Abstract

We prove that the Coleff-Herrera residue current, corresponding to a pair of holomorphic functions defining a complete intersection, can be obtained as the unrestricted weak limit of a natural smooth $(0, 2)$-form depending on two parameters. Moreover, we prove that the rate of convergence i Hölder. This result is in contrast to the fact, first discovered by Passare and Tsikh, that the residue integral in general is discontinuous at the origin. We also generalize our regularization results to pairs of so called Bochner-Martinelli, or more generally, Cauchy-Fantappiè-Leray blocks in the case of a complete intersection.

We generalize the classical Cayley transform to tuples of unbounded operators by using Taylor's analytic functional calculus. We give necessary and sufficient conditions on an n-tuple a of closed unbounded operators in order that a can be transformed to an n-tuple of bounded commuting operators by a projective transformation of $\mathbb{CP}^n$. The components of such tuples need not all have non-empty resolvent sets. The construction gives an analytic functional calculus, supported by a closed subset of $\mathbb{CP}^n$, for each such a. This subset is then a natural candidate for a joint spectrum of a. We provide an integral representation for this functional calculus. We also study "all" tuples of unbounded operators admitting a smooth functional calculus by considering multiplicative operator valued distributions A with an additional property meaning, in a weak sense, that $A(1)$ is the identity operator.

Keywords: residue integral, Coleff-Herrera current, Cauchy-Fantappiè-Leray current, regularization, division problem, Cayley transform, Taylor spectrum, functional calculus, integral representation, projective space

AMS 2000 Subject classification: 32A27, 32C30, 32A26, 47A13, 47A60, 47A10, 47A65