UNIVERSITY OF GOTHENBURG
SCHOOL OF BUSINESS, ECONOMICS AND LAW

WORKING PAPERS IN ECONOMICS

No 469

General Properties of Expected Demand Functions: Negativity (No Giffen Good) and Homogeneity

A Descriptive Non Utility Maximizing Approach

Lars-Göran Larsson

September 2010

ISSN 1403-2473 (print)
ISSN 1403-2465 (online)

Department of Economics
School of Business, Economics and Law at University of Gothenburg
Vasagatan 1, PO Box 640, SE 405 30 Göteborg, Sweden
+46 31 786 0000, +46 31 786 1326 (fax)
www.handels.gu.se   info@handels.gu.se

GENERAL PROPERTIES OF EXPECTED DEMAND FUNCTIONS: NEGATIVITY(NO GIFFEN GOOD) AND HOMOGENEITY
A DESCRIPTIVE NON UTILITY MAXIMIZING APPROACH.

BY LARS-GÖRAN LARSSON

Department of Economics and Statistics,

School of Business, Economics and Law

University of Gothenburg

Vasagatan 1 S 41180
Goteborg
Sweden

E-mail lars-goeran.larsson@economics.gu.se

September 2010

In this paper we assume that choice of commodities at the individual (household) level is made in the budget set and that the choice can be described by a probability density function. We prove that negativity ($\frac{\partial E(x)}{\partial p_x} < 0$ *) is valid for one(x) or two choice variables (x, y) (No Giffen good).Negativity at the market level is valid by summation. The expected demand functions are homogeneous of degree zero in prices and income* (p_x, p_y, m). *We use general positive continuous functions f(x), f(x, y) defined on the bounded budget set. We transform them into probability density functions to calculate E(x) and prove negativity. The present approach use simple assumptions and is descriptive in its nature.* ***Any choice behaviour*** *that can be described by a continuous density function gives the above results.*

Why not keep descriptions as simple as possible?

Entia non sunt multiplicanda praetor necessitatem
Beings ought not to be multiplied except out of necessity

"Occam´s razor"

Encyclopedia Brittannica

KEYWORDS: Negativity (No Giffen good) and other properties of consumer demand functions, Microeconomics, Consumer theory, Consumer behaviour, Choice described in random terms, Expected individual and market demand.
JEL classification: C60, D01, D11

1. INTRODUCTION

1.1. Utility maximization

In the traditional "rational" theory of choice based on maximization of a quasiconcave utility function (u(x ,y)) the derived demand functions($x(p_x, p_y, m), y(p_x, p_y, m)$) have certain testable properties. The demand functions are homogeneous of degree zero in the variables prices and income (p_x, p_y, m). The choice is mostly found on the budget line (the adding up property).Furthermore the substitution matrix of the demand function is symmetric and negative semi definite (see Jehle and Reny (2001)).

For one good x (p_x, p_y, m) we have $\frac{\partial x}{\partial p_x} + x(p_x, p_y, m)\frac{\partial x}{\partial m} \leq 0$

1.2. *Utility maximization and negativity.*

A negative slope of demand (negativity) in relation to own price ($\frac{\partial x(p_x, p_y, m)}{\partial p_x} < 0$) is not valid in traditional theory without making further assumptions.

That the traditional theory holds the negativity result for important can be seen as it is formulated as a "law "(see Varian (2006)). "If the demand for a good increases when income increases, then the demand for that good must decrease when its price increases" (see Varian (2006 p 147)). The normal goods assumption is sufficient to give negativity at the individual and market level (sum of individual demands**).**

The importance can also be seen in that papers are published in the highly ranked journal Econometrica. One paper (see Quah (2000)) introduce convex indirect utility functions (or concavity of the direct utility function) plus some numerical conditions. Focus on the expected law of demand at the aggregate (market) level can be found in (see Hildenbrand (1983) and Härdle, Hildenbrand and Jerison (1991)). The later aggregate approaches look at density functions of household income distribution and focus less on "rational" choice. For a little more details see appendix.

1.3 *A descriptive approach.*

In this paper we assume that consumer choice can be *described* in probabilistic terms. We start by assuming a general positive continuous function f(x, y) which we transform into a *general continuous probability density function p(x, y, a, b)* ($a = \frac{m}{p_x}$, $b = \frac{m}{p_y}$).This procedure is assumed to describe the choice of commodities in a bounded set (notably the budget set). We then calculate the expected demand function (the average demand). After the calculation we find the *general expected properties:*

Negativity at the individual and market level
(in other words there is no Giffen good)
Demand functions are homogeneous of degree zero.

The approach is *simple* in its *behavioural assumptions. Any choice behaviour* that can be described by a continuous density function gives the above results. To find negativity (no Giffen good) there is no need to use classification of goods into normal or inferior.

1.4 *Further results and examples.*

Since our general results are valid for any continuous function we note the great variety of functions that can be used. Convex, decreasing and constant functions are such examples (see section 3). There is no need to only use increasing density functions. We can have density functions with “peaks” inside the budget set (compare “bliss points” see Varian 2006)). Statistical distributions which fit “well” to choice behaviour might be used.

Traditional theory in expected form

To facilitate a comparison with traditional theory we note that it can be expressed in expected values as well since estimation and/or testing the theory mostly handle the same properties to E(x) as to x. In testing and/or estimation we have
E(x) $= x(p_x, p_y, m) + E(\nu) = x(p_x, p_y, m)$,
where ν is a random error term with E(ν)=0 . This assumption gives the result that the expected demand function has the same slope as the ordinary demand function
$Ex_{p_x} = x_{p_x}$, and so on for the other properties.

The independent properties of demand functions in expected form are:
Budget balancedness (adding up) E ($p_x x(p_x, p_y, m) + p_y y(p_x, p_y, m)) = m$
and negative semi definiteness of the symmetric substitution matrix.

$$\begin{bmatrix} Ex_{p_x} & Ex_{p_y} \\ Ey_{p_x} & Ey_{p_y} \end{bmatrix} + \begin{bmatrix} Ex \\ Ey \end{bmatrix} \begin{bmatrix} Ex_m & Ey_m \end{bmatrix}.$$

For details (see Jehle and Reny (1991 pp 82-83))

Budget balancedness and symmetry not valid.

The expected demand in the present approach is found *inside the budget set*, meaning that *budget balancedness (adding up) is not valid*
(E ($p_x x(p_x, p_y, m) + p_y y(p_x, p_y, m)) < m$.).

The budget balancedness (adding up) condition is however *not tested in econometric demand studies*. “ Deaton and Muellbauer point out that data sets used in expenditure analysis satisfy adding up by construction, so that these restrictions are not testable”(see Sabelhaus (1990 p 1472)). On another test of the neoclassical theory of consumer behaviour (see Sippel (1997))

We put in numerical values in example 1below and find that the substitution matrix is non symmetric and negative definite. Therefore *symmetry* of the substitution matrix is *not a general property* in our approach.

We calculate numerical elasticities (own price and cross price) to classify goods as gross substitutes, independent or gross complements. For the same frequency function we can find *changes in classification* from independent goods to gross complements by introducing lower bounds on choice. We can also study demand responses to changes in economic policy (taxes and subsidies). A number of examples are given in the chapter 3 table.

1.5 *On the descriptive value of utility maximization.*

There is no need to use a utility function and its maximization to find these general and specific results. We think maximization of utility functions is too complex and doubtful in a descriptive interpretation. That we are not alone in this belief can be seen in some critical remarks against the mainstream ideas, extensions and alternatives that have been given.

Samuelson (1947) ("Nobel" price winner) (Atheneum edition 1970 p 117). After he concludes his chapter 6 on theory of consumer`s behaviour by stating its empirical meaning in difference and differential form (p 116) he comments " Many writers have held the utility analysis to be an integral and important part of economic theory. Some have even sought to employ its applicability as a test criterion by which economics might be separated from the other social sciences. Nevertheless, I wonder how much economic theory would be changed if either of two conditions above were found to be empirically untrue. I suspect, very little".

Varian (1992) After writing the properties of demand functions on page 99 in Varian 1992, he gives on p 123 the following comment
"this is a rather nonintuitive result: a particular combination of price and income derivatives has to result in a negative semidefinite matrix. However it follows inexorably from the logic of maximizing behaviour."

Stigler (1961) ("Nobel" price winner) on price dispersion as one measure of ignorance about the market and the introduction of a probabilistic search theory.

Herbert Simon (1965) ("Nobel" price winner) on "bounded rational behaviour".

Barten (1969) on empirical tests of the traditional theory of demand.
According to Klevmarken (1979) the classical theory of demand is rejected.

Kahneman - Tversky (1974) (Kahneman "Nobel" price winner) on behavioural elements in economic choice.
We now have a new field of research in economics – behavioural economics.

Myerson (1999) ("Nobel" price winner) p 1069 "This assumption of perfect rationality is certainly imperfect as a description of real human behaviour"

1.6 *Separate description of behaviour from prescription of how to behave*

To separate the study of consumer choice into a descriptive part (consumer behaviour) and normative (prescriptive) part might be useful in the future. The normative part may contain improvement of consumer choice by better information of prices and quality, optimal search theories of prices and quality, utility maximization with more complex preferences and constraints, static or dynamic, deterministic or stochastic etc. The use of optimization methods in economic models reflects the important aspect of helping forming better decisions in the world but does not necessarily mean that they are good models for explaining (describing) actual economic behaviour. As a textbook of Robert H Frank puts it: “But even where economic models fail on descriptive grounds, they often provide useful guidance for decisions” (Frank 2008 p 6).

Contents

In section 2 we give our proof of negativity and homogeneity
Section 3 contains some examples.
Section 4 contains some reflections on extensions.
In the appendix we briefly present the neoclassical theory and some later modifications in attempts to prove law of demand (negativity).

2. THE GENERAL PROPERTIES OF EXPECTED DEMAND FUNCTIONS

We prove that expected demand functions have negative own price slope and are homogeneous of degree zero in prices and income. We use continuous density functions describing household choice of commodities in a bounded set.

2:1 One dimensional frequency function of choice p(x, a).

To find the frequency function p(x,a) we start by assuming a positive continuous function f(x)>0 defined on the interval I = (0, c) Let $a = \frac{m}{p_x} < c$. Define

$F(a) = \int_0^a f(x)dx$ and G(a)=$\int_0^a xf(x)dx$. We then have

$E(x) = \frac{G(a)}{F(a)} < a.$

F (a) is the area below the positive function f(x) in the interval (0, a) and the frequency function
p(x, a) = f(x)/F(a).Remember that a density function has the property $\int_A p(x)dx = 1$.

Note that the *parameter* (a) is part of the frequency function and that(a)is a *variable* in the expected value function.

Properties of E(x)

Next we want to find some properties of E(x). We can use the chain rule of differentiation $\frac{\partial E(x)}{\partial p_x} = \frac{dE(x)}{da}\frac{\partial a}{\partial p_x}$

We take

$$\frac{dE(x)}{da} = \frac{1}{F(a)^2}\left(G'(a)F(a) - F'(a)G(a)\right) = \frac{1}{F(a)^2}(af(a)F(a) - f(a)G(a))$$

$$= \frac{f(a)}{F(a)^2}\left(a\int_0^a f(x)dx - \int_0^a xf(x)dx\right) > 0$$

(1) $\frac{dE(x)}{da} > 0$

We have found that the expected demand function have negative own-price slope for all continuous choice frequency functions.

$$\frac{\partial E(x)}{\partial p_x} < 0$$

Homogeneity of degree zero in price and income is obvious since $a = \frac{m}{p_x}$ is homogeneous of degree zero in price and income.

Change of variable

For later use we *change variable***.** Let x=au. We then find

$E(x) = \frac{a^2 \int_0^1 uf(a,u)du}{a\int_0^1 f(a,u)du} = \frac{a\int_0^1 uf(a,u)du}{\int_0^1 f(a,u)du} = \frac{G(a)}{F(a)}$ where (a) is the Jacobian of the transformation.

For the change of variable formula (see Buck(1956 p244)).

We differentiate

(2) $\frac{dE(x)}{da} = \frac{1}{F(a)^2}\left(G'(a)F(a) - F'(a)G(a)\right) > 0$

following the result (1)) above.
The latter result (2) will be useful when we turn to two dimensional choices.

2.2 *Choice in two dimensions*

To find the frequency function p(x, y, a, b) we start by assuming a positive continuous function f (x, y)>0 defined on a set E, where $D \subset E$.
For the function f(x, y) we should integrate over the budget set
D=$\left\{(x,y) \text{ in } \mathrm{R}^2 : p_x x + p_y x_2 \le m \text{ and } x \ge 0, y \ge 0\right\}$

To make the calculations simpler we change variables
We put x=u$\frac{\mathrm{m}}{\mathrm{p_x}}$=au and y=v$\frac{\mathrm{m}}{\mathrm{p_y}}$=bv to integrate over the set

D′=$\left((\mathrm{u,v}) \in R^2 : \mathrm{u} \ge 0, \mathrm{v} \ge 0, \mathrm{u+v} \le 1\right)$

We then have $\iint\limits_D f(x,y)dxdy = ab\iint\limits_{D'} f(x(u),y(v))dudv$,

where the Jacobian (= ab)of the transformation is taken outside the integral on the right side. For the change of variable formulas (see Buck (1956 p 244)).

Calculating E(x)

To find E(x) we first integrate f(x, y) over the area D'

$$ab\iint_{D'} f(x(u), y(v))dudv = ab\iint_{D'} f(u,v,a,b)dudv = ab\int_0^1 du \int_0^{1-u} f(u,v.a,b)dv.$$

We now put the last integral $\int_0^{1-u} f(u,v,a,b)dv = F(u,a,b)$ and the volume below f(x,y) is $ab\int_0^1 F(u,a,b)du$ .The next integral to be calculated is

$$aba\iint_{D'} uf(u,v,a,b)dudv = aba\int_0^1 udu \int_0^{1-u} f(u,v.a,b)dv = aba\int_0^1 uF(u,a,b)du$$

We now find $E(x) = \dfrac{aba\int_0^1 uF(u,a,b)du}{ab\int_0^1 F(u,a,b)du} = \dfrac{a\int_0^1 uF(u,a,b)du}{\int_0^1 F(u,a,b)du} = \dfrac{G(a,b)}{F(a,b)}$

Finding the properties of expected demand E(x)

Expected demand is homogeneous of degree zero in prices and income (p_x, p_y, m).
This follows since $E(x) = \dfrac{G(a,b)}{F(a,b)}$ is a function of ($a = \dfrac{m}{p_x}, b = \dfrac{m}{p_y}$) both homogeneous of degree zero in prices and income.

The chain rule of differentiation helps us to find sensitivity in relation to price (p_x).

$$\frac{\partial E(x)}{\partial p_x} = \frac{\partial E(x)}{\partial a}\frac{\partial a}{\partial p_x}$$

Differentiate w r t a. and use the result (2) in one dimension
$\dfrac{\partial E(x)}{\partial a} > 0$ .This in turn gives us $\dfrac{\partial E(x)}{\partial p_x} < 0$.

Negativity is valid for all continuous frequency functions p (x, y, a, b).
The property is additive so the result is valid at the aggregate (market) level as well.
Homogeneity was mentioned above.

3 EXAMPLES

Even if law of demand is valid for all continuous density functions we also illustrate the stochastic approach by presenting some examples, many of them familiar to micro economic textbooks. At the same time we find other properties of expected demand functions and compare them with traditional theory. In an earlier working paper Larsson (2008) some examples and calculations can be found.

Each example is calculated as follows. For a given function f(x, y) its volume over the budget set is calculated. This volume is used to transfer f(x, y) into the density function p(x , y, a, b).Using the density function we calculate the expected demand E(x(a, b)) which then is differentiated to find various properties.

Example 1. A constant function.

Choosing a constant function can give us a uniform distribution.

Why use the uniform distribution?

The uniform distribution assumption might reflect our *ignorance* of what determines the choice (such as preferences) inside the bounded budget set. If we know that choice is taken place in the set we know that expected choice changes when the "walls" (budget line and lower bounds) of the set changes. A uniform distribution can also *approximate* a more complex function if we use a lower bound as in the example. The assumption also makes for *simple calculations* (used before in economics) and produces some results. To quote Krugman (2008): "express your ideas in the simplest possible model."

This example has *no unique solution in utility theory (a constant utility)* and most solutions (the demand functions) are *independent* of price and income. The following example shows *no such independence of economic variables* and is

$E(x) = \frac{1}{3p_x}(m - p_y y_0) + \frac{2}{3}x_0$ If no lower bound we have E(x)= $\frac{\mathrm{m}}{3\mathrm{p_x}}$, E(y)= $\frac{\mathrm{m}}{3\mathrm{p_y}}$

$E(y) = \frac{1}{3p_y}(m - p_x x_0) + \frac{2}{3}y_0$

the expected demand functions obtained by integrating a uniform distribution over a bounded set. The set is bounded by the budget constraint and lower bounds $x \geq x_0$ $\mathrm{y} \geq \mathrm{y}_0$ where x_0 and y_0 can be seen as a function of shiftvariables s if one wishes (one can then use the chain rule to obtain sensitivity wrt s).

We name these lower levels "subsistence" levels, *S levels* for short .One early example on the use of lower levels is Stone-Geary preferences (Hey 2003 p 80).

Given the expected demand we can study properties such as own price and cross price derivatives, income derivatives, homogeneity and symmetry properties as in ordinary theory. In the example given we have $E(x)'_{p_x} < 0, E(x)'_m > 0, E(x)'_{p_y} < 0, E(y)'_{p_x} < 0, E(x)'_{p_y} \neq E(y)'_{p_x}$

In words: Own prise derivative negative, cross price derivatives negative(x and y are gross complementary commodities), income derivative positive (normal good) . The own price elasticity of demand is inelastic $El_{Expx} = \frac{\partial E(x)\, p_x}{\partial p_x E(x)} > -1$.

If lower bounds are zero x and y are independent commodities. *Note that the introduction of a lower bound changes the character of the commodities from independent to complements.*

Symmetry of the substitution matrix is not a general property in the present approach as seen by introducing numerical values in this example.

Putting m=100, $p_x = 10$, $p_y = 5$, x_0=5 and y_0=6 we find

E(x)=$5\frac{2}{3}$, E(y)=$7\frac{1}{3}$

Average expenditure = 0,933m < m.
Note the relative closeness to m due to the tight lower border. Identify lower bounds and the choice and/or preferences inside the set matter less. The substitution matrix in the numerical example is *non symmetric and negative definite.*

$$\begin{bmatrix} \frac{-7}{30} & \frac{-1}{5} \\ \frac{-1}{3} & \frac{-2}{3} \end{bmatrix} + \begin{bmatrix} \frac{17}{3} \\ \frac{22}{3} \end{bmatrix} \begin{bmatrix} \frac{1}{30} & \frac{1}{15} \end{bmatrix} = \begin{bmatrix} \frac{-4}{90} & \frac{8}{45} \\ \frac{-8}{90} & \frac{-8}{45} \end{bmatrix}.$$

The own price elasticity $\mathrm{El}_{\mathrm{Exp_x}} = \frac{\partial E(x) p_x}{\partial p_x E(x)} = \frac{-7}{17}$

The cross price elasticity $\mathrm{El}_{\mathrm{Exp_y}} = \frac{\partial E(x) p_y}{\partial p_y E(x)} = \frac{-3}{17}$

The income elasticity $\mathrm{El}_{\mathrm{Exm}} = \frac{\partial E(x) m}{\partial m E(x)} = \frac{10}{17}$

Note that the homogeneity condition in elasticities hold (sum of elasticities =0).
For elasticity forms in traditional theory (see Shone (1975 p 91)).

Table of examples

In the following table we present more examples. The table contains the used function f(x, y). It also contains numerical values calculated for $(p_x, p_y, m, x_0, y_0) = (10,5,100,5,6)$ and c=a + b=30.

We give expected demand, the utility maximising solution for each function f(x, y), own price elasticity, cross price elasticity and type of commodity following classification by cross price (gross complements etc) and average expenditure.

The table is ordered following own price elasticities (from inelastic to elastic). Note especially the *change in type of goods from independent to complements* due to introduction of a lower bound for the same function. Example 1 and 2 plus example 3 and 4. Furthermore note the *change in type of goods from substitutes to complements* in example 10 and 6.Both examples have a linear form but example 6 has a decreasing density function- **“***low consumption***”** and example 10 has an increasing density function “high consumption”. Example 12 in one dimension give us a” Luxury good”.

Functional form of f(x, y)	Expected demand	Utility maximizing solution	Own price elasticity	Cross price elasticity. Type of commodities	Average expenditure
1 Constant-uniform with lower bounds f(x,y)=1	E(x)= 5,67 E(y)= 7,33	Infinite number of solutions	-0,41 Inelastic demand	-0,18 Comple-ments	0,93m
2 Constant-uniform with no lower bounds	E(x)= 3,33 E(y)= 6,67	Infinite number of solutions	-1 Unit elastic demand	0 Independent	0,67m
3 Quasi-concave with lower bounds.Cobb-Douglas form f(x,y)=xy	E(x)=5,8 E(y)=7,6	x=5 y=10	-0,48 Inelastic demand	-0,20 Comple-ments	0,96m
4 Quasi-concave with no lower bounds.Cobb-Douglas form f(x,y)= xy	E(x)=4 E(y)=8	x=5 y=10	-1 Unit elastic demand	0 Independent	0,8m
5 Linear proportions Perfect complements f(x,y)=min(x,y)	E(x)=4,17 E(y)= 6,67	x=6,67 y=6,67	-0,87 Inelastic demand	-0,13 Comple-ments	0,75m
6 Linear "low consumption" f(x,y)=c-x-y	E(x)= 3,33 E(y)= 5,83	x=0 y=0	-0,91 Inelastic demand	-0,08 Comple-ments	0,63m
7 "Quasilinear" f(x,y)= $\sqrt{x}+y$	E(x)= 2,86 E(y)=9,3	x=0,1 y=19,8	-0,93 Inelastic demand	0,08 Substitutes	0,74m
8 Quasi-concave Cobb Douglas form f(x.y)= x^2y	E(x)=5 E(y)= 6,67	x=6,67 y=6,67	-1 Unit elastic demand	0 Independent	0,83m
9 Concave f(x,y)= $\sqrt{x}+\sqrt{y}$	E(x)=3,45 E(y)= 7,39	x=3,33 y=13,33	-1,05 Elastic demand	0.05 Substitutes	0,71m
10 Linear "High consumption" f(x,y)=x+y	E(x)=3,3 E(y)= 8,33	x=0 y=20	-1,16 Elastic demand	0,17 Substitutes	0.75m

11 Convex f(x,y)= x^2+y^2	E(x)=2,8 E(y)= 10,4	x=0 y=20	-1,45 Elastic demand	0,46 Substitutes	0,8m
12 Exponential f(x)= e^x	E(x)=9	x=10	-1,11 Elastic demand	Income elasticity 1,11 Luxury good	0,9m

4 EXTENSIONS

As we noted there is no "Giffen good" in this expected demand approach. To allow for Giffen good in our approach some *new information is needed*. In traditional utility theory shift variables (s) are introduced in the utility functions u(x,y,s). Something similar could be used in the function f(x,y,s). If as an example, a theory is based on monetary variables like price dependent preferences in utility theory (see Kalman 1968) we can have price dependent functions $(f(x,y,p_x))$ and integrate them to find expected demand. The result can be demand functions which are non-homogenous in prices and income or have non-negative own price slopes.

Negativity might not be compatible with any choice behaviour, but if we know more then we know more.

Constraints such as *time restrictions* could also be used in the same manner perhaps together with the budget constraint. To find examples of expected demand and supply functions in consumer theory and theory of the firm where choice is made in bounded

Recently, behavioural sciences and *behavioural economics* have supplied economics with examples of choice behaviour. It this choice behaviour can be expressed in specific density functions these functions can be integrated over the budget set to relate them to economic variables. Help to integrate more complex density functions can be done by experts. In waiting for a "final theory", which of course is an illusion, probability formulated choice gives a link between traditional theory of negativity and homogeneity and theories of behaviour within the budget set.

APPENDIX

The neoclassical theory and some later modifications in attempts to prove law of demand (negativity).

A:1 *Traditional theory and the law of demand*

The traditional neoclassical theory assumes a utility description of consumer preferences and that

the consumer makes his choice of commodities (x,y)

by maximizing his utility function u(x,y) subject to a budget constraint.

$u(x,y)$ is maximized over the budget set D= $\left((x,y)\text{: } x \geq 0, y \geq 0,\ p_x x + p_y y \leq m\right)$

where p_x, p_y are prices of (x,y) and m is income(budget)

The result of this maximization gives the individual demand functions

$x(p_x, p_y, m), y(p_x, p_y, m)$

Assuming the utility function u(x, y) is an increasing quasi concave function it is proven that the demand function has the following property

$$\frac{\partial x}{\partial p_x} + x(p_x, p_y, m)\frac{\partial x}{\partial m} \leq 0$$

For more details (see Jehle and Reny (2001 pp 82-83))

It is not possible to exclude the Giffen case where

$\frac{\partial x}{\partial p_x} > 0$ since $\frac{\partial x}{\partial m} < 0$ is possible in the theory (inferior good)

The law of demand in this context is formulated

"If the demand for a good increases when income increases, then the demand for that good must decrease when its price increases" (see Varian (2006 p 147)).

$\frac{\partial x}{\partial p_x} < 0$ since $\frac{\partial x}{\partial m} > 0$ (normal good)

The normal goods assumption is sufficient to give law of demand at the individual and market level (sum of individual demands) and is used in economic literature.

A:2 *Some later approaches to obtain law of demand.*

In the literature Quah (2000), Hildenbrand (1983) and Härdle; Hildenbrand , and Jerison (1991) are different examples how to prove the law of demand.

A:2:1 *Utility maximization as a maintained hypothesis.*

To obtain negativity at the individual level stronger assumptions on utility functions can be used. Since law of demand is valid at the individual level it is valid at the market level by summation.

Quah (2000) identifies sufficient conditions on an agents indirect utility function $v(p_x, p_y, m)$ which guarantees law of demand at the individual level.

The conditions are *convexity* in prices of the indirect utility function and the *numerical condition* that the elasticity of the marginal utility of income $\varepsilon(p, w)$ with respect to income (w).

$$\varepsilon(p, w) \equiv \frac{wv_{ww}(p, w)}{v_w(p, w)} < 2 .$$

(See Quah (2000 p 916)).

Earlier studies (referred to in Quah (2000 p 912)) proved law of demand by using sufficient assumptions such as *concave* utility functions and a *numerical condition.*

A:2:2 *Properties of market (mean) expected demand.*

Distribution of household income is the focus of study. Utility maximization is not very important.

Hildenbrand (1983) takes a different route. Referring to Hicks "A study of individual demand is only a means to the study of market demand" (see Hildenbrand

(1983 p 997)), he proves that the "law of demand" holds for *the market(mean) expected demand function, i.e.,*

$\frac{\partial F_h(p)}{\partial p_h} < 0$ where $\mathrm{F}_\mathrm{h}(p) = \int_0^1 f_h(p,w)dw,\ 1 \leq \mathrm{h} \leq l$ where h is commodity h.

Hence, the partial market demand curve for every commodity is strictly decreasing. (see Hildenbrand (1983 p 998)). The paper extends the result for more general density functions $\rho(w)$ of income distribution than the uniform used in the text above.

Hildenbrand also points out:

"This remarkably simple result shows clearly that aggregating individual demand over a large group of individuals can lead to properties of the market demand function F which, in general, individual demand functions f do not posses. There is a *qualitative difference* in market and individual demand functions. This observation shows that the concept of a "representative consumer", which is often used in the literature, does not simplify the analysis ; on the contrary, it might be misleading."

Härdle; Hildenbrand , and Jerison (1991 p1529) takes a more general approach in proving Law of demand at the market level:"In conclusion, assuming that the mean Slutsky matrix S (p) is negative semi definite, a sufficient condition for monotonicity of F is that the mean income effect matrix M(p) is positive definite. This property does not follow from an assumption on "rational" individual behaviour." The goal of their analysis is to better understand the distributions μ that leads to a positive definite mean income effect matrix M (p) and to perform an empirical test.

Note the quotation "This property does not follow from an assumption on "rational" individual behaviour".

We have presented some papers and the main assumptions used in them to prove that the law of demand is valid at the individual and/or market level. We have not mentioned the proofs themselves but a look at them shows that they use *no simple mathematics.*

REFERENCES

BARTEN, A.P, (1969) Maximum likelihood estimation of a complete system of demand Equations, *European Economic Review*1, 7-73

BUCK, R. C. (1956): *Advanced calculus*, New York: McGraw-Hill.

FRANK,R.H. (2008): *Microeconomics and behaviour,* International Edition 2008: McGraw-Hill,

HEY, J. D. (2003).*Intermediate microeconomics*, McGraw-Hill, Maidenhead.

HILDENBRAND, W. (1983):"On the Law of Demand," *Econometrica*, 51, 997-1019.

HÄRDLE, W, HILDENBRAND,W and JERISON, M. (1991): "Empirical Evidence on the Law of Demand," *Econometrica*, 59, 1525-1549.

JEHLE, G. A and RENY, P, J (2001): *Advanced microeconomic theory,* Boston: Addison Wesley.

KAHNEMAN, D and TVERSKY, A (1974), " Judgement under uncertainty: Heuristics and biases," *Science*, *185*: 1124-1131.

KALMAN,P.J:"Theory of consumer behaviour when prices enter the utility function," *Econometrica, XXXVI(October, 1968),497-510.*

KLEVMARKEN, N A (1979) A comparative study of complete systems of demand functions, *Journal of econometrics*, Vol 16, No 2, 165-191.

KRUGMAN, P (2008): http://www.princeton.edu/ ~pkrugman/howiwork.html

LARSSON, L-G (2008): “Non-Utility Maximizing Behaviour”, Scandinavian Working Papers in Economics S-WoPEc, nr 293, 2008. http://swopec.hhs.se/
MYERSON, R.B.(1999): Nash equilibrium and the history of economic theory, *Journal of economic literature*, September 1999, 1067-1082.
QUAH, JOHN K.-H.(2000):”The monotonicity of individual and market demand”, *Econometrica*, 68,No4,911-930.
SABELHOUSE, J (1990): “Testing neoclassical consumer theory with aggregate and household data”, *Applied economics*,1990,22,1471-1478.
SAMUELSON, P.A (1947) (ed. 1970):*Foundations of economic analysis*, New York: Atheneum.
SHONE, R. (1975): *Microeconomics: a modern treatment*, MacMillan, London
SIMON, H. A. (1965). *Administrative Behavior*. The Free Press, New York
SIPPEL,R (1997): “An experiment on the pure theory of consumer`s behaviour”, *Economic Journal*, 107, 1441-1444.
STIGLER, G. J. (1961) “The economics of information”, *Journal of Political Economy*, vol. 69, 213-25.
VARIAN, H. (2006): *Intermediate microeconomics*, New York: Norton.
VARIAN, H. (1992). *Microeconomic analysis*, Norton, New York.