

Abstract

Algorithmic, intuitive and formal aspects of mathematics in dynamic interplay: A study of students' use of their conceptions in calculus

Focusing on the potentiality of students' ways of treating a mathematical material this thesis aims to investigate how students use their conceptual understanding when working with mathematical tasks in calculus. Two case studies were carried out to explore students' understanding of threshold concepts. The first study, an interview study, explored engineering students' understanding of limit and integral. The second study, a problem solving study, involved students within a mathematics programme, working on a challenging task including the concepts function and derivative, requiring proof by induction. Drawing on a theory of contextualisation data were analysed within a constructivist research framework following the principles of intentional analysis. The results reveal that the students in the mathematics programme expressed their understanding in a formal context in which also intuitive ideas played an important role. They used intuitive ideas and formal reasoning in a dynamic interplay with several functions: to control intuitive ideas, to offer a new basis of reasoning, to reduce the complexity of the problem and to push the problem solving process forward. The engineering students expressed their conceptions in an algorithmic context, in which procedural knowledge was predominant and the operations of the concepts were seen as defining features and a basis for understanding. However, faced with probing questions, the students appeared to shift to a contextualisation foregrounding ideas relating to conceptual dimensions of calculus. These contextual shifts display the transformative aspect of threshold concepts allowing the development of conceptions and students' awareness of ways of thinking and practising in mathematics.

Keywords: Algorithmic context; Calculus; Conceptual understanding; Contextualisation; Higher education; Intentional analysis; Learning potentiality; Procedural knowledge.