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Real Option Valuation in High-Tech Firm

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Abstract

In traditional financial theory, the discounted cash flow model (or NPV) operates as the basic framework for most analyses. In doing valuation analysis, the conventional view is that the net present value (NPV) of a project is the measure of the value that is the present value of expected cash flows added to the initial cost. Thus, investing in a positive (negative) net present value project will increase (decrease) value.

Recently, this framework has come under some fire for failing to consider the options which are the managerial flexibilities, which are the collection of opportunities.

A real-option model (Option-based strategic NPV model) is estimated and solved to yield the value of the project as well as the option value that is associated with managerial flexibilities. Most previous empirical researchers have considered the initial-investment decision (based on NPV model) but have neglected the possibility of flexible operation thereafter. Now the NPV must be compared with the strategic option value, by which investment is optimal while the NPV is negative. This leads investors to losing the chances to expand themselves.

In the paper we modify the NPV by taking into account real options—theme of this paper, or strategic interactions. R&D, Equity and Joint Ventures will be viewed as real options in practice of case studies of this paper.

Keywords: Discount rate, Net present value (NPV), Option(s) and valuation

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We are talking in a vacuum.
We are talking in a void.
We are talking in a state of quantal bliss.
Because everything's uncertain
We can't even tell the time.
It's a proble that defies analysis*

------B.K. Ridley

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* Proble—a problem without end

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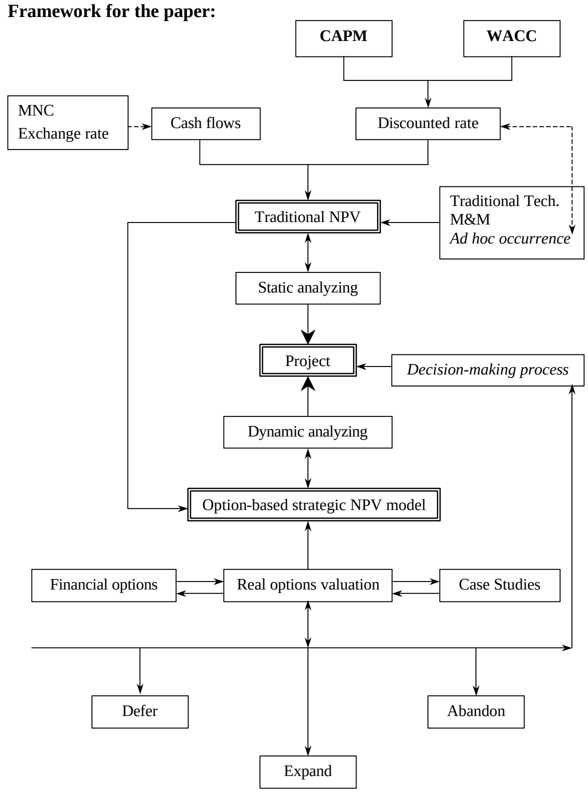
Framework for the paper:



Part I: Static Tools for Valuation

1 Introduction

If we must rethink in this paper the links between customary methods of valuation and the ones emerging currently, it is because we humans evolve from one stage to another.

People have found that there are some pitfalls in most traditional methods of valuation, NPV for example. Our objective is to introduce a new way to valuation, or in a certain sense repair traditional NPV to be strategic option-based NPV method.

It is thus necessary first to go back to the traditional valuation methods. Only then, against this background, may we be able to appreciate the evolution of knowledge in the course of time and see the turning point of valuation methods. This is precisely what we are going to do.

We shall first talk about static valuation tools, NPV which involves with discounted rate, cash flows, and so on. To which a couple of magnificent theories, CAPM and WACC will be covered.

In view of the static and passive feature of NPV, we shall bring in the more flexible financial option theory. And by combining financial option with NPV, an option-based NPV—Real Option theory might well appear to us. This above is just the second part of this paper: the expanding of financial option to real option.

For deeply understanding real options, the implementation of real option will be scrutinized in third part of the paper.

1.1 Background

Traditional financial theory states that the value of a firm is independent of its capital structure in the absence of taxes and bankruptcy. It has been demonstrated true in 4 prospects:

- The returns on the underlying assets are certain and investors can borrow at the risk free rate.
- The firm is part of an efficient market.
- There are two or more firms with different leverage in the same " risk class", and these firms will no go bankrupt with probability
- Investors can arrange "no course" loans with the stock as collateral (Ingeroll, 1987)

The main approach to value firm by traditional financial theory is discounted cash flow (DCF) model. DCF models are used for project evaluation by most companies, presumably because they are straightforward to apply and because they are intuitively appealing. One needs to forecast the future cash flows, choose the appropriate discount rate, and find the present value of the forecasted cash flows. The net present value (NPV) is defined as the difference between the present value of the future cash flows and the initial cost. If NPV is positive, then accepting the project adds value to firm. Given accurate estimates of future cash flows, the success of the discounted cash flow then will depend on how well you choose the discount rate. If you pick a rate that is too high, you will reject projects that have negative NPV; if you pick a rate that is too low, you will accept projects that have positive NPV.

However the DCF model values the firm with a certain deterministic discount rate that of course is unrealistic in an uncertain world. It ignores the value of management, growth, deferring, liquidation and abandonment value in real assets and other factors that may impact the value of the firm.

Recent financial literature points out that a firm's equity, debt and even itself can be treated as a real option for investors. For example, the implication of viewing equity as a call option is that equity will have value, even if the value of the firm falls well below the face value of the outstanding debt. While the firm will be viewed as troubled by investors, accountants, and analysts, its equity is not worthless. In fact just as deep out-of-the-money traded options command value because of the possibility that the value of the underlying asset may increase above the strike price in the remaining lifetime of the option, equity commands value because of the time premium on the option and the possibility that the value of the assets may increase above the face value of the bonds before they come due. Thus the real option approach to firm valuation can explain why highly levered, risky firms still have a high value in equity markets. Not only explaining by general option theory, but also the real option approach may derive value from the firm equity for relevant investors.

1.2 Problems Identification

The problem of this paper is mainly of examining the real option theory in both theory and case studies, to which strategic decision-making based on real options could presumably maximize the wealth of the shareholders of a corporation. In doing so, we shall divert our analyses from traditional static viewpoints to dynamic ones that integrate managerial flexibility (uncertainty) for instance, which were neglected in traditional valuation tools.

In theory part, we review proved academic research on real option. While in case studies, we give the quantitative analysis and theoretical explanations to the business events.

1.3 Objectives

One vital consequence of uncertainty is that the laws of social science have to be statistical in character. Real Option theory has to talk about expectation values rather than determined quantities, probabilities rather than certainties. Prediction is more uncertain, however, real option theory does quantify our model precisely, which can be remarkable.

The goal of this thesis is mainly about applying real options to real case, for example Ericsson Radio System. In other words, we want to make the above financial theories more applicable in life by transforming numerical outcomes into strategic decision making for a corporation.

1.4 Limitations

The thesis work is rather a theoretical one. Most parts of the thesis are the review of the empirical studies.

The results of case study are actually calculated upon some assumed inputs that certainly may cause some biases from real outcomes. Assumptions are made consistent with NPV method for the estimation of future value.

The interactive strategy of game theory is excluded.

1.5 Methodology

By comparing the different tools, we illustrate the advantages and disadvantages of different approaches, traditional NPV and Option-base strategic NPV, to valuation. The above approaches will be used to implement the objectives of the paper. Very beneficial for both authors and readers is to speak of two sorts of ways for contemplating the Real Options in strategic NPV valuation: Binomial trees and Black-Scholes methods. Traditional NPV will be improved for measuring the more flexible underlying assets, for instance a corporation. Strategic NPV tries to reasonably estimate a project (firm) for decision-makers.

2 Traditional Valuation Method

It is nearly impossible to discuss valuation without uncertainties. Valuation approach under certainty circumstances only exists in treasure issues buying. Investors may enjoy a risk free rate on their holding these kinds of issues. A simple discounted cash flow (DCF) model can be well used for certainty valuation.

However in the real world, uncertainties do affect the value. Bierman and Smidt (1993) pointed out that the traditional technique for dealing with uncertainty could be classified into two groups. One group of techniques attempts to consider explicitly all alternative sequences of cash flow. The state preference approach fits in this group. These techniques are attractive theoretically but are difficult to implement. The commonly used practical techniques are often methods of approximating the result that would be obtained if a theoretically correct approach were used.

A second group of techniques requires the decision maker to provide a concise summary description of the asset that can be used to make an estimate of its value. For example, the decision maker may estimate the expected cash flows of each period and discount these by an appropriate risk adjusted discount rate to estimate the value of the asset. In estimating values for bond, the promised (most likely) cash flows are used in place of the expected cash flows. In the capital asset pricing model (CAPM), it is assumed that the decision maker knows the asset's beta coefficient, which describes the relationship between the values of the asset (or of some closely related asset) following a particular probability distribution and must specify parameters of the distribution, such as its variance. With certainty equivalent approach, the uncertain cash flows of each period are collapsed into a single measure that reflects both probabilities and risk preferences. All of these techniques aim to produce an estimated market valuation for the investment proposal. This group approach is actually the extension of discounted cash flow- net present value (DCF-NPV) analysis. This group approach is widely used in today's capital valuation, probably because they are straightforward to apply and because they are intuitively appealing. You forecast the cash flows, choose the appropriate discount rate, and find the present value of the forecasted cash flows. If the NPV is positive, then accepting the project adds value to the firm.

We would like to introduce two approaches determining the discount rate before we have a review on DCF-NPV method in valuation.

2.1 Major Approaches to Determine the Discount Rate

2.1.1 Capital Asset Pricing Model (CAPM)

CAPM is an equilibrium model of asset pricing that states that the expected return on a security is a positive linear function of the security's sensitivity to changes in the market portfolio's return. (Sharpe, 1997) The key variable in the CAPM is called “beta”, a statistical measure of risk which has become as familiar as—and, indeed, interchangeable with—the CAPM itself. Financial managers have long realized that some projects were riskier than others, and that these projects require a higher rate of return. A risky investment is, of course, one whose return is uncertain in advance; and in such a case, it is only the expected or average rate of return that can be projected. To justify undertaking the risky project, a higher payout in the event of success is required.

The simple CAPM Model captures this perspective. According to the simple CAPM, an investment’s required rate of return increases in direct proportion to its beta. The CAPM also implies that investors, in pricing common stocks, are concerned exclusively with systematic risk. A security’s systematic risk, as measure by beta, is the sensitivity (or co-variance) of its return to movements in the economy as a whole. Asset with high betas exaggerate general market developments, performing exceptionally well when the market goes up and exceptionally poorly when the market goes down (Rosenburg and Rudd, 1998).

The simple CAPM model states:

$$E(R) = R_f + \lfloor E(R_m) - R_f \rfloor b$$

where: $E\ (R)$ = The required rate of return (or rate of return)

R_f = The Risk-free rate (the rate of return on a “risk-free investment”, like U.S. government treasury bonds)

b = Beta (see above)

$E\ (R_m)$ = the expected return on the overall stock market

In other words, the required rate of return is equal to the sum of two terms: the risk-free return and an increment that compensates the investor for accepting the asset's risk. The compensation for risk is expressed as the asset's beta multiplied by the expected excess return of the market, [E (R_m) – R_f]. This expected excess return is sometimes referred to as the "risk premium"(Rosenburg and Rudd, 1998).

2.1.2 Weighted Average Cost of Capital (WACC) Approach

The cost of capital to a firm may be defined as a weighted average of the cost of each type of capital. The weight of each type of capital is the ratio of the market value of the securities representing that source of capital to the market value of all securities issued by the company. The term security includes common and preferred stocks and all interest-bearing liabilities, including notes payable. It is sometimes stated that the weighted average cost capital of a firm may be used to evaluate investments whose cash flows are perfectly correlated with the cash flows from the firm's present assets. With perfect correlation between the two sets of cash flows, the risk is the same (Bierman and Smidt, 1993). The usual definition of the weighted average cost of capital is to weight the after-tax cost of debt by the percentage of debt in the firm's capital structure and add the result to the cost of equity multiplied by the percentage of equity. The equation is

$$WACC = K_b (1 - t_c) \frac{B}{B + C} + K_s \frac{S}{B + S}$$

The cost of capital combines in one discount rate an allowance for the time value of money and an allowance for risk. To apply the same cost of capital to cash flows that occur at different points in time, the magnitude of these allowances (i.e., the percent per unit of time) must remain constant over time.

A specific asset might have a smaller or larger amount of risk, thus should have a smaller or larger discount rate. The WACC is the correct discount rate only for one level of risk. For a given capital structure, the weighted average cost of capital to a firm reflects the characteristics of the firm's assets, and particularly their average risk, but also the timing of the expected cash proceeds.

Both CAPM and WACC approaches can be used to determine discount rate for valuing the future cash flow to the project and firm.

2.2 DCF-NPV Approach to Project Valuation

Most capital-budgeting investments, of course, involve discounting of cash flow over multiple future periods, so that using the single period CAPM for discounting one period at a time would necessitate certain additional assumptions concerning the evolution of its variables over time. Fama (1977 shows that the present value of a future net cash flow is its current expected value discounted at risk-adjusted discount rates K_s given by the CAPM. The discount rates must be known and non-stochastic (i.e., they must evolve in a deterministic fashion through time), and in general they will differ from period to period (and across cash flows for a given period). The NPV of an investment project is then simply the sum of the present values of all its future net cash flows.

$$NPV_j = \sum_{t=0}^{N} \frac{NCF_{jt}}{[1 + K_s]^t}$$

2.3 DCF-NPV Approach to Firm Valuation

Modigliani and Miller (1958, 1963) wrote a breakthrough working paper on cost of capital, corporate valuation, and capital structure based listed assumptions[1]:

- Capital markets are frictionless.
- Individuals can borrow and lend at the risk-free rate.
- There are no costs to bankruptcy.
- Firms issue only two types of claims: risk free debt and (risky) equity.
- All firms are in the same risk class.
- Corporate taxes are the only form of government levy (i.e., there are no wealth taxes on corporations and no personal taxes)
- All cash flow streams are perpetuities (i.e., no growth)
- Corporate insiders and outsiders have the same information (i.e., no signaling opportunities).
- Managers always maximize shareholder's wealth (i.e., no agency costs).

Although many of these assumptions are unrealistic, they do not really change the major conclusions of the model of firm behavior (Copland and Weston, 1992).

Suppose the assets of a firm return the same distribution of net operating cash flow each time period for an infinite number of time periods. This is a no growth situation because the average cash flow does not change over time. The value of the firm can be written as below:

[1] See Copeland/Weston "Financial Theory and Corporate Policy" third edition P439, Addison-Wesley Publishing Company.

$$V^U = \frac{E(FCF)}{r}$$

where: V^U = the present value of an unlevered firm (i.e., all equity)
E (FCF) = the perpetual free cash flow after taxes
r = the discount rate for an all-equity firm of equivalent risk.

This is the value of an un-levered firm because it represents the discounted value of a perpetual, non-growing stream of cash flows after taxes that would accrue to shareholders if the firm had no debt.

To value a levered firm, the equation below can be derived from M&M approach.

$$V^L = \frac{NOI}{WACC}$$

where: $V^L =$ The present value of a levered firm
NOI= Net Operating Income

The discount rate for NPV which determines the value of the (un-levered and levered) firm can be found in Table 2.1:

Table 2.1 Comparisons between CAPM and M&M
(Source: Copeland and Weston 1992)

Type of Capital	CAPM Definition	M&M Definition
Debt	$K_b = R_f + \lfloor E(R_m) - R_f \rfloor b_b$	$K_b = R_f , b_b = 0$
Un-levered Equity	$r = R_f + \lfloor E(R_m) - R_f \rfloor b_u$	$r = r$
Levered Equity	$K_s = R_f + \left[E(R_m) - R_f \right] b_s$	$K_s = r + (r - K_b)(1 - t_c)\frac{B}{S}$
WACC for the firm	WACC= $K_b (1 - t_c)\frac{B}{B+C} + K_s \frac{S}{B+S}$	WACC= $r(1 - t_c \frac{B}{B+S})$

2.4 Failure of DCF-NPV Method

The application of the DCF-NPV to the valuation of real risky assets is made possible by two almost tacit assumptions or conventions. The first is that uncertain future cash flow can be replaced by their expected values and that these expected cash flows can be treated as given at the outset. The second is that the discount rate is known and constant, and that it depends solely upon the risk of the project. Let us consider the limitations of an approach based on these assumptions and see why the underlying DCF-NPV analogy may be a poor one for some investment projects.

First, by assuming that the cash flows to be discounted are given at the outset, pre-supposes a static approach to investment decision-making—one which ignores the possibility of future management decisions that will be made in response to the market considerations encountered. Over the life of a project, decisions can be made to change the output rate (ad hoc discounted rate should be used), to expand or close the facility, or even to abandon it. The flexibility afforded by these decision possibilities may contribute significantly to the value of the project.

To introduce an analogy which we shall develop further below, the DCF-NPV approach may be likened to valuing a stock option contract while ignoring the right of the holder not to exercise when it is unprofitable. To some extent this drawback of the DCF-NPV approach may be overcome by employing a scenario or simulation approach in which alternative scenarios-involving for example different price outcomes and management responses-are generated and the resulting cash flows estimated. These cash flows are then averaged across scenarios and discounted to arrive at the present value.

Unfortunately this scenario or simulation approach gives rise to two further problems. First, it requires that the appropriate policy for each scenario be determined in advance. Of course sometimes this will be possible. For example, if the output rate can be adjusted without any costs, the simple rule of setting marginal cost equal to price may sometimes be optimal. But more generally this will not be possible. If it is costly to close or abandon a project, then the decision to close is itself an investment decision with uncertain future cash flows depending on commodity prices. The optimal closure policy must therefore be determined simultaneously with the original capital budgeting decision.

Even more fundamentally, the degree of managerial discretion in making future operating decisions will tend to affect the risk of the project under consideration. A project which can be abandoned under adverse circumstances will be less risky than one that cannot; it will be even less risky if part of the initial capital investment can be recovered in the event of abandonment. The classical approach offers no way of allowing for this risk effect except through some *ad hoc* adjustments of the discount rate.

In fact the tacit assumption concerning the discount rate is the second Achilles' heel of the classical approach. Given any set of expected cash flows, there almost always exists some discount rate, which will yield the correct present value. But the determination of this discount rate presents a quite difficult task, and current procedures cannot be regarded as any more than highly imperfect rules of thumb. Thus these procedures all assume that the discount rate is constant, which is equivalent to assuming that the risk of the project is constant over its life. And this is, of course, highly unlikely. Not only will the risk depend in general upon the remaining life of the project, it will almost certainly depend upon the current profitability of the project through an operating leverage effect. Hence, not only will the discount rate vary with time, it will also be uncertain.

Even if the appropriate discount rate were deterministic and constant, the problem of estimation would still be formidable. In principle the discount rate should depend upon the risk of the project, but how is this risk to be assessed? The generally approved procedure is to use the CAPM and to base the discount rate on the beta of the project as estimated from other firms with similar projects. In practice these other firms consist in effect of portfolios of projects, sometimes in unrelated industries, and this makes the assignment of betas to individual projects a hazardous undertaking. Transferring these betas to the project under consideration creates further problems, for a new project is likely to have a cost structure that differs in a systematic fashion from existing, mature projects. The problem is compounded by the consideration, mentioned above, that the latitude of future operating decisions inherent in a project will affect its risk, and is unlikely to be duplicated in existing projects.

Of course these problems are often ignored in practice and a single corporate discount rate based on the weighted average cost of capital is employed for all projects, regardless of risk. As is well known, however, the price of this simplification is a capital budgeting decision system which contains systematic biases as between projects with different risks and different lives. And, such a decision system will lead to the systematic under-valuation of projects with significant operating options.

A final practical difficulty with the classical approach is the necessity to forecast expected output prices for many years into the future. A wide range of possibilities for the path of expected future spot prices will appear plausible, and the calculated present value of the project will depend upon some arbitrary selection among them.

The above appears to constitute a fairly strong indictment of the classical discounted cash flow approach to capital budgeting. The limitations of the classical approach arise because it is based fundamentally on an analogy between a portfolio of risk-less bonds and a real investment project. In many cases this analogy may be useful; for example, in situations in which the scope for future managerial discretion is limited, and the fiction of other similar risk projects can be maintained.

For improving the static feature of traditional DCF-NPV approach, option theory could be taken into account, from which investor can really find an "option" to implement his idea.

Part II: Expanding of Financial Option to Real Option

The discounted cash flow method is a powerful and valuable tool for valuing such projects as routine projects, or projects that involve replacement of existing machinery with more efficient equipment.

However, many other projects require considerable intervention (flexibility) from management of a firm (project). The application of this type of intervention in management increases more opportunities for project (firm). Therefore, it is quite important to take these opportunities (options) into account when you value a project.

As indicated by the title of this part, we shall expand financial option theory to real option theory in this part.

3 Financial-Options and Option-Pricing Theory

A *financial derivative security* (derivative) is an instrument whose value depends on the price of its underlying variables, including stocks, stock indices, foreign currencies, debt instruments, commodities and future contracts and so on. The derivatives, (forward contracts, swaps, and options for instance), are also known as contingent claims for they can actually solve the *ad hoc* problems.

In this paper we mainly concentrate on options. A stock option for example, is a contract which conveys to its holder the right, but not the obligation, to buy or sell shares of the underlying asset at a specified price on or before a given date. This right is granted by the seller of the option.

3.1 Basic Concepts of Options[2]

In financial markets people may very often hear one word: options. Two types of options are in general used for either hedging or speculating: puts and calls, which are contracts that give the owner the right (no obligation) to do something. The options' contracts are different from futures. Just for option, it offers the holder of option the right to do something, leading to the prices being imposed on them (Option premier).

A call option is the right for holder of the option to buy the *Underlying* asset by a certain date at the pre-negotiated price. A put option is the right for holder of the option to sell the underlying asset by a certain date at the pre-negotiated price. For example, an American-style XYZ Corp. May 60 call entitles the buyer to purchase 100 shares of XYZ Corp. common stock at $60 per share at any time prior to the option's expiration date in May. Likewise, an American-style XYZ Corp. May 60 put entitles the buyer to sell 100 shares of XYZ Corp. common stock at $60 per share at any time prior to the option's expiration date in May.

Underlying Asset: The specific asset on which an option contract is based is commonly referred to as the underlying assets. The underlying of an option can be any asset at all, as long as it has a value upon which both sides of the contract can agree (Chriss, 1997). For example, the underlying can be a commodity (gold or silver), or a foreign currency (U.S. dollar-yen exchange rate), or stock indexes (Standard & Poor index) and so on. Options are categorized as derivative securities because their value is derived in part from the value and characteristics of the underlying asset (security for instance). A stock option contract's unit of trade is the number of shares of underlying stock which are represented by that option. Generally speaking, stock options

[2] Source: Chicago Board Option Exchange, Feb 1999.

have a unit of trade of 100 shares. This means that one option contract represents the right to buy or sell 100 shares of the stock (underlying asset).

Strike price: The strike price, or exercise price, of an option is the specified underlying asset price at which the underlying asset can be bought or sold by the holder, or buyer, of the option contract if he/she exercises his/her right against a writer, or seller, of the option. To exercise your option is to exercise your right to buy (in the case of a call) or sell (in the case of a put) the underlying asset at the specified strike price of the option.

The strike price for an option is initially set at a price which is reasonably close to the current underlying asset price. Additional or subsequent strike prices are set at the following intervals: 2½-points when the strike price to be set is $25 or less; 5-points when the strike price to be set is over $25 through $200; and 10 points when the strike price to be set is over $200. (New strike prices are introduced when the price of the underlying asset rises to the highest, or falls to the lowest, strike price currently available). The strike price, a fixed specification of an option contract, should not be confused with the premium, the price at which contract trades, which fluctuates daily.

If the strike price of a call option is less than the current market price of the underlying asset, the call is said to be in-the-money because the holder of this call has the right to buy the stock at a price which is less that the price he would have to pay to buy the stock, for example, in the stock market. Likewise, if a put option has a strike price that is greater than the current market price of the underlying asset, it is also said to be in-the-money because the holder of this put has the right to sell the stock at a price which is greater than the price he would receive selling the stock in the stock market. The converse of in-the-money is, not surprisingly, out-of-the-money. If the strike price equals the current market price, the option is said to be at-the-money.

Premium: Option buyers pay a price for the right to buy or sell the underlying asset. This price is called the option premium. The premium is paid to writer, or seller, of the option. In return, the writer of a call option is obligated to deliver the underlying asset to a call option buyer if the call is exercised. Likewise, the writer of a put option is obligated to take delivery of the underlying asset from a put option buyer if the put is exercised. Whether or not an option is ever exercised, the writer keeps the premium. Premiums are quoted on per underlying unit basis. Thus, a premium of 7/8 represents a premium payment of $87.50 per option contract ($0.875 • 100 units).

American, European and Capped styles: There are 3 styles of options: American, European and Capped. In the case of an American option, the holder of an option has the right to exercise his option on or before the expiration date of the option. A European option is an option which can only be exercised during a specified period of time prior to its expiration. A capped option gives the holder the right to exercise that option only during a specified period of time prior to its expiration, unless the option reaches the cap value prior to expiration, in which case the option is automatically exercised. The holder or writer of either style of option can close out his position at any time simply by making an offsetting, or closing, transaction. A closing transaction is the transaction in which, at some point prior to expiration, the buyer of an option makes an offsetting sale of an identical option, or the writer of an option makes an offsetting purchase of an identical option. A closing transaction cancels out an investor's previous position as the holder or writer of the option.

The Option Contract: An option contract is defined by the following elements: type (put or call), style (American, European and Capped), underlying asset, unit of trade (number of shares), strike price, and expiration date. All option contracts that are of the same type and style and cover the same underlying asset are referred to as a class of options. All options of the same class that also have the same unit of trade at the same strike price and same expiration date are referred to as an option series.

If a person's interest in a particular series of options is as a net holder (that is, if the number of contracts bought exceeds the number of contracts sold), then this person is said to have a long position in the series. Likewise, if a person's interest in a particular series of options is as a net writer (if the number of contracts sold exceeds the number of contracts bought), he is said to have a short position in the series. After making contract, the thing people face is to exercise the option.

Exercising the option: if the holder of an option decides to exercise his right to buy (in the case of a call) or to sell (in the case of a put) the underlying asset, the holder must direct his broker to submit an exercise notice to OCC (Options Clearing Corporation). In order to ensure that an option is exercised on a particular day, the holder must notify his broker before the broker's cut-off time for accepting exercise instructions on that day.

Based on these basic concepts, we can proceed more easily to the next section: pricing the options.

3.2 Pricing for Options

There are several factors that contribute value to an option contract and thereby influence the premium or price at which it is traded. The most important of these factors are the prices of the underlying assets; time remaining until expiration, the volatility[3] of the underlying asset price, cash dividends and interest rates.

[3] See Appendix 2

Factors affecting option value[4]: The value of an option is determined by a number of factors relating to the underlying asset and financial markets.

1) Underlying Asset Price: the value of an option depends heavily upon the price of its underlying asset. As previously explained, if the price of the stock is above a call option's strike price, the call option is said to be in-the-money. Likewise, if the stock price is below a put option's strike price, the put option is in-the-money. The difference between an in-the-money option's strike price and the current market price of a share of its underlying security is referred to as the option's intrinsic value. Only *in-the-money* options have intrinsic value.

For instance, if a call option's strike price is $45 and the underlying shares are trading at $60, then the option has intrinsic value of $15 because the holder of that option could exercise the option and buy the shares at $45. The buyer could then immediately sell these shares on the stock market for $60, yielding a profit of $15 per share, or $1,500 ($15*100) per option contract.

When the underlying share price is equal to the strike price, the option (either call or put) is *at-the-money*. An option that is not in-the-money or at-the-money is said to be *out-of the-money*. An at-the-money *or* out-of-the-money option has no intrinsic value, but this does not mean it can be obtained at no cost. There are other factors that give options value and therefore affect the premium at which they are traded. Together, these factors are termed time value. The primary components of time value are time remaining until expiration, volatility, dividends, and interest rates. Time value is the amount, by which the option premium exceeds the intrinsic value, i.e.,

Option Premium = Intrinsic Value + Time Value

[4] (Source: Chicago Board Option Exchange, Feb, 1999)

For in-the-money options, the time value is the excess portion over intrinsic value. For at-the-money and out-of-the-money options, the time value is the total option premium.

2) Time Remaining Until Expiration: Generally, the longer the time remaining until an option's expiration date, the higher the option premium because there is a greater possibility that the underlying asset price might move so as to make the option in-the-money. Time value drops rapidly in the last several weeks of an option's life.

3) Volatility: Volatility is the propensity of the underlying security's market price to fluctuate either up or down. Therefore, volatility of the underlying asset price influences the option premium. The higher the volatility of the stock, the higher the premium because there is, again, a great possibility that the option will move in-the-money.

4) Dividends paid on the Underlying Assets: Regular cash dividends are paid to the stockholder. Therefore, cash dividends affect option premiums through their effect on the underlying asset (stock/share) price. Because the stock price is expected to fall by the amount of the cash dividend, higher cash dividends tend to imply lower call premiums and higher put premiums.

Options customarily reflect the influences of stock dividends (e.g., additional shares of stock) and stock splits because the number of shares represented by each option is adjusted to take these changes into consideration.

5) Interest Rates: Historically, higher interest rates have tended to result in higher call premiums and lower put premiums.

Premiums (prices) for exchange-traded options are published daily in a large number of newspapers. A typical newspaper listing, as below, can give us a better understanding of option premium:

Option &	Strike		Calls-Last			Puts-Last	
NY Close	Price	5) May	Jun	Jul	May	Jun	Jul
1) XYZ	3) 105	4) 7½	9 1 4	10	1 4		1
2)112	110	3	4 3 4	6 1 4	1 3 16	1	2
112	115	13 16	2	3 1 2	4	4	5
112	120	3 16		1 3 4	8	8	8 3 4
112	125	1 16	s	13 16	r	s	r
112	130	s	s		s	s	18 3 4

1) stock identification — 4) closing option prices
2) stock closing price — 5) option expiration months
3) option strike prices — r = not traded s = no option listed

(Source: Chicago Board Option Exchange, Feb, 1999)

In this example, the out-of-the-money XYZ July 115calls closed at 3 1 2, or $350 per contract, while XYZ stock closed at 112 . The in-the-money July 120 puts closed at 8 3 4, or $875 per contract (Understanding stock options, Feb 1999, Chicago Board Option Exchange).

These factors and their predicted effects on call and put prices are also summarized below:

Table 3.1 Summary of Factors Affecting Call and Put Prices

Factors		Effect on	
		Call Value	Put Value
Increase in	underlying asset's value		
	strike price		
	volatility (variance)		
	time to expiration		
	interest rates		
	dividends paid		

3.3 Option Pricing Models

In this section, we introduce the most widely used model for options pricing—the *binomial model*. This model for option pricing is much more generalized than the Black-Scholes model which based on the geometric Brownian motion model. The former is a discrete time model while the latter is a continuous time one.

3.3.1 The Binomial Model

A paper entitled "Option Pricing: A Simplified Approach" appeared in 1979. Basically, the development of the binomial model was spurred by an attempt by Professors Cox, Ross and Rubinstein to find an easier way to teach their students how the Black and Scholes formula (which we will have an explanation in later chapters) works. Just they introduced the Binomial model to option pricing. And Binomial model is an extremely powerful tool for pricing a wide variety of options. So popular and very widespread a tool it is in today's world.

The binomial model has proved over time to be the most flexible, intuitive and popular approach to option pricing. It is based on the simplification that over a single period (of possibly very short duration), the underlying asset price can only move from its current price to two possible levels. The general formulation of a stock price process that follows the binomial is shown in Figure 3.1 below.

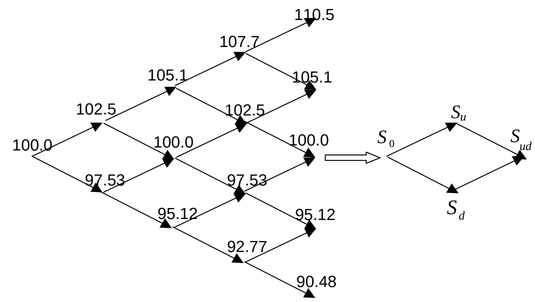


Figure 3.1 Binomial Trees from Specific Case to General Case

(General Formulation for Binomial Price Path)

In Figure 3.1, S_0 is the current stock price; the price moves up to S_u with probability p and down to the S_d with probability $(1 - p)$ in any time period.

For details on binomial formulation, it is necessary and convenient to bring in the ***Replicating portfolio*** used by Black and Scholes, which is a portfolio composed of the underlying asset and the risk-free asset that had the same cash flows as the option being valued. By following this “replicating portfolio” Black and Scholes came up with their final formulation, as we will see later. We introduce this portfolio here to derive the binomial formulation. The objective of establishing a replicating portfolio is to combine risk-free borrowing/lending and the underlying asset, with which we can create the same cash flows as the option being valued.

The tree in Figure 3.1 is named a recombining tree, of which the most important property is that, an up move followed by a down move is at any time exactly the same as a down move followed by an up movement (Chriss, 1997). We, in this paper, will mainly study these sorts of trees. The only assumption we need here is that there are no arbitrage opportunities for an investor. In terms of the definition of the replicating portfolio, the value of the option is equal to the value of the replicating portfolio. The replicating portfolio for a call with strike price *K* will involve borrowing $B and acquiring   of the underlying asset. In the general formulation above, where we can consider a stock price, which can either move up to S_u or down to S_d ($u > 1$, $d < 1$) in any time period. If the stock price moves up to S_u, we suppose that the payoff from the option is C_u; if the stock price moves down to S_d, we suppose the payoff from the derivative is C_d. This situation is illustrated in Figure 3.2.

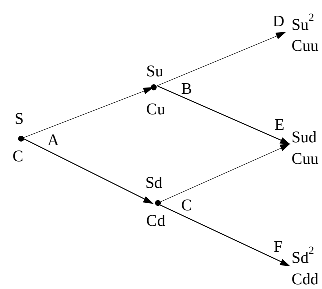


Figure 3.2 Stock and Option Prices in General Two-step Tree

We create a replicating portfolio consisting of a long position in shares and a short position in one option. We calculate the value of that makes the portfolio risk-free. If there is an up movement in the stock price, the value of the portfolio at the end of the life of the option is $(S_u \quad - C_u)$, and the value of the portfolio is $(S_d \quad - C_d)$ if a down movement. The two are equal only when

$$S_u \quad - C_u = S_d \quad - C_d \Rightarrow \Delta = \frac{C_u - C_d}{S_u - S_d}. \qquad (3.1)$$

Because of the time value, a risk-free interest rate will be considered in terms that the portfolio is risk-free.

We assume that the time at node B or C in Figure 3.2 is T and the risk-free interest rate is r, then the present value of the portfolio will be $(S_u \quad - C_u)\, e^{-rT}$.

The cost of setting up the portfolio is: $(S \quad - C)$

It follows that $(S_u \quad - C_u)e^{-rT} = S \quad - C \Rightarrow C = e^{-rT}[pC_u + (1- p)C_d]$ by substituting from equation (3.1) and simplifying, in which $p = \dfrac{e^{rT} - d}{u - d}$.

This is for one-step binomial process. By following the same logic, we can value an option in a two-step (two-period) binomial tree. In the process of a two-period tree, we start with the last time period and move backwards in time until the current point in time. Further, we can, in iterative way, proceed the valuation[5] for multi-period binomial tree form backwards in time to the start point. Since the calculation for multi-period binomial tree is too complicated and tedious, thus generating a general formula is rather necessary.

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[5] See Appendix 1 for better understanding.

Table 3.2 The Binomial Option Pricing Model

(Cox, Ross, Rubinstein Option Pricing Model)

The General Binomial Option Pricing Formula		Description of the inputs	
$C = S * q[a; n, p'] - E * e^{-n} q[a; n, p]$ Where $p' = \frac{u}{r} p$		*C*	Call option value
		è []	Binomial function (distribution)
		p	Risk neutral probability
a	Smallest non-negative integer greater than ln (E/Sd ^{n})/ln (u/d)		
e	Exponential function	*r*	Short term interest rate until expiration
E	Exercise price of option	*n*	Number of discrete periods until expiration
u	Possible upward movement	*d*	Possible downward movement in prices

Nonetheless, a binomial tree has several curious, and possibly limiting, properties. For example, all sample paths that lead to the same node in the tree have the same risk-neutral probability. The types of volatility – objective, subjective and realized – are indistinguishable; and, in the limit, its continuous-time sample path is not differentiable at any point.

3.3.2 Extending the Binomial Model to Continuous Time—Black-Scholes Option Pricing Model

The binomial pricing model can be extended to derive a continuous time equivalent—Black-Scholes model if we hold the amount of calendar time (one year for example) constant and divide it into more and more binomial tree's nodes. We will define *T* as the life of the option expressed as a fraction of a year and will divide *T* into *n* smaller time intervals. As *n* becomes larger, the calendar interval between the binomial tree's nodes becomes shorter and shorter until, at the limit, we have a continuous stochastic process[6].

Then we need to calculate the up and down movements, *u* and *d*, in a one-step binomial tree relative to the annual standard deviation of a stock's rate of return (equal to annual risk-free rate r_f). Cox, Ross, and Rubinstein in 1979 proved the following relationships:

$$u = e^{s\sqrt{T/n}} \text{ and } d = e^{-s\sqrt{T/n}}$$

where:ó is the annualized standard deviation of returns
T is the time to maturity (expressed as a fraction of a year)
n is the number of binomial steps

These two formulas are extremely useful because they construct a bridge between continuous time variables like the annualized standard deviation, σ in Black-Scholes model, and discrete time variables like *u* and *d* in binomial option pricing model.

The continuous-time option pricing formula[7] was derived by Black and Scholes in 1973 as follows:

$$c = SN(d_1) - Xe^{-rT}N(d_2)$$

[6] The binomial formula can also be used to model a jump stochastic process as a limiting case. See Cox, Ross, and Rubinstein [1979, 254-255] for the derivation. With a jump process the stock price will usually move in a smooth deterministic way but will occasionally experience sudden discontinuous jumps.
[7] In this paper we only cover the call option pricing model for simplicity. One who is interested in put, may derive put pricing model by combining put-call-parity formula with call option pricing model.

where: $$d_1 = \frac{\ln(S/X) + (r + s^2/2)T}{s\sqrt{T}}$$

$$d_2 = d_1 - s\sqrt{T}$$

c = Price of a call option
S = Current price of the asset
X = Exercise price
r = Risk free interest rate per annum at current time, with continuous compounding, for an investment maturing at time T
T = Time to expiration of the option
ó = Standard deviation of returns
N (•)[8] = Normal distribution function

The principal assumption behind Black-Scholes model is that returns are of lognormal distribution; besides, there are a number of other assumptions which it relies upon[9]:

- The underlying asset can be bought and sold freely, even in fractional units
- The underlying asset can be sold short, and the proceeds are available to the short seller
- The underlying asset pays no dividends or other distributions before maturity
- Lending and borrowing is possible at the same riskless interest rate, which accrues continuously in time
- The option is European style, and cannot therefore be exercised prior to maturity
- There are no taxes, transactions costs, or margin requirements

[8] See Appendix 3.
[9] See Galitz L. (1994) "Financial Engineering: Tools and Techniques to Manage Financial Risk" P216, PITMAN PUBLISHING.

- The underlying price is continuous in time, with no umps or discontinuities
- Variability of underlying asset prices and interest rates remain constant throughout the life of the option

In view of the limitations of Black-Scholes, the model above was extended later by Merton, Black, and some others. We summarize them below.

Table 3.3 Summary for Modified Black-Scholes Model

	Option Pricing Formulas
Option on a stock with a *continuous dividend yield* at rate q[10] (Note: $d_2 = d_1 - s\sqrt{T}$)	$c = Se^{-qT}N(d_1) - Xe^{-rT}N(d_2)$ where $d_1 = \frac{\ln(S/X) + (r - q + s^2/2)T}{s\sqrt{T}}$
Options on currency[11] (we define S as the *spot exchange rate*, and r_f is riskfree *interest in base foreign currency, while r in the pricing currency*) (Note: $d_2 = d_1 - s\sqrt{T}$)	$c = Se^{-r_f T}N(d_1) - Xe^{-rT}N(d_2)$ where $d_1 = \frac{\ln(S/X) + (r - r_f + s^2/2)T}{s\sqrt{T}}$
Options on future[12] (S replaced by *future price* F) (Note: $d_2 = d_1 - s\sqrt{T}$)	$c = e^{-rT}\left[FN(d_1) - XN(d_2)\right]$ Where $d_1 = \frac{\ln(F/X) + (s^2/2)T}{s\sqrt{T}}$

[10] See R. Merton, "Theory of Rational Option Pricing," *Bell Journal of Economics and Management Sccience*, 4 (Spring 1973), 141-83.
[11] This arrives at the so-called Garman-Kohlhagen model for currency options.
[12] See F. Black, "The Pricing of Commodity Contracts," *Journal of Financial Economics*, 3 (March, 1976), 167-79.

3.3.3 Comparison between Binomial Model and Black and Scholes Model

Instead of assuming a smooth lognormal distribution like Black and Scholes, Cox, Ross and Rubinstein assume this jagged Binomial distribution in their model. Their equation is often used for pricing American options on foreign exchange. While it initially looks different from the Black and Scholes formula presented above, it is really based upon similar assumptions. Once again, the model has the underlying price S minus the Strike price E which determines the intrinsic value just like the Black and Scholes model. The difference between the binomial and Black and Scholes models is the θ function in the binomial formula. This is just like the lognormal distribution in the Black and Scholes model but here the Binomial distribution with discrete time (assuming that the change in market prices is not continuous but can only vary by a minimum amount) is used. Essentially, one takes the binomial tree diagram turns it sideways, and puts it on a listed graph centered at the spot underlying price and the discounted exercise price. In fact as the binomial approach is extended to continuous time, the curve becomes identical to the lognormal distribution and the Binomial result is equal to the Black and Scholes result (Tompkins, P81, 1994)

Also, the Binomial model is often used for American style option where an early exercise feature exists. The Binomial model estimates the value of early exercise by assuming that at a particular price for the underlying asset, the option will be exercised. Thus, all subsequent branches in the tree diagram have been “ pruned”. Because the option after that point no longer exists, it is no longer necessary to continue the branching process. When the option price is calculated (assuming the possibility of early exercise), once again all the possible prices for the underlying asset are multiplied by their probability of occurrence to give us the option price adjusted for the exercised event.

After comprehending the basic concepts of options, after grasping the approaches of option-pricing, it is necessary and useful to simply introduce the payoffs from basic position to option. This will be the basis for the payoffs of real options in later sections.

3.3.4 Payoffs from Options

One of the parties to an option assumes to take a *long potion*, i.e., buy the option. The other party assumes to take a *short position*, i.e., sell or *write* the option. Thus the benefits (losses) of writer of an option corresponds the losses (benefits) of purchaser of the option.

Four basic combinations are composed of the long/short position and put/call option:

- A long position in a call option.
- A long position in a put option.
- A short position in a call option.
- A short position in a put option.

As mentioned above we for simplicity take European option into account to analyze the payoff to investor at maturity in terms of these 4 kinds of positions. Then the initial cost of the option is not included in the calculation.

If X is the strike price and S_T is the final price of the underlying asset, the payoff from a long position in a European call option is:

$$\textbf{Max}\ (S_T - X, 0)$$

This reflects the fact that the option will be exercised if $S_T > X$ and will not be exercised if S_T X. Similarly, we can calculate the payoffs for other 3 kinds of positions and for having a distinct comparison among them please see the Table 3.4:

Table 3.4 Payoffs from Positions in European Options

Positions	*Payoffs*
Long Call	Max ($S_T - X$, 0)
Short Call	– Max ($S_T - X$, 0)
Long Put	Max ($X - S_T$, 0)
Short Put	– Max ($X - S_T$, 0)

Much more straightforwardly, we illustrate these payoffs graphically:

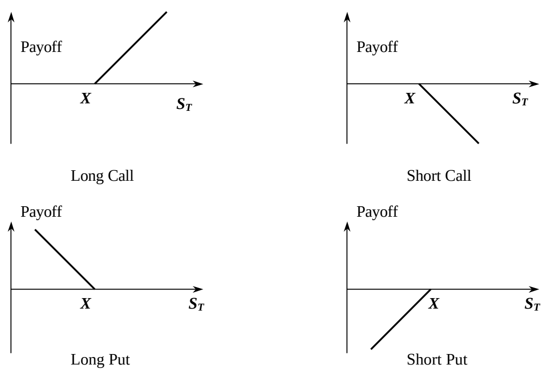


Figure 3.3 Payoffs from Positions in European Options

(Hull, 1997)

Until now we almost have reviewed financial option from basic concepts to option pricing model as well as the comparison between binomial model and Black-Scholes model and the payoffs from options. After mastering the bases above on financial option, it is quite naturally to transfer our attention to a more flexible but sophisticated field—Real Options.

4 Real Options

This chapter offers the general conceptual framework—*strategic NPV* model, which based on real option theory. A real option is the option on real assets, which takes managerial flexibility of a corporation into account, which is the collection of real investment opportunities. In "real options", the real is because it is an investment in operating as opposed to financial capital, and the option because it need never be exercised[13]. We integrate both strategic points—how to occupy the dominant position in competitive market and operating points—we can either defer, abandon, and expand a project. Concretely, the real option theory can be implemented in corporate finance—"To me, all kinds of business decisions are options", said by CFO Judy Lewent of Merck. In this paper, 3 topics below will be covered.

- A Project or a firm as real option
- Capital-budgeting, capital-structure decisions as real options
- Joint Ventures as real options

We will first introduce real option theory, and then analyze some real cases.

4.1 Survey of Real Option

The distinctive characteristic of an option contract is its asymmetric payoff profile. Real option develops this feature of option further. It offers us more

[13] See Myers (1984) for an interesting qualitative discussion of real options and Mason and Merton (1985) for an extensive analytical treatment.

chances to make a rather sensible choice. Customarily, some basic ideas are mentioned below when talking about real options.

- Options to defer
- Options to expand
- Options to abandon

4.1.1 The Option to Defer a Project

When investing a project, real option theory judges the time value of a project more noticeably than the traditional investment analysis method. Thus in calculating the NPV we, in addition to considering the discount rate and the cash flow, should also think over the time value—the value generated by deferring the project maturity time. We can illustrate this in terms of a call option.

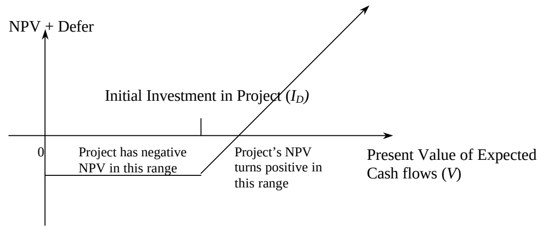


Figure 4.1 The Option to Defer a Project

(Damodaran,…)

In Figure 4.1, the underlying asset is the project, the strike price of the option is the investment in the project, and the life of the option is the period, prior to which the firm has rights to take on the project. Just prior to the expiration (opportunity disappears), the opportunity (real option) value, *V*, will be analogous to an American call option, **Max (*V-I_D*, *0*).** Therefore, by integrating the real option analyses, the firm value should be modified to be Expanded NPV, which equals the NPV of the firm plus the value of the option to defer a project. We can make a formula in terms of this idea below:

NPV of the firm < NPV Project (firm) + Opt (growth options or opportunity)

where: Opt is the NPV of options.

4.1.2 The Option to Expand a Project

When the situation of a project turns out favorable, investor (management) can accelerate the pace of investment step (expand their project). In this case, a firm should accept the negative NPV of the initial project to obtain the much higher positive NPV in the coming future. This is similar to a call option to acquire an additional part (*x%)* of the base-scale project.

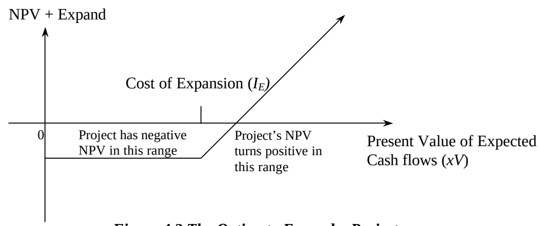


Figure 4.2 The Option to Expand a Project

In Figure 4.2, the underlying asset is the additional part (*x%)* of the base-scale project (V), and the strike price of option is the follow-on cost of expansion (I_E). Then, the total value of a project with opportunity option will be expanded as the base-scale value of a project plus the NPV of the options, i.e.,

V **+ max (*xV* - I_E, *0*)**. This expand option is of strategic significance for capturing the future growth opportunities. This expand option diverts traditional static NPV (unprofitable) to dynamic option-based NPV. This expand option makes a seemingly unprofitable project worth undertaking (attracting more opportunities).

4.1.3 The Option to Abandon a Project

One will abandon a project for salvage value (i.e., the resale value of its capital equipment and other assets on the secondhand market) when its cash flows do not measure up to expectations. By contrasting the options to defer or abandon, the option to abandon can be viewed as an American put option as below:

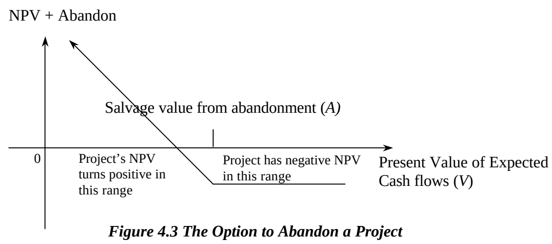


Figure 4.3 The Option to Abandon a Project

In Figure 4.3, the underlying asset is the project's current value (*V*), strike price is the salvage value from abandonment (*A*). In this way, the option-based NPV of the project is: ***V*** **+ max (*A* – *V*, *0*) Þ max (*V*, *A*).**

4.1.4 Managerial Flexibility, Asymmetry, and Strategic (Expanded) NPV

In traditional NPV analysis, the expected cash flows and terminal project value would be discounted at an appropriate risk-adjusted rate. In the results of calculations one very often encounters the negative NPV so that the one finally forgoes the project. And we can plot for this case with normal distribution (see Figure 4.4). In view of the absence of option in traditional valuation, it is rather necessary to look through the option-based strategic valuation, which accounted for adaptabilities of management, which comprised psychological factors, which involved with political elements and so on.

As depicted in financial strategy part, the financial option can improve, for investors, the upside potential profits while limiting downside losses. So does the real option. Reinforcing the management's flexibility can heighten the management expectations. Thus, a firm can maximize its wealth by flexibly adapting new uncertainty, applying new information and so on while reducing its limitations. We can introduce this asymmetry of management flexibility by applying the probability distribution of NPV.

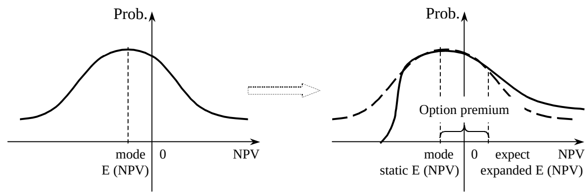


Figure 4.4 Managerial Flexibility or Options Introduce an Asymmetry into the Probability Distribution of NPV

(Trigeorgis, 1996)

In Figure 4.4, Left: Symmetric distribution of NPV in absence of managerial flexibility. Static expected NPV coincides with the mode (or most likely estimate). Right: Asymmetric (skewed to right) distribution of NPV caused by managerial flexibility (e.g., options to defer or abandon). The (expanded) expected net present value exceeds the mode (= static expected NPV):

Expanded NPV = Static NPV + "option premium"
(Trigeorgis, 1996)

The formula above implies the shiftiness of capital budgeting from traditional NPV to options-based NPV, from the static to dynamic, from passive to active. This formula involves a couple of interactions between a firm and its competitors, with operating methods, with advanced managerial skills and so on.

For instance, in 1998, Ericsson initiated development of a new business model called Ericsson Global Business Model (EGBM), which is intended to increase efficiency and reduce lead-times through standardized and improved routines and processes. This model, if used successfully, will confer manager effective skills or thoughts to improve the managerial flexibility so as to bring the firm a fruitful harvest.

Now we have had the formula for strategic NPV model, however, how can we measure and use this formula? That is to say, what are the components included in this formula?

4.2 Inputs for Real Options

We have primarily drawn a picture for a real option on valuing a project. However, what about the inputs compared to the financial option?

The owner of a project (an investment opportunity) has the right—but not the obligation—to acquire the present value of expected cash flows from an investment prior to the anticipated date when the investment opportunity will cease to exist. Thus, we can illustrate a close analogy between such real options and call options on stocks.

Table 4.1 Comparison of Inputs between a Call Option and a Real Option
(Trigeorgis, 1996)

Call option on a stock	***Real option on a project***
Current value of a stock	(Gross) PV of expected cash flows
Exercise price	Investment cost
Time to expiration	Time until opportunity disappears
Stock value uncertainty	Project value uncertainty
Riskless interest rate	Riskless interest rate

We will analyze the concrete cases in reality by scrutinizing these inputs in the Table 4.1 in the case studies sections. For instance, the variance (volatility) of the project under uncertainty, the risk-free interest, and cost of defer (expand) will be estimated for the strategic valuation.

4.3 Valuing a Firm as a Real Option

We shall cover this section from the basic features of a firm to the newest understanding of them from the angle of options.

4.3.1 Stocks and Bonds

As we stated, in traditional discounted cash flow models, a firm is valued by estimating cash flows over a long time horizon (often over an infinite period) and discounting the cash flows back at a discount rate that reflects the riskiness of the cash flow. The value of equity is obtained by subtracting the value of debt from value. However, the discounted cash flow models understate the value of equity in firms with high financial leverage and negative operating income, since they do not reflect the option that equity investors have to liquidate the firm's asset.

Black and Scholes [1973] (Copeland and Weston 1992) suggest that the equity in a levered firm can be thought of as a call option. When shareholders issue bonds, it is equivalent to selling the assets of the firm (but not control over those assets) to the bondholders in return for cash (the proceeds of the bond issues) and a call option.

To reduce the analogy to its simplest form, we assume:

- The firm issues zero-coupon bonds that prohibit any capital disbursements (such as interest payments) until after the bonds mature T time periods hence
- M&M theorem holds
- There is a known non-stochastic risk-free rate of interest.
- There are homogeneous expectations about the stochastic process that describes the value of the firm's assets.

The equity in a firm is a residual claim, that is, equity holders lay claim to all cash flows left over after other financial claim holders (debt, preferred stock etc.) have been satisfied. If a firm is liquidated, the same principle applies; equity investors receive whatever is left over in the firm after all outstanding debt and other financial claims are paid off. The principle of limited liability protects equity investors in publicly traded firms if the value of the firm is less than the value of the outstanding debt, and they cannot lose more than their investment in the firm. The payoff to equity investors, on liquidation can therefore be written as:

Payoff to equity on liquidation = V-D if V > D
= 0 if V D

where:V = Liquidation Value of the firm

D = Face Value of the outstanding debt and other external claims

A call option, with a strike price of K, on an asset with a current value of S, has the following payoffs:

Payoff on exercise = S-K if S > K
= 0 if S K

Equity can thus be viewed as a call option the firm, where exercising the option requires that the firm be liquidated and the face value of the debt (which corresponds to the exercises price) paid off, as shown in below Figure 4.5.

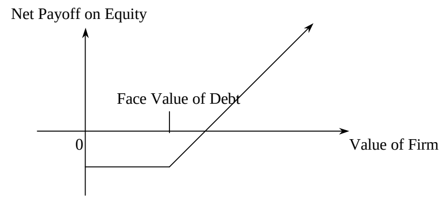


Figure 4.5 Payoffs on Equity as Option on a Firm

In other words, shareholders own a call on the firm with exercise as well as bondholders own the firm and have sold a call on the firm to shareholders. Of course, it is also possible to explain the situation in term of put option. That is,

For shareholders,

- Shareholders own the firm.
- Shareholders owe K in interest and principal to bondholders.
- Shareholders own a put option on the firm with exercise price, K.

For bondholders,

- Bondholders are owed K in interest and principal.
- Bondholders have sold a put on the firm to the shareholders.

The option-pricing model offers a great deal of insight into the way that capital structure changes may affect shareholders and bondholders. Shareholders and bondholders have different objective functions, and this can lead to agency problems, whereby shareholders expropriate wealth from bondholders. The conflict can manifest itself in a number of ways. For instance, shareholders have an incentive to take riskier projects than bondholders, and to pay more out in dividends than bondholders would like them to.

4.3.2 M&M vs. Option Approach

In the following cases, we rely on M&M assumptions holding. Furthermore, we also assume that any changes that affect the systematic risk of various securities, or their expected rate of return, are unanticipated changes. To the extent that changes in the value of securities are unanticipated, it is possible that there may be a redistribution of wealth from one class of security holder to another.

We also assume that two-fund separation does not apply. Two-fund separation implies, among other things, that all individuals hold the same portfolio of risky assets, namely, the market portfolio. For individuals holding both the equity and risky debt of a firm, any offsetting change in the market value of the debt and equity claims against the firm will not change their wealth position. Therefore they would be indifferent to the redistribution effects that we are about to discuss. It is necessary, then, to rule out two-fund separation and discuss the wealth of shareholders and bondholders as if they were separate and distinct. If shareholders are not constrained by the indenture provisions of debt from issuing new debt with an equal claim on the assets of the firm, then current bondholders will experience a loss of wealth when new debt is issued. It is possible to increase the book value debt-to-equity ratio by issuing new debt and using the proceeds to repurchase equity. In this way the assets of the firm remain unchanged. If the new debt has equal claim on those assets, then the current bondholders end up with only a partial claim to the assets of the firm, whereas before the new debt was issued, they had a complete claim on the assets. Clearly, this approach puts current bondholders in a riskier position, and they are unable to charge more for the extra risk because the discounted value of their bonds has already been paid (i.e., they cannot raise their coupon payments once the bonds have been issued). Consequently, the market value of their bond will fall. At the same time, the value of the firm remains unchanged, and new bondholders pay a fair market price for their position. Therefore the value that is expropriated from current bondholders must accrue to shareholders, who are the residual claimants of the firm. Their wealth increases. This is called the bondholder wealth expropriation hypothesis.

The theory of option pricing argues that in a world with no transactions costs or taxes the wealth of shareholders is increased by greater financial leverage. Meanwhile, the M&M theorem argues that under same set of assumptions the value of shareholders' wealth is unaffected by changes in capital structure. The crucial difference is that option pricing assumes that unanticipated redistributions of wealth are possible. To the extent that bondholders can appropriately assess the probability of shareholders' ability to expropriate their wealth, they can charge a rate of return that adequately compensates them for their risk or they can carefully write bond indenture provisions that restrict the actions of shareholders. Either way they can protect themselves against anticipated redistribution effect. A more detailed mathematical explanation can be found in Ingersoll's (1987) work.

4.3.3 About the Assumptions of Option Approach on Equity Valuation

We have made some assumptions on the real option approach to capital structure changing. Among them are the following:

- There are in general two claims in a firm — debt and equity
- There is only one issue of debt outstanding, and it can be retired at face value.
- The debt has a zero coupon and no special features (convertibility, put clause etc.)
- The value of the firm and variance in that value can be estimated.

Each of these assumptions is made for a reason. First, by restricting the claimholders to two, the problem is made more tractable by introducing other claimholders, such as preferred stock makes it more difficult to arrive at a result, albeit not impossible. Second, by assuming only one zero-coupon debt issue that can be retired at face value anytime prior to maturity, the features of the debt are made to correspond closely to the features of the strike price on a standard option. Third, if the debt is coupon debt, or more that one debt issue is outstanding, the equity investors can be forced to exercise (liquidate the firm) at these earlier coupon dates if they do not have the cash flows to meet their coupon obligations. Finally, knowing the value of the firm and the variance in that value makes the option pricing possible, but it also raises an interesting question about the usefulness of option pricing in the valuation context. If the bonds of the firm are publicly traded, the market value of the debt can be subtracted from the value of the firm to obtain the value of equity much more directly. The option pricing approach does have its advantages, however. Specifically, when a firm's debt is not publicly traded, option-pricing theory can provide an estimate of value for the equity in the firm. Even when the debt is publicly traded, the bonds may not be correctly valued, and the option-pricing framework can be useful in evaluating the values of debt and equity. Finally, relating the values of debt and equity to the variance in a firm's value provides some insight into the re-distributive effects of actions taken by the firm.

4.3.4 Other Corporate Financial Claims

Not only equity and bonds can be implemented by the real option approach, but also other corporate financial claims. We will further show two major financial claims issued by corporations can be valued by option approach.

------Warrant

A warrant gives the holder the right to buy common stock for cash. In this sense, this is very much like a call. Warrants are generally issued with privately placed bonds, though they are also combined with new issues of common stock and preferred stock. In the case of new issues of common stock, warrants are sometimes given to investment bankers as compensation for underwriting services.

The differences in contractual features between warrants and the call option are that warrants have longer maturity periods and some of them are actually perpetual, meaning they never expire at all.

To prevent arbitrage the warrant price, W, will be a fraction of the call price, C, as shown below:

$$W = \frac{1}{1+q} C$$

where: q= the ration of warrants to shares outstanding

Because the warrant and the call are perfectly correlated, they will have exactly the same systematic risk and therefore the same required rate of return.

The value of warrant is as follows:

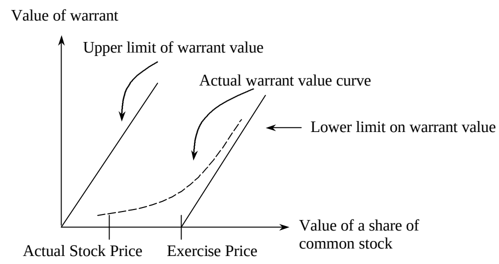


Figure 4.6 Value of Warrant

(Ross, Westerfield and Jaffe, 1993)

------Convertible Bonds

A convertible bond gives the holder the right to exchange the bond for common stock.

The value of a convertible bond can be described in terms of three components: straight bond value, conversion value, and option value.

The straight bond value is what the convertible bonds would sell at if they could not be converted into common stock. It will depend on the general level of interest rates and the default risk.

Conversion value is what the bonds would be worth if they were immediately converted into the common stock at current prices. Typically, conversion value is computed by multiplying the number of shares of common stock that will be received when the bond is converted by the current price of the common stock.

The value of a convertible bond will be generally both the straight bond value and the conversion value. This occurs because holders of convertibles need not convert immediately. Instead, by waiting they can take advantage of whichever is greater in the future, the straight bond value or the conversion value. This option to wait has value, and it raises the value over both the straight bond value and conversion value.

When the value of the firm is low, the value of convertible bonds is most significantly influenced by their underlying value as straight debt. However, when the value of the firm is very high, the value of convertible bonds is mostly determined by their underlying conversion value. This is illustrated in the bottom portion of Figure 4.7.

The bottom portion of Figure 4.7 implies that the value of a convertible bond is the maximum of its straight bond value and its conversion value, plus its option value:

Value of convertible bond =

The greater of (Straight bond value, Conversion Value)+ option value

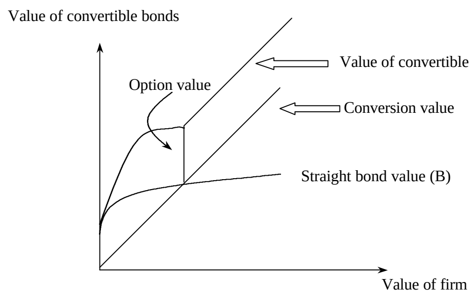


Figure 4.7 Valuations for Convertible Bonds

(Ross, Westerfield and Jaffe, 1993)

So far, we have had a general review of the real option approach applicable to financial claims in capital structure and we also would like to point out that beside that discussed above, the option approach can be also applicable to other financial claims, i.e. subordinated debt, callable bonds.

And also a brief review of a firm decision on a project that will definitely influence the value the firm was given. In later sections, we are to explain the real option approach implemented in project and equity numerically by case study.

Up to now, we have talked either about a stand-alone firm or a stand-alone project, however, the things we very often meet in reality are interrelated together, joint ventures for instances.

4.4 Compound Options Correspond to Interrelated Projects

A project that can be evaluated as a stand-alone investment opportunity is referred to as a ***simple option***. Standard equipment-replacement and maintenance projects are examples of independent opportunities whose value upon exercise is limited to the underlying project in and of itself. ***Compound options*** are options in which the underlying security is another option. Compound options are also known as ***mother-and-daughter options*** and ***options on options***. A simple option is one that stands alone, and which is out of consideration for interaction with other options. Compound options are composed of a couple of options which interact among each other. In doing so, the strategies applied by each option holder in compound options interact with each other. This leads to the complexity of the compound options. In terms of the characteristics of financial options, the deeper the strategies interrelate, interact with each other, the higher the value of compound options are.

Compound options provide investors with the benefit of a guaranteed price for the option at a date in the future. Thus, Compound options are more expensive to purchase than the underlying option, as the purchaser has received a price guarantee and effectively extended the life of the option. Similarly to the types of financial option, 4 types of compound options are as follows:

1) Call on a call
2) Put on a call
3) Call on a put
4) Put on a put

Rooted in above option theories, we can as expected go to the next part—implementation of real options. Before implementation, it is rather necessary to keep in mind the advantages and disadvantages of real options.

4.5 Advantage and Disadvantage of Real Option Approach

Like most financial analytic tools, the real option approach has its own advantages and disadvantages that were pointed out by a lot of empirical studies. Bierman Harold and Seymour Smidt (1993) point out the both-aspects of real option approach.

4.5.1 Advantage of Real Option Approach

1) Less chance of Overlooking Future Decision Strategies: Advocates of the option valuation approach claim that conventional capital budgeting procedures may result in overlooking important relevant values, in particular the strategic values of future investment opportunity that will be opened up by a current project that may not be profitable taken by itself. The option-valuation approach helps overcome this possibility.

2) Better valuation Procedures: Option-Valuation procedures fall into two main groups. The most commonly used valuation procedure in securities markets is valuation by arbitrage, as discussed earlier in this thesis. This requires finding an existing asset or assets whose values are known and in which it is possible to hold either long or short positions, and constructing a portfolio whose values and cash payoffs replicate those of the option. Replicating the payoff of a call option, a common stock with a portfolio consisting of the stock and a risk-less bond is an example. This arbitrage approach has proved successful in many situations in which the option is a derivative security-that is, one whose value is tied by contract to the value of some other security or securities. It is possible to construct arbitrage

arguments to value real assets. The most commonly successful situations in which this has been attempted are for assets that produce uniform commodities for which there are well-established markets.

Most financial economists would agree that valuation by market arbitrage is more accurate than valuation by projecting future cash flows, when it is applicable. But arbitrage arguments cannot be easily be used for most industrial investments, so even if options are identified, they must be evaluated using the present-value approaches.

The cash flows associated with many real options have quite different profiles than the cash flows associated with ordinary investments. Critics doubt if analysts using conventional present-value techniques will properly value these options. This is especially true if the expected cash flows are discounted by a weighted average cost of capital. For example, the appropriate cost of capital to use for a call option varies from year to year and changes as the value of the option changes. Also, volatility increases the value of options, but is conventionally considered to decrease the value of traditional investment.

3) Fewer Cash Flow Strategies to Be Considered. If an option has to be evaluated on the biases of its future cash flows, there may be a great many-possibly, infinitely many-strategies. The value of an option takes all of these strategies into account without the necessity of explicitly considering each one.

4.5.2 Disadvantages of Real Option Approach

1) Less of a Standard to Guide Future Operations. If an investment has option-like characteristics, these options must be considered in order to decide whether the investment is acceptable. If the investment is accepted, it is important that the correct operating decisions be made, so that options are

exercised when appropriate. For this purpose it may be helpful to have an operating rule, such as exercising the option at expiration if it is in the money. Sometimes, valuing the option provides an operating rule as a by-product of the valuation procedure. If this is not the case, then care should be taken to provide operating management with some guidelines about what decisions are optimal.

2) Hidden Assumptions. A large option value may swing the decision, but the implicit economic (cash flow assumptions may be hidden, preventing management from effectively evaluating the assumption.

As we have seen the both sides of the real option approach, now it is time to go to the next part—Implementation of Real Option approach, in which we shall perform case studies.

Part III: Implementation of Real Options

In this part, we shall go on with our case studies, in which real option theory will be incorporated. We mainly discuss the cases by means both of calculation and theoretical explanation.

In numerical work, most researchers assume that the value of a completed project follows a continuous-time stochastic process, and apply the Black-Scholes option-pricing model (with appropriate extensions) to derive the stochastic partial differential equations which govern the option value. In particular, it is assumed as in the Black-Scholes case that a hedge can be created between the option and a "twin security" whose return is perfectly correlated with the option pay off. We followed this rule in our R&D, equity valuation, and joint ventures cases.

In explanation work, we extend the real option approach by explaining the business event recently happened.

5 Case Studies

Not only a project but also a corporation can be viewed as a real option. Particularly joint ventures are created as real options, said Bruce Kogut (1994). In this part we scrunize the real options from part of corporation (a project) to integration (joint ventures). In below sections, we are to implement the real option approach into these three aspects.

5.1 Projects as Real Options

By taking real options into account, the project can be strategically analyzed and predicted.

5.1.1 Valuation for R&D

A firm, which performs the R&D, cherishes a prospective future, and its R&D highly relates to its strategy. Thus how to measure R&D becomes quite significant. We shall apply the strategic NPV in the following cases.

R&D Project as a Real Option: Ericsson's new organization, involves the establishment of a new corporate function for technology. It will coordinate Ericsson's Research and Development (R&D) and deal with standardization matters, IPR (Intelligence Property Rights) and patents and issues related to strategic partnerships.

Ericsson invested SEK 25,189 millions, 13.7% of sales, in research and development in 1998. A firm will undertake the R&D project only if the present value (*V*) of expected cash flow from R&D exceeds the costs (I) of R&D. Ericsson should be no exception. So what about the R&D in 1998?

A new product potentially generated by R&D can be valued as a call option illustrated as Figure 5.1, where the product generated by R&D is the underlying asset.

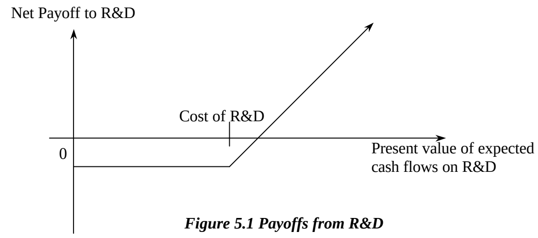


Figure 5.1 Payoffs from R&D

In terms of the formula of call option,

Payoff from owning a product generated by R&D = V – I if V > I
= 0 if V   I

R&D Project Analyses: AXD 301 is a new generation high-performance IP (Internet Protocol) & ATM (Asynchronous Transfer Mode) Switching System designed for use in public or large enterprise networks. AXD 301 has full ATM functionality and also supports telephony (together with AXE) and IP using Multi Protocol Label Switching (MPLS).

A present value of cash flows from AXD 301, based on the contract with BT (British Telecom) was valued at SEK 3,400 million. The initial cost of fulfilling this contract was estimated as SEK 4,200 million if the first switch went to live in June 1999. (*Here we assume that AXD 301 would first be applied for BT*). Thus, if only from this contract, the product AXD 301 will generate a negative NPV of SEK 800 million.

However, assume that by developing this new product, the Ericsson can also apply same technology to other orders. This will lead to option to expand into more quantities contracts over corresponding next 5 years. For instance, Ericsson may acquire the profits from following orders from Canadian BridgePoint (valued SEK 530 million) in March 1999, and from RomTelecom (Romania, valued SEK 830 million) in November 1999, and so forth. The total cost of R&D for AXD 301 will be SEK 4,000 million, and it will be rational and sensible to undertake the R&D for AXE 301 only if the PV of the expected cash flow exceeds SEK 4,000 million. But at the present, the PV from the expansions is estimated to be only SEK 3,600 million. Otherwise, Ericsson will expand the AXD 301 without more ado. Ericsson doesn't know exactly if there will be newer products coming up by other companies, and there should be a substantial uncertainty about this estimation. The variance is 0.16 in terms of the characteristics of high technology industry. In addition, we assume that 5-year risk-free interest rate is 5.5%.

That is to say, we can calculate the Strategic NPV below in terms of modified Black-Scholes formula[14]—Option on future by using data since the underlying asset, product AXD 301, is based on the future contracts between Ericsson and other firms.

$$c = e^{-rT}\left[FN(d_1) - XN(d_2)\right]$$

where: $d_1 = \frac{\ln(F/X) + (s^2/2)T}{s\sqrt{T}}$

$d_2 = d_1 - s\sqrt{T}$

[14] See summary for modified Black-Scholes model in previous part.

Value of underlying asset = S = F = PV of cash flows = 3,600 m SEK
Strike Price = Cost of product generated by R&D = K = 4,000 m SEK
Variance in underlying asset's value = $\sigma^2 = 0.16$
Time to expiration = Period of contract = T = 5 years
5-year risk-free interest rate = $r = 5.5\%$

The factors above could yield the following estimates for the d and N (d).

$d_1 = 0.3294$ $N(d_1) = 0.6293$

$d_2 = -0.5650$ $N(d_2) = 0.2877$

Then, the value of the option (product) can be estimated as below:

Call value = $e^{-0.275}$ (1114.68) = SEK 846.68 million

Adding this value to the NPV of the original project from R&D will derive strategic NPV:

NPV of original project = 3,400 - 4,200 = -800 million
Value of Option to expand = 846.68 million
Strategic NPV = -800 + 846.68 = 46.68 million

Figure 5.2 Calculations for Strategic NPV on R&D[15]

Consequence for R&D: A firm that spends large quantities of money on R&D has, in general, a pessimistic NPV when it evaluates the cash flows (expenses) since payoffs from R&D are highly involved with future perspectives of the project. As previously analyzed, R&D can be viewed as a call option—the R&D itself is underlying asset, and the expense spent on R&D are the strike price of the call option. The new project coming up from R&D provides the payoffs for the firm. The R&D yields in general high returns since it is of high-risk industry (in volatile environments). In other words, R&D should offer high expectation value since its variance is positively correlated with the value of the call option[16].

[15] In the calculations, one can obtain the value N(d) in Appendix 3.
[16] See Table 3.1

The fact that there may exist some competitors taking on the rival R&D projects accelerates the decrease of payoffs from the R&D. Thus, inputs estimation for real options (project analyses) is of biases. This leads the firm to adjusting variance for valuations in different stages of project development, application, and transformation.

As a consequence particularly for this case, it is sensible for Ericsson to perform this R&D even though it has a negative NPV because of its future perspectives. As advise, Ericsson should strategically improve its market study in accordance with its R&D's diligence.

5.1.2 Valuation for Equity (Capital Structure)

Equity actually can be seen as an option. In accordance with real option definition "investment opportunity as a real option", the firm's equity is a real option for equity investors.

Euro-Tunnel Case Views

------The implication of viewing equity as a call option is that equity will have value, even if the value of firm falls well below the face value of the outstanding debt.

First let's have a look at the Euro-tunnel case provided by Aswath Damodaran (Source: see References).

Euro-tunnel was the firm that was created to build and ultimately profit from the tunnel under the English Channel, linking England and France. While the tunnel was readied for operations in the early 1990's, it was never a commercial success and reported significant losses each year after opening. In early 1998, Euro-tunnel had a book value of equity of -£ 117 million, and in

1997, the firm had reported earning before interest and taxes of -£56 million and net income of 685 million. By any measure, it was a firm in financial trouble.

Much of the financing for the tunnel had come from debt, and at the end of 1997, Euro-tunnel had debt obligation in excess of £8,000 million, including expected coupon payments. The following table summarizes the outstanding debt at the firm, with Damodaran's estimates of the expected duration for each class of debt:

Table 5.1 Euro-Tunnel Debt Value and Duration

Debt Type	Face Value (including cumulated coupon)	Duration
Short term	£935	0.50
10 year	£2435	6.7
20 year	£3555	12.6
Longer	£1940	18.2
Total	£8,865 million	10.93 years

The firm's only significant asset is its ownership of the tunnel and the value of the asset is calculated by certain assumptions:

- Revenue will grow 5% a year in perpetuity.
- The cost of goods sold which was 85% of revenues in 1997 will drop to 65% of revenues by 2002 and stay at that level.
- Capital spending and depreciation will grow 5% a year in perpetuity.
- There are no working capital requirements.
- The debt ratio, which was 95.35% at the end of 1997, will drop to 70% by 2002. The cost of debt is 10% in high growth period and after that.
- The beta for the stock will be 1.10 for the next five years, and drop to 0.8 after the next 5 years (as the leverage decreases).

The long-term bond rate at the time of the valuation was 6%. Base on the above assumptions, the asset value of £2,312 was calculated. The standard deviation in firm value was calculated 0.0335.

In summary, the inputs to the option-pricing model were as follows:

Value of the firm = S = £2,312 million
Exercise price = K =Face value of outstanding debt = £8,865
Life of the option = t = Weighted average duration of debt = 10.93 years
Variance in the value of the underlying asset = σ^2 =0.0335
Riskless rate = r = Treasury bond rate corresponding to option life = 6%

Based upon these inputs, the Black-Scholes model provides the following value for the call:

$d_1 = -0.8337$	$N(d_1) = 0.2023$
$d_2 = -1.4392$	$N(d_2) = 0.0751$

Value of the call=2,312(0.2023) – 8,865exp$^{(-0.06)(10.93)}$(0.0751)=£122 million
Euro-tunnel's equity was trading at £150 million in 1997.

Figure 5.3 Calculations for Euro-Tunnel Case

The option-pricing framework in addition to yielding a value for Euro-tunnel equity also yields some valuable insight into the drivers of value for this equity. While it is certainly important that the firm try to bring costs under control and increase operating margins, the two most critical variables determining its value are the life of the options and variance in firm value. Any action that increases (decreases) the option life will have a positive (negative) effect on equity value. For instance, when the French government put pressure on the bankers who had lent money to Euro-tunnel to ease restrictions and allow the firm more time to repay its debt, equity investors benefited, as their options became long term.

Through this case, we may explain why so many companies running with high financial leverage and negative operating income still have value in their equity.

Ericsson's Equity (1998) Valuation: While implementing the real option approach to a healthy running company, we may have a deeper view on its capability for future growth, especially its equity option embedded value can be much more over market price.

At the end of 1998, the equity in Ericsson is SEK m. 63,112 and the long-term liability is SEK m. 13,068 as well as current liability SEK m.66, 941. To value its equity, we have made the assumptions as below:

Base upon an average annual growth rate at 29% in the last 10 years, we assume the equity will grow 20% in the next 5 years.

The financial structure is treated unchanged and dividend policy is excluded.

- A discount rate on inflation is around 5%.
- Short Debt rate is 3.25%. Long-term bond rate is 6.87%. They are zero-coupon type (this is unrealistic in the real world, but it is easy for our calculations).
- A variance of 0.04 in firm weighted value is assumed based upon Ericsson's new technology features.
- Risk-free rate is 5.5%.

The debt situation is shown in the Table 5.2.

Table 5.2 Ericsson Debt Value and Maturity Period

Debt Type	Face Value (SEK m.)	Maturity Period	Future Value (SEK m.)
Short Term	66,941	0.5	69,116.58
Long Term	4,470	4.5	6,027.79
Long Term	725	2	828.04
Long Term	5,516	5	7689.59
Long Term	1,898	5	2645.91
Long Term	459	5	639.87

Thus an average duration for all the debt is estimated at 1.48 years. Also a sum of debt is SEK m. 118,216.275.

The present equity value is calculated as:

$$\frac{63112(1.2)^5}{1.05^5} = 123047.18$$

So we have got the necessary input for our option-pricing model:

Value of underlying asset = S = SEK m. 123047.18
Exercise price = K = 118216.275
Life of the option = Weighted average duration of debt = 1.48 years
Variance in the value of the underlying asset = 0.2
Risk-free rate = 5.5%

Puting these key issues to the Black-Scholes equation, we get the call value –SEK m 19,495.1. However at the end of Dec. 1998, the Ericsson's capital stock was only SEK m. 4878.

Figure 5.4 Calculations for Ericsson Equity 1998

It is obvious that the market did not appreciate the exact value and Ericsson's equity trading has great potential for growth. This can also explain why the

Ericsson's stock share has been growing significantly in the recent half year despite the transition of top management.

5.2 Joint Ventures as Real Options

In many industries, joint ventures not only share risks, but also decrease the total investment. Because the parties bring different capabilities, the venture no longer requires the full development costs. Due to its benefits of sharing risk and of reducing overall investment costs, joint ventures serve as an attractive mechanism to invest in an option to expand in risky markets.

Joint venture are investments, which bring firm the asymmetry—to discretionally expand in favorable environments while to avoid some of the losses from downside risk. This asymmetry supports strongly that joint ventures are designed as options[17].

5.2.1 Microsoft and Ericsson Alliance on Wireless Access

Before we nearly finish our thesis, news related with Ericsson and Microsoft alliance became the best-case material for real option theory explanation.

Below is the news partly quoted from Washingtonpost.

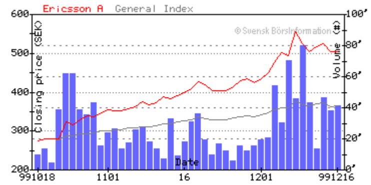
Thursday, December 9, 1999; Page E02

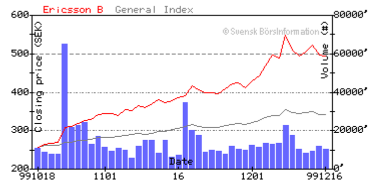
> *STOCKHOLM, Dec. 8—Microsoft and Sweden's LM Ericsson AB are forming a joint venture to develop products for consumers to access the Internet and send e-mail from any wireless device, they said today.*

[17] For details analysis that joint ventures can be viewed as real options, see "Joint Ventures and The Option to Expand and Acquire", Kogut, B. 1989.

> *The new company will use Ericsson's mobile communications technology, Microsoft's Windows operating system and a new Microsoft Mobile Explorer software platform introduced in conjunction with the announcement.*
>
> *Ericsson will hold a majority share in the venture, which comes amid a fierce scramble by technology firms to take a lead role in the potentially explosive market for wireless Internet devices.*
>
> *The Microsoft alliance parallels a recent deal by Finnish rival Nokia to adapt the software that runs the popular Palm handheld computer for mobile phones with Internet access.*
>
> *Notably, however, both Nokia and Ericsson are already partnered with Motorola to develop an operating system named Epoc for mobile phones with Web access.*
>
> *In a separate release, Ericsson said the agreement with Microsoft would not affect its involvement in the consortium with Motorola and Nokia.*
>
> *The Ericsson deal is important for Microsoft because the dominant maker of software for desktop and laptop computers wants to ensure a strong foothold with mobile phones after being trounced by the Palm system for handheld computers.*
>
> *Earlier this year, Microsoft announced that it would work with British Telecommunications on handheld wireless devices for sending e-mail and browsing the Internet.*
>
> *"Mobile Internet access and services are crucial for realizing Microsoft's vision of empowering knowledge workers and consumers through software any time, anywhere and on any device," Microsoft President Steve Ballmer said in the statement.*

Following this news, the Ericsson outstanding share raised sharply around a rate of 12%. In below charts, we may find a sharp jump in Ericsson's equity price. As our previous theory part states, the sharp rise in stock value was caused by the growth option embedded in the Ericsson's equity.





Growth or strategic options are opportunities that are made available in the future by undertaking a project but are not part of the initial project. For instance, launching a product into a new market may have a negative net present value (NPV) itself, but the project may provide exposure for the firm in the new market, opening up the way for future opportunities. Moreover, while high uncertainty in the new market may diminish the present value of the initial project's cash flows, it may mean access to a market with higher potential allowing the firm to make a discretionary follow-up investment. In our case, the value of growth option soon appeared on Ericsson's outstanding share because the equity investors believed the joint venture would bring more net cash flow added to the Ericsson's equity value. The joint venture can be treated as a growth option then. A general equation is listed below:

Strategic (Option-based) NPV of Ericsson's equity
= Growth Option Value + NPV of Ericsson's equity

Not only growth option can be joint venture, but also options of waiting to invest and expansion. The first one is that it pays to wait before committing resources, and the second is that investment commitment is necessary in order to have the right to expand in the future.

It is often the case that an investment decision involves a comparison of both options. Committing engineers or product planners to a risky project incurs the possibility that the market does not develop; it also draws resources from other projects. Clearly, there is a value in waiting before the technology or market is proven. But is there is a benefit in investing today in order to gain experience with the technology or to establish a brand image with customers, then investing generates the valuable option to expand in the future. Due to there is no a very strong brand in Mobile phone operating system, obviously, Ericsson chooses the options to expand. The joint venture serves as a way to bridge these options through pooling resources of Ericsson and Microsoft while it not only shares the investment burden, but sometimes reduce it, as the parties may bring their advantage skills, thereby lowering the total investment cost.

It seems contradicted that Ericsson took two project options which are rival to each other. However, option analysis provides a more flexible approach to explain this phenomenon. Ericsson expands both projects and waits to see what will the market reaction will be. When the windows based explorer succeed in the market, Ericsson might consider the options to acquire joint venture. Through the joint venture, Ericsson can acquire the skills of the Microsoft and no longer needs to invest in the development of the requisite capability to expand into the target market. Of course, Microsoft's willingness to sell depends, one, it realizes capital gains, and two, it may also not have the downstream asset to bring the technology to market. In the other hand, if the market prefers Epoc, Ericsson could take the options to abandon the joint venture with Microsoft due to a strategic NPV potential might be acquired. In one word, Ericsson's strategy is going to be as an earlier brand bird in mobile phone operating system market by taking the rival projects.

A more complicated option pricing numerical work on project budget can be for investment directing but there is no further investment data publicly available for us to have an even simple calculation.

The Ericsson and Microsoft joint venture case can be well explained by the real option theory. We have generally explained the joint ventures as growth options in equity and the abandonment option and expansion option embedded in joint ventures which reflect the flexibility of management. Also we found these options are serving for Ericsson's strategy of being leading position in mobile phone operating system.

6 Conclusions

Our thesis began with the DCF-NPV method for valuing a project, and then we simply introduced the financial options, upon which we expand real options. Afterward, we made an inquiry into real option approach in case studies in order to have a better understanding on real options.

In the R&D case, we found that the R&D project should go on even with negative traditional NPV since the project can be treated as a real option (opportunity) whose value should be added value to the R&D project. This leads R&D project to a strategically positive NPV; this echoes the prospective futures of R&D project.

In the equity valuation case, examinted that a poor-running firm's equity can still have stable value to trade by extending its debt maturity period. When a firm's equity goes largely over its debt future matured, the equity's market value will increase continuously even with the transition of top management. Therefore, we understand that flexibility embedded in capital structure deeply affects on the eqity's market value.

In the joint venture case, we generally examined that joint ventures as growth options embedded in equity can cause equity rising. When running joint venture, the party firm can use defer option, expansion option and abandonment option for its management flexibility and further these options can be extended to game theory, which is considered in competitive markets.

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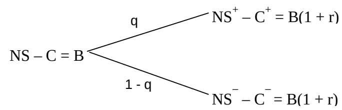
Appendices

1. The Basic Valuation Idea: Risk-Neutral Valuation

Suppose now we construct a portfolio consisting of a) buying N shares of the underlying stock at its current price, S, financed in part by b) borrowing an amount of \$B at the risk-free interest rate (e.g., selling short Treasury bills), for a net out-of-pocket cost of NS – B. That is,

Call option ≈ (buy N shares at S & borrow \$B at r) $\Rightarrow$ C ≈ (NS – B)

By arranging the equation above, we get NS – C = B, i.e., we creating a portfolio consisting of a) buying N shares of the underlying stock and b) selling (writing) one call option would provide a certain amount of (1 + r)*B next period, regardless of whether the stock moves up or down (see figure below):



2. Calculation for Volatility

We now show explicitly how to compute the volatility of a stock from historical data. This is crucial for our study of the Black-Scholes formula, as we will need to know the volatility of a stock in order to use the formula.

The main idea is to do what we have been saying all along: compute the standard deviation of short-term returns. This can be done in the following steps.

1) Fix a standard time period Δt (e.g., one day, one week, etc.), and express it in terms of years. For example, if we are using closing prices, then, Δt is equal to one day, which, expressed in years, is 1/365.
2) Collect price data on the stock for each time period; for example, collect the daily closing prices for 10 straight weeks.
3) Compute the return from the beginning to the end of each period. If the closing price on day i is denoted S_i and the closing price at day i + 1 is denoted S_{i+1}, then the one-day return is given by the formula: $r_i = \log (S_{i+1}/S_i)$, where r_i means " return number i". Note that the daily return on the stock, and we have not annualized it.
4) Compute the average value of the sample returns. If the sample returns are $r_0, r_1, \ldots, r_N$, so that there are a total of N + 1 returns, then the average return is:

$$\bar{r} = \frac{1}{N+1}(r_0 + r_1 + \cdots + r_N).$$

5) Compute the standard deviation using the formula

$$s = \frac{1}{\sqrt{\Delta t}}\sqrt{\frac{1}{N}\left[(r_0 - \bar{r})^2 + (r_1 - \bar{r})^2 + \cdots + (r_N - \bar{r})^2\right]}$$

The reason we divide by $\sqrt{\Delta t}$ in the formula for σ is because the standard unit of time in options pricing is one year. All measurements, such as interest rates and volatilities, are expressed in units of one year. The expression for σ ignoring the $\frac{1}{\sqrt{\Delta t}}$ is the standard deviation of returns for the actual time period studied (Chriss, 1997).

3. Cumulative Probability for the Standard Normal Distribution

Second digit of Z

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
-3.5	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002
-3.4	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0002
-3.3	0.0005	0.0005	0.0005	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0003
-3.2	0.0007	0.0007	0.0006	0.0006	0.0006	0.0006	0.0006	0.0005	0.0005	0.0005
-3.1	0.0010	0.0009	0.0009	0.0009	0.0008	0.0008	0.0008	0.0008	0.0007	0.0007
-3.0	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010
-2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
-2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
-2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
-2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
-2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
-2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
-2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
-2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
-2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
-2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
-1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
-1.8	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
-1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
-1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
-1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
-1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
-1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
-1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
-1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
-1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
-0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
-0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
-0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
-0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
-0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
-0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
-0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
-0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
-0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
-0.0	0.5000	0.4960	0.4920	0.4880	0.4840	0.4801	0.4761	0.4721	0.4681	0.4641

	Second digit of Z									
z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998
3.5	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998

The above table show values of $N(z)$ for z 0, and z 0 respectively. The table should be used with interpolation.

When z 0, for example,

$$
\begin{aligned}
N(-0.1234) &= N(-\,0.12) - 0.34\,[N(-\,0.12) - N(-\,0.13)] \\
&= 0.4522 - 0.34 \cdot (0.4522 - 0.4438) \\
&= 0.4509
\end{aligned}
$$

When z 0, for example,

$$
\begin{aligned}
N(0.6278) &= N(0.62) + 0.78\,[N(0.63) - N\,(0.62)] \\
&= 0.7234 + 0.78 \cdot (0.7357 - 0.7324) \\
&= 0.7350
\end{aligned}
$$