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**Hotelling's T<sup>2</sup> Method in Multivariate  
On-Line Surveillance.  
On the Delay of an Alarm**

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# HOTELLING'S T2 METHOD IN MULTIVARIATE ON-LINE SURVEILLANCE. ON THE DELAY OF AN ALARM

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## Abstract

A system for detecting changes in an on-going process is needed in many situations. On-line monitoring (surveillance) is used in early detection of disease outbreaks, of patients at risk and of financial instability. By continually monitoring one or several indicators, we can, early, detect a change in the processes of interest. There are several suggested methods for multivariate surveillance, one of which is the Hotelling's T2. Since one aim in surveillance is quick detection of a change, it is important to use evaluation measures that reflect the timeliness of an alarm. One suggested measure is the expected delay of an alarm, in relation to the time of change ( $\tau$ ) in the process. Here we investigate a delay measure for the bivariate situation. Generally, the measure depends on both change times (i.e.  $\tau_1$  and  $\tau_2$ ). We show that, for a bivariate situation using the T2 method, the delay only depends on  $\tau_1$  and  $\tau_2$  through the distance  $\tau_1 - \tau_2$ .

**Key words:** Monitoring; On-line; Surveillance; T2; Timeliness.

## 1. Introduction

In many situations it is important to monitor one or several processes in order to detect important changes as soon as possible. In on-line monitoring we have repeated decisions: at each new time point, a new observation becomes available and a new decision has to be made in order to decide whether the process has changed or not. In this situation the methodology of statistical surveillance is appropriate. Examples of different areas where statistical surveillance has been used is turning point detection (see (Neftci 1982), (Hamilton 1989), (Royston 1991), (Baron 2002), (Bock, Andersson and Frisén 2007), (Andersson 2004) and (Andersson, Bock and Frisén 2004)). Another area is detection of growth retardation of fetuses, see (Petzold, Sonesson, Bergman and Kieler 2004). Yet another is detection of an increased level, emerging from a source and spreading spatially ((Järpe 1999)).

In industrial quality control, statistical surveillance has been used in the form of control charts, such as xbar-charts, s-charts and R-charts. These charts were developed for the univariate case and often only the last observation is used (the Shewhart method ((Shewhart 1931))).

The time scale can differ from one application to another (daily, weekly, monthly), but common to all surveillance is the repeated decisions: at each new time point another observation becomes available and once more we have to decide whether the change has occurred or not. There is always a risk of a false alarm. A “good” alarm system should not have too many false alarms. But we must also consider that we want a system with high detection ability if there really has been a change – we want to detect a change quickly. When evaluating surveillance systems, we often use other measures than size and power, instead there is a trade off between false alarms and delay of motivated alarms (see e.g. (Frisén 1992) and (Frisén 2007)). The false alarms are often measured by the average run length, ARL0. For the motivated alarms it is important to consider how long it took until a signal was called, for example measured by the expected delay time.

In many situations we monitor several processes, which can change at the same time or at different times. There are several approaches to multivariate surveillance, see (Sonesson and Frisén 2005). One approach is to reduce the multidimensional data at each time point, to a scalar. The Hotelling’s T2 is an example of this approach ((Hotelling 1947)).

In this paper we present a delay measure for the multivariate situation. We also show a result regarding the delay of the T2 method, when monitoring a bivariate process, where the change times are not equal.

### 1.1 Model and method

First we study the univariate surveillance situation. At some unknown time tau there is a change in  $\mu$  (the expected value of X). In the simplest case the change can be a shift in the mean, i.e.

$$\begin{cases} \mu(s)=\mu^0, & s < \tau \\ \mu(s)=\mu^1, & s \geq \tau \end{cases}$$

exemplified in Figure 1.

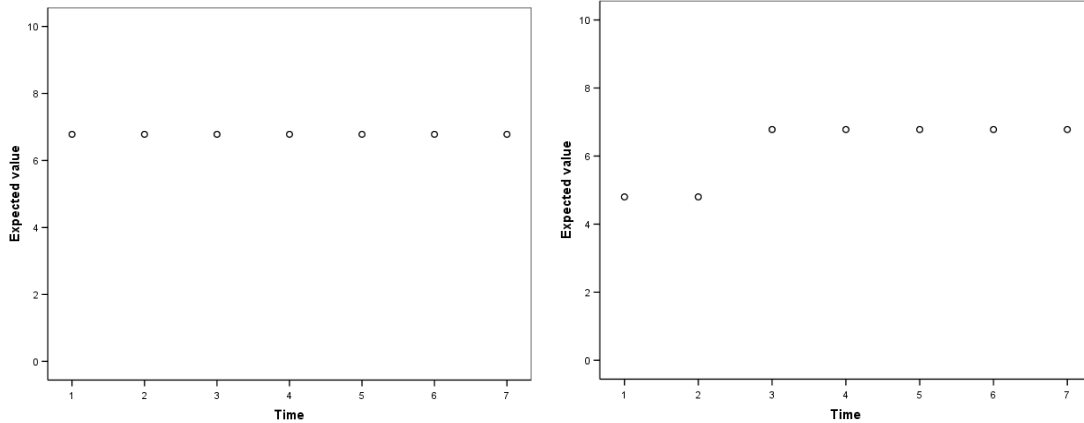


Figure 1: The vector  $\mu$ , as a function of time, for  $\tau=1$  (left) and  $\tau=3$  (right).

Another example is a change from a constant level to an increasing, unspecified function so that the vector  $\mu$  is

$$\begin{cases} \mu(1)=\dots=\mu(s-1)=\mu(s)=\mu^0, & s < \tau \\ \mu(1)=\dots=\mu(\tau-1)=\mu^0 \text{ and } \mu^0 < \mu(\tau) < \dots < \mu(s), & s \geq \tau \end{cases}$$

which is exemplified in Figure 2.

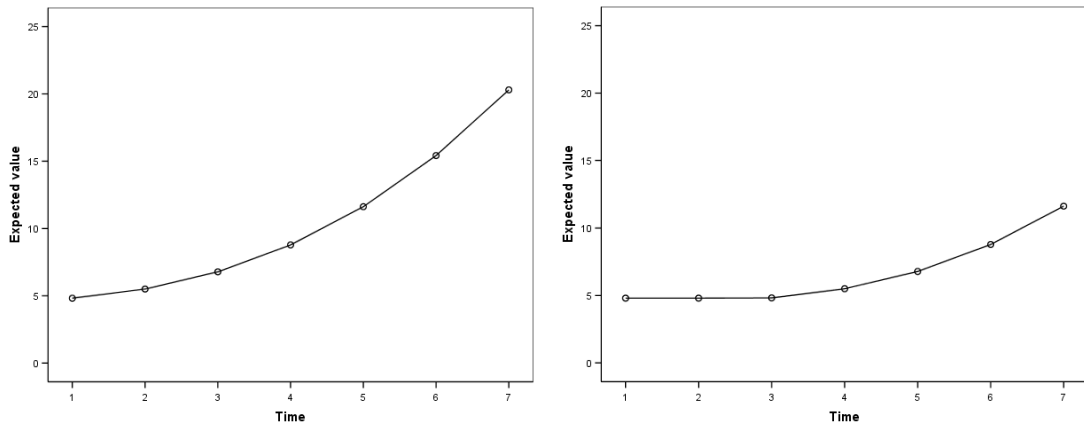


Figure 2: The vector  $\mu$ , as a function of time, for  $\tau=1$  (left) and  $\tau=3$  (right).

Now we turn to the bivariate case. At each decision time, a new bivariate observation becomes available, and at decision time  $s$  we have the observations  $(X, Y)$ . At an unknown time  $\tau_X$  there is a change in  $\mu_X$  and correspondingly for the process  $Y$ .

The two processes  $X$  and  $Y$  have the same variance (i.e.  $\text{Var}[Y(t)] = \text{Var}[X(t)] = \sigma^2$ ) and have covariance  $\rho \cdot \sigma^2$  (i.e.  $E[(X(t) - \mu_X(t))(Y(t) - \mu_Y(t))] = \rho \cdot \sigma^2$ ). The variables  $X(s)$  and  $Y(s)$  are (possibly) dependent but not  $X(s)$  and  $X(s-j)$  or  $Y(s)$  and  $Y(s-j)$  or  $X(s)$  and  $Y(s-j)$ .

At an unknown time  $\tau_X$  there is a change in  $\mu_X$  and correspondingly for the process  $Y$ . Thus, for the same value of  $\tau$  ( $\tau_X = \tau_Y$ ),  $X$  and  $Y$  have the same distribution. When  $\rho=0$ ,  $X$  and  $Y$  are independent, conditional on  $\tau$ . The aim is to detect the first change in either  $\mu_X$

or  $\mu_Y$ , when these processes may change at different time points  $\tau_X$  and  $\tau_Y$ . We study the situation when the  $\tau$  values are not identical or have known lags).

An early multivariate surveillance method is the T2 method of (Hotelling 1947). The covariance matrix is assumed to be known, see e.g. (Alt 1985).

$$T2(s) = \frac{(x(s) - \mu^0(s))^2}{(1 - \rho^2)\sigma^2} + \frac{(y(s) - \mu^0(s))^2}{(1 - \rho^2)\sigma^2} - \frac{2\rho(x(s) - \mu^0(s))(y(s) - \mu^0(s))}{(1 - \rho^2)\sigma^2} > k.$$

The alarm limit,  $k$ , is chosen to give a specified false alarm property, e.g. a specific  $ARL^0$ . The time of alarm,  $t_A$ , is defined as

$$t_A = \min\{s: T2(s) > k\}.$$

### ***1.2 A measure of delay in multivariate surveillance***

For an on-line system, the ability to detect a change quickly is important, i.e. we want a short delay of a motivated alarm. For most surveillance methods, the delay of an alarm depends on when the change did occur, in relation to the start of the surveillance. In the univariate situation, the delay can be measured by the conditional expected delay, defined as

$$CED(t) = E[t_A - \tau | t_A \geq \tau, \tau = t]. \quad (1)$$

Many evaluations are made using only  $\tau=1$ , e.g.  $CED(1)$  which is equivalent to  $ARL^1-1$ . However it is important to consider other change point times also.

In the multivariate situation where we want to detect the first change,  $\tau_{(1)}$ , the delay depends on both change points,  $\tau_X$  and  $\tau_Y$ . In (Wessman 1999) and (Andersson 2007) the following delay measure was suggested

$$CED(t_1, t_2) = E[t_A - \tau_{(1)} | t_A \geq \tau_{(1)}, \tau_X = t_1, \tau_Y = t_2]. \quad (2)$$

### ***1.3 Results***

A simulation study reveal the following, regarding the  $CED(t_1, t_2)$  of the T2 method (the complete study is presented in (Andersson 2007)). Below, CED curves for  $\rho = \{0, 0.5\}$  are presented.

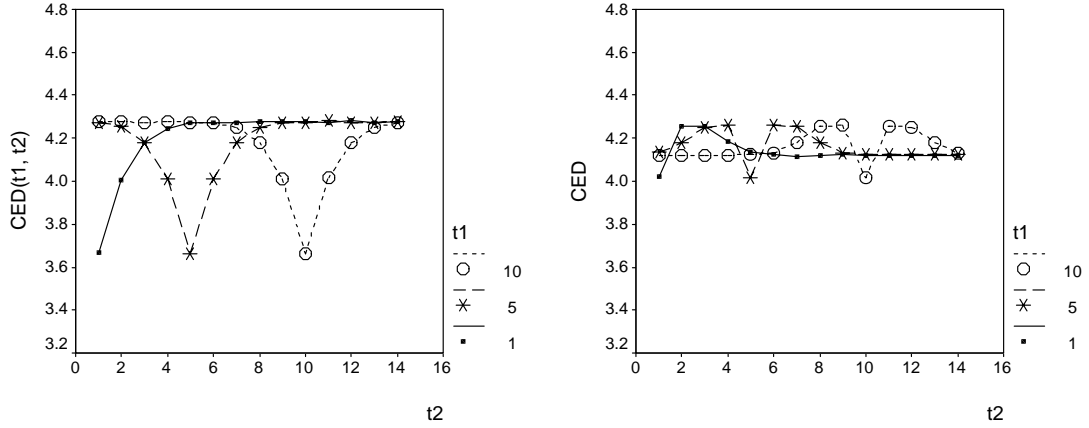


Figure 3:  $CED(t_1, t_2)$  for  $T_2$  when  $t_1 = \{1, 5, 10\}$ . Left:  $\rho=0.0$ , right:  $\rho=0.5$ .

The graphs above indicate that, for  $T_2$ , the  $CED(t_1, t_2)$  only depends on the distance  $(t_1 - t_2)$ . This will be generally proven below.

### 1.3.1 The delay of the $T_2$ method for a bivariate process

The  $CED(t_1, t_2)$  in (2) is based on the motivated alarms, i.e. alarms after time  $\tau_{(1)}$ , so that

$$CED(t_1, t_2) = \sum_{i=\tau_{(1)}}^{\infty} (i - \tau_{(1)}) \cdot P(t_A = i | t_A \geq \tau_{(1)}) = \sum_{i=\tau_{(1)}}^{\infty} (i - \tau_{(1)}) \cdot \frac{P(t_A = i)}{P(t_A \geq \tau_{(1)})}.$$

#### 1.3.1.1 Simultaneous change points

First we consider the situation with simultaneous changes,  $\tau_X = \tau_Y = t$ . Then  $\tau_{(1)} = t$  and the conditional expected delay equals

$$\begin{aligned} CED(t, t) &= \\ &= \frac{1}{P(t_A \geq t)} \left( (t - t) \cdot P(t_A = t) + (t + 1 - t) \cdot P(t_A = t + 1) + \dots \right) = \\ &= \frac{1}{P(t_A \geq t)} \left( \sum_{i=t}^{\infty} (i - t) \cdot P(t_A = i) \right). \end{aligned}$$

The probability in the denominator,  $P(t_A \geq t)$ , equals

$$\begin{aligned} P(t_A \geq t) &= P(t_A > t - 1) = \\ &= P(T^2(1) < k \cap \dots \cap T^2(t - 1) < k) = \\ &= \prod_{i=1}^{t-1} P(T^2(i) < k). \end{aligned}$$

The probability is independent of time, so we denote  $P(T^2(i) < k)$  by  $p_0$ . Thus

$$P(t_A \geq t) = \prod_{i=1}^{t-1} P(T^2(i) < k \mid \mu_X(i) = \mu_Y(i) = \mu^0) = (p_0)^{t-1}.$$

For the probability in the nominator,  $P(t_A = i)$ , we have

$$P(t_A = i) = \left( \prod_{j=1}^{i-1} P(T^2(j) < k) \right) \cdot P(T^2(i) > k).$$

We are interested in  $\text{CED}(t,t)$ . For a value  $i$ , the probability is divided into time points before  $t$  and time points after  $t$

$$\begin{aligned} & \left( \prod_{j=1}^{t-1} P(T^2(j) < k \mid \mu^0, \mu^0) \right) * \\ & \left( \prod_{j=t}^{i-1} P(T^2(j) < k \mid \mu^1(j-t+1), \mu^1(j-t+1)) \right) * \\ & P(T^2(i) > k \mid \mu^1(i-t+1), \mu^1(i-t+1)). \end{aligned}$$

We denote  $P(T^2(j) < k \mid \mu^1(j-t+1), \mu^1(j-t+1))$  by  $p_{11}(j-t+1)$  and

$P(T^2(j) > k \mid \mu^1(j-t+1), \mu^1(j-t+1))$  by  $q_{11}(j-t+1)$ . Then

$$P(t_A = i) = (p_0)^{t-1} \cdot \left( \prod_{j=t}^{i-1} p_{11}(j-t+1) \right) \cdot q_{11}(i-t+1).$$

Thus, the conditional expected delay equals

$$\begin{aligned} \text{CED}(t,t) &= \frac{1}{P(t_A \geq t)} \sum_{i=t}^{\infty} (i-t) \cdot P(t_A = i) = \\ & \frac{1}{(p_0)^{t-1}} \sum_{i=t}^{\infty} \left[ (i-t) \cdot (p_0)^{t-1} \cdot \left\{ \prod_{j=t}^{i-1} p_{11}(j-t+1) \right\} \cdot q_{11}(i-t+1) \right] = \\ & \sum_{i=0}^{\infty} i \cdot \left( \prod_{j=1}^i p_{11}(j) \right) \cdot q_{11}(i+1). \end{aligned}$$

Hence  $CED(t,t)$  is independent of  $t$ . If  $\mu^1$  is constant (i.e.  $\mu^1(s-\tau+1) = \mu^1$  for  $s \geq \tau$ ), the probabilities are constant over time

$$P(T^2(j) < k \mid \mu^1, \mu^1) = p_{11},$$

$$P(T^2(i) > k \mid \mu^1, \mu^1) = q_{11},$$

and for this situation the  $CED(t,t)$  equals

$$\sum_{i=0}^{\infty} i \cdot (p_{11})^i \cdot q_{11}.$$

### 1.3.1.2 Different change points

Second, we look at different change times,  $\tau_X = t_1$  and  $\tau_Y = t_2$ , where  $t_2 > t_1$  and hence  $\tau_{(1)} = t_1$ ,  $\tau_{(2)} = t_2$ . The conditional expected delay equals

$$\begin{aligned} CED(t_1, t_2) &= \\ & \sum_{i=\tau_{(1)}}^{\infty} (i - \tau_{(1)}) \cdot P(t_A = i \mid t_A \geq \tau_{(1)}) = \\ & \frac{1}{P(t_A \geq t_1)} \sum_{i=t_1}^{\infty} (i - t_1) \cdot P(t_A = i) = \\ & \frac{1}{P(t_A \geq t_1)} \left( \sum_{i=t_1}^{t_2-1} (i - t_1) \cdot P(t_A = i) + \sum_{i=t_2}^{\infty} (i - t_1) \cdot P(t_A = i) \right). \end{aligned}$$

Using the same notation as above, we have  $P(t_A \geq t_1) = (p_0)^{t_1-1}$ . For the probability  $P(t_A = i)$ , we have, for  $t_1 \leq i < t_2$

$$\begin{aligned} P(t_A = i) &= \\ & \prod_{j=1}^{t_1-1} P(T^2(j) < k \mid \mu^0, \mu^0) * \\ & \prod_{j=t_1}^{i-1} P(T^2(j) < k \mid \mu^1(j-t_1+1), \mu^0) * \\ & P(T^2(i) > k \mid \mu^1(i-t_1+1), \mu^0), \end{aligned}$$



and for  $i \geq t_2$

$$\begin{aligned}
P(t_A = i) &= \\
&\prod_{j=1}^{t_1-1} P(T^2(j) < k \mid \mu^0, \mu^0)^* \\
&\prod_{j=t_1}^{t_2-1} P(T^2(j) < k \mid \mu^1(j-t_1+1), \mu^0)^* \\
&\prod_{j=t_2}^{i-1} P(T^2(j) < k \mid \mu^1(j-t_1+1), \mu^1(j-t_2+1))^* \\
&P(T^2(i) > k \mid \mu^1(i-t_1+1), \mu^1(i-t_2+1)).
\end{aligned}$$

Denote the probabilities by

$$\begin{aligned}
P(T^2(j) < k \mid \mu^1(j-t_1+1), \mu^1(j-t_2+1)) &= p_{11}(j-t_1+1, j-t_2+1), \\
P(T^2(j) < k \mid \mu^1(j-t_1+1), \mu^0) &= p_{10}(j-t_1+1), \\
P(T^2(j) > k \mid \mu^1(j-t_1+1), \mu^1(j-t_2+1)) &= q_{11}(j-t_1+1, j-t_2+1), \\
P(T^2(j) > k \mid \mu^1(j-t_1+1), \mu^0) &= q_{10}(j-t_1+1).
\end{aligned}$$

Thus, for  $t_1 \leq i < t_2$  and for  $i \geq t_2$  we have the two following expressions

$$P(t_A = i) = (p_0)^{t_1-1} \cdot \left( \prod_{j=t_1}^{i-1} p_{10}(j-t_1+1) \right) \cdot q_{10}(i-t_1+1)$$

and

$$\begin{aligned}
P(t_A = i) &= \\
&(p_0)^{t_1-1} \cdot \left( \prod_{j=t_1}^{t_2-1} p_{10}(j-t_1+1) \right) \cdot \left( \prod_{j=t_2}^{i-1} p_{11}(j-t_1+1, j-t_2+1) \right) \cdot q_{11}(i-t_1+1, i-t_2+1).
\end{aligned}$$

The expression for  $CED(t_1, t_2)$  can be partitioned into one sum for  $t_1 \leq i < t_2$  and another sum for  $i \geq t_2$ . The first sum can be expressed as

$$\begin{aligned} & \left( \sum_{i=t_1}^{t_2-1} (i-t_1) \cdot P(t_A = i) \right) = \\ & \sum_{i=t_1}^{t_2-1} (i-t_1) \cdot (p_0)^{t_1-1} \cdot \left( \prod_{j=t_1}^{i-1} p_{10}(j-t_1+1) \right) \cdot q_{10}(i-t_1+1) = \\ & \sum_{i=0}^{t_2-t_1-1} i \cdot (p_0)^{t_1-1} \cdot \left( \prod_{j=1}^i p_{10}(j) \right) \cdot q_{10}(i+1). \end{aligned}$$

The second sum can be expressed as

$$\begin{aligned} & \left( \sum_{i=t_2}^{\infty} (i-t_1) \cdot P(t_A = i) \right) = \\ & \sum_{i=t_2}^{\infty} (i-t_1) \cdot (p_0)^{t_1-1} \cdot \left( \prod_{j=t_1}^{t_2-1} p_{10}(j-t_1+1) \right) * \\ & \left( \prod_{j=t_2}^{i-1} p_{11}(j-t_1+1, j-t_2+1) \right) \cdot q_{11}(i-t_1+1, i-t_2+1) = \\ & \sum_{i=t_2-t_1}^{\infty} i \cdot (p_0)^{t_1-1} \cdot \left( \prod_{j=1}^{t_2-t_1} p_{10}(j) \right) \cdot \left( \prod_{j=1}^{i-1} p_{11}(t_2-t_1+j, j) \right) \cdot q_{11}(i+1, i-(t_2-t_1)+1). \end{aligned}$$

The complete expression for the conditional expected delay is

$$CED(t_1, t_2) = \frac{1}{P(t_A \geq t_1)} \left( \sum_{i=t_1}^{t_2-1} (i-t_1) \cdot P(t_A = i) + \sum_{i=t_2}^{\infty} (i-t_1) \cdot P(t_A = i) \right) =$$

$$\frac{\sum_{i=0}^{t_2-t_1-1} i \cdot (p_0)^{t_1-1} \cdot \left( \prod_{j=1}^i p_{10}(j) \right) \cdot q_{10}(i+1)}{(p_0)^{t_1-1}} +$$

$$\frac{\sum_{i=t_2-t_1}^{\infty} i \cdot (p_0)^{t_1-1} \cdot \left( \prod_{j=1}^{t_2-t_1} p_{10}(j) \right) \cdot \left( \prod_{j=1}^{i-1} p_{11}(t_2-t_1+j, j) \right) \cdot q_{11}(i+1, i-(t_2-t_1)+1)}{(p_0)^{t_1-1}}$$

which equals

$$\sum_{i=0}^{t_2-t_1-1} i \cdot \left( \prod_{j=1}^i p_{10}(j) \right) \cdot q_{10}(i+1) +$$

$$\sum_{i=t_2-t_1}^{\infty} i \cdot \left( \prod_{j=1}^{t_2-t_1} p_{10}(j) \right) \cdot \left( \prod_{j=1}^{i-1} p_{11}(t_2-t_1+j, j) \right) \cdot q_{11}(i+1, i-(t_2-t_1)+1).$$

Denote  $t_2-t_1$  by  $c$ . Then the conditional expected delay equals

$$\text{CED}(t_1, t_2) =$$

$$\sum_{i=0}^{c-1} (i) \cdot \left( \prod_{j=1}^i p_{10}(j) \right) \cdot q_{10}(i+1) +$$

$$\sum_{i=c}^{\infty} (i) \cdot \left( \prod_{j=1}^c p_{10}(j) \right) \cdot \left( \prod_{j=1}^{i-1} p_{11}(c+j, j) \right) \cdot q_{11}(i+1, i-c+1),$$

which is independent of  $t_1$  and  $t_2$ , and only depends on the distance  $(t_2-t_1)=c$ .

For a constant  $\mu^1$  (i.e.  $\mu^1(s-\tau+1) = \mu^1$  for  $s \geq \tau$ ), the probabilities are constant over time

$$P(T^2(j) < k \mid \mu^1(j-t_1+1), \mu^1(j-t_2+1)) = p_{11},$$

$$P(T^2(j) < k \mid \mu^1(j-t_1+1), \mu^0) = p_{10},$$

$$P(T^2(j) > k \mid \mu^1(j-t_1+1), \mu^1(j-t_2+1)) = q_{11},$$

$$P(T^2(i) > k \mid \mu^1(i-t_1+1), \mu^0) = q_{10},$$

and then

$$\text{CED}(t_1, t_2) = \sum_{i=0}^{c-1} (i) \cdot (p_{10})^i \cdot q_{10} + \sum_{i=c}^{\infty} (i) \cdot (p_{10})^c \cdot (p_{11})^{i-c} \cdot q_{11} \cdot$$

Thus when X is independent over time and likewise with Y, the CED( $t_1, t_2$ ) for T2 only depends on the distance between the change times,  $t_2 - t_1 = c$ .

## 2. Summary

On-line monitoring of multivariate data is considered and the situation when the processes under surveillance change at different time points is studied. In on-line monitoring, the delay of an alarm is an important evaluation measure. A measure of the expected delay is suggested, for the multivariate situation.

One approach to multivariate surveillance is to reduce the data at each time point, to a scalar and then monitor this scalar by univariate surveillance. Here we study one reduction, namely the Hotellings T2. We prove that the conditional expected delay for T2, in the situation with two processes, only depends on the distance between the change times. By using the T2 at each time point, we only include the information from the current time point. A univariate correspondence is the Shewhart method (for time independent data), where only the current observation is used. It has been shown, e.g. in (Frisén and Wessman 1999), that the conditional expected delay for the Shewhart method, is independent of the time of change (i.e. CED(i) in (1) is constant over different values of i).

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