today's investment.

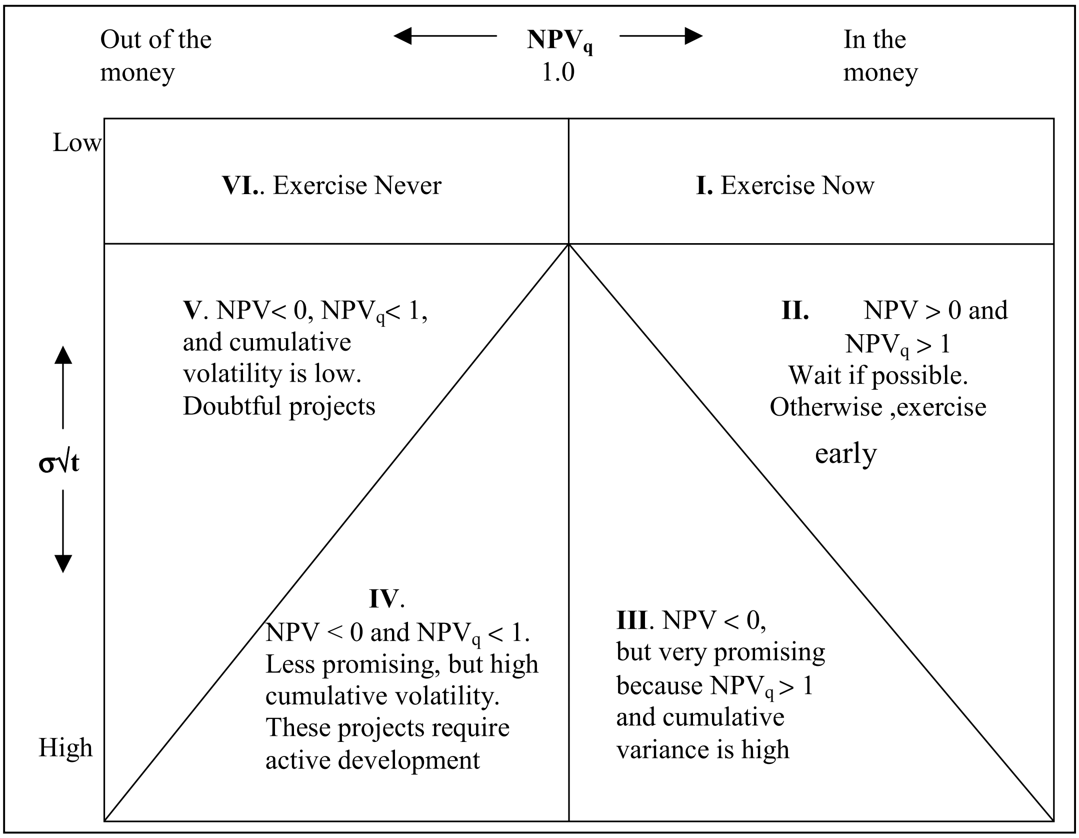


**Figure 9-4** Stylized Mapping of Projects Into Call-Option Space. Adopted from Luehrman (1994).

Figure 9.4 presents three different managerial prescriptions for options with NPVq > 1, each corresponding to a different region in the right half. Both scenarios I and II are positioned in section II in the figure indicating to management to wait with the investment if possible but otherwise exercise the option early.

Scenario III is positioned in sector IV as the NPVq < 1 and NPV < 0. The cumulative volatility however, is high which gives the project high potential while at the same time it requires active development.

## 10 Conclusion

In this paper we present a simple and intuitive real option based framework for analyzing and valuing capital investment opportunities. Our analysis showed that its usefulness varied depending on the project characteristics. In the case when there was no possibility of postponing the investment decision and the project had very limited strategic value, our results showed that the real option framework did not add any value to the capital budgeting decision. However, in the case when the investment decision could be postponed over a period of time the real option based valuation framework gave a result superior to a simple NPV analysis. The expanded valuation framework captured the extra value that postponing the investment added to the total project value. This was also true in the case when the project was assumed to have strategic value in the sense the investment could be expanded considerably 5 years after the initial investment was made.

In spite of the limitations of the ROV, presented in this paper, in some cases it is still able to compensate for many of the major shortcomings DCF valuation methods faces. The framework is able to incorporate the value inherent in strategic opportunities imbedded in many capital projects. It is also able to value the flexibility given by the opportunity to defer an investment over a period of time in which valuable information may become available as uncertainty unfolds.

A major advantage of the approach used in the case study is that it is simple and easily implementable as most of the information needed for the valuation is already present in the traditional DCF spreadsheet used by most corporations. At the same time as simplicity is an advantage, it is also a drawback, as it requires some liberties being taken which lead to an outcome that is more of an approximation than an exact answer.

Using option-pricing models to analyze capital projects presents some practical problems. Comparatively few of these have completely satisfactory solution; on the other hand, some insight is gained just from formulating and articulating the problems. Still more, perhaps, is available from approximations. When interpreting an analysis, it helps to remain aware of whether it represents an exact answer to an approximated problem, or an approximate answer to an exact problem. Either may be useful.

We believe that the results from the case study show sufficient evidence to support a recommendation to Gothenburg Energy AB to implement a real option based valuation framework to their capital budgeting process in addition to their existing valuation methods.

**Appendix I**

The following is a simple example of a decision tree analysis. The result is compared to a basic NPV calculation.

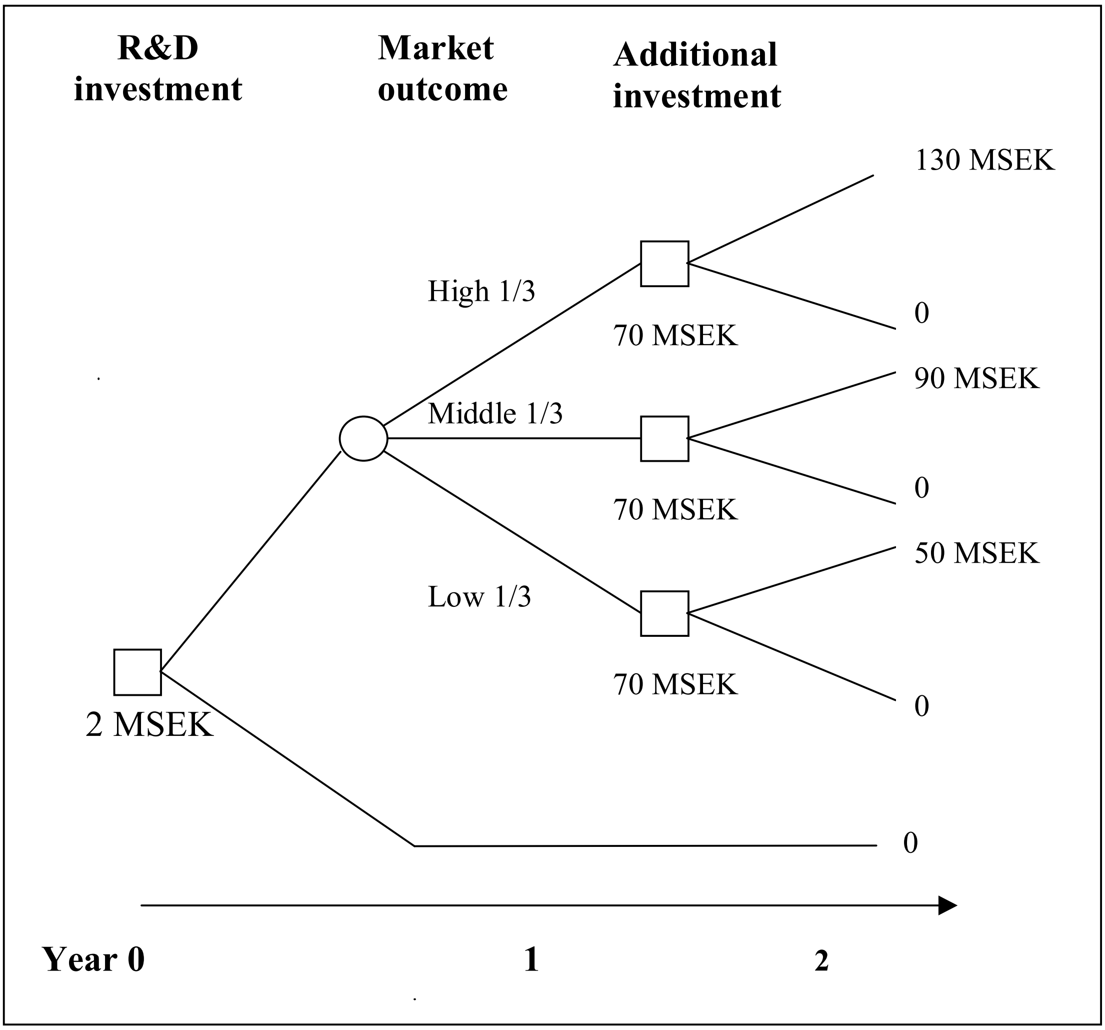


**Figure I**. Two step decision tree

Suppose that, in year 0, the decision is between making an initial investment of 2 million SEK in R&D or not making the investment at all. In year 2, if the project is going to be continued, an additional investment of 70 Million SEK has to be made. For the revenues there are three possible scenarios depending on the market outcome: low (50 million SEK), middle (90 million SEK), and high (130 million SEK). To keep it simple each scenario is assumed to be equally likely, which means a 1/3 probability of occurring. Assume also that the market uncertainty is resolved in year 1. Then, there is a possibility to abandon the project if the market outcome is low. The risk-adjusted discount rate is assumed to be 10%. The problem is illustrated in a decision tree depicted in figure I. The squares indicate decision nodes i.e. where a decision is made and the circles indicate outcome nodes, i.e. where the market outcome is resolved.

If the market outcome is low the project is abandoned since the operating profit is below zero. The additional investment is only made if market outcome is high or middle. The calculation of net present value using the decision tree is shown below.

Expected revenue in year 2 is:

*1/3(130-70)+1/3(90-70)+1/3(0) = 26,67 MSEK*

$$NPV = -2 + \frac{26.67}{1.1^2} = 20MSEK$$

The calculation with basic NPV method would be slightly different since the possibility to abandon the project if market conditions turn out to be low will not be incorporated into the analysis.

Expected revenue in year 2 will now be:

*1/3(130-70)+1/3(90-70)+1/3(50-70) = 20 MSEK.*

$$NPV = -2 + \frac{20}{1.1^2} = 14.52MSEK$$

Which is lower and less accurate than the NPV given by the decision tree analysis.

## Appendix II

### Stochastic Processes

Stock prices and gross project values are assumed to follow a stochastic process, which means that their value changes over time in an uncertain manner. Stochastic processes can be "continuous-time" or "discrete-time". One stochastic process is the Markov process, where only the present state of the process is relevant for predicting the future and the history of the process is irrelevant.

### Wiener process or Brownian motion

A specific type of Markov process is the Wiener process or Brownian motion. If a variable $z(t)$ follows a Wiener process, then changes in z, Δz, must satisfy two properties:

Δz over small time periods are independent, which means that the process can be viewed as the continuous limit of discrete random-walk.

Δz are normally distributed with a mean E (Δz)=0 and a variance follows a linear increase with the time interval, i.e., Var (Δz)= Δt. specifically, $\Delta z = \varepsilon_t \sqrt{\Delta t}$, where $\varepsilon_t$ is a variable that follows a standard normal distribution. In continuous time, as Δt→0, the increment of a standard Wiener process becomes $dz = \varepsilon_t \sqrt{dt}$ with $E(dz) = 0$ and $Var(dz) = dt$ (Hull, 1997).

Although stock prices seem to satisfy the first Markov property, price changes do not follow a normal distribution, in which case we would be observing negative prices. Instead, stock prices are closer to a lognormal distribution, so it is more reasonable to assume that the natural logarithm of price follows a Wiener process. Stock prices also appear to have a non-zero drift and some

volatility other than 1, so a more generalised Wiener process would be more appropriate. This can be presented as follows:
$dS = \alpha(S,t)dt + \sigma(S,t)dz$, where $dz$ is the increment of a standard Wiener process, with mean 0 and variance $dt$ and where $\alpha(S,t)$ and $\sigma(S,t)$ are the drift and variance of the coefficients expressed as function of the current state and time. The continuous time stochastic process S is called *Ito's process*. Its mean and variance are $E(dS) = \alpha(S,t)dt$ and $Var(dS) = \sigma^2(S,t)dt$ (Hull, 1997).

**Geometric Brownian Motion**

A special case is the geometric Brownian motion with drift, or the standard diffusion Wiener process. In this case $\alpha(S,t) = \alpha S$ and $\sigma^2(S,t) = \sigma^2 S^2$ (α and σ are constant) given by

$$dS = \alpha S dt + \sigma S dz$$

or by,

$$\frac{dS}{S} = \alpha dt + \sigma dz$$

where α is the instantaneous expected return on the stock, σ is the constant instantaneous deviation of stock returns and $dz$ is the differential of a standard Wiener process. The above equation is a widely used model for stock-price behaviour (Trigeorgis, 1996). Note that $E(dS) = aSdt$ and $Var(dS) = \sigma^2 S^2 dt$, therefore, the expected stock price drift as a proportion of the current stock price is assumed to be constant. With a constant instantaneous expected stock return, α, the expected increase in stock price within a small time interval, Δt, is αSΔt.
The discrete-time version of the above model is:

$$\frac{\Delta S}{S} = \alpha \Delta t + \sigma \varepsilon \sqrt{\Delta t} \ ,$$

where ΔS is the change in the stock price in a small time interval, Δt,ε is a random sample from a standardised normal distribution, α is the expected stock return per unit of time, and σ is the volatility of stock price.

**Ito's lemma**

Before the above process can be used in the derivation of a call option's value, we need to make use of Ito's lemma. Consider an option or a contingent claim, F(S,t), as a function of an underlying variable, S, and time, t, only. To value the contingent claim, we need to determine how it changes in a small interval of time as a function of the underlying variable. Ito's lemma is easier to understand as a Taylor-series expansion:

$$F(S+\Delta S, t+\Delta t) = F(S,t) + \frac{\partial F}{\partial t}\Delta t + \frac{\partial F}{\partial S}\Delta S + \frac{1}{2}\frac{\partial^2 F}{\partial S^2}(\Delta S)^2 + \cdots, \text{ or}$$

$$\Delta F \equiv F(S+\Delta S, t+\Delta t) - F(S,t)$$

$$= \frac{\partial F}{\partial t}\Delta t + \frac{\partial F}{\partial S}\Delta S + \frac{1}{2}\frac{\partial^2 F}{\partial S^2}(\Delta S)^2 + \cdots.$$

In the limit as higher, as higher-order terms disappear,

$$dF = \frac{\partial F}{\partial t}dt + \frac{\partial F}{\partial S}dS + \frac{1}{2}\frac{\partial^2 F}{\partial S^2}(dS^2).$$

if S follows the standard diffusion Wiener (Ito) process, then

$\frac{dS}{S} = adt + \sigma dt$ and $(dS)^2$ behaves like $\sigma^2 S^2 dt$, so that Ito's lemma becomes:

$$dF = \frac{\partial F}{\partial S}dt + \frac{\partial F}{\partial S}dS + \frac{1}{2}\frac{\partial^2 F}{\partial S^2}(\sigma^2 S^2 dt) \text{ (Hull, 1997)}$$

## Appendix III

**Real options**

| Category | Description | Important in |
|---|---|---|
| Option to defer | Management holds a lease on (or an option to buy) valuable land or resources. It can wait x years to see if output prices justify constructing a building or a plant or developing a field. | All natural-resource extraction industries; real-estate development; farming and paper products |
| Time to build option (staged investment) | Staging investment as a series of outlays creates the option to abandon the enterprise in midstream if new information is unfavorable. Each stage can be viewed as an option on the value of subsequent stages and valued as a compound option. | All R&D intensive industries, especially pharmaceuticals; long development capital intensive projects (e.g. Large scale construction on energy-generating plants); startup ventures |
| Option to alter operating scale (e.g. To expand; to contract; to shut down and restart) | If market conditions are more favorable than expected, the firm can expand the scale of production or accelerate resource utilization. Conversely, if conditions are less favorable than expected, it can reduce the scale of operation. In extreme cases, production may be halted and restarted. | Natural-resource industries (e.g. mining);facilities planning and construction in cyclical industries; fashion apparel; consumer goods; commercial real estate. |
| Option to abandon | If market conditions decline severely, management can abandon current operations permanently and realize the resale value of capital equipment and other assets on secondhand markets. | Capital-intensive industries (e.g. Airlines, railroads); financial services; new-product introductions in uncertain markets. |

| | | |
|---|---|---|
| Option to switch (e.g. outputs or inputs) | If prices or demand change, management can change the output mix of the facility (product flexibility). Alternatively, the same outputs can be produced using different types of inputs (process flexibility). | Output shifts: any good sought in small batches or subject to volatile demand (e.g. Consumer electronics); toys; specialty paper; machine parts; autos. Input shifts: all feedstock-dependent facilities; electric power; chemicals; crop switching; sourcing. |
| Growth options | An early investment (e.g. R&D, lease on undeveloped land or oil reserves, strategic acquisition, information network) is a prerequisite or a link in a chain of interrelated projects, opening up future growth opportunities (e.g. New product or process, oil reserves, access to new market, strengthening of core capabilities). Like inter- project compound options | All infrastructure-based or strategic industries - especially high-tech, R&D, and industries with multiple product generations or applications (e.g. Computers, pharmaceuticals); multinational operations; strategic acquisitions. |
| Multible interacting options | Real-life projects often involve a collection of various options. Upward potential-enhancing and downward protection options are present in combination. Their combined value may differ from the sum of their separate values; i.e. They interact. They must also interact with financial flexibility options. | Real-life projects in most industries listed above. |

Source: Trigeorgis, 1996

## Appendix IV[28]

## Option Pricing Table

NPVq = (Underlying asset value) / PV (exercise price)

$\sigma\sqrt{t}$

| | 0.05 | 0.10 | 0.15 | 0.20 | 0.25 | 0.30 | 0.35 | 0.40 | 0.45 | 0.50 | 0.55 | 0.60 | 0.65 | 0.70 | 0.75 | 0.80 | 0.85 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **0.50** | * | * | * | * | 0.0003 | 0.0015 | 0.0044 | 0.0094 | 0.0167 | 0.0261 | 0.0375 | 0.0506 | 0.0651 | 0.0808 | 0.0976 | 0.1151 | 0.1333 |
| **0.60** | * | * | * | 0.0004 | 0.0024 | 0.0070 | 0.0144 | 0.0243 | 0.0366 | 0.0506 | 0.0661 | 0.0827 | 0.1003 | 0.1185 | 0.1373 | 0.1565 | 0.1761 |
| **0.70** | * | * | 0.0005 | 0.0035 | 0.0103 | 0.0204 | 0.0333 | 0.0482 | 0.0645 | 0.0820 | 0.1003 | 0.1191 | 0.1384 | 0.1580 | 0.1778 | 0.1977 | 0.2176 |
| **0.75** | * | 0.0001 | 0.0018 | 0.0077 | 0.0178 | 0.0310 | 0.0463 | 0.0632 | 0.0810 | 0.0997 | 0.1188 | 0.1383 | 0.1580 | 0.1779 | 0.1978 | 0.2178 | 0.2377 |
| **0.80** | * | 0.0005 | 0.0050 | 0.0148 | 0.0283 | 0.0442 | 0.0615 | 0.0799 | 0.0989 | 0.1183 | 0.1380 | 0.1578 | 0.1777 | 0.1977 | 0.2176 | 0.2374 | 0.2572 |
| **0.82** | * | 0.0010 | 0.0072 | 0.0186 | 0.0334 | 0.0502 | 0.0682 | 0.0870 | 0.1063 | 0.1259 | 0.1457 | 0.1657 | 0.1856 | 0.2055 | 0.2254 | 0.2452 | 0.2648 |
| **0.84** | * | 0.0018 | 0.0099 | 0.0230 | 0.0390 | 0.0566 | 0.0752 | 0.0943 | 0.1139 | 0.1337 | 0.1536 | 0.1735 | 0.1935 | 0.2133 | 0.2331 | 0.2528 | 0.2724 |
| **0.86** | * | 0.0031 | 0.0133 | 0.0280 | 0.0450 | 0.0633 | 0.0824 | 0.1019 | 0.1216 | 0.1415 | 0.1614 | 0.1814 | 0.2013 | 0.2211 | 0.2408 | 0.2604 | 0.2798 |
| **0.88** | 0.0001 | 0.0051 | 0.0175 | 0.0336 | 0.0516 | 0.0705 | 0.0899 | 0.1096 | 0.1295 | 0.1494 | 0.1693 | 0.1892 | 0.2091 | 0.2288 | 0.2484 | 0.2679 | 0.2872 |
| **0.90** | 0.0003 | 0.0079 | 0.0225 | 0.0399 | 0.0586 | 0.0779 | 0.0976 | 0.1175 | 0.1374 | 0.1573 | 0.1772 | 0.1971 | 0.2168 | 0.2364 | 0.2559 | 0.2752 | 0.2944 |
| **0.92** | 0.0010 | 0.0118 | 0.0283 | 0.0467 | 0.0660 | 0.0857 | 0.1055 | 0.1255 | 0.1454 | 0.1653 | 0.1852 | 0.2049 | 0.2245 | 0.2440 | 0.2634 | 0.2825 | 0.3016 |
| **0.94** | 0.0027 | 0.0169 | 0.0349 | 0.0542 | 0.0738 | 0.0937 | 0.1136 | 0.1336 | 0.1535 | 0.1773 | 0.1931 | 0.2127 | 0.2322 | 0.2515 | 0.2707 | 0.2898 | 0.3086 |
| **0.96** | 0.0060 | 0.0232 | 0.0424 | 0.0622 | 0.0821 | 0.1020 | 0.1219 | 0.1418 | 0.1616 | 0.1813 | 0.2010 | 0.2204 | 0.2398 | 0.2590 | 0.2780 | 0.2969 | 0.3156 |
| **0.98** | 0.0116 | 0.0309 | 0.0507 | 0.0707 | 0.0906 | 0.1105 | 0.1304 | 0.1501 | 0.1698 | 0.1894 | 0.2088 | 0.2282 | 0.2473 | 0.2664 | 0.2852 | 0.3039 | 0.3224 |
| **1.00** | 0.0199 | 0.0399 | 0.0598 | 0.0797 | 0.0995 | 0.1192 | 0.1389 | 0.1585 | 0.1780 | 0.1974 | 0.2167 | 0.2358 | 0.2548 | 0.2737 | 0.2923 | 0.3108 | 0.3292 |
| **1.02** | 0.0311 | 0.0501 | 0.0695 | 0.0891 | 0.1086 | 0.1281 | 0.1476 | 0.1670 | 0.1862 | 0.2054 | 0.2245 | 0.2434 | 0.2622 | 0.2809 | 0.2994 | 0.3177 | 0.3358 |
| **1.04** | 0.0445 | 0.0613 | 0.0799 | 0.0988 | 0.1180 | 0.1372 | 0.1563 | 0.1754 | 0.1945 | 0.2134 | 0.2323 | 0.2510 | 0.2696 | 0.2880 | 0.3063 | 0.3244 | 0.3424 |
| **1.06** | 0.0595 | 0.0734 | 0.0907 | 0.1090 | 0.1276 | 0.1463 | 0.1651 | 0.1839 | 0.2027 | 0.2214 | 0.2400 | 0.2585 | 0.2769 | 0.2951 | 0.3132 | 0.3311 | 0.3489 |
| **1.08** | 0.0754 | 0.0863 | 0.1020 | 0.1193 | 0.1373 | 0.1556 | 0.1740 | 0.1925 | 0.2109 | 0.2293 | 0.2477 | 0.2659 | 0.2841 | 0.3021 | 0.3200 | 0.3377 | 0.3552 |
| **1.10** | 0.0914 | 0.0996 | 0.1136 | 0.1299 | 0.1472 | 0.1649 | 0.1829 | 0.2010 | 0.2191 | 0.2372 | 0.2553 | 0.2733 | 0.2912 | 0.3090 | 0.3267 | 0.3442 | 0.3615 |
| **1.12** | 0.1073 | 0.1132 | 0.1255 | 0.1407 | 0.1572 | 0.1743 | 0.1918 | 0.2095 | 0.2273 | 0.2451 | 0.2629 | 0.2806 | 0.2983 | 0.3158 | 0.3333 | 0.3506 | 0.3677 |
| **1.14** | 0.1229 | 0.1270 | 0.1376 | 0.1516 | 0.1672 | 0.1837 | 0.2007 | 0.2180 | 0.2354 | 0.2529 | 0.2704 | 0.2878 | 0.3052 | 0.3226 | 0.3398 | 0.3569 | 0.3738 |
| **1.16** | 0.1380 | 0.1407 | 0.1497 | 0.1626 | 0.1773 | 0.1932 | 0.2096 | 0.2264 | 0.2435 | 0.2606 | 0.2778 | 0.2950 | 0.3121 | 0.3292 | 0.3462 | 0.3631 | 0.3798 |
| **1.18** | 0.1525 | 0.1544 | 0.1619 | 0.1736 | 0.1874 | 0.2026 | 0.2185 | 0.2349 | 0.2515 | 0.2683 | 0.2852 | 0.3021 | 0.3190 | 0.3358 | 0.3525 | 0.3692 | 0.3857 |
| **1.20** | 0.1667 | 0.1679 | 0.1741 | 0.1846 | 0.1975 | 0.2120 | 0.2273 | 0.2432 | 0.2595 | 0.2759 | 0.2925 | 0.3091 | 0.3257 | 0.3423 | 0.3588 | 0.3752 | 0.3916 |
| **1.25** | 0.2000 | 0.2004 | 0.2040 | 0.2119 | 0.2227 | 0.2353 | 0.2492 | 0.2639 | 0.2791 | 0.2946 | 0.3104 | 0.3262 | 0.3422 | 0.3581 | 0.3741 | 0.3900 | 0.4058 |
| **1.30** | 0.2308 | 0.2309 | 0.2329 | 0.2385 | 0.2473 | 0.2583 | 0.2707 | 0.2842 | 0.2983 | 0.3129 | 0.3278 | 0.3429 | 0.3582 | 0.3735 | 0.3888 | 0.4042 | 0.4194 |
| **1.35** | 0.2593 | 0.2593 | 0.2604 | 0.2643 | 0.2713 | 0.2806 | 0.2916 | 0.3039 | 0.3169 | 0.3306 | 0.3447 | 0.3591 | 0.3736 | 0.3883 | 0.4031 | 0.4178 | 0.4326 |
| **1.40** | 0.2857 | 0.2857 | 0.2863 | 0.2889 | 0.2944 | 0.3023 | 0.3120 | 0.3230 | 0.3351 | 0.3478 | 0.3611 | 0.3747 | 0.3886 | 0.4026 | 0.4168 | 0.4310 | 0.4453 |
| **1.45** | 0.3103 | 0.3103 | 0.3106 | 0.3124 | 0.3166 | 0.3232 | 0.3316 | 0.3416 | 0.3526 | 0.3645 | 0.3769 | 0.3898 | 0.4030 | 0.4165 | 0.4301 | 0.4438 | 0.4575 |
| **1.50** | 0.3333 | 0.3333 | 0.3335 | 0.3346 | 0.3378 | 0.3432 | 0.3506 | 0.3595 | 0.3696 | 0.3806 | 0.3923 | 0.4044 | 0.4170 | 0.4298 | 0.4429 | 0.4561 | 0.4693 |
| **1.75** | 0.4286 | 0.4286 | 0.4286 | 0.4287 | 0.4294 | 0.4313 | 0.4347 | 0.4395 | 0.4457 | 0.4530 | 0.4613 | 0.4703 | 0.4799 | 0.4900 | 0.5005 | 0.5112 | 0.5222 |
| **2.00** | 0.5000 | 0.5000 | 0.5000 | 0.5000 | 0.5001 | 0.5007 | 0.5022 | 0.5047 | 0.5083 | 0.5131 | 0.5188 | 0.5253 | 0.5326 | 0.5404 | 0.5488 | 0.5575 | 0.5666 |
| **2.50** | 0.6000 | 0.6000 | 0.6000 | 0.6000 | 0.6000 | 0.6001 | 0.6003 | 0.6009 | 0.6021 | 0.6041 | 0.6067 | 0.6101 | 0.6142 | 0.6190 | 0.6243 | 0.6301 | 0.6363 |

[28] Note: Values in the table represent percentages of underlying asset values: e.g., 39.3 denotes a call option worth 39,3% of the underlying asset value. Values in the table were computed from the Black-Scholes option pricing model. The table is adapted from Luehrman (1994).

# Option Pricing Table

NPVq = (Underlying asset value) / PV (exercise price)

$\sigma \sqrt{t}$

| | 1.05 | 1.10 | 1.15 | 1.20 | 1.25 | 1.30 | 1.35 | 1.40 | 1.45 | 1.50 | 1.75 | 2.00 | 2.25 | 2.50 | 2.75 | 3.00 | 3.50 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **0.50** | 0.2103 | 0.2301 | 0.2500 | 0.2700 | 0.2899 | 0.3098 | 0.3295 | 0.3491 | 0.3686 | 0.3878 | 0.4803 | 0.5651 | 0.6412 | 0.7080 | 0.7655 | 0.8143 | 0.8883 |
| **0.60** | 0.2555 | 0.2754 | 0.2953 | 0.3150 | 0.3346 | 0.3540 | 0.3731 | 0.0921 | 0.4109 | 0.4293 | 0.5174 | 0.5973 | 0.6684 | 0.7306 | 0.7840 | 0.8291 | 0.8973 |
| **0.70** | 0.2964 | 0.3165 | 0.3358 | 0.3550 | 0.3739 | 0.3926 | 0.4111 | 0.4293 | 0.4472 | 0.4649 | 0.5485 | 0.6239 | 0.6907 | 0.7490 | 0.7989 | 0.8410 | 0.9446 |
| **0.75** | 0.3162 | 0.3355 | 0.3545 | 0.3733 | 0.3919 | 0.4102 | 0.4283 | 0.4461 | 0.4636 | 0.4808 | 0.5723 | 0.6356 | 0.7005 | 0.7570 | 0.8054 | 0.8462 | 0.9077 |
| **0.80** | 0.3347 | 0.3535 | 0.3722 | 0.3906 | 0.4088 | 0.4268 | 0.4444 | 0.4618 | 0.4789 | 0.4957 | 0.5751 | 0.6464 | 0.7095 | 0.7643 | 0.8113 | 0.8509 | 0.9106 |
| **0.82** | 0.3418 | 0.3605 | 0.3790 | 0.3973 | 0.4153 | 0.4331 | 0.4506 | 0.4678 | 0.4847 | 0.5014 | 0.5799 | 0.6505 | 0.7129 | 0.7671 | 0.8135 | 0.8527 | 0.9116 |
| **0.84** | 0.3488 | 0.3674 | 0.3857 | 0.4039 | 0.4217 | 0.4393 | 0.4566 | 0.4737 | 0.4904 | 0.5069 | 0.5847 | 0.6554 | 0.7162 | 0.7698 | 0.8157 | 0.8544 | 0.9127 |
| **0.86** | 0.3557 | 0.3741 | 0.3923 | 0.4103 | 0.4279 | 0.4454 | 0.4625 | 0.4694 | 0.4960 | 0.5123 | 0.5893 | 0.6583 | 0.7194 | 0.7724 | 0.8178 | 0.8560 | 0.9137 |
| **0.88** | 0.3624 | 0.3807 | 0.3987 | 0.4165 | 0.4340 | 0.4513 | 0.4683 | 0.4850 | 0.5014 | 0.5176 | 0.5938 | 0.6621 | 0.7225 | 0.7749 | 0.8198 | 0.8577 | 0.9146 |
| **0.90** | 0.3690 | 0.3772 | 0.4050 | 0.4226 | 0.4400 | 0.4571 | 0.4739 | 0.4905 | 0.5068 | 0.5227 | 0.5981 | 0.6658 | 0.7255 | 0.7774 | 0.8218 | 0.8592 | 0.9156 |
| **0.92** | 0.3755 | 0.3935 | 0.4112 | 0.4287 | 0.4459 | 0.4628 | 0.4795 | 0.4958 | 0.5119 | 0.5278 | 0.6024 | 0.6693 | 0.7284 | 0.7798 | 0.8237 | 0.8607 | 0.9165 |
| **0.94** | 0.3719 | 0.3997 | 0.4172 | 0.4345 | 0.4516 | 0.4683 | 0.4848 | 0.5011 | 0.5170 | 0.5327 | 0.6066 | 0.6728 | 0.7313 | 0.7821 | 0.8256 | 0.8622 | 0.9174 |
| **0.96** | 0.3882 | 0.4058 | 0.4232 | 0.4403 | 0.4572 | 0.4738 | 0.4901 | 0.5062 | 0.5220 | 0.5375 | 0.6106 | 0.6762 | 0.7343 | 0.7844 | 0.8274 | 0.8636 | 0.9182 |
| **0.98** | 0.3944 | 0.4118 | 0.4290 | 0.4460 | 0.4627 | 0.4791 | 0.4953 | 0.5112 | 0.5268 | 0.5422 | 0.6146 | 0.6795 | 0.7368 | 0.7866 | 0.8292 | 0.8650 | 0.9191 |
| **1.00** | 0.4004 | 0.4177 | 0.4347 | 0.4515 | 0.4680 | 0.4843 | 0.5003 | 0.5163 | 0.5315 | 0.5467 | 0.6184 | 0.6827 | 0.7394 | 0.7887 | 0.8309 | 0.8664 | 0.9199 |
| **1.02** | 0.4064 | 0.4234 | 0.4403 | 0.4569 | 0.4733 | 0.4894 | 0.5053 | 0.5209 | 0.5362 | 0.5512 | 0.6222 | 0.6858 | 0.7420 | 0.7908 | 0.8325 | 0.8677 | 0.9207 |
| **1.04** | 0.4122 | 0.4291 | 0.4458 | 0.4623 | 0.4785 | 0.4944 | 0.5101 | 0.5255 | 0.5407 | 0.5556 | 0.6259 | 0.6889 | 0.7445 | 0.7928 | 0.8342 | 0.8690 | 0.9214 |
| **1.06** | 0.4179 | 0.4345 | 0.4512 | 0.4675 | 0.4835 | 0.4993 | 0.5149 | 0.5301 | 0.5452 | 0.5599 | 0.6295 | .6919 | 0.7469 | 0.7948 | 0.8375 | 0.8702 | 0.9222 |
| **1.08** | 0.4236 | 0.4401 | 0.4565 | 0.4726 | 0.4885 | 0.5041 | 0.5195 | 0.5346 | 0.5495 | 0.5641 | 0.6330 | 0.6948 | 0.7493 | 0.7967 | 0.8373 | 0.8715 | 0.9229 |
| **1.10** | 0.4291 | 0.4455 | 0.4617 | 0.4776 | 0.4933 | 0.5088 | 0.5241 | 0.5390 | 0.5538 | 0.5682 | 0.6364 | 0.6976 | 0.7517 | 0.7986 | 0.8388 | 0.8726 | 0.9236 |
| **1.12** | 0.4345 | 0.4508 | 0.4668 | 0.4826 | 0.4981 | 0.5134 | 0.5285 | 0.5434 | 0.5579 | 0.5722 | 0.6398 | 0.7004 | 0.7539 | 0.8005 | 0.8403 | 0.8738 | 0.9243 |
| **1.14** | 0.4399 | 0.4559 | 0.4718 | 0.4874 | 0.5028 | 0.5180 | 0.5329 | 0.5476 | 0.5620 | 0.5762 | 0.6431 | 0.7031 | 0.7562 | 0.8033 | 0.8417 | 0.8749 | 0.9250 |
| **1.16** | 0.4451 | 0.4610 | 0.4767 | 0.4922 | 0.5074 | 0.5224 | 0.5372 | 0.5517 | 0.5660 | 0.5801 | 0.6463 | 0.7058 | 0.7583 | 0.8040 | 0.8431 | 0.8760 | 0.9257 |
| **1.18** | 0.4503 | 0.4660 | 0.4815 | 0.4968 | 0.5119 | 0.5268 | 0.5414 | 0.5558 | 0.5699 | 0.5838 | 0.6495 | 0.7084 | 0.7605 | 0.8057 | 0.8445 | 0.8771 | 0.9263 |
| **1.20** | 0.4554 | 0.4709 | 0.4863 | 0.5014 | 0.5163 | 0.5310 | 0.5455 | 0.5598 | 0.5738 | 0.5876 | 0.6526 | 0.7110 | 0.7626 | 0.8074 | 0.8458 | 0.8782 | 0.9269 |
| **1.25** | 0.4677 | 0.4828 | 0.4978 | 0.5125 | 0.5271 | 0.5414 | 0.5555 | 0.5694 | 0.5831 | 0.5965 | 0.6600 | 0.7871 | 0.7676 | 0.8115 | 0.8490 | 0.8807 | 0.9284 |
| **1.30** | 0.4796 | 0.4943 | 0.5088 | 0.5231 | 0.5373 | 0.5513 | 0.5651 | 0.5786 | 0.5920 | 0.6051 | 0.6671 | 0.7230 | 0.7723 | 0.8153 | 0.8521 | 0.8831 | 0.9299 |
| **1.35** | 0.4909 | 0.5052 | 0.5193 | 0.5333 | 0.5471 | 0.5608 | 0.5742 | 0.5874 | 0.6005 | 0.6133 | 0.6739 | 0.7285 | 0.7768 | 0.8189 | 0.8550 | 0.8854 | 0.9312 |
| **1.40** | 0.5018 | 0.5157 | 0.5295 | 0.5431 | 0.5565 | 0.5698 | 0.5829 | 0.5958 | 0.6086 | 0.6211 | 0.6804 | 0.7338 | 0.7811 | 0.8224 | 0.8577 | 0.8875 | 0.9325 |
| **1.45** | 0.5123 | 0.5258 | 0.5392 | 0.5524 | 0.5656 | 0.5785 | 0.5913 | 0.6039 | 0.6163 | 0.6286 | 0.6865 | 0.7389 | 0.7852 | 0.8257 | 0.8603 | 0.8896 | 0.9337 |
| **1.50** | 0.5224 | 0.5355 | 0.5485 | 0.5614 | 0.5742 | 0.5869 | 0.5993 | 0.6116 | 0.6238 | 0.6357 | 0.6924 | 0.7437 | 0.7892 | 0.8288 | 0.8628 | 0.8916 | 0.9349 |
| **1.75** | 0.5674 | 0.5788 | 0.5902 | 0.6015 | 0.6128 | 0.6240 | 0.6350 | 0.6460 | 0.6568 | 0.6675 | 0.7186 | 0.7650 | 0.8064 | 0.8426 | 0.8738 | 0.9001 | 0.9400 |
| **2.00** | 0.6051 | 0.6151 | 0.6250 | 0.6350 | 0.6450 | 0.6549 | 0.6648 | 0.6746 | 0.6843 | 0.6939 | 0.7402 | 0.7826 | 0.8206 | 0.8540 | 0.8828 | 0.9072 | 0.9441 |
| **2.50** | 0.6645 | 0.6721 | 0.6798 | 0.6877 | 0.6956 | 0.7035 | 0.7115 | 0.7195 | 0.7242 | 0.7354 | 0.7740 | 0.8100 | 0.8427 | 0.8716 | 0.8967 | 0.9180 | 0.9505 |

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