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Ethnic diversity, economic performance and civil wars.

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Abstract

We develop a conflict model linking dissipation to the distribution of the population over an arbitrary number of groups. We extend the pure contest version of the model by Esteban and Ray (1999) to include a mixed public-private good. We analyze how the level of dissipation changes as the population distribution and the share of publicness of the prize change. First, we find that, in case of pure private goods, the dissipation-distribution relationship resembles the fractionalization index. This may explain the sensitiveness of empirical evidence on the impact of ethnic diversity with respect to outcome (growth, incidence of civil wars) and index (fractionalization, discrete polarization). Second, we find that, in case of pure private goods, smaller groups always contribute more and so the fractionalization index under-estimates their weight. Indeed, we find that the fractionalization index under-estimates the true level of dissipation.

Key words: ethnic diversity, public-private goods, polarization, fractionalization.

JEL Classification codes: D72, D73, D74, H42.

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1 Introduction

The empirical literature on ethnic diversity suggests two stylized relationships: ethnic diversity affects negatively steady-state GDP per capita (Easterly and Levine 1997) and affects positively the risk of civil wars (Montalvo and Reynal-Querol 2005b). The magnitude and significance of these reduced form relationships hinges on the measure used to capture ethnic diversity, either a fractionalization index either a discrete polarization one. This sensitiveness may inform us as to the mechanisms through which they work. The research questions we tackle in this paper are: which sorts of distributions are associated with dissipation? Does the dissipation-distribution relationship resemble the fractionalization or discrete polarization index? In order to answer these questions, we develop a behavioral model linking societal dissipation to the distribution of population across groups and we investigate how societal dissipation changes as the population distribution changes.

We conceive dissipation as a situation in which, in presence of weak institutions (absence of checks and balances, inefficiency of elections to discipline politicians or absence of elections overall) and in absence of a well-defined and agreed-upon collective decision rule, individuals incur costs to capture their most preferred outcome. The concept encompasses both inefficiency of economic policies and conflict. We choose such a broad concept because of two reasons. First, the paper is motivated by the empirical relationships between ethnic diversity on one side and (low) economic performance and likelihood of civil wars on the other. Second, our modelling strategy allows us to do so. We study a simple rent-seeking model with an arbitrary number of groups. The characteristic feature of this class of models is the diversion of resources from productive activities. This is why it is commonly used to explain rebel activities, open and latent conflict, lobbying and capture of the government and why it should capture not only the relationship between ethnic diversity and civil wars, but also the one between ethnic diversity and economic performance. The model borrows largely from the pure contest version of the model by Esteban and Ray (1999), who investigate the relationship between conflict and distribution. Since the properties of their model resemble closely those of the polarization index, one way to answer to our research questions is to extend it in a way that make the properties of the model resemble the fractionalization index for some parameter values, and those of the discrete polarization index for some others. By doing so, the model should suggest which features drive the change in the properties and which do not matter. The main novelty with respect to their model is the specification of the prize. Within the winning group, part of the outcome is a public good and is enjoyed in the same quantity by group members, no matter their number; another part is private, in the sense that it has to be shared among group members, which means that the per capita share shrinks with group size. If we were to consider both conflict and low economic performance as the outcome of the capture of government, then the public component could represent features like ideology and economic policies favoring members of the winning group, while the private component would represent not only any monetary component of the outcome, but also the capture of rent-seeking position whose value shrinks with the number of individuals having access to[1].

We have no general results for intermediate public-private goods. On the other hand, we do have general results for the pure private good case. We find that the model replicates rather well the properties of the fractionalization index. Indeed, it is possible to show that, if per-capita contributions were homogeneous across groups and the cost function was quadratic, then the model-based dissipation would be a monotonically increasing transformation of the fractionalization index (Esteban and Ray 2009). Therefore we ask: do all groups devote the same per-capita contributions or some contribute more than others? In order to answer to this question, we look at the pattern of contributions across groups within a given equilibrium. We find that, in case of pure private goods, smaller groups always contribute more. Thus, the

[1] An example could be the often mentioned overvaluation of the exchange rate in African countries (Bates 1981). The value of such access depends on the number of people having access to it.

fractionalization index systematically under-estimates the weight of smaller groups in the creation of dissipation. Indeed, we find that, for the special case of quadratic cost functions, this leads the fractionalization index to under-estimate the true level of dissipation.

The remainder of the paper is organized as follows. In section II we review the literature, present the empirical stylized facts and provide a short discussion of the fractionalization and polarization indexes. In section III we describe the model and derive some propositions. Section IV compare the predictions of the model with the properties of the two indexes. Section V concludes.

2 Literature review and stylized facts

More than ten years ago Easterly and Levine (1997) showed evidence suggesting that the degree of ethno-linguistic heterogeneity within a country affects negatively its growth prospects[2]. Later, Montalvo and Reynal-Querol (2005b) showed that ethnic diversity also affects the incidence of civil wars.

Notwithstanding the multitude of works following these papers[3], it is still not clear the way ethnic diversity is related to economic performance and social unrest. Easterly and Levine (1997) suggest three channels: political instability, rent-seeking policies and generalized corruption, low provision of public goods. However, they recognize that they cannot distinguish them in a useful way, since they are likely to be correlated. For example, they show that by including a measure of public infrastructures in the regression specification, the coefficient associated with ethnic diversity decreases greatly in magnitude and becomes insignificant. They interpret this result as evidence that an increase in ethnic diversity induces under-provision

[2] Before them, the index of ethnic fractionalization had been used by Canning and Fay (1993) and Mauro (1995).

[3] There is a considerable number of parallel works in political science, mainly focussed in explaining the onset and duration of civil wars. See the special issue in the Journal of Peace Research 2008.

of pure public goods. This has been argued also by La Porta et al. (1999), Alesina et al. (2003) and Montalvo and Reynal-Querol (2005a). However, it seems rather difficult to infer whether ethnic diversity reduces the demand for pure public goods (as rationalized by Alesina, Baqir and Easterly 1999) or increases corruption and so its equilibrium provision (Mauro 1995)[4]. More in general, the proxy used by Easterly and Levine (1997) to capture the provision of public goods (logarithm of telephones per thousand workers) is not a policy but an outcome variable, and so its correlation with ethnic diversity is not informative on the channels through which the latter works (Arcand et al. 2000). Montalvo and Reynal-Querol (2005a) try to address the issue by including measures of ethnic diversity into first stage regressions explaining government consumption and investments, but they obtain mixed results[5].

The strand of literature on ethnic diversity and likelihood of civil wars could be thought as concerning one of the above-mentioned channels (political instability). In this respect, Fearon and Laitin (2003) and Caselli and Coleman (2006) suggest that ethnicity may be a strategic way of grouping to avoid sharing the benefits from winning with non-members. More recently, Esteban and Ray (2008a) suggest a theory according to which conflicts along ethnic lines are more likely than conflicts along class lines.

Finally, there is an emerging strand of literature investigating whether ethnic diversity can really be taken as exogenous and fixed over time. The relevant level

[4] For example, Schleifer and Vishny (1993) suggest corruption motives may be so relevant that they may distort significantly the allocation of public spending away from schooling and health into infrastructures, given the lower level of transparency of the latter sector. This argument alone is sufficient to explain the previous empirical results. Notice also that inefficiency in the public sector (patronage and corruption) and targeted transfers could explain why there seems to be no effect of ethnic diversity on the size of the public sector notwithstanding the low provision of public goods (see Alesina et al. 2003 following the analysis pursued by La Porta et al. 1999).

[5] Their work seems not to be robust to a number of points. Their measures of ethnic diversity do not affect significantly government consumption when entererd one at the time (and there is no clear reason to include them both if the aim is predicting the dependent variable instead of assessing the relative explanatory power, as we do here). They do not distinguish private from public investment, while the common idea is that the two respond to very different mechanisms: public investment may suffer low provision of public goods as well as corruption; private investment suffers the low provision of complementary infrastructures and high political instability, which depress the marginal return of capital. Finally, their measure of ethnic diversity remains significant even after including consumption and investments, which means that they do not identify all the channels the relationship works through.

of diversity may be subject to manipulation by politicians in the short term (Posner 2000), evolve over time (Michalopulous 2008, Campos and Kuzenyev 2007) and may be the result of long-run human history (Ahlerup and Olsson 2008, Spolaore and Wacziarg 2006).

2.1 Sources of data

Easterly and Levine (1997) use the index of ethno-linguistic fragmentation computed by Taylor and Hudson (1972) with data from Atlas Norodov Mira (1964)[6]. The latter is the result of a twenty years research programme in the Soviet Union.

Alesina et al. (2003) consider language, religion and ethnic diversity separately. For the ethnic index, which involves both racial and linguistic characteristics[7], they use the Encyclopedia Britannica (124 on 190 countries; henceforth EB), CIA World Factbook (25 countries), Levinson (1998, 23 countries), Minority Rights Group International (1997, 13 countries). When more than one source provided information over the same country, they first computed the fractionalization index according to each source and then they chose among them according to the following rule: "if two or more sources for the index of ethnic fractionalization were identical to the third decimal point, we used these sources (..). If sources diverged in such a way that the index of fractionalization differed to the second decimal point, we used the source where ethnic groups covered the greatest share of the total population. If this was 100 percent in more than one source, we used the source with the most disaggregated data (i.e. the greatest number of reported groups)" (Alesina et al. 2003:160).

Montalvo and Reynal-Querol (2002, 2005a, 2005b) use the World Christian Encyclopedia (WCE), which provides more details on the way used to discriminate groups. First, it considers six different characteristics: race and color, culture and language,

[6] The same data were also used by Canning and Fay (1993) and Mauro (1995).

[7] For example, they find that the source refers mainly of race for south american countries; language for european ones (ex. Belgium, Switzerland); mixed for sub-saharan countries.

ethnic origin, nationality. Then, it combines them to provide a classification of ethno-linguistic diversity with several levels: 7 major races, 7 colors, 13 geographical races, 4 sub-race, 71 ethno-linguistic families, 432 major peoples, 7010 distinct languages, 8990 sub-peoples, 17000 dialects. Following Vanhanen (1999), they consider an intermediate level of disaggregation, ethno-linguistic families[8]. However, since the identification strategy used in the WCE is more flexible[9], this choice led the two researchers to aggregate proportions or groups of peoples or sub-peoples, when these are identified as the relevant cleavages.

Simple comparison of the two data collection processes then suggests that, if there is a bias in the data used by Alesina et al. (2003), it is in favor of disaggregation; on the contrary, if there is a bias in the data used by Montalvo and Reynal-Querol, it is in favor of aggregation.

2.2 Measures of ethnic diversity: fractionalization vs polarization

The measures commonly used to capture ethnic diversity are the fractionalization index, constructed by Taylor and Hudson (1972), and the discrete polarization index, adapted by Montalvo and Reynal-Querol (2002) from Esteban and Ray (1994). Both these measures aggregate data on group shares into an index ranging along the unit interval[10].

The fractionalization index is the probability that any two randomly chosen individuals belong to different ethnic groups. Let the size of a generic group be denoted

[8] Fearon (2003) also considers an intermediate level of disaggregation: he considers only groups whose population share is greater then one percent. His sources are the CIA World Factbook, the EB, the Library of Congress Country Study (LCCS), Morrison (1989), the Summer Institute of Language's Ethnologue, and Levinson (1998). In addition, he uses country-specific sources in case of significant discrepancies. Notice that the data provided by Alesina et al. (2003) and Fearon (2003) share most of the sources.

[9] Since the WCE relies on survey questions like the following "What is the first, or main, or primary ethnic or ethnolinguistic term by which persons identify themselves, or are identified by people around them?"

[10] Ideally, one would like to count for inter-group distances as well. However, to the best of my knowledge there are no convincing measures of ethnic hate. Indeed, this constitutes one of the main reason of interest for the recent research on genetic distances, which draws largely upon the seminal work by Cavalli-Sforza et al. (1994).

by n_i and the entire population be normalized to unity $\left(\sum_{i=1}^{G} n_i = 1\right)$, then the fractionalization index is[11]:

$$F = \sum_{i=1}^{G} n_i \left(1 - n_i\right) = 1 - \sum_{i=1}^{G} n_i^2.$$

It has the following properties:

1. for a given number of groups G, F is maximized at the uniform population distribution over these groups;

2. over the set of uniform distributions, F increases with the number of groups;

3. the split of any group into two new groups increases F;

4. any transfer of population to a smaller group increases F.

Since the impact of a split (3) on the index does not depend on the size of the group that splits nor it depends on the distribution of the other groups, the index is said to local. Properties 3 and 4 imply that it is always possible to break down a transfer in a sequence of smaller transfers, all changing the index in the same direction. For this reason the index is said to be monotonic.

The discrete polarization index is a simplified version of the polarization index introduced by Esteban and Ray (1994)[12]. The expression for its discrete version (Q) is:

$$Q = 4 \sum_{i=1}^{G} \left[n_i^2 \left(1 - n_i\right)\right]$$

where n_i denotes the population share for group i and the population is normalized to unity: $\sum_{i=1}^{G} n_i = 1$.

It has the following properties:

[11] The index has two theoretical backgrounds: one is the Gini coefficient (the fractionalization index can be seen as its semplification); the other is Herfindal index (the fractionalization index is its complement).

[12] Essentially, Montalvo and Reynal-Querol (2002, 2005a, 2005b) simplified the expression for the general index to exclude the use of ethnic distances, normalized the index to unity to make it easier to be interpreted, and chose a particular value of a polarization sensitiveness (see one of the paper for details). Notice that the main purpose of the latter was to provide an alternative to the Gini coefficient in the field of inequality measurement and that the fractionalization index constitutes a semplification of the Gini coefficient itself.

1. for a given number of groups G, Q is maximized when the population is concentrated on two equally sized groups only (bimodal symmetric distribution);

2. over the set of uniform distributions, Q decreases with the number of groups, provided there are at least two groups to begin with;

3. the split of a group in two increases Q if and only if the initial group size was at least 2/3;

4. a transfer of population to a smaller group increases Q if both groups are larger than 1/3. If both groups are smaller than 1/3, the transfer decreases Q.

Since the impact of a split (3) on the index depends on the size of the non-splitting population, which is not directly associated with the change, the index is said to global (Esteban and Ray 1994:829). Properties 3 and 4 imply that a population change cannot necessarily be broken down into a sequence of changes having the same effect on the index. For this reason the index is said to be non-monotonic (Esteban and Ray 1994:829).

Notice that in case $G = 2$, both measures reach their maximum in correspondence of the uniform distribution $(n_1 = n_2 = 1/2)$ and transfers from big to small groups increase both indexes[13]. The two indexes diverge more and more as the number of groups with positive population shares increase $(G \geq 3)$, since Q maintains its maximum in correspondence of the bimodal distribution (population concentrated in any two groups with equal population shares $n_i = n_j = 1/2$), while the maximum for F becomes the uniform distribution over all groups.

2.3 Ethnic diversity, economic performance and civil wars: empirical evidence

[13] Indeed, Montalvo and Reynal-Querol (2002) show that, within the two-group case, even when group sizes diverge, the two indexes continue to be proportional to each other.

What we want to investigate in this sub-section is whether the fractionalization and the discrete polarization indexes differ also in their explanatory power. In the following regression analysis the two indexes will be the explanatory variables of interest, while we will consider two types of dependent variables: growth rates of real GDP per capita and likelihood of civil wars. In order to make the comparison as rigorous as possible, we choose the same time period (1960-1989) and the same level of observation (country data averaged over decades: one observation per decade). The econometric technique, the treatment of standard errors and the set of control variables are those of our reference papers: Easterly and Levine (1997) for economic performance; Montalvo and Reynal-Querol (2005b) for civil wars. With respect to the former, we include both indexes at the same time in the specification. With respect to the latter, we only change the time period. We also run separate regressions corresponding to different data sources: Montalvo and Reynal-Querol (2005b), who use the World Christian Encyclopedia, and Alesina et al. (2003), who use the Encyclopedia Britannica[14].

In table 1 (Appendix) we show the results for economic performance. They suggest that the fractionalization index explains growth rates better than the discrete polarization index, no matter the source of data.

In table 2 and 3 (see Appendix) we present parallel evidence on the relationship between ethnic diversity and civil wars. There is no clear cut index measuring the likelihood of civil wars. There are several measures used extensively in the literature: broadly speaking, they are based on the number of deaths due to battles between the government and an opposing faction within a given year and they differ mainly for the threshold number of deaths above which a country is defined as going through a civil war. Here we present results for five measures: at least 25 deaths (PRIO25)[15], at

[14] The empirical analysis of the risk of civil wars differ in several dimensions: i) dependent variable (incidence, onset, duration); ii) definition of civil wars (threshold in terms of number of battle deaths; iii) unit of analysis (country-year, country-5 years). Here we explore sensitiveness with respect to different thresholds and data sources. See Schneider and Weisohmeier (2006) for a robustness analysis on onset and the time-dimension; Collier, Hoeffler and Soderbom (2004) and Montalvo and Reynal-Querol (forthcoming) study the duration of civil wars.

[15] Prio-Uppsala database.

least 25 deaths plus at least 1000 death over the course of the war (PRIOcw), at least 100 deaths on both sides plus 1000 deaths over the course of the war (FLcw)[16], at least 1000 deaths (PRIO1000), at least 1000 deaths plus it challenged the sovereignty of the State (SDcw)[17]. Table 2 reports the results associated with the use of data from Montalvo and Reynal-Querol (2005b). Ethnic polarization outperforms ethnic fractionalization in all but one measure.

In table 3 we run the same regression specifications using data from Alesina et al. (2003). Here the results are less clear-cut: the ethnic polarization index outperforms the fractionalization one only when using the most sensitive index (at least 25 deaths in a given year). Thus, the evidence on this relationship is ambiguous. However, given that the data used by Montalvo and Reynal-Querol (2005b) seem more accurate (the WCE provides the criteria used for the classification) and that they find the same results when considering five year averages, we rely on the findings in table 2 for the rest of the analysis.

With the caveat that sensitiveness to the indexes may be due also to the data source, we draw the following stylized facts from this literature:

- ethnic fractionalization explains (low) economic performance better than ethnic polarization;
- ethnic polarization explains likelihood of civil wars better than ethnic fractionalization.

3 The model

We provide a behavioral model linking dissipation to the distribution of the population over a set of groups.

[16] based on Fearon and Laitin (2003).
[17] based on Doyle and Sambanis (2000).

We consider the pure contest version of the model by Esteban and Ray (1999)[18]. Individuals belonging to different groups compete for the capture of a prize. We extend their model by specifying a mixed public-private prize. This feature introduces an additional channel through which group size determines the incentives of economic agents to contribute. Group size determines the per capita share of the private component: the bigger the group, the smaller the per capita share[19]. Whether this means introducing the Pareto-Olson argument into the model will be discussed later in the section. Esteban and Ray (2009) also introduce a mixed public-private good in their 1999 framework, along with varying intra-group cohesion and inter-group distances. They ask on which grounds one should choose among different indices of dispersion and find that the equilibrium level of conflict can be approximated by a linear function of the Gini coefficient, the fractionalization and the polarization index. Our paper is closer to Esteban and Ray (1999): we investigate the dissipation-distribution relationship for varying degrees of publicness of the prize and the implications for the pattern of per-capita contributions across groups. In addition, we ask whether the indexes suffer a systematic measurement error relative to the model-based relationship. In this respect, the paper is complementary to Esteban and Ray (2009) as we provide an analytic result explaining some of their numerical simulations.

The model is the has some limits too. first, we neglect the productive side of the economy. In this sense the relationship between dissipation and distribution is a very reduced form. Although the marginal cost of contributing is increasing and captures the rising opportunity cost of devoting resources to a non-productive activity, the prize is exogenous and independent from the level of dissipation in the society[20]. Second, we assume a specific ratio contest success function[21]. These

[18]We neglect the notion of distance between groups. In this respect, our model is less ambitious, but this is a precise choice. The full version of their model includes the distance between groups. Our paper is empirically motivated though, and there is no reliable measure of distance between groups in the literature on ethnic diversity.

[19]The mixed public-private prize has been used in a different framework by Esteban and Ray (2001). They investigate the group members' ability to overcome the collective action model for different types of prize at stake.

[20]There is a large conflict literature considering the endogeneity of the prize of the contest (Garfinkel and Skaperdas 2007 for an excellent survey).

[21]See Skaperdas (1996) for a general treatment and an axiomatization of contest success functions.

modelling choices are driven by reasons of tractability: allowing an arbitrary number of groups in the society complicates the analysis considerably and we had to simplify other aspects of the economy.

In section 3.1 we describe the model and how it differs from the literature. In section 3.2 we settle the existence and uniqueness of the equilibrium. In section 3.3 we analyze the relationship between equilibrium dissipation and population distribution.

3.1 Description of the model

Agents. There is a unit mass of individuals distributed over the unit interval, where i indicates the group and k indicates the individual. Individuals are distributed across G groups, each with population n_i, so that $n_i \in (0, 1]$ and $\sum_{i=1}^{G} n_i = 1$.

Actions. Society must choose the allocation of a prize. We model this prize directly in terms of the utility individuals receive from it (w_{ik}). We assume that individuals can influence the allocation of the prize by devoting resources into a non-productive activity. The decision process can be interpreted as a lottery, where the probability of receiving the prize is distributed over the population according to a vector of resources. Let $a_{ik} \in \mathbb{R}^+$ denote the resources devoted by individual k in group i. The aggregate amount of resources devoted by the entire population is $A \equiv \sum_{i=1}^{G} \sum_{h \in i} a_{ih}$, (where h indicates the generic individual in group i), where $A \in \mathbb{R}^+$. We will use A as a measure of societal dissipation in the non-productive activity.

Timing. The timing is the following: i) all individuals of all groups choose simultaneously their contributions; ii) nature chooses the winning group with probabilities π_i; iii) the prize is distributed across members of the winning group.

Information. The payoff structure of all individuals is common knowledge.

Payoffs. Let $c(a)$ denote the utility cost of a generic amount of resources. The cost function $c : \mathbb{R}^+ \to \mathbb{R}^+$ is homogeneous across all groups. Assume that

Assumption 1. c is continuous, increasing and twice differentiable with $c(0) = 0$, $c' > 0$, $c'' > 0$ for all $a > 0$, and $\lim_{a \longrightarrow 0^+} c'(a) = c'(0) = 0$.

Define the winning probability of individual k in group i (π_{ik}) as the share of resources[22] devoted by members (indexed by h) of group i :

$$\pi_{ik}(a_{ik}) = \frac{\sum_{h \in i} a_{ih}}{A}, \tag{1}$$

provided $A > 0$. By definition (1) individuals belonging to the same group have the same winning probability: $\pi_{ik} = \pi_{il} = \pi_i \ \forall (k,l) \in i, \ \forall i = 1,..,G$.

Let w_{ik} be the individual benefit from winning the prize. We specify the prize as a mixed private-public good. Let $\lambda \in [0,1]$ denote the share of publicness of the prize:

$$w_{ik} = w(\lambda, n_i) = \lambda + \frac{1-\lambda}{n_i}. \tag{2}$$

It is important to specify exactly the nature of the prize. Both the public component (λ) and the private component $(1-\lambda)$ are enjoyed exclusively by members of the winning group. The difference between the two is that the per capita benefit associated with the public component is constant, while the one associated with the private component shrinks with group size[23]. The public component can be interpreted in several ways: i) the good is non-excludable (all groups receive it), but only members of the winning group derive utility from it; ii) the good is non-excludable, members of all groups derive utility from it, but members of non-winning groups derive a lower utility than members of the winning group[24]; iii) the good is excludable to members of non-winning groups (and continues to be non-excludable among members of the

[22] The model is robust to the re-definition of the contest success function as $\pi_{ik}(a_{ik}) = \frac{\sum_{h \in i} a_{ih}^m}{A \equiv \sum_{j=1}^{G} \sum_{s \in i} a_{js}^m}$, where $m > 0$.

[23] The prize may also be interpreted as a basket of different prizes. In this case λ is the share of club prizes within the basket. This interpretation is convenient if we think about a multitude of contests in the same country. The model is consistent as long as the social cleavage remains the same.

[24] In this case w_{ik} constitutes a utility differential.

winning group). With respect to the first two cases, one may think about government policies valid for everybody, but enjoyed by one particular group[25]. With respect to the last case, one may think about government policies reserved to one particular group[26]. With this caveat in mind, we will refer to λ as the public component of the prize in the rest of the paper. A related point is that the prize does not need to be one good with both public and private features. It can also be interpreted as a basket of goods. In this case λ would be the average share of publicness of the prizes. This interpretation is useful also because the model is the stylized description not necessarily of one contest over one good, but possibly about several contest over several goods, as long as the cleavage distinguishing groups remains the same. For simplicity, we assume that the share of publicness of the prize λ is the same across groups. By definition (2), individuals of the same group receive the same benefit in case of capture of the prize: $w_{ik} = w_{il} = w_i \ \forall (k,l) \in i, \ \forall i$.

We assume a utility function for individual k in group i linear in the expected benefit from winning the prize net of the cost of contributions:

$$u_{ik}(a_{ik}) = \pi_i(a_{ik}) w_i - c(a_{ik}). \tag{3}$$

We assume that individual k in group i chooses his contribution so as to maximize his extended utility function (v_{ik}), which includes the ones of his fellow members:

$$v_{ik}(a_{ik}) = \sum_{l \in i} u_{il}(a_{il}) = u_{ik}(a_{ik}) + \sum_{l \in i, l \neq k} u_{il}(a_{il}) \tag{4}$$

By assuming that individuals maximize this extended utility, we abstract from within-group free-riding. Similar assumptions can be found in Esteban and Ray (1999, 2008) and Montalvo and Reynal-Querol (2002, 2005a)[27]. Esteban and Ray

[25] For example, an eventual extension of public health in the US will benefit disproportionally much more those without private health insurance relatively to those who already have one. Another example may be the regulation of access to the sea, which applies to any citizen but is enjoyed disproportionally by those living close to the seaside.

[26] Like reservation of political seats to women (Chattopadhyay and Duflo 2004) or to minorities (Pande 2003).

[27] This assumption can be grounded on either one of two theoretical background: either individual contributions

(2009) show that it is not very restrictive. Suppose we were to allow individuals to assign greater weight to their own utility than to their fellow members'. Then results would hold as long as they assigned a non-zero weight to their fellow members. Indeed, internalization of fellow members' preferences is thought to be one of the reasons why ethnicity is salient (Alesina and La Ferrara 2005). Even if they did assign zero weight to their fellow members' utilities, all results of the model would resemble the case of pure private goods, which is the main focus of the paper.

To complete the specification of the model, we describe the outcome when $A = 0$. We take this to be an arbitrary vector $\bar{\pi} = (\bar{\pi}_1, .., \bar{\pi}_G)$ [28].

The following table summarizes all variables and functions included in the model.

Table 4 - List of the variables in the model.		
a_{ik}	individual contribution of member of individual k in group i	choice variable
n_i	size of group i	exogenous
w_i	utility for any member of group i for outcome $i : \lambda + \frac{1-\lambda}{n_i}$	exogenous
λ	share of publicness of the prize: $\lambda \in [0, 1]$	exogenous
π_i	winning probability for any member of group $i : \sum_{i=1}^{G} \pi_i = 1$	endogenous
A	dissipation: $A = \sum_{i=1}^{G} \sum_{h \in i} a_{ih}$	endogenous
$c()$	cost of effort $c : \mathbb{R}_+ \longrightarrow \mathbb{R}_+$ and $c(.) : c'(.) > 0, c''(.) > 0$	
a^*	vector of individual contributions $a^* \equiv \left(a^*_{11}, .., a^*_{1n_1}, .., a^*_{G1}, .., a^*_{Gn_G}\right)$	equilibrium
π	vector of winning probabilities $\pi \equiv (\pi_1, .., \pi_G) : \sum_{i=1}^{G} \pi_i = 1$	
N	vector of group sizes $N \equiv (n_1, .., n_G) : \sum_{i=1}^{G} n_i = 1$	

are really determined by a group leader, like in Esteban and Ray (2008a), because of coercion or group ideology, either individuals maximize an extended utility, which includes the utility of fellow members (this paper, Esteban and Ray 2009). Esteban and Ray (1999) amd Montalvo and Reynal-Querol (2002, 2005a) assume absence of free-riding, but they leave implicit the theoretical background to support it.

[28] Esteban and Ray (1999) provide a similar assumption to complete the specification of their model.

3.2 Agents' Behavior and Equilibrium

All the proofs to the propositions henceforth are relegated in the Appendix.

Notice that individuals belonging to the same group have exactly the same payoff structure. Therefore, they will devote the same per capita contributions in equilibrium: $a_{ik} = a_{il} = a_i \; \forall (k,l) \in i$, $\forall i = 1, .., G$, where a_i denotes the per capita contribution of members of group i.

Proposition 1 *Suppose that assumption 1 holds. Provided $a_j > 0$ for some $j \neq i$, the amount of resources devoted members of groups i are strictly positive and completely described by the first-order condition (FOC)*

$$\pi_i (1 - \pi_i) w_i (\lambda, n_i) = c' (a_i) a_i. \tag{5}$$

In addition, there exists an equilibrium and it is unique.

The first part of proposition 1 states that the solution to the individual's maximization problem is always interior. Thus, any equilibrium must involve positive contributions by all individuals. Equation (5) provides an intuition of the influence of the mixed prize specification. A larger groups means more fellow members (π_i), but also less opponents $(1 - \pi_i)$ and, above all, a greater dissipation of the private component of the prize, and so reduced incentives to contribute (smaller benefit w_i). This latter force is more relevant the greater the share of the private component within the prize. This is why we expect both level and pattern of dissipation to vary with the level of this parameter.

The second part of the proposition states that there is one and only one vector of optimal contributions $a^* \equiv (a_1^*, .., a_G^*)$ such that a_{ik}^* solves the maximization of (3) subject to (1). This imply existence and uniqueness of equilibrium dissipation $A^* = \sum_{i=1}^{G} n_i a_i^*$ and equilibrium winning probabilities $\Pi = (\pi_1^*, .., \pi_G^*)$.

3.3 Dissipation and distribution: levels and patterns

In this section we analyze the properties of the model. First, we look at how equilibrium dissipation (A) varies as we let population distribution (N) for each share of publicness (λ) of the prize (comparative statics). Second, we look at how per capita contributions (a_i) vary across groups within a given equilibrium (A fixed).

Recall that our model is an extension of the pure contest version of Esteban and Ray (1999) to mixed public-private goods. With respect to our model, their results cover the case of pure public goods $(\lambda = 1)$. Throughout the analysis, we refer to their results as a benchmark against which evaluate ours $(\lambda \in [0, 1))$.

3.3.1 Dissipation and distribution: levels

We start our analysis by looking at the global maxima of the dissipation-distribution relationship.

In case of two groups, we would expect the uniform distribution to be the global maximum (Tullock 1980). This is how both the fractionalization (F) and discrete polarization index (Q) behave and what Esteban and Ray (1999) find for pure public goods. The next proposition extends their result to all other cases.

Proposition 2 *Suppose that assumption 1 holds. Then in the two groups case the uniform distribution is the strict global maximum.*

Proposition 2 implies that any departure from the uniform distribution, which corresponds to increased population inequality, lowers the level of dissipation. The result is consistent both with the fractionalization and discrete polarization indexes and extends earlier findings by Esteban and Ray (1999).

In case of an arbitrary number of groups $G \geq 3$, Esteban and Ray (1999) find that the uniform distribution over two groups (symmetric bimodal distribution) continues to be the global maximum. This is consistent with the discrete polarization index. Let us make the following assumption:

Assumption 2. The cost function is three times differentiable and $c''' \geq -\frac{2c''(a)}{a}$.

This assumption is not very restrictive. For example, all iso-elastic cost functions $c(a) = \beta a^{\alpha}$ satisfying assumption 1 ($\alpha > 1$) always satisfy this assumption. The next proposition summarizes what we know about the global maximum in case of an arbitrary number of groups:

Proposition 3 *Suppose that assumptions 1 and 2 hold. Then in the case of purely private goods the uniform distribution over all groups is the strict global maximum.*

Proposition 3 shows that, in case of pure private goods, the global maximum is consistent with the fractionalization index (F) and stands in stark contrast with the discrete polarization index (Q) and the finding by Esteban and Ray (1999).

We now investigate the comparative statics with respect to the set of uniform distributions. Over this class of distributions, Esteban and Ray (1999) find that equilibrium dissipation decreases with the number of groups, provided there are at least two groups to begin with. This is exactly in line with the second property of the discrete polarization index. We investigate whether this continues to be true for all types of goods.

Proposition 4 *Suppose that assumptions 1 holds. Then over the set of uniform distributions, equilibrium dissipation increases with the number of groups up to a threshold* $G(\lambda)$*, and decreases thereafter. The number of groups maximizing dissipation increases as the prize becomes more private* $\left(\frac{\partial G(\lambda)}{\partial \lambda} < 0\right)$*, and approaches infinity as the prize is half public half private* ($\lambda = 1/2$)*.*

Proposition 4 shows that Esteban and Ray's finding is not robust over all types of goods. Most importantly, the dissipation-distribution relationship does not resemble the property of the discrete polarization index anymore. On the contrary, for a large set of goods ($\lambda \in [0, \frac{1}{2}]$), dissipation increases with the number of groups, thus resembling the second property of the fractionalization index[29].

Next, we ask whether there exists a sequence of changes providing unidirectional impacts on dissipation. First, we explore the possibility of groups merging together. Besides knowing the direction of the change, we want to know whether it is conditional on factors not directly associated with the change, like the size of the non-merging groups. If it is, then the dissipation-distribution relationship is said to be global. If it is not, it is said to be local. Esteban and Ray (1999) find that, in case of pure public goods, the repeated merge of the two smallest groups or the one-step merge of the smallest $G-1$ groups increase dissipation. Since the groups need to be the smallest, their dissipation-distribution relationship is global and so broadly consistent with the third property of the discrete polarization index. A related point is that, starting from a uniform distribution over two groups, the split of one group always causes dissipation to fall. They comment on this as consistent with the well-known strategy of "*divide and conquer*" (p.397). The next proposition summarizes our findings for the case of purely private goods.

Proposition 5 *In case of pure private goods any merge lowers equilibrium dissipation.*

Proposition 5 says that splits (the opposite of merges) increase dissipation no matter the distribution of the other groups (locality), which corresponds to the third property of the fractionalization index. The idea that dissipation increases as groups become smaller (split) is counters the "divide and conquer" conflict-strategy, while it is consistent with the hypothesis that many independent corrupted agencies are worse that few ones (Scheleifer and Vishny 1993).

[29] Even the Esteban and Ray's finding that the symmetric bimodal distribution is the global maximum is not robust to our extension. In fact, although we do not identify the global maximum for each possible degree of publicness, we can rule the symmetric bimodal distribution out of the potential candidates for a large set of goods. In order to establish this, it is enough to notice that ER's global maximum is a uniform distribution. Since over the set of uniform distributions dissipation is greatest in correspondence of the three-point uniform distribution for $\lambda = \frac{3}{4}$, then the two-points uniform distribution can be ruled out for that and for smaller values: $\lambda \in [0, \frac{3}{4}]$.

Next, we extend the analysis to any kind of population transfer. We ask whether all population changes can be broken down into sequences of smaller changes, all having the same impact on dissipation. In section 2 we have seen that this property, which Esteban and Ray (1999) call monotonicity, distinguishes the fractionalization index from the discrete polarization index. From their paper we know that, in case of pure public goods, the dissipation-distribution relationship is non-monotonic. This is consistent with the discrete polarization index. We consider a subset of all other cases. Consider the set of goods for which $\lambda \in \left[0, \frac{1}{2}\right]$ and a sequence of transfers moving from the $G-1$ point uniform distribution to a G-point uniform distribution. In order to pass from the former to the latter with a series of transfers "in the same direction" we carry on the following thought exercise: first, we split one of groups so that there is a new group with very small size; second, we transfer population from all other groups to this small new group. By continuity, the new G-point distribution must have a level of dissipation close to the $G-1$ uniform distribution. From proposition 5 we know that such level of dissipation must be lower that that associated with the G-point uniform distribution[30]. The next proposition adds something to what we know about other kinds of population transfers. Let α denote the elasticity of the marginal cost of contribution $c'(a)$ with respect to the contribution itself $a : \alpha(a) = \frac{c''(a)a}{c'(a)}$. We make the following regularity assumption on such elasticity:

Assumption 3. c is three times differentiable and $\alpha'(a) : -[\alpha(a)+1]\alpha(a)+\delta <$

$\alpha'(a)\, a < [\alpha(a)+1]\alpha(a) - \delta$

The intuition behind this assumption is that we want the cost function is be "convex enough". It is not very restrictive though. For example the entire set of iso-elastic cost functions $c(a) = \beta a^{\alpha}$ satisfying assumption 1 $(\alpha > 1)$ is included. In

[30] From proposition 6 we also know that, in case of pure private goods, the new level of dissipation must be slightly greater than that associated with the $G-1$ uniform distribution.

fact, the derivative of the elasticity of an iso-elastic function is zero[31]. We find that

Proposition 6 *Suppose that assumptions 1 and 3 hold. Then in case of purely private goods, any uniform distribution is always a strict local maximum.*

Proposition 6 says that, in case of pure private goods, the uniform distribution over G groups is also a local maximum, which means that transfers close to it will be also dissipation-increasing. Therefore we cannot reject that the sequence of transfers leading to it does not affect dissipation in the same direction as the aggregate one.

Overall, in case of pure private goods , the dissipation-distribution relationship resembles closely the fractionalization index:

1. for a given number of groups G, both A and F are maximized at the uniform distribution over these groups;
2. over the set of uniform distributions, both A and F increase with the number of groups;
3. the split of any group into two new groups increases both A and F;
4. any transfer of population to a smaller group increases F; we cannot reject the hypothesis that this is true even for A.

On the other hand, Esteban and Ray (1999) consider the case of pure public goods and find that the dissipation-distribution relationship resembles the discrete polarization index (Q). This may suggest that the higher weight assigned to population frequency in the discrete polarization index does not reflect intra-group homogeneity (Esteban and Ray 1999) or the sense of identification (Esteban and Ray 1994), but rather the difference in the prize at stake. Indeed, if we were to include varying intra-group cohesion like Esteban and Ray (2009), we would still find that the properties

[31] It is possible to show that assumption 3 is more restrictive than assumption 2 if $\alpha(a) \in (0,1)$, exactly equal if $\alpha(a) = 1$ and less restrictive if $\alpha(a) > 1$.

of the model are close to the Q in case of pure public goods and close to F in case of pure private goods as long as intra-group cohesion was positive[32].

3.3.2 Dissipation and distribution: patterns

We now look at how per capita contributions (a_i) vary across groups within a given equilibrium (A fixed). In particular, we compare per capita contributions (a_i) to the average contribution across the entire population (A). Define the ratio between the two ($\frac{a_i}{A}$) as intensity of lobbying. Define activism any equilibrium such that at least two groups differ in their intensity of lobbying: $a_i \neq a_j$ for some i, j.

In case of pure public goods, "contests with two groups can never involve activism. On the other hand, contests with more than two groups display activism whenever all groups are not equal sized, and larger groups always lobby more than smaller groups" (Esteban and Ray 1999: 398). This is how results change once we allow the prize not to be a pure public good.

Proposition 7 *Suppose assumption 1 holds. Then*

[1] In the two-group case, contests involve activism whenever the prize is not a pure public good and the two groups are not equal sized. In this case, the larger group always lobbies less intensively than the smaller one.

[2] In case of three or more groups and pure private goods, larger groups always lobby less intensively than smaller ones.

Proposition 7 (part 1) illustrates clearly the forces at work described in section 3.2: a larger group means a greater number of contributions (greater incentive to contribute), but also a smaller opponent (lower incentive to contribute) and lower per capita benefit from the private component of the prize. In case of pure public goods, the latter component does not exist, the first two forces exactly cancel each other out

[32] Esteban and Ray (2009) model individuals' extended utility function as a weighted average between one's own utility and the fellow members' utilities. The weight represents the degree of intra-mgroup cohesion.

and individuals contribute the same no matter the population distribution. For all intermediate cases though, the additional incentive created by the private component of the prize plays a role and individuals belonging to the smaller group contribute more than the opponents. In case of an arbitrary number of groups ($G \geq 3$), the second force we listed becomes weaker, but the third one still dominates. Notice that this does not mean that the share of resources devoted by the larger group is smaller than the share of resources devoted by the smaller group. Indeed, the larger group continues to have a greater winning probability (see Lemma 8.1), but not as much as it would have had in case of pure public goods. Therefore, whether we may say that the Pareto-Olson argument plays a role in the model depends on the definition of the latter. According to Esteban and Ray (2001), the Pareto-Olson argument dominates when larger groups have smaller winning probability than smaller groups, which is not the case here[33].

Proposition 7 also unveils one difference between the model and the fractionalization index: members of different groups behave differently. This constitutes a new prediction to be tested empirically. It also has some implications for existing empirical evidence:

Proposition 8 *Suppose assumption 1 holds. Then, in case of pure private goods and a quadratic iso-elastic cost function, the fractionalization index always under-estimates the true level of dissipation.*

Proposition 8 shows that neglecting the pattern of contributions is not without consequences: the fractionalization index suffers a systematic measurement error.

4 Conclusions

In this paper we asked which population distributions are associated with a high

[33] If we relax the assumption of no free-riding, this is not necessarily true (Esteban and Ray 2001).

level of dissipation and whether the dissipation-distribution relationship resembles the fractionalization or the discrete polarization index. In order to answer these questions we developed a conflict model linking dissipation to the distribution of the population across an arbitrary number of groups. The model is an extension of the pure-contest model by Esteban and Ray (1999). They consider only pure public goods and find that the dissipation-distribution relationship[34] resembles the discrete polarization index. Here, the prize is allowed to vary from pure public good to pure private good.

We find that, in case of pure private goods, the dissipation-distribution relationship resembles the fractionalization index. This result may explain why cross-country regressions associating ethnic diversity to economic performance and likelihood of civil wars are sensitive to the index used to capture the former. To the extent both reflect competition for the capture of the State, our results suggest that the latter is perceived as a public good in case of open conflict, while it is perceived as a private good in case of lobbying and generalized corruption. It could also be the case that open conflict increases the ability to deliver public goods after the conflict.

The analysis of the per-capita contributions across groups suggests that, in case of pure private goods, individuals belonging to smaller groups always contribute more. This suggests that the fractionalization index may systematically under-estimate the weight of smaller groups in the creation of dissipation. Indeed, we find that, for the special case of quadratic cost functions, the fractionalization index under-estimates the level of dissipation. This confirms the pattern in the numerical simulations run by Esteban and Ray (2009) for the case of pure contests, quadratic costs, a large population and pure private goods. Their simulations are based on random draws for the population vector (over five groups). In this case the divergence between the model-based and index-based levels of dissipation appears negligible. Future work

[34] In their paper they consider the concept of conflict whereas here we consider the concept of dissipation to better interpret the model in light of the empirical stylized facts. However, the modelling strategy is neutral with respect to the concept used.

should confirm this with real-world data.

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Appendix

Proof. Proposition 1.

Given that individuals within the same group, then maximizing (4) subject to (1), (2) and (3) becomes maximizing

$$\frac{n_i a_i}{\sum_{j=1}^{G} n_j a_j} w_i - c(a_i). \tag{6}$$

Equation (6) is well-defined for every a_i since we have assumed that $a_j > 0$ for some $j \neq i$. The end-point restriction on c in assumption 1 and the observation that the existence of a positive lower bound on the benefit from winning the prize $(w_i \geq \lambda > 0)$ ensure that the solution to the maximization problem is interior (the FOC must hold with equality). Differentiation of (6) with respect to a_i provides exactly (5). Since the expected benefit $(\pi_i w_i)$ is strictly concave in a_i and assumption 1 ensures that the cost function is strictly convex, then the individual utility function is strictly convex, which means that equation (4) is also sufficient to define the solution.

In order to establish the existence and uniqueness of the equilibrium, define a function $\phi : [0,1]^2 \times \mathbb{R}^+ \to \mathbb{R}$ such that the single element $\phi(\pi_i, A, n_i)$ is defined by the first order derivative of the maximization problem in terms of winning probability, dissipation and group size $\left(a_i = \frac{\pi_i A}{n_i}\right)$:

$$\begin{aligned} & \frac{n_i}{A}(1-\pi_i) w_i - c'\left(\frac{\pi_i A}{n_i}\right) \\ = \; & \phi(\pi_i, A, n_i). \end{aligned}$$

Re-define the equilibrium as any combination of winning probabilities $\pi^* = (\pi_1^*, .., \pi_G^*)$ and total effort A^*, such that $\phi(\pi_i^*, A^*, n_i) = 0 \; \forall i$, and $\sum_{i=1}^{G} \pi_i^* = 1$.

The determination of the equilibrium can be shown in two step: first, by making reference to the individual optimality condition (FOC); second, by making reference to the population consistency condition $\left(\sum_{i=1}^{G}\pi_i = 1\right)$.

Suppose A (and N) fixed, and consider the behavior of the first derivative $\phi\left(\pi_i, A, n_i\right)$ as the winning probability (π_i) varies along its domain $[0,1]$:

- $\frac{\partial\phi(\pi_i,A,N_i)}{\partial\pi_i} = -\frac{n_i}{A}w_i - \frac{A}{N_i}c''\left(\frac{\pi_i A}{n_i}\right) < 0$ (strictly decreasing);
- $\lim_{\pi\longrightarrow 0^+}\phi\left(\pi_i, A, n_i\right) = \frac{n_i}{A}w_i > 0$;
- $\lim_{\pi\longrightarrow 1^-}\phi\left(\pi_i, A, n_i\right) = -c'\left(\frac{A}{n_i}\right) < 0$;

The intermediate value theorem ensures the existence and uniqueness of a winning probability satisfying the equilibrium condition: $\exists!\pi_i^* : \phi\left(\pi_i^*, A, n_i\right) = 0$. This value can be thought of as a function depending on the remaining variables: $\pi_i^* = \pi\left(A, n_i\right)$.

Aggregate consistency requires the sum of these winning probabilities to equal unity: $\sum_{i=1}^{G}\pi\left(A, n_i\right) = 1$. Suppose N fixed and consider the behavior of the sum of winning probabilities $\left(\sum_{i=1}^{G}\pi\left(A, n_i\right)\right)$ as total dissipation (A) varies along its domain $[0,+\infty)$. Since we have not derived an explicit expression for the equilibrium winning probability, we refer to the implicit function theorem to study it. Re-write the FOC function $\phi_i \equiv \phi\left(\pi_i, A, n_i\right)$, then we know:

$$\frac{\partial\phi_i}{\partial\pi_i}\frac{d\pi\left(A, n_i\right)}{dA} + \frac{\partial\phi_i}{\partial A} = 0,$$

which means

$$\frac{d\pi\left(A, n_i\right)}{dA} = -\frac{\frac{\partial\phi_i}{\partial A}}{\frac{\partial\phi_i}{\partial\pi_i}}.$$

Since $\frac{\partial\phi_i}{\partial A} = -\frac{n_i}{A^2}\left(1-\pi_i\right)w_i - \frac{\pi_i}{n_i}c''\left(\frac{\pi_i A}{n_i}\right) < 0$, and $\frac{\partial\phi_i}{\partial\pi_i} = -\frac{n_i}{A}u - \frac{A}{N_i}c''\left(\frac{\pi_i A}{n_i}\right) < 0$, then

$$\frac{d\pi\left(A, n_i\right)}{dA} < 0\forall i.$$

which implies

$$\sum_{i=1}^{G} \frac{d\pi(A,n_i)}{dA} < 0 \Longrightarrow \frac{d\left[\sum_{i=1}^{G} \pi(A,n_i)\right]}{dA} < 0.$$

Again, we derive the behavior of this function as total dissipation approaches the limits of its domain. In order to do so, we focus on the single winning probability $\pi(A,n_i)$. In order to determine the behavior of the winning probability for any member of group i as total dissipation shrinks to zero, fix such winning probability and consider the behavior of the first derivative ϕ as total dissipation shrinks to zero:

$$\begin{aligned}
\lim_{A\longrightarrow 0^+} \phi(\pi_i, A, n_i) &= \lim_{A\longrightarrow 0^+} \frac{n_i}{A}(1-\pi_i)\, w_i - \lim_{A\longrightarrow 0^+} c'\left(\frac{\pi_i A}{n_i}\right) \\
&= \infty - c'(0) = \infty.
\end{aligned}$$

If the first order condition $(\phi_i = 0)$ is to continue to hold, the winning probability must approach unity as total dissipation $\left(\lim_{A\longrightarrow 0^+} \pi(A,n_i) = 1\right)$. This implies that the sum of winning probabilities will exceed unity: $\lim_{A\longrightarrow 0^+}\left[\sum_{i=1}^{G} \pi(A,n_i)\right] = G(>1)$. In order to determine the behavior of the winning probability for any member of group i as total dissipation increases to infinity, fix such winning probability and consider the behavior of the first derivative ϕ as total dissipation increases to infinity:

$$\begin{aligned}
\lim_{A\longrightarrow +\infty} \phi(\pi_i, A, n_i) &= \lim_{A\longrightarrow +\infty} \frac{n_i}{A}(1-\pi_i)\, w_i - \lim_{A\longrightarrow +\infty} c'\left(\frac{\pi_i A}{n_i}\right) \\
&= 0 - \infty = -\infty.
\end{aligned}$$

If the first order condition $(\phi_i = 0)$ is to continue to hold, the winning probability must shrink to zero $\left(\lim_{A\longrightarrow +\infty} \pi(A,n_i) = 0\right)$. This implies that the sum of winning probabilities will shrink to zero as well: $\lim_{A\longrightarrow +\infty}\left[\sum_{i=1}^{G} \pi(A,n_i)\right] = 0$.

Given the last three results $\left(\frac{d\left[\sum_{i=1}^{G} \pi(A,n_i)\right]}{dA} < 0,\ \lim_{A\longrightarrow 0^+}\left[\sum_{i=1}^{G} \pi(A,n_i)\right] = G,\ \lim_{A\longrightarrow +\infty}\left[\sum_{i=1}^{G} \pi(A,n_i)\right] = 0\right)$, the intermediate value theorem ensures the existence and uniqueness of a value of total dissipation satisfying the equilibrium condition: $\exists! A^* : \sum_{i=1}^{G} \pi(A^*, n_i) = 1$. Such value can be thought as depending on the vector of group sizes $N = (n_1, .., n_G)$: $A^* = A(N)$.

In summary, for any vector of group sizes N there is one and only one level of total effort and vector of winning probabilities satisfying the equilibrium conditions. ■

Proof. Proposition 2.

Consider the case of two groups $(G = 2)$. Proposition 3 states that the uniform distribution $(\bar{N} = (\frac{1}{2}, \frac{1}{2}))$ is the strict global maximum. Since there are only two groups (1,2), and their sizes (n_1, n_2) must add to unity, we can just re-define their sizes as $n_1 = n$ and $n_2 = 1 - n$. The dissipation function $A(N)$ can be re-defined accordingly $A(n)$. Re-define the group's winning probability $\Pi_i(n) \equiv \pi_i(A(n), n)$. Since winning probabilities also to unity, then $\Pi_1 = \Pi$ and $\Pi_2 = 1 - \Pi$. Re-define the first-order derivative accordingly: $\phi(\pi, A, n) = \Phi(\Pi(n), A(n), n) \equiv \Phi_1$, where . The first-order derivative of this function with respect to n is:

$$\frac{d\Phi(\Pi, A, n)}{dn} = \frac{\partial \Phi_1}{\partial \Pi}\frac{d\Pi}{dn} + \frac{\partial \Phi_1}{\partial A}\frac{dA}{dn} + \frac{\partial \Phi_1}{\partial n} = 0. \tag{7}$$

Explicit the derivative of the winning probability with respect to n :

$$\frac{d\Pi}{dn} = -\frac{\frac{\partial \Phi_1}{\partial A}\frac{dA}{dn} + \frac{\partial \Phi_1}{\partial n}}{\frac{\partial \Phi_1}{\partial \Pi}}.$$

Since population is normalized to unity an infinitesimal change in the size of group 1 (n) directly affects also the size of group 2 $(1 - n)$. Let the first-order derivative for the generic member of group 2 be: $\Phi_2 \equiv \Phi(1 - \Pi, A, 1 - n)$. There will be another direct and indirect effect to count for. However, we know that the sum of winning probabilities must be equal unity before and after the shift. Therefore, the two aggregate changes in winning probabilities must compensate each other: $\sum_{i=1}^{2} \frac{d\Pi_i}{dn} = 0$. Then we can explicit the total derivative of dissipation A with respect to the population parameter $\left(\frac{dA}{dn}\right)$:

$$\frac{dA}{dn} = -\frac{\sum_{i=1}^{2}\left[\frac{\partial\Phi_i/\partial n}{\partial\Phi_i/\partial\Pi}\right]}{\sum_{i=1}^{2}\left[\frac{\partial\Phi_i/\partial A}{\partial\Phi_i/\partial\Pi}\right]}.$$

The two initial first-order derivatives Φ_i are: $\Phi_1 = \frac{n}{A}(1-\Pi)\,w(n) - c'\left(\frac{\Pi A}{n}\right)$ and $\Phi_2 = \frac{1-n}{A}\Pi w(1-n) - c'\left(\frac{(1-\Pi)A}{(1-n)}\right)$. Differentiation of these two expressions and some manipulation provides the following expression, where $\alpha_1 = \alpha\left(\frac{\Pi A}{n}\right)$ and $\alpha_2 = \alpha\left(\frac{(1-\Pi)A}{(1-n)}\right)$:

$$\frac{dA}{dn} = \frac{A}{n(1-n)}\frac{(\alpha_1+\theta_1)(1-n)\left[\Pi\alpha_2+(1-\Pi)\right]+(\alpha_2+\theta_2)\,n\left[(1-\Pi)\,\alpha_1+\Pi\right]}{(1-2n)(\alpha_1\alpha_2-1)}.$$

It follows that:

$$sign\left\{\frac{dA}{dn}\right\} = sign\left\{(1-2n)\right\},$$

which means that $A(n)$ is increasing in n for $n \in \left(0,\frac{1}{2}\right]$ and decreasing afterwards. Therefore, $A(n)$ attains its maximum at $n=\frac{1}{2}$, which corresponds to the uniform distribution over the two groups. ■

Proof. Proposition 3.

Consider the case of an arbitrary number of groups $(G \geq 3)$. Re-consider equation (5) in case of pure private goods $(\lambda = 0)$:

$$\pi_i(1-\pi_i)\frac{1}{n_i} = c(a_i)\,a_i. \tag{8}$$

Define a new function $f:\mathbb{R}_+ \to \mathbb{R}_+$ such that $f(a) \equiv c'(a)a$. This let us re-write equation (8) as:

$$\pi_i(1-\pi_i) = n_i f(a_i).$$

Aggregate over groups to obtain:

$$\sum_{i=1}^{G}\left[\pi_i(1-\pi_i)\right] = \sum_{i=1}^{G}\left[n_i f(a_i)\right].$$

Assumption 1 ensures that the $f(.)$ is strictly increasing: $f'(a) = c''(a)\,a + c'(a) > 0.$ Assumption 2 ensures that $f(.)$ is convex: $f''(a) = c'''(a)a + 2c''(a) \geq 0$. This let us use Jensen inequality theorem: $\sum_{i=1}^{G}[n_i f(a_i)] \geq f\left(\sum_{i=1}^{G} n_i a_i\right)$, where $f\left(\sum_{i=1}^{G} n_i a_i\right) = f(A)$. In turn, we know that:

$$f(A) \leq \sum_{i=1}^{G}[\pi_i(1-\pi_i)].$$

Maximizing the right hand side subject to the constraint that the sum of winning probabilities must be equal to unity $\left(\sum_{i=1}^{G}\pi_i = 1\right)$ provides the uniform distribution $\bar{\pi} = (\pi, .., \pi)$:

$$\sum_{i=1}^{G}[\pi_i(1-\pi_i)] \leq G[\pi(1-\pi)],$$

with equality only if $\pi_i = \pi\ \forall i$.

From the proof of existence and uniqueness of the equilibrium we know that there is only one population vector that corresponds to the uniform winning probability vector, and that it is the uniform population vector $\bar{N} = (n, .., n)$. Let A^* denote the dissipation level corresponding to this maximum, then we know that:

$$f(A) \leq f(A^*).$$

Since f is strictly increasing, this implies $A \leq A^*$, with equality if and only if $N = \bar{N}$. ■

Proof. Proposition 4.

As we restrict our attention to uniform distributions $(n_i = n\ \forall i)$, the maximization problem becomes identical for individuals across all groups. Per capita contributions are identical $(a_i = a_j = a\ \forall i, j)$ and so are winning probabilities $(\pi_i = \pi = n\ \forall i)$. Therefore, equilibrium contributions across individuals. Given the normalization of total population to unity, equilibrium contributions will also equal total dissipation

$(a = A)$. Equation (5) reduces to:

$$n\,(1-n)\,w\,(n) = c'\,(A)\,A.$$

Define a new function $f : \mathbb{R}_+ \to \mathbb{R}_+$ such that $f\,(a) \equiv c'(a)a$. This let us re-write the previous equality as:

$$n\,(1-n)\,w\,(n) = f\,(A)\,.$$

Assumption 1 ensures that the $f\,(.)$ is strictly increasing: $f'\,(A) = c''\,(A)\,A + c'(A) > 0$. This means that f is invertible and the dissipation-maximizing problem reduces to maximizing the LHS:

$$\begin{aligned} & \max_n \{n\,(1-n)\,w\,(n)\} \\ = \;& \max_n \left\{n\,(1-n)\left(\lambda + \frac{1-\lambda}{n}\right)\right\} \\ = \;& \max_n \{n\,(1-n)\,\lambda + (1-n)\,(1-\lambda)\}\,, \end{aligned}$$

$$FOC : (1-2n)\,\lambda - (1-\lambda) \leq 0 \;(= 0 \;if\; n > 0).$$

If the share of publicness of the prize (λ) is equal or smaller than $\frac{1}{2}$, the solution is corner $(n = 0)$. Otherwise the solution is interior and equal to:

$$n = 1 - \frac{1}{2\lambda} \equiv n\,(\lambda)\,.$$

The number of groups corresponding to these solutions is $G(\lambda) = \frac{1}{n(\lambda)}$, which means $G(\lambda) = +\infty \; \forall \lambda \in \left[0, \frac{1}{2}\right]$ and $G(\lambda) = \frac{2\lambda}{2\lambda - 1}$. In particular, notice that

$$\frac{\partial G\,(\lambda)}{\partial \lambda} < 0 \quad \forall \lambda \in \left(\frac{1}{2}, 1\right]\,,$$

and $G\,(1) = 2$. ■

Proof. Proposition 5.

In order to clarify the exposition, we drop the subscripts. The following definition will be used frequently throughout the proof. Define the subjective share of publicness of the prize (θ) as the ratio between the share of publicness of the prize (λ) and the benefit from winning the prize (w):

$$\theta = \frac{\lambda}{w} = \frac{\lambda}{\lambda + (1-\lambda)/n}. \tag{9}$$

The following lemma describes properties that will be needed in the proof of proposition 5, 6 and 7. ■

Lemma 9 *Suppose assumption 1 holds. Then*

[1] the function $\pi(.)$ is strictly increasing and twice continuously differentiable;

[2] provided $\lambda = 0$, $\left(\frac{\pi}{n}\right)$ is strictly decreasing;

[3] provided $\lambda = 0$, if $(a, b) >> 0$, then $\pi(a+b) > \pi(a) + \pi(b)$.

Proof. Recall that $\pi(.)$ is implicitly defined by equation (5), which we can re-write in terms of (π, A, n):

$$\frac{n}{A}(1-\pi)\, w(n) = c'\left(\frac{\pi A}{n}\right). \tag{10}$$

Let $\alpha(a)$ denote the the elasticity of the marginal cost of effort $c'(a)$ with respect to effort $a : \alpha(a) = \frac{ac''(a)}{c'(a)}$. Set A fixed and differentiate equation (10) with respect to n to obtain:

$$\pi'(n) = \frac{\pi}{n} \frac{(1-\pi)[\alpha(a) + \theta]}{(1-\pi)\,\alpha(a) + \pi}. \tag{11}$$

Assumption 1 ensures $\alpha(a) > 0\ \forall a > 0$.Therefore $\pi'(.) > 0\ \forall n > 0$. So part 1 is established.

Using (11) we can derive the derivative of the ratio between winning probability and group size $\left(\frac{\pi}{n}\right)$ with respect to size (n):

$$\frac{\partial\left(\frac{\pi}{n}\right)}{\partial n} = \frac{\pi'(n)}{n} \frac{\theta - (\theta+1)\pi}{(1-\pi)[\alpha(a)+\theta]}. \tag{12}$$

Equation (12) shows that

$$sign\left\{\frac{\partial\left(\frac{\pi}{n}\right)}{\partial n}\right\}=sign\left\{\theta-(\theta+1)\,\pi\right\}.$$

In case of pure private goods $\theta=0$, so $\frac{\partial\left(\frac{\pi}{n}\right)}{\partial n}<0\ \forall n,\forall\lambda$. So part 2 is established.

Consider $(a,b)>>0$. From part 2 we know that $\frac{\pi(a+b)}{a+b}<\frac{\pi(a)}{a}$ and $\frac{\pi(a+b)}{a+b}<\frac{\pi(b)}{b}$. It follows that:

$$\begin{aligned}\pi\left(a+b\right) &= \frac{a+b}{a+b}\pi\left(a+b\right)=a\frac{\pi\left(a+b\right)}{a+b}+b\frac{\pi\left(a+b\right)}{a+b}\\ &< a\frac{\pi\left(a\right)}{a}+b\frac{\pi\left(a\right)}{a}=\pi\left(a\right)+\pi\left(a\right).\end{aligned}$$

So part 3 is established. ■

Proof. We return to the main proof.

Sort groups according the their winning probabilities (π_i). Consider any sub-set M of the G groups. From Lemma 9.3 we know that

$$\pi\left(A,\sum_{i\in M}n_i\right)<\sum_{i\in M}\pi\left(A,n_i\right).$$

Add the winning probabilities of all remaining groups $(j\neq M)$, evaluated at the initial level of dissipation $A:$

$$\pi\left(A,\sum_{i\in M}n_i\right)+\sum_{j\neq M}\pi\left(A,n_j\right)<\sum_{i\in M}\pi\left(A,n_i\right)+\sum_{j\neq M}\pi\left(A,n_j\right)=1.$$

For the sum of winning probabilities to equal unity also in the final distribution, the level of dissipation must decrease: $A'<A$. Therefore any merge must decrease the level of dissipation (and any split must increase it). ■

Proof. Proposition 6.

Consider the G-point uniform distribution $\bar{N}_G=(n,n,..)$. Call the corresponding level of dissipation $\bar{A}_G$. Set $\bar{A}_G$ fixed and differentiate (11) with respect to n. After some manipulation, obtain the following expression:

$$\pi''(n) = \frac{[\pi'(n)]^2}{\pi(1-\pi)[\alpha(a)+\theta][(1-\pi)\alpha(a)+\pi]} \left\{ \begin{array}{c} [\theta-(\theta+1)\pi][(1-\pi)\alpha(a)+\pi] + \frac{\theta(1-\theta)}{\alpha(a)+\theta}[(1-\pi)\alpha(a)+\pi]^2 + \\ -[\alpha(a)+\theta]\pi - \frac{\alpha'(a)a}{\alpha(a)+\theta}[\theta-(\theta+1)\pi]^2 \end{array} \right\},$$

where $\theta = \theta(\lambda, n)$.

Define the expression in curly brackets as $\varphi(\lambda, n)$. Clearly, $sign\{\pi''(n)\} = sign\{\varphi(\lambda, n)\}$.

The case of purely private goods corresponds to setting $\lambda = 0$, which means $\theta(0, n) = 0$. By substitution we find:

$$\begin{aligned} \varphi(0,n) &= \left\{-\pi[(1-\pi)\alpha(a)+\pi] - \alpha(a)\pi - \frac{\alpha'(a)a}{\alpha(a)}\pi^2\right\} \\ &= -\pi\left\{(1-\pi)\alpha(a)+\pi+\alpha(a)+\frac{\alpha'(a)a}{\alpha(a)}\pi\right\} \\ &= -\pi\left\{2(1-\pi)\alpha(a)+\left[\alpha(a)+1+\frac{\alpha'(a)a}{\alpha(a)}\right]\pi\right\}. \end{aligned}$$

By assumption 3 the equation becomes

$$\varphi(0,n) = -\pi\{2(1-\pi)\alpha(a)+\delta\pi\}$$

where $\frac{\alpha'(a)a}{\alpha(a)} = -[\alpha(a)+1]+\delta$. Since $\delta > 0$, then $\varphi(0,n) < 0 \ \forall n > 0$. The winning probability $\pi(.)$ is locally strictly concave in an open neighborhood around the point combination $(\bar{A}_G(\lambda), n)$. Pick any G-point non-uniform distribution $\tilde{N}_G = (\tilde{n}_{G1}, .., \tilde{n}_{GG})$ such that the combination $(\bar{A}_G(\lambda), \tilde{n}_{Gi})$ lies in the open neighborhood of $(\bar{A}_G(\lambda), n)$ for every i. By local strict concavity and the equilibrium condition $\sum_{i=1}^{G} \pi(A, n_i) = 1$,

$$1 = G\pi\left(\bar{A}_G(\lambda), n\right) > \sum_{i=1}^{G} \pi\left(\bar{A}_G(\lambda), \tilde{n}_{Gi}\right).$$

Let $\tilde{A}_G(\lambda)$ be the equilibrium dissipation associated with $\tilde{N}_G$. Recall that $\pi(.)$ is strictly decreasing in A : $\frac{d\pi(A_G(\lambda),\tilde{n}_{Gi})}{dA} < 0$ $\left(\text{as well as } \frac{d\left[\sum_{i=1}^{G}\pi(A_G(\lambda),\tilde{n}_{Gi})\right]}{dA} < 0\right)$ $\forall i$. This, joint to the previous inequality, implies $\tilde{A}_G(\lambda) < \bar{A}_G(\lambda)$. ■

Proof. Proposition 7.

Notice that the ratio between group's per capita contribution and average contribution $\left(\frac{a_i}{A}\right)$ is exactly equal to the ratio between winning probability and group size $\left(\frac{\pi_i}{n_i}\right)$.

Consider the case $G = 2$. Let n be the size of group 1 and $(1-n)$ the size of group 2. Let π be the winning probability of group 1 and $(1-\pi)$ the winning probability of group 2. Consider the ratio between the FOC of two individuals belonging to different groups:

$$\begin{aligned}\frac{c'(a_1)\,a_1}{c'(a_2)\,a_2} &= \frac{w(\lambda,n)}{w(\lambda,1-n)}\\ &= \frac{1-n}{n}\frac{\lambda n+1-\lambda}{\lambda(1-n)+1-\lambda}.\end{aligned}$$

If the RHS is greater then unity, group 1 lobbies more intensively than group 2. If the two groups have equal size $\left(n=\frac{1}{2}\right)$, the RHS is equal to unity, which means absence of activism. Consider the general case:

$$\frac{1-n}{n}\frac{\lambda n+1-\lambda}{\lambda(1-n)+1-\lambda} \geq 1.$$

After some manipulations, we find that

$$(1-2n)(1-\lambda) \geq 0.$$

If the good is purely public $(\lambda = 1)$, then the inequality is satisfied for any value of n. If the good is intermediate or purely private $(\lambda < 1)$, then the inequality is satisfied for $n \leq \frac{1}{2}$ (with strict inequality if $n < \frac{1}{2}$). This means exactly the the bigger group lobbies less intensively than the smaller one.

Consider the case $G \geq 3$ and the special case $\lambda = 0$. Sort groups with respect to their size. Recall from Lemma 9.1 that the ratio $\left(\frac{\pi}{n}\right)$ is decreasing in n, which means that bigger groups lobby less intensively than smaller ones. ■

Proof. Proposition 8.

In case of pure private goods $(\lambda = 0)$, and an iso-elastic cost-function $c(a) = \frac{a^2}{2}$, we get $A^2 = \sum_{i=1}^{G} [n_i (1 - \pi_i)]$. Recall the formula for the fractionalization index: $F = \sum_{i=1}^{G} [n_i (1 - n_i)]$. Proposition 8 says that the dissipation A^2 is always greater than fractionalization $(F) : A^2 > F$. This can be written as:

$$
\begin{aligned}
A^2 - F &= \sum_{i=1}^{G} [n_i (1 - \pi_i)] - \sum_{i=1}^{G} [n_i (1 - n_i)] \\
&= \sum_{i=1}^{G} \{[(1 - \pi_i) - (1 - n_i)] n_i\} \\
&= \sum_{i=1}^{G} [(n_i - \pi_i) n_i]
\end{aligned} \tag{13}
$$

Sort groups so that: $n_1 \leq .. \leq n_G$. Since $\pi'(n) > 0$, the same sorting applies to winning probabilities: $\pi_1 \leq .. \leq \pi_G$. Lemma 9.2 ensures that the ratio $\left(\frac{\pi}{n}\right)$ is decreasing in n: $\frac{\pi_1}{n_1} \geq .. \geq \frac{\pi_G}{n_G}$. Since $\frac{\pi_i}{n_i} = \frac{a_i}{A}$, and A is a weighted average of per-capita contributions (given that population is normalized to unity), then $A \in [a_1, a_G]$ (with equality only in case of uniform distribution). This implies that $\exists! n^* \in [n_1, n_G]$[35] : $\frac{\pi(n^*)}{n^*} = 1$, or, $\pi(n^*) = n^*$. n^* divides the groups in the following way: $\pi_i > n_i \; \forall i \in \{n_i < n^*\}$; $\pi_i < n_i$

[35] There is only one case in which $n^* = n_1$ or $n^* = n^G$. It corresponds to the uniform distribution $(n_1 = .. = n_G)$.

$\forall i \in \{n_i > n^*\}$; $\pi_i = n_i \ \forall i \in \{n_i = n^*\}$. So that

$$\begin{aligned}\sum_{i\in\{n_i<n^*\}} [(n_i - \pi_i)\, n_i] &< 0;\\ \sum_{i\in\{n_i>n^*\}} [(n_i - \pi_i)\, n_i] &> 0;\\ \sum_{i\in\{n_i=n^*\}} [(n_i - \pi_i)\, n_i] &= 0.\end{aligned}$$

Define $\hat{n} : \hat{n} \in \{n_i < n^*\} \wedge n_i < \hat{n} \ \forall i \in \{n_i < n^*\}$. This let us establish a lower bound to the first subset:

$$\sum_{i\in\{n_i<n^*\}} [(n_i - \pi_i)\, \hat{n}] \leq \sum_{i\in\{n_i<n^*\}} [(n_i - \pi_i)\, n_i]$$

Define $\check{n} : \check{n} \in \{n_i > n^*\} \wedge n_i > \check{n} \ \forall i \in \{n_i > n^*\}$. This let us establish a lower bound to the first subset:

$$\sum_{i\in\{n_i>n^*\}} [(n_i - \pi_i)\, \check{n}] \leq \sum_{i\in\{n_i>n^*\}} [(n_i - \pi_i)\, n_i]$$

In addition, notice that the two group size thresholds are ordered: $\hat{n} < \check{n}$.

Disaggregate equation (13) with respect to the subgroups and use these inequalities:

$$\begin{aligned}& \sum_{i\in\{n_i<n^*\}} [(n_i - \pi_i)\, n_i] + \sum_{i\in\{n_i>n^*\}} [(n_i - \pi_i)\, n_i]\\ \geq\ & \sum_{i\in\{n_i<n^*\}} [(n_i - \pi_i)\, n_i] + \check{n} \sum_{i\in\{n_i>n^*\}} [(n_i - \pi_i)]\\ \geq\ & \hat{n} \sum_{i\in\{n_i<n^*\}} (n_i - \pi_i) + \check{n} \sum_{i\in\{n_i>n^*\}} (n_i - \pi_i)\\ >\ & \hat{n} \sum_{i\in\{n_i<n^*\}} (n_i - \pi_i) + \hat{n} \sum_{i\in\{n_i>n^*\}} (n_i - \pi_i)\\ =\ & \hat{n} \sum_{i=1}^{G} (n_i - \pi_i) = 0\end{aligned}$$

The last equality comes from the fact that $\sum_i \pi_i = \sum_i n_i \Rightarrow \sum_i (\pi_i - n_i) = 0$. So we have established that $A^2 - F > 0$, which proves proposition 8. ■

Appendix 2

Table 1: Economic performance, ethnic fractionalization and discrete polarization

Dependent variable is growth of per capita GDP

	1	2
	coef/t	coef/t
	yes	yes
decade and regional dummies, initial income and schooling		
ETHNIC FRACTIONALIZATION	-0,023***	-0,024***
	(-2,589)	(-2,737)
ETHNIC POLARIZATION	0,001	0,004
	(0,132)	(0,506)
N	265	267

t-statistics are in parentheses

∗p < 0.05; ∗∗p < 0.01; ∗∗∗p < 0.001.

Estimated using Seemigly Unrelated Regression (SUR) following McDowell (2004).

Period: 1960-1989. One observation per decade.

Control variables: dummy for the 60s, dummy for the 70s, dummy for the 80s, dummy for Africa, dummy for Latin America and Caribbeans, log of real per capita GDP at the beginning of the decade (both in levels and squared), log of 1 plus average years of schooling at the beginning of the decade.

The two columns differ in the data used to measure ethnic diversity: column (1) is based on Montalvo and Reynal-Querol (2005a); column (2) is based on Alesina et al. (2003).

Table 2a: Civil wars, ethnic fractionalization and discrete polarization

Dependent variables are indexes for civil wars

	PRIO25	PRIOcw	FLcw	PRIO1000	SDcw
	1	2	3	4	5
	coef/t	coef/t	coef/t	coef/t	coef/t
controls	yes	yes	yes	yes	yes
ETHNIC POLARIZATION	3,65***	2,68*	1,67	4,03**	2,91*
	(3,22)	(1,73)	(1,04)	(2,42)	(1,74)
ETHNIC FRACTIONALIZATION	-0,52	0,91	1,79	0,30	0,34
	(-0,58)	(0,69)	(1,28)	(0,25)	(0,24)
N	255	255	255	255	255
Adjusted R2	0,181	0,176	0,336	0,207	0,278

t-statistics are in parentheses; ∗p < 0.05; ∗∗p < 0.01; ∗∗∗p < 0.001.
Standard errors clustered by country
Estimated using Seemigly Unrelated Regression (SUR) following McDowell (2004).
Period: 1960-1989. One observation per decade.
Dependent variables: indexes for civil wars by PRIO (Oslo), see discussion in the text; index used by Fearon and Laitin (FLcw); index used by Doyle and Sambanis (SDcw).
Control variables: Controls: log income per capita beginning of the decade; log population; primary exports; dummy for presence of mountains; dummy for non-contiguous states; dummy for democracy.
Data used to measure ethnic diversity based on Montalvo and Reynal-Querol (2005a).

Table 2b: Civil wars, ethnic fractionalization and discrete polarization

Dependent variables are indexes for civil wars

	PRIO25	PRIOcw	FLcw	PRIO1000	SDcw
	1	2	3	4	5
	coef/t	coef/t	coef/t	coef/t	coef/t
controls	yes	yes	yes	yes	yes
ETHNIC POLARIZATION	2,07*	0,36	0,11	1,89	1,14
	(1,95)	(0,24)	(0,07)	(1,07)	(0,68)
ETHNIC FRACTIONALIZATION	0,51	2,23	1,61	0,12	1,07
	(0,45)	(1,42)	(0,95)	(0,07)	(0,66)
N	255	255	255	255	255
Adjusted R2	0,159	0,154	0,296	0,166	0,249

t-statistics are in parentheses; ∗p < 0.05; ∗∗p < 0.01; ∗∗∗p < 0.001.
Standard errors clustered by country
Estimated using Seemigly Unrelated Regression (SUR) following McDowell (2004).
Period: 1960-1989. One observation per decade.
Dependent variables: indexes for civil wars by PRIO (Oslo), see discussion in the text; index used by Fearon and Laitin (FLcw); index used by Doyle and Sambanis (SDcw).
Control variables: Controls: log income per capita beginning of the decade; log population; primary exports; dummy for presence of mountains; dummy for non-contiguous states; dummy for democracy.
Data used to measure ethnic diversity based on Alesina et al. (2003).